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DEVIATORIC CONSTITUTIVE RELATIONSHIP FOR ANISOTROPIC MATERIALS

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US ARMY BALLISTIC RESEARCH LABORATORY
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An anisotropic constitutive formulation is presented in which the deviatoric terms have been completely separated from the hydrostatic terms. In this way, a non-linear equation of state is readily employed.

For code calculations employing an incremental strain approach, separating the hydrostatic terms from the deviatoric terms provides for a more accurate means of calculating pressure. This results because of the fact that the proposed deviatoric formulation allows evaluation of hydrostatic stress directly from dilatation. Such a scheme is less prone to cumulative integration error than existing formulations, which use stress increments as the basis for modifying hydrostatic pressure. The error is most severe when calculating pressure for materials with variable compressibility. Additionally, an error in the original formulation produces error in the pressure calculation when the material undergoes plastic flow. This flaw is eliminated in the present formulation.

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Under constraints of constant compressibility, and isotropic parameters, this formulation reduces to Hooke's Law and the Prandtl-Reuss flow rule. The scheme described is amenable for use in a variety of existing impact codes, and has been implemented in the DEFEL code.

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1. INTRODUCTION

It is desired to improve upon the ability to describe the behavior of anisotropic media subjected to large pressures, as is the case for hypervelocity impact. It is believed that expressing the anisotropic constitutive relationship in a form that makes use of the deviatoric stress and strain tensors provides for a better description of anisotropic materials whose compressibility is permitted to vary with volumetric strain. The deviatoric stress technique is used routinely in many impact codes for describing isotropic behavior[1-3], and is described in many books on elasticity and plasticity[4-5]. Anisotropic schemes have also been developed for various impact codes[6-7] which calculate a deviatoric stress. However, the deviatoric stress is expressed in terms of a total strain and the bulk modulus. In a true deviatoric formulation, deviatoric stress is expressed only in terms of deviatoric strain, and compressibility affects only the equation of state, not the deviatoric stress/strain relation.

An anisotropic formulation is proposed which satisfies the condition of reducing to Hooke's Law/Prandtl Reuss Flow Rule when employing the constraint of constant compressibility and isotropy, but which conveniently allows for anisotropy and variable compressibility. Additionally, the formulation is amenable for inclusion into existing impact codes which presently use the deviatoric stress technique for isotropic materials. A skeleton coding of the scheme is provided in Appendix A. The scheme also provides an improved technique for calculating hydrostatic pressure which is less prone to error than existing techniques. Finally, it is hoped that the formulation provides an enhanced physical interpretation on the behavior of anisotropic materials which might otherwise be lacking.

2. BACKGROUND

The constitutive relationship for any elastic material may be represented in contracted form as

$$\sigma_i = C_{ij}\,\epsilon_j \tag{1}$$

where σ_i and ϵ_j represent the six independent stress and strain components, and C_{ij} is the modulus matrix. The contracted form of the constitutive relation is used for the sake of simplicity, but the tensorial components of the contracted form are defined as follows:

$$\sigma_i = (\sigma_{11}\ \sigma_{22}\ \sigma_{33}\ \sigma_{23}\ \sigma_{13}\ \sigma_{12})$$

$$\epsilon_i = (\epsilon_{11}\ \epsilon_{22}\ \epsilon_{33}\ \epsilon_{23}\ \epsilon_{13}\ \sigma_{12})$$

In general, C_{ij} may be a function of σ, ϵ, $\dot{\epsilon}$, etc. However, it is somewhat unwieldy as such, and is sometimes considered to be constructed of constants, which produces the familiar Hooke's Law. One reason why the deficiency of Hooke's Law becomes apparent experimentally under large pressures is that the bulk modulus of the material is quite different from the material's stress free value.

For isotropic materials, this problem has been computationally

circumvented by the introduction of the deviatoric stress and strain tensors. These tensors differ from the absolute stress/strain tensors in that the normal components of stress and strain are decremented by the average of the normal stresses and strains respectively. In this way, the deviatoric quantities represent deviation from a hydrostatic condition, while the relationship existing between the average stress (negative of pressure) and average strain (volumetric dilatation) is an equation of state. Since experimental evidence reveals that the compressibility of many materials changes under large pressures, the deviatoric formulation suggests that while the simplicity of Hooke's Law (constant coefficients) might possibly be retained for computation of the deviatoric stresses and strains, a more accurate scalar equation of state should simultaneously be employed to account for non-linear compressibility effects.

3. ELASTIC DEVIATORIC ANISOTROPY

While the mathematics of the constant coefficient constitutive relationship for anisotropic materials is well understood, the casting of these rules into a deviatoric format is not nearly as straightforward as it is for isotropic materials. Difficulties arise because of two primary differences in the behavior of anisotropic materials with respect to that of isotropic materials: (a) under hydrostatic pressure, strain is not uniform in all three directions of the material coordinates, and (b) except under restrictive modulus conditions, deviatoric strain will produce volumetric dilatation (i.e., two different stress states with the same pressure will produce different dilatations in the material).

Decomposition of the stress and strain tensors into their hydrostatic and deviatoric components yields:

$$s_i = \sigma_i - \bar{\sigma}_i \tag{2}$$

$$e_j = \epsilon_j - \bar{\epsilon}_j \tag{3}$$

where $\bar{\sigma}_i$ are all equal to the components of hydrostatic stress ($\bar{\sigma} = (\sigma_1 + \sigma_2 + \sigma_3)/3$) for normal stress components and equal to zero for the shear stress components. The term $\bar{\epsilon}_j$ represents the normal strains due to hydrostatic stress, and are formulated in Appendix C. One may acquire upon substitution into equation (1):

$$(s_i + \bar{\sigma}_i) = C_{ij}\,(e_j + \bar{\epsilon}_j) \tag{4}$$

where barred quantities represent conditions resulting from a hydrostatic pressure, s_i and e_j are the deviatoric stresses and strains respectively, and C_{ij} is the modulus matrix. Unlike the isotropic materials in which a hydrostatic pressure produces a uniform dilatation in all three coordinate directions, hydrostatic strain for an anisotropic material is non-uniform. Therefore, if one defines the deviatoric components of stress and strain to be the total stress/strain components decremented by an amount which would result from a hydrostatic stress state, one can conclude (per condition "a" above) that there is a unique hydrostatic strain component associated with all three directions in the material coordinates (the coordinate system which produces no shear coupling). Equation (4) may be

decoupled to give a hydrostatic equation

$$\bar{\sigma}_i = C_{ij}\,\bar{\epsilon}_j \tag{5}$$

and a deviatoric relationship void of hydrostatic terms:

$$s_i = C_{ij}\,e_j \tag{6}$$

For the sake of clear visualization, the formulation will be described for transverse isotropy, though extension to orthotropy is straightforward[8]. Figure 1 depicts material elements from an anisotropic body whose material (preferred) coordinate systems differ from the laboratory frame of reference. The preferred coordinate system is the reference frame in which the constitutive relation reduces to its most simple form. Figure 2 shows properties of the preferred transversely isotropic material frame. Mechanical properties are invariant with respect to reference frame rotations that are confined to the plane of isotropy. As such, a certain symmetry of mechanical properties exist in transversely isotropic materials which are absent in orthotropic materials. The proposed model will be described in the material (preferred) coordinate system. Solutions of problems in which the laboratory frame and the material frame do not coincide pose no problem if one first transforms stress and strain to the material frame (see Appendix B).

Under the influence of a purely hydrostatic stress state (and assuming the moduli to be constant), there will be a constant ratio between the anisotropic (longitudinal) strain $\bar{\epsilon}_1$ and the transversely isotropic planar strain $\bar{\epsilon}_2$. Defining the ratio in terms of material compliances S_{ij} (where $S_{ij} = (C_{ij})^{-1}$):

$$K_\epsilon = \frac{\bar{\epsilon}_1}{\bar{\epsilon}_2} = \frac{S_{11} + 2S_{12}}{S_{22} + S_{12} + S_{23}} \tag{7}$$

it is seen that this parameter (K_ϵ) reduces to a value of unity for isotropy, where S_{11} will equal S_{22}, and S_{12} will equal S_{23}.

Using the definition that deviatoric stress is that part of the stress tensor which deviates from the hydrostatic stress condition, one can conclude that the deviatoric stress has no hydrostatic component

$$s_1 + s_2 + s_3 = 0 \tag{8}$$

One may substitute the deviatoric constitutive relation, equation (6), to acquire

$$K_\sigma\,e_1 + e_2 + e_3 = 0 \tag{9}$$

where K_σ physically represents the ratio of longitudinal and transverse stress under conditions of uniform strain ($\epsilon_1 = \epsilon_2 = \epsilon_3$), and is given by

$$K_\sigma = \frac{C_{11} + 2C_{12}}{C_{22} + C_{12} + C_{23}} \tag{10}$$

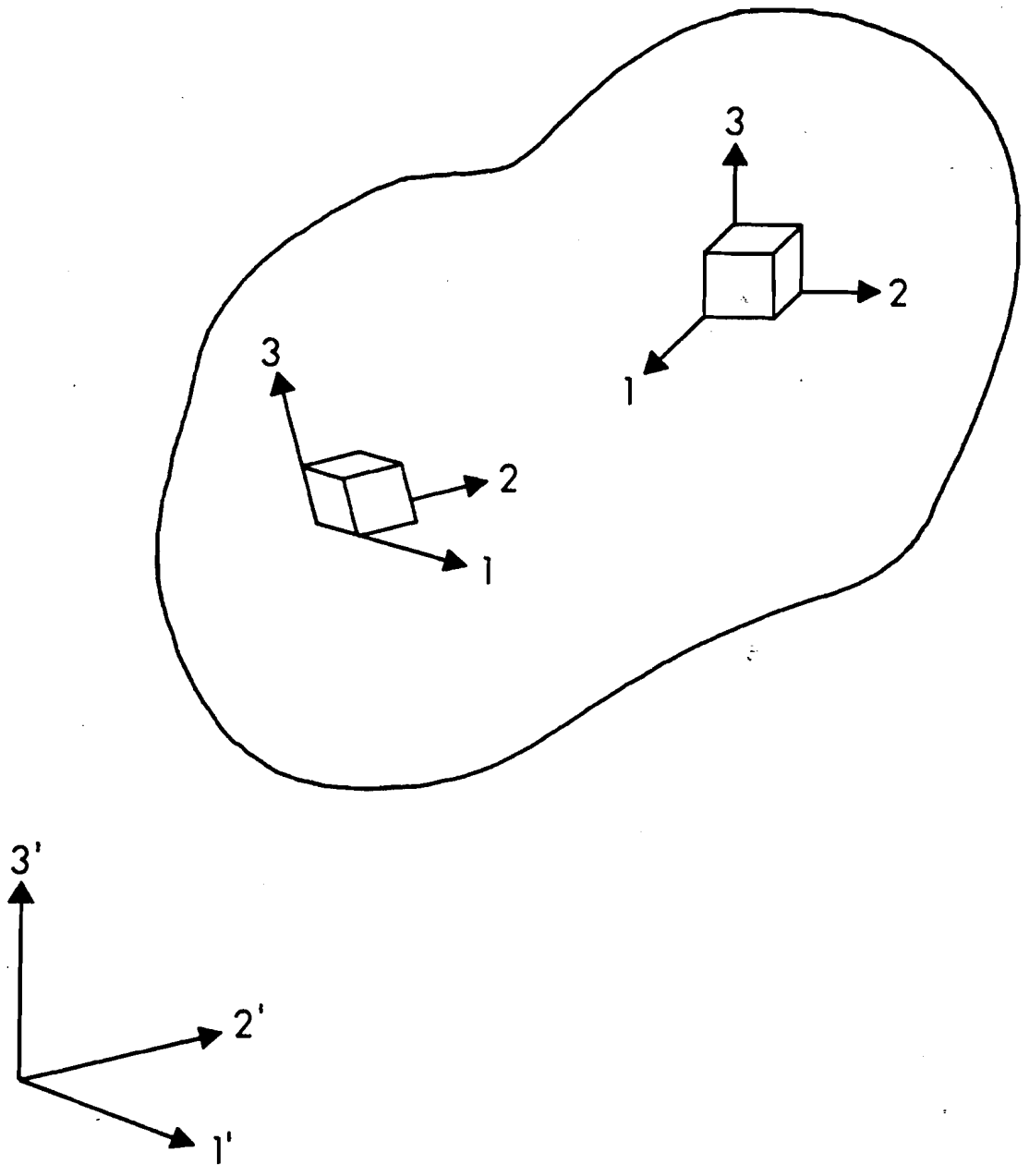
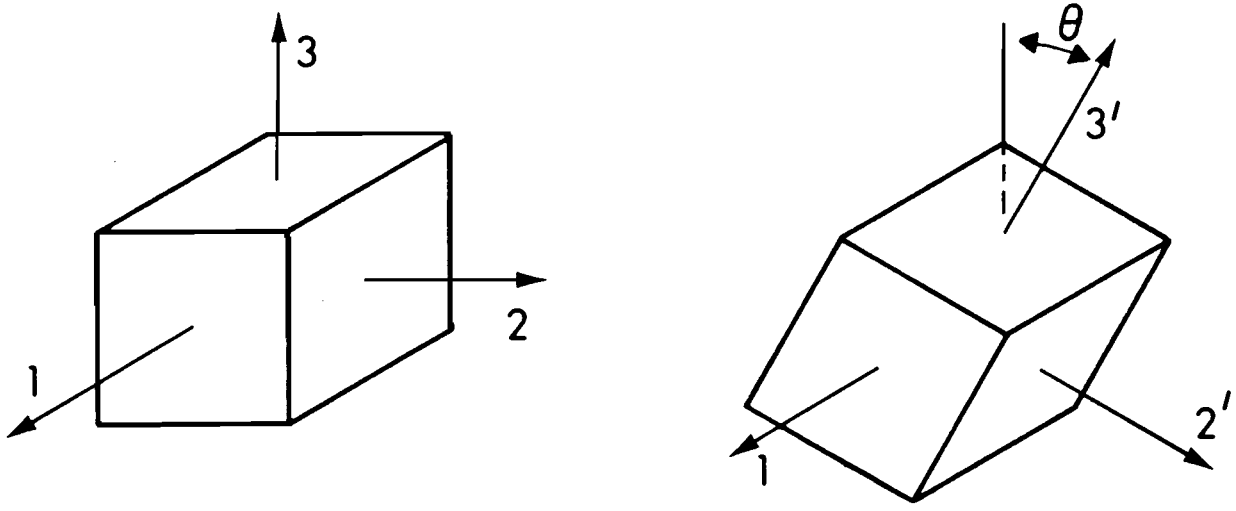


Figure 1. The Preferred Reference Frame of Material Elements May Not Coincide With the Laboratory Frame of Reference



$$\nu_{13} = \nu_{13'}\ ,\ G_{13} = G_{13'}\ ,\ E_3 = E_{3'}\ ,\ Y_3 = Y_{3'}\ ,\ Y_{13} = Y_{13'}$$

When $\theta = 90°$, it follows that

$$\nu_{12} = \nu_{13}\ ,\ G_{12} = G_{13}\ ,\ E_2 = E_3\ ,\ Y_2 = Y_3\ ,\ Y_{12} = Y_{13}$$

Figure 2. The Transversely Isotropic Material Reference Frame With Isotropy in the 2-3 Plane.

As a result, the sum of the three normal deviatoric strain increments is not generally zero, but rather equals a deviatoric dilatation ($\bar{e}$). The significance of this term is that a state of stress whose average normal value is zero can produce volumetric change on an element with respect to that element's stress free volume.

If one wishes to convert a given elastic strain state (ϵ_i) into the elastic deviators (e_i), elastic deviatoric dilatation ($\bar{e}$), and the hydrostatic strain components ($\bar{\epsilon}_j$), the following nine equations given below may be used for a transversely isotropic material (whose plane of isotropy is the 2-3 plane):

$$e_1 = \epsilon_1 - \bar{\epsilon}_1 \tag{3a}$$

$$e_2 = \epsilon_2 - \bar{\epsilon}_2 \tag{3b}$$

$$e_3 = \epsilon_3 - \bar{\epsilon}_2 \tag{3c}$$

$$e_4 = \epsilon_4 \tag{3d}$$

$$e_5 = \epsilon_5 \tag{3e}$$

$$e_6 = \epsilon_6 \tag{3f}$$

$$\bar{e} = e_1 + e_2 + e_3 \qquad \text{(Dilatation of Deviatoric Strain)} \tag{11}$$

$$\bar{\epsilon}_1 = K_\epsilon \bar{\epsilon}_2 \qquad \text{(Non-uniform hydrostatic strain)} \tag{7}$$

$$K_\sigma e_1 + e_2 + e_3 = 0 \qquad \text{(Assures that deviatoric stress has no hydrostatic component)} \tag{9}$$

A convenient solution of this set of equations is given in Appendix C. Finally, the use of the deviatoric constitutive relation, equation (6) hinged upon the satisfaction of equation (5). Inverting equation (5) into compliance form and summing the three equations for normal strain yields upon reduction:

$$\bar{\sigma} = \tilde{K} (\epsilon_1 + \epsilon_2 + \epsilon_3 - \bar{e}) \tag{12}$$

where $\tilde{K}$ is a true material property which will be called the effective bulk modulus of the material (it equals the reciprocal of the sum of the nine normal compliance matrix components), and $(\epsilon_1 + \epsilon_2 + \epsilon_3)$ is the total volumetric dilatation of the material element. This effective modulus, unlike the bulk modulus, is independent of deviatoric stress in anisotropic materials. The bulk modulus reduces to the effective bulk modulus only when the deviatoric dilatation $\tilde{e}$ equals zero. This condition occurs under either of the following conditions: the material is isotropic, or the loading is purely hydrostatic.

It was mentioned previously that the empirical relation between dilatation and pressure is not a linear one. One advantage of the deviatoric

formulation lies in the ability to arbitrarily make the hydrostatic relation non-linear while retaining the linear simplicity of Hooke's Law for the deviatoric portion of the constitutive relation. Though this ad hoc procedure does not theoretically follow as an extension to Hooke's Law, it does permit the code user to more flexibly model the empirical behavior of the material.

There are also codes employing the incremental strain approach which use a formulation employing deviatoric stress, though the formulation can not be termed deviatoric. The form of the relation used by the HELP code[6] is

$$\Delta s_i = \begin{cases} C_{ij}\,\Delta\epsilon_j - 3K\,(\Delta\epsilon_1 + \Delta\epsilon_2 + \Delta\epsilon_3), & i=1,2,3 \\ C_{ij}\,\Delta\epsilon_j, & i=4,5,6 \end{cases} \qquad (13)$$

where K is identified as the bulk modulus which presumably can be made dependent on dilatation (and therefore hydrostatic stress). In this way, the formulation may also provide the flexibility of a truly deviatoric formulation. However, equation (13) is not truly a deviatoric relation, since the deviatoric stress increment is not related to deviatoric strain increment, but rather is expressed in terms of the total strain increment. The system of equations presently proposed, equations (6 and 12), are thus more attractive in a theoretical sense. Similarly, it has already been pointed out that the bulk modulus (as opposed to the effective bulk modulus derived in equation (12)) is functionally dependent on deviatoric stress, and in this sense equation (13) will exhibit flawed behavior if the deviatoric variation in bulk modulus is not modeled. Finally, the flexibility afforded in equation (13) by allowing the bulk modulus to vary with hydrostatic stress has the disturbing effect that the resulting sum of the normal stress deviators is not generally zero. If this interpretation of the HELP algorithm as described in reference 7 is correct, the use of the term stress deviators to describe the left hand side of equation (13) does not even seem justified.

EPIC[7] use a form similar to equation (13) except that K is defined in such a way as to force the sum of the normal stress deviators to zero. This ad hoc procedure will coincidentally mimic the behavior of equation (6), though the formulation is in error during the subsequent hydrostatic stress calculation by not accounting for the deviatorically induced dilatation ($\bar{e}$).

To see additional advantages afforded by the proposed formulation when using a code which employs an incremental strain approach, compare the proposed algorithm specifics with that of the prior formulation used in HELP[6]. The proposed formulation takes strain increments, decomposes them into hydrostatic and deviatoric components. Equation (6) is used in an incremental way to update deviatoric stress. If the hydrostatic strain increments are summed and remembered, equation (12) may be used to evaluate the hydrostatic stress value directly. If the hydrostatic stress is a function of volumetric dilatation only, then errors introduced into the calculation of hydrostatic stress are machine precision dependent, but not algorithm dependent. That is to say, errors in the calculation of hydrostatic pressure are insensitive to the size of the hydrostatic strain increment.

On the other hand, an incremental stress formulation like that proposed for HELP[6] experiences errors which are dependent on hydrostatic strain increment size (which is proportional to the calculation timestep size), if

variable compressibility is employed. For example, use of equation (13) as described for materials with variable compressibility requires that some sort of average compressibility be calculated for the time increment in question. As shown in Figure 3, the average bulk modulus depends not only on the total element dilatation, but also on the size of the strain increment (since dilatation changes with strain increment). Therefore, the accuracy of such a scheme is limited by the integration step size regardless of machine precision. Presumably, this problem can be avoided if one replaces the modulus dilatation product at the end of equation (13) with a $\Delta\bar{\sigma}$ term, where the $\Delta\bar{\sigma}$ term is directly obtainable knowing the previous and present cycles' average stress.

However, many non-linear equations of state that are routinely employed in impact codes like HELP[6] show a dependence of hydrostatic pressure on internal energy. Under such conditions, this dependence of pressure on energy must effectively be reflected in equation (13) for consistency to be maintained. However, since internal energy is affected by the work done by the internal stresses (which include deviatoric stresses), a coupling of internal energy, pressure, and deviatoric stresses exists. No simple means exists to solve this set of equations simultaneously, and a lengthy iterative process becomes necessary. Since no mention of such coupling and/or iteration was made in reference 6, it is believed that none is performed. Thus, it can be seen that equation (13) suffers many drawbacks which make its use less desirable than the proposed method given by equations (6 and 12) in which the deviatoric relations are free of hydrostatic terms.

In summary, the steps proposed for deducing elastic anisotropic deviators in equations (6 and 12) follow closely those for isotropic materials in the following ways: (1) deviatoric stress is expressible totally in terms of deviatoric strain, and (2) pressure is expressible totally in terms of dilatations.

The differences from the isotropic formulation may also be noted: (1) the matrix relating deviatoric stress to deviatoric strain is not diagonal in the anisotropic case, and (2) the total volumetric dilatation must be modified by the deviatorically induced dilatation when calculating the pressure.

4. PLASTIC DEVIATORIC ANISOTROPY

The anisotropic equivalent to the Prandtl-Reuss flow rule of plasticity can be similarly cast into a deviatoric form. Stress behavior of yielding material is governed primarily by the nature of the yield surface, which defines the allowable stress states of the material and subsequent plastic flow properties (Appendix D). In general, only a portion of a post-elastic strain increment (Δe_j^t) contributes to changing the stress. That portion is designated the elastic strain increment (Δe_j). The remaining portion of the strain increment is designated the plastic strain increment (Δe_j^p). This decomposition of the strain increment is governed by two rules: (1) an infinitesimal plastic strain increment vector must be normal to the yield surface at the stress state under consideration, and (2) a stress increment vector tending to go outside of the yield surface can at most move tangentially to the yield surface at the stress state under consideration.

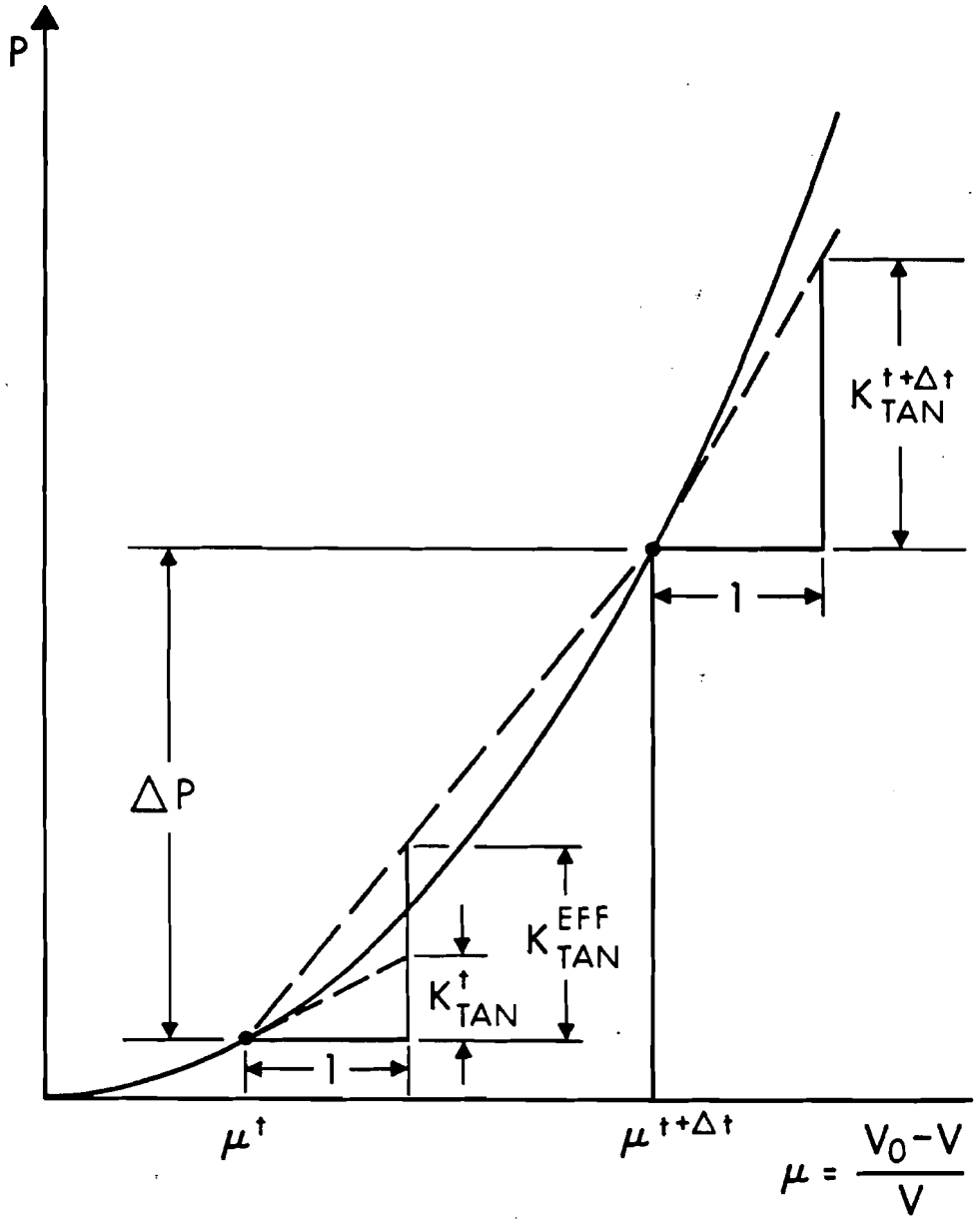


Figure 3. Hydrostatic Pressure Calculations Based on Dilatation Increments Have Their Accuracy Limited by the Size of the Dilatation Increment.

Because of the linearity of the equations governing the conversion from absolute elastic strain (ϵ_j) to deviatoric elastic strain (e_j, $\bar{e}$, $\bar{\epsilon}_j$), one is assured that by decomposing the elastic strain increment into any two arbitrary divisions, the sum of the two converted strain divisions equals the conversion of the strain division sum. This rule becomes handy for impact code implementation if the two strain divisions are taken as the total strain increment and the negative of the plastic strain increment (the sum of which add up to the elastic strain increment). In this way, the stress changes may be calculated on the assumption that the total stress increment is elastic. If it can then be determined that yield has been violated, a fictitious stress may be calculated from the plastic strain increment, and subtracted from the stress state which is in violation of yield to give the true stress state.

To see how this is employed in actuality, consider the deviatoric constitutive relation, equation (6), in which the deviatoric stress increment is calculated via the product of the modulus and elastic deviatoric strain increment. The linearity of the deviatoric conversion equations implies, for plastic deformation, that:

$$\Delta s_i = C_{ij} \, (\Delta e_j^{\,t} - \Delta e_j^{\,p}) \tag{14}$$

The deviatoric total strain increment ($\Delta e_j^{\,t}$) is calculated with the deviatoric conversion equations, based on the total strain increment. The plastic deviatoric strain increment ($\Delta e_j^{\,p}$) can be decomposed into its total plastic ($\Delta \epsilon_j^{\,p}$) and hydrostatic plastic ($\Delta \bar{\epsilon}_j^{\,p}$) components respectively.

The total plastic strain component is necessarily normal to the yield surface, and is given by:

$$\Delta \epsilon_j^{\,p} = \Delta\lambda \, \frac{\partial f}{\partial \sigma_j} \tag{15}$$

where f is the equation governing the yield surface, and $\Delta\lambda$ is a proportionality constant for the yield surface normal ($\partial f/\partial \sigma_j$), which has been evaluated at the stress state in question. If one assumes an anisotropic yield condition like Hill's[9] in which the yield criterion is independent of the hydrostatic pressure, then the yield surface normal may be evaluated with the use of the deviatoric stresses (e.g. $\partial f/\partial s_j$).

Similarly, the hydrostatic plastic component represents the three components of plastic deviatoric dilatation, and can be explicitly calculated knowing the elastic and plastic material constants and the same proportionality constant $\Delta\lambda$ required above.

As a side note, the usage "plastic dilatation" would seem to imply that plastic incompressibility does not hold. This is however not the case. Recall that equations (3,7,9 and 11) were proven valid only for elastic deformations. The concept of plastic strain was introduced to represent the difference between the elastic and total strain components. This term "plastic dilatation" in fact represents a portion of the total dilatation to

be subtracted off to yield the proper value of elastic deviatoric dilatation. The plastic incompressibility relation:

$$\Delta\epsilon_1^P + \Delta\epsilon_2^P + \Delta\epsilon_3^P = 0 \tag{16}$$

is still assumed to hold throughout all calculations derived here. Thus, expressing the plastic deviatoric dilatation term as

$$\Delta\bar{\epsilon}_j^P = \frac{d\bar{\epsilon}_j^P}{d\lambda}\,\Delta\lambda \tag{17}$$

the deviatoric constitutive relation may be expressed, using equations (14,15, and 17) as

$$\Delta s_i = C_{ij}\left\{\Delta e_j^t - \left(\frac{\partial f}{\partial s_j} - \frac{d\bar{\epsilon}_j^P}{d\lambda}\right)\Delta\lambda\right\} \tag{18}$$

Notice that the only term in this relationship which differs from the isotropic case is the last term involving $(d\bar{\epsilon}_j^P/d\lambda)$. This term is zero for the isotropic case because of the fact that there is no dilatation as a result of deviatoric stress. Similarly, this term can not generally be zero for the anisotropic case because equation (18) is a deviatoric stress relationship. The term $(d\bar{\epsilon}_j^P/d\lambda)$ is precisely the magnitude required to force the deviator stress to remain in the π plane (i.e. have no hydrostatic components). The derivation of this term $(d\bar{\epsilon}_j^P/d\lambda)$ is described in Appendix C.

The quantity $\Delta\lambda$ may be evaluated by taking the scalar product of equation (18) with $(\partial f/\partial s_i)$. Because Δs_i is tangential to the yield surface and $(\partial f/\partial s_i)$ is the yield surface normal, the scalar product is zero. Similarly the term $(d\bar{\epsilon}_j^P/d\lambda)$ as derived in Appendix C is of a form identical to that resulting from the purely hydrostatic stress state described in equation (7). Thus, it is the case that the quantity $C_{ij}(d\bar{\epsilon}_j^P/d\lambda)$ is parallel with the hydrostat vector. If one assumes an anisotropic yield condition like Hill's[9] in which the yield criterion is independent of the hydrostatic pressure, the scalar product of $C_{ij}(d\bar{\epsilon}_j^P/d\lambda)$ and $(\partial f/\partial s_i)$ is also zero. Thus the value for $\Delta\lambda$ may be calculated as:

$$\Delta\lambda = \frac{\dfrac{\partial f}{\partial s_i}\,C_{ij}\,\Delta\epsilon_j^t}{\dfrac{\partial f}{\partial s_i}\,C_{ij}\,\dfrac{\partial f}{\partial s_j}} \tag{19}$$

This expression for $\Delta\lambda$ is of a form identical to that obtained for the isotropic case, and can be used in equation (18) to calculate the elastic deviatoric stress increment.

Because of the curvature of the yield surface and the fact that $\Delta\lambda$ was calculated for the stress state existing at the beginning of the time cycle, the updated stress state resulting from equation (18) may in fact still lie slightly outside the yield surface. What is done at this point in both the existing models and the proposed one is to scale back all the stress components uniformly until the yield surface is exactly reached. Though this technique introduces some error on its own, it is believed that the error is not too great since the components of the increment of stress scale back are nearly normal to the yield surface in many cases. Also, ways have been devised by Vavrick and Johnson[7] to decrease the magnitude of this error. Their techniques employ subdivision of the time cycle. However, some anisotropic formulations use a deviatoric stress formulation in which elastic deviatoric stresses are defined in the following way[6]

$$\Delta s_i = \begin{cases} C_{ij}\,\Delta\epsilon_j - 3K\,(\Delta\epsilon_1 + \Delta\epsilon_2 + \Delta\epsilon_3), & i=1,2,3 \\ C_{ij}\,\Delta\epsilon_j, & i=4,5,6 \end{cases} \tag{13}$$

and additional error is introduced as a result. This occurs because the formulation in equation (13) does not guarantee that the sum of the deviatoric stresses will equal zero for an anisotropic material, and in fact they will generally not do so. As a result, any scale back of the stresses employed to meet the yield criterion will include a hydrostatic component. Such hydrostatic scale back violates basic rules of yield surface normality in a fundamental way. Furthermore, techniques proposed by Vavrick and Johnson which decrease the error resulting from stress scale-back will not decrease the amount of hydrostatic stress error introduced into the calculation as the result of using a formulation like that of equation (13).

5. CONCLUSIONS

An anisotropic formulation has been proposed which satisfies the condition of reducing to Hooke's Law/Prandtl Reuss Flow Rule when employing the constraint of constant compressibility and isotropy, but which conveniently allows for anisotropic material properties and variable compressibility.

The deviatoric stress technique which has been used routinely in the isotropic impact codes for describing isotropic behavior has been effectively combined with the anisotropic constitutive relations to produce a truly deviatoric anisotropic constitutive relation. In this deviatoric formulation, deviatoric stress is expressed only in terms of deviatoric strain, and compressibility does not influence the deviatoric relation.

Existing formulations suffer from drawbacks which have been eliminated in the present formulation. Some of the drawbacks of previous formulations may be enumerated as follows: (1) working with absolute stress and strain offers no simple way to perform calculations involving variable compressibility, (2) calculating hydrostatic pressure increments (instead of complete hydrostatic pressure) can introduce error associated with obtaining and averaging the tangent bulk modulus over a strain increment (this problem compounded by the fact that Hugoniot data is usually gathered in the form pressure versus dilatation, the slope of which is the tangent bulk modulus), and (3) use of a

"deviatoric" stress which includes a hydrostatic component will produce error in the pressure calculation if stresses are scaled back to satisfy the yield condition.

Additionally, the formulation can be simply coded into existing impact codes which presently use the deviatoric stress technique for isotropic materials. Finally, it is hoped that the formulation provides an enhanced physical interpretation on the behavior of anisotropic materials.

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APPENDIX A

SKELETON FORTRAN CODING OF THE DEVIATORIC TRANSVERSELY ISOTROPIC ELASTIC PLASTIC CONSTITUTIVE RELATION

In most explicit impact codes, stress is generally computed for a region of the mesh by providing a subroutine with the strain rates in that region of the mesh, the stresses in that region of the mesh at a previous time, and a timestep over which the strain rates act. Though each code's constitutive relation routine use their own unique notations, they all generally: (1) convert the strain rates into deviatoric strain rates, (2) calculate deviatoric stress increments based on the deviator strain rates and timestep, and increment the previous stress state by this increment, (3) check deviatoric stress state for material yielding, (4) modify the deviatoric stress state to account for plastic flow if necessary, and (5) calculate pressure based on the volumetric strain, and time increment, generally using a non-linear equation of state.

The coding required to modify isotropic constitutive subroutines is provided below, with all variables defined, with hopefully enough additional comments to clarify where in the old subroutine the new coding should be substituted. The variable notations used generally conform to those used in the EPIC code[2].

```
      SUBROUTINE ASTRES (REQUIRED ARGUMENTS)
c
c  Anisotropic  stress  increment  formulation:
c
c  REVISED  February-June  1985:  Deviator  Anisotropy
c
      REAL LAMBDA
      INCLUDE 'commons.file'

      COMMON /ELAST/ SIGK, EPSK
      COMMON  /YIELD/EPSBAR(1600),BN(3,3),BS(3),CN(3,3),CS(3),MODFLA
      DIMENSION  STR(6),  DFDS(6),  GI(6),  DE(6),  DSIG(6),  SIG(6),  RSG(6),
     &          DEDL(6)
      LAMBDA = 0.
c
c Generate required anisotropic parameters if they haven't been generated
c  already.
c
      IF (MODFLA .EQ. 0) CALL AGEN
c
c Calculate anisotropic deviator strains based on total strains
c
      CALL DEPS (I, ERDOT, EZDOT, ETDOT, EZTDOT, ERTDOT, ERZDOT,
     &           DE, DEPSB)
c
c Compute rotation and change in normal stresses because of rotation
c
      SPDT = SPINRZ*DT1
      DSTRN  =  2.*SPDT*SRZ(I)
c
c Obtain deviator stresses
c
      SBAR  =  (SR(I)  +  SZ(I)  +  ST(I))  /  3.
      SR1  =  SR(I)  -  SBAR
      SZ1  =  SZ(I)  -  SBAR
      ST1  =  ST(I)  -  SBAR
      SZT1  =  SZT(I)
      SRT1  =  SRT(I)
      SRZ1  =  SRZ(I)
c
c  Strength  variable  SEFF  is  constant  for  my  formulation
c
      SEFF = FU(M)
c
c Transform stress to LTT frame
c
      CALL THETA (I, TH)
      CALL  XFORM  (SR1  ,SZ1  ,ST1  ,SZT1  ,SRT1  ,SRZ1,
     &            SIG(1),SIG(2),SIG(3),SIG(4),SIG(5),SIG(6),TH)
c
c Calculate stress increment due to element rotation (rsg) in LTT frame
c
      CALL  XFORM  (-DSTRN,  DSTRN,  0.,  0.,  0.,  (SR(I)-SZ(I))*SPDT,
     &            RSG(1),RSG(2),RSG(3),RSG(4),RSG(5),RSG(6),TH)
```

```
c
c Calculate trial stress increment (dsig) due to strain changes (de)
c in LTT frame
c
      CALL CXE (DE, DSIG)
c
c Lump together strain induced stress (dsig) and rotation induced
c  stress (rsg)
c
      DO 31 K = 1, 6
   31 DSIG(K) = DSIG(K) + RSG(K)
c
c Calculate trial stress state
c
      DO 33 K = 1, 6
      STR(K) = SIG(K) + DSIG(K)
   33 CONTINUE
c
c Test for yielding
c
      TERM1 = 0.
      TERM2 = 0.
      DO 35 K = 1, 3
      DO 34 L = 1, 3
   34 TERM1 = TERM1 + BN(K,L) * STR(K) * STR(L)
   35 TERM2 = TERM2 + BS(K) * STR(K+3)**2
      VMISES = SQRT(TERM1/2. + 3.*TERM2)
      IF(VMISES.LE.SEFF) THEN
c
c Stress is elastic.  Transform stress back to RZT frame...
c
         SEFF = VMISES
         ICHECK(I) = 0
         DEPSBP = 0.
         CALL  XFORM  (STR(1),STR(2),STR(3),STR(4),STR(5),STR(6),
     &              SR2   ,SZ2   ,ST2   ,SZT2   ,SRT2   ,SRZ2,-TH)
         GO TO 310
      END IF
c
c Yield has occured: Determine ALF, the fraction of strain
c  that is pre-yield.
c
      IF (ICHECK(I) .EQ. 1) THEN
c
c  Deformation already plastic... elastic fraction (alf) = 0.
c
         ALF = 0.
      ELSE
c
c else must determine elastic fraction (alf) (see Vavrick, Johnson)
c
        ICHECK(I) = 1
        TERM1 = 0.
        TERM2 = 0.
```

```
        TERM3 = 0.
        TERM4 = 0.
        TERM5 = 0.
        TERM6 = 0.
        DO 41 K = 1, 3
        DO 40 L = K, 3
        IF (K .EQ. L) GOTO 40
        TERM1 = TERM1 - BN(K,L) * (DSIG(K)-DSIG(L))**2
        TERM3 = TERM3 - BN(K,L) * (DSIG(K)-DSIG(L))*(SIG(K)-SIG(L))
        TERM5 = TERM5 - BN(K,L) * (SIG(K)-SIG(L))**2
 40   CONTINUE
        TERM2 = TERM2 + BS(K) * DSIG(K+3)**2
        TERM4 = TERM4 + BS(K) * DSIG(K+3)*SIG(K+3)
        TERM6 = TERM6 + BS(K) * SIG(K+3)**2
 41   CONTINUE
        AAA = TERM1/2. + 3.*TERM2
        BBB = TERM3 + 6.*TERM4
        CCC = TERM5/2. + 3.*TERM6 - SEFF**2
        ALF = (-BBB + SQRT(BBB**2 - 4.*AAA*CCC)) / (2. * AAA)
      END IF
c
c Calculate transition stress (str) and post elastic strain increment (d
c
      DO 51 K = 1, 6
      STR(K) = SIG(K) + ALF*DSIG(K)
 51   DE(K) = (1. - ALF) * DE(K)
c
c Of this post-elastic strain increment, only that portion normal
c to the yield surface is plastic. Equation is
c
c  delta (epsilon plastic) = lambda * (df/d(sigma))
c
c where f=constant functionally defines the yield surface
c
      CALL DFDSIG(STR, SEFF, DFDS)
c
c Generate Cij (df/d(sigma)j) vector (otherwise known as Gi)
c
      CALL CXE(DFDS, GI)
c
c Generate the de/dlambda vector (based on the transition stress str)
c
      SFACTR = STR(1) / SEFF
      ETERM = -1.5 * (SIGK-1.) / (2.+SIGK*EPSK) * BN(1,2) * SFACTR
      DEDL(1) = ETERM * EPSK
      DEDL(2) = ETERM
      DEDL(3) = ETERM
      DEDL(4) = 0.
      DEDL(5) = 0.
      DEDL(6) = 0.
c
c Calculate lambda (happens to equal the equivalent plastic strain)
c
      TERM1 = 0.
```

```
      TERM2 = 0.
      DO 54 K = 1, 6
      TERM1 = TERM1 + GI(K) * DE(K)
   54 TERM2 = TERM2 + GI(K) * DFDS(K)
      LAMBDA = TERM1 / TERM2
c
c Calculate element dilitation resulting from plastic deviator increment
c
      DEPSBP = -ETERM * (2. + EPSK) * LAMBDA
c
c Since [lambda * (df/do)] is the plastic strain vector, the
c elastic part of the post elastic deviator strain vector (LHS) is :
c [post elastic strain vector (RHS)] - [lambda * {(df/do)-(de/dlambda)}]
c
      DO 56 K = 1, 6
   56 DE(K) = DE(K) - LAMBDA * (DFDS(K) - DEDL(K))
c
c Multiply this elastic part of the post elastic strain increment (de)
c by the modulus to find the change in stress after yielding (dsig)
c
      CALL CXE (DE, DSIG)
c
c Add this actual stress change (dsig) to the transition stress (str) in
c order to obtain the updated stress (sig)
c
      DO 58 K = 1, 6
   58 SIG(K) = STR(K) + DSIG(K)
c
c Because of the linear interpolation along the curved yield surface,
c a correction must be made to the stress to place the stress back onto
c the yield surface
c
      TERM1 = 0.
      TERM2 = 0.
      DO 60 K = 1, 3
      DO 59 L = 1, 3
   59 TERM1 = TERM1 + BN(K,L) * SIG(K) * SIG(L)
   60 TERM2 = TERM2 + BS(K) * SIG(K+3)**2
      VMISES = SQRT(TERM1/2. + 3.*TERM2)
c
c Correct stress (sig) to place it back on the yield surface
c
      DO 64 K = 1, 6
   64 SIG(K) = SIG(K) * SEFF/VMISES
c
c Transform stress back to RZT frame
c
      CALL   XFORM   (SIG(1),SIG(2),SIG(3),SIG(4),SIG(5),SIG(6),
     &           SR2   ,SZ2   ,ST2   ,SZT2   ,SRT2   ,SRZ2,-TH)
c
c  EFFECTIVE  PLASTIC  STRAIN
c
      EBAR(I) = EBAR(I) + LAMBDA
c
```

```
c Update dilitation from deviator elastic and plastic calculations
c
  310 EPSBAR(I) = EPSBAR(I) + (DEPSB - DEPSBP)
c
c modify dilitation to account for deviator stresses (for pressure
c  calculation)
c
      U = U + EPSBAR(I)
c
c   dW   =  σdε
c    =   σ̄dε̄    +    σ̄de    +    sdε̄    +    se
c
c The first two terms end up in energy equation as p dV.  Second two
c  terms  appear  below  as  sdε.
c
      SRBAR = (SR1 + SR2)
      SZBAR = (SZ1 + SZ2)
      STBAR = (ST1 + ST2)
      SZTBAR = (SZT1 + SZT2)
      SRTBAR = (SRT1 + SRT2)
      SRZBAR = (SRZ1 + SRZ2)
      FDVMT=DVDOT*DT1/2.
      EDEV =.5 * (SRBAR*ERDOT + SZBAR*EZDOT + STBAR*ETDOT
     & + SZTBAR*EZTDOT + SRTBAR*ERTDOT + SRZBAR*ERZDOT)
     & * (DVOLI - FDVMT +1.)*DT1
c
c PLASTIC  WORK  FOR  SYSTEM
c
      IF(ICHECK(I).GT.0)  THEN
        PLAST = PLAST + (SEFF * LAMBDA)*VOL(I)
      END IF
c
c      Calculate sound speed for eventual use in timestep calculation
c
      .
      .
      .
c
c INTERNAL ENERGY & PRESSURE (use corrected dilitation for pressure)
c
      .
      .
      .
c
c  NET  STRESSES
c
  440 SR(I) = SR2 - PRES - Q
      SZ(I) = SZ2 - PRES - Q
      ST(I) = ST2 - PRES - Q
      SRZ(I) = SRZ2
      SRT(I) = SRT2
      SZT(I) = SZT2
      RETURN
      END
```

```
C*********************************************************************
      SUBROUTINE DEPS (I, ERDOT, EZDOT, ETDOT, EZTDOT, ERTDOT, ERZDOT,
     &           DE, DEPSBT)
c
c Calculates the anisotropic deviator strain increment
c
      COMMON /ELAST/ SIGK, EPSK
      INCLUDE 'commons.file'
      DIMENSION DE(6)
c
c Define 6x1 tensorial total strain increment vector (in RZT frame)
c
   10 DER = ERDOT *DT1
      DEZ = EZDOT *DT1
      DET = ETDOT *DT1
      DEZT = EZTDOT *DT1 / 2.
      DERT = ERTDOT *DT1 / 2.
      DERZ = ERZDOT *DT1 / 2.
c
c Transform strain increment vector to LTT frame
c
      CALL THETA (I, TH)
      CALL XFORM(DER ,DEZ ,DET ,DEZT ,DERT ,DERZ,
     &           DE(1),DE(2),DE(3),DE(4),DE(5),DE(6),TH)
c
c Transform into deviator strains, determine depsbt (dilitation caused b
c total deviator strains, later to be modified by plastic deviatoric
c  dilitation)
c
      TERM = (SIGK - 1.) / (2. + SIGK*EPSK)
      DESUM = DE(1) + DE(2) + DE(3)
      DEPSBT = TERM * (EPSK * DESUM - (2.+EPSK) * DE(1))
      EPST = (DESUM - DEPSBT) / (2. + EPSK)
      EPSL = EPSK * EPST
      DE(1) = DE(1) - EPSL
      DE(2) = DE(2) - EPST
      DE(3) = DE(3) - EPST
      RETURN
      END
C*********************************************************************
      SUBROUTINE THETA (I, TH)
c
c Calculate orientation of element by any appropriate means
c
      .
      .
      .
      RETURN
      END
C*********************************************************************
      SUBROUTINE CXE (EE, SS)
c
c Multiplies on axis modulus by vector EE to obtain vector SS
c
```

```
      COMMON /YIELD/ EPSBAR(1600),BN(3,3),BS(3),CN(3,3),CS(3),MODFLA
      DIMENSION EE(6), SS(6)
      DO 20 I = 1, 3
         SS(I) = 0.
      DO 10 J = 1, 3
   10  SS(I) = SS(I) + CN(I,J) * EE(J)
   20  SS(I+3) = CS(I) * EE(I+3)
      RETURN
      END
C*********************************************************************
      SUBROUTINE AGEN
C
C Calculate the on-axis modulus matrix and yield parameters once only
C
      COMMON /ELAST/ SIGK, EPSK
      COMMON /ORIENT/ ANGLE, TPARAM(1600)
      COMMON /YIELD/ EPSBAR(1600),BN(3,3),BS(3),CN(3,3),CS(3),MODFLA
      COMMON/LUS/LUI,LUP,LUT,LUPR,LUST,LUFAST
      DATA LUA /13/
C
      OPEN (LUA, FILE='amat1.dat', STATUS='old')
      REWIND (LUA)
      MODFLA = 1
      WRITE (LUP, 505)
  505 FORMAT (//' Calculating Anisotropic Modulus'//)
C
C  Engineering Constants: (for trans.-isotropic material)
C
C Longitudinal Young's Modulus
      READ (LUA, *) EL
C Transverse Young's Modulus
      READ (LUA, *) ET
C Shear Modulus in Longitudinal-Transverse Plane
      READ (LUA, *) GLT
C Shear modulus in transverse (isotropic) plane
      READ (LUA, *) GTT
C Bulk Modulus:
      READ (LUA, *) FK
C Left to calculate: isotropic, LT, and TL Poisson Ratios
      VTT = ET/(2.*GTT) - 1.
      VLT = .25 + (1.-VTT)*EL/(2.*ET) - EL/(4.*FK)
      VTL = VLT * (ET/EL)
C
C  Modulus Matrix (transversely isotropic):
C
      DEL = (1 - 2*VLT*VTL - VTT**2 - 2*VLT*VTL*VTT) / (EL * ET**2)
      CL = (1 - VTT**2 ) / (ET**2 * DEL)
      CT = (1 - VTL*VLT) / (EL * ET * DEL)
      CLT = (VLT + VTT*VLT) / (EL * ET * DEL)
      CTT = (VTT + VLT*VTL) / (EL * ET * DEL)
      CG = GLT
C
      SL = 1. / EL
      ST = 1. / ET
```

```
      SLT = -VLT / EL
      STT = -VTT / ET
      SG = 1. / (2. * GLT)
      SGI = ST - STT
c
c Calculate Keps and Ksig (variables EPSK and SIGK respectively)
c
      EPSK = (SL + 2.*SLT) / (ST + SLT + STT)
      SIGK = (CL + 2.*CLT) / (CT + CLT + CTT)
c
      CN(1,1) = CL
      CN(1,2) = CLT
      CN(1,3) = CLT
      CN(2,2) = CT
      CN(2,3) = CTT
      CN(3,3) = CT
      CS(1) = (CT - CTT)
      CS(2) = 2. * CG
      CS(3) = 2. * CG
      CN(2,1) = CN(1,2)
      CN(3,2) = CN(2,3)
      CN(3,1) = CN(1,3)
      WRITE (LUP, *) 'Compliance Matrix:'
      WRITE (LUP, 10) SL,       SLT,    SLT
      WRITE (LUP, 10) SLT, ST,          STT
      WRITE (LUP, 10) SLT, STT, ST
      WRITE (LUP, 10) SGI, SG,          SG
      WRITE (LUP, 11)
      WRITE (LUP, *) 'Modulus Matrix: '
      WRITE (LUP, 10) CN(1,1), CN(1,2), CN(1,3)
      WRITE (LUP, 10) CN(2,1), CN(2,2), CN(2,3)
      WRITE (LUP, 10) CN(3,1), CN(3,2), CN(3,3)
      WRITE (LUP, 10) CS(1), CS(2), CS(3)
   10 FORMAT (3(E15.7,4X))
c
c Read Orientation of anisotropy
c
      READ (LUA, *) ANGLE
c
c Calculate Yield parameters (bn(i,j) , bs(i)) (transversely isotropic)
c
c Longitudinal Strength
      READ (LUA, *) SIGL
c Transverse Strength
      READ (LUA, *) SIGT
c LT Shear Strength
      READ (LUA, *) SIGLT
c
      SEFF = SIGT
      BN(1,1) = 2. * SEFF**2 / SIGL**2
      BN(2,2) = 2. * SEFF**2 / SIGT**2
      BN(3,3) = BN(2,2)
c
      BN(1,2) = -(+BN(1,1) + BN(2,2) - BN(3,3)) / 2.
```

```
      BN(1,3) = -(+BN(1,1) - BN(2,2) + BN(3,3)) / 2.
      BN(2,3) = -(-BN(1,1) + BN(2,2) + BN(3,3)) / 2.
      BN(2,1) = BN(1,2)
      BN(3,1) = BN(1,3)
      BN(3,2) = BN(2,3)
C
      FACTOR = SIGL / SIGT
      TAU2 = SIGT**2 / (4. - (1./FACTOR**2))
      BS(1) = SEFF**2 / (3. * TAU2)
      BS(2) = SEFF**2 / (3. * SIGLT**2)
      BS(3) = BS(2)
      WRITE (LUP, 11)
      WRITE (LUP, *) 'Strength Matrix:'
      WRITE (LUP, 10) BN(1,1), BN(1,2), BN(1,3)
      WRITE (LUP, 10) BN(2,1), BN(2,2), BN(2,3)
      WRITE (LUP, 10) BN(3,1), BN(3,2), BN(3,3)
      WRITE (LUP, 10) BS(1), BS(2), BS(3)
      WRITE (LUP, 11)
   11 FORMAT (/)
      RETURN
      END
C*********************************************************************
      SUBROUTINE XFORM (U1,U2,U3,U4,U5,U6,P1,P2,P3,P4,P5,P6,TH)
C
C Transforms stresses and strains:
C ui : stress or strain prior to transformation (unprimed frame)
C pi : stress or strain after transformation (primed frame)
C th : CCW angle of transformation (in RZ frame)
C
      COMMON/LUS/LUI,LUP,LUT,LUPR,LUST,LUFAST
C
      REAL M, N, M2, N2, MN
C
      M = DCOS(TH)
      N = DSIN(TH)
      M2 = M**2
      N2 = N**2
      MN = M*N
C
C All transformations are tensorial, so stress and strain are the same
C
      P1 = M2*U1      + N2*U2                     + (2.*MN)*U6
      P2 = N2*U1      + M2*U2                     - (2.*MN)*U6
      P3 =                          U3
      P4 =                              M*U4 - N*U5
      P5 =                              N*U4 + M*U5
      P6 = -(MN)*U1 + (MN)*U2                     + (M2-N2)*U6
C
      RETURN
      END
C*********************************************************************
      SUBROUTINE DFDSIG (S, SEFF, DFDS)
C
C Calculates df/d(sigma) for element in question
```

```
C
      COMMON /YIELD/ EPSBAR(1600),BN(3,3),BS(3),CN(3,3),CS(3),MODFLA
      DIMENSION S(6), DFDS(6)
C
      TWOS = 2.*SEFF
      DFDS(1) = (-BN(1,2)*(S(1)-S(2)) - BN(1,3)*(S(1)-S(3))) / TWOS
      DFDS(2) = ( BN(1,2)*(S(1)-S(2)) - BN(2,3)*(S(2)-S(3))) / TWOS
      DFDS(3) = ( BN(1,3)*(S(1)-S(3)) + BN(2,3)*(S(2)-S(3))) / TWOS
      DFDS(4) = 3.*BS(1)*S(4) / TWOS
      DFDS(5) = 3.*BS(2)*S(5) / TWOS
      DFDS(6) = 3.*BS(3)*S(6) / TWOS
      RETURN
      END
C*********************************************************************
```

APPENDIX B

THE EFFECT OF MATERIAL FRAME ON ANISOTROPIC COMPUTATIONS

Material frame is not a consideration in isotropic codes, because the constitutive relation is identical in all reference frames. As such, doing the calculations in the laboratory frame of reference is the logical choice. However, when anisotropy is involved, the material properties are different in different reference frames. For regular types of anisotropy (e.g. transverse isotropy, orthotropy, etc.), there are preferred directions in which the materials constitutive relations reduce to their most simple forms. In general, this material frame does not coincide with the laboratory frame of reference. Unfortunately, it is usually the laboratory frame in which system properties (stress, strain, etc.) are described. Two approaches may thus be taken to implement anisotropy into the codes: 1) transform laboratory stress and strain into the material frame, perform constitutive computations in the material frame, and transform the resulting stresses and strains back into the laboratory frame, or 2) transform the simple material frame constitutive relations into the laboratory frame of reference, and perform calculations with these new laboratory frame constitutive relations.

The following is a comparison of the pertinent relations as they would appear in both the material and laboratory frame coordinate systems. In Table B-1, primed values of stress and strain denote values in the laboratory frame, while unprimed values denote the material frame values. The relationship between material and laboratory frame stress and strain is

$$\sigma_i = T_{ij}\,\sigma_j' \tag{B-1}$$

$$\epsilon_i = T_{ij}\,\epsilon_j' \tag{B-2}$$

where T_{ij} is the appropriate transformation matrix between laboratory and material coordinate systems. Note that because the contracted stress and strain notations are being used, the transformation matrix T_{ij} is not symmetric.

Table B-1 shows the nature of the calculations when done in both the laboratory and material reference frames. In the Table B-1, the substitution:

$$\frac{\partial f}{\partial \sigma_i} = \varphi_{ij}\,\sigma_j \tag{B-3}$$

has been made for simplicity of transformation, where f is the function defining the yield surface and $\partial f/\partial \sigma_i$ is the vector normal to the yield surface in the material reference frame. Table B-2 contains the general form of the terms contained in Table B-1. For calculations done in the material frame, there is a constant "overhead" penalty of making the initial stress and strain transformations, which does not exist in the laboratory frame scenerio. However, it can be seen from Table B-1 that in the laboratory frame there is penalty of transformation for every calculation done.

Thus, if anything but the most trivial of calculations are required, then it computationally pays to first transform stress and strain to the material frame, perform the calculations there, and transform back at the conclusion of the computations.

Table B-1. Comparison of Governing Equations in the Material and Laboratory Coordinate Frames

	Material Frame	Laboratory Frame
Transformation to Desired Frame	$\{\sigma\} = [T]\{\sigma'\}$	N/A
	$\{\epsilon\} = [T]\{\epsilon'\}$	N/A
Elastic Constitutive Equation	$\{\sigma\} = [C]\{\epsilon\}$	$\{\sigma'\} = [T]^{-1}[C][T]\{\epsilon'\}$
Yield Equation	$f^2 = \{\sigma\}^T[\varphi]\{\sigma\}$	$f^2 = \{\sigma'\}^T[T]^T[\varphi][T]\{\sigma'\}$
Plastic Strain	$\{\Delta\epsilon^p\} = \Delta\lambda[\varphi]\{\sigma\}$	$\{\Delta\epsilon'^p\} = \Delta\lambda[T]^{-1}[\varphi][T]\{\sigma'\}$
Plastic Strain Parameter	$\Delta\lambda = \dfrac{\{\sigma\}^T[\varphi]^T[C]\{\Delta\epsilon\}}{\{\sigma\}^T[\varphi]^T[C][\varphi]\{\sigma\}}$	$\Delta\lambda = \dfrac{\{\sigma'\}^T[T]^T[\varphi]^T[C][T]\{\Delta\epsilon'\}}{\{\sigma'\}^T[T]^T[\varphi]^T[C][\varphi][T]\{\sigma'\}}$
Transformation to Original Frame	$\{\sigma'\} = [T]^{-1}\{\sigma\}$	N/A

where:

[ ] denotes a 6x6 matrix,
{ } denotes a 6x1 vector,
the superscript -1 denotes a matrix inverse,
the superscript T denotes a matrix transpose, and
the vectors and matrices used in this table are defined in Table B-2.

Table B-2. General Forms of Pertinent Orthotropic Terms

Constitutive Matrix $[C] = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix}$

Yield Normal Matrix $[\varphi] = 1/2 \begin{pmatrix} B_{11} & B_{12} & B_{13} & 0 & 0 & 0 \\ B_{12} & B_{22} & B_{23} & 0 & 0 & 0 \\ B_{13} & B_{23} & B_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3B_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 3B_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3B_{66} \end{pmatrix}$

where:
$B_{12} = (-B_{11} - B_{22} + B_{33}) / 2$
$B_{13} = (-B_{11} + B_{22} - B_{33}) / 2$
$B_{23} = (B_{11} - B_{22} - B_{33}) / 2$

Stress Vector $\{\sigma\}^T = \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{23} & \sigma_{13} & \sigma_{12} \end{pmatrix}$

Strain Vector $\{\epsilon\}^T = \begin{pmatrix} \epsilon_{11} & \epsilon_{22} & \epsilon_{33} & \epsilon_{23} & \epsilon_{13} & \epsilon_{12} \end{pmatrix}$

APPENDIX C

DERIVATION OF GOVERNING RELATIONS

I. Elastic Strain Decomposition:

For transversely isotropic material, with isotropy in the 2-3 plane, the decomposition of a given elastic strain state (ϵ_i) into the elastic deviatoric strains (e_i), elastic deviatoric dilatation ($\bar{e}$), and the hydrostatic strain components ($\bar{\epsilon}_j$) has been shown to require the solution of the following nine equations:

$$e_1 = \epsilon_1 - \bar{\epsilon}_1 \tag{3a}$$

$$e_2 = \epsilon_2 - \bar{\epsilon}_2 \tag{3b}$$

$$e_3 = \epsilon_3 - \bar{\epsilon}_2 \tag{3c}$$

$$e_4 = \epsilon_4 \tag{3d}$$

$$e_5 = \epsilon_5 \tag{3e}$$

$$e_6 = \epsilon_6 \tag{3f}$$

$$\bar{e} = e_1 + e_2 + e_3 \qquad \text{(Dilatation of Deviatoric Strain (11)}$$

$$\bar{\epsilon}_1 = K_\epsilon \, \bar{\epsilon}_2 \qquad \text{(Non-uniform Hydrostatic Strain) (7)}$$

$$K_\sigma \, e_1 + e_2 + e_3 = 0 \qquad \text{(Assures that deviatoric stress has no hydrostatic component) (9)}$$

Standard equation reduction techniques may be employed to obtain the following solution sequence:

$$\bar{e} = \frac{(K_\sigma - 1)}{(2 + K_\sigma K_\epsilon)} \left[K_\epsilon \, (\epsilon_1 + \epsilon_2 + \epsilon_3) - (2 + K_\epsilon) \, \epsilon_1 \right] \tag{C-1}$$

$$\bar{\epsilon}_2 = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 - \bar{e}}{(2 + K_\epsilon)} = \frac{K_\sigma \, \epsilon_1 + \epsilon_2 + \epsilon_3}{(2 + K_\sigma K_\epsilon)} \tag{C-2}$$

$$\bar{\epsilon}_1 = K_\epsilon \, \bar{\epsilon}_2 \tag{C-3}$$

Equations (3) are now directly solvable for e_j.

II. Derivation of $(d\bar{\epsilon}_j^P/d\lambda)$:

In determining the elastic strain components to be used for the calculation of stress after yielding, it was found to be convenient to decompose these elastic components into the total strain increment, and the negative of the plastic strain increment. The plastic flow relations required the knowledge of the term $\Delta\bar{\epsilon}_j^P$, which at the time was left only as $(d\bar{\epsilon}_j^P/d\lambda)\Delta\lambda$, the quantity $\Delta\lambda$ being determined through other means. The term $(d\bar{\epsilon}_j^P/d\lambda)$is acquired by employing the deviatoric conversion equations (C-1,C-2, C-3, and 3) on the plastic portion of the strain increment. Again, this is permitted because of the linearity of the conversion equations, the negative of the plastic strain increment being nothing more than a decomposed component of the elastic strain increment.

Employing equation (C-1) and making use of the plastic incompressibility relation (16), the dilative quantity $\Delta\bar{e}^P$ is determined to be:

$$\Delta\bar{e}^P = \frac{-(K_\sigma - 1)(2 + K_\epsilon)}{(2 + K_\sigma K_\epsilon)} \Delta\epsilon_1^P \qquad \text{(C-4)}$$

Employing the first order approximation to the plastic flow rule, one acquires $\Delta\epsilon_1^P = \Delta\lambda(\partial f/\partial\sigma_1)$. Using the relations of Appendix D under the constraints of a transversely isotropic material, one can show

$$\frac{\partial f}{\partial \sigma_1} = \frac{-3\, B_{12}\, s_1}{2} \qquad \text{(C-5)}$$

Thus, for the transversely isotropic material in question, the dilative quantity $\Delta\bar{e}^P$ may be cast completely in terms of available quantities (excepting $\Delta\lambda$) as:

$$\Delta\bar{e}^P = \frac{3(K_\sigma - 1)(2 + K_\epsilon)}{2(2 + K_\sigma K_\epsilon)} B_{12}\, s_1\, \Delta\lambda \qquad \text{(C-6)}$$

Equations (C-2,C-3 and C-6) may then be employed to ascertain the quantity $\Delta\bar{\epsilon}_j^P$ as

$$\Delta\bar{\epsilon}_j^P = \frac{-3(K_\sigma - 1)\, B_{12}\, s_1}{2\,(2 + K_\sigma K_\epsilon)} \begin{pmatrix} K_\epsilon \\ 1 \\ 1 \end{pmatrix} \Delta\lambda \qquad \text{(C-7)}$$

As a result of differentiating equation (C-7), the quantity $(d\bar{\epsilon}_j^P/d\lambda)$ is readily available for use in equations (17) and (18).

APPENDIX D

ANISOTROPIC YIELD AND FLOW RULE RELATIONS

The theory and computer code implementation of yield and plasticity rules for anisotropic materials has been detailed by others[6,7,9]. The theory extends the approach of the Von Mises yield criterion, which is used extensively for isotropic materials. A simple review of the pertinent points will be done just for clarity. Hill's original statement of the anisotropic yield criterion was given as:

$$2F = 1 = E(\sigma_2-\sigma_3)^2 + G(\sigma_3-\sigma_1)^2 + H(\sigma_1-\sigma_2)^2 + 2(L\sigma_4^2 + M\sigma_5^2 + N\sigma_6^2) \qquad (D\text{-}1)$$

By making the appropriate substitutions, this criterion was restated by Vavrick and Johnson as:

$$f = 1 = \left\{ .5(B_{11}\sigma_1^2 + B_{22}\sigma_2^2 + B_{33}\sigma_3^2) + B_{12}\sigma_1\sigma_2 + B_{13}\sigma_1\sigma_3 + B_{23}\sigma_2\sigma_3 + 3(B_{44}\sigma_4^2 + B_{55}\sigma_5^2 + B_{66}\sigma_6^2) \right\}^{1/2} \qquad (D\text{-}2)$$

The yield function f, when fixed at a value of unity, implies a perfectly plastic material. Uniform work hardening may be realized by letting the yield function f take on values greater than unity. The form of equation (D-2) makes it easy to define the material constants of the B matrix. If for example, Y_1 represents the tensile strength of the material in material direction 1, then considering the simple case of uniaxial tension in the 1 direction, substitution into (D-2) reveals directly that:

$$B_{11} = 2 f^2 / Y_1^2 \qquad (D\text{-}3)$$

Similarly, the other constants are generated easily from simple tension and shear data, or from linear combinations of the other constants. For transversely isotropic materials such as the ones being considered in this report, the yield matrix B_{ij} takes the form:

$$B_{ij} = \begin{pmatrix} B_{11} & B_{12} & B_{12} & 0 & 0 & 0 \\ B_{12} & B_{22} & B_{23} & 0 & 0 & 0 \\ B_{12} & B_{23} & B_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & B_{55} \end{pmatrix} \qquad (D\text{-}4)$$

where:

$$B_{11} = 2 f^2 / Y_1^2$$

$$B_{22} = 2 f^2 / Y_2^2$$

$$B_{12} = -B_{11} / 2$$

$$B_{23} = (B_{11} - 2B_{22}) / 2$$

$$B_{44} = f^2 / 3\, Y_4^2$$

$$B_{55} = f^2 / 3\, Y_5^2$$

Because of the fact that the 2-3 plane is isotropic, the yield criterion must be independent of rotations in that plane. By evaluating the yield equation (D-2) under conditions of pure shear in the 2-3 plane, and by then reevaluating yield in a coordinate frame rotated by 45 degrees in the isotropic plane, it can be shown that Y_4 is constrained for transversely isotropic materials to be:

$$Y_4^2 = \frac{Y_2^2}{4 - \left(\frac{Y_2}{Y_1}\right)^2} \qquad \text{(D-5)}$$

For calculations involving the yield surface normal, partial derivatives are taken on equation (D-2) with respect to each of the nine tensorial stress components, and evaluated at the stress state in question. Because the yield equation (D-2) is expressed in terms of a convenient (albeit non-tensorial) six dimensional contracted "stress vector" space, it must be realized that terms like σ_4 in reality represent the sum of two equal valued shear stresses (e.g. $\sigma_4 = .5(\sigma_{23} + \sigma_{32})$). As such the partial derivatives with respect to the shear stresses is one half that if calculated strictly on the basis of equation (D-2).

SYMBOLS

$( )'$ a primed variable is a quantity whose value is taken in an arbitrary laboratory reference frame. Unprimed quantities are those taken in the "material coordinate frame" of a transversely isotropic material.

$( )^t$ a superscript t denotes that a variable represents a total quantity, which is composed of an elastic part and a plastic part.

$( )^p$ a superscript p denotes that a variable represents a plastic quantity.

$\Delta( )$ a delta before a quantity signifies that the quantity is an increment. As with all models employing the Cauchy strain tensor, incremental constitutive relations must be employed with corrections for rotation in order to make the proposed model acceptable for computation of systems involving large strains.

C'_{ij} modulus matrix (6x6) which relates stress components σ_i' to strain components ϵ_j'. The "material coordinate frame" of a transversely isotropic material will be defined as the reference frame whose the modulus matrix (designated without the use of primes) is:

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{pmatrix}$$

σ_i elastic stress components in contracted notation; indices 1 to 3 are normal components, whereas 4 to 6 are the shear components 23, 13 and 12 respectively.

ϵ_j elastic strain components in contracted notation; indices 1 to 3 are normal components, whereas 4 to 6 are the shear components 23, 13 and 12 respectively.

$\bar{\sigma}_i$ average stress, by definition equal to the negative of the hydrostatic pressure.

s_i deviatoric elastic stress components (6 independent). In this report, the term "deviatoric" will imply a deviation from the stress state resulting from a condition of hydrostatic pressure.

e_j deviatoric elastic strain components (6 independent). In this report, the term "deviatoric" will imply a deviation from the strain state resulting from a condition of hydrostatic pressure. For anisotropic materials, strain is not uniform under conditions of hydrostatic pressure (i.e. the three principal components of strain are not

identical). As a result, the normal deviatoric strain components are NOT simply the difference between the total strain component and the average of the normal strain components.

$\bar{e}$ deviatoric dilatation ($e_1+e_2+e_3$). Though dilatation is only a function of pressure for isotropic materials, dilatation may vary in an anisotropic material just by varying the deviatoric stress (without changing the pressure). Thus, this dilatation associated with the deviatoric stress is is referred to as deviatoric dilatation.

$\bar{e}_j$ strain state resulting from hydrostatic pressure. For an isotropic material, the three normal "hydrostatic" strains would be equal. This is not the case for anisotropic material.

K_σ a parameter which represents the ratio of longitudinal to transverse strain (in the material reference frame) under conditions of hydrostatic pressure ($\sigma_1 = \sigma_2 = \sigma_3$).

K_ϵ a parameter which represents the ratio of longitudinal to transverse stress (in the material reference frame) under conditions of uniform strain ($\epsilon_1 = \epsilon_2 = \epsilon_3$).

$\partial f/\partial\sigma_j$ the vector normal to the yield surface, which is given by the function f.

$\partial f/\partial s_j$ is equivalent to $\partial f/\partial\sigma_j$ for a yield criterion like the Von Mises or Hill, where yielding is not a function of hydrostatic pressure.

$\Delta\lambda$ a proportionality constant between the yield surface normal vector, and the total plastic strain increment vector, which are parallel.

Y_1 axial flow stress along the longitudinal material direction (for normal stresses in the 1 direction).

Y_2 axial flow stress along the transverse material direction of a transversely isotropic material (for normal stresses in the 2 and 3 directions).

Y_4 shear flow stress in the isotropic (i.e. transverse-transverse) plane of a transversely isotropic material (i.e. for shear stresses in the 2-3 plane); contracted form of Y_{23}.

Y_5 shear flow stress in a plane normal to the isotropic plane of a transversely isotropic material, known as the longitudinal-transverse shear strength (i.e. for shear stresses in the 1-2 and 1-3 planes); contracted form of Y_{13}.

E_i Youngs modulus in direction i.

G_{ij} Shear modulus in i-j plane.

ν_{ij} Poisson's ratio in i-j plane.

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