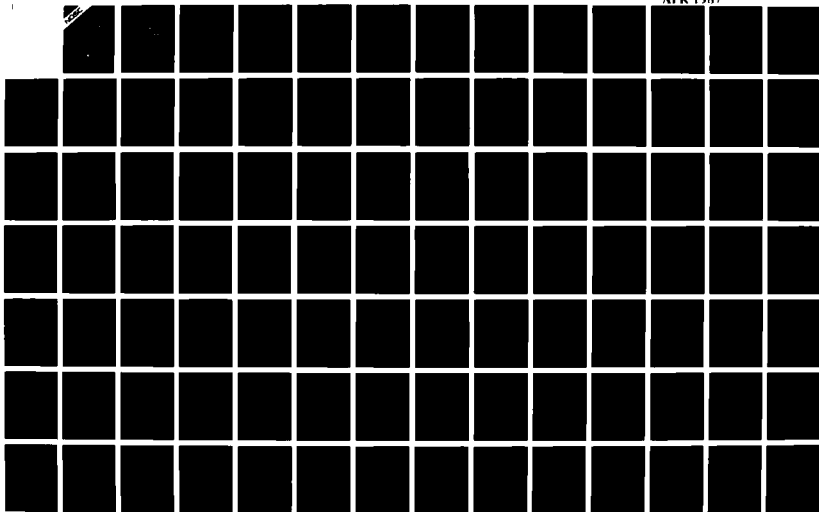


AD-A184487

NAVAL OCEAN SYSTEMS CENTER, SAN DIEGO, CA
OPERATOR MULTIPLE-TASKING STUDY FOR REMOTELY
OPERATED PLATFORMS BY: KS HAALAND, DD SWORDER

1 OF 2
NOSC TD 1084
UNCLASSIFIED
APR 1987



NOSC
 NAVAL OCEAN SYSTEMS CENTER San Diego, California 92152-5000

Technical Document 1084

April 1987

Operator Multiple-Tasking Study for Remotely Operated Platforms

K. S. Haaland

D. D. Sworder

**Dept. of Applied Mechanics and Engineering Sciences
 University of California, San Diego**



Approved for public release; distribution is unlimited.

The views and conclusions contained in this report are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Naval Ocean Systems Center or the U.S. government.

NAVAL OCEAN SYSTEMS CENTER

San Diego, California 92152-5000

E. G. SCHWEIZER, CAPT, USN
Commander

R. M. HILLYER
Technical Director

ADMINISTRATIVE INFORMATION

This task was performed for the Office of Naval Technology, Washington, DC 20360. Work on Contract N66001-85-D-0203 was carried out by the Department of Applied Mechanics and Engineering Sciences, University of California, San Diego, La Jolla, CA 92093. The Contracting Officer's Technical Representative was G. Clapp, Code 808, Naval Ocean Systems Center, San Diego, CA 92152-5000.

Released by
G.A. Clapp, Head
Techbase Program Management Office

Under authority of
H.D. Smith, Head
Communications Department

FM

UNCLASSIFIED
SECURITY CLASSIFICATION OF THE PAGE

REPORT DOCUMENTATION PAGE				
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESPECTIVE NUMBERS		
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE				
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S) NOSC TD 1084		
6a. NAME OF PERFORMING ORGANIZATION UCSD, Dept. of Applied Mechanics and Engineering Sciences	6b. OFFICE SYMBOL (if applicable)	7a. NAME OF MONITORING ORGANIZATION Naval Ocean Systems Center		
6c. ADDRESS (City, State and ZIP Code) La Jolla, CA 92093		7b. ADDRESS (City, State and ZIP Code) San Diego, CA 92152-5000		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Technology Technical Division	8b. OFFICE SYMBOL (if applicable) ONT	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N64001-85-D-0203		
8c. ADDRESS (City, State and ZIP Code) Washington, DC 20360		10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. PROJECT NO. TASK NO. AGENCY ACCESSION NO. 62744N C12134 CC12134 ICSYO 600		
11. TITLE (Include Security Classification) Operator Multiple-Tasking Study for Remotely Operated Platforms				
12. PERSONAL AUTHOR(S) K.S. Hasland and D.D. Sworder				
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM Oct 1985 TO Aug 1986	14. DATE OF REPORT (Year, Month, Day) April 1987	15. PAGE COUNT 100	
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES FIELD GROUP SUB-GROUP		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Radar detection Teleoperators Robotics Remotely operated platforms EO Surveillance		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) This report provides the equations of evolution of an encounter involving a teleoperated vehicle. The global model contains interconnected submodels describing the conventional external primitives of the encounter (base states), suddenly occurring events (feature states), and a dynamic description of the remote operator (the generalized operator model). This model is phrased as a set of stochastic differential equations that can accommodate both linear and nonlinear effects. The final section of the report places these results within the context of the multitask problem and indicates the direction of future research that will yield a quantitative description of vehicle performance in a rapidly changing environment.				
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL G. Clapp		22b. TELEPHONE (Include Area Code) 619-225-2044	22c. OFFICE SYMBOL Code 808	

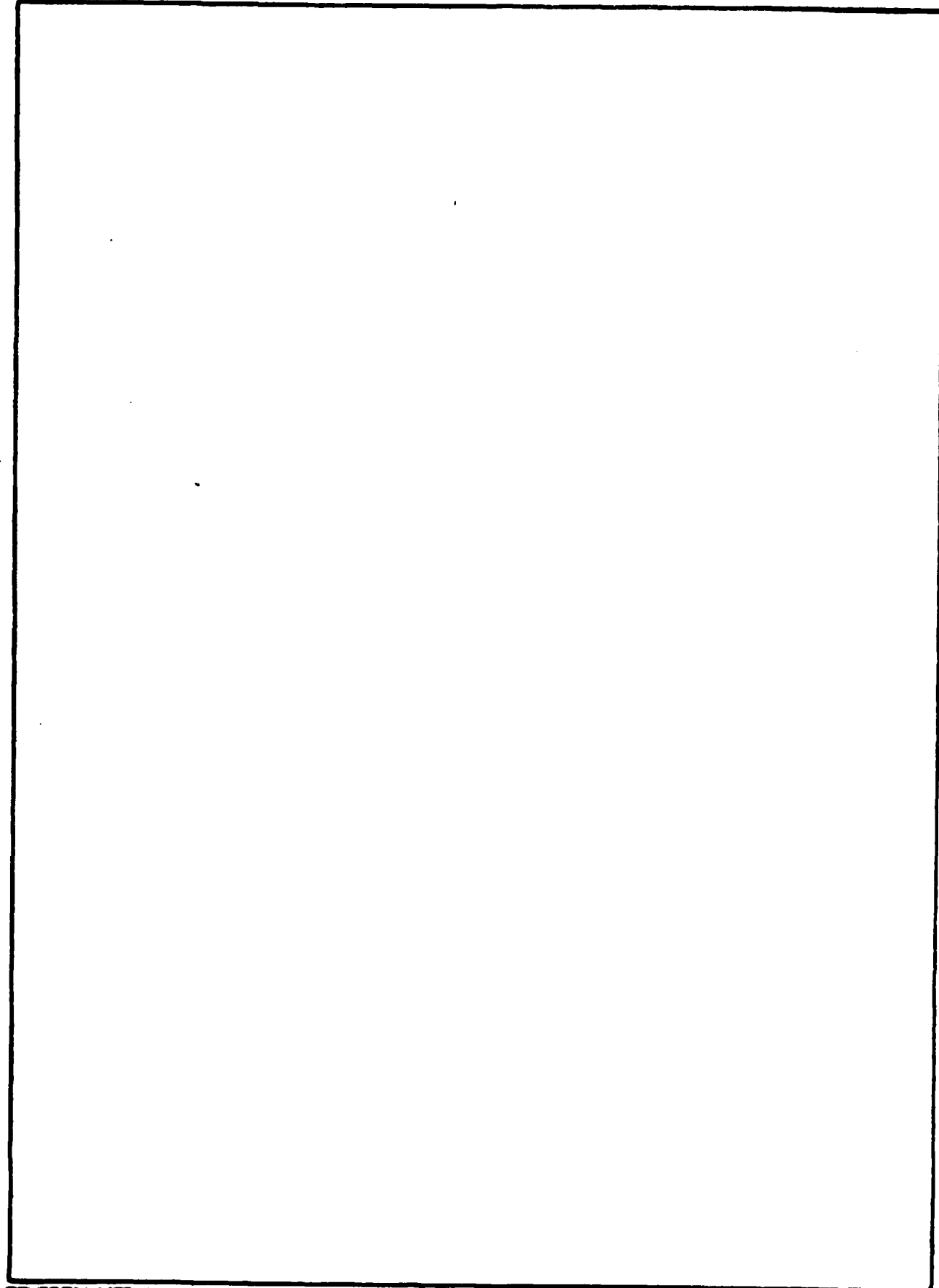
DD FORM 1473, 84 JAN

83 APR EDITION MAY BE USED UNTIL EXHAUSTED
ALL OTHER EDITIONS ARE OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THE PAGE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)



DD FORM 1473, 84 JAN

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
1. INTRODUCTION.....	2
2. MODELLING THE HUMAN OPERATOR	7
A. Hierarchy of Human Operator Activities	8
B. Review of Human Controller Modelling Theory	19
3. THE GENERALIZED OPERATOR MODEL.....	63
4. SIMPLIFICATION OF THE OPERATOR MODEL.....	75
5. CONCLUSIONS AND DIRECTION OF FUTURE EFFORT.....	77
REFERENCES	82
APPENDIX	86

ABSTRACT

This report provides the equations of evolution of an encounter involving a teleoperated vehicle. The global model contains interconnected submodels describing the conventional external primitives of the encounter (base states), suddenly occurring events (feature states), and a dynamic description of the remote operator (the generalized operator model). This model is phrased as a set of stochastic differential equations that can accommodate both linear and non-linear effects. The final section of the report places these results within the context of the multi-task problem, and indicates the direction of future research which will yield a quantitative description of vehicle performance in a rapidly changing environment.

1. INTRODUCTION

This report presents the initial results of an investigation seeking to determine the performance capability of a "pointing-and-tracking" system incorporating a teleoperated vehicle (TOV) when the remote operator is required to perform several tasks simultaneously. Determining performance in a multitask environment is significantly more complicated than it is in the single task setting. Not only must the encounter dynamics be delineated in each of the alternative modes of operation, but the transition properties of the system as it moves between tasks must be described in a compatible manner.

An encounter involving a TOV may have several primitive elements, and an analytical description is required for each. The TOV is itself an electromechanical device with lags, gains, etc. Its primary properties are conveniently phrased in terms of a set of ordinary differential equations. These equations relate the actuating signals arising from action by the remote operator to the dynamic variables of the TOV; e.g., position, velocity, orientation, etc. To the extent that there is uncertainty in the way that the TOV will respond to direct commands, a random forcing may be included in the dynamic equation at the TOV.

The tasks which engage the operator's attention often involve objects that are much less predictable than is the TOV. For example, if the operator seeks to track an evasive target, uncertainty in target motion reduces the incentive to utilize a high-order dynamic model of target evolution. Instead, a simple model driven by a large amplitude exogenous process is a more appropriate description of target behavior.

Similarly, if the TOV is required to follow a prespecified path, the path behavior can be phrased in terms of an ordinary differential equation, and the operator causes the TOV states to match the associated path states. When the path becomes less predictable, a stochastic model of the path becomes expedient.

The use of stochastic forcing terms is an attractive way to quantify the uncertainty that exists in how an encounter will evolve. A model of low dimension will frequently suffice to describe such complex dynamic objects as tanks, APCs, etc. Indeed, the more uncertainty that surrounds the motion of one of the primitives, the lower the dynamic order needs to be.

In order that this procedure yield useful results, the behavior of the model should duplicate that observed in tests of the object being modelled. Frequently such empirical data is phrased in terms of a power spectral density, and the resulting model takes the form of a simple Gauss-Markov process. First or second order models often suffice to give a close approximation to the power spectral density.

While this spectrally-based approach is justified as long as only "linear-quadratic" analysis is required, it may yield a model whose sample function behavior bears little resemblance to those of the object. A linear Gauss-Markov model has continuous sample paths of great local volatility. Such a model does not have the inherent flexibility required to produce sample functions having discrete changes at isolated points in time. For example, while a target operating in a benign environment may be well described by Gauss-Markov process, a target in a hostile environment may execute evasive maneuvers that involve sudden and unpredictable changes in acceleration. Another example of a discrete event which abruptly influences the evolution of an encounter is that of the sudden appearance or disappearance of a target.

Changes of the type described above can be thought of as a variation in the mode of evolution of the encounter. A modal descriptor or feature fixes the equations of motion which currently govern the basic system elements. The advantage of a TOV is that it injects a human intelligence into the loop dynamics. The human operator has a unique ability to discern the fundamental features of a time varying and spatially cluttered sequence of images, and as a consequence, the control becomes contingent on the observed feature process.

While it is desirable to construct a model that matches the behavioral qualities of the external portion of the encounter as experienced by the operator of the TOV, it is essential that the method of description be simple in delineation, and lead to tractable analytical problems. By

"simple in delineation" it is meant that the model has few parameters, and that these parameters correspond to readily identifiable properties of sample behavior. Note that these two attributes do not always occur together.

In order to lead to "tractable analytical problems," the model type must have a well structured "calculus" or rules of manipulation. Transfer function models are in this category, as are the conventional Gauss-Markov models; but neither of these model types is sufficiently compliant to permit the inclusion of the relevant feature variation.

In this report, the elementary constituents of the encounter will be described by stochastic differential equations. Such equations have the requisite properties noted above. Further, the conventional transfer function and Gauss-Markov models are special versions of a model given in terms of a stochastic differential equation. As with the more prosaic models, addition or deletion of elementary components of the encounter is easily accomplished by changing the order of the model. Further, in contrast to the previous models, it is possible to investigate both linear and nonlinear operation.

The ingredient that gives a system containing a TOV its unique character is the remote operator himself. The remote operator provides the intermediary through which the disparate elements are coupled. To provide a complete encounter model then, the operator's dynamic behavior must be quantified. Clearly, the operator model must satisfy the desiderata outlined above, and it must be compatible with the model of the external portions of the encounter.

Operator models are fundamentally more diverse than are models of electromechanical objects. The human is capable of so many dissimilar patterns of action that it is quite difficult to capture all of his attributes in a single, simple model. Yet, an approximation to such a model is required to complete the overall system description.

The issue of selecting a suitable operator model is of primary concern in this report. This is a topic with an interesting history, and it is explored in much more detail in the next section. Suffice it to say here that there is a convenient trichotomy of human action based upon the time scale or planning horizon of the activity. Models based upon this trichotomy are referred to as

knowledge based (long horizon), rule based (intermediate horizon) or skill based (short horizon) depending upon the type of activity which is being modelled.

The first two of the above model classes require an accurate description of the operator's behavior in relatively ambiguous environments. Such models are quite difficult to construct, and even more difficult to verify since the development of a suitable experimental protocol is a formidable task.

The third model category is most applicable to the work reported here. These short horizon models are also called reflexive. This latter appellation will be used here because it more clearly identifies the reactive nature of an operator engaged in the type of tasks being studied. Reflexive models are based upon the assumption that the trained operator is functioning in a familiar environment, and has a well defined objective or objectives. This is a situation often encountered in pointing-and-tracking tasks. The resulting model includes both response delays and the clutter suppression that are inherent in the operator response to external events. Such models are pseudo input-output models in the sense that the input to the operator model is a derived variable which is not the neurological stimulus the operator actually receives. In this sense the operator's attributes precede the specific operator model in the system description.

Reflexive operator models have been successfully used in several pointing-and-tracking systems. As will be detailed in the sequel, transfer function models are useful in stationary environments, and Gauss-Markov models in nonstationary environments. The parameters of the former are frequently obtained empirically, while the latter may be based on an additional calculation; e.g., the minimization of a performance functional.

Both of the conventional model types - transfer function and Gauss-Markov - satisfy the requisite properties of simplicity and analytical compatibility with the external portion of the encounter state. Unfortunately, neither readily admits the feature variation so much a part of the TOV application. Feature dependence in the operator model can be introduced on an ad hoc basis by indexing a set of stationary models to the external features. While this is a satisfactory

approach when the feature changes are infrequent and unambiguous, it does not suffice for the situations of most interest here.

This report develops a reflexive model of the remote operator of a TOV. This model is expressed as a (nonlinear) stochastic differential equation, and thus fits naturally with the other encounter primitives to yield the total system model. The development of the model follows the approach used in creating the optimal control model, but generalizes this earlier work by assuming that the features that determine the realized time evolution of the encounter are both random and time varying.

Permitting the system equations to depend upon a changing mode of operation aids in the investigation of multitask operation of the TOV. Indeed, as an initial approximation, multitask operation can be placed within the framework created in this report by simply making the operator's observation of the scene, situation dependent. The discrete nature of task selection fits naturally within the bounds of the generic encounter model.

The next section provides a review of the operator models which have proven useful in other applications with an emphasis on reflexive models. Section 3 develops the operator model of choice. It is a nonlinear dynamic equation that is responsive to feature variation. The model is not complete in a sense made precise in Section 3, and Section 4 provides a useful approximation that overcomes this deficiency in part. Section 5 reviews the development as well as providing suggestions as to how the model should be completed, and some implications of the model characteristics.

With the full encounter model discussed above, it is now possible to begin a study of multitask operation of the TOV. Because there is no way to verify the model parameters at present, the full system description provided here should be thought of as a preliminary approximation to actual system behavior. Nevertheless, a careful study of this model can be expected to yield useful indications of performance sensitivities and limitations.

2.0 MODELING THE HUMAN OPERATOR

A systematic analysis of the performance of the teleoperated vehicle (TOV) requires a model for the entire system. The model must include a mathematical description of all elements which significantly influence preselected measures of performance. From a top-down point of view, important elements are the vehicle dynamics, the characteristics of the environment (e.g. terrain and target dynamics), the observation system (stereoscopic video and stereophonic audio at present) and the human operator's control. In this section, methods for quantitatively describing human performance of representative TOV piloting tasks are investigated.

Section A discusses general man-machine systems and provides an introduction to the issues involved in modeling such systems. A hierarchy of human functions, organized to reflect the degree of intellectual involvement required, is identified based on the formalism suggested in previous works by psychologists and systems theorists. In this framework, the role of the TOV operator is assessed and the features of the TOV environment which shape it are identified.

In a typical TOV mission, the remote operator drives the vehicle, monitors the local environment and responds to anticipated stimuli in a prespecified fashion. The local environment is unpredictable; it may include, for example, path obstacles, sharp turns and multiple targets. The high quality of the operator's visual information (stereoscopic video transmitted via fiber optical link) enables him to discern these features and respond to them. The result is a problem which must integrate existing manual control results with a model for the human's detection and response capabilities. A historical summary of relevant manual control theory is therefore presented in Section B. The review is followed by a detailed description of those models which have received the most exposure and enjoyed the greatest success in application to TOV-related tasks (e.g. driving, target tracking). In particular, quasi-linear describing function models (DFM) and the Optimal Control Model (OCM) are presented. The emphasis in this section is on previous research and its relevance to the manual control aspects of TOV operation.

A. Hierarchy of Human Operator Activities

A theoretical framework within which the teleoperated vehicle can be systematically analyzed would complement the ongoing experimental testing of the TOV. For this purpose, a mathematical model of the composite man-machine-environment system is being developed. Possible uses for this model include:

- the assessment of current system performance capabilities in multiple environmental or tracking scenarios;
- the evaluation of the sensitivity of the TOV to various system parameters;
- the assessment of the usefulness of system enhancements such as driver aids or additional sensors (e.g. motion sensors).

The ultimate use of the system model is an important consideration in the selection of a modeling methodology. For the present aspect of TOV analysis, the emphasis is on the gross system performance as a result of the *interaction* of all system elements, and not on the individual performance of any single component. The submodels are thus selected, for a specific system configuration, to reflect the influence of each component on the overall success or failure of the system. The submodel of interest here is that of the human operator. Its development requires a careful evaluation of the role of the human operator, a precise definition of the tasks he must perform, and an evaluation of the importance of each. A review of some perspectives on human task analysis provides a framework within which these issues can be addressed.

Johannsen [1982] identifies all human tasks as falling in one of two categories, controlling or problem solving. Tasks in the first category include classical continuous control tasks as well as any other action oriented activities which produce system outputs. Models for these tasks are numerous and well documented. Some are presented in the next subsection. The second category of tasks, the problem solving tasks, tend to be internally rather than physically demanding and require a higher level of cognitive involvement. Examples include: the formulation and

modification of plans; the assessment of alarm situations; and the development and initiation of control procedures to combat such situations. Problem solving tasks generally involve the development, modification and utilization of the broad knowledge base characteristic of human beings. Models for these tasks are much more difficult to develop since they must necessarily include many of the psycho-social factors which govern human behavior. Such factors are difficult to describe in the quantitative manner generally desired by mathematical modelers.

In an attempt to bridge the gap which existed between modelers who were addressing specific manual control problems and others who viewed human behavior from a more broad psychological perspective, Johannsen and Rouse [1978] proposed a framework within which human activities could be organized. Their hierarchical perspective, illustrated in Figure 2.1, is amenable to a quantitative computer-like interpretation of human functions, but at the same time accounts for higher level psychological and intellectual activities such as reflecting and planning. At the lower level of the diagram, the activities correspond to essentially automatic behaviours. In highly trained operators, such behaviors, once learned, become reflex-like and are probably performed at the level of the cerebellum. Johannsen and Rouse point out that events which necessitate these activities tend to occur more frequently than those which activate higher level processes. The implication is, however, that although the time horizons involved in the low-level processes are much shorter, they are not considered more frequently by the high level processor (in this case the cerebrum with its enormous knowledge base and reasoning capabilities). In fact, they are viewed as essentially autonomous. The authors draw an illustrative analogy to a time sharing computer system in which certain programs are executed by peripheral devices and rarely require intervention by the operating system.

Johannsen and Rouse additionally acknowledge that human *planning* itself can be viewed as a hierarchical procedure. At the highest level, a broad plan is developed to address major goals. This plan may not involve specific activities or require immediate implementation. As it is executed, however, goals are partitioned into lower and lower subgoals. Eventually the times between subgoal identification, corresponding plan development, and ultimate plan execution

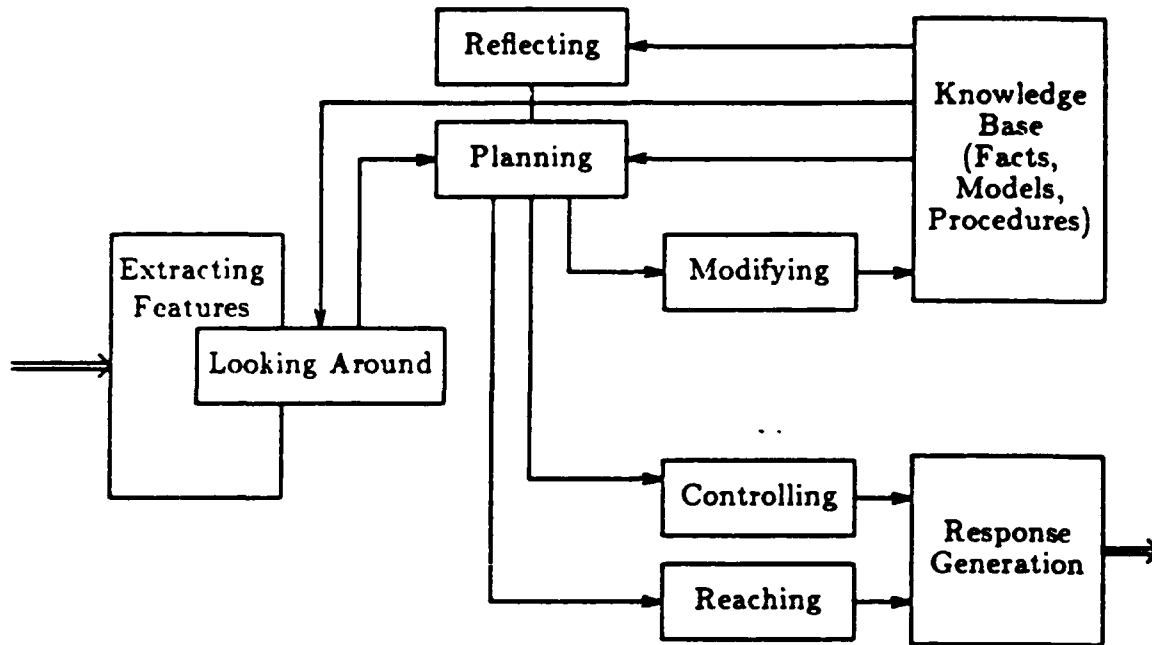


Figure 2.1: Hierarchical Multi-Level Structure of Human Activities

become very short. The authors assert that at this point the planning is probably *unconscious* and that a concise system dynamics model, in terms of quantitative state transitions, "probably provides a reasonable description of human behavior." This issue is discussed again later in this subsection with reference to the autopilot of Nitao and Parodi.

Rasmussen [1980] continued the trend toward a hierarchical representation of human controlling and problem solving behaviors with the more precise breakdown illustrated by the block diagram in Figure 2.2 (Adopted from Phatak [1983]). He phrases the behaviors commonly identified in the field of behavioral psychology in system theoretic terms. The interesting feature of Rasmussen's structure is the inclusion of "shunts" or shortcuts which allow the bypass of unneeded intellectual processes as the state of the system demands. The path indicated by the first shunt represents the lowest level of human activity, what Rasmussen calls "skill based" behavior. This type of behavior is exhibited in situations where the operator is familiar with the observed state of the system; the features he observes have been experienced before and evoke an

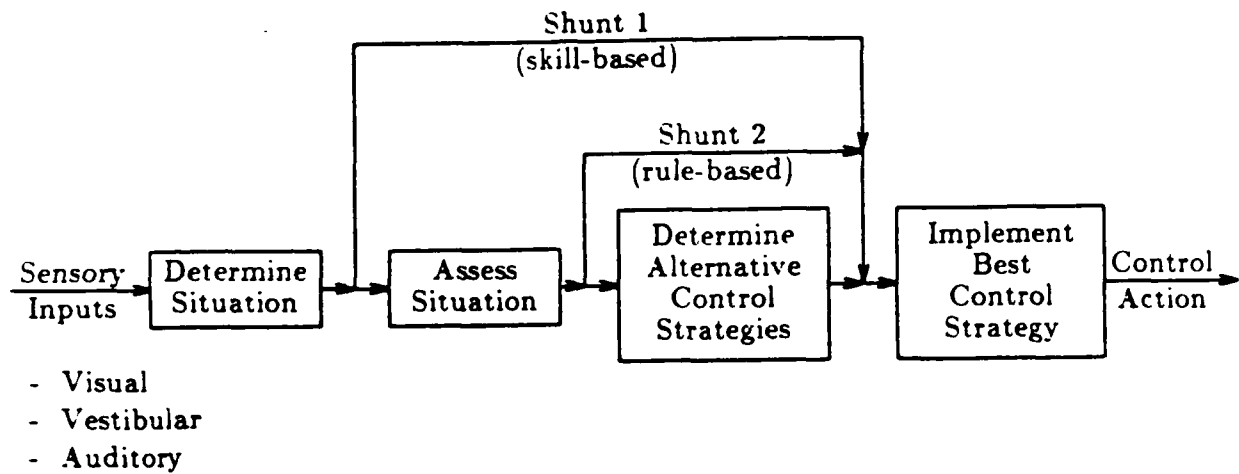


Figure 2.2: Functional Tasks Performed by the Human.

immediate response. As an example, an experienced driver continuously and automatically adjusts his steering and acceleration to maintain his desired position on the road. Even in the event of an abrupt curve in his path, his response is essentially automatic.

The path indicated by the second shunt represents the execution of "rule based" behaviors. These involve higher cognitive facilities than do skill based behaviors because a conscious assessment of the situation is required prior to the initiation of an appropriate action or procedure. However, once the situation is assessed, the appropriate action is assumed clear. Rule-based behaviors, therefore, do not require the intellectual capacities involved in the on-line development of new plans to handle unanticipated situations.

Finally, the highest level of human behavior, indicated by the path without shunts in Figure 2.2, Rasmussen terms "knowledge based" behavior. At this level, the human utilizes his basic knowledge of the system and accumulated expertise to generate plans and procedures to be used in the accomplishment of major goals. Due to the lack of detailed information, these plans may

be broad and sketchy, perhaps subject only to vague verbal interpretation. Behaviors of this type are generally exhibited in the presence of previously unexperienced circumstances. Such situations are necessarily characterized by a low frequency of occurrence.

Rasmussen's human behavioral structure of Figure 2.2 can be included as the feedback element in a closed loop system. The result is the configuration of nested feedback loops illustrated in Figure 2.3. Consistent with the previous observation that changes in the system which require global replanning are infrequent (or at least slow to evolve), the outer (knowledge-based) planning loop has a long time constant. Similar time scale interpretations apply to the inner loops.

In their work on an autonomous land vehicle (ALV), Nitao and Parodi, [1985] take the ideas of "frequency of critical events" and the associated time scale interpretations one step further. They propose a hierarchy of autopilot functions which are in fact *characterized* in terms of the time horizons involved in the feedback loops. Although these functions are performed by hardware and software modules rather than a human, the authors' analytical perspective on the functions required to drive a vehicle in an uncertain and cluttered environment provides a framework within which the role of the human teleoperator can be analyzed.

Figure 2.4 illustrates the space-time hierarchy of the ALV piloting functions. In the outer loop, the "Planner" software uses a broad view of the world (e.g. terrain and elevation information) to generate a global plan. An example is "proceed along the road until landmark 'x' is passed, then ...". Such a plan is developed initially, and due to its generality, requires revision only in the event of a drastic change in the world view. Such changes are assumed to occur either infrequently or very slowly. The loop is characterized by a time horizon on the order of $t > 10^3$ sec, presumably on the order of that of the entire mission. For representative TOV missions of the type considered in the present analysis, it is assumed that the type of planning modeled by this slow outer loop is performed *a priori*. Thus, the overall online effect of this loop is eliminated. Missions in which high-level knowledge-based planning behavior influence the performance are considered anomalous. Examples of situations not considered here are:

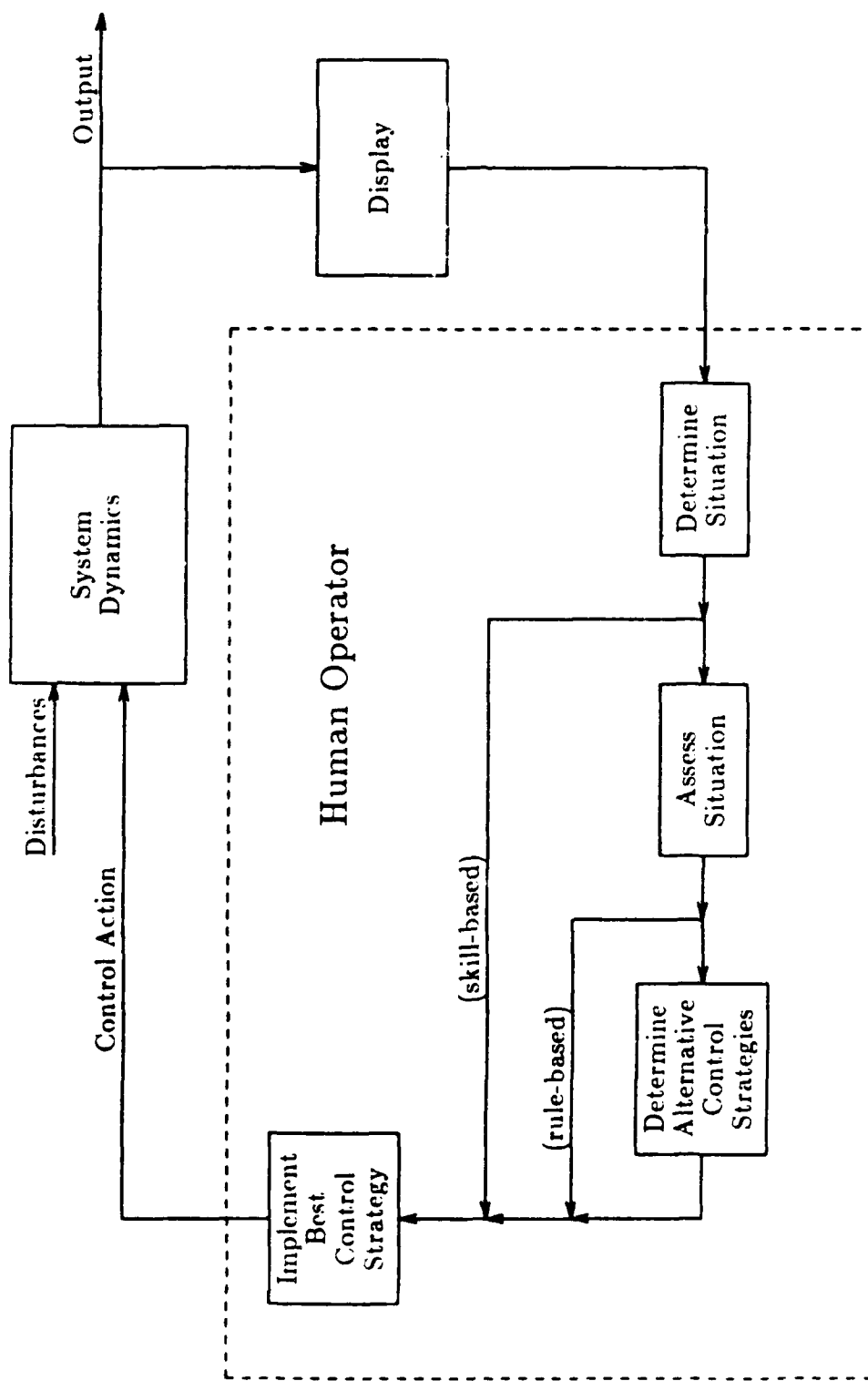


Figure 2.3: Functions of the Human Operator (Closed Loop)

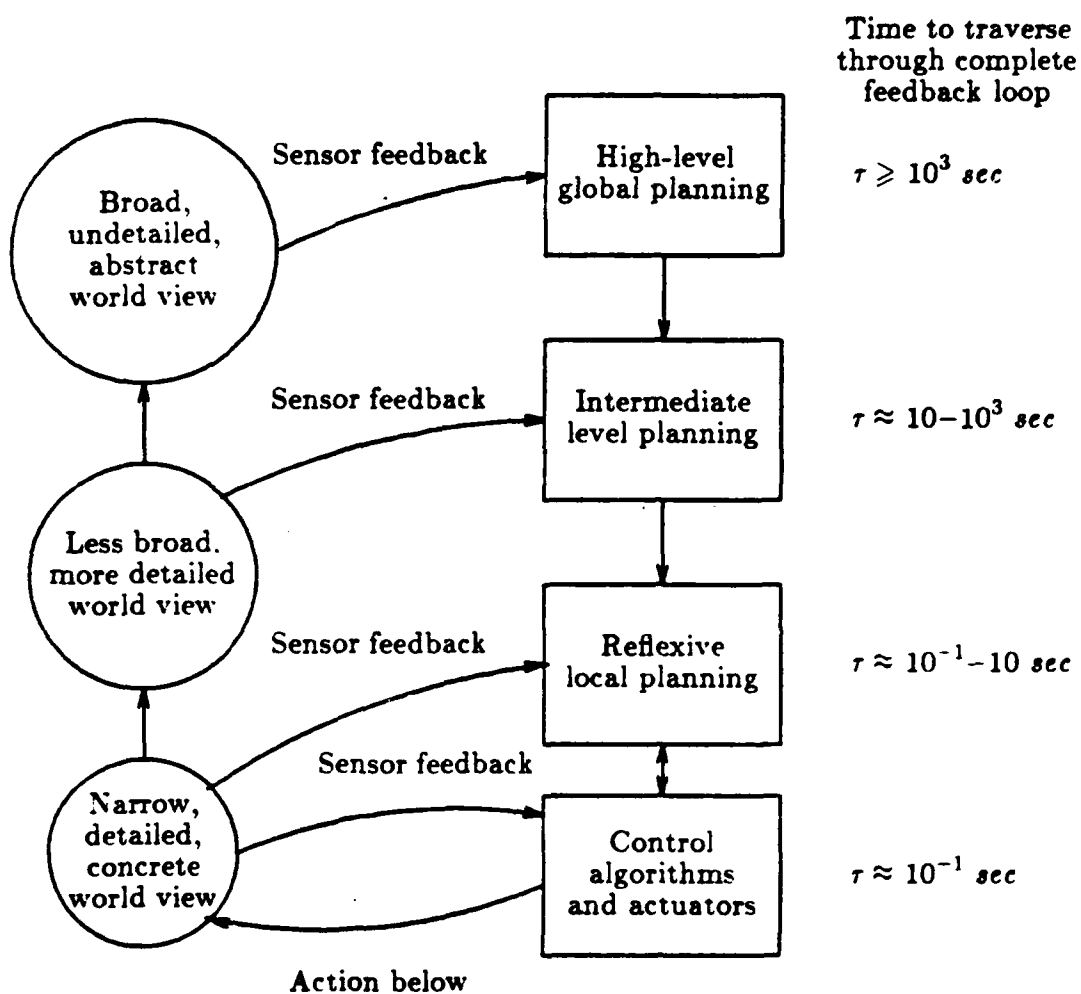


Figure 2.4: Space-Time Relationship for a Hierarchical Planning System

- loss of brakes or other mechanical failures;
- drastic terrain alteration caused by an event on the scale of a natural disaster;
- a change in overall strategic objective.

At the intermediate level of Figure 2.4, a software module called the "Observer" interprets the general plan in light of additional information collected en route by the on board sensors. The situation is assessed and the abstract plan converted into a concrete form. The result is a feasible path, specified in coordinates meaningful to the pilot and sensors, which extends into the visible future. A localized version of the path is sent to the "mapmaker" which generates a detailed map of the path in the immediate vicinity of the vehicle. The map includes the broad path borders, sensor visibility limits, and obstacle information. The latter are generated by a sonic imaging sensor. This map and additional vehicle velocity data are sent to the next loop, the functional component of which is called the "reflexive" pilot.

It is the role of the reflexive pilot to guide the vehicle along a dynamically feasible route within the planned path while avoiding previously undetected obstacles. In Johannsen's terminology the pilot must "execute" the plan passed down from the observer. This involves two distinct levels of processing. In the higher of the two, the reflexive pilot utilizes the detailed *local* map generated by the mapmaker to formulate and select possible subgoals. At this level, subgoals are defined as feasible directions in which the vehicle could proceed so as to stay on the path, avoid nearby obstacles and make progress in the overall goal direction. One of these subgoals is then selected based on a weighting of the factors above and vehicle dynamics. In this sense, the reflexive pilot is a low level planner. The constraints that govern its activity, however, are so strict (temporally and spatially) that straight forward mathematical algorithms perform the subgoal generation and selection tasks adequately. This activity is comparable to the low level planning Johannsen and Rouse described as "unconscious" and "automatic" in the human being. The time horizons associated with this reflexive planning loop are on the order of 1 sec. This is consistent with both the relatively high frequency of events which change the relevant local view (e.g. appearance of an obstacle, movement of an obstacle due to erroneous sensing, or

appearance of a curve in the road), and the need for rapid control action to respond to these features.

The inner loop in Figure 2.4 contains the control algorithms and actuators. Its primary functions are to respond to small perturbations in the desired path (i.e. to reduce noise), and to execute control commands generated by the reflexive pilot. The loop is characterized by a very short time scale ($\tau < .1$ sec). The functions performed in this loop are on the approximate level of those modeled by a vast majority of manual control algorithms.

In light of the development above, the role of the teleoperator in representative TOV missions is now made more explicit. Assumptions about the mission, tasks, human, and environment which determine the structural requirements of the operator model are presented.

As mentioned above, it is assumed that a global plan for operation is developed *a priori* and that for the current analysis, online revisions on the part of the operator are not required. Execution of this plan is assumed to involve such manual control tasks as traversing a smooth or tortuous path, or tracking a target for the purpose of identification, designation or eventual weapons release. Simultaneous activities may be required. The human operator is assumed to be well trained in accomplishing the relevant tasks; he is familiar with the dynamics of the vehicle and control system, and has performed similar tasks before.

Under these assumptions, the human's behavior and its effect on the system, are characterized by the nature of the task environment. His behavior largely reflects properties of the environment in light of the current goals [Newell and Simon (1972)]. For example, when the road is relatively straight and the tracked target is well-defined and exhibits only benign or predictable maneuvers, the human's control behavior consists of simple automatic responses which are well modeled in concise control theoretic terms. Given a goal and system constraints, the control methodology is relatively unambiguous and the operator's primary function is that of noise reduction. His function is characterized by the inner most loop in Figure 2.4.

The TOV pilot, however, is expected to perform the types of behaviors discussed above in a natural environment composed of multiple, ambiguous stimuli which can change from moment to moment. Examples are:

- sharp turns in the road or steep grades;
- the appearance or disappearance of obstacles or targets;
- sudden changes in target acceleration or orientation.

A specific combination of any of these we call a "feature" of the environment. For the present analysis, we assume that the features of interest to the driver can be enumerated, and that he has a notion of how they might evolve. Details are presented in Section 3.

The properties of the environment described above can be compared to those which are input to the reflexive pilot module in the hierarchy of Nitao and Parodi. They are characterized by a high frequency of transition relative to the time scale of the mission. In the ALV autopilot, the mapmaker generates the local detailed map which the reflexive pilot uses to define its control behavior. In the teleoperated vehicle, the human performs both these tasks. His vision enables him to generate a map of the immediate vicinity of interest. In this case the "map" includes estimates of features and their uncertainties, and the "vicinity of interest" may be in his path or that of a target. Based on the map, the teleoperator performs the role of the reflexive pilot; that is, he generates vehicle or tracking control commands which are responsive to features in his map. As in the familiar case of an experienced driver who, when he encounters a turn in the road, automatically adjusts his steering and acceleration, it is assumed that due to experience and training, the teleoperator's response requires little reflection and is essentially automatic.

Recall the comparison of reflexive planning as performed by the ALV autopilot to skill based behavior in the human being. In both cases, a connection was made between the time horizons of changes in the world view, and the level of automation of the response. In the human it was proposed that such behaviors are triggered at the level of the cerebellum and that models for these behaviors can be developed without taking into account the psycho-social aspects of

humanity. In the ALV autopilot, sequential software algorithms executed in real time proved adequate for the accomplishment of this type of function.

From the perspective of the hierarchies presented in this section, it is postulated that the majority of human behavior exhibited during the execution of typical TOV missions is "reflexive" or "skill based". Although in a human these classifications can never be absolute or distinct, the implications associated with them, recast the general problem in a more tractable form. With reference to this somewhat restricted view of the TOV mission and teleoperator's role, a model for the system is proposed. The model accounts for a higher level of human control behavior than most earlier models. The increment is illustrated by the outer loop of the reflexive pilot. In the next subsection earlier models are reviewed. In Section 3, the present model is developed.

B. Review of Human Controller Modelling Theory

The use of mathematical modeling as a tool for the analysis of manned system performance has been the subject of considerable research for the past 40 years. As a function of the needs, point of view and background of the researcher, models have been developed based on physiology [Johannsen (1971)], psychology [Siegel and Wolf (1969)], cognitive science [Newell and Simon (1972)] and systems theory [Tustin (1947), McRuer and Krendel (1959), Kleinman, and Levison (1969)].

Much of the impetus behind modern manual control research came from the pioneering work of feedback control engineers during and immediately following World War II. Tustin [1947] was among the first to compare the control behavior of a human to that of an inanimate feedback device, thus laying the groundwork for what has come to be known as the control theoretic approach to human performance modeling. The earliest research was dictated by the development of complex weapons systems (e.g. power driven guns); more recently models for aircraft piloting [McRuer and Graham (1963), Kleinman and Killingsworth (1974)], ship piloting [Veldhuyzen and Stassen (1977)], automobile steering and following [McRuer and Weir (1969), (1977), Bekey *et al.* (1977)], and modern artillery system operation [Phatak *et al.* (1977), Kleinman (1981)] have been developed based on the control theoretic perspective.

The driving factor in the investigations referenced above was the existence of a technological system which could only operate in concert with a human being acting in a manual control capacity. Overall, the approach has been particularly successful in quantitatively modeling human performance in tasks which involve rapidly responding systems with severe constraints on human performance. For these systems, the models have been successful largely because the operator is faced with a task which demands his constant attention and response, and allows little reflective thought. Consequently, his performance is dominated by his control behavior rather than his reasoning powers or problem solving capabilities. The TOV system, represents a version of such a system and is thus a candidate for application of control theoretic methodologies.

Briefly, the TOV system is unlike previous applications in that it operates in a an unpredictable environment that exhibits various features which the teleoperator tries to identify. In the case that his detection of these features enhances his ability to perform, his role is no longer that of a simple controller performing a single manual control task in a predictable yet noisy environment. He must also adapt his response to those environmental features which may result in changes in the system dynamics. The key observation, moreover, is that his behaviors remain in the class of "reflexive" behaviors described in section 2.A, and his primary responsibilities in the realm of manual control (i.e. vehicle guidance and/or target tracking). In particular, it is assumed that as he becomes aware of features in the environment he reacts according to some *predetermined* plan or set of rules. That is, he has experienced the feature before and his response is essentially automatic.

In the remainder of this section, a discussion of the perspectives underlying the control theoretic approach and a review of the most popular models which have resulted are presented. It is concluded with a critical appraisal of their utility in modeling the TOV pilot.

The Control Theoretic Approach to Human Operator Modeling

The generic block diagram for a manual control system is shown in Figure 2.5. A few observations illustrate several perspectives which characterize the control theoretic approach to human operator modeling. First, the primary goal is a model which is useful for predicting/analyzing *total* system performance. The human is viewed simply as one of several system elements, the input/ output behavior of which must be mathematically described in order that the performance of the integrated man-machine-environment system may be analytically investigated. Note that the existence of compatible models for the direct task environment as well as the controlled element are thus implicitly assumed. With this approach, the analysis begins with system considerations, (e.g. task goals and human limitations) rather than a direct analysis of the human element. The human is modeled from a functional or behavioral standpoint rather than by the more traditional approach in which his performance is synthesised from a sequence of models for elementary physiological, neurological and/or cognitive activities (e.g. eyeball motions, knob turns, memory recalls). The resulting models tend to be less task specific than those previously obtained.

Another idea underlying the control theoretic approach is the characterization of the human as an element in a feedback loop who correspondingly adopts characteristics such that the closed loop system dynamics approximate those of a "good" feedback system. The exact definition of "good" is of course dependent on the type of model which is used. This is discussed more fully in the individual model descriptions below.

Based on the control theoretic perspectives established above, numerous methods for representing and evaluating human performance in a wide range of tasks have been proposed. The model structures vary but can, in general, be classified into three groups. The first group comprises those models which rely on linear system theory in the frequency domain to describe and evaluate the human's control behavior. [Tustin, (1947), McRuer *et al.* (1967), Anderson (1970)]. The description is based on stability of the entire man-system control loop. Of all the

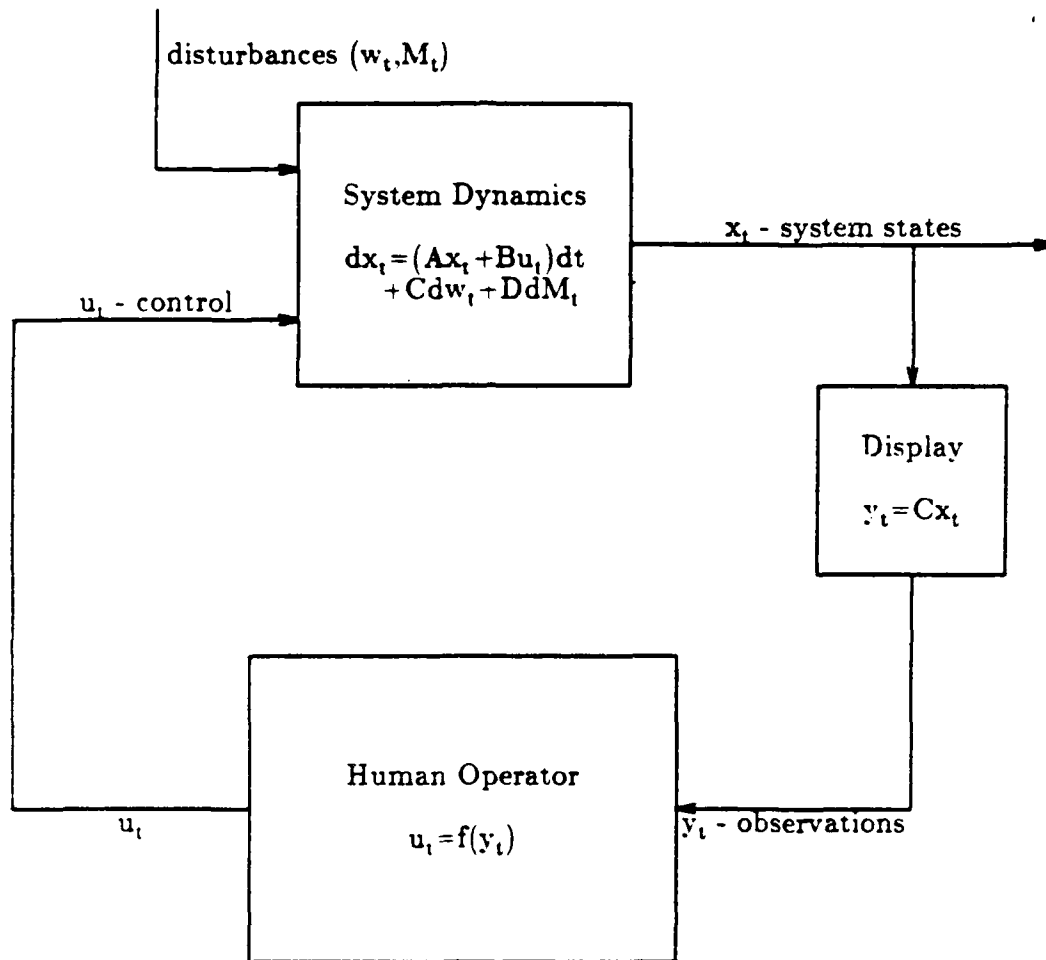


Figure 2.5: Composite Human/Vehicle/Environment System

models in this class, the quasi-linear describing function model and, in particular, the "cross-over" model of McRuer and Jex [1967] have emerged as the most dominant. These are described in more detail in the next subsection. As is often the case in applications fields, the second class of models emerged to reflect the 1960's trend in systems analysis from the use of frequency domain toward the use of time domain techniques. These models rely heavily on state space methods to represent human limitations, perceptual processes, and information processing and control/decision capabilities. The most sophisticated and well validated model in this class is the Optimal Control Model (OCM) of Kleinman *et al.* [(1969), (1971)]. Since its formulation, this model has, in varying forms, enjoyed considerable attention and multiple applications. It is described in detail below. The last class of models includes an enormous number of nonlinear, finite state and discrete models. The motivation, success, and applications of a few of these types of models are discussed briefly at the end of this subsection.

Quasilinear Describing Function Method (DFM)

Quasilinear Describing Function models are the most widely used and well-validated human operator models in the class of models which apply frequency domain methods to represent and evaluate the system. They have been highly successful in modeling human behavior in the limited but important class of stationary compensatory tracking tasks. In these tasks, the operator observes the error between desired and actual output and by manual means acts to null or "compensate" for the error. The majority of applications have involved automobile steering and aircraft piloting [see e.g. Ashkenas and McRuer (1962), McRuer and Graham (1963), McRuer and Wier (1969)]. Models based on the describing function method are unstructured; they attempt to describe human input/output response by the adoption of a model form and the selection of model parameters which give the best fit to data available for a given task. During the development stage there was no attempt to mimic the human's physiological structure, although certain analogs have been identified since. These are discussed later in this subsection.

The structure of the compensatory tracking systems typically modeled with describing function models is illustrated in the block diagram of Figure 2.6. The characteristics of the controlled vehicle and control actuator are lumped into the block labeled "controlled element dynamics". The human operator block may include nonlinearities. To the extent, however, that the man-machine system operates under stationary conditions, and that a linear model can account for a significant portion of the human's control action, a quasi-linear approach to modeling the operator's response is appropriate.

For the quasi-linear approach, the human's control response, c_t , is represented as the sum of two components;

$$c_t = l_t + \eta_t \quad (2.1)$$

where l_t is the response of an "equivalent" linear element and η_t is the "remnant". In the

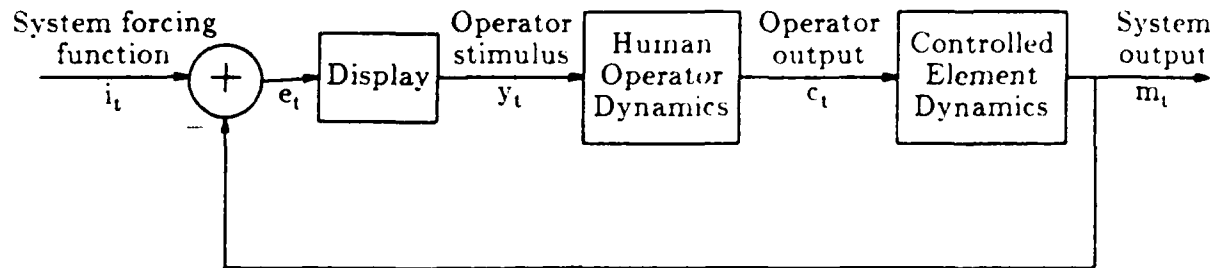


Figure 2.6: Single Loop Compensatory Tracking System

frequency domain

$$C(j\omega) = G(j\omega)E(j\omega) + N(j\omega) \quad (2.2)$$

where $G(j\omega)$ is the "describing function" and $E(j\omega)$ the transform of the input to the human. Figure 2.7 illustrates the equivalent block diagram. Signals are represented in the frequency domain to emphasize the stationarity requirement. The display dynamics are lumped in the controlled element block.

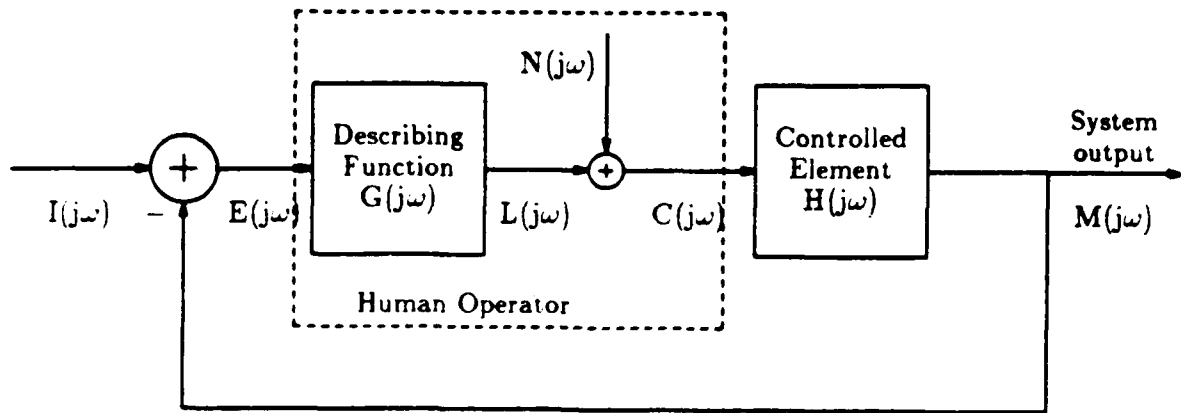


Figure 2.7: Equivalent Block Diagram of the Human Operator in a Continuous Control Task

The determination of a describing function for a specific non-linear element depends on the nature of the input. Commonly used input types are periodic (in particular sinusoidal) or random with specified stochastic properties. The case of sinusoidal inputs provides a simple, illustrative example of the technique.

Denote the response of a nonlinear element by c_t and the input by e_t . For a memoryless system their relationship is

$$c_t = f(e_t) \quad (2.3)$$

If e_t is sinusoidal, i.e. $e_t = E \sin \omega t$, the response is likely to be non-sinusoidal but periodic with the same period as the input. It thus has a Fourier series expansion which is the sum of a fundamental and all higher order harmonics. The fundamental is related to the input by an amplitude ratio and a phase shift. This relationship defines the describing function of the element in the same manner that it defines the transfer function for a linear system. If the nonlinear element is memoryless, the output is in phase with the input and the describing function is simply the Fourier coefficient of the fundamental; i.e. the ratio of the amplitude of the fundamental to that of the input.

The describing function derived for sinusoidal inputs has another interpretation which plays an important role in the derivation of the D.F. for random inputs. It is a well-known result of the theory of Fourier series for L^2 periodic functions that the Fourier coefficient of each term minimizes the mean-squared error between the associated basis element and the original function. For example, if $f(t)$ has the Fourier sine series

$$f(t) = \sum_{k=1}^{\infty} C_k E \sin k\omega T \quad (2.4)$$

then

$$C_k = C: \min \| f - C E \sin k\omega T \|_2 \text{ is achieved; } k = 1, 2, \dots \quad (2.5)$$

where the norm $\| \cdot \|_2$ is the $L^2(T)$ norm

$$\| g \|_2 = \left(\int_0^T g(t)^2 dt \right)^{1/2} \quad (2.6)$$

Thus, in the memoryless case, the describing function is the equivalent gain, K_{eq} , and

$$K_{eq} = K: \min \left(\int_0^T (c_t - K e_t)^2 dt \right) \text{ is achieved,} \quad (2.7)$$

where $e_t = E \sin \omega t$.

In practical manual control systems, the types of inputs most commonly encountered by the human are random or random appearing. These inputs have no interpretation in terms of a Fourier fundamental and higher harmonics, but are instead specified in terms of their statistical properties. An extension of the interpretation of the describing function as a linear approximation which minimizes mean squared error is now presented for random inputs.

The problem [Booton (1954)] is to find a linear element with impulse response $g(\tau)$ such that

$$\min \| c_t - \int_0^{\infty} g(\tau) e(t-\tau) d\tau \|_2 \quad (2.8)$$

is achieved. The function c_t again represents actual operator response, and e_t his input. The

norm for a stationary second order random process is defined in terms of the probability density function of the underlying random variables as

$$\|g(x)\|_2 = \left(E |g(x)|^2 \right)^{1/2} = \left(\int_{-\infty}^{\infty} g(x)^2 p(x) dx \right)^{1/2} \quad (2.9)$$

The calculus of variations yields the defining equation for the describing function, $G(j\omega)$, as the solution of (2.8):

$$R_{ee}(\tau) = \int_0^{\infty} g(s) R_{ee}(\tau - s) ds \quad \tau > 0$$

or

(2.10)

$$\Phi_{ee}(j\omega) = G(j\omega)\Phi_{ee}(\omega)$$

where R_{ee} , and Φ_{ee} are the cross-correlation function and the power spectral density of the operator inputs and outputs. This usual result for a linear system provides, in fact, the defining relation for the equivalent linear element in a non-linear system. This is not unexpected since correlations measure the *linear* relationship of signals. Notice that (2.10) does not provide a computational procedure for obtaining the describing function, $G(j\omega)$. In particular, $\Phi_{ee}(\omega)$ probably has no analytical representation but must be empirically derived based on experimentation.

Finally, observe that equation (2.10) was derived via an open loop analysis. In a feedback system like the one of interest here, the analysis is more complicated because the input to the nonlinearity depends on the response. In this case it is usually assumed that the input to the nonlinearity is Gaussian. This is reasonable because if the output of the nonlinearity is non-Gaussian, the lowpass characteristics of the controlled element tend to make it more Gaussian. Similarly, in the case of a feedback loop containing sinusoidal signals, the lowpass controlled element tends to filter the higher harmonics of the output of the nonlinearity and thus restores the sinusoidal nature of the input signal.

With these observations and the equation (2.10), the closed loop analysis outlined below [McRuer (1959)] yields a representational equation for $G(j\omega)$ in terms of measurable quantities. The cross spectral density between the operator output and the system input is

$$\begin{aligned}\Phi_{ue} &= \frac{G}{1 + GH} \Phi_{uu} + \frac{1}{1 + GH} \Phi_{u\eta} \\ &= \frac{G}{1 + GH} \Phi_{uu}\end{aligned}\quad (2.11)$$

where the second equality follows because the remnant is uncorrelated with the input. The cross spectral density between the error and the input is

$$\Phi_{ue} = \frac{1}{1 + GH} \Phi_{uu}\quad (2.12)$$

Thus, dividing (2.11) by (2.12),

$$G(j\omega) = \frac{\Phi_{ue}(j\omega)}{\Phi_{ue}(j\omega)}\quad (2.13)$$

Again, (2.13) permits the empirical determination of $G(j\omega)$ from experimental data for $\Phi_{ue}(j\omega)$ and $\Phi_{ue}(j\omega)$.

Based on a series of empirical studies involving aircraft pilots, McRuer 1959| concluded that most operator behavior could be well-fitted by the generic describing function

$$G(j\omega) = K \left(\frac{1 + j\omega\tau_L}{1 + j\omega\tau_I} \right) \frac{1}{1 + j\omega\tau_N} e^{-j\omega\tau_v}\quad (2.14)$$

with an additive remnant, where the time delay τ_v , gain K , and time constants τ_L , τ_I , τ_N are estimated. In general it is thought that τ_v and τ_N (which has come to be known as the "neuromuscular lag"), are essentially inherent physiological quantities, whereas the operator's static gain, K , and lead and lag time constants, τ_L , τ_I , reflect the equalization adopted by the human to achieve good closed loop performance.

The generic describing function (2.14) has been successful in describing human transfer characteristics for a variety of controlled element dynamics, but is by no means fixed. McRuer, Graham, and Krendel [1967] note, for example, that for low frequencies τ_N acts essentially to increase the time delay and thus combined τ_L and τ_N to yield an effective time delay τ_e and the simpler form of the $G(j\omega)$;

$$G(j\omega) = K \left(\frac{1 + j\omega\tau_L}{1 + j\omega\tau_I} \right) e^{-j\omega\tau_e} \quad (2.15)$$

This form has been used in concert with multiple conventional stable controlled elements with low input frequencies [see, in addition, Levison and Elkind (1967)]. On the other hand, in some cases, such as in the presence of an unstable controlled element or higher input frequencies, more complicated structures have been required to yield adequate matches with experimental data. One of the more common refinements has been the inclusion of a catchall increment in the low frequency phase angle to account for the low frequency lags observed in operator data. McRuer *et al.* [(1967), (1969)] discuss these more fully and present gain-phase plots for numerous controlled elements, all of which are matched by some version of the describing function.

Complete specification of a describing function is a two step procedure, the first of which is the specification of a form such as (2.14) or (2.15). Next, a strategy for the selection of the parameters K , τ_L , τ_I etc. such that the closed loop system exhibits "good" closed loop performance is required. A good system in the classical sense should [see for example Dorf, (1974)]:

- suppress disturbances;
- reduce the sensitivity of the system to variations and uncertainty in the elements of the system;
- provide good servo response over the bandwidth of the inputs;
- provide adequate gain and phase margins.

These properties are classically analyzed in terms of the open-loop frequency response.

$$H_{OL}(j\omega) \triangleq G(j\omega)H(j\omega) \quad .$$

where $H(j\omega)$ is the transfer function of the controlled element, and $G(j\omega)$ is the describing function. In principle, the goals set out above, or at least a compromise thereof, can be achieved by choosing the parameters so that the open loop gain is high for low (input) frequencies, and low for high (noise) frequencies. Adequate stability margins must be simultaneously maintained. In the "crossover" model of McRuer *et al.* [1967] these requirements are met by the selection of τ_I , τ_L , and K so that the open loop gain behaves as an integrator in the region where $H_{OL}(j\omega) \cong 1$. This 0 dB frequency is called the crossover frequency, ω_c . Mathematically,

$$H_{OL}(j\omega) = G(j\omega)H(j\omega) \cong \frac{\omega_c}{j\omega} e^{-j\omega\tau_c} \quad \text{near } \omega = \omega_c \quad . \quad (2.16)$$

This equation specifies what H_{OL} should look like, but does not indicate an automatic procedure for the adjustment of τ_N , τ_L , τ_I , K and τ_c to achieve the crossover behavior. McRuer and Jex [1967] summarize a series of what they call "verbal adjustment rules" for adjusting the parameters to achieve (2.16). They are not neat, sequential rules but rather guidelines which have been applied and have led to successful results for controlled elements with K , K/s , K/s^2 , and $K/s(s + \tau)$ dynamics.

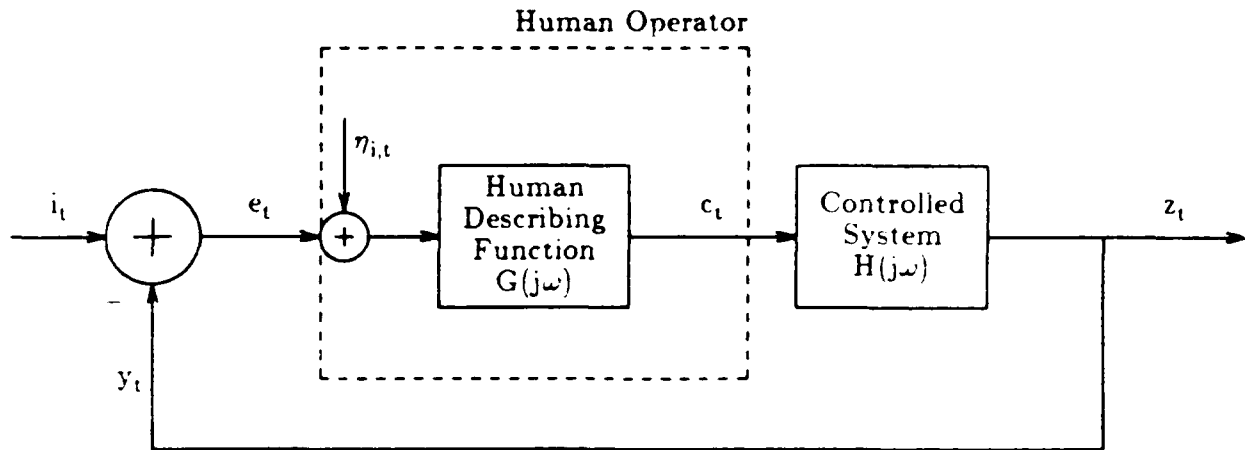
Recall, the second component of operator response which results from the quasi-linearization decomposition process is the remnant. The remnant is that portion of the operator's total response which is linearly unrelated to the input. Its existence is attributed to such factors as:

- non-linear human input/output response due, for example, to indifference thresholds;
- non-steady pilot behavior;
- intentional noise injection by the human to probe or linearize the system;
- stochastic variation intrinsic in human response

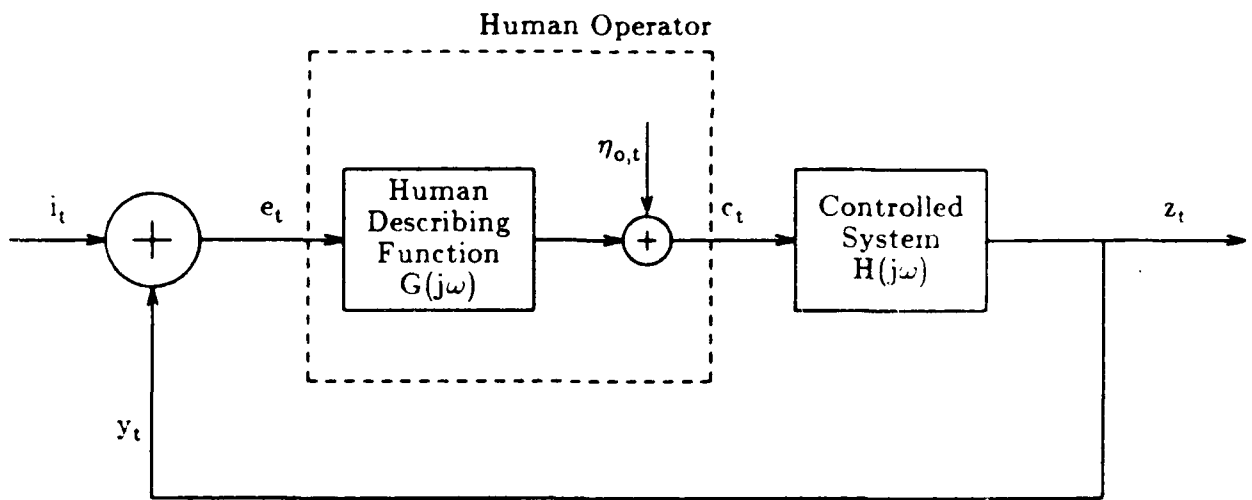
Methods for modeling remnant are not as well-developed as for the linear portion of the DFM. The existing models generally consist of empirically derived (1st order) noise spectra injected at the operator's input or output (see Figure 2.8). The point of insertion is arbitrary and is not intended to indicate the physical source. The equivalent closed loop remnant, however, is uncorrelated with the input (and thus any linear transformation thereof). Thus, the power spectrum of the output, $\Phi_{CC}(\omega)$, is given as the sum

$$\Phi_{CC}(\omega) = \left| \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)} \right|^2 \Phi_{nn}(\omega) + \Phi_{nn}(\omega) \quad (2.17)$$

where $\frac{G}{1+GH}$ is the equivalent closed loop describing function. In the optimal control model described in the next section, the model for that portion of remnant due to the stochastic nature of the human's response is made more explicit.



a) Remnant injected at operator's input



b) Remnant injected at operator's output

Figure 2.8: Remnant Models

$$\Phi_{\eta\eta}(\omega) = \frac{\Phi_{\eta\eta_s}(\omega)}{|1 + GH|^2} = \frac{\Phi_{\eta\eta_s}(\omega) |G|^2}{|1 + GH|^2}$$

Discussion

Describing function models have been used to explain a large body of manual control data, thus justifying the approximation of man as a linear element in some situations. The best results have been obtained for systems involving a compensatory tracking task with a single display and single manipulator. As previously mentioned, excellent agreement has been obtained between measured operator/system describing functions and those obtained via the crossover model. Bode plot comparisons for a variety of controlled element dynamical structures appear throughout the literature [see in particular McRuer *et al.* (1959), (1969)].

Outside the realm of strict laboratory conditions, describing function models have also been used to predict human-vehicle performance, to study stability or other problems associated with a particular manned system, and to generate insight into the mechanisms of human perception and control. As an example, McRuer and Weir [1967] report the use of the crossover model to investigate the importance of several visual cues in a freeway driving task. They modelled driver steering during overtaking and passing maneuvers under good and degraded vehicle and environmental conditions. The baseline case involved nominal vehicle conditions. The second involved a reduction of air pressure in the rear tires which resulted in notably different vehicle dynamics. Both systems were second order, but the second was unstable in the open loop and thus required constant attention by the driver to produce a conditionally stable system. Four crossover models were developed for each system; one each to model human response to heading angle, heading rate, path angle, and path rate. The resulting driver-vehicle Bode plots revealed that heading rate as an input yielded the best system, especially under degraded conditions. Experimental data collected for the degraded system also led the investigators to conclude that "the driver's dominant response (under these conditions) is to heading rate". This corresponded to their interpretation of the modeling results.

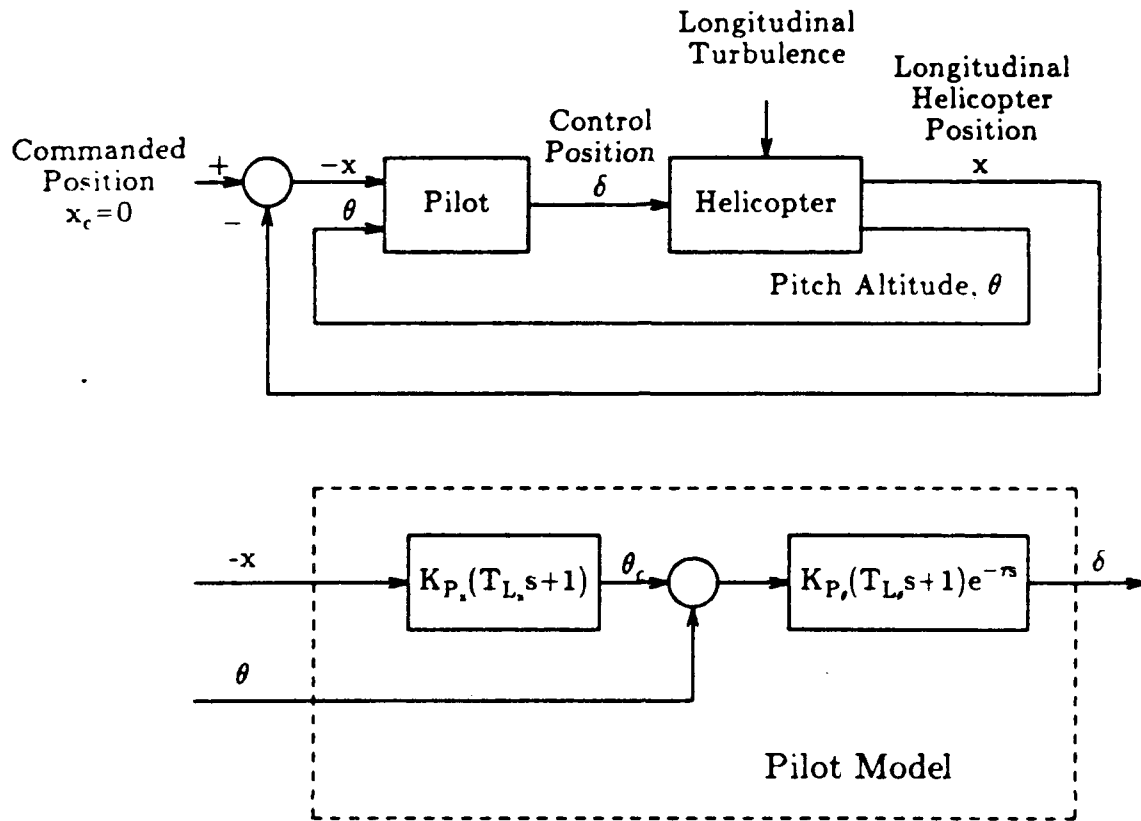
Another example of the type of analysis afforded by models of this type is reported in McRuer and Weir [1967]. In this case the crossover model was used in concert with a fourth order air frame model to predict pilot dynamics, to derive a correlation between these dynamics

and the pilot's rating of the aircraft, and to demonstrate how stability problems can be predicted and better understood so that corrective measures can be taken. Details may be found in the reference.

Extensions of describing function methods to multi-input/multi-output tasks have been developed [McRuer and Jex (1969)], but in general, application of the approach to such systems is difficult. This is because there are no systematic rules for selecting loop structures (i.e. inner loops and outer loops), describing function forms within a loop, or model parameters. Thus, any attempt at application would involve a comprehensive research program supported by extensive experimentation.

Even when the task of interest is of the single input/single output type, there are some problems associated with the application of the DFM to problems outside the domain within which it was developed. The basic difficulty lies in the absence of systematic principles for choosing model structures. The model structures given by (2.14) and (2.15) worked well for the tasks referenced, but since they have no real physical or rational basis, they provide little guidance to a modeler addressing a new task. Additionally, the verbal adjustment rules used by McRuer *et al.* 1967 to find the parameters of the model which achieved crossover requirements are not easily adapted to new situations. Thus, the modeler is faced with a non-trivial parameter selection task even if he has a model structure which he has reason to believe will work. A new application of a model of this type, would therefore probably require a large scale experimental program before it could be used to describe even the gross behaviour of the system with any certainty.

In an attempt to overcome the problems identified above, Anderson [1970] proposed another frequency domain approach for application to V/STOL aircraft analysis. The model, called "paper pilot", is a fixed form model. It is based on the hypothesis that the pilot adopts an equalization strategy which maximizes his impression of the vehicle handling qualities. Mathematically, a model structure is postulated (that chosen by Anderson for the V/STOL hovering task is shown in Figure 2.9), then the parameters are selected to minimize some rating function which weights pilot workload as well as system performance. The problem of choosing model



- K_{P_i} = Pilot gain for outer control loop
- T_{L_i} = Pilot lead time constant for outer control loop
- θ_c = Pilot internally generated pitch command for inner control loop
- K_{P_i} = Pilot gain for inner control loop
- T_{L_i} = Pilot lead time constant for inner control loop
- τ = Pilot effective time delay

Figure 2.9: Block Diagram of Helicopter Longitudinal Hover Task

parameters is thus replaced with that of selecting a cost functional and then performing a parameter optimization (which may itself be computationally burdensome). Anderson's approach was applied to some pre-existing data for which pilot parameters were available and yielded good matches to the pilot ratings of their aircraft. On the other hand, when the method was applied to predict pilot parameters and pilot rating, the predictions of rating were good, but the matches with measured pilot parameters questionable. The utility of this approach on a different problem thus remains unverified.

Another limitation of the frequency domain models described above is their inability to account for operator response in the absence of stimulation. As an example, if a target disappears temporarily, the human continues to track, based perhaps on velocity estimates, until it reappears. These methods have no mechanism for handling such a situation.

Finally, all frequency domain methods have the ultimate limitation that they are strictly valid only under stationary operating conditions. An interesting aspect of TOV analysis, however, involves the operator's ability to respond to time varying (in particular suddenly changing) features of the environment which have a direct influence on the dynamical structure of the total system. Related situations have been addressed by several researchers in the past [Phatak and Bekey (1969), Elkind and Miller (1967)]. Their models were motivated by a desire to model human adaptive capabilities in the event of a system failure which results in a sudden change in the system dynamics. The model of Elkind and Miller, for example, addresses a situation in which the human operator controls a system which is subject to sudden, random changes in process dynamics. They assumed that the controlled system consists of a finite number of linear, time invariant subsystems, and that the operator has an internal model for each. At a random time, the system switches from one mode to another. They assume that the operator is aware of possible transitions and is well trained in dealing with them. The authors' proposed model relies on the crossover model to describe the human's behavior between changes, and methods of statistical decision theory to model his adaptive behavior when a change occurs. The model for his adaptation consists of three stages: detection, identification and modification. In the first stage

he detects changes based on a comparison of expected and actual error rate. In the second, he identifies the type of change which has occurred. Finally, he selects his new control behavior and rapidly adjusts to the new steady-state crossover model. The model emphasizes the detection and identification phases; the experimental program was correspondingly designed to verify predictions of the subjects' times of detection and identification of changes. The control dynamics which result after identification has occurred are not addressed by this model, but since transitions are assumed to occur infrequently, the dynamics during the transition period have little effect on overall system performance. In a TOV encounter, system changes occur frequently and the operator's response during transitions is more critical. Weir and Phatak [1966] addressed the transition problem with the addition of another stage between the "identification" and "post-transition steady state" control stages. During this intermediate period, they modeled the operator as a time optimal controller who acts to null the error which has accumulated during the detection identification stage before assuming a new steady-state control strategy. The model explains the bang-bang control behavior exhibited by controllers in situations where large errors have accumulated.

The frequency domain methods presented in this subsection are not, in general, easily adapted to time variable systems. A typical TOV encounter, however, is characterized by very pronounced time-variability; that is by frequent, sudden changes in the dynamical system structure. The environmental features which indicate these changes (to the operator) may be ambiguous and hidden in noise. Due to their moderately high frequency, the operator's control behavior during detection and transition stages is reflected in overall system performance. For a system such as this, a time domain approach derived from that embodied by the Optimal Control Model (OCM), may provide a more natural avenue for modeling the system.

The Optimal Control Model

The representative model from the second class of control theoretic human operator models is the Optimal Control Model (OCM) of Baron, Kleinman and Levison [(1969), (1971)]. Since its development, the model has been applied to a wide variety of manual control problems with considerable success. An extensive backlog of empirical data validates the ability of the model to mimic human performance of manual control tasks. These include, for example, car following [Bekey (1977)], Remotely Piloted Vehicle (RPV) operation [Grunwald and Merhav (1976), (1978)], AAA tracking [Phatak (1977)], tank tracking [Kleinman (1981)], and V/STOL hovering [Baron and Kleinman (1971), Kleinman and Killingsworth (1981)]. More recent research is devoted to the extension of the OCM methodology to supervisory and multiple task control problems which involve increased decision making and reduced manual participation on the part of the human. For example, models have been developed for the DEMON-multiple RPV operation task [Mulradin and Baron (1980)] and the AAA flight crew (AAACRU) [Zacharias *et al.* (1982)]. These models remain to be empirically validated.

The success of the OCM is attributed primarily to the flexibility of the modeling technique in treating multi-dimensional, time variable, nonlinear and nonsteady state stochastic control problems within the well-developed theory of state variable optimal control. Multiple tasking, monitoring and attention sharing are easily incorporated into the model structure [Pattipati *et al.* (1983), Kleinman and Curry (1977), Levison *et al.* (1971)] as long as the system can be described by linear stochastic differential equations. A description of the model, a review of some applications, and a discussion of the model follow.

The Optimal Control Model is based on the assumption that the well-trained, well motivated human operator behaves in a near optimal fashion subject to his internal limitations and understanding of the task. This underlying assumption is not unique to this model. In fact, the "crossover model" described in the last section is based on a similar point of view. What

differentiates this model is the method for representing human limitations and the structuring of the model to include components which compensate for them.

A block diagram of the standard optimal control model is given in Figure 2.10. At the top of the loop is the model for the physical system (actuators/ machine/environment). It is assumed to be described by stochastic differential equations, in particular, differential equations driven by *continuous* random inputs. The state variables in such a model are hereafter termed "base states". These could include, for example, position, velocity, pitch or angular rate.

At the bottom of the diagram is the block containing the model for the human operator. The inputs are the sensory information available to the human. These could include visual, auditory and or vestibular data as a function of the application. These "displayed" variables are assumed to be linear combinations of the base state variables.

In contrast to the unstructured describing function model of the last section, the OCM is a structured model. It reflects a homomorphic map of the three psycho-motor functions of perception, information processing and control actuation. The block representing perception transforms displayed sensory data into that which is perceived by the human. The information processing block mathematically describes the human's ability to deduce information about the current state given the perceived variables. Finally, the actuation block models the human's generation of commanded and realized controls as a function of the estimated base states. It should be emphasized that although these blocks are organized to structurally match human functions, there is no attempt to define the actual mechanism by which these functions are accomplished. Indeed, the mathematical techniques employed to model the performance of these functions simply provide a model structure and a means for parametrically describing his total response. This has proved to be adequate in many applications. The system model and mathematical forms contained in each block are now presented.

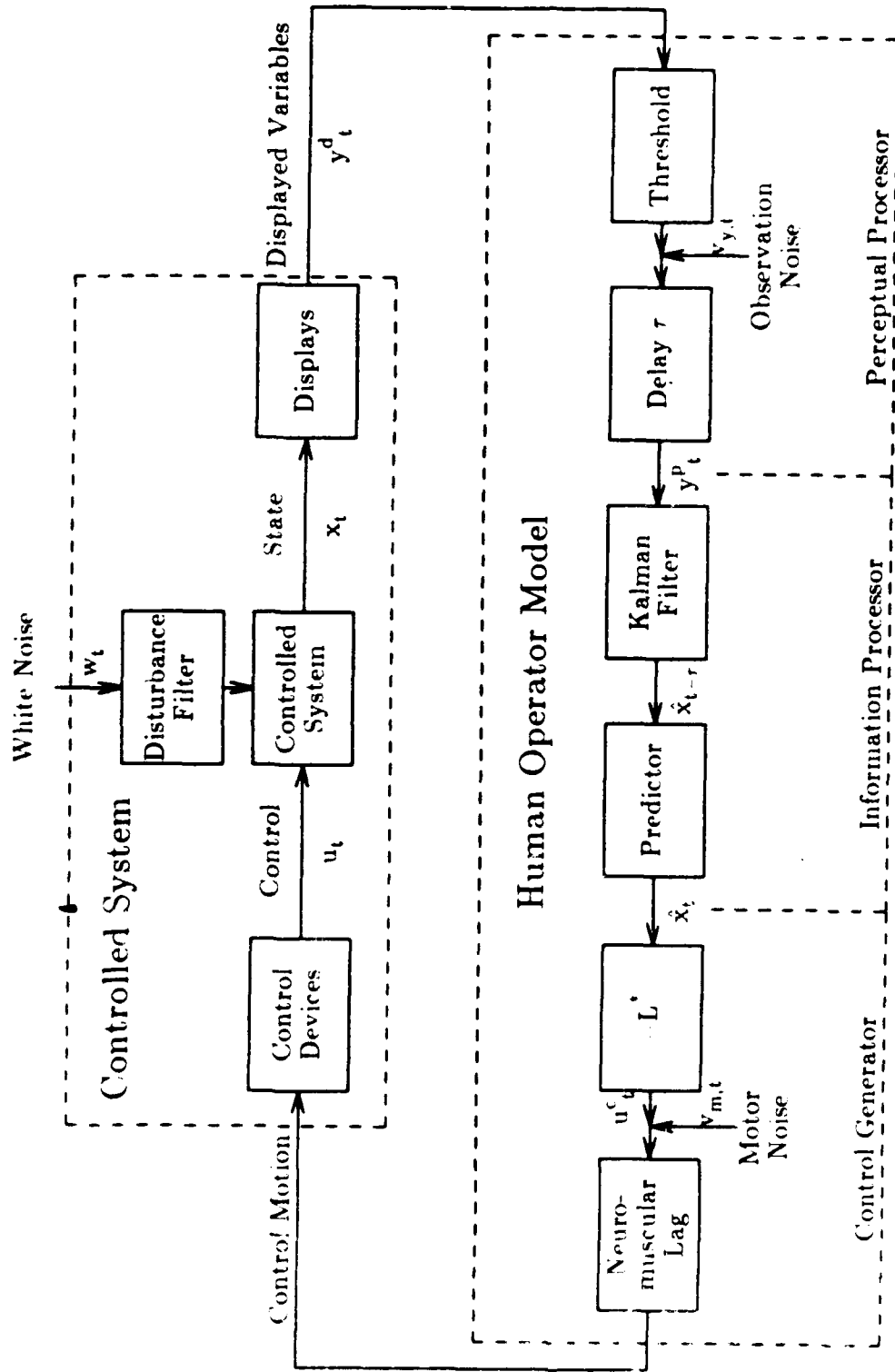


Figure 2.10: Optimal Control Model

System Model

The system model is given by a set of linear stochastic differential equations which comprise the linearized dynamics of the controlled system and environment. Dynamics associated with the measurement and control subsystems are also assumed to be included here. The equation for the dynamic model is

$$dx_t = \bar{A}x_t dt + \bar{B}u_t dt + dw_t \quad (2.18)$$

where x_t is an n -vector of dynamic base states, u_t is an r -vector of control variables and w_t is an n -vector of Brownian motion with intensity W ;

$$(dw_t)(dw_t') = \bar{W}dt \quad (2.19)$$

$\bar{A}$ and $\bar{B}$ are $(m \times m)$ and $(m \times r)$ linear transformation matrices, respectively. They are assumed time invariant for convenience, but the method applies to the time variable case directly.

The displayed variables are assumed to be linear combinations of the base state variables;

$$y_t^d = Dx_t \quad (2.20)$$

where D is a linear transformation matrix and y_t^d a p -vector. The vector y_t^d should contain all sensory information available to the human which might aid his understanding of the system state. This could include visual, auditory and vestibular information as a function of the system configuration. For example, in an *in situ* car driving task, the driver observes his relative position (with respect to a car ahead or the center of the road), relative velocity and perhaps other "visual" information. These are easily expressed as linear combinations of position and velocity states. Additionally, he might physically sense the acceleration of the car as he traverses a sharply winding road, i.e. "vestibular" information. This information is again expressible in terms of linear combinations of acceleration states. It is clear that an expedient form of the state vector relative to the sensed information should be selected.

The structure of y_i^d , and the associated problem of modeling it, vary in complexity. In the case that the display is an instrument panel or a simple display composed of well defined data (such as one might encounter in a tracking or laboratory control task), the structure of y_i^d is more or less self-evident. When the display is a cluttered visual scene, however, the composition of the vector y_i^d is not as obvious. One method for constructing it involves the augmentation of the standard display vector (i.e. a vector composed of such things as centerline displacement and relative velocity in a tracking or driving task) with an observation of a "feature" vector which is composed of additional clues about the system and environment. In some applications, the informational content of the additional clues can be significant and their inclusion in the model for the display thus mandatory. In fact, it is the ability of a human driver to quickly extract (with varying degrees of certainty) these features which distinguish him from the current autonomous pilots and enables him to perform more capably in a cluttered environment. This method is discussed more completely as part of the generalized operator model in Section 3.

Human Operator Model

The mathematical models used in the OCM to model the three functions of perception, information processing and control generation are now presented.

Perceptual Processor:

A human operator has certain limitations which interfere with the process of perception, thus preventing him from making perfect, instantaneous observations and interpretations of the system display. The OCM accounts for these limitations by the inclusion of an equivalent perceptual time delay and lumped observation noise. The association of the perceived variables, y_t^p with those displayed, is thus given by the equation

$$y_t^p = y_{t-\tau}^d + v_{t-\tau} \quad (2.21)$$

where v_t is a vector of Gaussian white noise which is independent of all other noise processes and has intensity R_t , and τ is the equivalent perceptual delay. The noise is generally attributed to such things as errors in perceptual resolution or central processing, but the mechanism for its generation is not important to the model. The model is completely specified by the selection of the noise intensity, R_t , the delay τ , and the identification of any dynamics associated with the perceptual process or display.

R_t reflects the quality of the information in the display vector as perceived by the human. The value is, of course, display dependent. There are no set rules available for its selection, but certain guidelines have been suggested. In general, R_t is chosen to be proportional to the mean squared value of y_t^d ;

$$\left(R_t \right)_1 = P_n \left(y_t^d \right)_1^2 \quad (2.22)$$

which defines P_n as the noise/signal ratio of the displayed variable type. A value of P_n corresponding to -20 dB [Kleinman *et al.* (1971)] has been appropriate for a variety of single axis tracking tasks. If indifference thresholds are important, they can be accommodated at this point

by the use of a more complex form of (2.22). Another guideline for the selection of R_i in complex multivariable tasks is given by the attention sharing model of Levison *et al.* [1971]. The model is based on the assumption that P_o is essentially constant and that P_i , the noise/signal ratio associated with the i th display variable may be selected as

$$P_i = \frac{P_o}{f_{c_i} f_c} \quad (2.23)$$

where f_c is the fraction of attention devoted to the task, and f_{c_i} is the subfraction of the operator's attention given to $(y_i^d)_i$.

Values for τ have been shown to be essentially constant and on the order of $.2 \pm .05$ sec. [Kleinman *et al.* (1971)].

Some systems exhibit perceptual bandwidth limitations as a result of the dynamic properties of the display or internal processing by the human. In these cases, the associated lags can be incorporated directly into the model for the system dynamics, (2.18), and need not be treated separately here.

Information Processor/Control Generator:

The assumptions about the human operator's task comprehension and control strategy which admit the formulation of the OCM are now made precise. The operator's control objective is to apply an input to u_t to the dynamic system (2.18) so as to minimize a cost functional of the form

$$J_t(u) = E \left\{ \int_t^T (x_s' M x_s + u_s' N u_s) ds \mid \underline{Y}_t \right\} \quad (2.24)$$

where $\underline{Y}_t$ is the filtration generated by the observations of the system, $\underline{Y}_t = \sigma \{ y_s^p; s \leq t \}$, y_t^{hp} is as in equation (2.21), and M and N are appropriately selected, positive semidefinite and positive definite, respectively, weighting matrices which should reflect relative costs associated with the various states and controls. In applications, the parameters of these matrices are the primary means by which the modeled operator response is shaped to match actual data. In some applications, a weighting on control rate, $\dot{u}_t$, is also included instead of or in addition to that on u_t . This issue is addressed in the discussion.

The control selected to minimize (2.24) must be chosen from the class of corresponding *admissible* controls, $\underline{u}$. This class of functions is defined by the following properties [Tse (1971)]: First, the function at any time must depend only on *past* observations. Mathematically, $u(t, y)$ must be $\underline{Y}_t$ measurable. It must thus have the form

$$u(t, y) = \bar{\gamma}(t, y, s \leq t) \quad (2.25)$$

Such a control is termed non-anticipative. Equation (2.25) is stated equivalently in terms of a stopped function

$$y_s^t = \begin{cases} y_s & t_0 \leq s \leq t \\ y_t & t < s \leq T \end{cases}$$

Then,

$$u(t, y) = \gamma(t, y^t) \quad (2.26)$$

where $\gamma(\cdot, \cdot)$ is a mapping from $R \times C^P[t_0, T] \rightarrow R^N$, and $C^P[t_0, T]$ is the class of continuous functions from $[t_0, T] \rightarrow R^P$. Second, a condition which assures the existence of a unique (in probability law) solution to (2.18), (2.21), such that

$$E \{ \|x_t - \underline{Y}_t\|^k \} < M_{k,y}$$

and

$$E \left\{ \int_{s_0}^T \|u_t\|^k dt \mid \underline{Y}_s \right\} < \infty \quad s \leq t \leq T, \quad k > 0 \quad (2.27)$$

is required. A uniform Lipschitz condition on γ

$$\|\gamma(t, g) - \gamma(s, f)\| \leq \alpha \|f - g\|_C; \quad f, g \in C^P(t_0, T) \quad (2.28)$$

where $\|\cdot\|_C$ is the usual sup norm

$$\|\cdot\|_C = \sup_{t \in [t_0, T]} \|\cdot\|$$

and $\|\cdot\|$ is the Euclidean norm, provides a sufficient condition.

Thus, the problem is summarized as finding a control of the form (2.25), satisfying (2.28), which minimizes (2.24) subject to the dynamic constraints (2.18), under the additional assumption that the human has an internal model for the system. The separation principle of stochastic control provides the framework for its solution.

THEOREM 1 - The Separation Theorem, adapted from Fleming and Rischel, Ch. V 1975.

1. Equation for the conditional mean: Suppose x_t and y_t are stochastic processes which satisfy

$$dx_t = (Ax_t + Bu_t) dt + dw_t \quad (2.29)$$

$$dy_t = Dx_t dt + dv_t \quad (2.30)$$

where y_t is observed, and w_t and v_t are independent Brownian motions. Let $\underline{Y}_t$ denote the observation σ -algebra, $\underline{Y}_t = \sigma \{y_s, s \leq t\}$. Then, the conditional mean $\hat{x}_t = E(x_t \mid \underline{Y}_t)$, obeys the linear stochastic differential equation

$$d\hat{x}_t = (A\hat{x}_t + Bu_t) dt + P_t D d\nu_t \quad (2.31)$$

where ν_t is the "innovations process" defined by

$$d\nu_t \triangleq dy_t - D\hat{x}_t dt \quad (2.32)$$

and P_t is the error covariance, $E \{ (x_t - \hat{x}_t) (x_t - \hat{x}_t)' \}$, and is non-random.

2. The cost functional (2.24) can be rewritten in terms of the conditional mean as

$$J_t(u) = E \left\{ \int_t^T (\hat{x}_s' M \hat{x}_s + u_s' N u_s) ds \mid \underline{Y}_t \right\} + Z_t \quad (2.33)$$

where

$$Z_t = E \left\{ \int_t^T \tilde{x}_s' M \tilde{x}_s ds \mid \underline{Y}_t \right\} = E \left\{ \int_t^T \text{tr}(M P_s) ds \mid \underline{Y}_t \right\}$$

3. Let ν denote the class of admissible controls. The original stochastic control problem with partial observations,

$$\min_{u \in \nu} E \left\{ \int_t^T (x_s' M x_s + u_s' N u_s) ds \mid \underline{Y}_t \right\} \text{ s.t. (2.29), (2.30)}$$

thus has the equivalent formulation as a stochastic control problem with complete observations

$$\min_{u \in \nu} E \left\{ \int_t^T (\hat{x}_s' M \hat{x}_s + u_s' N u_s) ds \mid \underline{Y}_t \right\} + Z_t \text{ s.t. (2.31)}$$

Furthermore, since Z_t is independent of the control, the problem may finally be expressed as the standard stochastic linear regulator problem

$$\min_{u \in U} E \left\{ \int_0^T (\dot{\hat{x}}_s' M \dot{\hat{x}}_s + u_s' N u_s) ds - Y_T \right\} \text{ s.t. (2.31) } \quad (2.34)$$

The problem has thus been transformed into a linear stochastic control problem with accessible state and quadratic cost. The solution to this problem (2.34) is well-known e.g. Fleming and Rischel Ch. VI, 1975 :

$$u^*(t, \hat{x}_t) = -N^{-1} B' K_t \hat{x}_t = L^* \hat{x}_t \quad (2.35)$$

where $\hat{x}_t$ is as above, and K_t is the solution of the matrix Riccati equation

$$\dot{K}_t = -K_t A - A' K_t - K_t B N^{-1} B' K_t - M \quad (2.36)$$

with boundary condition $K_T = 0$.

The proposition is stated for the case of nondelayed observations. Kleinman [1969] showed that in the time delayed case, the separation property still holds with the mmse filtered estimate, $\hat{x}_t = E \{ x_t | Y_t \}$ replaced by the mmse predicted estimate, $\hat{x}_t^* = E \{ x_t | Y_{t-T} \}$.

The significance of the proposition is as follows: The original problem (2.18) (2.21) (2.24) can be solved as two separate problems, one of estimation and the other of control. First, find the mmse estimate of x_t given the observations, and second, solve the equivalent stochastic control problem with accessible state via equations (2.35), (2.36). In accordance with the block diagram in Fig. 2.10, the two phases of solution are now addressed in the individual blocks termed "information processor" and "control generator," respectively. The equations associated with each by the OCM are now presented.

Information Processor

The information processing block, the heart of the OCM, consists of a Kalman-Bucy filter cascaded with an optimal predictor. These generate the mmse estimates of the system states given the delayed, noisy observations y_t^p . Estimates of the mean squared uncertainty in the state estimates are also generated, thus enabling computation of the optimal cost via Equation (2.33).

The use of the Kalman-Bucy filter implicitly assumes that the operator knows the linearized state variable representation of the system dynamics and external stochastic disturbances; he has an internal model. This assumption is strong, but experience has shown small errors in the model can usually be compensated by the introduction of noise into the model at the process and motor levels. The human's model for the system is thus given by

$$dx_t = (A x_t + B u_t) dt + dw_t \quad (2.37)$$

where

$$E \{ dw_t dw_t' \} = W dt,$$

all other quantities have the same interpretations in Eqn. (2.18), and A and B are "close" to $\bar{A}$ and $\bar{B}$

The human's observation of the system is given by the output of the perception block;

$$y_t^P = D x_{t-r} + v_{t-r} \quad (2.38)$$

where equations (2.20) and (2.21) have been combined. Equation (2.38) has the equivalent representation

$$dy_t = D x_{t-r} dt + d\eta_{t-r} \quad (2.39)$$

where y_t is the solution of the equation

$$\frac{dy}{dt} = y_t^P,$$

and η_t is the Brownian motion process from which the white noise v_t is derived. Formally,

$$v_t = \frac{d\eta_t}{dt}$$

and

$$d\eta_t d\eta_t' = R_t dt$$

Given the pair of equations (2.37), (2.39), an equation for the evolution of the mmse estimate of x_t given the observations $\{y_s, s \leq t\}$ is desired. Kleinman [1969] showed that the solution of

this problem is obtained as a cascade combination of a Kalman-Bucy filter and an optimal (least mean-square) predictor. The derivation in the reference is complete and only the equations for each stage are presented here.

In the first stage, the best mmse estimate of the delayed state, x_{t-r} , given the observations up to time t , $\{y_s: s \leq t\}$, is determined. Since $dy_t = Dx_t dt + d\eta_{t-r}$ define a new observation

$$z_t = y_{t+r}.$$

Then

$$dz_t = dy_{t+r} = Dx_t dt + d\eta_t. \quad (2.40)$$

The problem now assumes a more standard form and can be restated in more standard terms. That is, find the best mmse estimate of x_t given the observations $\{y_s: s \leq t\}$. The solution to this problem is known to be given by the conditional mean,

$$\hat{x}_{t-r}^{\text{mmse}} \triangleq \hat{x}_{t-r} = E(x_{t-r} | Z_{y-r})$$

where Z_{t-r} is the observation σ -algebra, $\sigma\{z_s: s \leq t\}$. The well known Kalman-Bucy filter gives the equation of evolution for this quantity via the pair of equations

$$d\hat{x}_{t-r} = (A\hat{x}_{t-r} + Bu_{t-r})dt + P_{t-r}D'R^{-1}d\nu_{t-r} \quad (2.41)$$

$$\dot{P}_t = AP_t + P_tA' + W - P_tD'R^{-1}DP_t \quad (2.42)$$

where ν_t is the "innovations process" given by

$$d\nu_t = dz_t - \hat{C}x_t dt,$$

and P_t is the covariance matrix of the error, $\hat{x}_t = x_t - \hat{x}_t$. Observe that the equation for the error covariance matrix, P_t , is an ordinary differential equation, and P_t itself a non-random process. This is a direct consequence of the assumption that the state vector is composed strictly of "base" states. That is, the disturbances are strictly of the Gaussian white noise type.

In the second stage, the optimal linear predictor generates the present state, $\hat{x}_t^r$ from $\hat{x}_{t-\tau}$, according to

$$\begin{aligned}\hat{x}_t^r &= \zeta_t + e^{A\tau} [\hat{x}_{t-\tau} - \zeta_{t-\tau}] \\ \dot{\zeta}_t &= A \zeta_t + B u_t\end{aligned}\quad (2.43)$$

where $\hat{x}_t^r$ denotes the least-mean square prediction of x_t given observations delayed by τ , $\{y_s, s \leq t-\tau\}$.

This completes the solution of the first problem associated with the implementation of Proposition 2.1. Notice that the K-B filter reflects compensation on the part of the operator for his perceptual limitations as modeled by the lumped observation noise, v_t . The optimal predictor compensates optimally for the delay.

Control Generator:

This block models the operator's generation of the control. A "commanded" control is generated by the equation

$$u_t^c = -L^* \hat{x}_t \quad (2.44)$$

where L^* is the matrix of optimal gains generated as the solution of the pair of equations (2.35), (2.36). To account for the human's inability to generate perfect control responses, an equivalent "motor" noise is added to u_t^c . The result is filtered to account for possible bandwidth limitations. The motor model is thus given as

$$T_N \dot{u}_t + u_t = u_t^c + v_t^m \quad (2.45)$$

where v_t^m is assumed to be Gaussian white noise with intensity S , and T_N is the so called neuromuscular lag which has been found to be an essentially inherent parameter on the order of .08 - .1 sec. S_t is defined by

$$S_t = P_m E \left\{ \left(u_t^c \right)^2 \right\}$$

where Kleinman [1971] reports a typical value of $P_m = -25$ dB. The neuro-muscular dynamics reflected by equation (2.45), can be incorporated directly into the dynamic model (2.37) by the augmentation of the state vector, x_t , with u_t to yield

$$d \begin{pmatrix} x \\ u \end{pmatrix}_t = \left[\begin{pmatrix} A & B \\ 0 & -\frac{1}{T_N} \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix}_t + \begin{pmatrix} 0 \\ \frac{1}{T_N} \end{pmatrix} u_t^c \right] dt + \begin{pmatrix} I & 0 \\ 0 & \frac{1}{T_N} \end{pmatrix} \begin{pmatrix} du \\ dv_m \end{pmatrix} \quad (2.46)$$

which has the same form as (2.37). Observe it is the commanded control u_t^c which is actually selected to minimize (2.24).

Discussion

The Optimal Control Model provides a mechanism for describing human controller response in a *non-stationary* environment. The model is a simple parametric one: the structure is fixed and the parameters are then selected to shape the operator's modeled response to match experimental data. In this sense, the OCM is not far from the describing function models discussed above. Recall the two step procedure for obtaining the DFM: first, a describing function structure is selected; and second, the parameters of the model are chosen to give the best match with experimental data. In fact, one would hope that in a stationary environment, the two models would converge to yield a single human operator model. The above expectation, although theoretically reasonable, may not be strictly realizable with the OCM in its current form. This is because the OCM is not necessarily parsimonious. Other structures could exist which are equally adept at matching measured human response data. Phatak [1977], for example, proposed alternate optimal control structures which involved some simplifications to the standard model. Examples of modifications and simplifications used by him and others include:

- elimination of the perceptual delay;

- aggregation of the motor and observation noise;
- revision of the terms in the cost functional;
- reduction of the number of displayed variables the human is assumed to perceive.

An argument for the elimination of the perceptual delay is as follows. If the environment is predictable and the operator well-trained, he is able to compensate for the delay and minimize its effect. The same function is accomplished mathematically by the optimal predictor in the information processing block of the OCM. For this reason Phatak and others (e.g. Baron and Levison 1973) have eliminated the delay from the model. Other researchers [Hess (1977)] have chosen instead to approximate the delay with a Padé approximation which is then incorporated directly into the system dynamics. The elimination of the predictor from the model in either case greatly reduces the computational burden in application. Aggregation of the motor and observation noise is similarly justified with an argument that the operator is well-trained.

In the OCM presentation above, a weighting on control (u_t) was included. This was the case in the original development, but in this early work, [Kleinman (1969)], the neuromuscular lag was not included in the model. Subsequently, the model's developers, Kleinman *et al.* [1971], included instead a weighting on control *rate*. Then,

$$J_t = E \left\{ \int_t^T (x', Mx_t + \dot{u}', N\dot{u}_t) ds \mid Y_t \right\} \quad (2.47)$$

This formulation gave better matches with experimental data. Additionally it was noted that highly trained pilots rarely make rapid control movements and thus the inclusion of this term in the cost functional is physically reasonable. The result of this revised cost functional is a control law of the form

$$r_N \dot{u}_c(t) + u_c(t) = -L^* \hat{x}_t + l_m^* \dot{u}_m(t), \quad (2.48)$$

where $\hat{u}_m(t)$ represents the best estimate of the motor noise $u_m(t)$, l_m^* is a matrix of optimal

gains, and all other notation is as previously defined. τ_N is functionally related to the weighting in (2.47). The motor noise is assumed to be a (wide band) first order noise process generated by

$$\dot{u}_m(t) + \gamma u_m(t) = \gamma v_m(t), \quad (2.49)$$

where $v_m(t)$ is Gaussian white noise with covariance S . At this point, in the interest of simplification, two assumptions were made. First, based on evidence reported to exist by Kleinman *et al.* 1971, it was assumed that the bandwidths of $u_m(t)$ and $u_c(t)$ were approximately equal. Then, $\gamma \cong 1/\tau_N$. Second, it was assumed that $l_m^* \dot{u}_m(t) \ll L^* \dot{x}_t$ and may thus be neglected. This is a reasonable assumption since $u_m(t)$ is a wide band process and at any time $\dot{u}_m(t) \cong 0$. These assumptions yield the simplified sub-optimal control law

$$\tau_N \dot{u}_t + u_t = -L^* \dot{x}_t + v_m(t), \quad (2.50)$$

which "introduces" the neuromuscular lag discussed earlier. In this formulation, the weighting on $\dot{u}$ (i.e. the matrix N) must be adjusted to yield an appropriate value for τ_N .

Since the evolution of the model outlined above, the existence of the control dynamics or neuromuscular lag given by (2.50) has been fairly well acknowledged. Many researchers since Bekey (1977), Hess (1977), among others have opted to include the lag explicitly. Correspondingly, they introduce a weighting on actual control in the cost functional (u_t). This approach is expedient because:

- τ_N is a relatively invariant parameter from task to task. Its explicit inclusion thus allows it to be selected *a priori* and then modeled directly in the system dynamics.
- The inclusion of a weighting on control rather than control rate simplifies the initial parameter selection process. In this configuration M and N may be chosen to reflect maximum allowable deviations in the important variables. These are based on physical considerations and often indicate a good initial estimate of the parameters.

The parameter selection process is discussed further in the applications section which follows.

The information processor block of the OCM produces, in addition to the best estimate of system state, $\hat{x}_t$, the error covariance P_t , and the innovations process ν_t . These quantities have important implications beyond those discussed above, and their availability contributes to the flexibility of the OCM approach. For example, the innovations process provides a key quantity used in signal processing to detect events (such as system failures). Additionally, $\hat{x}_t$ and P_t together represent a sufficient statistic (in the case of Gaussian white noise disturbances) for describing the human's understanding of the system state. They thus provide the key variables upon which decisions about system operation can be made. For example, in a situation involving multiple tasks or a cost on monitoring, a strategy for monitoring is required. This strategy would be determined based on $\hat{x}_t$ and P_t . Similarly, consider a problem in which the human has the option of operating in one of several modes (for example in a computer aided mode or one in which additional sensors or tracking aids are exploited). Then again, his choice of mode should be based on $\hat{x}_t$ and P_t . More basically, in a simple two-task control situation, these quantities determine which task he should address. These ideas are suggested and expanded upon by Baron [1984] and White [1981] among others.

Applications

The Optimal Control Model has been used both for prediction of human behavior in known dynamical systems and as a model for the human element during the design and evaluation phases of system development. A review of some of the applications which verify the model structure and are relevant to the pointing and tracking aspects of the current task, provides a perspective on the status and utility of the OCM.

The baseline verification and validation studies were performed and reported by Kleinman, Baron and Levison [1971]. The experiments consisted of a compensatory tracking task in which the human was given an explicit display of tracking error, e_t . It was assumed that the operator

could extract error rate $\dot{e}_t$, and $\dot{e}_t$ was thus included in the model display vector y_t^d . Velocity ($\frac{K}{s}$) and acceleration ($\frac{K}{s^2}$) control were investigated. The results for both sets of vehicle dynamics yielded good agreement between experimental and model predictions of mean-squared closed loop performance quantities (e_t , $\dot{e}_t$, and u_t). Additionally, plots of equivalent human describing functions derived from the OCM vs. those measured, and of computed vs. measured human remnant spectra were in excellent agreement. It should be noted, however, that the OCM parameters in this study were adjusted on line to yield the best matches with the data. Based on these initial results, it cannot be concluded that the OCM is predictive nor that the parameters are independent of the body of data. The significance of this set of experiments is simply that the OCM is capable of reproducing many aspects of human response. The human limitation parameters used in this set of experiments are tabulated in Table I.

TABLE I
Human Operator Parameters for Base Line OCM V and V Studies

dynamics	τ	τ_N	P_v	P_m
$k \cdot s$	.1 sec.	.08 sec	-20dB	-25dB
$k \cdot s^2$	.21 sec.	.1 sec	-20dB	-25dB

Kleinman *et al.* [1971] postulated that the parameter values shown in Table I are typical for many systems and are not in general task dependent. This notion is consistent with results reported previously by McRuer *et al.* [1959, 1967] for associated variables in the DFM. They reported $\tau = .15$ sec. ($\pm .05$ intrasubject) and $\tau_N = .1 - .3$ sec. with .1 being typical for many tasks and input types. Other data support this hypothesis. Baron, Kleinman and Levison [1971] for example, used the values $\tau = .15$, $\tau_N = .1$, $P_v = -20$ dB and $P_m = -25$ dB to predict the effects

of changes in aircraft stability derivatives on rms hovering performance in a VTOL vehicle. The other parameters in the OCM (i.e the weighting matrices in the cost functional) were adjusted to reflect the nature of the task. The pilots were instructed to minimize position error, so a weighting was included on that state variable. Additionally, as a result of the knowledge that well-trained pilots avoid excessive altitude changes, a weighting was included on pitch rate. The relative weighting of the two is somewhat subjective. In this case it was based on some existing experimental data which reflected the balance of position error vs. pitch rate error in a nominal flight configuration. The results of this model were compared to measured data obtained from a wide range (in terms of aircraft parameters and disturbance conditions) of simulator experiments. In most cases, the model's predicate performance agreed quite well (within $\pm 1\sigma$) with the data. Note that a fixed set of operator parameters was used for all conditions. A complete set of plots for the various air frame characteristics are presented in the reference along with a discussion of a few anomalous points which occurred at the extremes of the (airframe) parameter changes. This example illustrates the OCM parameter selection process, provides an example of its utility, and supports the hypothesis that the parameters associated with human limitations may be relatively invariant for many task types.

As another example in which the OCM was applied in a predictive manner, Kleinman and Perkins [1974] used the OCM in an anti-aircraft (Vulcan Air Defense-VADS) tracking loop which exhibited rapidly varying system dynamics. The pilot was required to track both azimuth and elevation. For each axis, a five dimensional state vector reflecting second order target motion and a third order gun-sight-hand controller model comprised the system dynamical model. The human operator model was the OCM with limitation parameters selected *a priori* based on previous data as $\tau_N = .1$ sec, $\tau = .2 \pm .05$ sec., $P_y = -20$ dB and $P_m = -25$ dB. The cost functional weighted tracking error. Again, the model predictions of tracking error covariances throughout the trajectories matched well with human tracking error obtained from experiments.

The OCM has been used in many other tracking and weapon systems applications which are not detailed here (Harvey and Dillow (1974), Kleinman (1977)). The latter reference presents a particularly thorough statistical comparison of three anti-tank systems. The OCM was used to generate an ensemble of tracking error time histories for each system and each of several target trajectories. The model-generated data were compared against ensembles of equivalent data sets obtained in field tests. Several methods of statistical comparison (e.g. ensemble mean and standard deviation analysis, temporal analysis, and for stationary target trajectories, frequency domain analysis) were used to compare the results. Additionally subjective comparisons of individual runs were impressive. The values for the OCM parameters used by Kleinman in this application were not reported. He did comment, however, that the only human limitation parameter which required adjustment from one tracking system to another was the motor noise parameter (P_m). This need is attributed to the great variation in the manipulator characteristics of the TOW, DRAGON and TLV systems.

As a final application of interest in the current discussion, Grunwald and Merhav (1976), (1978) used the OCM to model manual visual field control of a (low-flying) remotely piloted vehicle (RPV). The model was originally developed to model the manual lateral control of the RPV along a nominally straight reference path. A five degree of freedom simulator was available for experimental validation of the models. It was used initially, however, to investigate the effect constraints on the visual field on pilot behavior. Experiments were performed in which the pilot was permitted to view the road at only a single distance, at multiple distances, and finally in an unconstrained fashion. The data indicated that his behavior in the two looking-distance scenario closely matched his behavior in the unconstrained viewing scenario. For this reason, one- and two- looking-distance models were developed. These were incorporated into the OCM framework for investigation. The VFI model included such state variables as lateral deviation from the reference path, yaw and slip angles between the desired path and vehicle axis, and velocity. In the first stage of the investigation a parametric study was performed to determine the sensitivity of various model responses to variations in model parameters. Such sensitivities are required if

the parameters are to be accurately estimated via experimental procedures. In the second stage, the resulting one- and two-distance models were compared to experimental data. The investigators concluded that the two-distance VFI model is a valid representation of the visual field control task with unconstrained viewing.

In the second reference, the same investigators determined the extent to which the human operator used the higher order information in the state (e.g. lateral velocity and acceleration). This information was used to evaluate the effectiveness of augmenting the display with explicit indicators of such quantities. These display aids were evaluated for various dynamical conditions. Again, the model results were matched with experimental data to validate the model. The authors concluded that "the analytical model proves to be a convincing representation of actual man-machine visual field control," and that "this model proves to be an effective research tool for the prediction of system performance in the development and evaluation of display aids." The human operator parameter values determined in these studies are summarized in the reference and a complete discussion of the types of display aids considered is presented there.

The references discussed above are just a few of the many which indicate that the OCM can be a valuable tool for the analytical investigation of the performance of systems which utilize a human controller. The teleoperated vehicle is such a system, but its nature is such that the existing forms of the OCM do not result in an adequate representation for the human teleoperator. This is due to the unpredictable qualities of the environment, in concert with the availability of a (visual field) display which enables the man's detection of them. Although visual field displays were used in some of the applications discussed or mentioned above, in these cases the ability of the human to extract information from the scene which was not directly related to the base states was not relevant. In the case of the TOV, however, this aspect of the human's abilities is critical. A model based on the OCM methodology, but which accounts for these higher human capabilities is presented in Section 3.

Nonlinear, Discrete, Sampled Data, Finite State Models

A wide variety of models have been developed to address particular types of human control tasks, or specific aspects of human behavior which are not well-handled by the standard models discussed above. Bekey (1962) for example, proposed a sampled data model for human response. The model essentially used the DFM to model the human's control, but assumed additionally that the human's behavior was characterized by sampling, data reconstruction and extrapolation operations at his input. This model was motivated by evidence in some research that human sampling is intermittent. The introduction of a sampler explains the observed presence of frequencies at the human's output which are not in the input signal. Additionally, the inclusion of a first order hold models the human's ability to extrapolate in the absence of stimulation (note that this ability is also modeled by the OCM). This model structure is supported by experimental results when the input frequency is high (> 1 cps), but in general for lower frequencies a continuous model is adequate.

In some control systems it has been observed that humans display bang-bang behavior. This may be due to the nature of the controller, or some aspect of the controlled element. For example, Young and Meiry (1965) note that "when the human operator is placed in a control task with a difficult high order controlled element requiring considerable lead compensation on his part for stable closed-loop operation, his tracking becomes quite non-linear even with a continuous control stick." Systems of this type motivated their development of a simple on-off model for human behavior. This non-linear model has proven to be useful for certain controlled elements in systems in which minor perturbations are not important, and it is rather the operator's task to establish a limit cycle to keep the system within allowable bounds.

Finite state models have also been proposed to model human control in some tasks. Bekey and Angel (1968) proposed a model in which the operator is modeled as a finite state machine which switches among the states based on the states of the system. Burnham and Bekey (1976) used this general approach to model the human driver in a single-lane car following task. The

states of the system were quantitized to yield a four-state system model. A decision logic was associated with each. Examples of states they used are:

- the car is at a desired velocity but is too close to the lead car
- the car is at the desired relative position but is moving up on the lead car.

Details of the decision logic are presented in the reference. This approach may be useful when meaningful quantization of all or part of the system is possible.

These are just a few of nonlinear, discrete approaches to human controller modeling. There are many more each developed for a very specific reason. Johannsen [1976], for example, proposed a model which incorporated threshold elements and decision elements to account for human physiological characteristics. In summary, these models have led to promising results for the specific tasks or situations for which they were developed. They are not, however, generally suited to or easily adapted to new systems. They tend to be dynamically dependent: a change in system dynamics can result in a change in model structure. The non-linear nature of the models additionally introduces other drawbacks for many tasks. Parameter identification and closed loop performance analyses of the type to be performed in the TOV-analysis program, are much more difficult to accomplish when non-linearities are in the loop. This makes the models more difficult to develop and to use. If feasible, a standard approach for which a well developed theory exists is thus preferable for the current TOV modeling task.

3. THE GENERALIZED OPERATOR MODEL

The dynamic model of an encounter involving a TOV contains several interconnected sub-models. Some of these have been discussed in greater or lesser detail in the preceding sections of this report. The vehicle, etc. are conveniently described by stochastic differential equations. The completed encounter model requires a compatible model for the remote operator.

The previous section reviews several alternative ways in which a human operator has been modelled when engaged in tracking and control tasks. Because it permits the inclusion of both time variability and randomness, the formalism leading to the optimal control model (OCM) is attractive. The classical OCM is a linear stochastic differential equation, and is therefore, easily included with the other sub-models to form the full encounter state. The weighting parameters in the performance index can be selected to cause the OCM to mimic the behavior of an actual operator.

While the OCM has been used successfully in diverse applications, it is a "short time" model of human response. It has been found to be most suitable when a trained operator is performing a well-defined task in a familiar environment. His primary function is noise reduction. There is little opportunity to use his decision making capability in the context of his assigned task.

The remote operator of a TOV must respond to more varied stimuli than does his counterpart assigned a conventional pointing-and-tracking task. He must utilize the capabilities of the TOV in an unpredictable environment. This charge requires more of the operator's ability to identify the relevant characteristics of a time varying and ambiguous scene. Thus, he is required not only to follow a target as it meanders within his field of view, but he must additionally identify sudden changes in target motion, or any other events which influence the dynamic structure of the encounter.

In this study the operator still acts reflexively in the sense described earlier. The remote operator is assumed to have a good understanding of the current scenario, and to have plan of the appropriate actions which he should take. His uncertainty about the current state of the

encounter has two distinct components. On the one hand, he uses his observations to estimate the state of the primary constituents of the encounter; e.g. the center-line of the path to be followed, the position and velocity of the target, etc. An analogous functional block is to be found in the conventional OCM.

The second component of the TOV operator model results from the fact that certain dynamic properties of the primitives of the encounter may change abruptly. Because the operator is assumed to be cognizant of the possible changes which may take place, it will be supposed that the mode of evolution of the encounter is indicated by a random process $\{r_t\}$ with state space $\{1, \dots, N\}$. The process $\{r_t\}$ will be thought of as delineating the current status of the encounter, and $\{r_t\}$ will be referred to as the feature process.

The inclusion of a feature process is not common in the literature on operator models. If there is but one environment, the notion of a feature indicator is superfluous. Alternatively, if the features change infrequently and are sufficiently unambiguous, the operator can be thought of as adaptively changing his own behavior in concert with the exogenous process.

This section considers an intermediate situation in which the feature changes are sufficiently frequent and equivocal that the operator must accomplish his desiderata in the presence of both uncertainty in $\{r_t\}$ and significant modal transients. The resulting operator model is still reflexive, but the time scale of the human intervention is extended beyond that of the OCM. To distinguish these models, the description of the operator of the TOV will be termed the generalized operator model (GOM).

To be more specific, denote by z_t the conventional dynamic state of the encounter including components related to the targets, the path and the TOV. This portion of the system description will be called the base state. Let ϕ_t be an N-vector which indicates the current value of the feature process;

$$(\phi_t)_i = \begin{cases} 1 & \text{if } r_t = i \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

Then the encounter dynamics will be given by the joint dynamics of $\{z_t\}$ and $\{\phi_t\}$.

Define an augmented encounter state $\{\zeta_t\}$ by

$$\zeta_t = \begin{bmatrix} x_t \\ \phi_t \end{bmatrix} \quad (3.2)$$

It will be assumed that the dynamics of the base state are given by an equation of the form

$$dx_t = (Ax_t + Bu_t) dt + \rho' d\phi_t + dw_t \quad (3.3)$$

where ρ' is a fixed $n \times N$ -matrix. The other variables in (3.3) have the same interpretation they had in (2.37).

Before continuing it is well to review the implications of (3.3) and to contrast it with the equation which gives rise to the OCM (see (2.37)). If there were no feature dependence, then $\rho = 0$. In this event (3.3) becomes identical to (2.37). Alternatively, if the features are unchanging, $d\phi_t \equiv 0$, then $\rho' \phi_t \equiv \rho_0$ and (3.3) is equivalent to (2.37) with an additive bias. Such dynamic structures are easily accommodated by the OCM.

Equation (3.3) differs from (2.37) in a fundamental way when $\{\phi_t\}$ is variable. Suppose the target suddenly accelerates. This would be indicated by a change in the component of $\{\phi_t\}$ which corresponds to target acceleration, i.e.,

$$\rho' \phi_t \neq \rho' \phi_{t-} \quad (3.4)$$

The base state contains a component (target acceleration) which experiences an abrupt change.

The feature process $\{\phi_t\}$ will be assumed to be a Markov process with transition matrix $Q = [q_{ij}]$:

$$\text{Prob}(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} 1 + q_{ii}\Delta + o(\Delta) & ; \quad i = j \\ q_{ij}\Delta + o(\Delta) & ; \quad i \neq j \end{cases} \quad (3.5)$$

The elements of Q have a simple, intuitive interpretation. The mean lifetime in state i is $-q_{ii}^{-1}$. The probability that $\{r_t\}$ will make an $i \rightarrow j$ transition is $-q_{ij}/q_{ii}$. Consequently, the Markov process hypothesis leads to a model whose parameters can be estimated from easily discernible sample function characteristics of $\{r_t\}$.

To combine the dynamics of $\{z_t\}$ and $\{\phi_t\}$ into a suitable model, let $(\Omega, \underline{F}, P)$ be the probability space on which the exogenous process of the encounter model are defined. Let $\{\underline{F}_t\}$ be the filtration on $[0, \infty]$ generated by $\{w_t, r_t\}$. Then $\{\phi_t\}$ can be described by the stochastic differential equation

$$d\phi_t = Q' \phi_t dt + dm_t \quad (3.6)$$

where $\{m_t\}$ is a purely discontinuous $\{\underline{F}_t\}$ -martingale. Equations (3.3) and (3.6) can be combined to form the external portion encounter dynamic model;

$$d \begin{pmatrix} z_t \\ \phi_t \end{pmatrix} = \begin{pmatrix} A & \rho' Q' \\ 0 & Q' \end{pmatrix} \begin{pmatrix} z_t \\ \phi_t \end{pmatrix} dt + \begin{pmatrix} B \\ 0 \end{pmatrix} u_t dt + \begin{pmatrix} I \\ 0 \end{pmatrix} dw_t + \begin{pmatrix} \rho' \\ I \end{pmatrix} dm_t \quad (3.7)$$

or more compactly

$$d\zeta_t = (F\zeta_t + Gu_t)dt + F_w dw_t + F_m dm_t \quad (3.8)$$

where the composite factors in (3.8) are identifiable in (3.7). It is frequently convenient to write matrix relations in block form without the additional comment; e.g., if $F_t = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$, F_{11} would be interpreted as A , $\{F_{12}\}$ as $\{\rho' Q'\}$, etc.

Equation (3.8) gives the intrinsic model of the external elements of the encounter. Note again that (3.8) differs from (2.37) because of the discontinuous term $\{F_m dm_t\}$. The human operator ties the disparate parts of the total system together through his reaction to observations of the evolving engagement. The observation structure will be assumed to have a decomposition that conforms to the state decomposition indicated in (3.2). Denote the observation vector by $\{y_t\}$. Then, it will be assumed that

$$\begin{aligned} dy_t &= \begin{pmatrix} D & 0 \\ 0 & b' \end{pmatrix} \begin{pmatrix} z_t \\ \phi_t \end{pmatrix} dt + d\eta_t \\ &= H\zeta_t dt + d\eta_t \end{aligned} \quad (3.9)$$

where b is an N -vector, $\{\eta_t\}$ is a vector Brownian motion, independent of $\{w_t, m_t\}$, and with

intensity $R > 0$;

$$E\{\eta_t \eta_t'\} = Rt \quad (3.10)$$

Let the filtration generated by $\{y_t\}$ be denoted by $\{\underline{Y}_t\}$.

The observation model given by (3.9), has the generic form that was used in the conventional OCM (see (2.40)). Again the salient difference resides in the operator's reaction to the feature vector. Equation (3.9) indicates that the operator observes a noisy version of the current feature state. Feature i ($r_t = i$) is represented by a signal b_i , contaminated by an additive, wide-band noise. The speed with which the operator can detect changes in features is related to the size of b_i (strength of the stimuli) and the size of the associated element of R (noise intensity). These are parameters which will be used to tailor the model to the empirical response characteristics of the operator. The OCM uses no feature information, and it is thus represented by (3.9) without the $\{\phi_t\}$ component.

It is well to note that the observation model given by (3.9) is an intermediary for describing the input-output behavior of the operator. It is not intended to describe the physiological processes that occur within the operator. The ostensible "observation" in (3.9) characterizes an illusory knowledge state in the operator which is generated after he performs a stage of physiological scene processing. Thus, while $\{y_t\}$ is an intermediary in formulating the GOM, it is not an observation in a literal sense. The validity of (3.9) is based, therefore, on indirect measurements of operator response, e.g. delays, false alarm rates etc., rather than on direct measurement of scene stimuli.

With the formalism described in the previous section, the explicit description of the GOM can be derived. The operator is assumed to act to minimize a quadratic performance index J , given by

$$J_t = E \left\{ \int_t^T (\zeta_r' M \zeta_r + u_r' N u_r) dr - \underline{Y}_t \right\}; \quad M \geq 0, \quad N > 0 \quad (3.11)$$

In the previous section, it was shown that the conventional OCM consists of a cascade combination of two blocks, a time-dependent processor creating an estimate of the base state, and a gain block which creates the actuating signal therefrom. This latter is independent of the peculiarities of the exogenous disturbances. Indeed, as will be shown, this block in the GOM is identical to the corresponding block of the OCM.

The structure of the time-dependent processing is more complicated in this application. The dynamic behavior of the GOM is governed by the equations which delineate the first two conditional moments of $\{\zeta_t\}$;

$$\hat{\zeta}_t = E\{\zeta_t \mid \underline{Y}_t\} \quad (3.12)$$

$$P_t = \text{Var}\{\zeta_t \mid \underline{Y}_t\}$$

The requisite dynamics of the GOM are the stochastic equations which characterize the evolution of $\{\hat{\zeta}_t\}$ and $\{P_t\}$.

The equations of the time-dependent processor are derived in this section as a sequence of propositions. Because the development is rather convoluted, the analytical details are provided in the appendix. The most important of these results is given in Proposition 2. This proposition provides a representation of the GOM.

PROPOSITION 1. The exogenous processes $\{m_t\}$ and $\{w_t\}$ in (3.7) have the local moments;

$$E\{dw_t \mid \underline{F}_t\} = 0 \quad (3.13a)$$

$$E\{dm_t \mid \underline{F}_t\} = 0 \quad (3.13b)$$

$$(dw_t)(dw_t)' = W dt \quad (3.13c)$$

$$E(dm_t dm_t' \mid \underline{F}_t) = \sum_{i=1}^N \tilde{Q}_i \phi_{t-1} dt \quad (3.13d)$$

$$= V(\phi_t) dt$$

$$\tilde{Q}_i = -\phi_i q_i - q_i \phi_i' + \text{diag}(q_{ii})$$

with ϕ_i the indicator of mode i , and q_i the i th row of Q . ■

As indicated above, $\{m_t\}$ and $\{w_t\}$ are both $\{\underline{F}_t\}$ martingales. Interestingly, the quadratic variance of $\{m_t\}$ is a linear function of $\{\phi_t\}$ while the quadratic variance of $\{w_t\}$ is constant.

The next proposition gives a representation of the estimation portion of the operator model. It is phrased in terms of the innovations process $\{\nu_t\}$. Let

$$d\nu_t = dy_t - d\hat{y}_t \quad (3.14)$$

From (3.9), this can be written as

$$\begin{aligned} d\nu_t &= H_{\zeta_t} dt + d\eta_t - H_{\hat{\zeta}_t} dt \\ &= H_{\hat{\zeta}_t} dt + d\eta_t \end{aligned} \quad (3.15)$$

where $\hat{\zeta}_t$ is the estimation error process:

$$\hat{\zeta}_t = \zeta_t - \hat{\zeta}_t \quad (3.16)$$

The fundamental result of this section is given next.

PROPOSITION 2. The conditional mean process satisfies the equation

$$d\hat{\zeta}_t = (F_\zeta \hat{\zeta}_t + Gu_t) dt + P_t H' R^{-1} d\nu_t \quad (3.17)$$

■

Equation (3.17) gives the explicit form of the information processing block in the operator model. It has the functional form of a Kalman filter, although as will become apparent, there are fundamental differences in these two estimators. These dissimilarities are made clearer in the next proposition.

PROPOSITION 3. The error covariance satisfies the stochastic differential equation

$$dP_t = (F_\zeta P_t + P_t F_\zeta' + F_w W F_w' + F_m \sum_{i=1}^N \tilde{Q}_i \tilde{\phi}_{t,i} F_m' - P_t H' R^{-1} H P_t) dt + d\rho_t \quad (3.18)$$

$$P_0 = E\{\tilde{\zeta}_0 \tilde{\zeta}_0'\}$$

where $\{\rho_t\}$ is given by the equation

$$d\rho_t = (\widehat{\tilde{\zeta}\tilde{\zeta}'}_{\tilde{\zeta}_1} H' R^{-1} d\nu_t, \dots, \widehat{\tilde{\zeta}\tilde{\zeta}'}_{\tilde{\zeta}_{N+n}} H' R^{-1} d\nu_t) \quad (3.19)$$

$$\rho_0 = 0$$

■

The relationship between the GOM and the conventional OCM becomes clearer by reference to Proposition 3. As mentioned earlier, the time-dependent processing done by the operator is delineated by (3.17). This equation is identical for both operator models, although the implications are profoundly different. The OCM uses the Kalman gain which utilizes a covariance matrix $\{P_t\}$ given by the solution to

$$\dot{P}_t = F_\zeta P_t + P_t F_\zeta' + F_w W F_w' - P_t H' R^{-1} H P_t \quad (3.20)$$

Equation (3.20) differs from (3.18) in two fundamental ways, the first rather obvious and the second more subtle. The discrete martingale $\{m_t\}$ has no relevance to the OCM, and hence makes no contribution to $\{P_t\}$ in (3.20). If (3.8) is compared to a linear Gauss-Markov,

stochastic differential equation as given in (2.37), the most obvious difference is that the Brownian motion term in (3.8) is augmented by an orthogonal term: i.e.

$$F_w dw_t \rightarrow F_w dw_t + F_m dm_t$$

Hence, it might be conjectured that the quadratic variance term in (3.20) should be replaced by a sum of terms in (3.18);

$$E\{(F_w dw_t)(F_w dw_t)' | \underline{Y}_t\} \rightarrow E\{(F_w dw)(F_w dw)' + (F_m dm_t)(F_m dm_t)' | \underline{Y}_t\}$$

or

$$F_w W F_w' \rightarrow F_w W F_w' + F_m V(\hat{\phi}_t) F_m'$$

This heuristic reasoning is validated by (3.18). The trend term in $\{P_t\}$ is given by an equation that is identical to that which obtains in the Gaussian case. The only property of the feature process that influences the trend in $\{P_t\}$ is its quadratic variance.

The forcing term in (3.18) has no analog in (3.20). The conditional covariance in the Gaussian problem is independent of the observation process. In this application, however, the conditional covariance $\{P_t\}$ is a random process, and hence the gain in the estimator given in (3.17) is random as well. Furthermore, (3.17) and as a consequence, the GOM are nonlinear functions of the $\{y_t\}$ process. This again contrasts with both the OCM and the transfer function models described earlier.

Equation (3.18) can be viewed as a Riccati equation driven by the observation process. To gain insight into its behavior, suppose that $\hat{\zeta}_t$ were conditionally Gaussian. If it has been a long time since a feature change, this would be a good approximation of the actual distribution. Because ζ_t is symmetric about its mean,

$$\widehat{\zeta \zeta'}_{\hat{\zeta}_t} = 0 \quad \text{for all } t \quad (3.21)$$

Hence, (3.18) is equivalent to (3.20) with the substitution indicated in (3.21)

Suppose, alternatively, that $\tilde{\zeta}_t$ is not symmetric about its mean. This will certainly be the case when a feature transition is suspected. The forcing term $\{\rho_t\}$ will now cause the conditional covariance to change. Interestingly, but not surprisingly, this change tends to be toward higher covariances and thence toward faster dynamics in the GOM. To see this, suppose that all of the system variables were scalar. Then

$$\widehat{\tilde{\zeta}\tilde{\zeta}'} H' R^{-1} d\nu_t = \widehat{\tilde{\zeta}^2} H^2 R^{-1} dt + \text{noise}$$

The asymmetry in $\tilde{\zeta}$ is such that $\tilde{\zeta}^2$ tends to have the same sign as $\tilde{\zeta}$ with the result that $\{P_t\}$ tends to increase.

In the GOM proposed here, the operator dynamic gain, $PH'R^{-1}$, is a random process because $\{P_t\}$ is driven by the innovations process $\{\nu_t\}$. The responsiveness of the operator changes as his evaluation of the situation changes. Hence, the operator model is nonlinear as well as being time varying. It is the situation-dependent aspect of the model which makes this generalization so attractive. By contrast to other studies, the feature dependence of the GOM is not ad hoc, but is instead a consequence of the feature dynamics.

The information processing portion of the GOM given by (3.17)-(3.18) is not complete. The factors which multiply $d\nu_t$ in (3.19) are conditional third moments of $\tilde{\zeta}_t$. To finish the description of the estimation block, the structure of this conditional moment must be displayed.

As pointed out earlier, if $\tilde{\zeta}$ were Gaussian (the conventional OCM) this would be a trivial - $\widehat{\tilde{\zeta}\tilde{\zeta}'} \tilde{\zeta}_t = 0$ if $\tilde{\zeta}$ is Gaussian. Unfortunately, in this application the evolution of $\{\rho_t\}$ is more complex.

PROPOSITION 4 Denote the conditional third moment of $\tilde{\zeta}_t$ by Π_t ; specifically

$$\Pi(k) = \widehat{\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k} \quad (3.22)$$

Then the equation for $\{P_t\}$ can be written as

$$dP_t = (F_t P_t + P_t F_t' + F_w W F_w' + F_M \sum_i \tilde{Q}_i \tilde{\phi}_i F_m' - P_t H' R^{-1} H P_t) dt \\ + (\Pi(1) H' R^{-1} d\nu_1 + \dots + \Pi(N+n) H' R^{-1} d\nu_l) \quad (3.23)$$

Further

$$d\Pi(k)_l = \hat{f}_3(k)_l dt + (\Xi(l, k) + \Lambda_k H') R^{-1} d\nu_l; \quad l = 1, 2, \dots, N \quad (3.24)$$

where $\hat{f}_3(k)$ is given by (A.4.23), $\Xi(l, k)$ is given by (A.4.29) and

$$\Lambda_k = \widehat{(\tilde{\zeta} \tilde{\zeta}' \tilde{\zeta}_k)_l \tilde{\zeta}'} \quad (3.25)$$

Equation (3.17), (3.18) and (3.24) give the dynamic equations of the GOM. This model is expressed as the solution to a set of three, coupled, stochastic differential equations. Unfortunately, the system of equations is not closed in the sense that a mechanism for evaluating Λ_k has not been presented. Issues arising from the dependence of the GOM on Λ will be addressed in the next section.

The action block of the GOM is a relation between $\hat{\zeta}_t$ and u_t .

PROPOSITION 5.

$$u_t = K_{GOM} \hat{\zeta}_t \quad (3.26)$$

where

$$K_{GOM} = -N^{-1} G' \Sigma_t \quad (3.27)$$

and $\{\Sigma_t\}$ is given by

$$\Sigma_t = -F_t \Sigma_t - \Sigma_t F_t' + \Sigma_t G N^{-1} G' \Sigma_t - M \quad (3.30)$$

$$\Sigma_T = 0$$

Proposition 5 completes the representation of the GOM. As was the case in the OCM, the model takes the form of a linear function of the conditional mean of the encounter state. This "gain" is independent of both the form and intensity of the exogenous processes. The dynamics of the GOM are those of the estimation process.

Before leaving this topic, it is well to note explicitly the way in which the GOM is related to the OCM. In most cases, the state penalty matrix M provides a weighting on only the base-state components:

$$M = \begin{bmatrix} M_{11} & 0 \\ 0 & 0 \end{bmatrix} \quad (3.31)$$

From (3.27)

$$u_t = -N^{-1}B'(\Sigma_{11}\hat{x}_t + \Sigma_{12}\hat{\phi}_t) \quad (3.32)$$

where

$$\Sigma_t = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \quad (3.33)$$

Under condition (3.31) though

$$\dot{\Sigma}_{12} = - \left[(\Sigma_{11}A' + \Sigma_{12})Q' + (A' + \Sigma_{11}BN^{-1}B'\Sigma_{12}) \right] \quad (3.34)$$

$$\Sigma_{12}(T) = 0$$

If Q' is sufficiently small, then Σ_{12} will be small with the result that

$$u_t \approx -N^{-1}B'\Sigma_{11}\hat{x}_t \quad (3.35)$$

To first approximation then, the GOM is identical with the OCM. The term $\Sigma_{12}\hat{\phi}_t$ in (3.32) is a correction to the OCM which accounts for the dynamics of the feature process¹

¹ Note that Σ_{11} is also dependent on the properties of the feature process

4.0 SIMPLIFICATION OF THE OPERATOR MODEL

The GOM is given by (3.17), (3.23), (3.24) and (3.26). The first three of these equations are stochastic matrix equations of fairly high dimension. P_t for example, is in an $(N + n) \times (N + n)$ matrix where n is the dimension of the base state. Higher moments become even more numerous. To make this model useful in predicting TOV performance, certain simplifications must be made. These reductions in complexity take the form of judicious approximations to the exact expressions which delineate the GOM.

The encounter state contains two clearly discernible sub-vectors: the base state x_t and the feature state ϕ_t . The observation vector has the same decomposition. Although the full observation is useful in principle, in evaluating it, the operator tends to determine the features of the encounter from an observation of the features themselves. To be specific, denote the feature-observation vector by y_t :

$$dy_t = b(\phi_t)dt + I(1)dy_t \quad (4.1)$$

It will be supposed that B is block diagonal with lower right element $b(\phi_t)$. Denote the filtration generated by $\{y_t\}$ by $\mathcal{Y}_t$. It will be assumed that $\mathcal{Y}_t$ can be taken as a negligible error for

$$x_t = E[\phi_t | \mathcal{Y}_t] \quad (4.2)$$

Equation (4.2) quantifies the hypothesis that the operator processes observational data in a hierarchical manner. He uses the tree structure of the scene to identify the current mode of system operation. He then uses this partially processed data to infer the true value of the encounter state. He then processes his observations of the base state as

$$dx_t = F(x_t)dt + I(0)dy_t \quad (4.3)$$

where $\mathcal{Y}_t$ is generated by y_t

$$dy_t = I(2)dt + I(0)dy_t \quad (4.4)$$

This sequencing of the operator's signal processing has implications with respect to the form of the equation for $\{\hat{\phi}_i\}$. Using (4.2), the equation for $\{\hat{\phi}_i\}$ can be written directly. This equation is also given in such references as Elliott (1982). To illustrate the utility of (3.17), the equation is derived from the general equation for $\hat{\phi}_i$.

PROPOSITION 6 The dynamic equation for $\{\hat{\phi}_i\}$ is given by

$$d\hat{\phi}_i = Q^{-1}\hat{\phi}_i dt + (\hat{B} - \hat{b}_i I)\phi_i R_c^{-1} O^T J^{-1} dv_i \quad (4.5)$$

where

$$\hat{B} = \text{diag}(b_1, \dots, b_N) \quad (4.6)$$

$$\hat{b}_i = b_i \hat{\phi}_i$$

■

Equation (4.5) provides an important portion of the operator dynamics. It is a closed system of equations in the sense used earlier, but nonlinear because of the product dependence of the second term on the right hand side of (4.5).

The equation for $\{\hat{x}_i\}$ can be written as

$$d\hat{x}_i = (A\hat{x}_i + \phi_i Q^{-1}\hat{\phi}_i + Bv_i) dt + I(O^T P_i H R^{-1} dv_i)$$

The equation for $\{P_i\}$ still remains to be solved, although the lower right hand block of $\{P_i\}$ is known (see (A.6.3)).

$$P_i = \begin{bmatrix} \hat{P}_i & \hat{P}_i \hat{\phi}_i \\ \hat{P}_i \hat{\phi}_i & \text{diag}(\hat{\phi}_1^2, \dots, \hat{\phi}_N^2) + \phi_i \hat{\phi}_i \end{bmatrix} \quad (4.7)$$

5. CONCLUSIONS AND DIRECTION OF FUTURE EFFORT

This report presents the development of a system model which will be used to study the operation of a TOV in a multitask environment. Models of those parts of the encounter that are external to the remote operator (targets, TOV, etc.) are combined to form the base state. A modal indicator vector or feature state augments the base state to form the encounter state.

In order to quantify the interaction between the TOV and its surroundings, a suitable model for the operator is required. The methodology employed in the generation of the conventional OCM is used here to create a dynamic structure (the GOM) whose response mimics that of a remote operator engaged in guidance and tracking tasks in an abruptly changing environment.

The GOM has attractive properties in this application. The full state model, containing both the operator and the encounter states, is phrased in terms of a set of stochastic differential equations. Such equations lend themselves to digital simulation, thus permitting an investigation of specific scenarios. Further, such useful statistical measures of performance as the error covariance $\{P_i\}$ are computed as part of the analysis procedure.

In the forthcoming period, effort will be devoted to completing the GOM and to investigating multitask applications. Completion of the GOM requires the dynamic equation for $\{\dot{x}_i\}$. The equation given in (3.17) is a representational result rather than a computational recipe because $\{P_i\}$ is not provided in a computationally feasible form. Even under the hypotheses delineated in Section 4, only the lower right hand block is known explicitly.

To provide the requisite equations, some simplification of the third moment equation (3.24) is necessary. It is proposed that this be done by assuming that the conditional distribution of ζ_i has both Gaussian and nonGaussian parts. As mentioned earlier, the Gaussian part plays no role in $E\{\zeta_i \zeta_j^T\}$ or $E\{\zeta_i \zeta_j^T \zeta_k\}$ if ζ_i were Gaussian. An analytically tractable approximation to the nonGaussian part can then be used to create a term which can be used to replace $\{A_i\}$ in (3.24). This partitioning of ζ_i into separate parts will result in a finite dimensional operator model.

Performance analysis of the TOV system is based upon the GOM. However, the imposition of several simultaneous task requirements creates a distinctly different sort of dynamic structure from that encountered when there is but one activity. To place this situation within the model structure defined here, it will be assumed that the operator monitors his diverse tasks by sequencing his foveal direction. Specifically, suppose that the operator must perform two antithetical tasks simultaneously, e.g follow an irregular path while simultaneously tracking an evasive target at a large angular distance from the path. It will be supposed that the operator can focus his attention in one of two directions;

$$dy_t = H_{p_t} \zeta_t dt + d\eta_t \quad (5.1)$$

where

$$p_t = \begin{cases} 1 & \text{if current foveal direction is along the path} \\ 2 & \text{if current foveal direction is toward the target.} \end{cases} \quad (5.2)$$

The indexing of the observations gives another dimension to the actuating signal; i.e. the operator's action is now the pair $\{u_t, p_t\}$ rather than only $\{u_t\}$ as in the GOM.

Without further restriction, (5.2) would lead rather quickly to degeneracies in the choice of $\{p_t\}$. Rapid switching in direction of observation would give rise to performance that is indistinguishable from that which obtains from dual foveal observations; a physical impossibility if only one operator is permitted. This anomaly can be avoided if every $1 \rightarrow 2$ or $2 \rightarrow 1$ transition in $\{p_t\}$ must pass through another state $p_t = 3$ such that

$$H_3 = 0 \quad (5.3)$$

The rate at which an intelligent operator will make changes in his direction of observation is regulated by the lifetime in state 3.

This behavior is made most apparent if the properties of $\{P_t\}$ are explored. Partition $\{P_t\}$

as

$$P_i = \left[\begin{array}{c|c|c} P_{111} & & P_{12} \\ \hline & & \\ \hline \dots & P_{112} & \\ \hline & & \\ \hline & P_{21} & P_{22} \end{array} \right] \quad (5.4)$$

where P_{22} is the conditional error variance of the feature state (see (4.8)), P_{111} is the error variance of those components of the base state most relevant to task 1, and P_{112} is the same quantity for the portion of the base state relevant to task 2.

The inherent flexibility of the GOM provides the means to quantify the behavior of the operator even in the presence of variations in observation direction. To see this, note that the dynamics of $\{\hat{\zeta}_i\}$ depend upon the matrix $\{P_i\}$ with "faster" response associated with "greater" uncertainty. To model the operator's reaction in the two-task application, it could be supposed that the line-of-sight direction is changed whenever the performance of the unattended task *degrades to an unacceptable degree*.

As an example of how this idea might be stated more precisely, denote by $\{U_i\}$ the cost in the unattended task and suppose that

$$U_i = \begin{cases} \text{Tr } M_{111} P_{111} & \text{if } P_{i-} = 2 \\ \text{Tr } M_{112} P_{112} & \text{if } P_{i-} = 1 \end{cases} \quad (5.5)$$

where M_{111} is a decomposition of M compatible with the decomposition of $\{P_i\}$ shown in (5.4). Note that $\{U_i\}$ need not be defined for $\{p_i = 3\}$ since this is a transition state.

The discrete portion of the operation algorithm can then be given by an inequality of the form

$$dp_i = 0 \quad \text{if} \quad U_i \leq U \quad (5.6)$$

where U is a threshold selected to distinguish acceptable from unacceptable performance. To the extent that this line of research proves fruitful, the question then arises as to how small can U be chosen in order that the operator model not become indeterminate; i.e. subject to (5.6).

$$\lim_{t \rightarrow \infty} \text{Tr } M_{11i} P_{11i} > U; \quad i = 1, 2 \quad (5.7)$$

Switching procedures like that indicated in (5.6) have been used by other investigators in the LQG context. In such references the switching sequence can be precomputed since $\{P_i\}$ is not random. In the case under study, the evolution of $\{P_i\}$ is situation dependent, and the switching times cannot be computed *a priori*. Further the statistical properties of the error variance are not yet clearly comprehended. Hence, the best form of the switching criterion cannot be defined with certainty at this time.

Multiple task situations in which operator behavior is indeterminate arise when the workload is such that a single operator is unable to perform all tasks satisfactorily. Such a situation presents another interesting problem which can be investigated from the present perspective. One method of maintaining operator workload at an acceptable level involves the introduction of automation which is capable of performing some of the human's tasks. The multitask model for human performance, when used with compatible models for proposed aids, will be of use in predicting the performance of the augmented system. Various types of automation, deployed in multiple configurations could be analyzed in terms of their utility in overall system performance enhancement. Mode switching criteria similar to those proposed in the absence of automation could be investigated. Such an avenue would provide an interesting application of the multitask model now being developed.

The work proposed for the next year can be partitioned into two main areas of activity.

AREA 1. The GOM developed in the previous reporting period must be completed, simplified and related to such empirical data as exists. The "completion" of the GOM has been discussed in earlier sections. The model of the remote operator must be expressed as a finite dimensional set of stochastic differential equations. This will be accomplished by approximating the probability law of the base state as a sum of two specific parametric distributions.

The OCM has been observed to be rather more complicated than necessary. The analytical approach leading to both the OCM and GOM is not a procedure which is parsimonious in the number of free parameters in the model. If system simulation was the only use to be made of the

model, this would merely be a nuisance. For a careful study of the TOV system, more is required.

To simplify the GOM, the "relevant" portion of the operator model must be distinguished from the peripheral dynamics. The aim of this study is to predict system performance, not to provide a highly accurate operator model. Hence, those aspects of the GOM which have marginal influence on system performance should be deleted.

The indicated simplification and reduced parameterization of the GOM is essential if the empirical data from the test facility is to be incorporated into the final model determination. The operator can not be tested apart from the rest of the system. Further, the variables that delineate the GOM are mathematical artifices. Hence, the model parameters can only be indirectly determined from experimental data. To reduce the ambiguity of the relationship between the model and the empirical data, the model must be reduced to a small set of physically substantive parameters. Such a representation makes more meaningful any sensitivity study based upon the GOM.

AREA 2. The system description must be expanded to a multitask environment using the procedures discussed earlier. Both nonrandom, time dependent attention switching as was done with the OCM - and situation dependent attention switching based on $\{P_i\}$ will be investigated. The latter is clearly the most apropos, but to arrive at definitive conclusions, the dynamic behavior of the covariance equations must be better understood.

It is expected that the proposed work will complement the ongoing experimental activity of the TOV test facility. The modelling activity proposed here will be enriched by an interaction and technical interchange with the test personnel. The results of this effort will also aid them in the development of test protocols.

REFERENCES

- Anderson, R. O., "A New Approach to the Specification and Evaluation of Flying Qualities," AFDL-TR-69-120, WPAFB, June 1970.
- Angel, E. S. and Bekey, G. A., "Adaptive Finite State Models of Manual Control Systems," *IEEE Trans. on Man Machine Systems*, Vol. MMS-9, 1968, pp. 15-20.
- Ashkenas, I. L. and McRuer, D. T., "A Theory of Handling Qualities Derived from Pilot-Vehicle System Considerations," *Aerospace Engineering*, February 1962.
- Baron, S., "A Control Theoretic Approach to Modelling of Human Supervisory Control of Dynamic Systems," in *Advances in Man Machine Systems*, Ed. W. B. Rouse, Vol. 1, 1984.
- Baron, S. and Kleinman, D., "The Human as an Optimal Controller and Information Processor," *IEEE Trans. on Man Machine Systems*, Vol. MMS-10, March 1969, pp. 9-17.
- Baron, S. and Kleinman, D. L., "Prediction and Analysis of Human Performance in a VTOL Hovering Task," *Proceedings of the 7th Annual Conference on Manual Control*, University of Southern California, 1971, pp. 247-256.
- Bekey, G., "The Human Operator as a Sampled Data System," *IRE Trans. on Human Factors in Electronics*, Vol. HFE-3, No. 2, 1962, pp. 43-51.
- Bekey, G. A., Burnham, G. O. and Seo, J., "Control Theoretic Models of Human Drivers in Car Following," *Human Factors*, Vol. 19, No. 4, August 1977, pp. 399-413.
- Boonton, R. C., "Nonlinear Control Systems with Random Inputs," *Trans. IRE PGCT*, Vol. CT-1, 1954, pp. 9-17.
- Burnham, G. O. and Bekey, G. A., "A Heuristic Finite State Model of the Human Driver in a Car Following Situation," *IEEE Trans. on Systems, Man and Cybernetics*, Vol. SMC-6, No. 8, 1976, pp. 554-562.
- Dorf, R. C., *Modern Control Systems*, Addison Wesley, 1974.
- Elkind, J. I. and Miller, D., "The Adaptive Response of the Human Controller to Sudden Changes in Controlled Process Dynamics," *IEEE Trans. Human Factors in Electronics*, Vol. HFE-8, No. 3, September 1967.
- Elliott, R. J., *Stochastic Calculus and Applications*, Springer-Verlag, New York, 1982.
- Fleming, W. and Rischel, R., *Deterministic and Stochastic Optimal Control*, Springer Verlag, New York, 1975.
- Grunwald, A. J. and Merhav, S. J., "Effectiveness of Display Augmentation in Vehicular Control by Visual Field Cues," *IEEE Trans. on Systems, Man and Cybernetics*, Vol. SMD-8, No. 9, 1978.

Grunwald, A. J. and Merhav, S. J., "Vehicular Control by Visual Field Cues - Analytical Model and Experimental Validation," *IEEE Trans. on Systems, Man and Cybernetics*, Vol. SMC-6, No. 12, December 1976.

Harvey, T. and Dillow, J., "Fly and Fight: Predicting Piloted Performance in Air-to-Air Combat," *Proceedings of the Tenth Conference on Manual Control*, 1974.

Hess, R., "Prediction of Pilot Opinion Ratings Using an Optimal Pilot Model," *Human Factors*, Vol. 19, No. 5, October 1977, pp. 459-475.

Johannsen, G., "The Design of a Non-Linear Multi-Parameter Model for the Human Operator," in *Displays and Controls, Proceedings of the Adv. Study Inst., Berchtesgaden*, pp. 249-367.

Johannsen, G., "Man-Machine Systems - Introduction and Background," in *Proceedings of the IFAC Conference on Analysis, Design and Evaluation of Man-Machine Systems*, Ed. G. Johannsen and J. E. Rijnsdorp, 1982.

Johannsen, G. and Rouse, W., "Prospects on a Mathematical Theory of Human Behavior in Complex Man-Machine Systems Tasks," *Fourteenth Conference on Manual Control*, 1978.

Kleinman, D. L., "Modeling the Human Tracking Error in Several Different Anti-Tank Systems," *Seventeenth Conference on Manual Control*, June 1981, pp. 379-392.

Kleinman, D. L., "Optimal Control of Linear Systems with Time-Delay and Observation Noise," *IEEE Transactions on Automatic Control*, October 1969.

Kleinman, D. L., Baron, S. and Levison, W. H., "A Control Theoretic Approach to Manned Vehicle Systems Analysis," *IEEE Trans. on Automatic Control*, Vol. AC-16, December 1971, pp. 824-832.

Kleinman, D. L., Baron, S., and Levison, W. H., "An Optimal Control Model of Human Response, Part I: Theory and Validation," *Automatica*, Vol. 6, 1970, pp. 357-369.

Kleinman, D. L. and Curry, R. E., "Some New Control Theoretic Models for Human Display Monitoring," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. SMC-7, No. 11, 1977, pp. 778-784.

Kleinman, D. L. and Killingsworth, W. R., "A Predictive Pilot Model for STOL Aircraft Landing," NASA CR-2374, March 1974.

Kleinman, D. L. and Perkins, T. R., "Modeling Human Performance in a Time-Varying Anti-Aircraft Tracking Loop," *IEEE Transactions on Automatic Control*, Vol. AC-19, No. 4, August 1974.

Krishnan, V., *Nonlinear Filtering and Smoothing*, John Wiley, 1984.

Levison, W. H. and Elkind, J. I., "Two Dimensional Manual Control Systems with Separated Displays," *IEEE Trans. on Human Factors in Electronics*, Vol. HFE-8, No. 3, 1967, pp. 202-209.

Levison, W. H., Elkind, J. I. and Ward, J. L., "Studies of Multivariable Manual Control Systems: A Model for Task Interference." NASA CR-1746.

McRuer, D. T., Allen, R. W., Weir, D. and Klein, R., "New Results in Driver Steering and Control Models," *Human Factors*, Vol. 19, No. 4, 1977.

McRuer, D. T., and Graham, D., "Pilot-Vehicle Control Systems Analysis," *AIAA Guidance and Control Conference*, Cambridge, Mass., August 1963.

McRuer, D. T., Graham, P. and Krendel, E. S., "Manual Control of Single Loop Systems. Parts I and II," *Journal of the Franklin Institute*, Vol. 283, 1967, pp. 1-29 and 145-168.

McRuer, D. T. and Jex, H. R., "A Review of Quasi-Linear Pilot Models," *IEEE Trans. on Human Factors in Elec.*, Vol. HFE-8, No. 3, September 1967.

McRuer, D. T. and Krendel, E. S., "The Human Operator as a Servo System Element," *J. Franklin Institute*, Vol. 267, 1959.

McRuer, D. T. and Weir, D. H., "Theory of Manual Vehicular Control," *Ergonomics*, Vol. 12, No. 4, 1969.

Mulradin, R. and Baron, S., "DEMON: A Human Operator Model for Decision Making, Monitoring and Control," *Journal of Cybernetics and Information Science*, Vol. 3, 1980, pp. 97-122.

Newell, A. and Simon, H. A., *Human Problem Solving*, Prentice Hall, 1972.

Nitao, J. and Parodi, A., "A Real Time Reflexive Pilot for an Autonomous Land Vehicle," *IEEE Control Systems Magazine*, February 1986.

Pattipati, K., Kleinman, D. L. and Ephrath, A., "From OCM to ODM - An Optimal Decision Model of Human Task Sequencing Performance," *Proceedings of the IEEE Systems, Man, and Cybernetics Conference*, 1980.

Phatak, A., "Investigation of Alternate Human Operator Optimal Control Model Structures," *Proceedings of the Fifteenth Conference on Manual Control*, 1979.

Phatak, A. V., "Review of Model-Based Methods for Pilot Performance and Workload Assessment," NASA Contract NAS2-11318, April 1983.

Phatak, A. V. and Bekey, G. A., "Models of the Adaptive Behavior of the Human Operator in Response to Sudden Change in the Control System," *IEEE Trans. on Man Machine Systems*, Vol. MMS-10, No. 3, September 1969.

Phatak, A. V., Kessler, K. M., "Modelling the Human Gunner in an Anti-Aircraft Artillery (AAA) Tracking Task," *Human Factors*, Vol. 19, No. 9, October 1977.

Phatak, A. and Kleinman, "Current Status of Models for the Human Operator as a Controller and Decision Maker in Manned Aerospace Systems," *AGARD Proceedings*, Dayton, Ohio, No. 114, October 1972.

- Rasmussen, J., "The Human as a Systems Component," in *Human Interaction with Computers*, Ed. H. T. Smith and T. R. Green, Academic Press, London, 1980.
- Siegel, A. and Wolf, J., *Man-Machine Simulation Models*, John Wiley and Sons, New York, 1969.
- Stapleford, R. L., McRuer, D. T. and Magdaleno, R. F., "Pilot Describing Function Measurements in a Multi-Loop Task," *IEEE Trans on Human Factors in Electronics*, Vol. HFE-8, No. 2, 1967.
- Tustin, A., "The Nature of the Operator's Response in Manual Control and Its Implications in Controller Design," *Journal of the Institution of Electr. Engineers*, Vol. 94, Part IIA,(2), 1947.
- Tse, E., "On the Optimal Control of Stochastic Linear Systems," *IEEE Transactions on Automatic Control*, Vol. AC-16, No. 6, December 1971.
- Veldhuyzen, W. M. and Stassen, H. G., "The Internal Model Concept: An Application to Modeling Human Control of Large Ships," *Human Factors*, Vol. 19, No 4, August 1977.
- White, T., "Supervision of Dynamic Systems: Monitoring, Decision Making and Control," *Proceedings of the 17th Conference on Manual Control*, 1981.
- Weir, D. H. and Phatak, A. V., "Model of Human-Operator Response to Step Transitions in Controlled Element Dynamics," 2nd Annual Conference on Manual Control, 1966.
- Young, L. R. and Meiry, J. L., "Bang-Bang Aspects of Manual Control in High Order Systems," *IEEE Trans. Automatic Control*, Vol. AC-10, No. 3, 336-431.
- Zacharias, G. L., Baron, S., Muralidharam, R., "A Supervisory Control Model of the AAA Crew," *Proceedings of the 1982 American Control Conference*.

APPENDIX

PROPOSITION 1. Since $\{m_t\}$ and $\{w_t\}$ are $\{F_t\}$ martingales, (3.13a)-(3.13b) follow immediately. Further $\{w_t\}$ is Brownian motion, and hence (3.13c). To show (3.13d), observe that if $\{r_t\}$ makes an $i \rightarrow j$ transition at time t ,

$$dm_k dm_l = \begin{cases} 1 & \text{if } k \neq l \text{ and } k, l \in \{i, j\} \\ 1 & \text{if } k = l \in \{1, j\} \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.1.1})$$

Hence if $r_{t-} = i$

$$E \{ dm_k dm_l \mid F_{t-} \} = \begin{cases} -q_{ik} dt & \text{if } l = i \\ -q_{il} dt & \text{if } k = i \\ q_{ii} dt & \text{if } k = l \neq i \\ -q_{ll} dt & \text{if } k = l = i \end{cases} \quad (\text{A.1.2})$$

Let q_i be the i 'th row of the Q matrix. Then if $\phi_{t-,i} = 1$

$$\begin{aligned} E \{ dm_t dm_t' \mid F_{t-} \} &= (-\phi_t q_i - q_i' \phi_t' + \text{diag}(q_i)) dt \\ &= \tilde{Q}_i dt \end{aligned} \quad (\text{A.1.3})$$

where $\tilde{Q}_i$ is the matrix indicated in (A1.3). It then follows that

$$\begin{aligned} E \{ dm_t dm_t' \mid F_{t-} \} &= \sum_{i=1}^N \tilde{Q}_i \phi_{t-,i} dt \\ &= V(\phi_t) dt \end{aligned} \quad (\text{A.1.4})$$

The quantity on the right side of (A1.4) is called $d\langle m, m \rangle_t$. Note that

$$\begin{aligned} dm_t dm_t' &= d\langle m, m \rangle_t + (dm_t dm_t' - d\langle m, m \rangle_t) \\ &= d\langle m, m \rangle_t + dm_{1,t} \end{aligned}$$

where $\{m_{1,t}\}$ is a matrix martingale, i.e.

$$E \{ (dm_{1,t})_{ij} \mid \underline{F}_{t-} \} = 0 \quad (\text{A.1.5})$$

PROPOSITION 2. As indicated in (3.3)

$$d\zeta_t = (F_\zeta \zeta_t + Gu_t) dt + \text{martingale increment} \quad (\text{A.2.1})$$

From Krishnan (1984), Thm. 8.5.1]

$$d\hat{\zeta}_t = (F_\zeta \hat{\zeta}_t + Gu_t) dt + (\widehat{\zeta_t - \zeta_{t-}} H') R^{-1} d\nu_t \quad (\text{A.2.2})$$

where it has been noted that $\{u_t\}$ is adapted to $\{\underline{Y}_t\}$ and that $\{w_t\}$ and $\{\eta_t\}$ are independent. Equation (A.2.2) is equivalent to (3.17).

PROPOSITION 3. The derivation of (3.18) is as follows. From (3.3) and (3.12) the error process $\tilde{\zeta}_t$ satisfies

$$d\tilde{\zeta} = F_\zeta \tilde{\zeta} + F_m dm + F_w dw - P_t H' R^{-1} d\nu_t \quad (\text{A.3.1})$$

since $P_t = \widehat{\zeta_t \zeta_t'}$, it is first necessary to obtain an equation for $\tilde{\zeta}_t \tilde{\zeta}_t'$. It is known that

$$d(\tilde{\zeta}_t \tilde{\zeta}_t') = (d\tilde{\zeta}_t) \tilde{\zeta}_t' + \tilde{\zeta}_t (d\tilde{\zeta}_t') + (d\tilde{\zeta}_t)(d\tilde{\zeta}_t') \quad (\text{A.3.2})$$

Expanding (A.3.1), and using the fact that

$$(d\nu_t)(d\nu_t') = R dt \quad (\text{A.3.3})$$

it follows that

$$\begin{aligned}
 d\tilde{\zeta}\tilde{\zeta}' = & \left[(F_{\zeta} - PH'R^{-1}H)\tilde{\zeta}\tilde{\zeta}' + \tilde{\zeta}\tilde{\zeta}'(F_{\zeta} - PH'R^{-1}H)' + F_w W F_w' + F_m V(\phi)F_m' \right. \\
 & \left. + PH'R^{-1}HP \right] dt + F_m dm_1 F_m' + F_m d\tilde{m}\tilde{\zeta}' - PH'R^{-1}d\tilde{\eta}' + F_w dw\tilde{\zeta}' \\
 & + \tilde{\zeta}dm'F_m' + \tilde{\zeta}dw'F_w' - \tilde{\zeta}d\eta'R^{-1}HP
 \end{aligned} \tag{A.3.4}$$

where

$$dm_1 = dm_t dm_t' - V(\phi_t) dt$$

Note the $\tilde{\zeta}\tilde{\zeta}'$ is a symmetric matrix. Let us consider the k 'th column at $\tilde{\zeta}\tilde{\zeta}'$;

$$\begin{aligned}
 \tilde{\zeta}\tilde{\zeta}' &= (\tilde{\zeta}\tilde{\zeta}_1, \dots, \tilde{\zeta}\tilde{\zeta}_k, \dots, \tilde{\zeta}\tilde{\zeta}_N) \\
 &= [(\tilde{\zeta}\tilde{\zeta}')_{\cdot k}]
 \end{aligned} \tag{A.3.5}$$

The equation of evolution of $\tilde{\zeta}\tilde{\zeta}_k$ is given by the k 'th column of (A.3.4). Hence,

$$\begin{aligned}
 d\widehat{\tilde{\zeta}\tilde{\zeta}_k} = & \left[((F_{\zeta} - PH'R^{-1}H)P)_{\cdot k} + (P(F_{\zeta} - PH'R^{-1}H)')_{\cdot k} + (F_w W F_w')_{\cdot k} \right. \\
 & \left. + (F_m V(\phi)F_m')_{\cdot k} + (PH'R^{-1}HP)_{\cdot k} \right] dt + (\widehat{\tilde{\zeta}dm'})_{\cdot k} \\
 & + (F_m \widehat{dm\tilde{\zeta}'})_{\cdot k} + \widehat{\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k} H'R^{-1}d\nu_t
 \end{aligned} \tag{A.3.6}$$

where the fact that $(F_m dm_1 F_m')_{\cdot k}$ is a martingale increment and that

$$E \left\{ (PH'R^{-1}d\tilde{\eta}')_{\cdot k} d\tilde{\eta}' \mid \underline{Y}_t \right\} = 0 \tag{A.3.7}$$

has been used. Combining the columns of $\widehat{\tilde{\zeta}\tilde{\zeta}'}$, and noting that $\widehat{dm\tilde{\zeta}'} = 0$, it follows that

$$dP_t = (F_{\zeta}P + PF_{\zeta}' - PH'R^{-1}HP + F_w W F_w' + F_m \hat{V} F_m') dt + d\rho_t \tag{A.3.8}$$

where

$$d\rho_t = (\widehat{\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_1} H'R^{-1}d\nu_t, \dots, \widehat{\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_{N+N}} H'R^{-1}d\nu_t)$$

PROPOSITION 4. Beginning as in Proposition 3, consider first the dynamic equation of the conditional product $\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k$. As indicated previously

$$d(\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k) = (d\tilde{\zeta}\tilde{\zeta}')\tilde{\zeta}_k + \tilde{\zeta}\tilde{\zeta}'(d\tilde{\zeta}_k) + (d\tilde{\zeta}\tilde{\zeta}')(d\tilde{\zeta}_k) \quad (\text{A.4.1})$$

The term $d\tilde{\zeta}\tilde{\zeta}'$ is given in (A.3.4) and

$$d\tilde{\zeta}_k = (F_{\zeta}\tilde{\zeta})_k dt + (F_m dm)_k + (F_w dw)_k + (PH'R^{-1}d\nu)_k \quad (\text{A.4.2})$$

There are many terms in expansion (A.4.1), and it is well to look at them individually

$$\begin{aligned} (d\tilde{\zeta}\tilde{\zeta}')\tilde{\zeta}_k &= (F_{\zeta} - PH'R^{-1}H)\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k + \tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k(F_{\zeta} - PH'R^{-1}H)' dt \\ &\quad + (F_m dm + F_w dw + PH'R^{-1}d\eta)\tilde{\zeta}'\tilde{\zeta}_k + \tilde{\zeta}\tilde{\zeta}_k(F_m dm + F_w dw + PH'R^{-1}d\eta)' \\ &\quad + [(F_w WF_w' + F_m VF_m' + PH'R^{-1}HP) dt + F_m dm_1 F_m']\tilde{\zeta}_k \end{aligned} \quad (\text{A.4.3})$$

$$\begin{aligned} \tilde{\zeta}\tilde{\zeta}'(d\tilde{\zeta}_k) &= \tilde{\zeta}\tilde{\zeta}'[(F_{\zeta}\tilde{\zeta})_k dt + (F_m dm)_k + (F_w dw)_k + (PH'R^{-1}(H\tilde{\zeta}dt + d\eta_t))_k] \\ &= \tilde{\zeta}\tilde{\zeta}'((F_{\zeta} - PH'R^{-1}H)\tilde{\zeta})_k dt + \tilde{\zeta}\tilde{\zeta}'(F_m dm + F_w dw + PH'R^{-1}d\eta)_k \end{aligned} \quad (\text{A.4.4})$$

Finally, if we neglect trivial terms

$$\begin{aligned} (d\tilde{\zeta}\tilde{\zeta}')(d\tilde{\zeta}_k) &= (F_m dm_1 F_m' + F_m dm\tilde{\zeta}' + \tilde{\zeta}dm'F_m')(F_m dm)_k \\ &\quad + (F_w dw\tilde{\zeta}' + \tilde{\zeta}dw'F_w')(F_w dw)_k \\ &\quad + (PH'R^{-1}d\eta\tilde{\zeta}' + \tilde{\zeta}d\eta'R^{-1}HP)(PH'R^{-1}d\eta)_k \end{aligned} \quad (\text{A.4.5})$$

Equations (A.4.3)-(A.4.5) give us the basic terms that we need. It is, however, convenient to simplify some of the terms involved before combining them into (A.4.1). Consider the second order terms in (A.4.5).

$$\begin{aligned}
 (F_m dm \tilde{\zeta}')_{ij} (F_m dm)_k &= \left(\sum_l F_{mil} dm_l \tilde{\zeta}_j \right) \left(\sum_\lambda F_{mk\lambda} dm_\lambda \right) \\
 &= \sum_{l,\lambda} F_{mil} F_{mk\lambda} \tilde{\zeta}_j dm_l dm_\lambda \\
 &= \sum_{l,\lambda} F_{mil} F_{mk\lambda} \tilde{\zeta}_j \left(d(m_l m_\lambda)_{l\lambda} + dm_{l\lambda} \right) \quad (A.4.6)
 \end{aligned}$$

Similarly

$$\begin{aligned}
 (F_m dm_1 F_m')_{ij} (F_m dm)_k &= \sum_{\alpha,\beta} F_{mi\alpha} dm_{1\alpha\beta} F_{mj\beta} \sum_\gamma F_{mk\gamma} dm_\gamma \\
 &= \sum_{\alpha,\beta,\gamma} F_{mi\alpha} F_{mk\gamma} F_{mj\beta} dm_{1\alpha\beta} dm_\gamma \quad (A.4.7)
 \end{aligned}$$

But

$$\begin{aligned}
 dm_{1\alpha\beta} dm_\gamma &= dm_\alpha dm_\beta dm_\gamma - V_{\alpha\beta} dm_\gamma dt \\
 &= d\lambda_\alpha dm_\beta dm_\gamma \quad (A.4.8)
 \end{aligned}$$

Hence

$$(F_m dm_1 F_m')_{ij} (F_m dm)_k = \sum_{\alpha,\beta,\gamma} F_{mi\alpha} F_{mk\gamma} F_{mj\beta} dm_\alpha dm_\beta dm_\gamma \quad (A.4.9)$$

Combining (A.4.6) and (A.4.9)

$$\begin{aligned}
 (F_m dm_1 F_m' + F_m dm \tilde{\zeta}' + \tilde{\zeta} dm F_m')_{ij} (F_m dm)_k &= \sum_{l,\lambda} F_{mil} F_{mk\lambda} \tilde{\zeta}_j dm_l dm_\lambda \\
 &\quad + F_{mj\lambda} F_{mk\lambda} \tilde{\zeta}_i dm_l dm_\lambda + \sum_{\alpha,\beta,\gamma} F_{mi\alpha} F_{mk\gamma} F_{mj\beta} dm_\alpha dm_\beta dm_\gamma \quad (A.4.10)
 \end{aligned}$$

Similarly

$$\begin{aligned}
 (F_w dw \tilde{\zeta})_{ij} (F_w dw)_k &= \left(\sum_l F_{wli} dw_l \tilde{\zeta}_j \right) \left(\sum_\lambda F_{wk\lambda} dw_\lambda \right) \\
 &= \sum_{l,\lambda} F_{wli} F_{wk\lambda} \tilde{\zeta}_j dw_l dw_\lambda \\
 &= \sum_{l,\lambda} F_{wli} F_{wk\lambda} \tilde{\zeta}_j W_{l\lambda} dt
 \end{aligned} \tag{A.4.11}$$

Hence

$$(F_w dw \tilde{\zeta}' + \tilde{\zeta}' dw' F_w')_{ij} (F_w dw)_k = \left(\sum_{l,\lambda} F_{wli} F_{wk\lambda} \tilde{\zeta}_j W_{l\lambda} + F_{wj}, F_{wk\lambda} \tilde{\zeta}_i W_{\lambda l} \right) dt \tag{A.4.12}$$

Finally

$$\begin{aligned}
 (PH'R^{-1}d\eta \tilde{\zeta}')_{ij} (PH'R^{-1}d\eta)_k &= \left(\sum_l (PH'R^{-1})_{il} d\eta_l \tilde{\zeta}_j \right) \left(\sum_\lambda (PH'R^{-1})_{k\lambda} d\eta_\lambda \right) \\
 &= \sum_{l,\lambda} (PH'R^{-1})_{il} (PH'R^{-1})_{k\lambda} \tilde{\zeta}_j d\eta_l d\eta_\lambda \\
 &= \sum_{l,\lambda} (PH'R^{-1})_{il} (PH'R^{-1})_{k\lambda} \tilde{\zeta}_j R_{l\lambda} dt \\
 &= (PH'R^{-1}HP)_{ik} \tilde{\zeta}_j dt
 \end{aligned} \tag{A.4}$$

Consider next the expectation of the second order terms. Since

$$E \left\{ \begin{pmatrix} \tilde{\zeta}' \\ \tilde{\zeta} \end{pmatrix} \right\} = 0$$

it follows that

$$E \left\{ \int_0^t F_w dv \right\} = 0$$

$$E \left\{ \int_0^t F_H dv \right\} = 0$$

Define a matrix A by

AD-A 184 487

NAVAL OCEAN SYSTEMS CENTER, SAN DIEGO, CA
OPERATOR MULTIPLE TASKING STUDY FOR REMOTELY
OPERATED PLATFORMS BY: KS HAALAND, DD SWORDER

2 OF 2
NOSC TD 1084
UNCLASSIFIED
APR 1987

END
DATE
FILMED

$$\begin{aligned}
 (\Lambda(k))_{ij} dt = & \sum_{l, \lambda} F_{mil} F_{mk\lambda} \overline{\zeta_j dm_l dm_\lambda} + F_{mjl} F_{mk\lambda} \overline{\zeta_i dm_l dm_\lambda} \\
 & + \sum_{\alpha, \beta, \gamma} F_{mi\alpha} F_{mk\gamma} F_{mj\beta} \overline{dm_\alpha dm_\beta dm_\gamma}
 \end{aligned} \tag{A.4.16}$$

Then

$$\overline{(d\zeta\zeta') (d\zeta_k)} = \Lambda(k) dt \tag{A.4.17}$$

From the above it is possible to evaluate the third conditional moment. First note that

$$\begin{aligned}
 d(\zeta\zeta' \zeta_k) = & \zeta\zeta' ((F_\zeta - PH'R^{-1}H)' \zeta_k + ((F_\zeta - PH'R^{-1}H)\zeta)_k) \\
 & + (F_\zeta - PH'R^{-1}H) \zeta\zeta' \zeta_k + \zeta(\zeta' (F_m dm + F_w dw)_k \\
 & + \zeta_k (F_m dm + F_w dw)') + ((F_m dm + F_w dw)_k \zeta + (F_m dm + F_w dw)' \zeta_k) \zeta' \\
 & - \zeta\zeta' PH'R^{-1}d\eta)_k + \zeta_k (PH'R^{-1}d\eta)' - ((PH'R^{-1}d\eta)_k \zeta + (PH'R^{-1}d\eta) \zeta_k) \zeta' \\
 & + F_m dm dm' F_m' \zeta_k + \Lambda(k) dt + \text{martingale increment terms}
 \end{aligned} \tag{A.4.18}$$

The expectation of the right hand side of (A.4.18) can be produced as follows. Define

$$E(\zeta\zeta' \zeta_k | \underline{Y}_t) = \Pi(k) \tag{A.4.19}$$

Then

$$\begin{aligned}
 \overline{\zeta\zeta' ((F_\zeta - PH'R^{-1}H)\zeta)_k} &= \Pi(k) (F_\zeta - PH'R^{-1}H)_k \\
 \overline{\zeta\zeta' ((F_\zeta - PH'R^{-1}H)\zeta)_k} &= \sum_l \overline{\zeta\zeta' (F_\zeta - PH'R^{-1}H)_{kl} \zeta_l} \\
 &= \sum_l \Pi(l) (F_\zeta - PH'R^{-1}H)_{kl}
 \end{aligned} \tag{A.4.20}$$

Similarly

$$\overline{\zeta(\zeta' (F_m dm + F_w dw)_k + \zeta_k (F_m dm + F_w dw)')} = \overline{\zeta\zeta' (F_m dm)_k} \tag{A.4.21}$$

$$\overline{\zeta\zeta_k dm' F_m'} = 0 \tag{A.4.21}$$

$$\overline{\zeta(\zeta' (PH'R^{-1}d\eta)_k + \zeta_k (PH'R^{-1}d\eta)')} = 0 \tag{A.4.22}$$

Hence the expectation of the right side of (A.4.18) becomes

$$\begin{aligned} \widehat{d(\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k)} &= (\Pi(k)(F_\zeta - PH'R^{-1}H)' + (F_\zeta - PH'R^{-1}H)\Pi(k) \\ &\quad + \sum_l \Pi(l)(F_\zeta - PH'R^{-1}H)_k + \Lambda(k)dt + F_m \widehat{dm dm' F_m' \tilde{\zeta}_k} \\ &= \widehat{f_3}(k) \end{aligned} \quad (A.4.23)$$

To obtain the gain term in the dynamic equation for $\Pi(k)$, consider the l 'th column of $\Pi(k)$.

$$\Pi(k) = (\Pi(k)_{\cdot 1}, \dots, \Pi(k)_{\cdot n})$$

Then

$$d\Pi(k)_{\cdot l} = (\dot{f}_3(k))_{\cdot l} dt + K(\Pi, k, l) R^{-1} d\nu_l \quad (A.4.24)$$

where

$$K(\Pi, k, l) dt = d\langle (\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k)_{\cdot l}, \eta' \rangle_l + \widehat{(\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k)_{\cdot l} \tilde{\zeta}' H'} dt \quad (A.4.25)$$

Only the terms involving η in (A.4.18) contribute to $d\langle \cdot, \eta' \rangle_l$. Again, there are many terms which must be evaluated.

$$\begin{aligned} ((\tilde{\zeta}\tilde{\zeta}'(PH'R^{-1}d\eta)_k)_{\cdot l} d\eta')_{ij} &= \sum_\alpha \tilde{\zeta}_i \tilde{\zeta}_l d\eta_j (PH'R^{-1})_{k\alpha} d\eta_\alpha \\ &= \sum_\alpha \tilde{\zeta}_i \tilde{\zeta}_l (PH'R^{-1})_{k\alpha} R_{\alpha j} dt \\ &= \tilde{\zeta}_i \tilde{\zeta}_l (PH')_{kj} dt = \tilde{\zeta}_l (\zeta(HP)_{\cdot k})_{ij} dt \end{aligned} \quad (A.4.26)$$

$$\begin{aligned}
 ((\tilde{\zeta}(PH'R^{-1}d\eta))'_{.i} \tilde{\zeta}_k d\eta')_{ij} &= \sum_{\beta} \tilde{\zeta}_i d\eta_{\beta} (PH'R^{-1})'_{\beta l} \tilde{\zeta}_k d\eta_j \\
 &= \sum_{\beta} \tilde{\zeta}_i \tilde{\zeta}_k (PH'R^{-1})_{l\beta} R_{\beta j} dt \\
 &= \tilde{\zeta}_i \tilde{\zeta}_k (PH')_{lj} dt \\
 &= \tilde{\zeta}_k (\tilde{\zeta}'(HP)_{.l})_{ij} dt
 \end{aligned} \tag{A.4.27}$$

Consequently

$$\begin{aligned}
 &\overbrace{((\tilde{\zeta}\tilde{\zeta}'(PH'R^{-1}d\eta)_k)'_{.i} d\eta')_{ij}} + \overbrace{((\tilde{\zeta}(PH'R^{-1}d\eta)')'_{.l} \tilde{\zeta}_k d\eta')_{ij}} \\
 &= P_{.l}(PH')_{kj} dt + P_{.k}(PH')_{lj} dt
 \end{aligned} \tag{A.4.28}$$

Combining these terms

$$d \langle (\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k)_{.l}, \eta' \rangle dt = (P_{.l}(PH')_{.k} + P_{.k}(PH')_{.l} + (HP)_{.k}P_{.l} + (HP)_{.l}P_{.k}) dt \tag{A.4.29}$$

Hence

$$\begin{aligned}
 d\Pi(k)_{.l} &= \dot{f}_3(k)_{.l} dt + \left[(P_{.l}(PH')_{.k} + P_{.k}(PH')_{.l} + (HP)_{.k}P_{.l} \right. \\
 &\quad \left. + (HP)_{.l}P_{.k}) + \Lambda_M H' \right] R^{-1} d\nu_l
 \end{aligned} \tag{A.4.30}$$

where

$$\Lambda_M = \overbrace{(\tilde{\zeta}\tilde{\zeta}'\tilde{\zeta}_k)_{.l} \tilde{\zeta}'} \tag{A.4.31}$$

PROPOSITION 5. This proposition follows directly from a decomposition of costs like that performed in [Tse (1971)]. Note that if $\tau \geq t$

$$\begin{aligned} E \{ \zeta_r' M \zeta_r \mid \underline{Y}_t \} &= E \left\{ E \{ \zeta_r' M \zeta_r \mid \underline{Y}_r \} \mid \underline{Y}_t \right\} \\ &= E \left\{ E \{ \tilde{\zeta}_r' M \tilde{\zeta}_r \mid \underline{Y}_r \} \mid \underline{Y}_t \right\} + E \{ \hat{\zeta}_r' M \hat{\zeta}_r \mid \underline{Y}_t \} \\ &= E \{ \hat{\zeta}_r' M \hat{\zeta}_r + T_r M P_r \mid \underline{Y}_t \} \end{aligned} \quad (\text{A.5.1})$$

Hence

$$J_t = E \left\{ \int_t^T (\hat{\zeta}_r' M \hat{\zeta}_r + u_r' N u_r) d\tau \mid \underline{Y}_t \right\} + E \left\{ \int_t^T M P_r d\tau \mid \underline{Y}_t \right\} \quad (\text{A.5.2})$$

Equation (A.3.8) indicates that $\{P_r\}$ is independent of the control policy. Consequently, $\{u_r\}$ should be selected to minimize the first term in (A.5.2).

Some care must be exercised in the indicated minimization because the dynamics of $\hat{\zeta}$ are not linear, e.g. $\{P_t\}$ depends upon $\{\nu_t\}$ in (A.2.2). Define

$$H_t = E \left\{ \int_t^T (\hat{\zeta}_r' M \hat{\zeta}_r + u_r' N u_r) d\tau \mid \underline{Y}_t \right\} \quad (\text{A.5.3})$$

$$H_t = (\hat{\zeta}_t' M \hat{\zeta}_t + u_t' N u_t) dt + dH_t \quad (\text{A.5.4})$$

Suppose that $\{H_t\}$ has the form

$$H_t = \hat{\zeta}_t' \Sigma_t \hat{\zeta}_t + s_t \quad (\text{A.5.4})$$

with $\{\Sigma_t, s_t\}$ satisfying the usual assumptions

$$\begin{aligned} dH_t &= 2 \hat{\zeta}_t' \Sigma_t d\hat{\zeta}_t + \hat{\zeta}_t' \dot{\Sigma}_t \hat{\zeta}_t + \dot{s}_t + T_r \Sigma_t (d\hat{\zeta}_t) (d\hat{\zeta}_t)' \\ &= 2 \hat{\zeta}_t' \Sigma_t ((F_\zeta \hat{\zeta}_t + G u_t) dt + P_t H' R^{-1} d\nu_t) + (\hat{\zeta}_t' \dot{\Sigma}_t \hat{\zeta}_t + \dot{s}_t) dt \end{aligned}$$

$$\begin{aligned}
 & + \text{Tr } \Sigma_t P_t H' R^{-1} H P_t \\
 & = (2 \dot{\hat{\zeta}}' (\Sigma_t F_{\zeta} + \dot{\Sigma}_t) \hat{\zeta}_t + 2 \dot{\hat{\zeta}}' \Sigma_t G_t u_t + \dot{s}_t + \text{Tr } \Sigma_t P_t H' R^{-1} H P_t) dt \\
 & + 2 \dot{\hat{\zeta}}' \Sigma_t P_t H' d\nu_t
 \end{aligned} \tag{A.5.6}$$

But the last term in (A.5.6) is a martingale increment, and the representation of $\{H_t\}$ is in the form developed in [Tse, 1971]. The solution to the regulation problem is well known:

$$u_t = -N^{-1} B' \Sigma_t \tilde{\zeta}_t \tag{A.5.7}$$

where

$$\dot{\Sigma}_t = -F_{\zeta}' \Sigma_t - \Sigma_t F_{\zeta} + \Sigma_t B R^{-1} B' \Sigma_t - M \tag{A.5.8}$$

$$\Sigma_T = 0$$

PROPOSITION 6. Specializing (A.2.2) to the $\{\phi_t\}$ subsystem, it follows that

$$d\hat{\phi} = Q' \hat{\phi} dt + P_{\phi} b R_{\phi}^{-1} d\nu_{\phi} \tag{A.6.1}$$

where

$$d\nu_{\phi} = dy_{\phi} - b \hat{\phi} dt \tag{A.6.2}$$

But

$$\begin{aligned}
 P_{\phi} & = E \{ \phi \phi' - \underline{Y}_{\phi,t} \} - \hat{\phi} \hat{\phi}' \\
 & = \text{diag} \{ \hat{\phi}_1, \dots, \hat{\phi}_N \} - \hat{\phi} \hat{\phi}'
 \end{aligned} \tag{A.6.3}$$

Define

$$\tilde{B} = \text{diag} (b_1, \dots, b_N) \tag{A.6.4}$$

$$\hat{b} = \hat{\phi}' b$$

Then

$$d\hat{\phi}_t = Q' \hat{\phi}_t dt + (\tilde{B}_t - \hat{b}_t I) \hat{\phi}_t R_{\phi}^{-1} d\nu_{\phi} \tag{A.6.5}$$

**END
DATE
FILMED**

7-27-87

E.A