AD-A184 493

NAVAL OCEAN SYSTEMS CENTER, SAN DIEGO, CA
POROSITY INFLUENCE ON THE STRENGTH AND ELASTICITY
OF FIRST-YEAR SEA ICE BY: JH BROWN

1 OF 1
NOSC TD 1109
UNCLASSIFIED
JULY 87

END
DATE
FILMED

NOSC TD 1109

NOSC
NAVAL OCEAN SYSTEMS CENTER San Diego, California 92152-5000

Technical Document 1109
July 1987

Porosity Influence on the Strength and Elasticity of First-Year Sea Ice

J. H. Brown
Integrated Systems Analysts, Inc.

NOSC TD 1109

NAVAL OCEAN SYSTEMS CENTER
San Diego, California 92152-5000

E. G. SCHWEIZER, CAPT, USN
Commander

R. M. HILLYER
Technical Director

ADMINISTRATIVE INFORMATION

This work was performed for the Naval Sea Systems Command, Washington, DC 20362-5101, under program element 63522N. Contract N66001-86-MB-416 was carried out by Integrated Systems Analysts, Inc., Marina Gateway, 740 Bay Boulevard, Chula Vista, CA 92010-5254, under the direction of M.A. Beal, Code 1901 of Naval Ocean Systems Center, San Diego, CA 92152-5000.

Released by
M.A. Beal
Deputy Director,
Arctic Submarine Laboratory

Under authority of
M.H. Dorman, CAPT, USN
Director, Arctic
Submarine Laboratory

MA

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b RESTRICTIVE MARKINGS:

2a SECURITY CLASSIFICATION AUTHORITY:

2b DECLASSIFICATION/DOWNGRADING SCHEDULE:

3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited.

4 PERFORMING ORGANIZATION REPORT NUMBER(S):

5 MONITORING ORGANIZATION REPORT NUMBER(S): NOSC TD 1109

6a NAME OF PERFORMING ORGANIZATION: Integrated Systems Analysts, Inc.

6b OFFICE SYMBOL (if applicable):

7a NAME OF MONITORING ORGANIZATION: Naval Ocean Systems Center

6c ADDRESS (City, State and ZIP Code):
Marina Gateway
740 Bay Boulevard
Chula Vista, CA 92010-5254

7b ADDRESS (City, State and ZIP Code):
Arctic Submarine Laboratory
San Diego, CA 92152-5000

8a NAME OF FUNDING SPONSORING ORGANIZATION: Naval Sea Systems Command

8b OFFICE SYMBOL (if applicable): NSEA-63D

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N66001-86-MB-416

8c ADDRESS (City, State and ZIP Code): Washington, DC 20362-5101

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	AGENCY ACCESSION NO
63522N	S1739	190-MR01	DN234 790

11 TITLE (include Security Classification): Porosity Influence on the Strength and Elasticity of First-Year Sea Ice

12 PERSONAL AUTHOR(S): J.H. Brown

13a TYPE OF REPORT: Final

13b TIME COVERED: FROM Jun 86 TO Sep 86

14 DATE OF REPORT (Year, Month, Day): July 1987

15 PAGE COUNT: 18

16 SUPPLEMENTARY NOTATION:

17 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number):
sea ice
salinity
elasticity
ice platelets
porosity
brine layers
flexural strength

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This review of theoretical models of elasticity and flexural strength of sea ice is to establish a baseline for future efforts in experimental and theoretical work in these areas. For example, brine content is not the total porosity of sea ice. Total porosity of sea ice is brine plus gas content. Other factors which might influence the models and should be considered in any theoretical models for sea ice strength and elasticity are:

- Visco-elastic and plastic effects along the failure plane as well as slippage.
- The distribution and geometry of the pores.
- Gas porosity must be taken into consideration.
- Consider other configurations for theoretical models.
- Surface tension effects on the cohesion of the ice platelets.
- Effects of solid salts on cold ice.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL: M.A. Beal

22b TELEPHONE (include Area Code): 619-225-6851

22c OFFICE SYMBOL: Code 1901

DD FORM 1473, 84 JAN

83 APR EDITION MAY BE USED UNTIL EXHAUSTED
ALL OTHER EDITIONS ARE OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

DD FORM 1473, 84 JAN

ASL-3

POROSITY INFLUENCE ON THE STRENGTH AND ELASTICITY OF FIRST-YEAR SEA ICE

Report for
Arctic Submarine Laboratory
Code 19

Prepared by
J. H. Brown

September 1986

Prepared for
Naval Ocean Systems Center
San Diego, California 92152-5000

Contract P.O. No. N66001-86-MB-416

Integrated Systems Analysts, Inc.
Marina Gateway
740 Bay Boulevard
Chula Vista, California 92010-5254

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1.0 INTRODUCTION

At the present time, there is no generally acceptable model of sea ice based upon the physics of the material. Two models have been proposed in the past, and both models fall short of adequately explaining the strength or elasticity of sea ice.

2.0 MODELS OF SEA ICE

2.1 STRENGTH MODEL

The first model was developed by Anderson and Weeks {1}. This model was based upon the experimental observations of Weeks and Anderson {2}. From their observations they deduced that the salinity and temperature were the controlling parameters for a sea ice strength model. Their experimental observations showed them that brine was trapped between crystal boundaries and subcrystalline platelets in vertical layers. With the nucleation and formation of sea ice, and then with further decrease in temperature, the brine layers start to decrease in cross-sectional area. Initially, with decrease in temperature, the brine layers start to neck, then form vertical cylinders, and finally separate into elliptical cylinders and circular cylinders. This sequence of events with the sea ice brine layers is shown in Figure 1.

Anderson and Weeks reasoned that if

$$\beta_e = d/l_o \tag{1}$$

where β_e is brine in a unit area, d is the diameter of the brine cylinders, and l_o is the separation of cylinders in the failure plane (see Figure 1), then the strength of sea ice σ will be a function having the following form

$$\sigma \sim \sigma_p(1 - \beta_e) \tag{2}$$

where σ_p is the strength of pure ice.

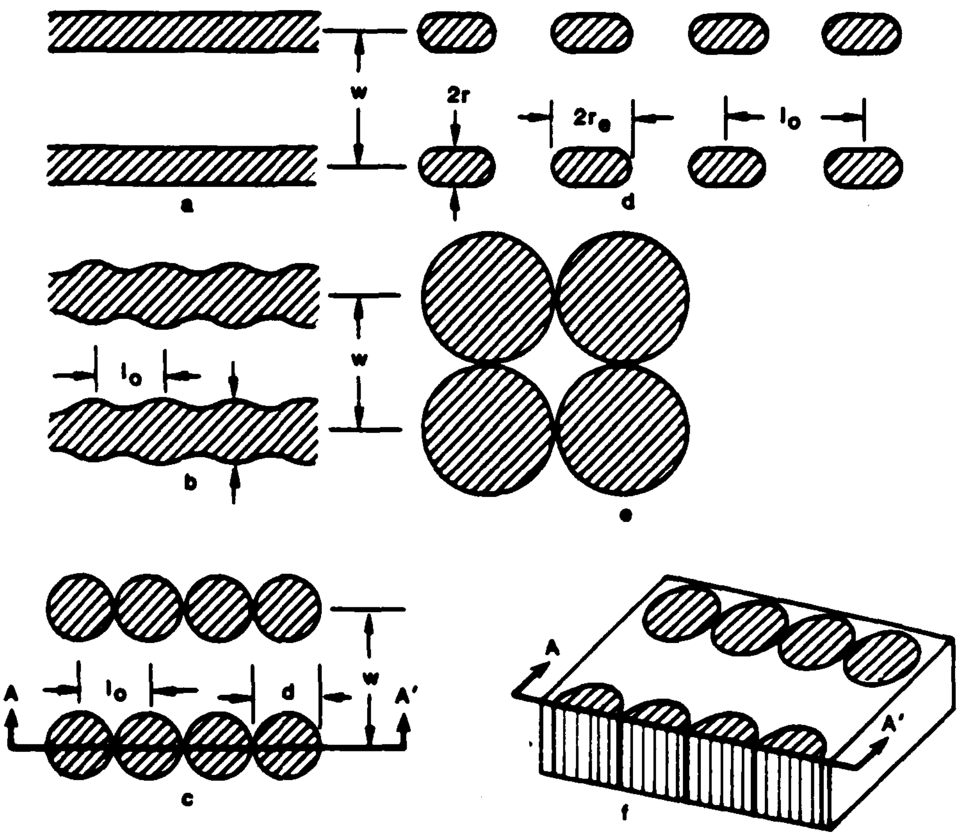

Figure 1. Schematic diagrams showing how a brine layer splits into cylinders. (From: Anderson and Weeks', see reference {1})

The stress concentration factor k is defined as the maximum stress to the average stress in the minimum cross section {3, 4}.

$$k = \frac{\sigma_p}{\sigma/(1 - \beta_e)} \tag{3}$$

or for the sea ice strength we will have

$$\sigma = \sigma_p(1 - \beta_e)/k \quad . \tag{4}$$

Next Anderson and Weeks considered round cylinders and the geometrical properties of sea ice to develop the following relation for the brine volume β, i.e., the volume of the brine to the total volume

$$\beta = \pi\beta_e^2/4a \tag{5}$$

where $W = l_o a$ and W is the platelet width. Hence, from Eqs. (3) and (4) the relation for sea ice strength can be written as

$$\sigma = \frac{\sigma_p}{k}\left[1 - \left(\frac{4a\beta}{\pi}\right)^{1/2}\right] \tag{6}$$

where a is a function of brine pocket separation. This approach considered the sea ice strength to be a function only of brine volume, brine pocket separation, pure water ice strength, and a nondefined stress concentration factor. Brown {5} shows a comparison of his experimental data, shown in Figure 2, with the Anderson and Weeks' model. In Figure 2, σ_p is the flexural strength of pure ice and k is the stress concentration factor. The comparison shows there is considerable room for improvement.

2.2 ELASTICITY AND STRENGTH MODEL

Following Anderson and Weeks' work, Brown {5} considered a perforated plate model as is shown in Figure 3 for a sea ice model. The plate is infinite in extent, and the extensions in x and y are uniform, then any stresses produced around any one hole are harmonic with respect to those of the other holes. For

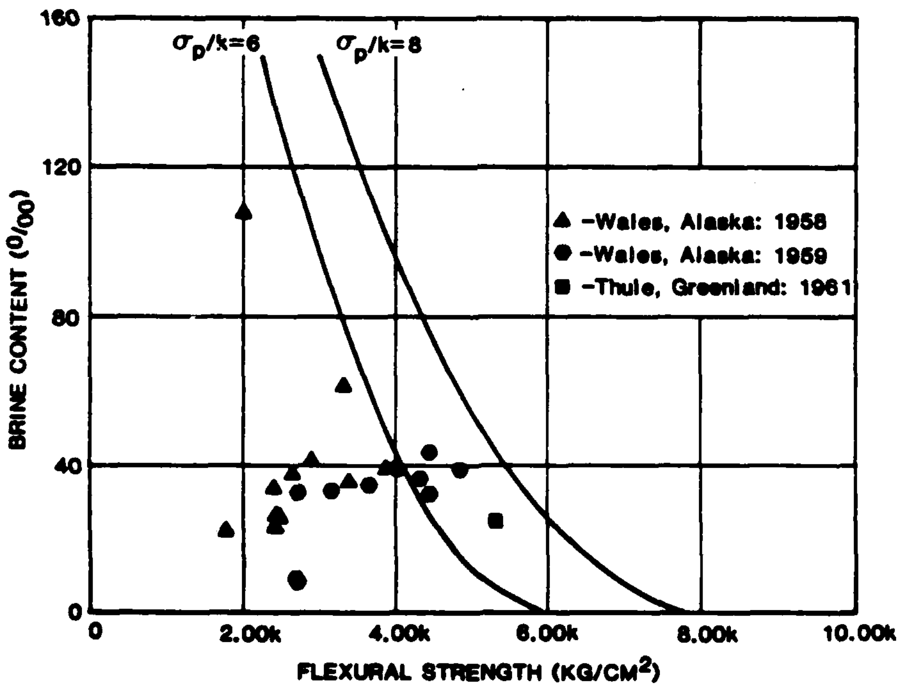

Figure 2. Comparison of experimental flexural strength versus brine content in low-temperature layer with the Anderson and Weeks' theoretical model.

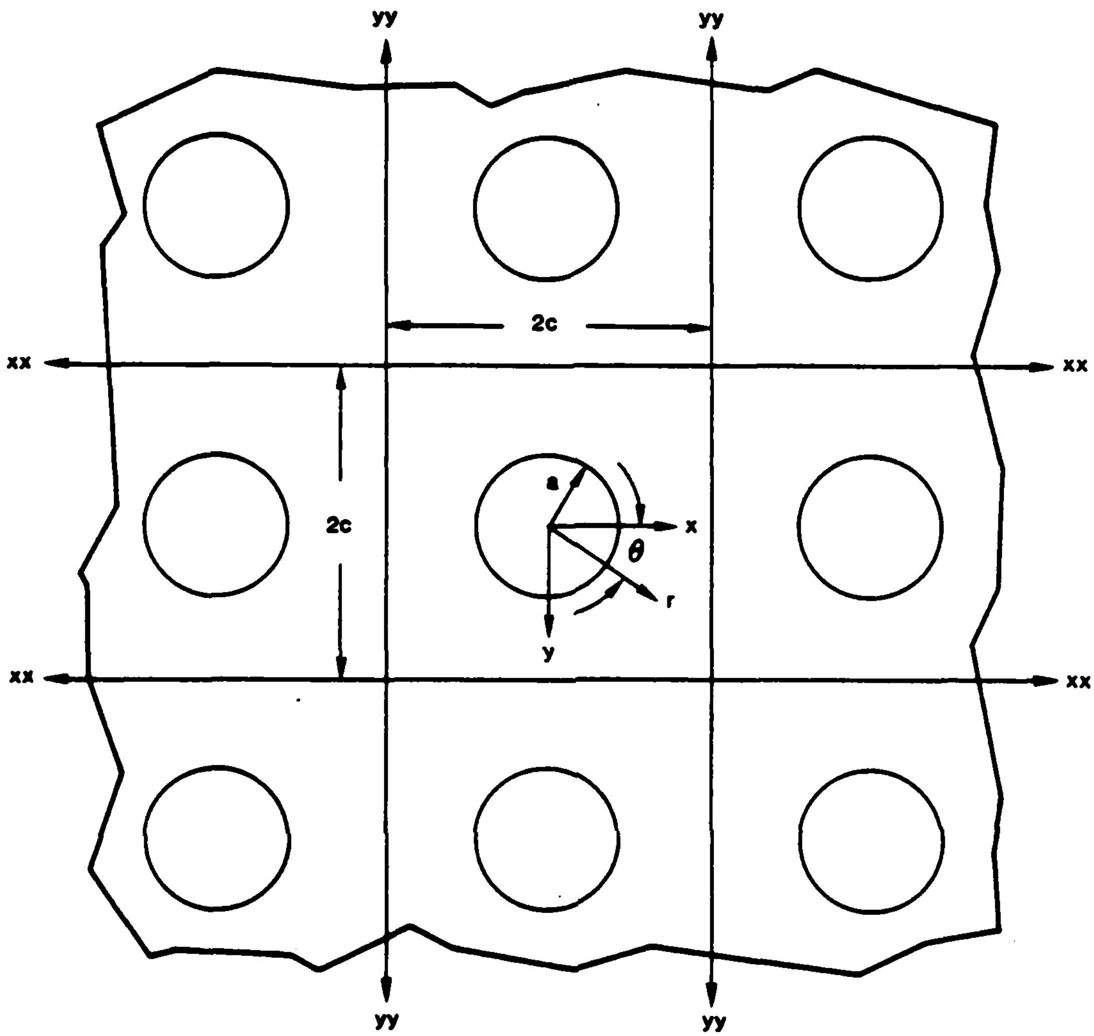

Figure 3. Plan view of a section of a perforated plate with a square array of holes. [From: Bailey, R. and R. Hicks, J. Mech. Eng. Sci., 2(2), 143 (1960).]

analysis, one need consider only one square element with the coordinates originating at the center of the hole. In approximating a perforated plate, it is assumed the internal properties of sea ice have brine cylinders in which the diameters are very small as compared to the lengths of the cylinders. In addition, the lengths of the cylinders are normal to the plane of the sea ice sheet, and the cylinders at different levels of the ice sheet add to become perforations of uniform extent throughout the ice sheet. Hence, we assume Bailey and Hicks {6} perforated plate theory is valid for sea ice. From Bailey and Hicks' theory, let's consider the perforated plate as shown in Figure 3 to be in tension. Then from Bailey and Hicks' theory, we have the following for Young's modulus E and Poisson's ratio ν:

$$\frac{E}{E'} = \frac{2\Sigma_1\Sigma_2}{\Sigma_1 + \Sigma_2} \tag{7}$$

and

$$\nu = \frac{\Sigma_1 - \Sigma_2}{\Sigma_1 + \Sigma_2} \tag{8}$$

where E' refers to Young's modulus of a material without any holes (pure ice in this case), and where

$$\Sigma_1 = \frac{\overline{\widehat{xx}}}{\overline{EU_x}/c} \text{ ; for } n = 0, 4, 8, \cdots \tag{9}$$

$$\Sigma_2 = \frac{\overline{\widehat{xx}}}{\overline{EU_x}/c} \text{ ; for } n = 2, 6, 10, \cdots \tag{10}$$

where (all symbols are defined in the appendix)

$$\begin{aligned}
\widehat{xx} = 2B_0 &\left[1 - \left(\frac{a}{r}\right)^2 \cos 2\theta\right] \\
&+ \sum_{n=2}^{\infty} (n+1)B'_n \left\{\left(\frac{r}{a}\right)^n \left[2 \cos n\theta - n \cos (n-2)\theta\right]\right. \\
&\left. + (n-1)\left(\frac{r}{a}\right)^{n-2} \cos (n-2)\theta - \left(\frac{a}{r}\right)^{n+2} \cos (n+2)\theta\right\} \\
&+ \sum_{n=2}^{\infty} (n-1)B_n \left\{-\left(\frac{a}{r}\right)^n \left[2 \cos n\theta + n \cos (n+2)\theta\right]\right. \\
&\left. + (n+1)\left(\frac{a}{r}\right)^{n+2} \cos (n+2)\theta + \left(\frac{r}{a}\right)^{n-2} \cos (n-2)\theta\right\}
\end{aligned} \tag{11}$$

and

$$\frac{EU_x}{c} = 2B_0 \left[(1 + \nu)\frac{a}{r} + (1 - \nu)\frac{r}{a}\right] \cos \theta$$

$$- \sum_{n=2}^{\infty} B'_n \left\{(1 + \nu)(n + 1)\left(\frac{r}{a} - \frac{a}{r}\right)\left(\frac{r}{a}\right)^n \cos (n - 1)\theta\right.$$

$$\left. - \left[(3 - \nu)\left(\frac{r}{a}\right)^{n+1} \quad (1 + \nu)\left(\frac{a}{r}\right)^{n+1}\right] \cos (n + 1)\theta\right\}$$

$$+ \sum_{n=2}^{\infty} B_n \left\{(1 + \nu)(n - 1)\left(\frac{r}{a} - \frac{a}{r}\right)\left(\frac{a}{r}\right)^n \cos (n + 1)\theta\right.$$

$$\left. + \left[(3 - \nu)\left(\frac{a}{r}\right)^{n-1} + (1 + \nu)\left(\frac{r}{a}\right)^{n-1}\right] \cos (n - 1)\theta\right\} \qquad (12)$$

Consider seismic waves being propagated through an Arctic Ocean sea ice sheet. Then from the plate wave velocity V_p and the transverse wave velocity V_t, we can obtain Young's modulus E, shear modulus μ, and Poisson's ratio ν for the ice by use of the following relations {7}:

$$E = \rho V_p^2(1 - \nu^2) \qquad (13)$$

$$\mu = \rho V_t^2 = \frac{E}{2(1 + \nu)} \qquad (14)$$

$$\nu = 1 - 2\left(\frac{V_t}{V_p}\right)^2 \qquad (15)$$

where ρ is the sea ice density. Now from Eq. 13, the relative plate wave velocity becomes

$$\frac{V_p}{V'_p} = \left[\frac{E\rho'(1 - \nu'^2)}{E'\rho(1 - \nu^2)}\right]^{1/2} \qquad (16)$$

where the primes refer to a material without holes. Assume, as Anderson and Weeks did, that sea ice failure in tension occurs across a row of brine cylinders that are in the minimum cross-sectional plane (the ligament between cylinders) passing through the circular cylinders normal to the applied stress. If the failure stress is the average stress across the ligaments at failure and if that failure occurs with an elastic break (i.e., on the linear portion of the deformation curve), then from Bailey and Hicks {6} we have the ratio of the average stress across the perforated plate $\bar{\sigma}$ to the stress of a plate without any holes σ' as given by

$$\frac{\bar{\sigma}}{\sigma'} = \bar{k} = \frac{1}{2}\left(\frac{\overline{\widehat{\theta\theta}}_1/EU_x/c}{\Sigma_1} + \frac{\overline{\widehat{\theta\theta}}_2/EU_x/c}{\Sigma_2}\right) \tag{17}$$

in which $\bar{k}$ is the average stress concentration across the ligament and, with an angle of $\theta = \pi/2$, $\widehat{\theta\theta}$ becomes

$$\begin{aligned} \widehat{\theta\theta} = -A_0\left(\frac{a}{r}\right)^2 + 2B_0 + \sum_{n=2}^{\infty} \Bigg\{ & n(n-1)A_n'\left(\frac{r}{a}\right)^{n-2} \\ & + (n+2)(n+1)B_n'\left(\frac{r}{a}\right)^{n} + n(n+1)A_n\left(\frac{a}{r}\right)^{n+2} \\ & + (n-2)(n-1)B_n\left(\frac{a}{r}\right)^{n}\Bigg\} \cos n\theta \end{aligned} \tag{18}$$

and wherein $\overline{\widehat{\theta\theta}}_1/(EU_x/c)$ has values of $n = 0, 4, 8, \cdots$ and $\overline{\widehat{\theta\theta}}_2/(EU_x/c)$ has values of $n = 2, 6, 10, \cdots$.

Bailey and Hicks have calculated the coefficients for Eqs. 11, 12, and 17 and have graphed most of the functions. The bars over several of the functions indicate averaging which has been done along the boundaries and ligaments of the unit plate.

From Eqs. 16 and 17 it is possible to determine the relative plate wave velocities and the relative flexural strengths with the conditions assumed here.

A comparison of the perforated plate theory with experimental field data of relative plate wave velocity (V_p/V'_p), for elasticity, is shown in Figure 4. It should be noted that the ratio c/a is the ratio of one-half the unit cell side to the radius of the brine cell. Figure 4 shows the comparison between theory and experimental data. While the comparison is not good, it does show that elasticity is a function of brine content.

A comparison of the perforated plate theory with the flexural strength of sea ice is shown in Figure 5. Again, the comparison indicates that flexural strength is a function of brine content; however, there must be other factors influencing the flexural strength.

A comparison of theory and experimental values of plate wave velocity with flexural strength is shown in Figure 6. The comparison between theory and experiment appears to be relatively good for geophysical data.

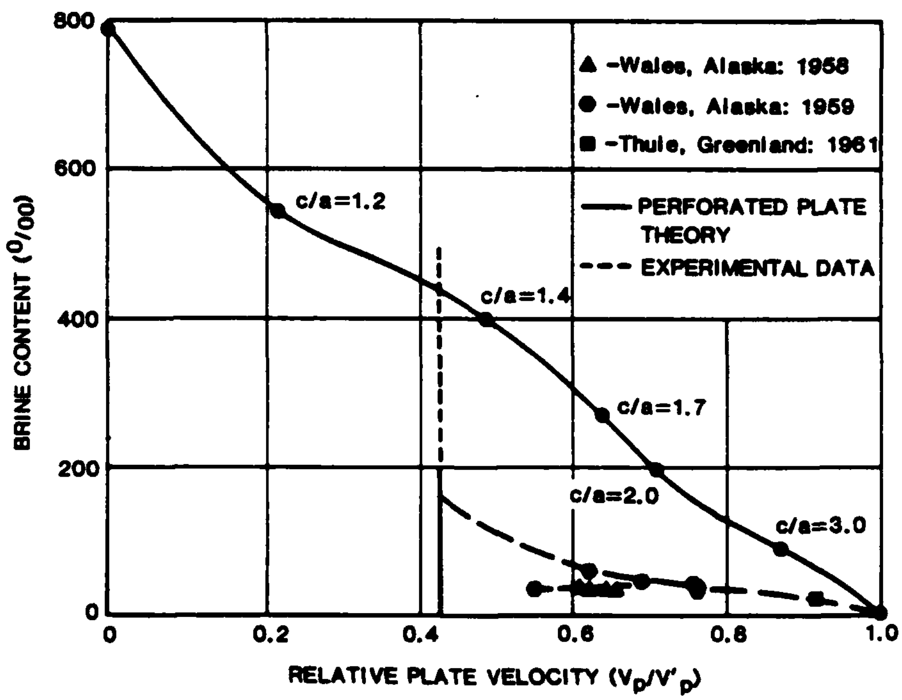

Figure 4. Comparison of experimental data with perforated plate theory for brine content versus the relative plate wave velocity, with V'_p = 3,410 m/sec.

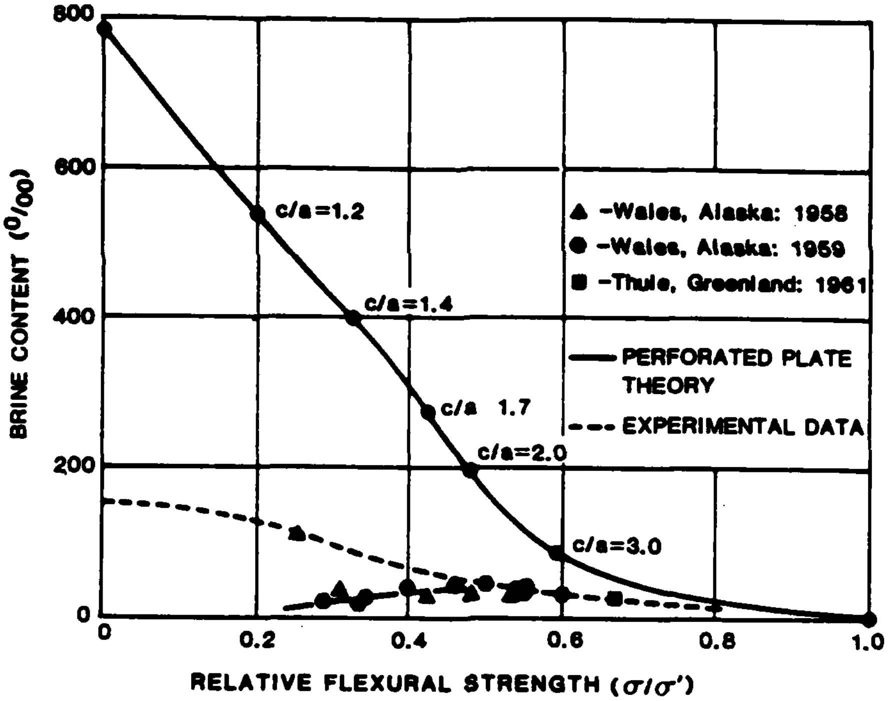

Figure 5. Comparison of experimental data with perforated plate theory for brine content versus the inverse of the relative flexural strength, with σ' = 8.0k kg/cm^2.

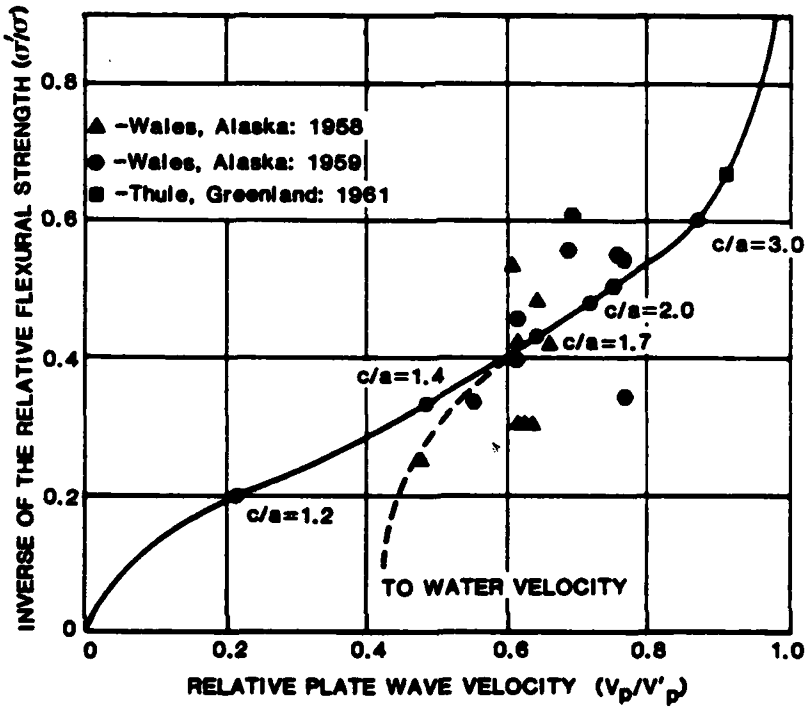

Figure 6. A comparison of plate wave velocity versus flexural strength from theory of perforated plates, with the experimental values, letting V_p' = 3,410 m/sec and σ' = 8.0k kg/cm^2.

3.0 SUMMARY

This review of theoretical models of elasticity and flexural strength of sea ice is to establish a baseline for future efforts in experimental and theoretical work in these areas. For example, brine content is not the total porosity of sea ice. Total porosity of sea ice is brine plus gas content. Other factors which might influence the models and should be considered in any theoretical models for sea ice strength and elasticity are:

1. Visco-elastic and plastic effects along the failure plane as well as slippage.
2. The distribution and geometry of the pores.
3. Gas porosity must be taken into consideration.
4. Consider other configurations for theoretical models.
5. Surface tension effects on the cohesion of the ice platelets.
6. Effects of solid salts on cold ice.

4.0 REFERENCES

{1} Anderson, D. L. and W. F. Weeks, "A theoretical analysis of sea-ice strength," Trans. Amer. Geophys. Union 39, 632-640, 1958.

{2} Weeks, W. F. and D. L. Anderson, "An experimental study of strength of young sea ice," Trans. Amer. Geophys. Union 39, 641-647, 1958.

{3} Timoshenko, S., "Strength of Materials, Pt. II," p. 307, Van Nostrand, Princeton, New Jersey, third edition, 1956.

{4} Timoshenko, S. P. and J. N. Goodier, "Theory of elasticity," p. 93, McGraw-Hill Book Co., New York, third edition, 1951.

{5} Brown, J. H., "Ice and snow," editor W. D. Kingery, p. 79-106, The M.I.T. Press, Cambridge, Mass., 1963.

{6} Bailey, R. and R. Hicks, "Behavior of perforated plates under plane stress," J. Mech. Eng. Sci., 2(2), 143, 1960.

{7} Ewing, M., A. Crary and A. Thorne, Jr., "Propagation of elastic waves in ice," Pt. I. Physics, 5, 165, 1934.

APPENDIX

DEFINITION OF SYMBOLS

A_n, B_n, etc	Constants in stress function
α	Radius of hole
c	Pitch of hole
E	Young's modulus
E_E	Effective Young's modulus
r, θ	Polar coordinates
$\widehat{rr}$, $\widehat{\theta\theta}$, $\widehat{r\theta}$	Stress components in polar coordinates
U'_x, V'_y	Displacements in cartesian coordinates for a plate with a diagonal system of holes
x, y	Cartesian coordinates
$\widehat{xx}$, $\widehat{yy}$, $\widehat{xy}$	Stress components in cartesian coordinates
ν	Poisson's ratio
$\Sigma_1 \ldots \Sigma_n$	Numerical integrations as defined

END
DATE
FILMED

8-12-87