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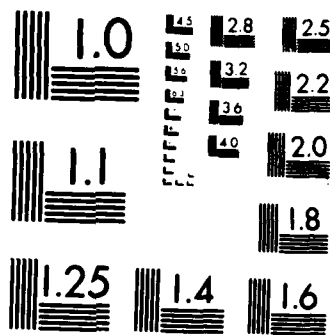
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An Exact Analysis and Performance Evaluation of Framed ALOHA with Capture

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AN EXACT ANALYSIS AND PERFORMANCE EVALUATION OF FRAMED ALOHA WITH CAPTURE

1. INTRODUCTION

We consider the problem of channel random access in a system characterized by a slotted frame structure, as shown in Figure 1. As in typical slotted systems, the slot duration is equal to the length of a packet and all packet transmissions start at the beginning of a time slot. There is no coordination among users; thus, there is the possibility of collision, i.e., the transmission of two or more packets in a slot. In this paper we present an exact analysis and performance evaluation of the Framed ALOHA protocol* for the case of a finite number of users. A new combinatorial technique, first introduced in [5], is used for this analysis. This technique permits the analysis of both uncontrolled and dynamically controlled versions of this protocol. It is initially assumed that all collisions result in the destruction of all packets that are involved, but in Section 4 we incorporate into our analysis a general model for capture, in which one packet may be successfully received despite the transmission by other users in the same slot.

2. SYSTEM MODEL

The system consists of M unbuffered users, each of which can hold at most one packet at a time. Typically, the arrival process is bursty. All newly generated packets are transmitted in the frame in which they are generated, except for packets generated in the last slot of a frame which are transmitted in the next frame. It is assumed that acknowledgment information is available before the end of the current frame and without overhead penalty. A packet suffering a collision will be retransmitted in a subsequent frame; we initially assume that all such packets are retransmitted in the next frame with probability one, but later we address the case in which these packets are transmitted in the next frame with probability X that can be chosen to increase throughput. A user whose packet has suffered a collision is known as a *blocked* user. The precise retransmission strategy and the arrival process description are discussed later on. Now we define:

* Framed ALOHA [1,2] and similar schemes [3,4] have been studied by several investigators.
Manuscript approved June 19, 1987.

- T_i = number of packets that are transmitted in frame i ,
 = number of users transmitting in frame i ,
 L_i = length of frame i (in slots),
 C_i = number of packets that suffer collisions in frame i ,
 S_i = number of successful transmissions in frame i ,
 A_i = number of new arrivals in frame i .*
 ϕ = probability that an arbitrary non-blocked user generates a new arrival
 sometime in frame.

Note that

$$T_i = C_{i-1} + A_i = S_i + C_i. \quad (1)$$

The colliding packet process C_i , also known as the *backlog process*, is of fundamental importance in the modeling of system dynamics, as we shall discuss in Section 5.

Capture in multiple access systems [6-8] is the ability to receive one packet correctly, despite the presence of other signals transmitted simultaneously over the same channel. A system mathematically equivalent to Framed ALOHA without capture was investigated in [4]. Framed ALOHA with perfect capture (i.e., the case in which one packet is successful in every slot chosen by one or more users) is straightforward to analyze, as will be demonstrated in Section 4. However, for non-perfect capture the analysis is complicated. The combinatorial technique that we propose is useful for analyzing more general capture models in which the probability of one user capturing a slot is dependent on the number of other users transmitting in that slot. Thus, although our mathematical formulation is perhaps more powerful than is actually necessary for the non-capture and perfect capture cases, it permits the analysis of general models for capture, as well. Furthermore, it can handle the cases of uncontrolled and controlled versions of the protocol.

* A_i is actually the number of new arrivals that are transmitted in frame i , and it includes packets that arrive during the last slot of frame $(i-1)$, but not those that arrive in the last slot of frame i .

3. THE BASIC ANALYSIS TECHNIQUE—NO CAPTURE

We begin by considering a constant frame length L , although later we will consider schemes in which the frame length varies in response to channel traffic. We can for the moment neglect consideration of the arrival process and examine the distribution of the number of successful and colliding packets (S_i and C_i respectively), given the total number of packets, T_i , transmitted in the frame. We do note, however, that since the users are unbuffered the arrival process at a user must be turned off whenever a new packet is generated. We assume it is turned back on in the frame following that in which the user's packet is successfully transmitted. Thus, each user transmits at most one packet in any frame. We assume that each of the T_i packets transmitted in frame i is equally likely to be transmitted in any of the L slots of the frame.

If each user were allowed to transmit more than a single packet in one frame, then the number of packets transmitted would be greater than the number of users transmitting in that frame. Clearly, the transmissions of any given user would not be independent, because he would not transmit two or more packets in the same slot. Our analysis here, based on the simpler model of each user transmitting at most a single packet in each frame, provides a lower bound for the throughput of the other case just discussed if T_i is the same in both cases. In this sense our analysis here, exact for the case of at most a single packet transmitted per frame per user, is also applicable, albeit pessimistic, for the other case.

To simplify notation, for now we suppress the subscripts that represent frame number. These subscripts will be reintroduced when we discuss the dynamic behavior of the system, i.e., the evolution of system behavior from one frame to the next.

We define the *state* of the system in a given frame in which T users transmit as

$$\mathbf{n}_T = (n_T(1), n_T(2), \dots, n_T(T)), \quad (2)$$

where $n_T(j)$ = number of slots in each of which exactly j users transmit. For example, consider the case of $T = 5$. The state $\mathbf{n}_T = (3, 1, 0, 0, 0)$ corresponds to a realization in which one slot is occupied by two users and three slots are singly occupied.

In general, the states \mathbf{n}_T that are realizable must satisfy two physical constraints. First, since there are a total of T packets in the frame we must have, for any state \mathbf{n}_T ,

$$\sum_{j=1}^T j n_T(j) = T. \quad (3)$$

Also, the total number of occupied slots, which we denote by m , can not be greater than either the frame size (L) or the number of users transmitting in the frame (T), i.e.,

$$m = \sum_{j=1}^T n_T(j) \leq \min(L, T). \quad (4)$$

We want to determine $P(\mathbf{n}_T)$, which is the conditional probability distribution for the states, for a given T . This distribution is necessary for the study of the system dynamics, because the throughput is directly related to some of the components of the state \mathbf{n}_T . For example, in the present case of no capture the number of packets transmitted successfully in a frame is simply given by

$$S = n_T(1), \quad (5)$$

and the number of colliding packets by

$$C = T - n_T(1). \quad (6)$$

In addition to the evaluation of $P(\mathbf{n}_T)$ for any given state, we must be able to enumerate the states \mathbf{n}_T that can occur for any particular values of T and L , i.e., all states that satisfy eqs. (3) and (4), in order to implement the calculations that will be presented later on. The detailed state description and enumeration are not strictly necessary when dealing with either noncapture or perfect capture. They are needed, however, to analyze general capture models. We first present a procedure for the enumeration of states, and we then present the derivation for the probability distribution for the states.

The Enumeration of States

The enumeration of the states \mathbf{n}_T that can occur for a fixed value of L (frame length) and for any particular value of T , i.e., all states for which eqs. (3) and (4) are satisfied, is equivalent to the construction of Young's lattice [9]. One approach is to proceed iteratively as follows:

Assume we know all states for a given value of T . (For example, we may start with the trivial case of $T = 1$, for which the only possible state is $\mathbf{n}_T = (1)$). For each state \mathbf{n}_T that is consistent with the presence of T users transmitting in the frame, we determine the states \mathbf{n}_T that can be generated as one additional user is added to the system. To do so we first consider the case in which the new $(T+1)^{\text{th}}$ user has chosen a slot not previously chosen by any of the first T users. The number of singly occupied

slots thus increases by 1 while the number of slots containing i users ($i = 2, 3, \dots, T$) remains unchanged. The new states are thus generated by the following procedure:

for each state \mathbf{n}_T , set

$$\begin{aligned} n_{T+1}(1) &= n_T(1) + 1, \\ n_{T+1}(i) &= n_T(i), \quad i = 2, 3, \dots, T. \end{aligned} \quad (7)$$

We now consider the case in which the new $(T+1)^{\text{th}}$ user has chosen one of the slots already chosen by one of the first T users. If that slot already contained i users, then it would now contain $i+1$ users, thus decrementing the number containing i users by 1 while incrementing the number containing $i+1$ by 1. The new states are thus generated by the following procedure:

for each state \mathbf{n}_T and every $n_T(i) \neq 0$, set

$$\begin{aligned} n_{T+1}(i) &= n_T(i) - 1 \\ n_{T+1}(i+1) &= n_T(i+1) + 1. \end{aligned} \quad (8)$$

Note that duplicate identical states are generated by this process, because two different \mathbf{n}_T states can evolve to the same \mathbf{n}_{T+1} successor state. Such duplicate states are easily recognized and thinned out in this iterative procedure. Also with this procedure, states for which the number of occupied slots (m) is greater than L may be generated. Such states are discarded.

As an example, we show in Table 1 the resulting states for $T < 5$ (and $L > T$). The number of states increases dramatically with increasing T , as shown in Table 2.

Table 1 — Enumeration of states for $T < 5$ and $L > T$.

$T = 1:$	(1)
$T = 2:$	(2,0), (0,1)
$T = 3:$	(3,0,0), (1,1,0), (0,0,1)
$T = 4:$	(4,0,0,0), (2,1,0,0), (0,2,0,0), (1,0,1,0), (0,0,0,1)
$T = 5:$	(5,0,0,0,0), (3,1,0,0,0), (1,2,0,0,0), (2,0,1,0,0), (0,1,1,0,0), (1,0,0,1,0), (0,0,0,0,1)

Table 2 — Number of states for $L = 10$ slots, and several values of T .

T	number of states
10	42
20	530
25	1455
30	3590
32	5013
40	16928
45	33401
50	62740
60	195491
65	327748

An alternative method of enumerating the states is to perform a search for each value of T . This method has the distinct advantage of not requiring the storage of the list of states and their probabilities. Later in this section, we indicate how performance can, in fact, be evaluated without the storage of these huge arrays.

The State Probability Distribution: $P(\mathbf{n}_T)$

We now determine the probability distribution $P(\mathbf{n}_T)$. We first observe that we are not interested in *which* of the L slots in a frame are occupied by a specific number of packets, but only in the *number* of such slots. Thus, each state \mathbf{n}_T corresponds to several different specific slot occupancy realizations, all of which are equally likely.

Our approach is to consider T users, each of which places a packet into one of the L slots of the frame; each slot is chosen by each user in equally likely fashion with probability $1/L$. We now consider the sequence in which some subset of the L slots is filled as we examine the T users, which are numbered from 1 to T . We want to realize the state $\mathbf{n}_T = (n_T(1), n_T(2), \dots, n_T(T))$.

There are numerous ways to do so. Let us consider a realization in which the first $n_T(1)$ users all choose different slots; these are followed by $n_T(2)$ pairs of users, such that the two members of each pair choose the same slot as each other, but different from all those previously chosen; these are followed by $n_T(3)$ groups of three users, such that the three members of each triplet choose the same slot as each other, but different from all those previously chosen, etc. We note that $n_T(T)$, and in fact $n_T(j)$ for $j > T/2$, cannot be greater than 1.

As we consider the users numbered from 1 to T , we evaluate the probability that the conditions corresponding to the specific realization of \mathbf{n}_T , which we denote \mathbf{I}_T , are satisfied. We now demonstrate that:

Proposition

$$Pr(\mathbf{I}_T) = \frac{L!}{L^T(L-m)!}, \quad (9)$$

where m is the total number of occupied slots, as defined in eq. (4). We note that the probability of any other specific realization corresponding to the same state \mathbf{n}_T is identical to that which we derive here.

Proof

We first consider the $n_T(1)$ singly occupied slots. The first user chooses a slot at random; thus, the probability that he picks a previously unchosen slot is simply $L/L = 1$. The second user also chooses a slot at random; the probability that he picks a previously unchosen slot is $(L-1)/L$. Continuing in the same manner, user $n_T(1)$ chooses a slot that was not previously chosen with probability $[L-(n_T(1)-1)]/L$. Thus,

$$\begin{aligned} &Pr(\text{first } n_T(1) \text{ users choose different slots}) \\ &= \frac{L}{L} \frac{(L-1)}{L} \frac{(L-2)}{L} \dots \frac{[L-(n_T(1)-1)]}{L}. \end{aligned} \quad (10)$$

Given that the first $n_T(1)$ users have all chosen different slots, we now evaluate the probability that we then have $n_T(2)$ pairs of users that choose the same slot as each other, but different from previously chosen slots. Thus, user $n_T(1)+1$ must choose a slot that is different from that of the first $n_T(1)$ packets, an event that occurs with probability $[L-n_T(1)]/L$. User $n_T(1)+2$ must choose the *same* slot as user $n_T(1)+1$, an event that occurs with probability $1/L$. Similarly, user $n_T(1)+3$ must choose a slot not previously chosen (resulting in a probability of $[L-n_T(1)-1]/L$), and user $n_T(1)+4$ must choose the same slot as user $n_T(1)+3$ (resulting in $1/L$). Continuing in the same manner, we finally obtain

$$\begin{aligned}
Pr(\mathbf{I}_T) &= \left(\frac{L}{L}\right) \left(\frac{L-1}{L}\right) \left(\frac{L-2}{L}\right) \dots \left(\frac{L-[n_T(1)-1]}{L}\right) \\
&\quad \left\langle \text{--- } n_T(1) \text{ factors ---} \right\rangle \\
&\times \left(\frac{L-n_T(1)}{L^2}\right) \left(\frac{L-n_T(1)-1}{L^2}\right) \dots \left(\frac{L-n_T(1)-[n_T(2)-1]}{L^2}\right) \\
&\quad \left\langle \text{--- } n_T(2) \text{ factors ---} \right\rangle \\
&\times \left(\frac{L-n_T(1)-n_T(2)}{L^3}\right) \dots \left(\frac{L-n_T(1)-n_T(2)-[n_T(3)-1]}{L^3}\right) \\
&\quad \left\langle \text{--- } n_T(3) \text{ factors ---} \right\rangle \\
&\quad \vdots \\
&\quad \vdots \\
&\times \frac{\left(L-n_T(1)-n_T(2)-\dots-[n_T(T)-1]\right)}{L^T} \\
&\quad \left\langle \text{--- } n_T(T) \text{ factors } (\leq 1) \text{ ---} \right\rangle \\
&= \frac{L(L-1)(L-2) \dots [L-(n_T(1)+n_T(2)+n_T(T)-1)]}{L^{n_T(1)}(L^2)^{n_T(2)} \dots (L^T)^{n_T(T)}} \\
&= \frac{L!}{L^T(L-m)!} \tag{11}
\end{aligned}$$

We have

$$P(\mathbf{n}_T) = N(\mathbf{n}_T)Pr(\mathbf{I}_T), \tag{12}$$

where,

a) $Pr(\mathbf{I}_T)$ = the conditional probability of one particular realization of \mathbf{n}_T , like the one described above

and,

b) $N(\mathbf{n}_T)$ = the number of the equally likely realizations of \mathbf{n}_T .

We now determine $N(\mathbf{n}_T)$. This determination coincides with finding the number of different partitions of a set of T objects into classes of $n_T(j)$ groups, each group having j objects, for $j = 1, 2, \dots, T$. This number is given in [9] by,

$$N(\mathbf{n}_T) = \frac{T!}{\prod_{j=1}^T (j!)^{n_T(j)} n_T(j)!} \quad (13)$$

Combining this result with the probability of a specific realization results in

$$P(\mathbf{n}_T) = \frac{L! T!}{L^T (L-m)! \prod_{j=1}^T (j!)^{n_T(j)} n_T(j)!} \quad (14)$$

We have demonstrated a procedure to enumerate all possible states \mathbf{n}_T corresponding to a given number of time slots (L) and packets transmitted (T) in the frame. We have also determined the probability of any arbitrary state \mathbf{n}_T , given L and T . For the case of no capture that we are now considering, the conditional distributions (given L and T) of the number of successful packets and colliding packets can be evaluated numerically as follows:

$$\begin{aligned} Pr(S = s | T) &= \sum_{\mathbf{n}_T} Pr(n_T(1) = s | \mathbf{n}_T) P(\mathbf{n}_T) \\ &= \sum_{\mathbf{n}_T \in Z_S} P(\mathbf{n}_T). \end{aligned} \quad (15)$$

where Z_S is the set of states for which $n_T(1) = s$.* Similarly,

$$Pr(C = T-s | T) = Pr(S = s | T). \quad (16)$$

As noted in the introduction, our mathematical formulation is not strictly necessary for the case of systems without capture. In fact, Szpankowski's analysis of S/V-ALOHA, or "Slotted ALOHA with V Subchannels," [4], can be applied to Framed ALOHA for noncapture channels. In S/V-ALOHA the channel is divided into V parallel

* Note that it is not necessary to store the list of states and their probabilities. As each state is enumerated, its probability can be evaluated using eq. (14), and the corresponding contribution can be incorporated into the probability distributions for the success process or the backlog process. Thus, despite the large number of states, performance evaluation is not limited to small systems. The largest system for which we have obtained results consists of $M = 50$ users.

sub-channels. The time axis is divided into multi-slots, each containing V slots, i.e., one in each sub-channel. The number of successful packets in a multislot is thus equal to the number of slots chosen by exactly one user. The choice of a slot within a multislot is therefore analogous to the choice of a slot within a frame in a Framed ALOHA model. The mathematical model of [4] can thus be applied to the case of Framed ALOHA in the case where there is no capture. It yields:

$$Pr(S = s | T) = \left(\frac{(-1)^s L! T!}{L^T s!} \right) \sum_{j=s}^{\min(L, T)} \frac{(-1)^j (L-T)^{T-j}}{(j-s)! (L-j)! (T-j)!}. \quad (17)$$

The technique introduced in this paper can be used not only for the case of Framed ALOHA without capture, but also for the case in which capture is allowed, in which case the other methods of analysis are not applicable.

4. FRAMED ALOHA WITH CAPTURE

We consider now the case in which one of the users captures the channel, despite the presence of other users' packets in the same time slot. Depending upon the implementation, the capture property may be based on either relative signal magnitudes (i.e., the stronger signal captures the channel) or on time of arrival. In frequency hopped systems, one signal can capture the transmitter provided that later signals are delayed by more than one dwell time. This may occur in practical slotted systems, either as a result of different propagation delays by users at different communication ranges or by an intentional randomization of transmission times (within a guard time established for this purpose at the beginning of each slot [6]). It is possible that two geographically separated receivers may capture the packets of different users. In our analysis, we assume that the same packet (if any) is captured by all users, or equivalently, in the mathematical sense, that there is a single destination.

Perfect Capture

The simplest, and most optimistic, model for capture is that of perfect capture, i.e., in every slot in which two or more users transmit one user captures the channel, and is therefore successful; the other users are unsuccessful and must be retransmitted in the next frame. In this case the number of successful packets in the frame, S , is simply equal to the number of occupied slots. From eq. (4) we thus obtain,

$$S = m = \sum_{j=1}^T n_T(j). \quad (18)$$

The probability distribution function for n_T is evaluated as presented earlier in this paper. We must in this case, however, reinterpret C as the **number of unsuccessful packets in the frame**, rather than the number transmitted in slots with one or more others. For the present case of perfect capture, we thus have,

$$C = T - m. \quad (19)$$

The distribution for the number of unsuccessful packets is therefore obtained as,

$$\begin{aligned} Pr(C=T-m | T,L) &= Pr(\text{exactly } m \text{ slots are occupied} | T,L), \\ &= \sum_{n_T \in Z_m} P(n_T), \end{aligned} \quad (20)$$

where Z_m is the set of states such that m slots are occupied.

The case of perfect capture can actually be analyzed without making use of the powerful combinatorial technique of this paper, since it only requires the statistics of the number of slots chosen by one or more users. This is equivalent to the classical occupancy problem in which T balls (packets) are distributed among L cells (slots). The probability that m slots are occupied is simply the probability that $(L-m)$ slots are empty, which is given in [10] by,

$$Pr(m \text{ slots are occupied} | T,L) = \binom{L}{L-m} \sum_{\nu=0}^m (-1)^\nu \binom{m}{\nu} \frac{(m-\nu)^T}{L}. \quad (21)$$

More General Capture Models

The full force of the combinatorial procedure presented in this paper is needed to accommodate other models for capture, such as those considered in [6-8]. For any such model the conditional probability distribution of the number of successful packets, given n_T , must be determined.

For example, consider a deterministic model in which one packet captures the receiver whenever the number of users in the slot is less than or equal to some threshold r . We then obtain,

$$S = \sum_{j=1}^r n_T(j), \quad (22)$$

which is simply a truncated version of eq. (18). We thus have,

$$Pr(C = T-S | T,L) = Pr(S \text{ successes} | T,L).$$

$$= \sum_{\mathbf{n}_T \in Z_r} P(\mathbf{n}_T), \quad (23)$$

where Z_r is the set of states such that eq. (22) is satisfied.

We may also consider the more general and more realistic case in which, with probability $Q(j)$, the receiver will be captured by one of the users whenever a total of j users transmit in the slot.* The expected value of the number of successes in the frame is then

$$E(S|\mathbf{n}_T) = \sum_{j=1}^T Q(j)n_T(j). \quad (24)$$

To determine equilibrium performance, we need the conditional probability distribution of S for a given state \mathbf{n}_T . To evaluate this distribution we note that if we are given the number of users transmitting in any arbitrary slot, then the success or failure of a packet to capture the receiver in that slot is independent of that in any other slot. Since there are m occupied slots, we have m independent Bernoulli trials, $n_T(j)$ of which have success probability $Q(j)$ for $j = 1, 2, \dots, T$. In each slot of the $n_T(j)$ slots (in each of which exactly j users are transmitting) there can be at most one successful transmission, an event that occurs with probability $Q(j)$. Therefore the total number of successful transmissions, $S(j)$, in that entire class of slots is binomially distributed with parameters $n_T(j)$ and $Q(j)$. Its generating function is thus given by

$$G_j(z) = [1 - Q(j) + zQ(j)]^{n_T(j)}. \quad (25)$$

The total successful packet process is the superposition of the successful transmissions $S(j)$ over all values of $j = 1, \dots, T$. Thus the total number of successes is given by

$$S = \sum_{j=1}^T S(j), \quad (26)$$

and, owing to the independence amongst the $S(j)$'s, the generating function of S is given by

$$G(z) = \prod_{j=1}^T [1 - Q(j) + zQ(j)]^{n_T(j)}. \quad (27)$$

Expanding this expression yields a polynomial form for the generating function:

* The effects of channel noise on the probability of correct reception can also be incorporated into the quantity $Q(j)$.

$$G(z) = \sum_{s=1}^m B_s z^s, \quad (28)$$

in which

$$B_s = Pr(s \text{ successes in frame} | \mathbf{n}_T). \quad (29)$$

The probability distribution of unsuccessful transmissions given \mathbf{n}_T is easily found from that of successful transmissions since, given a particular state, the probability of k failures is simply equal to the probability of $T-k$ successes. The distributions for the number of successes and failures, given T and L , are found by summing the distributions for each state while weighting them by the probability of that state. They are given as follows:

$$Pr(S = s | T) = \sum_{\mathbf{n}_T} B_s P(\mathbf{n}_T) \quad (30)$$

$$Pr(C = c | T) = \sum_{\mathbf{n}_T} B_{T-c} P(\mathbf{n}_T), \quad (31)$$

where the summations are performed over all states consistent with the values of L and T , as discussed in Section 3, using the expression for $P(\mathbf{n}_T)$ presented in eq. (14) and the expressions for B_s that follow from the expansion in eqs. (24) and (25).*

5. DYNAMIC SYSTEM MODEL: UNCONTROLLED FRAMED ALOHA

We now consider the dynamics as the system evolves from frame to frame. The subscripts representing frame number must therefore be reinserted into the notation. The system consists of M unbuffered users, and the frame length is kept constant at L slots in the present discussion. We first assume that all unsuccessful packets are retransmitted in the next frame. The number of packets transmitted in frame i is therefore

$$T_i = C_{i-1} + A_i. \quad (32)$$

The next step in the analysis is to find the distribution of T_i given C_{i-1} . This is denoted as $P_T(T_i | C_{i-1})$ and depends on the new arrival process A_i , where A_i is the number of new packets transmitted in frame i . Since it is assumed that blocked users do not produce new packets, and the number of users is finite, A_i depends on C_{i-1} .

* As in the non-capture case, it is not necessary to store the array of states and their probabilities.

Since each non-blocked user transmits with probability ϕ in the current frame, A_i is binomially distributed with parameters $M-C_{i-1}$ and ϕ ; thus, the expected total number of new arrivals during frame i is $(M-C_{i-1})\phi$. Since $T_i = A_i + C_{i-1}$, it is also binomially distributed (given C_i). Therefore, we have the following:

$$P_A(A_i | C_{i-1}) = \binom{M-C_{i-1}}{A_i} \phi^{A_i} (1-\phi)^{M-C_{i-1}-A_i}, \quad 0 \leq A_i \leq M-C_{i-1}. \quad (33)$$

$$P_T(T_i | C_{i-1}) = \binom{M-C_{i-1}}{T_i-C_{i-1}} \phi^{T_i-C_{i-1}} (1-\phi)^{M-T_i}, \quad C_{i-1} \leq T_i \leq M. \quad (34)$$

$$P_C(C_i | C_{i-1}) = \sum_{T_i=1}^M P_C(C_i | T_i) P_T(T_i | C_{i-1}), \quad (35)$$

where $P_C(C_i | T_i)$, which is independent of ϕ , is obtained from eq. (31).

The quantities $P_C(C_i | C_{i-1})$, for $0 \leq C_i, C_{i-1} \leq M$, are the elements of the transition probability matrix that represents the system dynamics in terms of the backlog of packets measured at the beginning (or equivalently at the end) of each frame. The system we have represented thus far is uncontrolled in the following senses: (a) the frame length is kept fixed at L slots; (b) the arrival process at each non-blocked user is independent of system backlog or channel traffic; (c) all blocked users transmit their packets in some slot in the frame with probability 1. The steady state distribution of backlog can be evaluated by starting with an arbitrary probability vector for $P_C(C_{i-1})$ and iterating until convergence is achieved, or by solving the Markov transition equations for a given initial condition. We have used the former approach. In all of our examples, the iteration has converged, regardless of whether the initial state is the empty state or if all M users are blocked.

Once the equilibrium distribution for the channel backlog has been determined, it is possible to determine the expected values of traffic, throughput, and backlog as follows:

$$\begin{aligned} E(S) = E(S_i) &= \sum_{C_{i-1}=0}^M E(S_i | C_{i-1}) P_C(C_{i-1}) \\ &= \sum_{C_{i-1}=0}^M \left[\sum_{S_i=0}^M S_i P_S(S_i | C_{i-1}) \right] P_C(C_{i-1}) \end{aligned} \quad (36)$$

$$\begin{aligned}
E(T) = E(T_i) &= \sum_{C_{i-1}=0}^M E(T_i | C_{i-1}) P_C(C_{i-1}) \\
&= \sum_{C_{i-1}=0}^M \left(\sum_{T_i=0}^M T_i P_T(T_i | C_{i-1}) \right) P_C(C_{i-1}) \quad (37)
\end{aligned}$$

$$E(C) = E(C_i) = \sum_{C_{i-1}=0}^M C_{i-1} P_C(C_{i-1}), \quad (38)$$

where $P_C(C_{i-1})$ is the steady state distribution of C . For the present case of constant frame length, the throughput (which has been defined as the expected number of packets successfully delivered over the channel per time slot) is simply,

$$\bar{S} = \frac{E(S)}{L}. \quad (39)$$

Similarly, the expected channel traffic (in packets per time slot), is

$$\bar{T} = \frac{E(T)}{L}. \quad (40)$$

This procedure is carried out for each value of ϕ .

Performance: No Capture

Figures 2 - 4 show the performance of a Framed ALOHA system without capture for a frame length of $L = 10$ slots. The number of users is varied between $M = 10$ and 50. Figure 2 shows \bar{T} vs \bar{S} , i.e., channel traffic vs throughput. Figure 3 shows throughput vs ϕ , the Bernoulli packet arrival rate (per frame) at each of the non-blocked users. The greatest throughput, which is achieved for $M = L = 10$ and $\phi = 1.0$, is slightly greater than $1/e$. As M is increased past 10, the maximum throughput decreases slightly and occurs for decreasing values of ϕ . For large values of M , the throughput is very sensitive to ϕ and decreases rapidly for ϕ greater than the value that produces the maximum throughput. These curves show that throughput decreases to unacceptably low levels when the arrival rate of new packets is too high, as is expected in ALOHA-type systems. However, since we are considering only finite systems, there is no stability problem; by lowering the input rate for new packets, a heavily backlogged system can always be brought to a better operating point. Figure 4 shows expected channel backlog vs ϕ as M is varied between 10 and 50. For large values of M , most users are backlogged, except for small values of ϕ .

Performance: Spread-Spectrum Capture

We have considered the following capture model, which is based on [6]

$$\begin{aligned} Q(1) &= 1 \\ Q(j) &= \alpha^j \quad j \geq 2. \end{aligned} \quad (41)$$

Larger values of α thus represent a greater capture capability. Figure 5 shows traffic versus throughput for several values of α in a system with $L = 10$ slots and $M = 30$ users. All of the subsequent examples we present in this paper will be for the same values of L and M . Note that maximum throughput increases as α increases, and that the effects of overload are decreased, as evidenced by the fact that the curves do not bend back as sharply. Figure 6 shows throughput performance as a function of ϕ , for the same values of α . For large values of α there is little decrease in throughput as ϕ is increased past the value that results in maximum throughput.* Figure 7 shows expected backlog vs ϕ for the same values of α .

6. A SYSTEM IN WHICH NOT ALL BLOCKED PACKETS ARE TRANSMITTED

Thus far, we have assumed that all unsuccessful packets are retransmitted in the following frame. Such systems, when heavily loaded, can suffer from unacceptably low values of throughput, as we have just shown. To alleviate this problem, we consider the case in which each blocked user retransmits its packet sometime in the next frame with some probability X_i . Such a scheme helps to increase throughput at large values of ϕ .

The following quantities describe the retransmission process:

X_i = probability of retransmission of blocked packet in frame i

U_i = number of blocked packets not retransmitted in frame i .

In the present discussion, X_i is held constant at X over all frames. In Section 7, the case of a dynamically varying X_i is discussed. The number of packets transmitted in frame i is now (see eq. (1))

$$T_i = C_{i-1} + A_i - U_i \quad (42)$$

* The curve labeled X_i^* , which represents the use of dynamically controlled retransmission probability in a system with $\alpha = 0.9$, will be discussed in Section 7.

C_i now includes both unsuccessfully transmitted packets from frame i as well as previously blocked packets which are not retransmitted in frame i . Also, we have

$$C_i = T_i - S_i + U_i. \quad (43)$$

To find the conditional distributions $P_C(C_i | T_i, U_i)$ and $P_T(T_i | C_{i-1}, U_i)$, the previously found distributions for the case of $U_i = 0$ (the case of $X=1$) must be shifted by the amount U_i . Therefore:

$$Pr(C_i=c-U_i | T_i, U_i) = Pr(C_i=c | T_i, U_i=0) \quad (44)$$

$$Pr(T_i=t+U_i | C_{i-1}, U_i) = Pr(T_i=t | C_{i-1}, U_i=0). \quad (45)$$

The new distributions are:

$$P_C(C_i | T_i, U_i) = \sum_{n_T} B_{T_i+U_i-C_i} P(n_T) \quad (46)$$

$$P_T(T_i | C_{i-1}, U_i) = \left[\begin{matrix} M-C_{i-1} \\ T_i+U_i-C_{i-1} \end{matrix} \right] \phi^{T_i+U_i-C_{i-1}} (1-\phi)^{M-T_i-U_i}. \quad (47)$$

$P_S(S_i | T_i, U_i)$, which depends only on T_i , is unchanged from the case of $U_i = 0$. The distribution of U_i depends on C_{i-1} and is binomial with parameters C_{i-1} and X . Its distribution is given by:

$$P_U(U_i | C_{i-1}) = \left[\begin{matrix} C_{i-1} \\ U_i \end{matrix} \right] (1-X)^{U_i} X^{C_{i-1}-U_i}. \quad (48)$$

The probability distribution of T_i given C_{i-1} is therefore:

$$P_T(T_i | C_{i-1}) = \sum_{U_i=0}^{C_{i-1}} P_T(T_i | C_{i-1}, U_i) P_U(U_i | C_{i-1}). \quad (49)$$

We now have everything that is needed to evaluate the transition probability matrix for the backlog process. Thus, we have

$$\begin{aligned} P_C(C_i | C_{i-1}) &= \sum_{U_i=0}^{C_{i-1}} \sum_{T_i=0}^{M-U_i} P_C(C_i | T_i, U_i, C_{i-1}) P_T(T_i, U_i | C_{i-1}) \\ &= \sum_{U_i=0}^{C_{i-1}} \left[\sum_{T_i=0}^{M-U_i} P_C(C_i | T_i, U_i, C_{i-1}) P_T(T_i | U_i, C_{i-1}) \right] P_U(U_i | C_{i-1}). \end{aligned} \quad (50)$$

Note that $P_C(C_i | T_i, U_i, C_{i-1})$ is actually independent of C_{i-1} , given that T_i and U_i are known. $P_S(S_i | C_{i-1})$ is found by replacing C_i by S_i in eq. (50).

Performance: No Capture

Figure 8 shows traffic vs throughput for $X = 0.4, 0.7,$ and 1.0 (again for the case of $L = 10$ slots and $M = 30$ users). Use of a *good* value of X not only increases the maximum value of throughput that can be achieved, but in addition cuts the curve short so that low throughput values are no longer obtained for large values of ϕ .^{*} Figure 9 shows throughput vs ϕ for values of X between 0.4 and 1.0 . For this range of values, lowering X results in improved throughput performance, except for small values of ϕ . Figure 10 shows throughput vs ϕ for X between 0.1 and 0.4 . The curves for $0.2, 0.3,$ and 0.4 intersect; the best value of X depends on ϕ . We see from the curves of Figures 9 and 10 that a value of $X = 0.3$ or 0.4 would provide near-optimal performance over a wide range of ϕ ; 0.3 would be better for ϕ greater than about 0.3 . If ϕ is known, then the best value of X can be chosen, resulting in a performance curve that is the upper envelope of curves for fixed values of X . Figure 11 shows throughput vs X for several fixed values of ϕ . For ϕ greater than about 0.3 , the best value of X is approximately 0.3 ; there is little change as X is varied from 0.2 to 0.4 .

7. DYNAMICALLY CONTROLLED SYSTEMS

In the previous section, we considered a system in which X_i , the probability that a blocked user retransmits its packet in frame i , is held constant at a value of X . We now consider a system in which X_i can be chosen as a function of channel backlog, and the frame length is again fixed at L slots. Our goal is to choose X_i such that S_i is maximized. We initially assume that in frame i all users know C_{i-1} , the number of blocked users from the previous frame, as well as ϕ , the probability that a non-blocked user transmits in the frame. Although channel backlog is not a directly observable quantity, it can be estimated, based on channel feedback information. Some techniques for traffic estimation are discussed in [2,3,11].

Although the events in the L slots of a frame are not independent (since a user can transmit at most one packet in a frame), the statistics of the events in each of the slots are identical. Thus, to maximize S_i it is sufficient to maximize the probability that there is a successful transmission in any particular slot. We first consider the case of no capture. We have,

$$\begin{aligned} Pr(\text{success in any particular slot}) &= Pr(\text{exactly 1 user in any particular slot}) \\ &= Pr(1 \text{ new user, no blocked users}) + Pr(\text{no new user, 1 blocked user}). \end{aligned} \quad (51)$$

^{*} Since it is not clear from Figure 8 where the curve for $X = 0.7$ stops, we note that it does so at the point $\bar{T} = 2.173$ and $\bar{S} = 0.2454$ packets per slot.

To simplify the notation, we let

$$p = \frac{\phi}{L} = \text{probability a particular non-blocked user transmits in an arbitrary given slot}$$

$$q = \frac{X}{L} = \text{probability a particular blocked user transmits in an arbitrary given slot.}$$

Thus, in an arbitrary slot of frame i , $(M-C_{i-1})$ users each transmit with probability p , and C_{i-1} users each transmit with probability q . Therefore, we have,

$$\begin{aligned} &Pr(\text{exactly 1 user in any particular slot}) \\ &= (M-C_{i-1})p(1-p)^{(M-C_{i-1}-1)}(1-q)^{C_{i-1}} + (1-p)^{M-C_{i-1}}C_{i-1}q(1-q)^{C_{i-1}-1}. \end{aligned} \quad (52)$$

To maximize with respect to q , we simply differentiate with respect to q and set the result equal to 0, which yields,

$$X_i^* = L \left[\frac{\phi(M-C_{i-1}+1) - L}{M\phi - C_{i-1}L} \right]. \quad (53)$$

Note that when $X_i = X_i^*$, the point at which throughput is maximized, the expected number of packets transmitted per slot is not equal to 1. This is a consequence of the fact that the superposition of two Bernoulli sequences with different parameters does not result in a binomial distribution.

Equation (51) will yield values for X_i^* which are greater than one as well as values which are negative. However, since X_i is a probability, it must be between 0 and 1. The upper and lower bounds on X_i are therefore set at 1 and 0 respectively. For each value of C_{i-1} between 0 and M , the optimum value of X_i^* is found, and the new distribution for U_i , given C_{i-1} , is calculated. The calculation is as in Section 6, except that now X_i is a function of C_{i-1} .

Performance results for $X_i = X_i^*$ were included in the plots of throughput vs ϕ shown in Figure 10. The throughput for $X_i = X_i^*$ is better than the upper envelope of throughput curves for fixed values of X , because X_i^* is chosen as a function of actual channel backlog. However, we see that the performance of a system that uses a *good* fixed value of X is almost as good as that of a system that uses X_i^* . This result is consistent with those of the early slotted ALOHA studies, in which it was shown that performance is relatively insensitive to the packet retransmission probability [11]. This result also suggests that inconsistencies in channel backlog estimates are not expected to have a severe impact on performance. A traffic vs throughput curve for $X_i = X_i^*$ was included in Figure 8. The use of the optimal retransmission probability maintains the throughput at its maximum value, and prevents channel traffic from increasing past

approximately 1 packet per slot. In Figure 5 we included a curve of traffic vs throughput for a system with capture probability $Q(j) = 0.9^j$ and $X_i = X_i^*$. Use of the optimal retransmission probability once again maintains the throughput at its maximum value as ϕ approaches 1. There is very little improvement in throughput, however; for the system parameters under consideration, $X = 1$ provides almost as good performance as X_i^* . The curve for $X_i = X_i^*$ is essentially a truncated version of that for $X = 1$ (at a maximum traffic value of $\bar{T} = 2.27$), with a very slight (almost imperceptible) improvement in throughput for ϕ greater than 0.6 (see Figure 6).

We also note that when the retransmission probability is chosen as a threshold-based function of the channel backlog, although not necessarily optimal, it stabilizes slotted ALOHA [12].

For a system with capture, the probability of successful packet transmission in an arbitrarily chosen slot is

$$Pr(\text{success} | C_{i-1}) = \sum_{j=1}^M Pr(j \text{ users transmit in slot} | C_{i-1}) Q(j), \quad (54)$$

where

$$\begin{aligned} & Pr(j \text{ transmit} | C_{i-1}) \\ &= \sum_{b=0}^j Pr(b \text{ blocked users transmit, } j-b \text{ non-blocked users transmit} | C_{i-1}) \\ &= \sum_{b=0}^j \left[\binom{C_{i-1}}{b} q^b (1-q)^{C_{i-1}-b} \right] \left[\binom{M-C_{i-1}}{j-b} p^{j-b} (1-p)^{M-C_{i-1}-j+b} \right]. \end{aligned} \quad (55)$$

X_i^* , the value of $X = qL$ that maximizes the probability of successful packet transmission (eq. (54)) for each value of C_{i-1} and ϕ , can be determined computationally using search techniques. Although this would be a tedious calculation, it could be performed once, offline, and the results stored in a lookup table.

We can also consider a system in which the frame length, L_i , is varied as a function of channel backlog, while X_i is held fixed at some value X . Varying frame length affects the transmission process not only at blocked users but also at unblocked users as well. Since each user can transmit at most one packet per frame, the transmission of an unblocked user in an arbitrarily chosen slot is ϕ/L_i , and that of a blocked user is X/L_i . Thus, increasing (decreasing) frame length results in a reduced (increased) transmission rate at all users (per slot) for a given value of ϕ .

Since L_i can only assume integer values, the best way to determine L_i may be simply to search for the value of L that maximizes throughput per time slot (eq. (52) for the no-capture case and eq. (54) for a system with capture). For the no-capture case, a good value to start the search is that for which the expected channel traffic per slot is approximately equal to 1 packet. Choosing L_i such that $E(T_i | C_{i-1}) = L_i$ yields:

$$L_i^* = (M - C_{i-1})\phi + C_{i-1}X. \quad (56)$$

Since L_i must be an integer, the result from the above equation is rounded to the nearest integer value. $L_i = 0$ is not allowed since it would mean skipping frame i . The conditional probability distribution for C_i given C_{i-1} is now calculated using the distributions $P_S(S_i | T_i, L_i^*)$ and $P_C(C_i | T_i, L_i^*)$, where L_i^* is determined for each (C_{i-1}, ϕ) pair; the resultant frame length is denoted as $L_i^*(C_{i-1})$, where we have suppressed the dependence on ϕ , as we have done with the other quantities. $E(S_i | C_{i-1})$ and $E(T_i | C_{i-1})$ are each divided by $L_i^*(C_{i-1})$ in the evaluation of throughput and traffic per slot. Modification of eqs. (36) and (37) yield:

$$\bar{S} = \sum_{C_{i-1}=0}^M \left(\frac{\sum_{S_i=0}^M S_i P_S(S_i | C_{i-1}, L_i^*)}{L_i^*(C_{i-1})} \right) P_C(C_{i-1}) \quad (57)$$

and

$$\bar{T} = \sum_{C_{i-1}=0}^M \left(\frac{\sum_{T_i=0}^M T_i P_T(T_i | C_{i-1}, L_i^*)}{L_i^*(C_{i-1})} \right) P_C(C_{i-1}). \quad (58)$$

The expected frame length is

$$E(L_i^*) = \sum_{C_{i-1}=0}^M L_i^*(C_{i-1}) P_C(C_{i-1}). \quad (59)$$

Figure 12 shows throughput vs ϕ for a system with $M = 30$ users in which $X = 1$ (all backlogged users retransmit their packets) and $L_i = L_i^*$. A throughput of somewhat greater than $1/e$ is obtained, except for very small ϕ . Figure 13 shows expected traffic vs ϕ for the same example. The "oscillation" for small values of ϕ results from the fact that L_i can take on only integer values. (The jaggedness could have been eliminated if results had been obtained for additional values of ϕ .) Finally, in Figure 14 we show the expected frame length as a function of ϕ . As ϕ approaches 1, the expected frame length approaches 30.

It would be more difficult to implement a system with variable frame length than one with variable retransmission probability. We have noted that an accurate estimate of channel backlog may not be available. Moreover, since this is a distributed system, not all users would have the same estimate of backlog, and so they may be operating with different frame lengths. Our mathematical model for Framed ALOHA does not accommodate such asynchronous operation, and it is not clear at this point how it would affect performance. However, as noted earlier, inconsistencies in channel backlog estimates are not expected to affect performance adversely in a system with fixed frame length and variable retransmission probability.

8. A SYSTEM WITH MULTIPLE RECEIVERS

Thus far, we have assumed that there is a single spread-spectrum code used by all network members, and that the destination can successfully receive at most one packet in a time slot. It is straightforward to extend our model to the case in which each network member uses a distinct spread-spectrum code, and in which the destination monitors all of these codes simultaneously. With some signaling schemes, it is in fact possible to monitor many codes simultaneously without a massive proliferation of hardware [13]. If the codes were truly orthogonal, it would be possible for all network members to transmit successfully at the same time. However, since in practice only a quasi-orthogonality can generally be maintained, the probability that a packet is successful depends on the number of other packets transmitted in the same time slot. We define

$$Q(m, j) = \text{Pr}(m \text{ packets are received successfully} | j \text{ packets transmitted in slot}).$$

This probability depends on the characteristics of the spread spectrum signaling scheme that is being used. The generating function of S (see eq. (27)) is now given by

$$G(z) = \prod_{j=1}^T \left[\sum_{m=0}^j z^m Q(m, j) \right]^{n_T(j)}. \quad (60)$$

The rest of the derivation is identical to that for the single-code case presented earlier. Note that the performance results for the case in which all codes can be simultaneously monitored would provide upper bounds for the more realistic case in which only a few codes can be so monitored. Such bounds would represent the maximum throughput that can be achieved under the Framed ALOHA protocol for a given signaling format (i.e., modulation scheme, spread spectrum channel bandwidth, and forward error control scheme) in the absence of equipment limitation constraints.

With some signaling structures, a distinct receiver is needed for each spread-spectrum code that is to be monitored. In such cases, the destination would be able to monitor only a limited number of codes at a time. We can consider a system in which the transmitting network members first choose a slot at random and then choose one of the codes at random to use in that slot. To analyze such a system, we would have to determine the distribution of the number of users that choose the same code, given the number that have chosen the same slot. To do so would require two applications of the technique used in this paper, i.e., first to determine the occupancy distribution of packets transmitted in slots, and then to determine the distribution of the number of transmitters that use the same code. The probability that a packet is successful would depend on both the total number of packets transmitted in the same slot, as well as the number of packets that were transmitted using the same code. This is a topic for future study.

9. CONCLUSIONS

In this paper we have presented an exact analysis and performance evaluation of Framed ALOHA for systems in which the number of users is finite. Our model can accommodate a general model for capture, in which the probability that one packet is received successfully depends on the number of packets involved in the collision. We have considered uncontrolled systems, in which all backlogged users retransmit in the current frame, as well as those in which a fixed fraction X retransmit. We have also considered dynamically controlled systems in which the retransmission probability is chosen as a function of channel backlog. Although this quantity is not directly observable, it can be estimated from channel observations; our results have shown that use of a *good* fixed value of retransmission probability results in performance that is almost as good as that of the optimal dynamically chosen value.

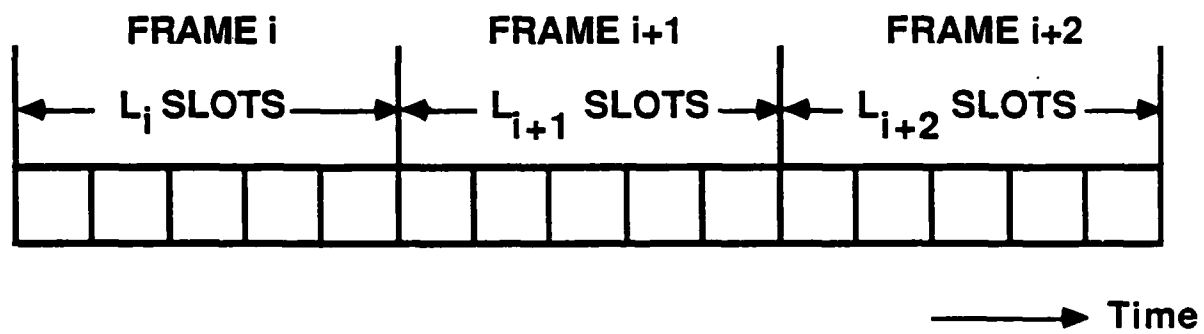


Fig. 1 Slotted frame structure.

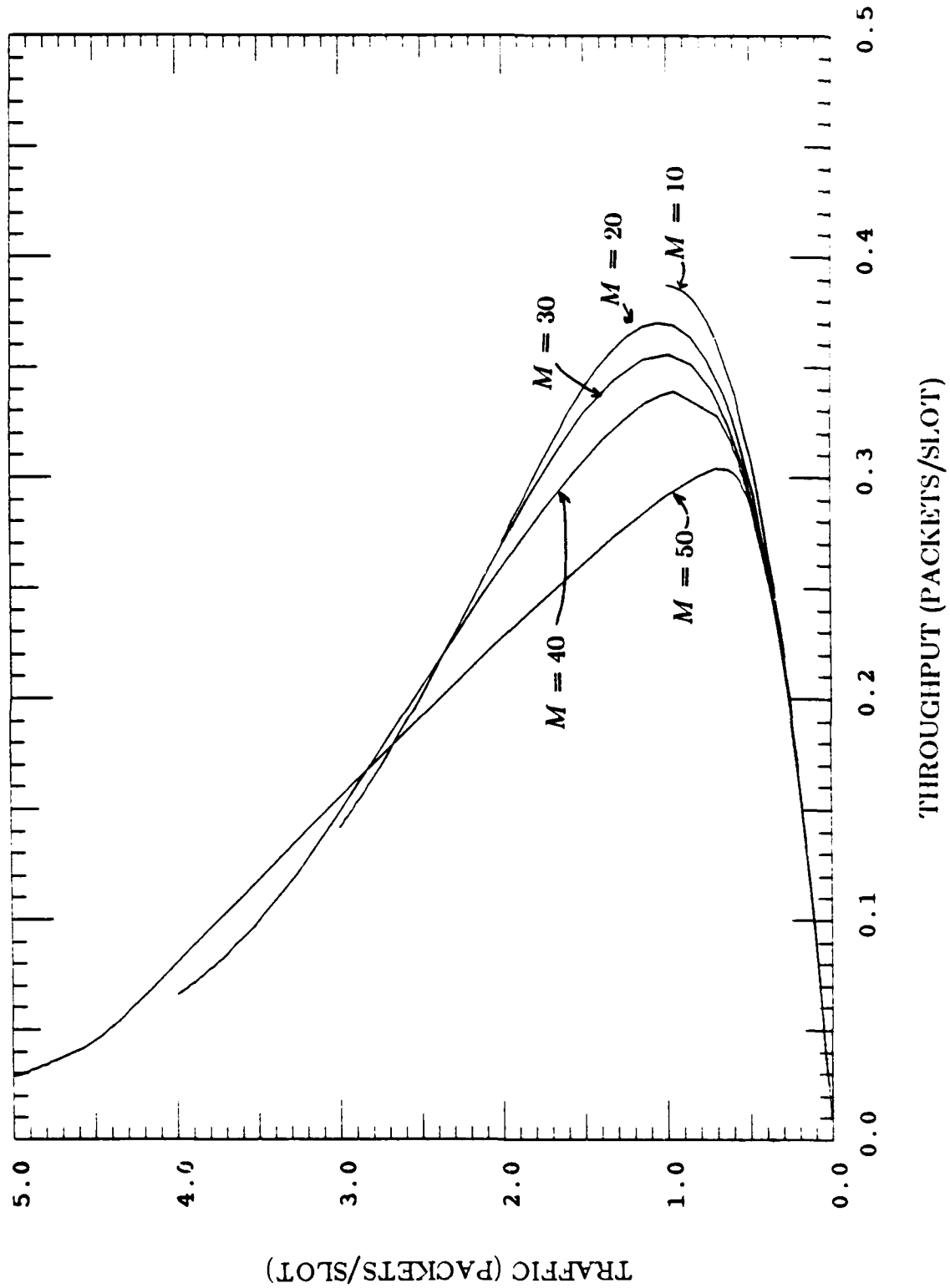


Fig. 2 Traffic vs throughput for Framed ALOHA; No Capture; $L = 10$.

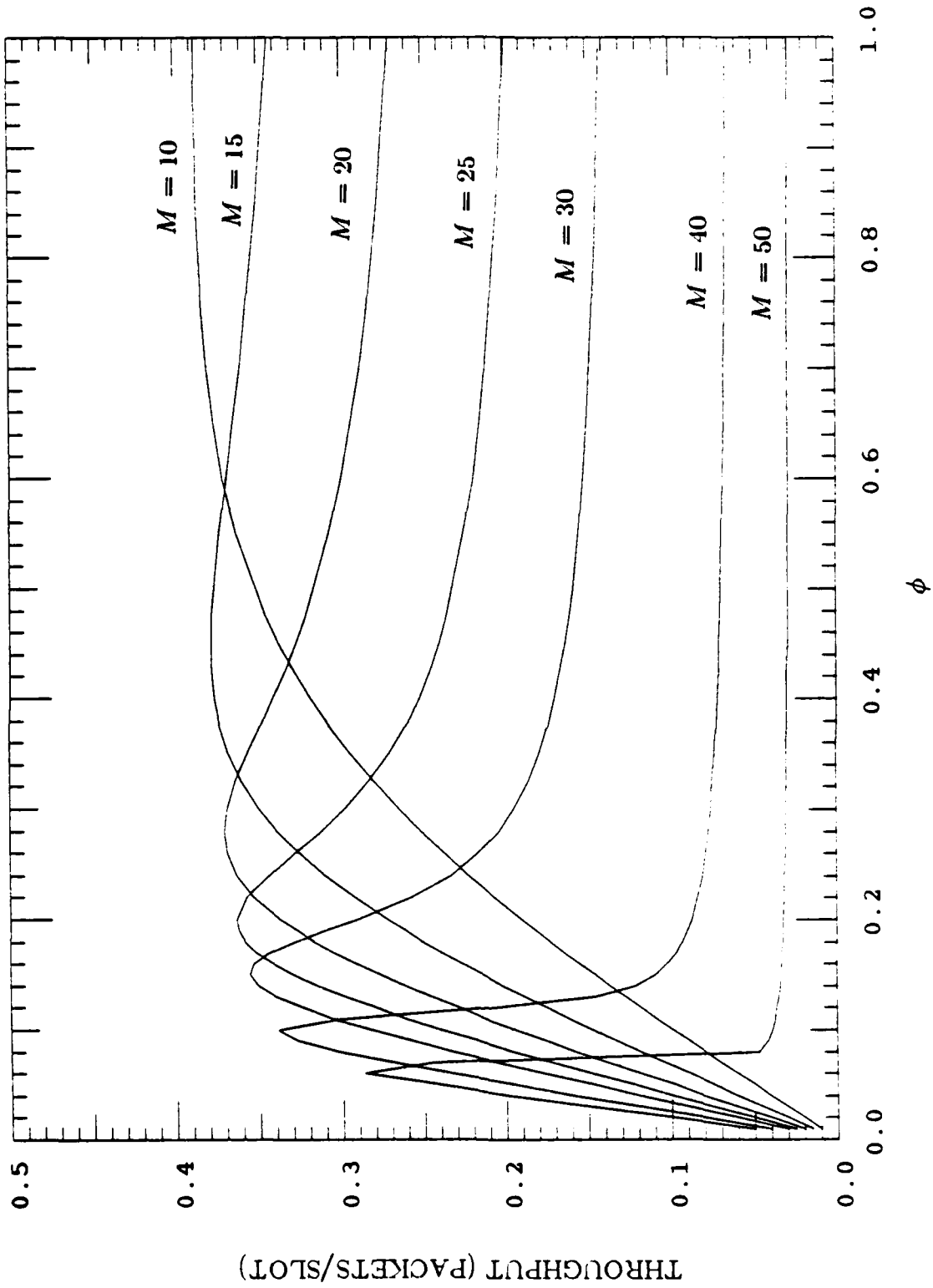


Fig. 3 Throughput vs ϕ for Framed ALOHA; No Capture; $L = 10$.

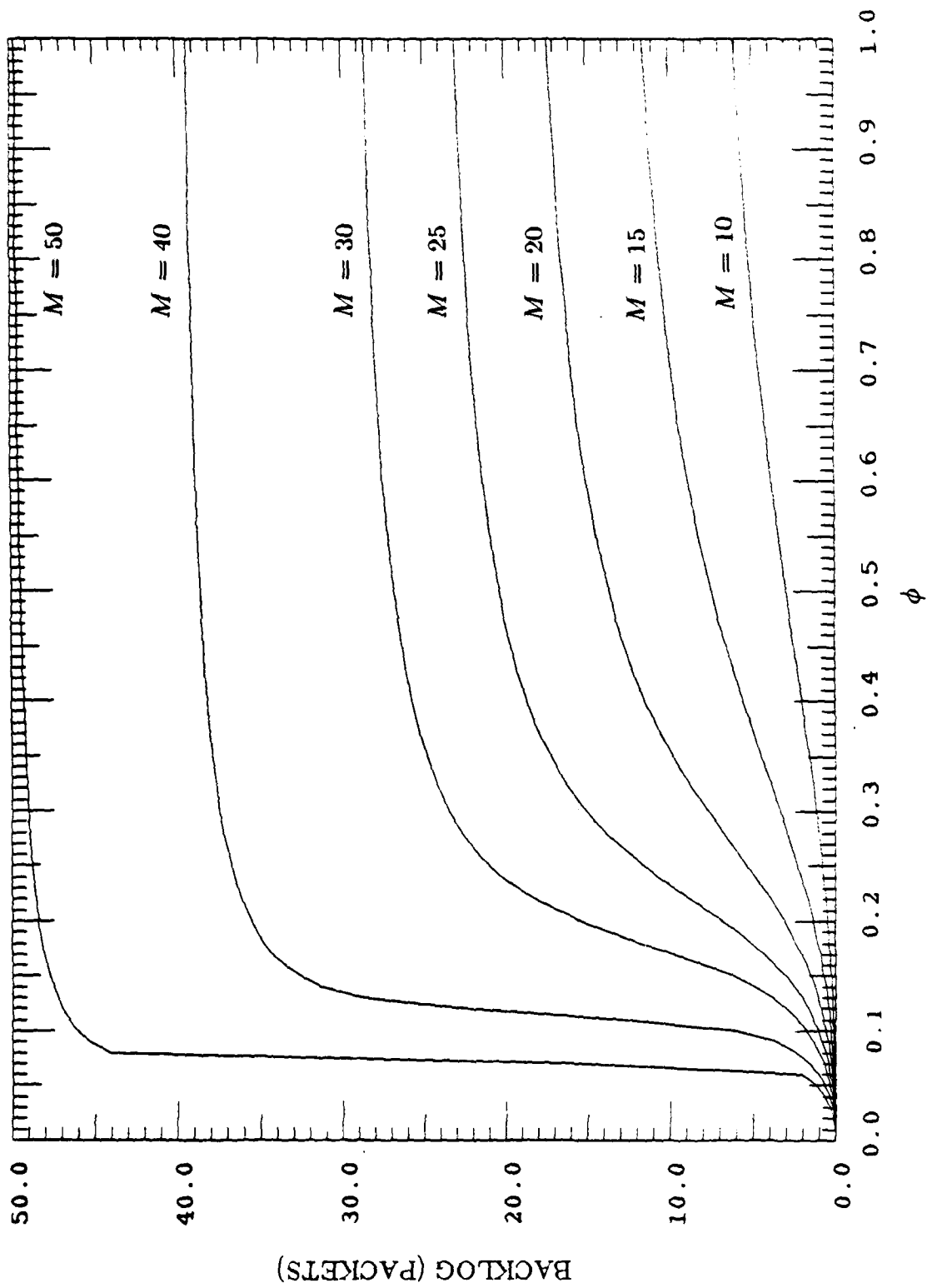


Fig. 4 Backlog vs ϕ for Framed ALOHA; No Capture; $L = 10$.

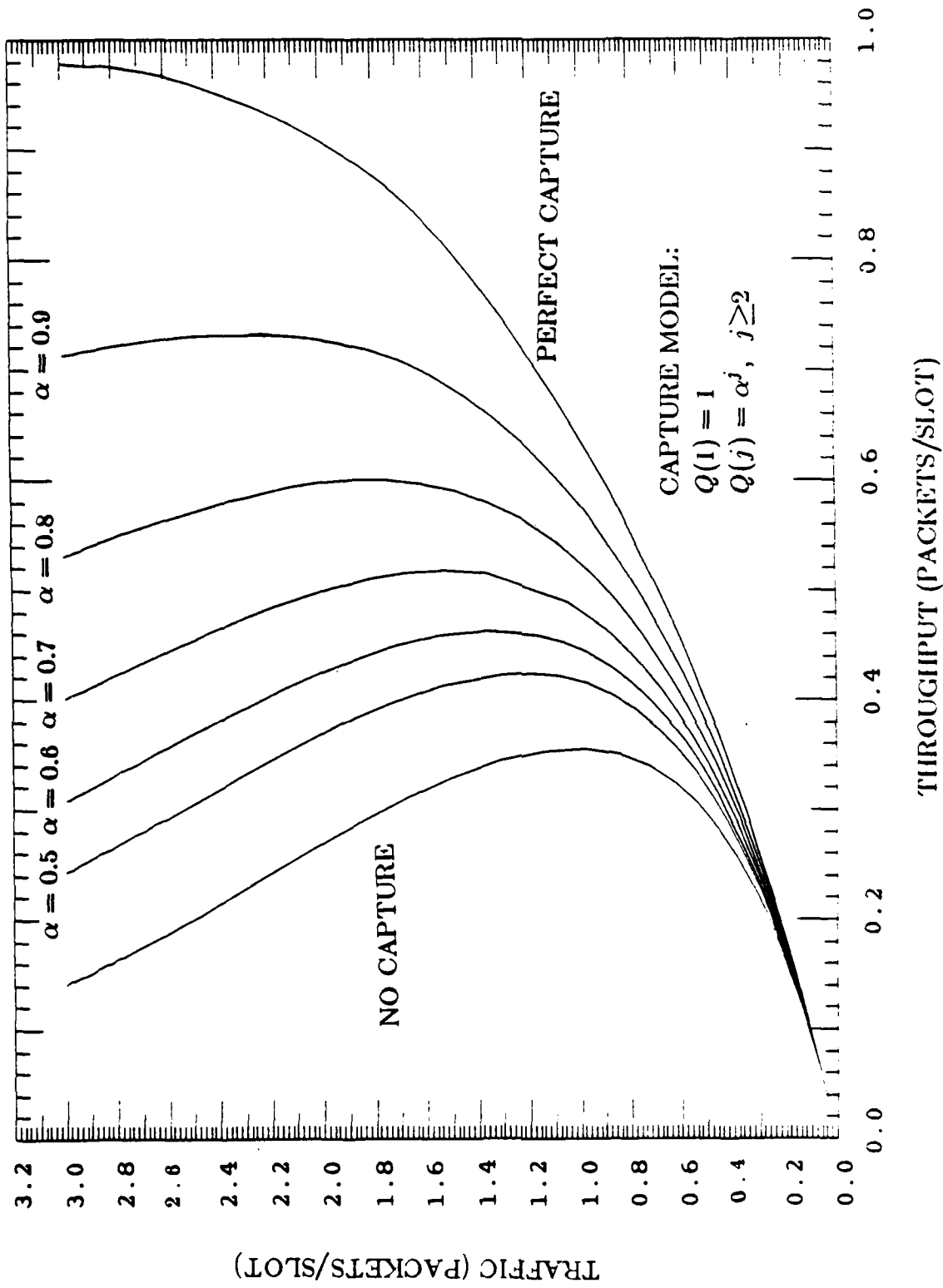


Fig. 5 Traffic vs Throughput for Framed ALOHA with Capture;
 $L = 10, M = 30$.

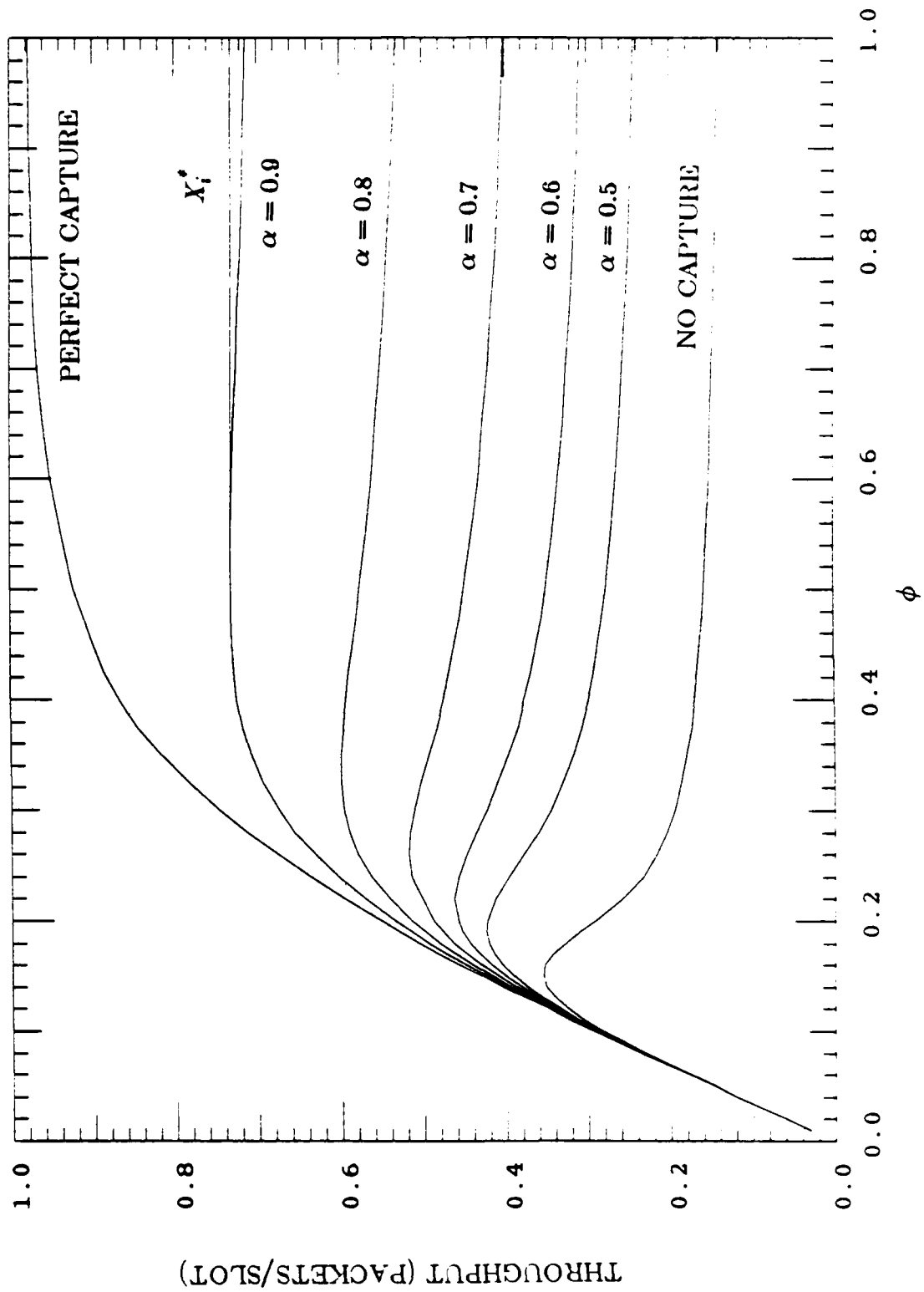


Fig. 6 Throughput vs ϕ for Framed ALOHA with Capture; $L = 10$, $M = 30$.

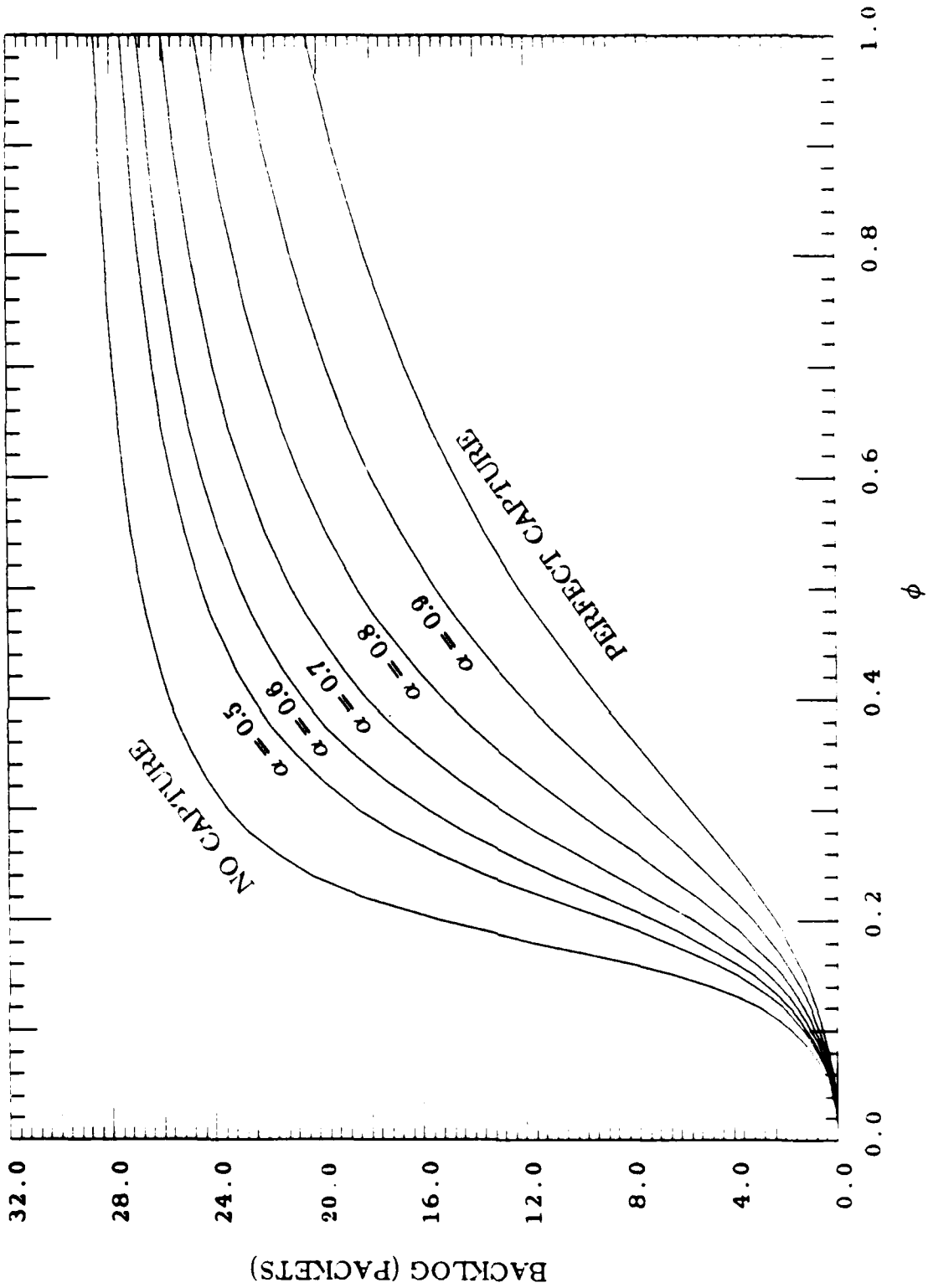
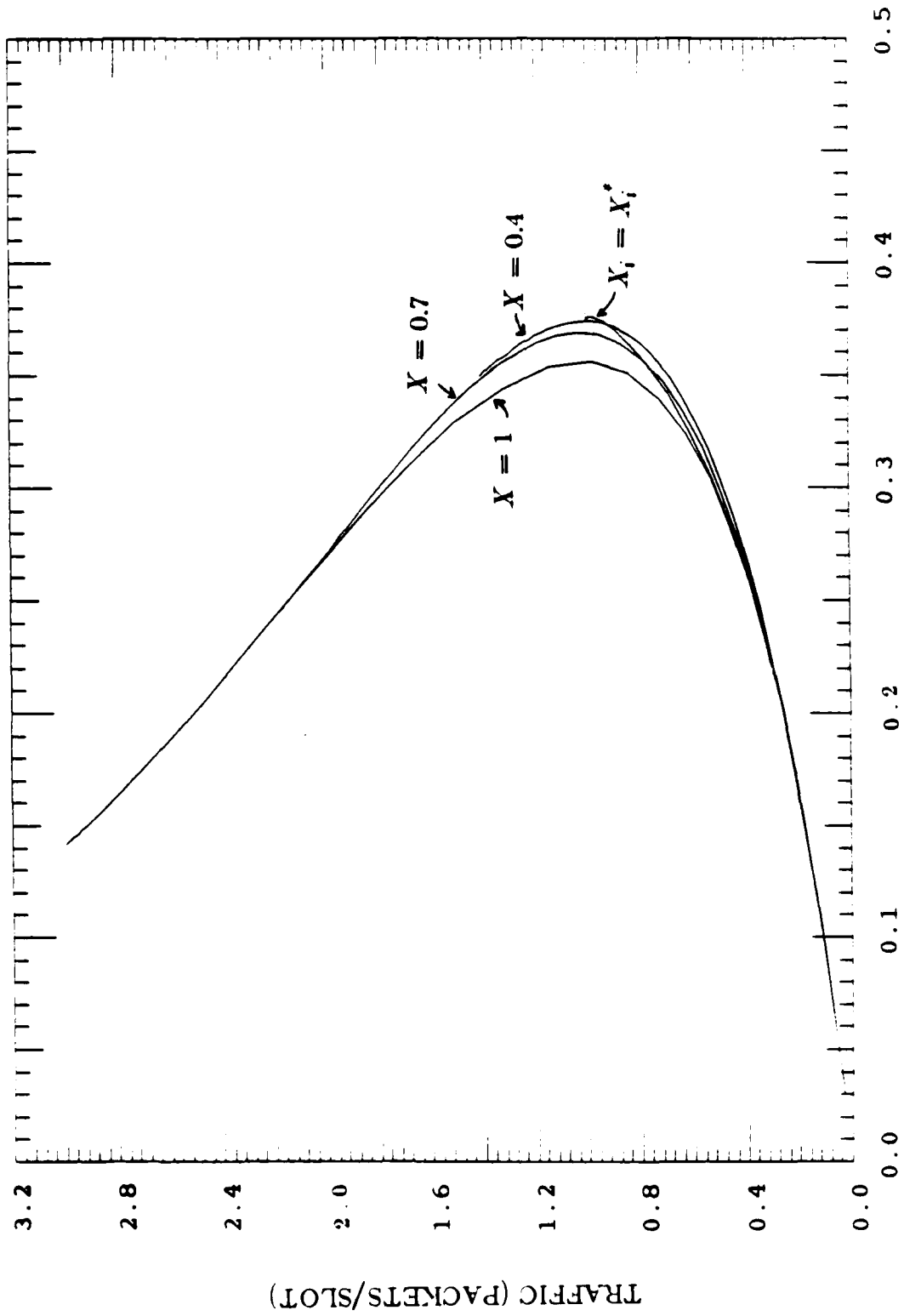


Fig. 7 Backlog vs ϕ for Framed ALOHA with Capture; $L = 10$, $M = 30$.



THROUGHPUT (PACKETS/SLOT)

Fig. 8 Traffic vs throughput for Framed ALOHA; No Capture;
 $L = 10, M = 30$, Several values of X .

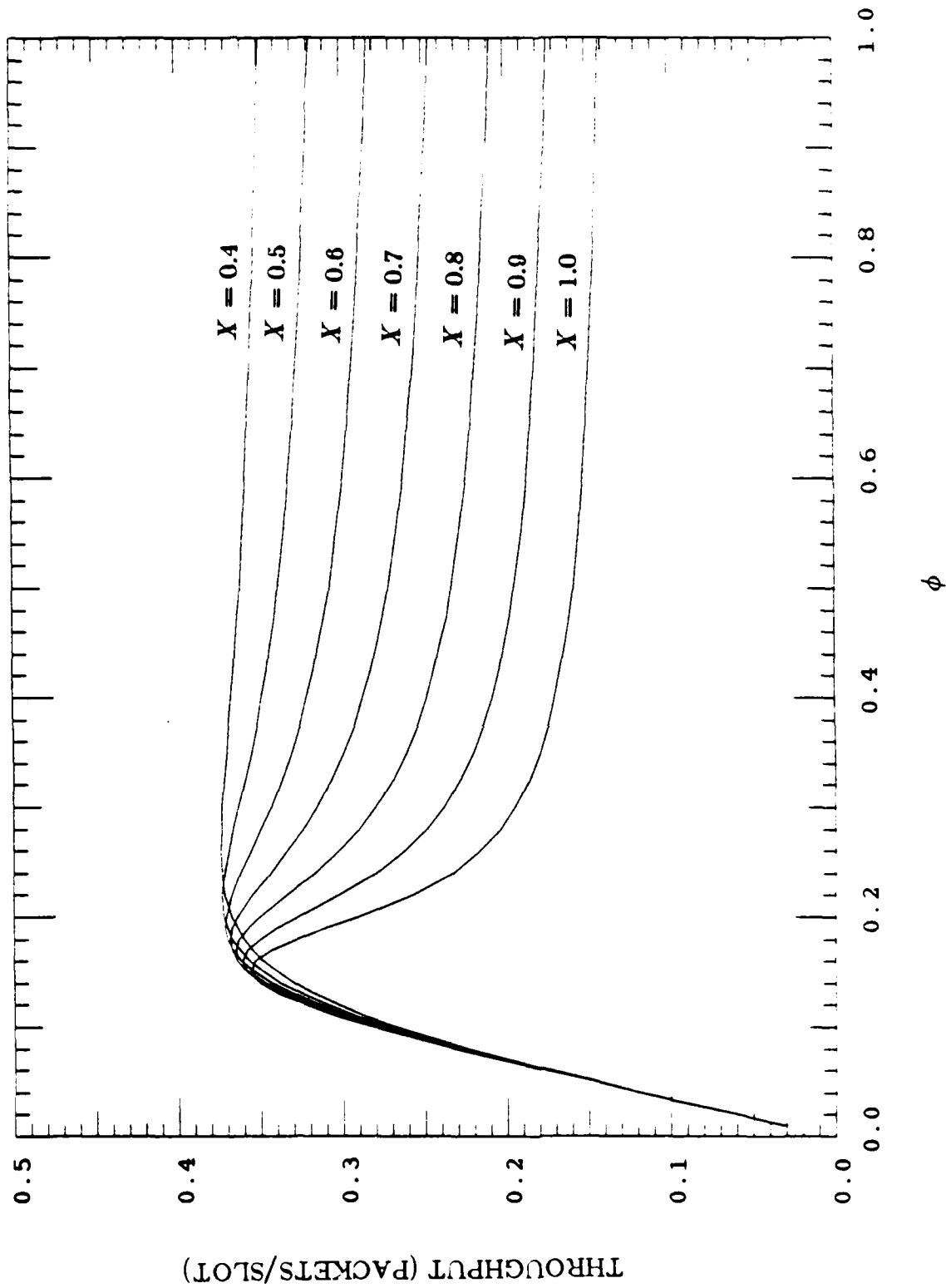


Fig. 9 Throughput vs ϕ for Framed ALOHA; No Capture;
 $L = 10, M = 30, 0.4 \leq X \leq 1.$

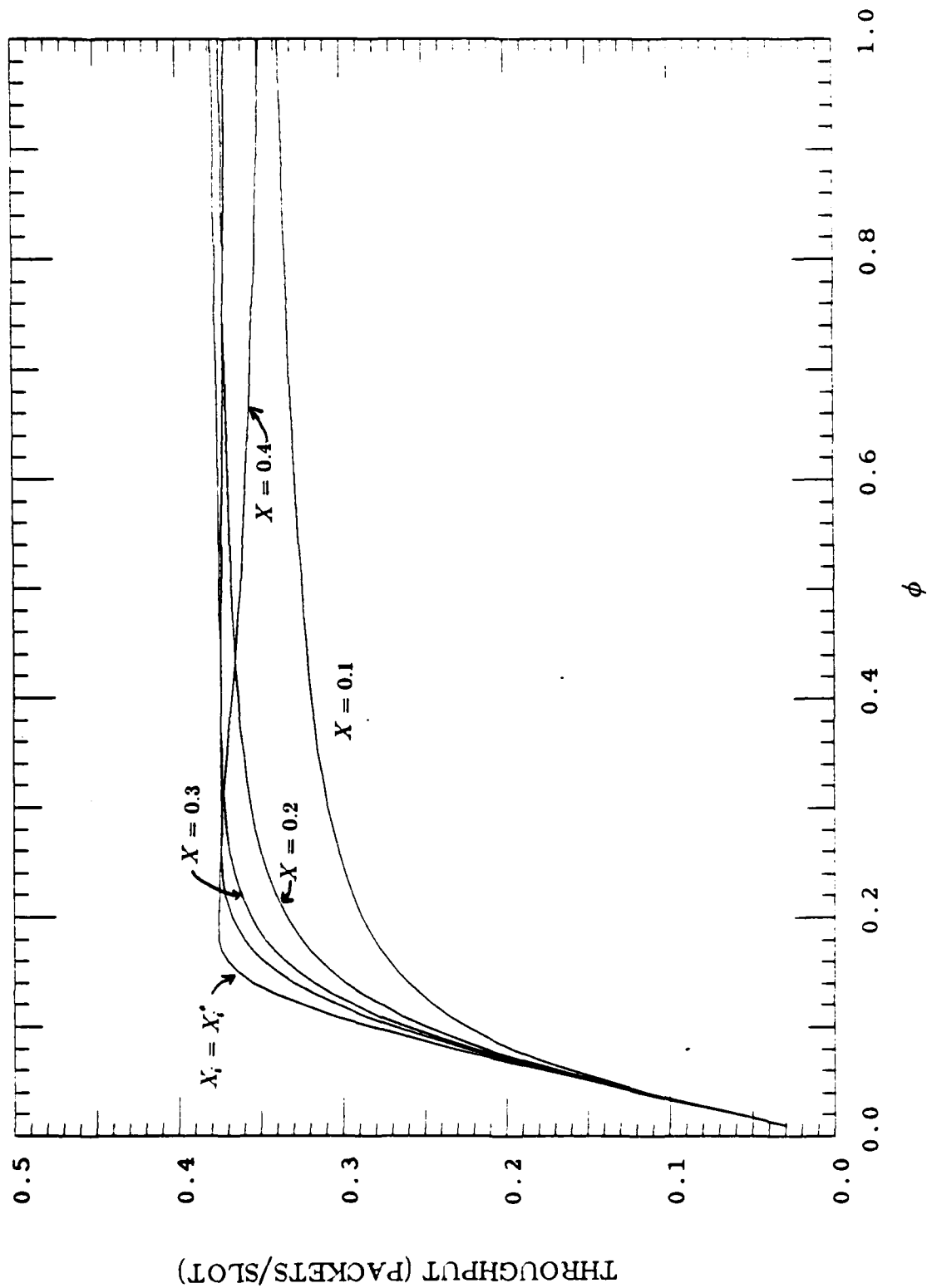


Fig. 10 Throughput vs ϕ for Framed ALOHA; No Capture;
 $L = 10, M = 30, 0.1 \leq X \leq 0.4$.

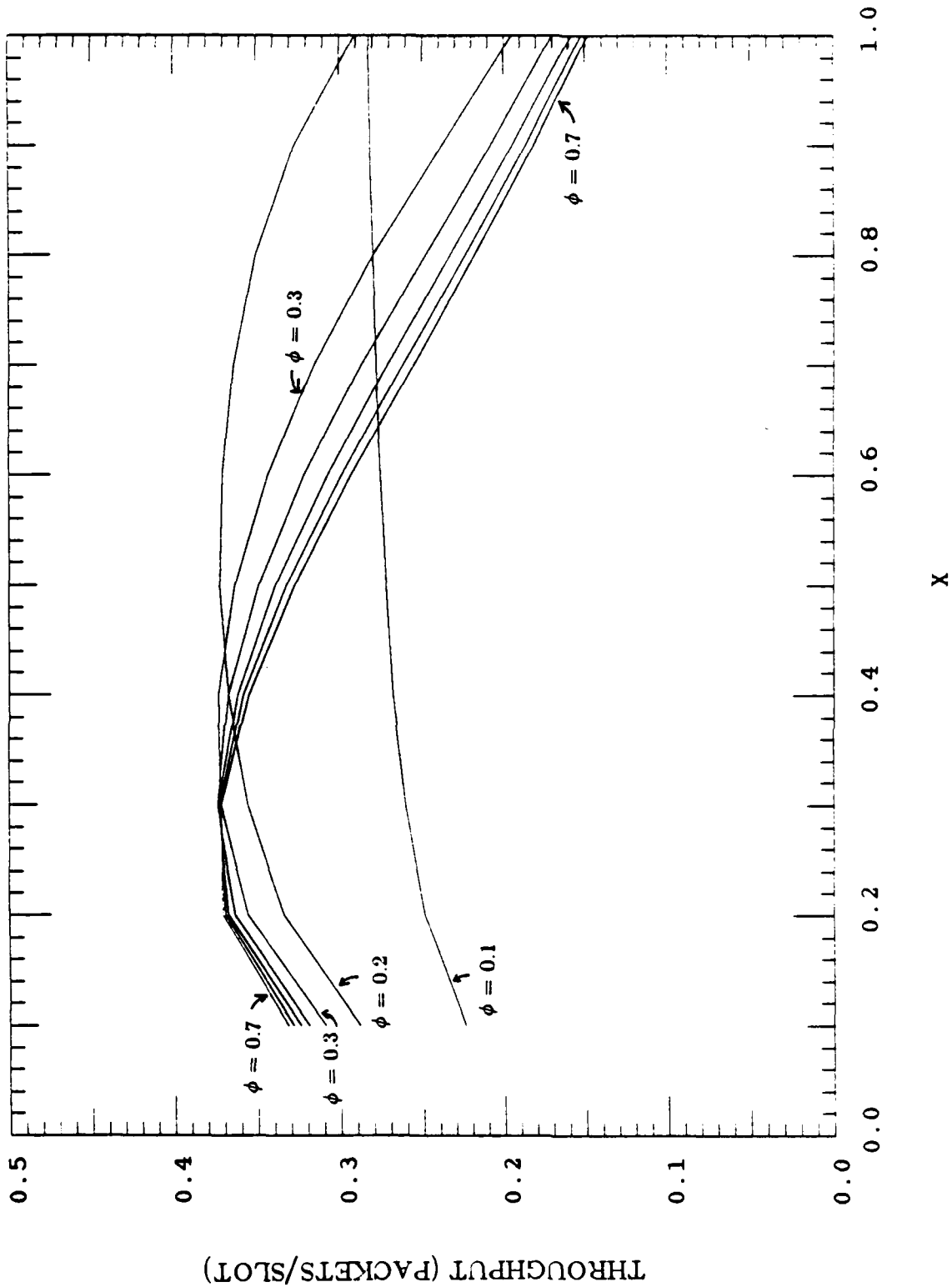


Fig. 11 Throughput vs X for Framed ALOHA; No Capture;
 $L = 10$, $M = 30$, several values of ϕ .

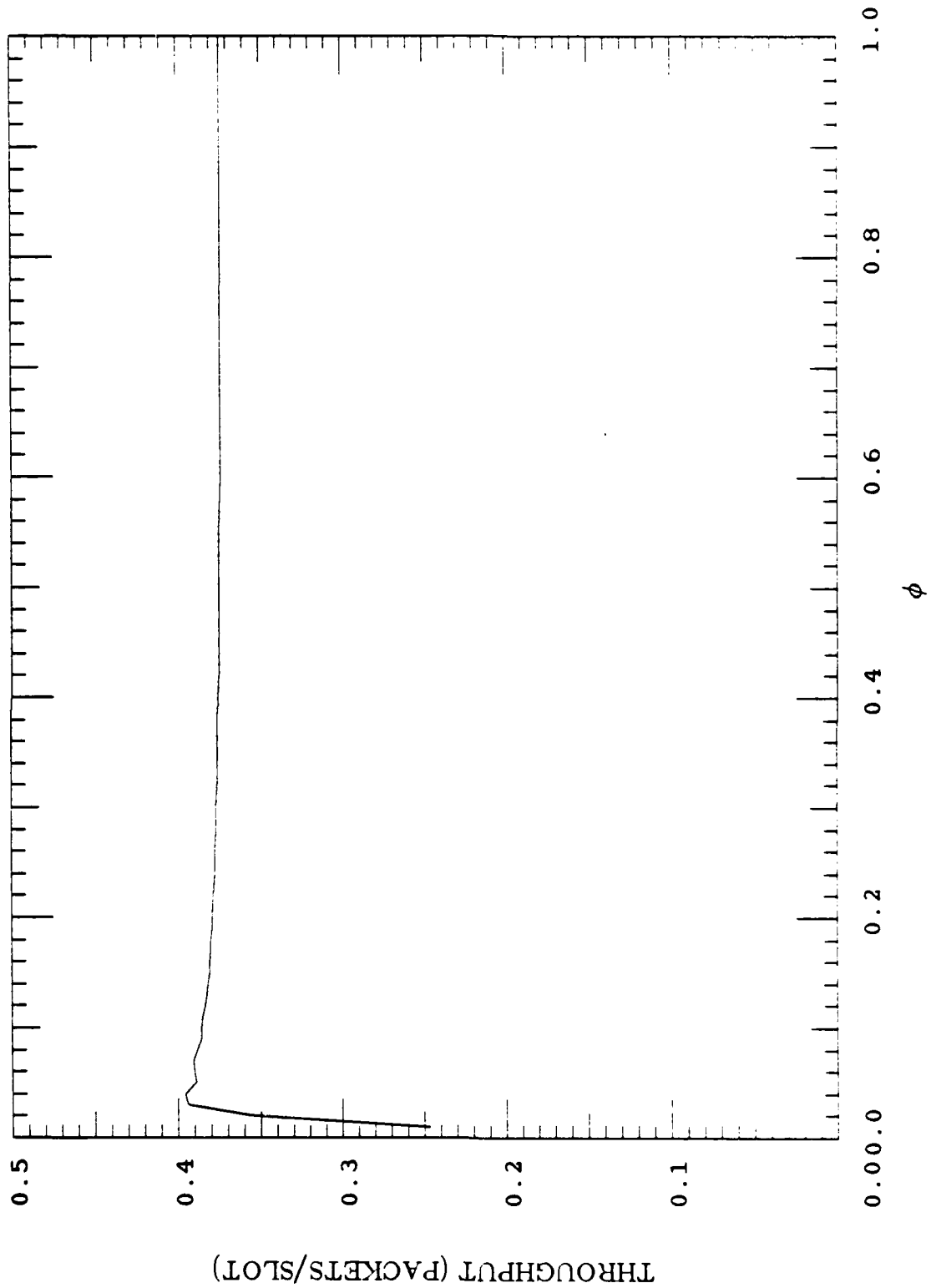


Fig. 12 Throughput vs ϕ for Framed ALOHA; No Capture; $M = 30$;
 L_i is chosen to maximize throughput.

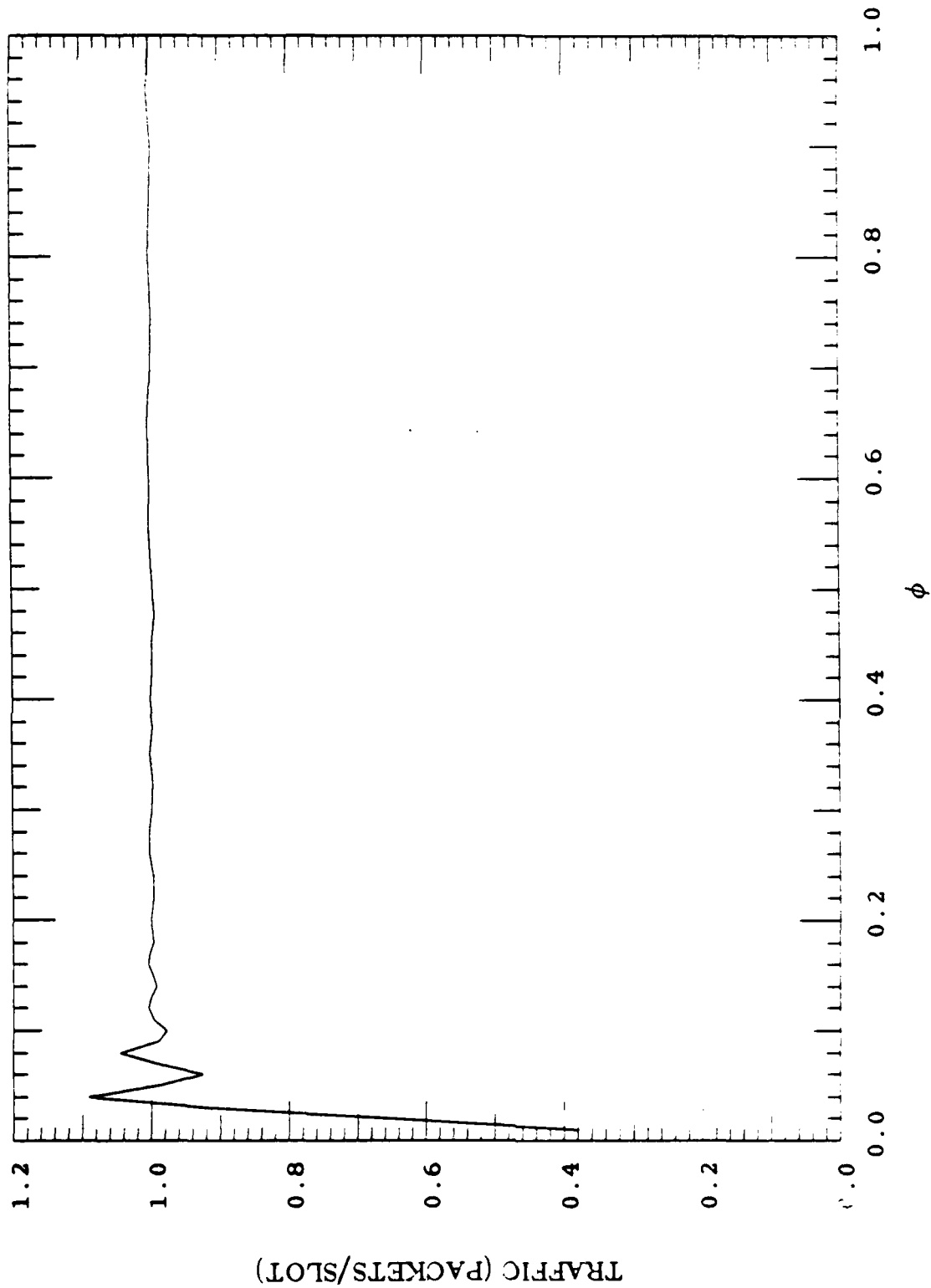


Fig. 13 Traffic vs ϕ for Framed ALOHA; No Capture; $M = 30$;
 L_i is chosen to maximize throughput.

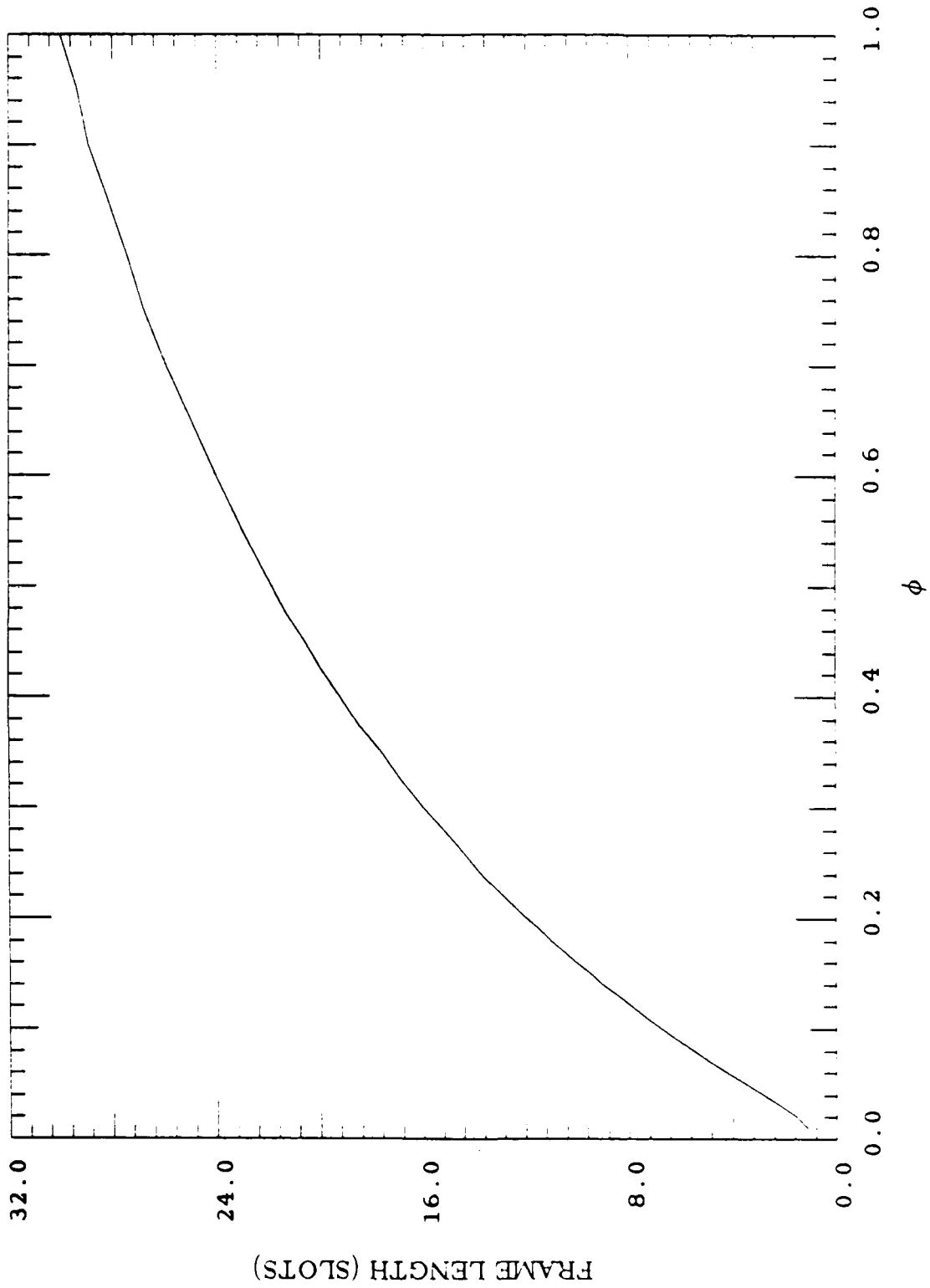


Fig. 14 Expected frame length vs ϕ for Framed ALOHA; No Capture; $M = 30$; L_i is chosen to maximize throughput.

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