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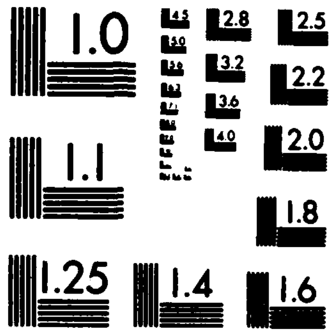

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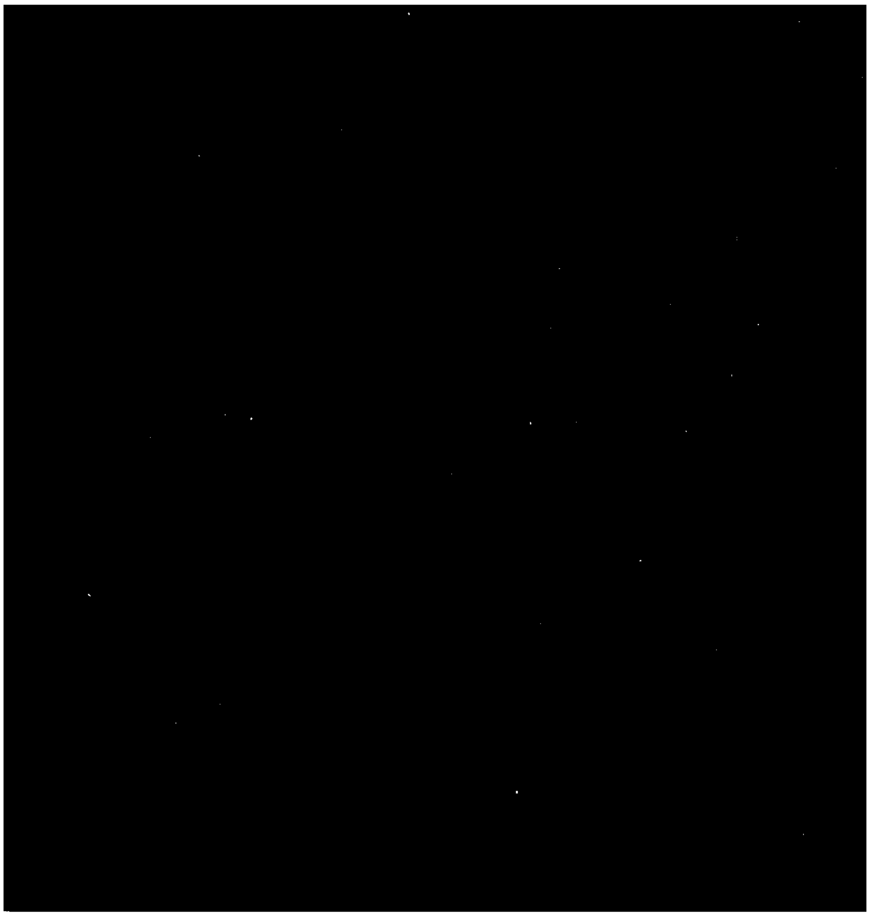

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Progress in the theory and applications of Vandermonde decomposition is briefly summarized. The broadband direction finding application is outlined. A step-by-step description of the factorization algorithm is given. Efforts to localize broadband acoustic transients are summarized.

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I. INTRODUCTION AND EXECUTIVE SUMMARY

A. Objective, Tasks, and Phases

The objective of this project was to study signal processing methods for broadband direction finding and transient waveform analysis based upon the technique of Vandermonde matrix factorization. There were two main tasks.

Task 1 Study the properties and behavior of the Vandermonde factorization method for finding the directions of distant targets that emit transient waves.

Task 2 Investigate the use of Vandermonde methods to resolve modal components of acoustic transient signals, e.g., echoes from mechanically resonant targets insonified by a broadband, active sonar.

Research efforts to carry out these tasks were begun in June 1985 and continued until October 1986. Those efforts were organized into the following five phases.

Phase 1 Formalize the theory of Vandermonde factorization, with particular emphasis upon the specific applications named above.

Phase 2 Groom the mechanics of the Vandermonde matrix factorization for matrices of the type likely to arise in those applications.

Phase 3 For the broadband direction finding application, generate data sets to test direction finding performance as a function of physical parameters and system order. (As further noted below, this phase was terminated after it was determined that the array dimensions, number of floating point operations required, and number of weeks needed to carry out the work were too large to be cost-effective.)

Phase 4 Test the use of Vandermonde factorization for modal analysis of transient waveforms of the type seen in acoustic echoes from resonant objects excited by broadband active sonar transmissions.

Study the effect of using realistic transmissions (i.e., not ideal impulses) whose durations are not infinitesimal as compared with the decay times of the target resonances, and whose spectral characteristics might therefore interact with those of the target.

Phase 5 Demonstrate the application of the Vandermonde factorization to directional analysis of isolated broadband transients using realistic parameter sets, using the lowest order version of the broadband direction finder.

The technical results of each of these phases of research are summarized below in Section II.

B. Personnel

Primary personnel engaged in the research efforts were the following.

T. L. Henderson - Principal Investigator
R. S. Bailey - Research Engineer
S. G. Lacker - Student

C. Publication of Results

The results of the research effort were presented to the scientific and technical community as follows.

(1) One technical paper was published in full in the Proceedings of the Nineteenth IEEE Asilomar Conference on Circuits, Systems, and Computers, after having been presented at that conference on November 7, 1985.[1]

(2) One paper was presented at the Fall 1986 meeting of the Acoustical Society of America.[2]

(3) One technical paper was prepared for submission to The Journal of the Acoustical Society of America.[3]

D. General Summary

The Vandermonde factorization was found to be useful not only as a concept but as a computational tool for applications involving spectral analysis of transients. Application to the analysis of transient responses of resonant objects to broadband active sonar pulses (pulse-compressed FM chirps, uncompressed FM chirps, and ideal bandpass filter transient responses) was investigated, with interesting and, in some cases, unexpected results, as summarized below in the summary of Phase 5.

The "signal subspace" approach that is implicit in the Vandermonde factorization method of transient analysis does have some known limitations. One of them is the requirement that the signal dimensionality K (the number of spectral components or "poles") must be finite; indeed, K must be of modest size, in theory, to permit the necessary matrix processing. Another drawback is one that afflicts all of the "modern spectral analysis" methods: The inherent nonlinearity of the analysis technique makes it difficult to predict performance against signals for which the method has not been explicitly tested, even when the new signals are linear combinations of those for which the performance is already well understood.[4]

The broadband direction finding application was seen to be useful, at least for the case of a single source direction of arbitrary spectral output. Unfortunately, when the number of sources K becomes large one must increase the dimension of the signal processor correspondingly, and that can place severe demands upon the aperture weighting functions and the frequency responses required for signal conditioning. However, for multiple sources that emit isolated, non-simultaneous broadband transients (so that K=1 at any given time and a processor of low dimensional order is therefore adequate), the results were very encouraging. Those results were incorporated in one of the technical papers that was submitted for publication.[3]

The results of each of the phases of research are summarized below, following a tutorial introduction to the basic theory.

II. TUTORIAL INTRODUCTION

A. Vandermonde Factorization

Given a set $\{z_k : k=1,\ldots,K\}$ of nonzero, distinct numbers (real or complex) we define the M×K Vandermonde matrix **V** whose (m,k)th element is $(z_k)^{m-1}$. This definition extends the classical definition of a square Vandermonde matrix, which is well known to be nonsingular. It follows that a Vandermonde matrix can never have deficient rank since it contains a nonsingular, square, Vandermonde submatrix having the same number of rows or columns (whichever is smaller). An M×N matrix **A** is said to admit a Type 2 Vandermonde matrix factorization of order K if **A**=**VC**, where **V** is an M×K Vandermonde matrix and **C** is a K×N constant matrix. An M×1 vector function **a**(t) is said to admit a Type 1 Vandermonde matrix factorization of order K if **a**(t)=**Vc**(t) over some specified time domain, where **c**(t) is a K×1 vector function.

B. Application to Transient Spectral Analysis

The Vandermonde factorization is useful in certain problems where observed "signals" (e.g., sonar hydrophone outputs) are known to lie within a signal subspace of finite dimensionality which, when identified, reveals useful information about the source(s) of the signals. For example, suppose a transient waveform a(t) admits a spectral decomposition of order K,

$$a(t) = \sum_{k=1}^{K} d_k \exp(s_k t) \quad , \tag{1}$$

where the unknown parameters d_k's and s_k's are allowed to be complex. Then if a *data matrix* **A** is constructed from a set of uniformly spaced data samples {a(t): t=1,2,..., etc.} as $[\mathbf{A}]_{m,n} = a(m+n-2)$, then **A** satisfies the Type 2 factorization **A**=**VC**, where **V** is the M×K Vandermonde matrix generated from the set $\{z_k=\exp(s_k):k=1,2,\ldots,K\}$ and **C** is the $K \times N$ matrix whose (k,n)th element is $d_k(z_k)^{n-1}$. It follows that when **A** is constructed from sampled values of any observed transient waveform a(t) of the form of Eq. (1) and its Vandermonde factorization is then determined, the set $\{z_k\}$ will be revealed in the second row of V. (It is required that M>K in order to achieve a unique factorization that reveals the z_k's.) The waveform's resonant "poles" s_k, whose imaginary parts express the resonance frequencies and whose real parts specify the exponential decay rates, can then be computed as $s_k=\log(z_k)$. The amplitude/phase coefficients d_k of the individual modal resonances can then be computed as $d_k=[C]_{k,n}\, z_k{}^{1-n}$ for any selected value of n.

As applied to the analysis of transient waveforms, the Vandermonde factorization amounts to an extended formalization of classical Prony/Hankel techniques, and leads to some useful insights regarding implementation (as exemplified by the Vandermonde factorization procedure that is briefly described below in the results summary of Phase 1).

C. Application to Broadband Direction Finding

The broadband direction finding application is based upon the use of a line hydrophone that is equipped to provide simultaneously a set of M different outputs $\{b_m(t):m=1,2,\ldots,M\}$, each of which is extracted with a different aperture weighting function $w_m(z)$ and a different signal conditioning filter whose impulse response is $h_m(t)$, i.e.,

$$b_m(t) \quad = \quad h_m(t) * \int_{-\infty}^{+\infty} w_m(z)\; p(z,t)\, dz \quad , \qquad (2)$$

where p(z,t) denotes the incident acoustic pressure at time t and position z along the line hydrophone's axis. To guarantee the desired direction finding behavior, the $w_m(z)$'s (which have to be functions of bounded support if the hydrophone aperture is finite) and the $h_m(t)$'s must obey

$$w_{m+1}(z) = d[w_m(z)]/dt \qquad (3)$$

$$-c\,h_m(t) = d[h_{m+1}(t)]/dt \qquad (4)$$

for m=1,...,M-1, where c denotes the speed of sound. It follows that

$$b_{m+1}(t) = h_{m+1}(t) * \int_{-\infty}^{+\infty} \{d[w_m(z)]/dz\}\, p(z,t)\, dz \qquad (5)$$

$$= -h_{m+1}(t) * \int_{-\infty}^{+\infty} \{\partial[p(z,t)]/\partial z\}\, w_m(z)\, dz \ , \qquad (6)$$

where integration by parts is used in the last step (capitalizing upon the fact that $w_m(z)$ has bounded support). If the incident sound consists of a single plane wave coming from a very distant source whose direction cosine is u (relative to the positive z axis), then the acoustic pressure p(z,t) can be expressed as a function of the single variable (t+uz/c). This means that

$$\partial[p(z,t)]/\partial z = (u/c)\times \partial[p(z,t)]/\partial t \ , \qquad (7)$$

which when substituted into Eq. (6) gives the result

$$b_{m+1}(t) = (u/c)\, h_{m+1}(t) * (d/dt) \int_{-\infty}^{+\infty} p(z,t)\, w_m(z)\, dz \quad . \qquad (8)$$

Differentiation and convolution are commutative, so the time differentiation can be applied to $h_{m+1}(t)$ instead of the integral. Using Eq. (4) and Eq. (2), the following simple result is then obtained:

$$b_{m+1}(t) = u\, b_m(t) \qquad (9)$$

for all t, for m=1,2,...,M-1. This result can be put into vector form: Let **b**(t) denote the M×1 vector $[b_1(t), b_2(t),...,b_M(t)]^T$ so that

$$\mathbf{b}(t) = [1, u, u^2, \ldots, u^{M-1}]^T\, b_1(t) \, . \qquad (10)$$

It follows by linear superposition that if there are K different sound sources with direction cosines $u_1, u_2, \ldots, u_K$, and whose individual contributions to $b_1(t)$ are denoted as $c_1(t), c_2(t), \ldots, c_K(t)$, then

$$\mathbf{b}(t) = \sum_{k=1}^{K} [1, u_k, (u_k)^2, \ldots, (u_k)^{M-1}]^T\, c_k(t) \, , \qquad (11)$$

i.e.,

$$\mathbf{b}(t) = \mathbf{V}\mathbf{c}(t) \quad , \qquad (12)$$

where **V** is the M×K Vandermonde matrix whose (m,k)th element is $(u_k)^{m-1}$ and $\mathbf{c}(t)=[c_2(t), \ldots, c_K(t)]^T$. Therefore, the Type 1 Vandermonde factorization of the output vector **b**(t) reveals the direction cosines of all K wave sources (the u_k's appear in the second row of **V**). This direction finding process works for waves of arbitrary spectra and bandwidths. It is required that M>K; i.e., M must be at least as large as K+1, where K is the number of wave sources that are present.

III. RESEARCH SUMMARIES FOR EACH PHASE

A. Phase 1: Formalization of the Theory of Vandermonde Factorization

The basic theory of Vandermonde factorization was formalized in terms of block-Hankel and unit-parallelogramic matrices. The details are presented in Ref. 1. The mathematical developments were quite successful, and led to a self-contained set of useful rules and formulas for manipulating the matrices and understanding the underlying structure of the factorization. Unfortunately, time did not permit the development of theory to predict performance in the presence of noise. Because of the inherent nonlinearity involved in solving $\mathbf{A}=\mathbf{VC}$ for a constrained $\mathbf{V}$ and $\mathbf{C}$, and the dependence of the solution upon null-eigenvectors of singular matrices, for which perturbation theory is not very well developed, such an analysis would have been a major undertaking beyond the scope of the project.

B. Phase 2: Mechanics of Factorization

1. Procedure for Type 2 Factorization

Reference 1 describes the procedure that we ultimately developed and tested for obtaining the Type 2 Vandermonde factorization of order K of a given M×N matrix $\mathbf{A}$. It utilizes the singular value decomposition (SVD), and is an improved and extended version of an SVD-based procedure proposed in Ref. 5. The procedure consists of the following steps.

Step 1: Solve the homogeneous equation $\mathbf{A}^H\mathbf{X}=\mathbf{0}$ for an M×(M-K) matrix $\mathbf{X}$ whose columns are linearly independent. ($\mathbf{A}^H$ denotes the conjugate transpose of A.) In practice we perform the SVD of $\mathbf{X}$ and take the singular vectors corresponding to the smallest M-K singular values, and use them as the columns of $\mathbf{X}$.

Step 2: Find the (unique) unit-parallelogramic matrix **P** (a unit-parallelogramic matrix is one whose main diagonal elements all have unit value, and for which all elements above the main diagonal or below the bottom diagonal are zero) whose column space matches that of **X**. In practice this is carried out by performing the following substeps.

Step 2a: Perform forward, Gaussian elimination by columns (not by rows as is usually done) with partial pivoting (in accordance with Algorithm 2.12 of Ref. 6, modified for column-wise rather than row-wise elimination), to reduce all elements above the main diagonal to zero.

Step 2b Perform backward, simple Gaussian elimination by columns, without column interchange, to produce zeroes below the bottom diagonal without disturbing the zeroes above the main diagonal. The pivot elements are guaranteed to be nonzero (see Appendix of Ref. 5), as are the main diagonal elements of the resulting matrix, which furthermore preserves the column space of **X** by virtue of its having been created entirely by column operations.

Step 2c Scale the columns of the resulting matrix to produce unit values along the main diagonal. The resulting unit-parallelogramic matrix is identified as **P**. This completes Step 2.

Step 3 "Toeplitz-ize" the matrix **P** by averaging its element values along diagonals; i.e., by replacing **P** by $\hat{P}$, where $[\hat{P}]_{i,j}$ is the average of all elements $[P]_{m,n}$ such that $m-n=i-j$. If the original matrix **A** admits a Type 2 factorization *exactly*, i.e., if **A**=**VC** with no allowance for error due to noisy data, then according to Ref. 1 **P** will already be a Toeplitz matrix (in the sense that the value of its (i,j)th element depends only upon the value of i-j) . In practice, however, **P** will only approximate a Toeplitz matrix, and the Toeplitz-izing step is necessary.

Step 4 Define the Kth order polynomial

$$P(z) = 1 + p_1 z + p_2 z^2 + \ldots + p_K z^K \quad , \qquad (13)$$

whose coefficients are $p_i = [P]_{i,1}$. Solve for its roots $\{z_k : k=1,2,\ldots,K\}$.

Step 5 Construct V from the set $\{z_k : k=1,2,\ldots,K\}$ as

$$[V]_{m,k} = (z_k)^{m-1} \qquad (14)$$

for $m=1,2,\ldots,M$ and $k=1,2,\ldots,K$.

Step 6 Compute $C=(V^H V)^{-1} V^H A$. (15)

The Type 2 factorization of **A** is thus complete.

2. Procedure for Type 1 Factorization

Our procedure for performing a Type 1 factorization is to transform it into a Type 2 factorization that has the same Vandermonde matrix V, using samples of the data vector $\mathbf{a}(t)$ at $t=t_1,t_2,\ldots$, etc. In particular, $\mathbf{a}(t)=\mathbf{V}\mathbf{c}(t)$ implies $\mathbf{A}=\mathbf{V}\mathbf{C}$ if we either define

$$[\mathbf{A}]_{i,j} = [\mathbf{a}(t_j)]_i \, ; \quad [\mathbf{C}]_{i,j} = [\mathbf{c}(t_j)]_i \qquad (16)$$

or define

$$\mathbf{A} = \sum_{j=1}^{J} \mathbf{a}(t_j)\mathbf{a}^H(t_j) \quad ; \quad \mathbf{C} = \sum_{j=1}^{J} \mathbf{c}(t_j)\mathbf{c}^H(t_j)\mathbf{V}^H \ . \qquad (17)$$

Therefore, to solve a Type 1 factorization we construct the data matrix **A** from sampled values of the data vector $\mathbf{a}(t)$ in accordance with either Eq. (16) or Eq. (17), and then compute the Type 2 factorization **A=VC**. This gives **V**, and **c**(t) can be computed as

$$\mathbf{c}(t) = (\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H \mathbf{a}(t) \ . \tag{18}$$

3. General Summary of Factorization Experiences

These factorization methods were tested in a variety of cases and appeared to work quite well. However, neither the budget nor the scope of work of the project supported the development and publication of a complete, self-contained, well documented, and thoroughly debugged software package for general distribution. Instead, the method was tested for software parameters (e.g., array dimensions) tailored to the particular examples being investigated, and the procedure was conducted on a step-by-step basis to monitor performance and watch for difficulties such as arithmetic underflow/overflow. Standard software libraries were used for the SVD and polynomial root-solving operations (which had to accommodate complex variables in both the matrix elements and the polynomial coefficients).

C. Phase 3: Parameter Study of Broadband Direction Finding for Steady State Sources

It was intended to develop a procedure for generating case study data for the broadband direction finding application, assuming a modest number of wave sources that emitted broadband, random processes of relatively long duration, within a noisy background of a much larger number of similar but weaker sources in random directions. (We knew of no other way to generate realistic background noise for the special multi-output hydrophone required by our construction.) The task turned out to be much more difficult than expected, due to the proverbial "curse of dimensionality" that sometimes plagues signal processing efforts. The data storage and file handling requirements, as well as the number of floating point operations required, were finally determined to be beyond the scope of the project. Attempts to incorporate approximations and limit the observation interval were not successful enough to reduce the computational effort to an acceptable level, and the effort was abandoned. (Ironically, an approximation technique that might have solved the problem was conceived shortly after the end of the project term.)

D. Phase 4: Studies of Transient Analysis for Active Sonar with Resonant Targets

The Vandermonde factorization procedure was tried out as a means of identifying target resonances. In particular, a sonar target impulse response of the form

$$h(t) = U(t) \times \sum_{k=1}^{K/2} A_k \exp(-\alpha_k t) \cos(2\pi f_k t + \varphi_k) \qquad (19)$$

was assumed, where U(t) denotes the unit step. The resonance frequencies f_k and decay rates α_k would be determined by observing the sonar system output

$$a(t) = \delta(t-\tau) * p(t) * h(t) \quad , \qquad (20)$$

where p(t) denotes either the transmitted waveform (if a broadband impulsive pulse of extremely short duration is used) or its autocorrelation function (if a time-stretched pulse is transmitted and the hydrophone output is matched filtered for purposes of pulse compression) and τ denotes the two-way travel time. Implicit in this equation are several simplifying assumptions:

(1) the target has zero Doppler,

(2) the medium is stable enough that it can be modeled as a time-invariant linear system, and

(3) spreading loss and absorption factors are assumed to be absorbed into the definition of p(t).

For analysis purposes it can be assumed that the echo delay, as expressed by $\delta(t-\tau)$, will have been removed in practice, at least approximately. Any residual delay that is not removed can be absorbed into the definitions of A_k and φ_k anyway, and does not influence the resonance parameters f_k and α_k that are to be determined. If the transmitted sonar pulse has sufficient bandwidth then p(t) will approximate a delta function, so that a(t) will have the form of Eq. (19) to a very good approximation; at least it should have the same set of resonance frequencies f_k and decay rates α_k , perhaps with altered amplitudes A_k and phases φ_k. Indeed, the error that is incurred in making this approximation can be regarded as a form of "noise" in the data a(t), and the primary purpose of our study was to determine the robustness of the Vandermonde factorization technique when applied directly to the sonar output a(t).

At the risk of being redundant we shall elaborate further: Unless the transmitted pulse is extremely impulsive (i.e., very intense and of extremely short duration) to begin with, one would like to deconvolve the transmitted sonar pulse from the sonar echo to recover an unadulterated version of the target impulse response h(t). However, in practice the transmitted pulse must have limited bandwidth and true deconvolution is therefore impossible. If the active sonar system is sophisticated (and expensive) enough, then one can use classical pulse compression as a substitute for true deconvolution; i.e., one can apply a matched filter. The matched filter convolves the sonar echo with the time-reversed transmitted pulse. Mathematically, the effect is as if the autocorrelation function p(t) had been transmitted; hence, the assertion of Eq. (20). The failure of the matched filtering process to perform a true deconvolution has the result that a(t) is a "noisy", i.e., distorted, version of h(t). If the transmitted pulse has sufficient bandwidth to encompass the target resonances adequately, then this distortion should be small, and can be modeled by a perturbation of the amplitude and phase parameters A_k and α_k, with a small amount of additive noise to account for the residual error. The usual reason for transmitting a "stretched pulse" instead of transmitting p(t) in the first place is to maximize signal energy. If it is feasible to transmit a very intense impulse of extremely short duration then matched filtering is unnecessary. In either case, the sonar output is thus *approximately* of the form of Eq. (19), and the Vandermonde factorization method can be used to estimate the target resonance parameters.

Although the form of Eq. (19) appears different from that for which the Vandermonde factorization was applied, it is actually the same as Eq. (1) with $s_k = -\alpha_k + j2\pi f_k$, $s_{k+K/2} = s_k^*$, $d_k = A_k \exp(j\varphi_k)$, and $d_{k+K/2} = d_k^*$ for $k = 1,2,\ldots,K/2$. A target with K/2 resonances thus has K spectral components or "poles", which appear in conjugate pairs. In accordance with the procedure described in Section II.B above, we formed the data matrix A whose (m,n)th *element is defined to be* a(m+n-2). In so doing, we were assuming that the time axis has been scaled so that it measures t in units of the data sampling interval. In the examples to be described below we assumed a sampling interval of 1 ms; i.e., t expresses time in units of milliseconds. The sampled values of a(t) can be expressed as

$$a(n) = \sum_{k=1}^{K} d_k z_k^n \quad , \tag{21}$$

where

$$z_k = \exp(s_k) = \exp(-\alpha_k + j2\pi f_k) \quad . \tag{22}$$

The data matrix A must admit the Type 2 factorization $A = VC$, where the z_k's appear in the second row of the M×K Vandermonde V. The target resonance parameters α_k and f_k are thus determined entirely by $|z_k|$ and $\arg(z_k)$, and the accuracy with which one has determined them can be measured in terms of the relative error in $|z_k|$ and the angular error in $\arg(z_k)$.

To have some basis of comparison with existing techniques the z_k's were also determined by a least-squares version of the Prony method; specifically, the equation $AA^T\mathbf{x}=0$ was solved for an $M\times 1$ vector $\mathbf{x}$ (in practice, by finding the unit vector that minimized the norm of $AA^T\mathbf{x}$; see Ref. 5) and the z_k's were then identified as a subset of the M-1 roots of the polynomial $(1,z,z^2,\ldots,z^{M-1})\mathbf{x}$. In selecting the proper roots it was, of course, helpful to know the correct answers (a circumstance that was only possible because the data were artificially generated with known resonance parameters).

Several different waveforms parameter sets were used for study purposes, using even values of K up to 6. To summarize the results we shall discuss the performance in selected cases that used a target transient response h(t) comprising two strongly damped resonances (K=4) at $f_1 = 430$ Hz and $f_2 = 280$ Hz, with decay times of $(\alpha_1)^{-1} = 2$ ms and $(\alpha_2)^{-1} = 3.333$ ms, and with initial amplitudes of 1.0 and 1.33, respectively.

To test the computational methods a 9×12 data matrix A was first constructed from h(t) directly, with no distortive convolution. As expected, the Vandermonde factorization method identified the four z_k's (in two conjugate pairs) with great precision. The least-squares Prony method produced eight z_k's, of which half were at the correct locations. Then white noise was added to the sampled data, in an amount adjusted to perturb the computed z_k's slightly. The perturbations in the z_k's were virtually the same for both the Vandermonde factorization and least-squares Prony methods. (The observed perturbations in the higher resonance frequency were proportionately larger than those of the lower resonance frequency.)

These results were in agreement with the proffered performance advantage of the Vandermonde factorization method: It determines the z_k's with a degree of precision comparable to that obtained with a least-squares Prony analysis of higher order, but without the requirement for selecting the correct z_k's from among the extraneous "noise poles".

Following this initial test, the performance was tested for convolved versions of h(t), using a 15×36 form of the data matrix A. The results for three special cases are summarized below. In each case, convolution was performed by sampling h(t) and p(t) at a much higher frequency (10 kHz) to avoid finite-sampling-rate effects, and using 409.6 ms segments of data to avoid windowing effects. Convolution was actually performed by using an 8192-point FFT.

<u>Case 1</u> It was assumed that the sonar transmitted a linear FM "chirp" of 61.5 ms duration, sweeping from 150 Hz to 800 Hz (time×bandwidth = 40). The autocorrelation function of this chirp (i.e., the result of convolving the chirp with its time-reversed self) became p(t); its approximate duration was 1.5 ms (since the matched-filter-compressed pulse always has a time×bandwidth product of approximately one). This p(t) was convolved with h(t). To avoid contaminating the data with the echo components still being generated while the effective transmitted pulse p(t) was "active", the initiation of data sampling was delayed for 1.5 ms beyond the onset of signal.

The Vandermonde factorization method identified the higher resonance frequency well (within 2%) but overestimated its decay time constant by over 30%. On the other hand, the decay time constant of the lower resonance was accurately estimated, but its frequency was underestimated by about 15%.

The seven conjugate pairs of z_k's produced by the least-squares Prony method were scattered around the left half of the unit circle in the complex z-plane; however, those seen to be closest to the actual, known values were closer than the z_k's measured by the Vandermonde factorization method.

Case 2 For the next exercise it was assumed that the sonar system "forgot" to perform the matched filtering operation, so that a(t) was, itself, a 61.5 ms chirp with two very broad resonance peaks (broad because of the high degree of damping) at the two resonance frequencies. Thus a(t) did not even remotely approximate Eq. (19), being dominated by the FM character of the transmitted pulse rather than by the system resonances, and one would expect that the z_k's would be improperly identified. Indeed, the least-squares Prony method gave a looping pattern of z_k's (as viewed in the complex plane) that bore no relation to the actual system resonances. Surprisingly, however, the Vandermonde factorization method identified both resonance frequencies (within 3%), although it failed to identify the decay rates (indeed, it perceived the resonances to be undamped).

The ability of the Vandermonde factorization method to find the resonances under these extraordinary conditions is noteworthy, but unexplained. The performance continued to be unexpectedly good (although not always *as* good) even when the relationship between sampling rate and resonance frequencies was varied, and the number of resonances was increased to three.

Case 3 To test the ability of the Vandermonde factorization to accommodate sharp-cutoff bandpass filtering it was assumed that p(t) was the impulse response of an ideal bandpass filter whose cutoff frequencies were 150 Hz and 800 Hz, the same as the limits of the FM chirp used before. Since the energy spectrum of the chirp is almost flat over this frequency range and small elsewhere, its autocorrelation function (which is the inverse Fourier transform of the energy spectrum) is very nearly the same as the impulse response of an ideal bandpass filter (which is the inverse Fourier transform of a perfectly flat, band limited, frequency response; i.e., it is a modulated "sinc" function). Therefore, it was expected that the results would be very similar to those of Case 1.

However, the Vandermonde factorization failed entirely to "see" the higher resonance frequency, and estimated both of its resonance frequencies to be near the lower of the two actual resonance frequencies; furthermore, both estimates were too highly damped (one was very highly damped). On the other hand, the least-square Prony method did spectacularly well in the sense that a subset of its z_k's fell almost exactly upon the correct values.

Perhaps the following interpretation explains this unexpected behavior: Apparently there were enough extraneous poles (there were 10) to model the characteristics of the bandpass filter, and to take the "stress" off the target poles. On the other hand, since it did not have these extra degrees of freedom, the Vandermonde factorization was forced to compromise between modeling the target resonances and modeling the bandpass filter's characteristics, with disruptive results.

The examples of transient spectral analysis demonstrated that Vandermonde factorization could identify spectral components of transients of very short duration. However, as regards analysis of sonar target resonances, the scope of the project did not permit an extensive investigation of performance. Only a preliminary study of target resonance analysis could be made, and it raised as many questions as it answered. The surprising ability of the Vandermonde factorization method to extract target resonances buried in an FM chirp of long duration was even more curious than its poor performance in the presence of a sharp-cutoff bandpass filter.

E. Phase 5: Direction Finding for Isolated Broadband Transients

As the broadband direction finding application was studied it appeared more and more likely that the most useful applications were those of small order, in particular, M=2. For that case only one sonar target can be located (i.e., K=1), and the broadband direction finder has little to offer in the classical case of passive tracking of stationary targets emitting stationary, broadband random process of very low signal-to-noise ratio. For that environment a crosscorrelation receiver pair works well, and has the ability to resolve multiple targets.

However, for targets emitting brief, albeit intense, transient sounds the broadband direction finder is useful, since it can be configured for M=2 if it can be assumed that the target emissions are so brief that only one occurs at a given time (i.e., K=1 so that M=2 gives M>K as required).

For M=2 the Vandermonde Type 2 factorization is trivial. (The singular vectors of **A** are the eigenvectors of the 2×2 matrix $\mathbf{AA}^H$, and therefore a simple closed form solution for the 2×1 "matrix" **X** of Step 1. All that is required to get **P** is to scale **X** so that its top element is 1. The "polynomial" of Eq. (13) is therefore of first order. It follows that a closed form solution for z_1 ($=u_1$) can be written in terms of the three distinct elements of the Hermitian, 2×2 matrix $\mathbf{AA}^H$. The algebra is straightforward, if tedious.

However, for M=2 the structure is simple enough that an alternate method for determining the direction of acoustic transients then becomes attractive. It is based upon the observation that if the two outputs $b_1(t)$ and $b_2(t)$ are applied to the horizontal and vertical plates of a cathode ray tube (CRT), then the following behavior will be exhibited: In the absence of directional signals the spot will hover near the center of the screen, producing a "fuzzy spot" due to random noise. If an acoustic transient arrives from a target at direction cosine u, then the displayed spot will deflect along the line $b_2 = u b_1$ in accordance with Eq. (9). The length of the trace will be proportional to the incident wave's amplitude, but the incident wave's spectrum has little relevance; the slope u indicates target direction.

Suppose that the observed slope is "quantized"; i.e., the CRT is divided into a large number of uniformly spaced wedges, each covering a small span of values of the slope angle, $\tan^{-1}(u)$. Suppose that for each such wedge there is a separate power detector that produces no output unless the spot falls into the appropriate wedge, whereupon the detected power output is $b_1^2 + b_2^2$. If all of these *zonal power detectors* are connected through a bank of identical smoothing filters to a multichannel recorder it will provide a kind of histogram record of the directional energies of incident wave transients, as a function of time.

It is even possible to scale and/or transform the display coordinates to "circularize" the background noise's fuzzy spot so that the distribution of energies due to background noise will be equal in all of the zones. With this normalization, the presence of a weak target can be more easily perceived. This coordinate transformation has a mildly distortive effect upon the mapping from the wedge zones to actual target angle, but is not a serious problem.

In a rather extensive study, this concept was developed in detail in a full technical paper that was submitted for publication (see Ref. 3). The study also included tests of performance using computer-generated data.

The results were very encouraging, and indicated that this proposed signal processing system might provide an attractive alternative for interception of directional transients of arbitrary spectra and brief duration, with enough signal energy to give an instantaneous signal-to-noise level of sufficient size.

IV. CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

The basic principles of Vandermonde factorization were developed without encountering any fundamental flaws in the concept. A step-by-step factorization method was defined and tested successfully. The method worked when applied to determining target resonances in an active sonar application (using computer-generated data), but with some unexpected and unexplained behavior. Our limited efforts did not prove that Vandermonde factorization is the method of choice for extracting target resonances from echoes elicited by sonar transmissions of very brief (either before or after pulse compression) duration.

The ability of the Vandermonde method to extract target resonances from uncompressed echoes elicited by FM chirp transmissions of long duration were encouraging. Moreover, significant progress was made on the problem of observing brief acoustic transients of arbitrary spectra, using systems of such low order that the actual Vandermonde factorization became unnecessary, so that a directional-histogram approach was a feasible alternative. The results of this new approach to acoustic transient interception were very interesting.

Recommended for further research are the following.

(1) Analysis of the effects of noisy data upon the Vandermonde factorization.

(2) Development of a well documented Vandermonde factorization software package for general distribution.

(3) Further studies of performance of Vandermonde factorization in target resonance analysis for both compressed pulses and uncompressed FM chirps.

(4) Advanced testing of the directional-histogram approach to acoustic transient interception, using real acoustic data.

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