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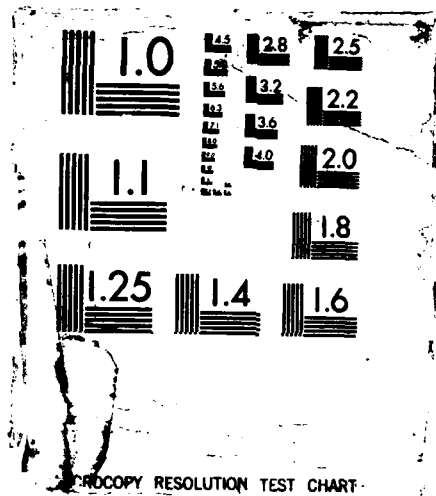
DEVELOPMENT OF SYMBOLIC COMPUTATION METHODS FOR
NONLINEAR DYNAMICS (U) CORNELL UNIV ITHACA NY DEPT OF
THEORETICAL AND APPLIED MECHAN. R H RAND 14 JUL 87
AFOSR-TP-87-1344 SAFOSR-84-0311 F 5 12/5

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Under the supervision of principal investigator R.H. Rand, software has been written in MACSYMA which automatically performs normal form computations for systems of nonlinear nonautonomous differential equations. Together with postdoctoral research associate W.L. Keith, we have produced a package which permits the user to perform Taylor expanded near-identity transformations with unevaluated coefficients on a system of autonomous ODE's (valid to terms of arbitrary order), and then to choose the transformation coefficients so that the resulting system is in normal form. A summary of this work, including the program listing has been published. This work has been applied to the nonlinear parametric stiffness control of flexible systems by Professors Moon and Rand, and to the dynamics of coupled van der Pol oscillators, by graduate student T. Chakraborty. This work is currently being extended to cover a class of nonautonomous periodic systems by graduate student J. Goon. In this case the computer algebra code performs both Taylor & Fourier					
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22a. NAME OF RESPONSIBLE INDIVIDUAL Brian W Woodruff Maj			22b. TELEPHONE (Include Area Code) 707-5027		22c. OFFICE SYMBOL NM

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Cornell University

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AFOSR-TR. 87-1344

20 May 1987

14 July 1987

Ms. Paulette Bowman
Air Force Office of
Scientific Research
Building 410
Bolling AFB, DC 20332

Dear Ms. Bowman:

RE: AFOSR-84-0311
R.H. Rand

Enclosed you will find a copy of the final technical report for the referenced grant. The report was transmitted to Captain D. R. McGhee on 21 March 1986.

Also enclosed are two copies of DD Form 882 Final Report of Inventions and Subcontracts. The report covers the period 30 July 1984 thru 29 December 1985.

Should you have questions, please feel free to contact me.

Sincerely,

Joseph W. Fitzgerald

Joseph W. Fitzgerald
Grant & Contract Officer

/j

Enclosures



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Applied Mechanics

hurston Hall
Ithaca, NY 14853-1503

607/256-5062

21 March 1986

TO: D.R.McGhee, Capt.,USAF

FROM: R.H.Rand

SUBJECT: Final Scientific Report for Grant No. AFOSR-84-0311

The above report is attached.

R. Rand

Final Report

Grant No. AFOSR-84-0311

"Development of Symbolic Computation Methods for Nonlinear Dynamics"

Principal Investigator: R.H. Rand

Dept. Theoretical & Applied Mechanics
Cornell University, Ithaca NY 14853
607-255-7145

This grant was used to purchase the following equipment:
SYMBOLICS 3670 Mainframe Computer (including 6 Mbytes of memory
and a 470 Mbyte hard disk)
Laser Graphics Printer
Ethernet board
and the following associated software:
MACSYMA
FORTRAN
PASCAL

The equipment has been used to facilitate computation on the following projects:

1. Normal Forms

Under the supervision of principal investigator R.H. Rand, software has been written in MACSYMA which automatically performs normal form computations for systems of nonlinear, nonautonomous differential equations. Together with postdoctoral research associate W.L. Keith, we have produced a package which permits the user to perform Taylor expanded near-identity transformations with unevaluated coefficients on a system of autonomous ODE's (valid to terms of arbitrary order), and then to choose the transformation coefficients so that the resulting system is in normal form. A summary of this work, including the program listing has been published [1].

This work has been applied to the the nonlinear parametric stiffness control of flexible systems by Professors Moon and Rand [2], and to the dynamics of coupled van der Pol oscillators by graduate student T. Chakraborty.

This work is currently being extended to cover a class of nonautonomous periodic systems by graduate student J. Goon. In this case the computer algebra code performs both Taylor and Fourier expansions.

2. Determinacy of Degenerate Equilibria

We have also used MACSYMA on the 3670 to study the problem of the determinacy of degenerate equilibria with linear part

$$(1) \quad x' = y, \quad y' = 0.$$

Takens [3] has shown that any nonlinear autonomous system with such a linear part can be put in the normal form:

$$(2) \quad x' = y + b_2 x^2 + b_3 x^3 + b_4 x^4 + \dots$$

$$(3) \quad y' = a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

Since this system is not structurally stable, it is not clear which terms can be truncated without affecting the topological nature of the flow. Takens showed that if $a_2 \neq 0$, then the flow is topologically equivalent to that of the truncated system:

$$(4) \quad x' = y, \quad y' = a_2 x^2.$$

We extended his result in the case that

$$(5) \quad a_2 = a_3 = a_4 = \dots = a_n = 0, \quad a_{n+1} \neq 0.$$

E.g. we showed that if $a_2 = 0$ but $a_3 \neq 0$, then the flow (2)-(3) is equivalent to the truncated system:

$$(6) \quad x' = y + b_2 x^2, \quad y' = a_3 x^3.$$

The procedure involved transforming from cartesian to polar coordinates in the neighborhood of the singular point and Taylor expanding the polar flow (called blowing up the singularity), then testing for determinacy. If the resulting system is not structurally stable, then we blow it up again. In some cases this required 9 consecutive transformations and Taylor expansions, a computation which could not have been done without a MACSYMA program. This work will soon appear in print [4].

3. Hilbert's 16th Problem

Professors Guckenheimer and Rand have been using MACSYMA to approach Hilbert's 16th problem from a bifurcation point of view. The problem is to find the maximum number of limit cycles exhibited by a system of 2 autonomous ODE's with quadratic right hand sides:

$$(7) \quad x' = a x^2 + b y^2 + c x y + d x + e y + f$$

$$(8) \quad y' = A x^2 + B y^2 + C x y + D x + E y + F$$

This problem has a long history including a paper by Petrovsky and Landis [5] which supposedly showed that the maximum number of limit cycles was 3, but was then claimed to

have errors in it by Moser [6]. The question was illuminated by the recent example of Songling [7], which exhibits 4 limit cycles.

We have been using bifurcation theory to investigate this problem in the following way: We look for the standard type of bifurcations which give rise to limit cycles (Hopfs and saddle connections), but require the bifurcations to be as singular as possible. E.g. in the case of the usual Hopf bifurcation, a single condition generates a single limit cycle. However, we have shown that requiring 2 additional conditions will give a bifurcation involving the birth of 3 concentric limit cycles. This computation pushes the 3670 to its limits, and certainly could not have been done on a smaller machine. Some algebraic expressions which result are 15 Kbytes in length.

4. High Precision Numerical Work on MACSYMA

Professors Parlange and Rand have worked on a problem in scattering theory [8] which involved a slowly convergent series. The leading terms were of order 10^{50} although the series summed to a value of order unity. In order to keep track of the differences between successive terms, we used MACSYMA's ability to work in arbitrary precision. We summed 170 terms, keeping 100 digits for each term. Note that even quadruple precision in FORTRAN would not have been sufficient here.

5. Liapunov-Schmidt Method in Dynamics Problems

Professor Rand and Dr. D. Armbruster have worked on formalizing the Liapunov-Schmidt method to problems involving Hopf and other bifurcations in systems of nonlinear partial differential equations. This method, based on the Fredholm alternative theorem, offers an alternative to traditional perturbation methods such as Linstedt's method (cf. [9]) or the method of averaging.

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R.H.Rand
Research Notes in Mathematics No.94, 181pp.
Pitman Publishing Inc. (1984)

REPORT OF INVENTIONS AND SUBCONTRACTS

(Pursuant to "Patent Rights" (Contract Clause) (See Instructions on Reverse Side.)

1. NAME OF CONTRACTOR/ SUBCONTRACTOR Cornell Univ.		2. NAME OF GOVERNMENT PRIME CONTRACTOR same as 1a.		3. CONTRACT NUMBER AFOSR-84-0311		4. AWARD DATE (YYMMDD) 840730		5. TYPE OF REPORT (check one) ☐ INTERIM ☒ FINAL		6. REPORTING PERIOD (YYMMDD) FROM 840730 TO 851229	
7. ADDRESS (include Zip Code) OSP, 120 Day Hall Ithaca, NY 14853		8. ADDRESS (include Zip Code)		9. AWARD DATE (YYMMDD)		10. AWARD DATE (YYMMDD)		11. AWARD DATE (YYMMDD)		12. AWARD DATE (YYMMDD)	

SECTION I - SUBJECT INVENTIONS

1. NAME OF INVENTOR(S) (Last, First, M.I.)	2. NAME OF EMPLOYER (Last, First, M.I.)	3. ADDRESS OF EMPLOYER (include Zip Code)	4. TITLE OF INVENTION(S)	5. DISCLOSURE NO. PATENT APPLICATION SERIAL NO. OR PATENT NO.		6. ELECTION TO FILE PATENT APPLICATIONS		7. CONFIRMATORY INSTRUMENT OF ASSIGNMENT TO CONTRACTING C.
				UNITED STATES	FOREIGN	UNITED STATES	FOREIGN	
NONE				YES	NO	YES	NO	

8. NAME OF INVENTOR (Last, First, M.I.)	9. NAME OF EMPLOYER	10. ADDRESS OF EMPLOYER (include Zip Code)	11. TITLE OF INVENTION	12. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED.	13. FOREIGN COUNTRIES OF PATENT APPLICATION

SECTION II - SUBCONTRACTS (Containing a "Patent Rights" clause)

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NONE								

SECTION III - CERTIFICATION

7. CERTIFICATION OF REPORT BY CONTRACTOR/SUBCONTRACTOR. (Not required if ☐ Small Business or ☐ Non-Profit organization.) (Check appropriate box)	
8. NAME OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL (Last, First, M.I.) Feocco, Frank J.	9. I certify that the reporting party has procedures for prompt identification and timely disclosure of "Subject Inventions" that such procedures have been followed and that all "Subject Inventions" have been reported
10. TITLE Associate Director Office of Sponsored Programs	SIGNATURE OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL <i>Frank J. Feocco</i>
	870520 DATE (YYMMDD)

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