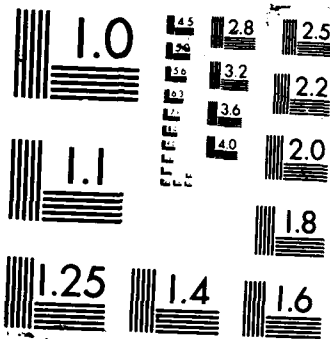


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ANNOTATED COMPUTER OUTPUT

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ANNOTATED COMPUTER OUTPUT FOR SPLIT PLOT DESIGN:
BMDP 2V

W.T. FEDERER, Z.D. FENG, AND N.J. MILES-MCDERMOTT



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ANNOTATED COMPUTER OUPUT FOR SPLIT PLOT DESIGN: BMDP 2V

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ABSTRACT

In order to provide an understanding of covariance analyses for split plot designs, analyses are first conducted on a data set without the covariate. Then an analysis with the covariate is performed. The third example has a different experiment design for the whole plots and the covariate is constant for all split plots within a whole plot. Computer packages may only give portions of the analysis correctly. Some packages require two procedural calls to obtain a portion of the correct results. BMDP 2V falls in the above category.

INTRODUCTION

This is part of a continuing project that produces annotated computer output for the analysis of balanced split plot experiments with covariates. The complete project will involve processing three examples on SAS/GLM, BMDP/2V, SPSS-X/MANOVA, GENSTAT/ANOVA, and SYSTAT/MGLH. Only univariate results are considered. We show here the results from BMDP 2V.

For Example 1, the data are artificial and were constructed for ease of computation; the experiment design for the whole plots is a randomized complete block and the split plot treatments are randomly allocated to the split plot experimental units within each whole plot. Example 2 is the same as Example 1 except that a

covariate varies from split plot to split plot. The data for Example 3 come from an experiment wherein the whole plot treatments were laid out in a completely randomized design and the split plot treatments were randomly allotted to the split plot experimental units within each whole plot. The value of the covariate varies from whole plot to whole plot but is constant for all split plots within a whole plot treatment.

We present the elementary computational steps. Simple hypothetical data are used for the first two examples so that it is easy to provide all detailed computations to illustrate how each number is obtained. Some readers may wish to skip the detailed computations. The third example comes from Winer (1971). The detailed computations are given in his book (p. 803).

Data SP-1

Split plot data with whole plots arranged in
randomized complete block design
(hypothetical data)

Block	Whole plot treatment									
	W1					W2				
	split plot treatment				Total	split plot treatment				Total
	S ₁	S ₂	S ₃	S ₄		S ₁	S ₂	S ₃	S ₄	
1	3	4	7	6	20	3	2	1	14	20
2	6	10	1	11	28	8	8	2	18	36
3	6	10	4	4	24	10	8	9	13	40
Total	15	24	12	21	72	21	18	12	45	96

Total and Means

Blocks (8 observations)			W(whole plots) (12 observations)			S(split plot) (6 observations)		
	Total	Mean		Total	Mean		Total	Mean
1	40	5	W1	72	6	S ₁	36	6
2	64	8	W2	96	8	S ₂	42	7
3	64	8				S ₃	24	4
Grand Total		168				S ₄	66	11
Grand Mean		7						

Model: $y_{ijk} = \mu + \rho_j + \tau_i + \delta_{ij} + \alpha_k + (\alpha\tau)_{ik} + \epsilon_{ijk}$

μ	= mean	τ_i	= effect of whole plot i
ρ_j	= effect of block j	α_k	= effect of split plot k
δ_{ij}	= error (a) effect	$(\alpha\tau)_{ik}$	= effect of interaction of
ϵ_{ijk}	= error (b) effect		whole plot i and split plot k

Analysis of Variance

Source	(*)	df	SS
B (Blocks)	$= R(\rho \mu, \tau, \alpha, \alpha\tau)$	2	48
W (whole plot treatments)	$= R(\tau \mu, \rho, \alpha, \alpha\tau)$	1	24
B×W (error (a))	$= R(\delta \mu, \rho, \tau, \alpha, \alpha\tau)$	2	16
S (split plot treatments)	$= R(\alpha \mu, \rho, \tau, \alpha\tau)$	3	156
S×W (interaction of S and W)	$= R(\alpha\tau \mu, \alpha, \tau, \rho)$	3	84
(**) S×B:W (error (b))	$= R(\epsilon \mu, \alpha, \tau, \alpha\tau, \rho)$	12	112
Total (Corrected for mean)	$= R(\rho, \tau, \delta, \alpha, \alpha\tau, \epsilon \mu)$	23	440
Mean	$= R(\mu)$	1	1176
Total (Uncorrected for mean)	$= R(\mu, \rho, \tau, \delta, \alpha, \alpha\tau, \epsilon)$	24	1616

(*) Notation follows that of Searle(1971). Since the design is balanced, $R(\rho|\mu, \tau, \alpha, \alpha\tau) = R(\rho|\mu)$, etc. The simpler notation is used later.

(**) $S \times B : W$ means $S \times B$ within W .

Calculations of sums of squares:

$$N = 2 \cdot 3 \cdot 4 = 24 \quad , \quad \bar{Y} = 7$$

$$R(\mu, \rho, \tau, \delta, \alpha, \alpha\tau, \epsilon) = \sum_{i=1}^2 \sum_{j=1}^3 \sum_{k=1}^4 y_{ijk}^2 = (3^2 + 6^2 + 6^2 + \dots + 18^2 + 13^2) = 1616$$

$$R(\mu) = N\bar{Y}^2 = 24 \cdot (7)^2 = 1176$$

$$R(\rho, \tau, \delta, \alpha, \alpha\tau, \epsilon | \mu) = 1616 - 1176 = 440$$

$$R(\rho|\mu) = R(\mu, \rho) - R(\mu) = \frac{(40^2 + 64^2 + 64^2)}{8} - 1176 = 1224 - 1176 = 48$$

$$R(\tau|\mu) = R(\mu, \tau) - R(\mu) = \frac{(72^2 + 96^2)}{12} - 1176 = 1200 - 1176 = 24$$

$$\begin{aligned} R(\delta|\mu, \rho, \tau) &= R(\delta, \mu, \rho, \tau) - R(\mu, \rho) - R(\tau, \mu) + R(\mu) \\ &= \frac{(20^2 + 28^2 + 24^2 + 20^2 + 36^2 + 40^2)}{4} - 1224 - 1200 + 1176 \\ &= 1264 - 1224 - 1200 + 1176 = 16 \end{aligned}$$

$$R(\alpha|\mu) = R(\alpha, \mu) - R(\mu) = \frac{(36^2 + 42^2 + 24^2 + 66^2)}{6} - 1176 = 1332 - 1176 = 156$$

$$\begin{aligned} R(\alpha\tau|\mu, \alpha, \tau) &= R(\alpha\tau, \mu, \alpha, \tau) - R(\mu, \alpha) - R(\mu, \tau) + R(\mu) \\ &= \frac{(15^2 + 24^2 + 12^2 + 21^2 + 21^2 + 18^2 + 12^2 + 45^2)}{3} - 1332 - 1200 + 1176 \\ &= 1440 - 1332 - 1200 + 1176 = 84 \end{aligned}$$

$$\begin{aligned} R(\epsilon|\mu, \rho, \delta, \alpha, \tau, \alpha\tau) &= R(\epsilon, \mu, \alpha, \rho, \delta, \tau, \alpha\tau) - R(\mu, \rho, \tau, \delta) - R(\mu, \alpha, \tau, \alpha\tau) + R(\tau, \mu) \\ &= 1616 - 1264 - 1440 + 1200 = 112 \end{aligned}$$

Data SP-2

Data SP-2: Data SP-1 with the following covariate Z which varies with split plot

Covariate (Z)

	whole plot									
	W1				Total	W2				Total
	S ₁	S ₂	S ₃	S ₄		S ₁	S ₂	S ₃	S ₄	
B ₁	1	2	1	2	6	2	0	2	4	8
B ₂	2	2	0	4	8	4	1	3	4	12
B ₃	3	5	2	0	10	3	2	4	7	16
Total	6	9	3	6	24	9	3	9	15	36

Totals and Means

blocks (8 observations)			W (whole plot) (12 observations)			S (split plot) (6 observations)		
	Total	Mean		Total	Mean		Total	Mean
1	14	14/8	1	24	2.0	1	15	2.5
2	20	20/8	2	36	3.0	2	12	2.0
3	26	26/8				3	12	2.0
Grand						4	21	3.5
Total	60	2.5						

$$\text{Model: } Y_{ijk} = \mu + \rho_j + \tau_i + \delta_{ij} + \alpha_k + (\alpha\tau)_{ik} + \beta_1(\bar{Z}_{ij.} - \bar{Z}...) + \beta_2(Z_{ijk} - \bar{Z}_{ij.}) + \epsilon_{ijk}$$

ρ_j = effect of jth block

τ_i = effect of ith whole plot

α_k = effect of kth split plot

β_1 = whole plot regression slope

β_2 = split plot regression slope

δ_{ij} = error a

ϵ_{ijk} = error b

Table of sum of squares and products

Source	df	YY	YZ	ZZ
B	2	48	18	9
W	1	24	12	6
B×W (error a)	2	16	4	1
S	3	156	33	9
S×W	3	84	33	21
S×B:W (error b)	12	112	17	20
Mean	1	1176	420	150
Total	24	1616	537	216

YY column is the same as in SP-1, ZZ column is computed in the same fashion. Thus, only computations for YZ column are illustrated.

$$\begin{aligned} \text{Total}_{YZ} &= \sum_{i=1}^2 \sum_{j=1}^3 \sum_{k=1}^4 Y_{ijk} \cdot Z_{ijk} \\ &= 3(1) + 6(2) + \dots + 14(4) + 18(4) + 13(7) = 537 \end{aligned}$$

$$\text{Mean}_{YZ} = N\bar{Y} \dots \bar{Z} \dots = \frac{168 \cdot 60}{24} = 420$$

$$\begin{aligned} B_{YZ} &= \frac{\sum_{j=1}^3 \left(\sum_{i=1}^2 \sum_{k=1}^4 Y_{ijk} \right) \left(\sum_{i=1}^2 \sum_{k=1}^4 Z_{ijk} \right)}{2 \cdot 4} - 420 = \frac{40(14) + 64(20) + 64(26)}{8} - 420 \\ &= 438 - 420 = 18 \end{aligned}$$

$$\begin{aligned} W_{YZ} &= \frac{\sum_{i=1}^2 \left(\sum_{j=1}^3 \sum_{k=1}^4 Y_{ijk} \right) \left(\sum_{j=1}^3 \sum_{k=1}^4 Z_{ijk} \right)}{3(4)} - 420 = 432 - 420 = 12 \end{aligned}$$

$$B \times W_{YZ} = \frac{\sum_{i=1}^2 \sum_{j=1}^3 (\sum_{k=1}^4 Y_{ijk}) (\sum_{k=1}^4 Z_{ijk})}{4} - 438 - 432 + 420$$

$$= 454 - 438 - 432 + 420 = 4$$

$$S_{YZ}: \sum_{k=1}^4 \frac{(\sum_{i=1}^2 \sum_{j=1}^3 Y_{ijk}) (\sum_{i=1}^2 \sum_{j=1}^3 Z_{ijk})}{2(3)} - 420 = 453 - 420 = 33$$

$$S \times W_{YZ}: \frac{\sum_{i=1}^2 \sum_{k=1}^4 (\sum_{j=1}^3 Y_{ijk}) (\sum_{j=1}^3 Z_{ijk})}{3} - 453 - 432 + 420$$

$$= 498 - 453 - 432 + 420 = 33$$

$$S \times B: W_{YZ}:$$

$$\sum_{i=1}^2 \sum_{j=1}^3 \sum_{k=1}^4 Y_{ijk} Z_{ijk} - 454 - 498 + 432$$

$$= 537 - 454 - 498 + 432 = 17$$

Analysis of Variance and Covariance

Source		df	SS
B (block)	$= R(\rho \mu, \tau)$	2	48
W (whole plot treatment)	$= R(\tau \mu, \rho, \beta_1)$	1	3.4286
Regression (a)	$= R(\beta_1 \mu, \rho, \tau)$	1	16.0
B x W (error (a))	$= R(\delta \mu, \rho, \tau, \beta_1)$	1	0.0
S (split plot treatment)	$= R(\alpha \mu, \rho, \tau, \alpha\tau, \beta_2)$	3	84.243
S x W (interaction of S and W)	$= R(\alpha\tau \mu, \rho, \tau, \alpha, \beta_2)$	3	37.474
Regression (b)	$= R(\beta_2 \mu, \rho, \tau, \alpha, \alpha\tau)$	1	14.450
S x B: W (error (b))	$= R(\epsilon \mu, \rho, \alpha, \tau, \alpha\tau, \beta_2)$	11	97.550

$$\hat{\beta}_1 = B \times W_{YZ} / B \times W_{ZZ} = 4/1 = 4$$

$$\hat{\beta}_2 = S \times B: W_{YZ} / S \times B: W_{ZZ} = 17/20 = 0.85$$

The SS's adjusted by regression on Z are illustrated below:

$R(\rho|\mu) = 48$, remains same since it is not of interest to adjust for Z on the blocks.

$$\begin{aligned} R(\tau, \delta | \mu, \rho, \beta_1) &= (W_{YY} + B \times W_{YY}) - \frac{(W_{YZ} + B \times W_{YZ})^2}{W_{ZZ} + B \times W_{ZZ}} \\ &= (24 + 16) - \frac{(12 + 4)^2}{6 + 1} = 40 - \frac{256}{7} = 3.4286 \end{aligned}$$

$$R(\delta | \mu, \rho, \tau, \beta_1) = B \times W_{YY} - \frac{(B \times W_{YZ})^2}{B \times W_{ZZ}} = 16 - \frac{4^2}{1} = 0$$

$$\begin{aligned} R(\tau | \mu, \rho, \beta_1) &= R(\tau, \delta | \mu, \rho, \beta_1) - R(\delta | \mu, \rho, \tau, \beta_1) \\ &= 40 - \frac{256}{7} - 0 = 3.4286 \end{aligned}$$

$$R(\beta_1 | \mu, \tau, \rho) = \frac{(B \times W_{YZ})^2}{B \times W_{ZZ}} = \frac{4^2}{1} = 16$$

$$\begin{aligned} R(\alpha, \epsilon | \mu, \rho, \tau, \alpha\tau, \beta_2) &= (S_{YY} + S \times B : W_{YY}) - \frac{(S_{YZ} + S \times B : W_{YZ})^2}{S_{ZZ} + S \times B : W_{ZZ}} \\ &= (156 + 112) - \frac{(33+17)^2}{9+20} \\ &= 268 - 86.207 = 181.793 \end{aligned}$$

$$\begin{aligned} R(\alpha\tau, \epsilon | \mu, \rho, \alpha, \tau, \beta_2) &= (S \times W_{YY} + S \times B : W_{YY}) - \frac{(S \times W_{YZ} + S \times B : W_{YZ})^2}{S \times W_{ZZ} + S \times B : W_{ZZ}} \\ &= 84 + 112 - \frac{(33+17)^2}{21+20} = 196 - 60.976 = 135.024 \end{aligned}$$

Note: $R(\alpha, \epsilon | \mu, \beta_2)$ and $R(\alpha\tau, \epsilon | \mu, \alpha, \tau, \beta_2)$ are intermediate steps for later use.

$$R(\beta_2 | \mu, \rho, \alpha, \tau, \alpha\tau) = \frac{(S \times B:W_{YZ})^2}{S \times B:W_{ZZ}} = \frac{17^2}{20} = 14.450$$

$$R(\epsilon | \mu, \rho, \alpha, \tau, \alpha\tau, \beta_2) = S \times B:W_{YY} - \frac{(S \times B:W_{YZ})^2}{S \times B:W_{ZZ}} = 112 - \frac{17^2}{20} = 112 - 14.45 = 97.55$$

$$R(\alpha | \mu, \rho, \tau, \alpha\tau, \beta_2) = R(\alpha, \epsilon | \mu, \rho, \tau, \alpha\tau, \beta_2) - \text{SS error } b = 181.793 - 97.55 = 84.243$$

$$R(\alpha\tau | \mu, \rho, \alpha, \tau, \beta_2) = R(\alpha\tau, \epsilon | \mu, \rho, \alpha, \tau, \beta_2) - R(\epsilon | \mu, \rho, \alpha, \tau, \alpha\tau, \beta_2) \\ = 135.024 - 97.55 = 37.474$$

Data SP-3

Split plot data with plots arranged in a completely randomized design and a covariate Z that is constant within the whole plot. (Winer, 1971, p. 803)

whole plot	Subject	Split plots		Z	Total
		B ₁	B ₂		
		Y	Y		Y
A ₁	1	10	8	3	18
	2	15	12	5	27
	3	20	14	8	34
	4	12	6	2	18
A ₂	5	15	10	1	25
	6	25	20	8	45
	7	20	15	10	35
	8	15	10	2	25
	Total	132	95	39	227
	Mean	16.5	11.9	4.88	

$$\text{Model: } Y_{ijk} = \mu + \tau_i + \delta_{ij} + \alpha_k + (\tau\alpha)_{ik} + \beta_1(Z_{ij} - \bar{Z}_{..}) + \epsilon_{ijk}$$

τ_i = A effect (whole plot)

δ_{ij} = error (a) ϵ_{ijk} = error (b)

α_k = B effect (split plot)

β_1 = whole plot regression slope

Analysis of variance and covariance

Source		df	SS
A (whole plot)	$= R(\tau \mu, \beta_1)$	1	44.492
Regression	$= R(\beta_1 \mu, \tau)$	1	166.577
Error (a)	$= R(\delta \mu, \tau, \beta_1)$	5	61.298
B (split plot)	$= R(\alpha \mu, \tau, \alpha\tau)$	1	85.563
A×B (interaction)	$= R(\tau\alpha \mu, \tau, \alpha)$	1	0.563
Error (b)	$= R(\epsilon \mu, \tau, \alpha, \tau\alpha)$	6	6.375

Table of SS and products

Symbol	Y^2	ZY	Z^2
W	68.06	12.38	2.25
E(a)	227.88	163.00	159.50
S	85.563	0	0
WS	0.563	0	0
E(b)	6.375	0	0

$$\hat{\beta}_1 = \frac{163.00}{159.50} = 1.02$$

Since the computations are illustrated in Winer (1971, p. 803-5) we have omitted them here.

SP-1: Control Language

Control Language is typed upper case and comments are in bold and lower case.

```

/PROBLEM TITLE IS 'SP-1: WHOLE PLOT ARRANGED IN RCB'.
/INPUT   VARIABLES ARE 6.
        FORMAT IS FREE.
/VARIABLE NAMES ARE BLOCK, WHOLE, Y1, Y2, Y3, Y4.  $\Rightarrow$  input variables
/DESIGN  GROUPING ARE BLOCK, WHOLE.  $\Rightarrow$  whole plot classification
        variables
        DEPENDENT ARE Y1, Y2, Y3, Y4.  $\Rightarrow$  specifies Y measured for
        four levels of split plot
        LEVEL IS 4.  $\Rightarrow$  specifies four levels of split plot
        NAME IS SUBPLOT.  $\Rightarrow$  split plot classification variable
        EXCLUDE IS 12.  $\Rightarrow$  model specification for whole plot: assumes
        full factorial. This statement requests the
        BLOCK (1) by WHOLE (2) interaction to be
        pooled with SSE(a)
        PRINT.  $\Rightarrow$  requests residuals and predicted values
/END
1 1 3 4 7 6  $\Rightarrow$  data organized with all split values for a partic-
2 1 6 10 1 11  $\Rightarrow$  ular BLOCK by WHOLE combination on the same line
3 1 6 10 4 4
1 2 3 2 1 14
2 2 8 8 2 18
3 2 10 8 9 13

```

SP-2: Control Language

Note: The whole plot and split plot analysis must be requested in separate runs. The first run gives the correct whole plot analysis but an incorrect split analysis (details follow in the annotations). The second run gives the correct split plot analysis.

Whole Plot Run

```
/PROBLEM  TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT    VARIABLES ARE 10.
          FORMAT IS FREE.
/VARIABLE  NAMES ARE BLOCK, WHOLE, Z1, Z2, Z3, Z4, Y1, Y2, Y3, Y4.
/DESIGN    GROUPING ARE BLOCK, WHOLE.
          DEPENDENT ARE Y1, Y2, Y3, Y4.
          COVARIATE ARE Z1, Z2, Z3, Z4.  ==> yields an analysis of
          LEVEL IS 4.                      covariance
          NAME IS SUBPLOT.
          EXCLUDE IS 12.
          RESIDUAL IS MEAN.
          PRINT.
```

/END

```
1 1 1 2 1 2 3 4 7 6
2 1 2 2 0 4 6 10 1 11
3 1 3 5 2 0 6 10 4 4
1 2 2 0 2 4 3 2 1 14
2 2 4 1 3 4 8 8 2 18
3 2 3 2 4 7 10 8 9 13
```

Split Plot Run

```
/PROBLEM  TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT    VARIABLES ARE 5.
          FORMAT IS FREE.
/VARIABLE  NAMES ARE BLOCK, WHOLE, SUBPLOT, Z, Y.
/DESIGN    GROUPING ARE BLOCK, WHOLE, SUBPLOT.
          DEPENDENT IS Y.
          COVARIATE IS Z.
          EXCLUDE IS 13,123. ==> model specification for split plot:
          PRINT                program assumes full factorial. This
                                statement specifies certain factors
                                to be pooled in SSE(b).
```

/END

```
1 1 1 1 3
1 1 2 2 4
1 1 3 1 7
1 1 4 2 6 ==> for split plot analysis data must be organized in a
:             similar arrangement with only one datum per line
:
:
3 2 4 7 13
```

SP-3: Control Language

Note: This example unlike the other two covariate examples requires only one run. This is because we are using a CRD and no special design specification is required.

```
/PROBLEM  TITLE IS 'SP-4: WHOLE PLOTS ARRANGED IN CRD WITH CONSTRAINT
              COVARIATE
/INPUT      VARIABLES ARE 4.
              FORMAT IS FREE.
/VARIABLE    NAMES ARE A, Z, Y1, Y2.
/DESIGN      GROUPING IS A.
              DEPENDENT ARE Y1, Y2.
              COVARIATE IS Z, Z.  => BMDP requires that two covariables be
              LEVEL IS 2.           specified here to match the number of
              NAME IS B.           levels of the split plot. Since Z is
              PRINT.              constant for both levels, its name is
                                   entered for both.
/END
1 3 10 8
1 5 15 12
1 8 20 14
1 2 12 6
2 1 15 10
2 8 25 20
2 10 20 15
2 2 15 10
```

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1PAGE 1 BMDP2V

BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

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(VM/CMS)

Manual Edition: 1983, 1985 reprint. State NEWS in the PRINT
paragraph for a summary of new features.

APRIL 20, 1987 AT 16:43:45

PROGRAM CONTROL INFORMATION

/PROBLEM TITLE IS 'SP-1: WHOLE PLOT ARRANGED IN RCB'.

/INPUT VARIABLES ARE 6.

/VARIABLE NAMES ARE BLOCK, WHOLE, Y1, Y2, Y3, Y4.

/DESIGN GROUPING ARE BLOCK, WHOLE.
DEPENDENT ARE Y1, Y2, Y3, Y4.
LEVEL IS 4.

NAME IS SUBPLOT.

EXCLUDE IS 12.

PRINT.

/END

} Control Language (see page 9 for details)

Data Organization - all split plot values for a
particular BLOCK by WHOLE combination must be
in the same observation. See control language
on page 9 for the exact data layout.

PROBLEM TITLE IS
SP-1: WHOLE PLOT ARRANGED IN RCB

NUMBER OF VARIABLES TO READ IN. 6
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. 0
TOTAL NUMBER OF VARIABLES 6
NUMBER OF CASES TO READ IN. TO END
CASE LABELING VARIABLES
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. NEITHER
BLANKS ARE. MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. 32362

VARIABLES TO BE USED
1 BLOCK 2 WHOLE 3 Y1 4 Y2 5 Y3 → Each variable is assigned a number and
6 Y4 will be referred to by its number below

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS 80 CHARACTERS.

DESIGN SPECIFICATIONS → Interpretation of DESIGN paragraph

GROUP = 1 2 → Variables 1 and 2 (BLOCK & WHOLE) make up the "grouping" or whole plot variables
DEPEND = 3 4 5 6 → Variables 3,4,5 and 6 make up the dependent variable, Y
LEVEL = 4 → 4 levels of the split plot variable, SUBPLOT

VARIABLE NO. NAME	MINIMUM LIMIT	MAXIMUM LIMIT	MISSING CODE	CATEGORY CODE	NAME	INTERVAL RANGE	
						GREATER THAN	LESS THAN OR = TO
1 BLOCK							
					1.00000	*1.00000	
					2.00000	*2.00000	
					3.00000	*3.00000	
2 WHOLE							
					1.00000	*1.00000	
					2.00000	*2.00000	

→ BMDP assigned names for the levels of the two whole plot variables

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

SP-1, page 3

NUMBER OF CASES READ. 6

BMDP2V SP-1: WHOLE PLOT ARRANGED IN RCB

GROUP STRUCTURE

BLOCK	WHOLE	COUNT	→ Frequency of observations in each BLOCK by WHOLE combination for Y1-Y4
*1.00000	*1.00000	1	
*1.00000	*2.00000	1	
*2.00000	*1.00000	1	
*2.00000	*2.00000	1	
*3.00000	*1.00000	1	
*3.00000	*2.00000	1	

SUMS OF SQUARES AND CORRELATION MATRIX OF THE ORTHOGONAL COMPONENTS POOLED FOR ERROR 2 IN ANOVA TABLE BELOW

0.20000	1.000
3.00000	0.000
16.80000	-0.327
	1.000

SPHERICITY TEST APPLIED TO ORTHOGONAL COMPONENTS - TAIL PROBABILITY 0.0000

Tests assumption that the orthogonal components for the split plot, SUBPLOT are independent and have equal variances (sometimes referred to as symmetry assumption). The above information on the orthogonal components of the split plots are useful when the levels of the split plot are ordered and the investigator is concerned about the assumption of independence and equal variances on the levels of the split plot. However, it will be shown on SP-1 page 5 that the "error 2" or SSE(b) that BMDP computes for this design is incorrect making the above values also incorrect.

CELL MEANS FOR 1-ST DEPENDENT VARIABLE

J = BLOCK		SUBP		LOT = k		MARGINAL	
i	=	*1.00000	*2.00000	*1.00000	*2.00000	*3.00000	*3.00000
i	=	*1.00000	*1.00000	*2.00000	*2.00000	*1.00000	*2.00000
Y1				3.00000	6.00000	8.00000	10.00000
Y2				2.00000	10.00000	8.00000	8.00000
Y3				1.00000	1.00000	2.00000	9.00000
Y4				14.00000	11.00000	18.00000	13.00000
						4.00000	4.00000
						6.00000	10.00000
						10.00000	8.00000
						4.00000	9.00000
						4.00000	13.00000
						6.00000	6.00000
						8.00000	9.00000
						10.00000	10.00000
						11.00000	11.00000
						13.00000	13.00000
						18.00000	18.00000
						2.00000	2.00000
						8.00000	8.00000
						8.00000	8.00000
						10.00000	10.00000
						6.00000	6.00000
						10.00000	10.00000
						4.00000	4.00000
						4.00000	4.00000
						11.00000	11.00000
						7.00000	7.00000
						5.00000	5.00000
						7.00000	7.00000
						9.00000	9.00000
						6.00000	6.00000
						10.00000	10.00000
						5.00000	5.00000
						5.00000	5.00000
						7.00000	7.00000
						9.00000	9.00000
						10.00000	10.00000
						11.00000	11.00000
						13.00000	13.00000
						18.00000	18.00000
						2.00000	2.00000
						8.00000	8.00000
						8.00000	8.00000
						10.00000	10.00000
						6.00000	6.00000
						10.00000	10.00000
						4.00000	4.00000
						4.00000	4.00000
						11.00000	11.00000
						7.00000	7.00000
						5.00000	5.00000
						7.00000	7.00000
						9.00000	9.00000
						10.00000	10.00000
						11.00000	11.00000
						13.00000	13.00000
						18.00000	18.00000
						2.00000	2.00000
						8.00000	8.00000
						8.00000	8.00000
						10.00000	10.000

COUNT 1 1 1 0

BMDP2V SP-1: WHOLE PLOT ARRANGED IN ROB

$\rho_j = \text{block effect}$

ρ_j = block effect
 τ_i = variety effect (whole plot)
 α_k = fertilizer effect (split plot)

ANALYSIS OF VARIANCE FOR 1-ST

DEPENDENT VARIABLE - Y1

SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	GREENHOUSE GEISSER PROB.	HUYNH FELD T PROB.
MEAN	$SSM = N\bar{y}^2 = 24(7)^2 = 1176.00000$	1	1176.00000	147.00	0.0067		
BLOCK	$R(\rho \mu)$ 48.00000	2	24.00000	3.00	0.2500		
WHOLE	$R(\tau \mu)$ 24.00000	1	24.00000	3.00	0.2254		
ERROR	$R(\delta \mu, \rho, \tau)$ 16.00000	2	MSE(a) 8.00000			3.0 = 24.0/8.0	
SUBPLOT	$R(\alpha \mu)$ 156.00000	3	52.00000	15.60	0.0031	0.0544	0.0031
SB	92.00000	6	15.33333	4.60	0.0428	0.1719	0.0428
SW	$R(\alpha\tau \mu, \alpha, \tau)$ 84.00000	3	28.00000	8.40	0.0144	0.0964	0.0144
ERROR	20.00000	6	3.33333				

incorrect → see note below

SP-1, page 5

The "2 error" or SSE(b) reported here is incorrect. This means that Greenhouse-Geisser & Huynh-Feldt are adjusted p-values to be used when the symmetry assumptions on the split plot are not met (see sphericity test on SP-1 page 3). However, these values are incorrect for this design because BMDP has computed the SSE(b) incorrectly. See the note below.

The "2 error" or SSE(b) reported here is incorrect. This means the F statistics and associated tail probabilities for the split plot portion of the analysis are also incorrect. To obtain the correct SSE(b) it is necessary to pool the SS's labelled SB(SUBPLOT*BLOCK) with the SS's labelled "Z error". I.E. $92 \times 20 = 112$. The F statistics for SUBPLOT(F) and SW(SUBPLOT*WHOLE) need to be recalculated using this corrected SSE(b).

EPSILON FACTORS FOR DEGREES OF FREEDOM ADJUSTMENT

GREENHOUSE-GEISSER HUYNH-FELDT
0.3490 1.0000

ERROR TERM

2

$$F_{\text{SUBPLOT}} = \frac{156/3}{112/12} = 5.57$$

$$F_{\text{SUBPLOT*WHOLE}} = \frac{84/3}{112/12} = 3.0$$

OF SQUARES CORRESPONDING RESIDUALS

1 R0

2 R1, R2, R3, R4

BMDP2V SP-1: WHOLE PLOT ARRANGED IN RCB

CASE	BLOCK	WHOLE	P0 R3	P1 R4	P2	P3	P4	R0	R1	R2
1	*1.00000	*1.00000	$\hat{Y}_{11} = 4.00000$	-2.00000	0.00000	0.00000	2.00000	$R0_1 = 1.00000$	0.00000	-1.00000
2	*2.00000	*1.00000	2.00000	-1.00000	3.00000	-5.50000	3.50000	0.00000	0.00000	0.00000
3	*3.00000	*1.00000	7.00000	-0.50000	3.00000	-0.50000	-2.50000	-1.00000	0.00000	1.00000
4	*1.00000	*2.00000	-1.50000	0.50000	-4.00000	-2.00000	8.00000	-1.00000	0.00000	1.00000
5	*2.00000	*2.00000	9.00000	-1.00000	-1.00000	-7.50000	9.50000	0.00000	0.00000	0.00000
6	*3.00000	*2.00000	9.00000	0.00000	-1.00000	-2.50000	3.50000	1.00000	0.00000	-1.00000
			1.50000	-0.50000						

ERROR SUM OF RECOMPUTED RELATIVE

TERM	SQUARES	FROM RESIDUALS	ERROR
1	16.00000	16.00000	0.00000
2	20.00000	20.00000	0.00000

CPU TIME USED 0.183 SECONDS

$$\bar{Y}_{11} - \hat{Y}_{11} = 5.0 - 4.0 = 1.0 = R0_1$$

P0 & R0 are whole plot predicted values and residuals, respectively. P1-P4 and R1-R4 are supposed to be split plot predicted values and residuals respectively but are computed from an incorrect value for SSE(b) (see previous page) and should therefore be ignored.

SP-2. SP-1 data with a covariate added and varying from split plot to split plot, first run.

BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

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Data Organization - all split plot values for Z and Y or a particular BLOCK by WIDE combination must be in the same observation. See control language on page 10 for exact data layout.

Program Version: April 1985
(VM/CMS)
Manual Edition: 1983, 1985 reprint. State NEWS in the PRINT paragraph for a summary of new features.

APRIL 20, 1987 AT 16:44:22

PROGRAM CONTROL INFORMATION

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/PROBLEM TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT VARIABLES ARE 10.
FORMAT IS FREE.
/VARIABLE NAMES ARE BLOCK, WHOLE, Z1, Z2, Z3, Z4, Y1, Y2, Y3, Y4.
/DESIGN GROUPING ARE BLOCK, WHOLE.
DEPENDENT ARE Y1, Y2, Y3, Y4.
COVARIATE ARE Z1, Z2, Z3, Z4.
LEVEL IS 4.
NAME IS SURPLOT.
EXCLUDE IS 12.
RESIDUAL IS MEAN.
PRINT

```

/END

} Control Language (see page 10 for details)

PROBLEM TITLE IS
SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

NUMBER OF VARIABLES TO READ IN. 10
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. 0
TOTAL NUMBER OF VARIABLES 10
NUMBER OF CASES TO READ IN. TO END
CASE LABELING VARIABLES
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. NEITHER
BLANKS ARE. MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. 32362

VARIABLES TO BE USED
1 BLOCK 2 WHOLE 3 Z1 4 Z2 5 Z3
6 Z4 7 Y1 8 Y2 9 Y3 10 Y4

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS 80 CHARACTERS.

DESIGN SPECIFICATIONS

GROUP = 1 2
DEPEND = 7 8 9 10
COVAR = 3 4 5 6
LEVEL = 4

INTERVAL RANGE
GREATER LESS THAN
OR = TO

VARIABLE NO. NAME	MINIMUM LIMIT	MAXIMUM LIMIT	MISSING CODE	CATEGORY CODE	NAME
1 BLOCK					
				1.00000	*1.00000
				2.00000	*2.00000
				3.00000	*3.00000
2 WHOLE				1.00000	*1.00000
				2.00000	*2.00000

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ. 6
 RMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

GROUP STRUCTURE

BLOCK	WHOLE	COUNT
*1.00000	*1.00000	1
*1.00000	*2.00000	1
*2.00000	*1.00000	1
*2.00000	*2.00000	1
*3.00000	*1.00000	1
*3.00000	*2.00000	1

SUMS OF SQUARES AND CORRELATION MATRIX OF THE ORTHOGONAL COMPONENTS POOLED FOR ERROR 2 IN ANOVA TABLE BELOW

0.01156	1.000
3.07111	-0.907
16.65067	0.902
	-0.979
	1.000

INSUFFICIENT DEGREES OF FREEDOM FOR SYMMETRY TEST

Like in SP-1, these values on the orthogonal components are incorrect because they are based on "Error 2" or SSE(b) which is computed incorrectly. See SP-1 page 3 and 5 for details.

CELL MEANS FOR 1-ST COVARIATE

		MARGINAL			
j = BLOCK	=	*1.00000	*1.00000	*2.00000	*3.00000
i = WHOLE	=	*1.00000	*2.00000	*1.00000	*2.00000
SURP					
LOI = k					
Z1	1	1.00000	2.00000	2.00000	3.00000
Z2	2	2.00000	0.00000	1.00000	5.00000
Z3	3	1.00000	2.00000	0.00000	2.00000
Z4	4	2.00000	4.00000	4.00000	0.00000
		Z		ijk	
		2.50000		2.00000	
				2.00000	
				3.50000	

MARGINAL $\bar{Z}_{1j} = 1.50000 \quad 2.00000 \quad 2.00000 \quad 3.00000 \quad 2.50000 \quad 4.00000 \quad 2.50000 = \bar{Z}_{\dots}$

COUNT	1	1	1	1	1	6
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CELL MEANS FOR 1-ST DEPENDENT VARIABLE

MARGINAL

$j = \text{BLOCK}$	$=$	$\#1.00000$	$\#1.00000$	$\#2.00000$	$\#3.00000$
$j = \text{WHOLE}$	$=$	$\#1.00000$	$\#2.00000$	$\#1.00000$	$\#2.00000$

$i = \text{WHOLE}$	$= 1.00000$	$\times 2.00000$	$\times 1.00000$	$\times 2.00000$	$\times 1.00000$	$\times 2.00000$
--------------------	-------------	------------------	------------------	------------------	------------------	------------------

SUBP

LOT = k

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Y1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Y2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Y3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Y4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

y2	2	4.00000	2.00000	10.00000	8.00000	10.00000	8.00000	Y	7.00000	$\bar{Y}$
----	---	---------	---------	----------	---------	----------	---------	---	---------	-----------

	3	7.00000	1.00000	1.00000	2.00000	4.00000	9.00000	4.00000
Y3								
Y2								
Y1								
Y0								
Y-1								
Y-2								
Y-3								
Y-4								
Y-5								
Y-6								
Y-7								
Y-8								
Y-9								
Y-10								
Y-11								
Y-12								
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Y-57								
Y-58								
Y-59								
Y-60								
Y-61								
Y-62								
Y-63								
Y-64								
Y-65								
Y-66								

4	6.00000	14.00000	11.00000	18.00000	4.00000	13.00000	11.00000
Y4							

MARGINAL $\bar{Y}_{1j} = 5.00000$ 5.00000 7.00000 10.00000 7.00000 = $\bar{Y}_{...}$

COUNT	1	1	1	1	1	1	6
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BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y1

	Y2	Y3	Y4	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	GREENHOUSE GEISSER PROB.	HUYNH FELDT PROB.	REGRESSION COEFFICIENTS
SOURCE											
BLOCK $R(\rho \mu, \tau, \beta_1, \alpha, \sigma\tau)$				15.60000	2	7.80000*****	0.0000	0.0000			
WHOLE $R(\tau \mu, \rho, \beta_1, \alpha, \sigma\tau)$				3.42857	1	3.42857*****	0.0000	0.0000			
1-ST COVAR $R(\beta_1 \mu, \rho, \tau, \alpha, \sigma\tau)$				16.00000	1	16.00000*****	0.0000	0.0000			$\hat{\beta}_1$
1 ERROR SSE(a)				0.00000	1	0.00000					4.00000 = $\hat{\beta}_1$
SUBPLOT				105.22500	3	35.07500	8.89	0.0190	0.1192	0.0190	
SB	SEE NOTE BELOW			77.81667	6	12.96944	3.29	0.1063	0.2624	0.1063	
SW				50.23889	3	16.74630	4.24	0.0768	0.1992	0.0768	
1-ST COVAR				0.26667	1	0.26667	0.07	0.9053			0.13333
1 ERROR				19.73333	5	3.94667					

ERROR TERM EPSILON FACTORS FOR DEGREES OF FREEDOM ADJUSTMENT

2	GREENHOUSE-GEISSER	HUYNH-FELDT
	0.3372	1.0000

POOLED REGRESSION COEFFICIENTS → The BMDP manual states that this estimate is "obtained from the weighted pooled cross-product matrix" ... and is used to compute adjusted cells means". This is probably of no interest to the experimenter.

1-ST COVARIATE 4.00000

The "2 error" or SSE(b) reported here is incorrect. This causes the SS's for the covariate in the above analysis to be incorrect also, thereby causing the SS's for SUBPLOT and SUBPLOT*WHOLE labelled (SW) to be adjusted incorrectly. The reason "2 error" or SSE(b) is wrong is because the SS's for SUBPLOT*BLOCK (labelled SB) should be pooled with "2 error". Another call to BMDP is required to get the correct split plot portion of the analysis and this begins on SP-2 page 8.

ERROR SUM
OF SQUARES

1 = SSE(a) R0
2 = SSE(b) R1, R2, R3, R4
4 BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

$$\bar{Y}_{11\cdot} - \hat{Y}_{11\cdot} = 5.0 - 5.0 = 0.0 = R0_1$$

CASE	BLOCK	WHOLE	P0 R3	P1 R4	P2	P3	P4	R0	R1	R2
1	*1.00000	*1.00000	$\hat{Y}_{11\cdot} = 5.00000$	-2.03333	-0.03333	0.03333	2.03333	$R0_1 = 0.00000$	0.03333	-0.96667
2	*2.00000	*1.00000	1.96667	-1.03333	2.93333	-5.56667	3.70000	0.00000	0.06667	0.06667
3	*3.00000	*1.00000	-0.43333	0.30000	3.10000	-0.46667	-2.73333	0.00000	-0.10000	0.90000
4	*1.00000	*2.00000	6.00000	0.10000	-3.96667	-2.03333	7.96667	0.00000	-0.03333	0.96667
5	*2.00000	*2.00000	-1.53333	0.73333	-0.93333	-7.43333	9.30000	0.00000	-0.06667	-0.06667
6	*3.00000	*2.00000	5.00000	-1.96667	-1.10000	-2.53333	3.73333	0.00000	0.10000	-0.90000
			-1.96667	1.03333						
			9.00000	-0.93333						
			0.43333	-0.30000						
			10.00000	-0.10000						
			1.53333	-0.73333						

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	0.00000	0.00000*****	*****
2	19.73333	19.73333	0.00000

ADJUSTED CELL MEANS ARE NOT COMPUTED WHEN THE INCLUDE OPTION IS USED. → BMDP cannot compute adjusted means for anything other than straight factorials.

CPU TIME USED 0.213 SECONDS

P0 and R0 are whole plot predicted values and residuals, respectively. P1-P4 and R1-R4 are supposed to be split plot predicted values and residuals but are computed from an incorrect value for SSE(b) (see previous page) and should therefore be ignored (correct values are computed in the run that follows).

SP-2 Split Plot, second run

BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

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Program Version: April 1985
(VN/QMS)

Manual Edition: 1983, 1985 reprint. State NEWS in the PRINT
paragraph for a summary of new features.

APRIL 10, 1987 AT 15:36:42

PROGRAM CONTROL INFORMATION

/PROBLEM TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT VARIABLES ARE 5.
/VARIABLE NAMES ARE BLOCK, WHOLE, SUBPLOT, Z, Y.
/DESIGN GROUPING ARE BLOCK, WHOLE, SUBPLOT.
DEPENDENT IS Y.
COVARIATE IS Z.
EXCLUDE IS 13,123.
PRINT.

/END

Data Organization: Data arranged with only one Z and Y
measurement per observation. See control language on page
10 for exact data layout.

} Control Language (see page 10 for details)

PROBLEM TITLE IS
SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

NUMBER OF VARIABLES TO READ IN. 5
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. 0
TOTAL NUMBER OF VARIABLES 5
NUMBER OF CASES TO READ IN. TO END
CASE LABELING VARIABLES
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. NEITHER
BLANKS ARE. MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. 32362

VARIABLES TO BE USED
1 BLOCK 2 WHOLE 3 SUBPLOT 4 Z 5 Y

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS 80 CHARACTERS.

DESIGN SPECIFICATIONS

GROUP = 1 2 3
DEPEND = 5
COVAR = 4

VARIABLE NO. NAME	MINIMUM LIMIT	MAXIMUM LIMIT	MISSING CODE	CATEGORY CODE	CATEGORY NAME	INTERVAL RANGE	
						GREATER THAN	LESS THAN OR = TO
1 BLOCK							
				1.00000	*1.00000		
				2.00000	*2.00000		
				3.00000	*3.00000		
2 WHOLE							
				1.00000	*1.00000		
				2.00000	*2.00000		
3 SUBPLOT							
				1.00000	*1.00000		
				2.00000	*2.00000		
				3.00000	*3.00000		
				4.00000	*4.00000		

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ. 24

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y

ρ_j = block effect
 α_k = subplot effect (split plot)
 τ_1 = whole plot effect
 β_2 = slope for split plot

SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	REGRESSION COEFFICIENTS
BLOCK IGNORE*	20.20862	2	10.10431	1.14	0.3551	
WHOLE IGNORE*	6.10385	1	6.10385	0.69	0.4244	
SUBPLOT $R(\alpha \mu, \rho, \tau, \alpha, \beta_2)$	84.24310	3	28.08103	3.17	0.0678	
BW IGNORE*	9.45000	2	4.72500	0.53	0.6014	
WS $R(\alpha\tau \mu, \rho, \tau, \alpha, \beta_2)$	37.47439	3	12.49146	1.41	0.2923	
Z $R(\beta_2 \mu, \rho, \tau, \alpha, \alpha\tau)$	14.45000	1	14.45000	1.63	0.2281	$\hat{\beta}_2$
1 ERROR SSE(b)	97.55000	11	8.86818			

*These whole plot effects were estimated correctly in the 1st call to
BMDP on SP-2 page 6 and should be ignored here.

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

CASE	BLOCK	WHOLE	SUBPLOT	PREDICTD	RESIDUAL = split plot residuals
1	*1.00000	*1.00000	*1.00000	$\hat{Y}_{111} = 3.57500$	-0.57500 = 3.0 - 3.575 = -0.575
2	*1.00000	*1.00000	*2.00000	6.57500	-2.57500
3	*1.00000	*1.00000	*3.00000	3.42500	3.57500
4	*1.00000	*1.00000	*4.00000	6.42500	-0.42500
5	*2.00000	*1.00000	*1.00000	6.00000	0.00000
6	*2.00000	*1.00000	*2.00000	8.15000	1.85000
7	*2.00000	*1.00000	*3.00000	4.15000	-3.15000
8	*2.00000	*1.00000	*4.00000	9.70000	1.30000
9	*3.00000	*1.00000	*1.00000	5.42500	0.57500
10	*3.00000	*1.00000	*2.00000	9.27500	0.72500
11	*3.00000	*1.00000	*3.00000	4.42500	-0.42500
12	*3.00000	*1.00000	*4.00000	4.87500	-0.87500
13	*1.00000	*2.00000	*1.00000	4.00000	-1.00000
14	*1.00000	*2.00000	*2.00000	3.00000	-1.00000
15	*1.00000	*2.00000	*3.00000	1.00000	0.00000
16	*1.00000	*2.00000	*4.00000	12.00000	2.00000
17	*2.00000	*2.00000	*1.00000	8.85000	-0.85000
18	*2.00000	*2.00000	*2.00000	7.00000	1.00000
19	*2.00000	*2.00000	*3.00000	5.00000	-3.00000
20	*2.00000	*2.00000	*4.00000	15.15000	2.85000
21	*3.00000	*2.00000	*1.00000	8.15000	1.85000
22	*3.00000	*2.00000	*2.00000	8.00000	0.00000
23	*3.00000	*2.00000	*3.00000	6.00000	3.00000
24	*3.00000	*2.00000	*4.00000	17.84999	-4.85000

NOTE: If the correct error SSE(b) sums of squares are to be obtained, we need to use the line "EXCLUDE IS 13,123." However, this does not allow the means to be adjusted for covariance. If the above statement is deleted from the program, adjusted means will be obtained but they are incorrect because SSE(b) is computed incorrectly.

ERROR	SUM OF	RECOMPUTED	RELATIVE
TERM	SQUARES	FROM RESIDUALS	ERROR
1	97.55000	97.55000	0.00000

ADJUSTED CELL MEANS ARE NOT COMPUTED WHEN THE INCLUDE OPTION IS USED.

CPU TIME USED 0.216 SECONDS

SP-3. Whole Plots Arranged in CRD With Constant Covariate Within Whole Plot

BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

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BMDP Statistical Software, Inc.
1964 Westwood Blvd. Suite 202
Los Angeles, California 90025
Phone (213) 475-5700
Telex 4992203

Program Version: April 1985
(VM/CMS)

Manual Edition: 1983, 1985 reprint. State NEWS in the PRINT
paragraph for a summary of new features.

MAY 1, 1987 AT 14:21:52

PROGRAM CONTROL INFORMATION

/PROBLEM TITLE IS 'SP-3: WHOLE PLOTS ARRANGED IN CRD WITH
CONSTANT COVARIATE'.

/INPUT VARIABLES ARE 4.

FORMAT IS FREE.

/VARIABLE NAMES ARE A, Z, Y1, Y2.

/DESIGN GROUPING IS A.

DEPENDENT ARE Y1, Y2.

COVARIATE IS Z, Z.

LEVEL IS 2.

NAME IS B.

PRINT.

/END

Data Organization - all split plot values for a
particular SUBJECT by A combination must be in
the same observation. See Control Language on
Page 11 for the exact data layout

} Control Language (see page 11 for details)

PROBLEM TITLE IS
SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

NUMBER OF VARIABLES TO READ IN. 4
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. 0
TOTAL NUMBER OF VARIABLES 4
NUMBER OF CASES TO READ IN. TO END
CASE LABELING VARIABLES
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. NEITHER
BLANKS ARE. MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. 32362

VARIABLES TO BE USED
1 A 2 Z 3 Y1 4 Y2

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS 80 CHARACTERS.

DESIGN SPECIFICATIONS

GROUP = 1
DEPEND = 3 4
COVAR = 2 2
LEVEL = 2

VARIABLE NO. NAME	MINIMUM LIMIT	MAXIMUM LIMIT	MISSING CODE	CATEGORY CODE	CATEGORY NAME	INTERVAL RANGE	
						GREATER THAN	LESS THAN OR = TO
1 A						1.00000	*1.00000
						2.00000	*2.00000

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ. 8

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

GROUP STRUCTURE

A	COUNT
*1.00000	4
*2.00000	4

CELL MEANS FOR 1-ST COVARIATE

	A	=	*1.00000	*2.00000	MARGINAL
	B				
Z	1	4.50000	5.25000	4.87500	
Z	2	4.50000	5.25000	4.87500	
	MARGINAL	4.50000	5.25000	4.87500	
	COUNT	4	4	8	

STANDARD DEVIATIONS FOR 1-ST COVARIATE

	A	=	*1.00000	*2.00000
	B			
Z	1	2.64575	4.42531	
Z	2	2.64575	4.42531	

CELL MEANS FOR 1-ST DEPENDENT VARIABLE

	A		B		MARGINAL
		=	*1.00000	*2.00000	
Y1	1		14.25000	18.75000	16.50000
Y2	2		10.00000	13.75000	11.87500
	MARGINAL		12.12500	16.25000	14.18750
COUNT			4	4	8

STANDARD DEVIATIONS FOR 1-ST DEPENDENT VARIABLE

	A		B	
	= *1.00000		*2.00000	
Y1	1	4.34933	4.78714	
Y2	2	3.65148	4.78714	

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y1 Y2

	SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	REGRESSION COEFFICIENTS
1	A(whole plot) 1-ST COVAR ERROR	$R(\tau \mu, \beta_1, \alpha, \alpha\tau)$ 44.49160 $R(\beta_1 \mu, \tau, \alpha, \alpha\tau)$ 166.57690 61.29820	1 1 5	44.49160 166.57690 12.25964	3.63 13.59	0.1151 0.0142	$\hat{\beta}_1$ 1.02194
	B(split plot) BA ERROR	$R(\alpha \mu, \tau, \beta_1, \alpha\tau)$ 85.56250 $R(\alpha\tau \mu, \tau, \beta_1, \alpha)$ 0.56250 6.37500	1 1 6	85.56250 0.56250 1.06250	80.53 0.53	0.0001 0.4943	

ERROR SUM
OF SQUARES CORRESPONDING RESIDUALS

1	RO
2	R1, R2

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

$$= \bar{Y}_{11\cdot} - \hat{Y}_{11\cdot} = 9.0 - 10.6 = -1.6$$

CASE	A	P0	P1	P2	R0	R1	R2
1	*1.00000	$\hat{Y}_{11\cdot} = 10.59208$	2.12500	-2.12500	-1.59208	-1.12500	1.12500
2	*1.00000	12.63597	2.12500	-2.12500	0.86403	-0.62500	0.62500
3	*1.00000	15.70180	2.12500	-2.12500	1.29820	0.87500	-0.87500
4	*1.00000	9.57014	2.12500	-2.12500	-0.57014	0.87500	-0.87500
5	*2.00000	11.90674	2.50000	-2.50000	0.59326	0.00000	0.00000
6	*2.00000	19.06033	2.50000	-2.50000	3.43965	0.00000	0.00000
7	*2.00000	21.10422	2.50000	-2.50000	-3.60423	0.00000	0.00000
8	*2.00000	12.92868	2.50000	-2.50000	-0.42868	0.00000	0.00000

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	61.29820	61.29820	0.00000
2	6.37500	6.37500	0.00000

ADJUSTED CELL MEANS FOR 1-ST DEPENDENT VARIABLE

A	=	*1.00000	*2.00000
B			

Y1	1	14.63323	19.36677
Y2	2	10.38323	13.36677

P0 and R0 are whole plot predicted values and residuals, respectively. P1-P2 and R1-R2 are split plot predicted values and residuals.

$$Y_{ijk} = P0 + R0 + Pj + Rj$$

$$e.g., Y_{111} = 10.59208 - 1.59208 + 2.125 - 1.125 = 10.00$$

The means $\bar{Y}_{i\cdot k}$ are adjusted only for the whole plot covariate because the covariate is constant for all split plots within one whole plot. The formula for adjusting $\bar{Y}_{i\cdot k}$ means is

$$\bar{Y}_{i\cdot k} \text{ adj} = \bar{Y}_{i\cdot k} - \hat{\beta}_1(\bar{Z}_{i\cdot\cdot} - \bar{Z}_{\cdot\cdot\cdot})$$

An example for $\bar{Y}_{1\cdot 1}$

$$\bar{Y}_{1\cdot 1} \text{ adj} = 14.25 - 1.02(4.5 - 4.9) = 14.63$$

END

DATE
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DEC.

1987