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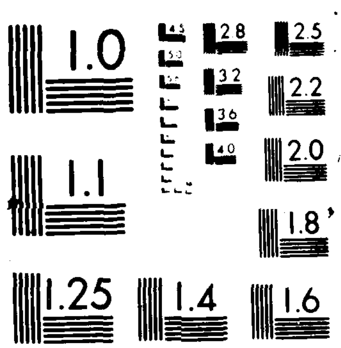
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

(2)

Naval Research Laboratory
Washington, DC 20375-5000

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TeXing the FORMULARY: A Collection of Examples of the Use of TeX to Produce Ruled Tables and Displayed Equations

DAVID L. BOOK

Laboratory for Computational Physics and Fluid Dynamics

TODD BRUN

Physics Department
Harvard University, Cambridge, MA 02138

ROBERT J. SCOTT

Berkeley Research Associates, Springfield, VA 22150

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The *NRL Plasma Formulary* is a collection of formulas and reference data in a handy pocketsized booklet. This report contains some sixty pages of text, tables, and displayed equations, together with the associated .TEX files. An enlarged copy (the output from an IMAGEN ink-jet printer) of each page of the *Formulary*, identical with that which was photographically reduced for the actual book, is reprinted facing the .TEX source code that generated it. Users can browse through the report until they come to a table or displayed equation similar to the one they want to produce, then extract from the facing page the control sequences they need.

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INTRODUCTION

The *NRL Plasma Formulary* is a collection of formulas and reference data in a handy pocket-sized booklet. Over thirty thousand copies of previous editions of the *Formulary* have been produced and distributed during the past decade and a half. Each revision before the present one precipitated a costly exercise in composition and layout. Finally the master became too dilapitated to use. We decided to retypeset the whole book using TEX, the computerized typesetting system developed by Donald Knuth of Stanford University. Although this was an arduous task, we believe that the difficulty of preparing future editions will thereby be greatly reduced.

In recent years TEX has been widely adopted by scientists, engineers, and others to produce documents containing technical information. The typical user, like us, is an author or scientific collaborator of the author, technically trained, computer-literate, and a clumsy typist. This user knows exactly how the final document should look and is strongly motivated to mine from *The TEXbook* the requisite nuggets of wisdom, and to grind away as long as necessary, in order to achieve it. Consequently, highly trained (and highly paid) people are spending a significant fraction of their time doing what used to be regarded as clerical work. It follows that any tool that can make the operation of using TEX more efficient is potentially valuable.

Knuth's comprehensive and very readable (if idiosyncratic) introductory manual, *The TEXbook*, devotes about the same amount of space to the production of ruled tables as to diacritical marks in Central European languages (two and a half pages). For most users, the former is by far the more important application. Although the two examples of tables Knuth provides are illuminating and the diligent student can learn a great deal from them, the process is time-consuming. Experience shows that most users want a portfolio of examples that they can use with a minimum of modification as templates for their own applications.

This report is intended to partially fill that need. It contains some sixty pages of text, tables, and displayed equations (the output from an IMAGEN 8/300 laser printer), together with the associated TEX source. An enlarged copy of each page of the *Formulary*, identical with that which was photographically reduced for the actual book, is reprinted facing the TEX source code that generated it. Users can browse through the report until they come to a table or displayed equation similar to the one they want to produce, then extract from the facing page the control sequences they need. The code is largely self-contained; however, many macros are taken from the file `PROLOG.TEX`. Whenever the user encounters an (apparently) undefined macro, its definition should be sought there. Additionally, the file `POINTSIZE.TEX` sets up all of the necessary font definitions (magnified seven-point fonts are used in place of the normal ten-point fonts in the *Formulary*, to improve readability after reduction). These files are listed immediately following this introduction.

No claim is made that the code reprinted here is optimum. We warrant only that the TEX input produces the output you see. Wizards and other supernatural beings could possibly find more flexible, transparent, and elegant ways of printing these tables and equations. Moreover, since three different individuals participated in the project, the programming style is highly nonuniform. Worse still, we were learning as we went along, so the sections typed at the beginning of the effort are rougher than those typed after we became more proficient. We attempted to go back and clean up after ourselves; but demands of space precluded extensive commenting, especially in the complex ruled tables which most needed it. To partly make up for this, we selected two tables as examples and explained them in some detail. These are found on pages 10 and 14. In spite of this, the user will probably have to discover by experimentation why we did much of what we did. This report *does*, however, contain a bonus that many older scientists (like the present senior author) will appreciate: a version of the *Formulary* with print big enough to read.

The *Formulary* used many TEX tricks and shortcuts in the interests of compactness and speed of implementation. Particularly irritating, but unavoidable, are the many "hard-wired" measure-

Manuscript approved May 9, 1987.

ments. We adjusted spacing, line widths, and positions until the output looked close to our mental picture. While this kind of built-in hack is not very elegant, it is quick and easy to implement, as opposed to trying to find exactly the right macro to handle every possible case. Also, we abbreviated commands wherever possible to conserve space. Originally, each page was printed from a separate file, as shown here, with separate calls to `PROLOG.TEX`. They have since been merged into one file, but the older version was used for better readability. In a number of places, we have made characters 'active'; for example, the vertical bar '|' was made to stand for two ampersands '&&' in tables. As mentioned above, most of these abbreviations are in `PROLOG.TEX`.

For convenience, users with DECNET access who can reach `LCP::` can copy the TeX file for any given page. These files, the names of which have the form `PAGExx.TEX`, where `xx` is the page number, as well as `PROLOG.TEX` and `POINTSIZE.TEX`, are currently located in the directory `LCP::SYS2:[GUEST.FORMULARY]`. A file containing the entire set of instructions used to compose the *Formulary*, `FORMULARY.TEX`, is also located in this directory. Users with Internet access can access the files through anonymous FTP (your default will be the `SYS2:[GUEST]` directory, so you need to get `[GUEST.FORMULARY]<filename>.TEX`). The host name is `NRL-LCP.ARPA`, soon to be a domain name `LCP.NRL.MIL`.

It is a pleasure to acknowledge the TEXnical assistance of Dr. Gopal Patnaik and Ken Laskey, two valuable suggestions made by Prof. Knuth, and the encouragement of Dr. Jay Boris.

```
%
% THIS FILE DEFINES THE MACROS USED THROUGHOUT THE FORMULARY.
%
% First, set up the default page sizes and magnification.
%
\magnification=1728
\hoffset=1.25truein
\voffset=1.0truein
\hsize=6.0truein
\vsize=9.0truein
\parindent=0pt
%
% Get the font definitions, and set the font to magnified sevenpoint.
%
\input pointsize
\sevenpoint
\font\headfont=cmbx5 scaled \magstep2
\font\tensorfont=cmssi8
\font\cs=cmsy7
%
% Now, set up all the commonly used macros for the various formulary pages.
%
\catcode'\|=\active
\def|{&&}
%
% Special characters.
%
\def\A{{\bf A}}
\def\AOB{{\alpha/\beta}}
\def\app{\displaystyle \approx}
\def\approxlt{\kern 0.35em \raise 0.6ex \hbox{$<$} \kern -0.77em \lower 0.6ex
  \hbox{$\sim$} \kern 0.35em}
\def\approxgt{\kern 0.35em \raise 0.6ex \hbox{$>$} \kern -0.77em \lower 0.6ex
  \hbox{$\sim$} \kern 0.35em}
\def\B{{\bf B}}
\def\C{{\bf C}}
\def\curl{\del\times}
\def\D{{\bf D}}
\def\del{\nabla}
\def\dint{\displaystyle\int}
\def\diver{\del\cdot}
\def\doint{\displaystyle\oint}
\def\E{{\bf E}}
\def\hbar{{\mathchar'26\mskip-9muh}}
\def\lambdabar{{\mathchar'26\mskip-10mu\lambda}}
\def\longrightarrow{\relbar\kern-0.5pt\joinrel\rightarrow}
\def\lra{\displaystyle \longrightarrow}
\def\part{\partial}
\def\R{{\bf R}}
\def\T{{\tensorfont T}}
\def\tfsigma{\sigma \kern-0.55em \sigma}
\def\ts{\thinspace}
%
% Special formats.
%
\def\leftdisplay#1{\medskip\leftline{\inndent$\displaystyle#1$}\bigskip}
\def\ital#1{{\it #1\/}}
```

```
\def\undertext#1{$\underline{\smash{\hbox{#1}}}$}
%
% Contractions and commonly used macros.
%
\def\bs#1{\noalign{\vskip-#1}}
\def\bsk{\bigskip}
\def\dfil{\dotfill\ }
\def\H{\hang}
\def\hang{\hangindent \oldparindent}
\def\indent{\hskip \oldparindent \spacefactor=1000}
\def\inn[illegible]\hskip 20pt}
\def\m{$\vphantom{\big(}$}
\def\msk{\medskip}
\def\N{\noindent}
\def\nocorr{\kern 0pt}
\def\oldparindent{20pt}
\def\om{\omit}
\def\ov{\bar}  %{\overline}
\def\ph{\phantom}
\def\s{\strut}
\def\sk{\noalign{\smallskip}}
\def\ssk{\smallskip}
\def\tablerule{\noalign{\hrule}}
\def\tf{\tensorfont}
\def\trule{\noalign{\hrule}}
\newcount\tscount
\def\tspart{|\om}
\def\tsend{&\cr}
\def\tska#1#2{\om&height#2&\om \tscount=#1 \ifcase\tscount \or\tsend
  \or\tspart\tsend
  \or\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tsend\else\bad\fi}
\def\tskb#1{height#1&\om|\om|\om|&\om|\om|\om&\cr}
\def\tskc#1#2{height#2&\om \tscount=#1 \ifcase\tscount \or\tsend
  \or\tspart\tsend
  \or\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tsend
  \or\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tspart\tsend\else\bad\fi}
\def\un{\hat}  %{\underline}
```

```
%
%     THIS FILE DEFINES MACROS TO SET THE POINT SIZE FOR THE FILE:
%
% \eightpoint  ==> eight point type
% \ninepoint   ==> nine point type
% \tenpoint    ==> ten point type (TeX default)
% \twelvepoint ==> twelve point type
%
\font\ninerm=cmr9   \font\eightrm=cmr8   \font\sixrm=cmr6
\font\ninei=cmmi9   \font\eighti=cmmi8   \font\sixi=cmmi6
\font\ninesy=cmsy9  \font\eightsy=cmsy8  \font\sixsy=cmsy6
\font\ninebf=cmbx9  \font\eightbf=cmbx8  \font\sixbf=cmbx6
\font\ninett=cmtt9  \font\eighttt=cmtt8  \font\seventt=cmtt10
\font\nineit=cmti9  \font\eightit=cmti8  \font\sevenit=cmti7
\font\ninesl=cmsl9  \font\eightsl=cmsl8  \font\sevensl=cmsl8
\font\sevenrm=cmr7  \font\seveni=cmmi7   \font\sevenbf=cmbx7
\font\twelverm=cmr12
\font\twelvei=cmmi12
\font\twelvesy=cmsy10 scaled 1200
\font\twelvebf=cmbx12
\font\tenex=cmex10
\font\twelvett=cmtt12
\font\twelveit=cmti12
\font\twelvesl=cmsl12

\skewchar\twelvei='177
\skewchar\twelvesy='60
\hyphenchar\twelvett=-1
\skewchar\ninei='177  \skewchar\eighti='177  \skewchar\sixi='177
\skewchar\ninesy='60  \skewchar\eightsy='60  \skewchar\sixsy='60
\hyphenchar\ninett=-1 \hyphenchar\eighttt=-1 \hyphenchar\tentt=-1
\catcode`@=11
\newskip\ttglue

\def\twelvepoint{\def\rm{\fam0\twelverm} % switch to 12 pt. type
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  \textfont1=\twelvei \scriptfont1=\ninei \scriptscriptfont1=\seveni
  \textfont2=\twelvesy \scriptfont2=\ninesy \scriptscriptfont2=\sevensy
  \textfont3=\tenex \scriptfont3=\tenex \scriptscriptfont3=\tenex
  \textfont\itfam=\twelveit \def\it{\fam\itfam\twelveit}
  \textfont\slfam\twelvesl \def\sl{\fam\slfam\twelvesl}
  \textfont\ttfam\twelvett \def\tt{\fam\ttfam\twelvett}
  \textfont\bffam\twelvebf \scriptfont\bffam=\ninebf
  \scriptscriptfont\bffam=\sevenbf \def\bf{\fam\bffam\twelvebf}
  \tt \ttglue=.5em plus .25em minus .15em
  \normalbaselineskip=15pt
  \setbox\strutbox=\hbox{\vrule height9.5pt depth4.5pt width0pt}
  \let\sc=\tenrm \let\big=\twelvebig \normalbaselines\rm }

\def\tenpoint{\def\rm{\fam0\tenrm}% switch to 10-point type
  \textfont0=\tenrm \scriptfont0=\sevenrm \scriptscriptfont0=\fiverm
  \textfont1=\teni  \scriptfont1=\seveni  \scriptscriptfont1=\fivei
  \textfont2=\tensy \scriptfont2=\sevensy \scriptscriptfont2=\fivesy
  \textfont3=\tenex \scriptfont3=\tenex   \scriptscriptfont3=\tenex
  \textfont\itfam=\tenit  \def\it{\fam\itfam\tenit}%
  \textfont\slfam=\tensl  \def\sl{\fam\slfam\tensl}%
  \textfont\ttfam=\tentt  \def\tt{\fam\ttfam\tentt}%
```

```
  \textfont\bffam=\tenbf  \scriptfont\bffam=\sevenbf
    \scriptscriptfont\bffam=\fivebf  \def\bf{\fam\bffam\tenbf}%
  \tt \ttglue=.5em plus.25em minus.15em
  \normalbaselineskip=12pt
  \setbox\strutbox=\hbox{\vrule height8.5pt depth3.5pt width0pt}%
  \normalbaselines\rm}

\def\ninepoint{\def\rm{\fam0\ninerm}% switch to 9-point type
  \textfont0=\ninerm \scriptfont0=\sixrm \scriptscriptfont0=\fiverm
  \textfont1=\ninei  \scriptfont1=\sixi  \scriptscriptfont1=\fivei
  \textfont2=\ninesy \scriptfont2=\sixsy \scriptscriptfont2=\fivesy
  \textfont3=\tenex  \scriptfont3=\tenex \scriptscriptfont3=\tenex
  \textfont\itfam=\nineit  \def\it{\fam\itfam\nineit}%
  \textfont\slfam=\ninesl  \def\sl{\fam\slfam\ninesl}%
  \textfont\ttfam=\ninett  \def\tt{\fam\ttfam\ninett}%
  \textfont\bffam=\ninebf  \scriptfont\bffam=\sixbf
    \scriptscriptfont\bffam=\fivebf  \def\bf{\fam\bffam\ninebf}%
  \tt \ttglue=.5em plus.25em minus.15em
  \normalbaselineskip=11pt
  \setbox\strutbox=\hbox{\vrule height8pt depth3pt width0pt}%
  \normalbaselines\rm}

\def\eightpoint{\def\rm{\fam0\eightrm}% switch to 8-point type
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  \textfont1=\eighti  \scriptfont1=\sixi  \scriptscriptfont1=\fivei
  \textfont2=\eightsy \scriptfont2=\sixsy \scriptscriptfont2=\fivesy
  \textfont3=\tenex  \scriptfont3=\tenex \scriptscriptfont3=\tenex
  \textfont\itfam=\eightit  \def\it{\fam\itfam\eightit}%
  \textfont\slfam=\eightsl  \def\sl{\fam\slfam\eightsl}%
  \textfont\ttfam=\eighttt  \def\tt{\fam\ttfam\eighttt}%
  \textfont\bffam=\eightbf  \scriptfont\bffam=\sixbf
    \scriptscriptfont\bffam=\fivebf  \def\bf{\fam\bffam\eightbf}%
  \tt \ttglue=.5em plus.25em minus.15em
  \normalbaselineskip=9pt
  \setbox\strutbox=\hbox{\vrule height7pt depth2pt width0pt}%
  \normalbaselines\rm}

\def\sevenpoint{\def\rm{\fam0\sevenrm}% switch to 7-point type
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  \textfont2=\sevensy \scriptfont2=\fivesy \scriptscriptfont2=\fivesy
  \textfont3=\tenex  \scriptfont3=\tenex \scriptscriptfont3=\tenex
  \textfont\itfam=\sevenit  \def\it{\fam\itfam\sevenit}%
  \textfont\slfam=\sevensl  \def\sl{\fam\slfam\sevensl}%
  \textfont\ttfam=\seventt  \def\tt{\fam\ttfam\seventt}%
  \textfont\bffam=\sevenbf  \scriptfont\bffam=\fivebf
    \scriptscriptfont\bffam=\fivebf  \def\bf{\fam\bffam\sevenbf}%
  \tt \ttglue=.5em plus.25em minus.15em
  \normalbaselineskip=8pt
  \setbox\strutbox=\hbox{\vrule height7pt depth2pt width0pt}%
  \normalbaselines\rm}
```

1987
REVISED

NRL PLASMA FORMULARY

DAVID L. BOOK
Laboratory for Computational Physics
Naval Research Laboratory
Washington, DC 20375

Supported by
The Office of Naval Research

```
\input prolog
\pageno=2
\centerline{\headfont CONTENTS}
\bigskip
\baselineskip=12pt
\parindent=0pt
Numerical and Algebraic \dfil \ 3 \par
Vector Identities \dfil \ 4 \par
Differential Operators in Curvilinear Coordinates \dfil \ 6 \par
Dimensions and Units \dfil 10 \par
International System (SI) Nomenclature \dfil 13 \par
Metric Prefixes \dfil 13 \par
Physical Constants (SI) \dfil 14 \par
Physical Constants (cgs) \dfil 16 \par
Formula Conversion \dfil 18 \par
Maxwell's Equations \dfil 19 \par
Electricity and Magnetism \dfil 20 \par
Electromagnetic Frequency/Wavelength Bands \dfil 21 \par
AC Circuits \dfil 22 \par
Dimensionless Numbers of Fluid Mechanics \dfil 23 \par
Shocks \dfil 26 \par
Fundamental Plasma Parameters \dfil 28 \par
Plasma Dispersion Function \dfil 30 \par
Collisions and Transport \dfil 31 \par
Approximate Magnitudes in Some Typical Plasmas \dfil 40 \par
Ionospheric Parameters \dfil 42 \par
Solar Physics Parameters \dfil 43 \par
Thermonuclear Fusion \dfil 44 \par
Relativistic Electron Beams \dfil 46 \par
Beam Instabilities \dfil 48 \par
Lasers \dfil 50 \par
Atomic Physics and Radiation \dfil 52 \par
References \dfil 58
\vfil\eject\end
```

CONTENTS

```
\input prolog
\hoffset=1.125truein
\voffset=1.0truein
\hsize=6.25truein
\vsize=9.0truein
\pageno=3
\centerline{\headfont NUMERICAL AND ALGEBRAIC}
\bigskip
\N Gain in decibels of {$P_2$\/} relative to {$P_1$\/}
$$G = 10 \log_{10} (P_2/P_1).$$
\medskip
\N To within two percent
$$(2\pi)^{1/2} \approx 2.5;\ \pi^2 \approx 10;\ e^3 \approx 2[illegible],
2^{10} \approx 10^3{.}$$
\medskip
\N Euler-Mascheroni constant$^1\ \gamma = 0.57722$
\bigskip
\N Gamma Function $\Gamma (x + 1) = x\Gamma(x)$:
$$\halign{\qquad\quad#\hfil&\ =\ #\hfil &\qquad#\hfil&\ =\ #\hfil\cr
$\Gamma(1/6)$&5.5663             &$\Gamma(3/5)$&1.4892\cr
$\Gamma(1/5)$&4.5908             &$\Gamma(2/3)$&1.3541\cr
$\Gamma(1/4)$&3.6256             &$\Gamma(3/4)$&1.2254\cr
$\Gamma(1/3)$&2.6789             &$\Gamma(4/5)$&1.1642\cr
$\Gamma(2/5)$&2.2182             &$\Gamma(5/6)$&1.1287\cr
$\Gamma(1/2)$&$1.7725=\sqrt{\pi}$  &$\Gamma(1)$&1.0\cr}$$
\medskip
\N Binomial Theorem (good for $\mid x\mid<1$ or $\alpha =$ positive integer[illegible]
$$(1+x)^\alpha=\sum^\infty_{k=0}{\alpha\choose k}x^k \equiv 1 + \alpha x +
{{\alpha(\alpha-1)}\over {2!}}x^2 + {{\alpha(\alpha-1)(\alpha-2)} \over {3!}}x^3
+ \ldots.$$
\medskip
\N Rothe-Hagen identity$^2\ $(good for all complex $x$, $y$, $z$ except when
singular):
$$[illegible]\sum[illegible] {x\over{x+kz}} {{x+kz} \choose k} {y\over{y+(n[illegible]
[illegible](n-k)z} \choose{n-k}} \cr &={{x+y}\over{x+y+nz}} {{x+y+nz}[illegible]
[illegible]
[illegible] summation formula$^3$ [good for $\mu$ nonintegral, [illegible]
[illegible] 1$]:
$$[illegible]n=-\infty}^{\infty} [(-1)^n J_{\alpha-\gamma n}(z) J_[illegible]
[illegible]mu} = {\pi \over \sin \mu\pi} J_{\alpha+\gamma\mu}(z)[illegible]
\gamma \mu}(z) \quad$$
\vfil\eject\end
```

NUMERICAL AND ALGEBRAIC

Gain in decibels of P_2 relative to P_1

$$G = 10\log_{10}(P_2/P_1).$$

To within two percent

$$(2\pi)^{1/2} \approx 2.5;\ \pi^2 \approx 10;\ e^3 \approx 20;\ 2^{10} \approx 10^3.$$

Euler-Mascheroni constant[1] $\gamma = 0.57722$

Gamma Function $\Gamma(x+1) = x\Gamma(x)$:

$$\begin{array}{ll}
\Gamma(1/6) = 5.5663 & \Gamma(3/5) = 1.4892 \\
\Gamma(1/5) = 4.5908 & \Gamma(2/3) = 1.3541 \\
\Gamma(1/4) = 3.6256 & \Gamma(3/4) = 1.2254 \\
\Gamma(1/3) = 2.6789 & \Gamma(4/5) = 1.1642 \\
\Gamma(2/5) = 2.2182 & \Gamma(5/6) = 1.1288 \\
\Gamma(1/2) = 1.7725 = \sqrt{\pi} & \Gamma(1) \quad = 1.0
\end{array}$$

Binomial Theorem (good for $|x| < 1$ or $\alpha =$ positive integer):

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \equiv 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots.$$

Rothe-Hagen identity[2] (good for all complex x, y, z except when singular):

$$\sum_{k=0}^{n} \frac{x}{x+kz}\binom{x+kz}{k}\frac{y}{y+(n-k)z}\binom{y+(n-k)z}{n-k} = \frac{x+y}{x+y+nz}\binom{x+y+nz}{n}.$$

Newberger's summation formula[3] [good for μ nonintegral, $\mathrm{Re}(\alpha+\beta) > -1$]:

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n J_{\alpha-\gamma n}(z) J_{\beta+\gamma n}(z)}{n+\mu} = \frac{\pi}{\sin\mu\pi} J_{\alpha+\gamma\mu}(z) J_{\beta-\gamma\mu}(z).$$

```
\input prolog
\hoffset=1.125truein\voffset=1truein\hsize=6.0truein\vsize=9.0truein
\pageno=4
\centerline{\headfont VECTOR IDENTITIES$^4$} \bsk
\N Notation: $f,\ g,$ are scalars; \A, \B, etc., are vectors; \T\ is a
tensor; {\tf I}\ is the unit dyad.
   %  '\T' IS SET A MAGNIFIED EIGHT-POINT CHARACTER, SINCE WE LACKED A
   %  SEVEN-POINT BOLD SANS-SERIF FONT.  IT IS FOLLOWED BY '\ ' TO LEAVE
   %  A SPACE FOLLOWING.
\msk\ssk
\N\quad $\ph{1}$(1)
$\A\cdot\B\times\C=\A\times\B\cdot\C=\B\cdot\C\times\A=\B
\times\C\cdot\A=\C\cdot\A\times\B=\C\times\A\cdot\B$ \msk
\N\quad $\ph{1}$(2)
$\A\times(\B\times\C)=(\C\times\B)\times\A=(\A\cdot\C)\B-(\A\cdot\B)\C$ \msk
\N\quad $\ph{1}$(3)
$\A\times(\B\times\C)+\B\times(\C\times\A)+\C\times(\A\times\B)=0$ \msk
\N\quad $\ph{1}$(4)
$(\A\times\B)\cdot(\C\times\D)=(\A\cdot\C)(\B\cdot\D)-
    (\A\cdot\D)(\B\cdot\C)$ \msk
\N\quad $\ph{1}$(5)
$(\A\times\B)\times(\C\times\D)=(\A\times\B\cdot\D)\C-
    (\A\times\B\cdot\C)\D$ \msk
\N\quad $\ph{1}$(6) $\del (fg)=\del (gf)=f\del g +g\del f$ \msk
\N\quad $\ph{1}$(7) $\diver(f\A)=f\diver\A+\A\cdot\del f$ \msk
\N\quad $\ph{1}$(8) $\curl(f\A)=f\curl\A+\del f\times\A$ \msk
\N\quad $\ph{1}$(9) $\diver(\A\times\B)=\B\cdot\curl\A-\A\cdot\curl\B$ \msk
\N\quad(10)
$\curl(\A\times\B)=\A(\diver\B)-\B(\diver\A)+(\B\cdot\del)\A-(\A\cdot\del)\B$
    \msk
\N\quad(11) $\A\times(\curl\B)=(\del\B)\cdot\A-(\A\cdot\del)\B$ \msk
\N\quad(12) $\del(\A\cdot\B)=\A\times(\curl\B)+\B\times(\curl\A)+
    (\A\cdot\del)\B+(\B\cdot\del)\A$ \msk
\N\quad(13) $\del^2f=\diver\del f$ \msk
\N\quad(14) $\del^2\A=\del(\diver\A)-\curl\curl\A$ \msk
\N\quad(15) $\curl\del f=0$ \msk
\N\quad(16) $\diver\curl\A=0$ \msk
\N If {\bf e$_1$, e$_2$, e$_3$} are orthonormal unit vectors, a second-order
tensor \T\ can be written in the dyadic form
    \msk
\N\quad(17) \T\ $=\sum_{i,j}\T_{ij}${\bf e$_i$e$_j$} \msk
\N In cartesian coordinates the divergence of a tensor is a vector with
components
\msk
\N\quad(18) $(\diver$\T\/)$_i=\sum_{j}(\partial T_{ji}/\partial x_j)$ \msk
\N[This definition is required for consistency with Eq. (29)].  In general
    \msk
\N\quad(19) $\diver(\A\B)=(\diver\A)\B+(\A\cdot\del)\B$ \msk
\N\quad(20) $\diver(f$\T\/$)=\del f\cdot$\T\/$+f\diver$\T
\vfil\eject\end
```

VECTOR IDENTITIES[4]

Notation: f, g, are scalars; $\mathbf{A}$, $\mathbf{B}$, etc., are vectors; T is a tensor; I is the unit dyad.

(1) $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{A}\times\mathbf{B}\cdot\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{B}\times\mathbf{C}\cdot\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} = \mathbf{C}\times\mathbf{A}\cdot\mathbf{B}$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{C}\times\mathbf{B})\times\mathbf{A} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$

(3) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

(4) $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$

(5) $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\times\mathbf{B}\cdot\mathbf{D})\mathbf{C} - (\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})\mathbf{D}$

(6) $\nabla(fg) = \nabla(gf) = f\nabla g + g\nabla f$

(7) $\nabla\cdot(f\mathbf{A}) = f\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla f$

(8) $\nabla\times(f\mathbf{A}) = f\nabla\times\mathbf{A} + \nabla f\times\mathbf{A}$

(9) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$

(10) $\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(11) $\mathbf{A}\times(\nabla\times\mathbf{B}) = (\nabla\mathbf{B})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$

(12) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(13) $\nabla^2 f = \nabla\cdot\nabla f$

(14) $\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}$

(15) $\nabla\times\nabla f = 0$

(16) $\nabla\cdot\nabla\times\mathbf{A} = 0$

If $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are orthonormal unit vectors, a second-order tensor T can be written in the dyadic form

(17) $T = \sum_{i,j} T_{ij}\mathbf{e}_i\mathbf{e}_j$

In cartesian coordinates the divergence of a tensor is a vector with components

(18) $(\nabla\cdot T)_i = \sum_j (\partial T_{ji}/\partial x_j)$

[This definition is required for consistency with Eq. (29)]. In general

(19) $\nabla\cdot(\mathbf{AB}) = (\nabla\cdot\mathbf{A})\mathbf{B} + (\mathbf{A}\cdot\nabla)\mathbf{B}$

(20) $\nabla\cdot(fT) = \nabla f\cdot T + f\nabla\cdot T$

```
\input prolog
\hoffset=1.125truein
\voffset=1truein
\hsize=6.0truein
\vsize=9.0truein
\pageno=5
\N Let ${\bf r}={\bf i}x+{\bf j}y+{\bf k}z$ be the radius vector of magnitude
$r$, from the origin to the point $x,y,z$.  Then
\medskip
\N\quad(21) $\diver{\bf r}=3$
\medskip
\N\quad(22) $\curl{\bf r}=0$
\medskip
\N\quad(23) $\del r={\bf r}/r$
\medskip
\N\quad(24) $\del(1/r)=-{\bf r}/r^3$
\medskip
\N\quad(25) $\diver({\bf r}/r^3)=4\pi\delta({\bf r})$
\medskip
\N\quad(26) $\del{\bf r} = $\ {\tf I}
\medskip
\N If $V$ is a volume enclosed by a surface $S$ and $d{\bf S}={\bf n}dS$,
where {\bf n} is the unit normal outward from $V,$
\medskip
\N\quad(27) $\dint_V dV\del f=\dint_S d{\bf S}f$
\medskip
\N\quad(28) $\dint_V dV\diver\A=\dint_S d{\bf S}\cdot\A$
\medskip
\N\quad(29) $\dint_V dV\diver$\T\ $ =\dint_S d{\bf S} \, \cdot$\T
\medskip
\N\quad(30) $\dint_V dV\curl\A=\dint_S d{\bf S\times A}$
\medskip
\N\quad(31) $\dint_V dV(f\del^2 g - g\del^2 f)=\dint_S d{\bf S}\cdot(f\del
g-g\del f)$
\medskip
\N\quad(32) $\dint_V dV(\A\cdot\curl\curl\B-\B\cdot\curl\curl\A)$
\vskip0.0001in \qquad\qquad\qquad\qquad\qquad
$=\dint_Sd{\bf S}\cdot (\B\times\curl\A-\A\times\curl\B)$
\medskip
\N If $S$ is an open surface bounded by the contour $C$, of which the line
element is $d{\bf l}$,
\medskip
\N\quad(33) $\dint_S d{\bf S}\times\del f=\doint_C d{\bf l}f$
\vfill\eject\end
```

Let $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ be the radius vector of magnitude r, from the origin to the point x, y, z. Then

(21) $\nabla \cdot \mathbf{r} = 3$

(22) $\nabla \times \mathbf{r} = 0$

(23) $\nabla r = \mathbf{r}/r$

(24) $\nabla(1/r) = -\mathbf{r}/r^3$

(25) $\nabla \cdot (\mathbf{r}/r^3) = 4\pi\delta(\mathbf{r})$

(26) $\nabla \mathbf{r} = \mathsf{I}$

If V is a volume enclosed by a surface S and $d\mathbf{S} = \mathbf{n}dS$, where $\mathbf{n}$ is the unit normal outward from V,

(27) $$\int_V dV \nabla f = \int_S d\mathbf{S} f$$

(28) $$\int_V dV \nabla \cdot \mathbf{A} = \int_S d\mathbf{S} \cdot \mathbf{A}$$

(29) $$\int_V dV \nabla \cdot \mathsf{T} = \int_S d\mathbf{S} \cdot \mathsf{T}$$

(30) $$\int_V dV \nabla \times \mathbf{A} = \int_S d\mathbf{S} \times \mathbf{A}$$

(31) $$\int_V dV (f\nabla^2 g - g\nabla^2 f) = \int_S d\mathbf{S} \cdot (f\nabla g - g\nabla f)$$

(32) $$\int_V dV (\mathbf{A} \cdot \nabla \times \nabla \times \mathbf{B} - \mathbf{B} \cdot \nabla \times \nabla \times \mathbf{A})$$
$$= \int_S d\mathbf{S} \cdot (\mathbf{B} \times \nabla \times \mathbf{A} - \mathbf{A} \times \nabla \times \mathbf{B})$$

If S is an open surface bounded by the contour C, of which the line element is $d\mathbf{l}$,

(33) $$\int_S d\mathbf{S} \times \nabla f = \oint_C d\mathbf{l} f$$

```
\input prolog
\hoffset=1.125truein
\voffset=1truein
\hsize=6.0truein
\vsize=9.0truein
\pageno=6
\N\quad(34) $\dint_S d{\bf S}\cdot\curl\A=\doint_C d{\bf l}\cdot\A$
\medskip
\N\quad(35) $\dint_S(d{\bf S}\times\del)\times\A=\doint_C d{\bf l}\times\A$
\medskip
\N\quad(36) $\dint_S d{\bf S}\cdot(\del f\times\del g)=\doint_C fdg=-\doint_C
gdf$
\bigskip\bigskip
\centerline {\headfont DIFFERENTIAL OPERATORS IN}
\vskip 1pt
\centerline {\headfont CURVILINEAR COORDINATES$^5$}
\bigskip
\N{\headfont Cylindrical Coordinates}
\medskip
\N Divergence
   %
   %  Note that the \leftdisplay macro from PROLOG.TEX is used to produce
   %  left-justified displayed equations.
   %
\leftdisplay{\diver\A={1\over r} {\partial\over{\partial r}} (rA_r) + {1\over r}
{{\partial A_\phi}\over{\partial\phi}} + {{\partial A_z}\over{\partial z}}}
\N Gradient
\leftdisplay{(\del f)_r = {{\partial f}\over{\partial r}};\quad (\del f)_\phi =
{1\over r}{{\partial f}\over{\partial\phi}};\quad (\del f)_z = {{\partial f}
\over{\partial z}}}
\N Curl
\leftdisplay{(\curl\A)_r={1\over r}{{\partial A_z}\over{\partial\phi}} -
{{\partial A_\phi} \over{\partial z}}}
\leftdisplay{(\curl\A)_\phi={{\partial A_r}\over{\partial z}}-{{\partial A_z}
\over{\partial r}}}
\leftdisplay{(\curl\A)_z={1\over r}{\partial\over{\partial r}} (rA_\phi) -
{1\over r} {{\partial A_r}\over{\partial\phi}}}
\smallskip
\N Laplacian
\leftdisplay{\del^2 f = {1\over r}{\partial\over{\partial r}}\left(r{{\partial
f}\over{\partial r}}\right) + {1\over{r^2}}{{\partial^2 f}\over{\partial\phi^2}
} + {{\partial^2 f}\over{\partial z^2}}}
\vfil\eject\end
```

$$(34)\quad \int_S d\mathbf{S} \cdot \nabla \times \mathbf{A} = \oint_C d\mathbf{l} \cdot \mathbf{A}$$

$$(35)\quad \int_S (d\mathbf{S} \times \nabla) \times \mathbf{A} = \oint_C d\mathbf{l} \times \mathbf{A}$$

$$(36)\quad \int_S d\mathbf{S} \cdot (\nabla f \times \nabla g) = \oint_C f\,dg = -\oint_C g\,df$$

DIFFERENTIAL OPERATORS IN CURVILINEAR COORDINATES[5]

Cylindrical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\phi = \frac{1}{r}\frac{\partial f}{\partial \phi}; \quad (\nabla f)_z = \frac{\partial f}{\partial z}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}$$

$$(\nabla \times \mathbf{A})_\phi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$$

$$(\nabla \times \mathbf{A})_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}$$

Laplacian

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

```
\input prolog
\hoffset=1.125truein
\voffset=1truein
\hsize=6.0truein
\vsize=9.0truein
\pageno=7
\N Laplacian of a vector
   %
   %  Note that the \leftdisplay macro from PROLOG.TEX is used to
   %  left-justify displayed equations.
   %
\leftdisplay{(\del^2\A)_r=\del^2 A_r - {2\over r^2}{{\partial A_\phi}\over
{\partial\phi}}- {{A_r}\over{r^2}}}
\leftdisplay{(\del^2\A)_\phi=\del^2 A_\phi + {2\over r^2}
{{\partial A_r}\over{\partial\phi}} - {{A_\phi}\over{r^2}}}
\leftdisplay{(\del^2\A)_z=\del^2 A_z}
\medskip
\N Components of $(\A\cdot\del)\B$
\leftdisplay{(\A\cdot\del\B)_r=A_r {{\partial B_r}\over{\partial r}}+{{A_\phi}
\over r}{{\partial B_r}\over{\partial\phi}}+A_z{{\partial B_r}\over{\partial z}
}-{{A_\phi B_\phi}\over r}}
\leftdisplay{(\A\cdot\del\B)_\phi=A_r{{\partial B_\phi}\over{\partial r}}+
{{A_\phi}\over r}{{\partial B_\phi}\over{\partial\phi}}+A_z{{\partial B_\phi}
\over{\partial z}}+{{A_\phi B_r}\over r}}
\leftdisplay{(\A\cdot\del\B)_z=A_r {{\partial B_z}\over{\partial r}}+{{A_\phi}
\over r}{{\partial B_z}\over{\partial\phi}}+A_z{{\partial B_z}\over{\partial
z}}}
\smallskip
\N Divergence of a tensor
\leftdisplay{(\diver\hbox{\tf T\/})_r={1\over r}{\partial\over{\partial r}}
(rT_{rr})+{1\over r}{{\partial T_{\phi r}\over{\partial\phi}}+{{\partial T_{zr}}
\over {\partial z}}-{T_{\phi\phi}\over r}}
\leftdisplay{(\diver\hbox{\tf T\/})_\phi={1\over r}{\partial\over{\partial r}}
(rT_{r\phi})+{1\over r}{{\partial T_{\phi\phi}}\over{\partial\phi}}+{{\partial
T_{z\phi}}\over{\partial z}}+{T_{\phi r}\over r}}
\leftdisplay{(\diver\hbox{\tf T\/})_z={1\over r}{\partial\over{\partial r}}
(rT_{rz})+{1\over r}{{\partial T_{\phi z}}\over{\partial\phi}}+{{\partial
T_{zz}}\over{\partial z}}}
\vfil\eject\end
```

Laplacian of a vector

$$(\nabla^2 \mathbf{A})_r = \nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2}$$

$$(\nabla^2 \mathbf{A})_\phi = \nabla^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial \phi} - \frac{A_\phi}{r^2}$$

$$(\nabla^2 \mathbf{A})_z = \nabla^2 A_z$$

Components of $(\mathbf{A} \cdot \nabla)\mathbf{B}$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_r = A_r \frac{\partial B_r}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_r}{\partial \phi} + A_z \frac{\partial B_r}{\partial z} - \frac{A_\phi B_\phi}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_\phi = A_r \frac{\partial B_\phi}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_\phi}{\partial \phi} + A_z \frac{\partial B_\phi}{\partial z} + \frac{A_\phi B_r}{r}$$

$$(\mathbf{A} \cdot \nabla \mathbf{B})_z = A_r \frac{\partial B_z}{\partial r} + \frac{A_\phi}{r}\frac{\partial B_z}{\partial \phi} + A_z \frac{\partial B_z}{\partial z}$$

Divergence of a tensor

$$(\nabla \cdot T)_r = \frac{1}{r}\frac{\partial}{\partial r}(rT_{rr}) + \frac{1}{r}\frac{\partial T_{\phi r}}{\partial \phi} + \frac{\partial T_{zr}}{\partial z} - \frac{T_{\phi\phi}}{r}$$

$$(\nabla \cdot T)_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rT_{r\phi}) + \frac{1}{r}\frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{\partial T_{z\phi}}{\partial z} + \frac{T_{\phi r}}{r}$$

$$(\nabla \cdot T)_z = \frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}) + \frac{1}{r}\frac{\partial T_{\phi z}}{\partial \phi} + \frac{\partial T_{zz}}{\partial z}$$

```
\input prolog
\hoffset=1.125truein\voffset=1truein\hsize=6.0truein\vsize=9.0truein
\pageno=8
\N{\headfont Spherical Coordinates} \msk
\N Divergence
   %
   %  Note that the \leftdisplay macro from PROLOG.TEX is used to
   %  left-justify displayed equations.
   %
\leftdisplay{\diver\A={1\over{r^2}}{\part\over{\part r}}(r^2 A_r) +
  {1\over{r\sin\theta}}{\part\over{\part\theta}}(\sin\theta{A_\theta}) +
  {1\over{r\sin\theta}}{{\part A_\phi}\over{\part\phi}}}

\N Gradient
\leftdisplay{(\del f)_r={{\part f}\over{\part r}};\quad
  (\del f)_\theta={1\over r} {{\part f}\over{\part\theta}};\quad
  (\del f)_\phi={1\over{r\sin\theta}} {{\part f}\over{\part\phi}}}

\N Curl
\leftdisplay{(\curl\A)_r={1\over{r\sin\theta}}{\part\over{\part\theta}}
  (\sin\theta A_\phi)-{1\over{r\sin\theta}}{{\part
  A_\theta}\over{\part\phi}}}
\leftdisplay{(\curl\A)_\theta={1\over{r\sin\theta}}{{\part
  A_r}\over{\part\phi}}-{1\over r}{\part\over{\part r}}(rA_\phi)}
\leftdisplay{(\curl\A)_\phi={1\over r}{\part\over{\part r}}
  (rA_\theta)-{1\over r}{{\part A_r}\over{\part\theta}}} \msk

\N Laplacian
\leftdisplay{\del^2 f={1\over{r^2}}{\part\over{\part r}}\left(r^2
  {{\part f}\over{\part r}}\right)+{1\over{r^2\sin\theta}}{\part
  \over{\part\theta}}\left(\sin\theta{{\part f}\over{\part\theta}}
  \right)+{1\over{r^2\sin^2\theta}}{{\part^2 f}\over{\part\phi^2}}}

\N Laplacian of a vector
\leftdisplay{(\del^2\A)_r=\del^2A_r-{{2A_r}\over{r^2}}-{2\over{r^2}}{{\part
  A_\theta}\over{\part\theta}}-{{2\cot\theta A_\theta}\over{r^2}}-
  {2\over{r^2\sin\theta}}{{\part A_\phi}\over{\part\phi}}}
\leftdisplay{(\del^2\A)_\theta=\del^2A_\theta+{2\over{r^2}}{{\part A_r}\over
  {\part\theta}}-{{A_\theta}\over{r^2\sin^2\theta}}-{{2\cos\theta}\over
  {r^2\sin^2\theta}}{{\part A_\phi}\over{\part\phi}}}
\leftdisplay{(\del^2\A)_\phi=\del^2A_\phi-{{A_\phi}\over{r^2\sin^2\theta}}+
  {2\over{r^2\sin\theta}}{{\part A_r}\over{\part\phi}}+{{2\cos\theta}\over
  {r^2\sin^2\theta}}{{\part A_\theta}\over{\part\phi}}}
\vfil\eject\end
```

Spherical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Gradient

$$(\nabla f)_r = \frac{\partial f}{\partial r}; \quad (\nabla f)_\theta = \frac{1}{r}\frac{\partial f}{\partial\theta}; \quad (\nabla f)_\phi = \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\phi}$$

$$(\nabla \times \mathbf{A})_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)$$

$$(\nabla \times \mathbf{A})_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial\theta}$$

Laplacian

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Laplacian of a vector

$$(\nabla^2\mathbf{A})_r = \nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{2\cot\theta A_\theta}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$(\nabla^2\mathbf{A})_\theta = \nabla^2 A_\theta + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{A_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$(\nabla^2\mathbf{A})_\phi = \nabla^2 A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}$$

```
\input prolog
\hoffset=1.125truein\voffset=1truein\hsize=6.0truein\vsize=9.0truein
\pageno=9

\N Components of $(\A\cdot\del)\B$
   %
   %  Note that the \leftdisplay macro from PROLOG.TEX is used to
   %  left-justify displayed equations.
   %
\leftdisplay{(\A\cdot\del\B)_r=A_r{{\part B_r}\over{\part r}}+{{A_\theta}
  \over r}{{\part B_r}\over{\part\theta}}+{{A_\phi}\over{r\sin\theta}}
  {{\part B_r}\over{\part\phi}}-{{A_\theta B_\theta + A_\phi B_\phi}\over r}}
\leftdisplay{(\A\cdot\del\B)_\theta=A_r{{\part B_\theta}\over{\part r}}+
  {{A_\theta}\over r}{{\part B_\theta}\over{\part\theta}}+
  {{A_\phi}\over{r\sin\theta}}{{\part B_\theta}\over{\part\phi}}+
  {{A_\theta B_r} over r} - {{\cot\theta A_\phi B_\phi}\over r}}
\leftdisplay{(\A\cdot\del\B)_\phi=A_r{{\part B_\phi}\over{\part r}}+
  {{A_\theta}\over r}{{\part B_\phi}\over{\part\theta}}+
  {{A_\phi}\over{r\sin\theta}}{{\part B_\phi}\over{\part\phi}}+
  {{A_\phi B_r}\over r} + {{\cot\theta A_\phi B_\theta}\over r}}

\N Divergence of a tensor
\leftdisplay{(\diver\hbox{\tf T\/})_r={1\over{r^2}}{\part\over{\part r}}
  (r^2 T_{rr})+{1\over{r\sin\theta}}{\part\over{\part\theta}}
  (\sin\theta T_{\theta r})}
\leftline{\hskip3.5truein$\displaystyle+{1\over{r\sin\theta}}{{\part
  T_{\phi r}}\over{\part\phi}}-{{T_{\theta\theta}+T_{\phi\phi}}\over r}$} \msk
\leftdisplay{(\diver\hbox{\tf T\/})_\theta={1\over{r^2}}{\part\over
  {\part r}} (r^2 T_{r\theta})+{1\over{r\sin\theta}}{\part\over{\part
  \theta}}(\sin\theta T_{\theta\theta})}
\leftline{\hskip3.5truein$\displaystyle+{1\over{r\sin\theta}}{{\part
  T_{\phi\theta}}\over{\part \phi}}+{T_{\theta r}\over r}-
  {{\cot\theta}T_{\phi\phi}\over r}$} \msk
\leftdisplay{(\diver\hbox{\tf T\/})_\phi={1\over{r^2}}{\part\over{\part r}}
  (r^2 T_{r\phi})+{1\over{r\sin\theta}}{\part\over{\part\theta}}
  (\sin\theta T_{\theta\phi})}
\leftline{\hskip3.5truein$\displaystyle+{1\over{r\sin\theta}}
  {{\part T_{\phi\phi}}\over{\part\phi}}+{{T_{\phi r}}\over r}+
  {{\cot\theta}T_{\phi\theta}\over r}$}
\vfil\eject\end
```

Components of $(\mathbf{A}\cdot\nabla)\mathbf{B}$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_r = A_r\frac{\partial B_r}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_r}{\partial\theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_r}{\partial\phi} - \frac{A_\theta B_\theta + A_\phi B_\phi}{r}$$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_\theta = A_r\frac{\partial B_\theta}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\theta}{\partial\theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\theta}{\partial\phi} + \frac{A_\theta B_r}{r} - \frac{\cot\theta A_\phi B_\phi}{r}$$

$$(\mathbf{A}\cdot\nabla\mathbf{B})_\phi = A_r\frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\phi}{\partial\theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\phi}{\partial\phi} + \frac{A_\phi B_r}{r} + \frac{\cot\theta A_\phi B_\theta}{r}$$

Divergence of a tensor

$$(\nabla\cdot T)_r = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{\theta r})$$
$$+ \frac{1}{r\sin\theta}\frac{\partial T_{\phi r}}{\partial\phi} - \frac{T_{\theta\theta} + T_{\phi\phi}}{r}$$

$$(\nabla\cdot T)_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{\theta\theta})$$
$$+ \frac{1}{r\sin\theta}\frac{\partial T_{\phi\theta}}{\partial\phi} + \frac{T_{\theta r}}{r} - \frac{\cot\theta T_{\phi\phi}}{r}$$

$$(\nabla\cdot T)_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta T_{\theta\phi})$$
$$+ \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi} + \frac{T_{\phi r}}{r} + \frac{\cot\theta T_{\phi\theta}}{r}$$

```
   %
   % See prolog.tex for macro definitions.
   %
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=10
\centerline{\headfont DIMENSIONS AND UNITS} \msk\indent
To get the value of a quantity in Gaussian units, multiply the value expressed
in SI units by the conversion factor.  Multiples of 3 in the conversion
fac\-tors result from approximating the speed of light
$c=2.9979\times10^{10}\,$cm/sec $\approx3 \times10^{10}\,$cm/sec.
   %
   % This is the first and most complex example of a ruled table.  Ruled
   % tables make use of \halign to position the information in the columns
   % and the vertical rules separating them. \vbox is used in double
   % dollar signs ($$) in order to center the table on the page.
   %
$$\vbox{\tabskip=0pt \offinterlineskip
   %
   % \tabskip=0pt is used to avoid a space to the left of the first vrule.
   % \offinterlineskip is used to allow the vertical rules to run
   % uninterrupted through the table.
   %
\halign to\hsize{#&\vrule#\tabskip=0.25em plus1em&#\hfil&\vrule#&
#\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#
&#\hfil&\vrule#&#\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr
   %
   % This \halign preamble alternates columns containing \vrules with columns
   % containing actual table entries.  The vertical bar '|' has been defined
   % as two ampersands '&&', and indicates columns containing
   % only a \vrule.  As is usual, \cr ends each line.  Note also that many
   % measurements (such as \bs{0.2truein}) were arrived at by trial and
   % error, to align the heading entries correctly.
   %
\bs{0.2truein}\cr
\om &height2pt &\om|\om|\multispan3|\om|\om|\om&\cr
&\om|\om|\cr|\multispan3\hfil Dimensions \hfil|\om|\om|\om & \cr
&\hfil Physical | \hfil Sym- \hidewidth | \multispan3|
\hfil SI | \hfil Conversion \hidewidth | \hfil Gaussian &\cr
\noalign{\vskip-1.5ex\moveright1.88truein\vbox{\hrule width1.545truein}}
   %
   % \multispan3 is used to allow "Dimensions" to span three columns: that is,
   % two columns with entries, separated by one containing a \vrule.  With
   % \noalign we add the extra horizontal rule under "Dimensions" in the
   % correct place. (The numbers were found by trial and error.) A somewhat
   % less hard-wired approach to placing these horizontal rules is found
   % in the tables on pp.49-50.
   %
```

```
\tska{7}{1ex}
\s|\hfil Quantity |\hfil bol |\om\hfil SI \hfil |\om\hfil Gaussian
   %
   %  \tska is a macro used as a tableskip. It skips a variable distance
   %  vertically in a table, continuing all rules, in order to improve the
   %  spacing, and is adjusted so that the spacing is pleasing to the eye.
   %  At the beginning of each line is the macro \m or \s.  These are struts,
   %  invisible hboxes of fixed height, which determine the height of each
   %  line.  \m is slightly taller than \s. Again, they were chosen by eye.
   %  \bs is defined as \noalign{\vskip-#1} in prolog.tex.  It enables us
   %  to "back up" in tables.
   %
\hfil | \hfil Units | \hfil Factor | \hfil Units &\cr
   %
   %  The preamble automatically puts an \hfil to the right of each entry,
   %  so we need add \hfil only on the left, to center them.
   %
\tska{7}{7pt} \trule \tska{7}{1pt} \trule \tska{7}{2pt}
   %
   %  We skip down, leaving a double rule beneath the table header.  The
   %  body of the table follows:
   %
\m| Capacitance \hidewidth | $C$ |{t^2q^2 \over ml^2} | 1 |
  farad | $9\times10^{11}$ | cm & \cr
   %
   %  Note the use of \hidewidth.  TeX allocated more space than necessary
   %  if we allowed it to position the text itself, so we use \hidewidth to
   %  conceal the length of the longer entries.  This prevents "overfull
   %  hbox" errors.
   %
\m| Charge | $q$ | q | {m^{1/2}l^{3/2} \over t} | coulomb
  \hidewidth | $3\times10^9$ | statcoulomb \hidewidth & \cr \tska{7}{2pt}
\s| Charge | $\rho$ | {q \over l^3} | {m^{1/2} \over l^{3/2}t} |
  coulomb \hidewidth | $3\times10^3$ | statcoulomb \hidewidth & \cr \bs{1.5ex}
\s| \quad density | | | | \quad /m$^3$ | | \quad /cm$^3$ & \cr
  %
  %  Charge density is spread out over two line .  \bs{1.5ex} is used
  %  to position the two lines more closely.
  %
\m| Conductance \hidewidth | | {tq^2 \over ml^2} | {l \over t} |
  siemens | $9\times10^{11}$ | cm/sec & \cr \tska{7}{2pt}
\m| Conductivity | $\sigma$ | {tq^2}\over{ml^3} | 1\over t |
  siemens | $9\times10^9$ | sec$^{-1}$ & \cr \bs{2.0ex}
\s| | | | |\quad /m | | & \cr
\m| Current | $I,i$ | q\over t | {m^{1/2}l^{3/2}}\over{t^2}|
  ampere | $3\times10^9$ | statampere & \cr \tska{7}{2pt}
\s| Current | ${\bf J},{\bf j}$ | q\over{l^2t} | {m^{1/2}} \over
  {l^{1/2}t^2} | ampere | $3\times10^5$ | statampere & \cr \bs{1.5ex}
```

```
\s| \quad density | | | |\quad /m$^2$ | | \quad /cm$^2$ & \cr
\m| Density | $\rho$ | m\over{l^3} | m\over{l^3} | kg/m$^3$ |
  $10^{-3}$ | g/cm$^3$ & \cr
\s| Displacement \hidewidth | {\bf D} | q\over{l^2} | {m^{1/2}}
  \over {l^{1/2}t} | coulomb \hidewidth | $12\pi\times10^5$ \hidewidth |
  statcoulomb \hidewidth & \cr \bs{1.75ex}
\s| | | | |\quad /m$^2$ | | \quad /cm$^2$ & \cr
\m| Electric field | {\bf E} |{ml}\over{t^2q} |{m^{1/2}}\over{l^{1/2}t} |
  volt/m | $\displaystyle{1\over3}\times10^{-4}$ | statvolt/cm & \cr\tska{7}{2pt}
\m| Electro- | {\cs E}, | {ml^2}\over{t^2q} | {m^{1/2}l^{1/2}}
  \over t | volt | $\displaystyle{1\over3}\times10^{-2}$ | statvolt & \cr
  \bs{2.0ex}
\s| \quad motance |Emf | | | | | & \cr
\m| Energy | $U,W$ \hidewidth | {ml^2}\over{t^2} | {ml^2} \over
  {t^2} | joule | $10^7$ | erg & \cr
\m| Energy | $w,\epsilon$ | m\over{lt^2} | m\over{l t^2} |
  joule/m$^3$ \hidewidth | $10$ | erg/cm$^3$ & \cr \bs{2.0ex}
\s| \quad density | | | | | | & \cr \tska{7}{2pt} \cr \trule}}$$
   %
   %  End of Table.
   %
\vfill\eject\end
```

DIMENSIONS AND UNITS

To get the value of a quantity in Gaussian units, multiply the value expressed in SI units by the conversion factor. Multiples of 3 in the conversion factors result from approximating the speed of light $c = 2.9979 \times 10^{10}$ cm/sec $\approx 3 \times 10^{10}$ cm/sec.

Physical Quantity	Symbol	Dimensions		SI Units	Conversion Factor	Gaussian Units
		SI	Gaussian			
Capacitance	C	$\frac{t^2q^2}{ml^2}$	l	farad	9×10^{11}	cm
Charge	q	q	$\frac{m^{1/2}l^{3/2}}{t}$	coulomb	3×10^9	statcoulomb
Charge density	ρ	$\frac{q}{l^3}$	$\frac{m^{1/2}}{l^{3/2}t}$	coulomb /m^3	3×10^3	statcoulomb /cm^3
Conductance		$\frac{tq^2}{ml^2}$	$\frac{l}{t}$	siemens	9×10^{11}	cm/sec
Conductivity	σ	$\frac{tq^2}{ml^3}$	$\frac{1}{t}$	siemens /m	9×10^9	sec^{-1}
Current	I, i	$\frac{q}{t}$	$\frac{m^{1/2}l^{3/2}}{t^2}$	ampere	3×10^9	statampere
Current density	$\mathbf{J}, \mathbf{j}$	$\frac{q}{l^2t}$	$\frac{m^{1/2}}{l^{1/2}t^2}$	ampere /m^2	3×10^5	statampere /cm^2
Density	ρ	$\frac{m}{l^3}$	$\frac{m}{l^3}$	kg/m^3	10^{-3}	g/cm^3
Displacement	$\mathbf{D}$	$\frac{q}{l^2}$	$\frac{m^{1/2}}{l^{1/2}t}$	coulomb /m^2	$12\pi \times 10^5$	statcoulomb /cm^2
Electric field	$\mathbf{E}$	$\frac{ml}{t^2q}$	$\frac{m^{1/2}}{l^{1/2}t}$	volt/m	$\frac{1}{3} \times 10^{-4}$	statvolt/cm
Electromotance	$\mathcal{E}$, Emf	$\frac{ml^2}{t^2q}$	$\frac{m^{1/2}l^{1/2}}{t}$	volt	$\frac{1}{3} \times 10^{-2}$	statvolt
Energy	U, W	$\frac{ml^2}{t^2}$	$\frac{ml^2}{t^2}$	joule	10^7	erg
Energy density	w, ϵ	$\frac{m}{lt^2}$	$\frac{m}{lt^2}$	joule/m^3	10	erg/cm^3

```
\input prolog \pageno=11
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\centerline{\ }\nointerlineskip
$$\vbox{\tabskip=0pt \offinterlineskip
\halign to\hsize{#&\vrule#\tabskip=0.25em plus1em&#\hfil&\vrule#&
#\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#
&#\hfil&\vrule#&#\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr\bs{0.2truein}\trule
\om &height2pt &\om | \om | \multispan3 | \om | \om | \om & \cr
\s|\om |\om |\multispan3\hfil Dimensions \hfil |\om |\om |\om & \cr
\s | \hfil Physical | \hfil Sym- \hidewidth | \multispan3 | \hfil SI |
  \hfil Conversion \hidewidth | \hfil Gaussian & \cr \bs{1.5ex}
\noalign{\moveright1.91truein \vbox{\hrule width1.45truein}} \tska{7}{1.0ex}
\s | \hfil Quantity | \hfil bol | \om \hfil SI \hfil | \om \hfil
  Gaussian \hfil | \hfil Units | \hfil Factor | \hfil Units & \cr
\tska{7}{7pt} \trule \tska{7}{1.0pt} \trule \tska{7}{2pt}
\m| Force | {\bf F} | {ml}\over{t^2} | {ml}\over{t^2} |
  newton | $10^5$ | dyne & \cr
\m| Frequency | $f,\nu$ | 1\over t | 1\over t | hertz | 1| hertz & \cr
\m| Impedance | $Z$ | {ml^2}\over{tq^2} | t\over l | ohm|
  $\displaystyle{1\over 9}\times 10^{-11}$ \hidewidth | sec/cm & \cr \tska{7}{2pt}
\m| Inductance | $L$ | {ml^2}\over{q^2} | {t^2}\over l | henry |
  $\displaystyle{1\over 9}\times 10^{-11}$\hidewidth |sec$^2$/cm &\cr \tska{7}{2pt}
\s| Length | $l$ | l | l | meter (m) | $10^2$ |
  centimeter \hidewidth & \cr \tska{7}{2pt} \bs{1.0ex}
\s| | | | | | | \quad (cm) & \cr
\s| Magnetic | {\bf H} | q\over{lt} | {m^{1/2}} \over{l^{1/2}t} |
  ampere-- | $4\pi\times10^{-3}$ \hidewidth | oersted & \cr \bs{1.5ex}
\s| \quad intensity | | | | \quad turn/m | | & \cr
\m| Magnetic flux | $\Phi$ | {ml^2}\over{tq} | {m^{1/2}l^{3/2}}
  \over t | weber | $10^8$ | maxwell & \cr \tska{7}{2pt}
\m| Magnetic | {\bf B} | m\over{tq} | {m^{1/2}}\over {l^{1/2}t} |
  tesla | $10^4$ | gauss & \cr \bs{1.75ex}
\s| \quad induction | | | | | | & \cr
\m| Magnetic | $m, \mu$ | {l^2 q}\over t | {m^{1/2}l^{5/2}}\over t |
  ampere--m$^2$ \hidewidth | $10^3$ | oersted-- & \cr \bs{1.75ex}
\s| \quad moment | | | | | |\quad cm$^3$ & \cr
\m| Magnetization \hidewidth | {\bf M} | q\over{lt} | {m^{1/2}}\over
  {l^{1/2}t} | ampere-- \hidewidth | $10^{-3}$ | oersted & \cr \bs{1.75ex}
\s| | | | | \quad turn/m | | & \cr
\m| Magneto- | {\cs M}, | q\over t | {m^{1/2}l^{1/2}}\over{t^2} |
  ampere-- | $\displaystyle{4\pi \over 10}$ | gilbert & \cr \bs{2.0ex}
\s| \quad motance |Hmf | | | \quad turn | | & \cr
\m| Mass | $m, M$ \hidewidth | m | m | kilogram |$10^3$| gram (g)&\cr\bs{1.75ex}
\s| | | | | \quad (kg) | | & \cr
\m| Momentum | ${\bf p}, {\bf P}$ | {ml}\over t | {ml}\over t |
  kg--m/s | $10^5$ | g--cm/sec & \cr
\m| Momentum | | m\over{l^2 t} | m\over{l^2 t} | kg/m$^2$--s |
  $10^{-1}$ | g/cm$^2$--sec & \cr \bs{1.75ex}
\s| \quad density | | | | | | & \cr
\m| Permeability | $\mu$ | {ml}\over{q^2} | 1 | henry/m |
  $\displaystyle{1 \over 4\pi}\times 10^{7\ph{0}}$ | \qquad --- & \cr
\tska{7}{2pt} \trule}}$$ \vfil\eject\end
```

Physical Quantity	Symbol	Dimensions SI	Dimensions Gaussian	SI Units	Conversion Factor	Gaussian Units
Force	**F**	$\frac{ml}{t^2}$	$\frac{ml}{t^2}$	newton	10^5	dyne
Frequency	f, ν	$\frac{1}{t}$	$\frac{1}{t}$	hertz	1	hertz
Impedance	Z	$\frac{ml^2}{tq^2}$	$\frac{t}{l}$	ohm	$\frac{1}{9} \times 10^{-11}$	sec/cm
Inductance	L	$\frac{ml^2}{q^2}$	$\frac{t^2}{l}$	henry	$\frac{1}{9} \times 10^{-11}$	$\mathrm{sec}^2/\mathrm{cm}$
Length	l	l	l	meter (m)	10^2	centimeter (cm)
Magnetic intensity	**H**	$\frac{q}{lt}$	$\frac{m^{1/2}}{l^{1/2}t}$	ampere turn/m	$4\pi \times 10^{-3}$	oersted
Magnetic flux	Φ	$\frac{ml^2}{tq}$	$\frac{m^{1/2}l^{3/2}}{t}$	weber	10^8	maxwell
Magnetic induction	**B**	$\frac{m}{tq}$	$\frac{m^{1/2}}{l^{1/2}t}$	tesla	10^4	gauss
Magnetic moment	m, μ	$\frac{l^2q}{t}$	$\frac{m^{1/2}l^{5/2}}{t}$	ampere–m^2	10^3	oersted cm^3
Magnetization	**M**	$\frac{q}{lt}$	$\frac{m^{1/2}}{l^{1/2}t}$	ampere-turn/m	10^{-3}	oersted
Magneto-motance	$\mathcal{M}$, Mmf	$\frac{q}{t}$	$\frac{m^{1/2}l^{1/2}}{t^2}$	ampere turn	$\frac{4\pi}{10}$	gilbert
Mass	m, M	m	m	kilogram (kg)	10^3	gram (g)
Momentum	**p**, **P**	$\frac{ml}{t}$	$\frac{ml}{t}$	kg m/s	10^5	g cm/sec
Momentum density		$\frac{m}{l^2t}$	$\frac{m}{l^2t}$	$\mathrm{kg/m^2}$ s	10^{-1}	$\mathrm{g/cm^2}$ sec
Permeability	μ	$\frac{ml}{q^2}$	1	henry/m	$\frac{1}{4\pi} \times 10^7$	

```
\input prolog \pageno=12
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\centerline{\ }\nointerlineskip
$$\vbox{\tabskip=0pt \offinterlineskip
\halign to\hsize{#&\vrule#\tabskip=0.25em plus1em&#\hfil&\vrule#&
#\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#&$\displaystyle{#}$\hfil&\vrule#
&#\hfil&\vrule#&#\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr\bs{0.2truein}\trule
\om &height2pt &\om | \cm | \multispan3 | \om | \om | \om & \cr
\s|\om |\om |\multispan3\hfil Dimensions \hfil |\om |\om |\om & \cr
\s | \hfil Physical | \hfil Sym- \hidewidth | \multispan3 | \hfil SI |
\hfil Conversion \hidewidth | \hfil Gaussian & \cr
\bs{1.5ex}\noalign{\moveright1.69truein \vbox{\hrule width1.45truein}}\tska{7}{1.0ex}
\s | \hfil Quantity | \hfil bol | \om \hfil SI \hfil | \om \hfil
Gaussian \hfil | \hfil Units | \hfil Factor | \hfil Units & \cr
\tska{7}{7pt} \trule \tska{7}{1.0pt} \trule \tska{7}{2pt}
\m| Permittivity | $\epsilon$ | {t^2q^2}\over{ml^3} | 1 | farad/m |
  $36\pi\times 10^9$ | \qquad --- & \cr
\m| Polarization | {\bf P} | q\over{1^2} | {m^{1/2}}\over{1^{1/2}t} |
  coulomb/m$^2$ \hidewidth |$3\times 10^5$| statcoulomb\hidewidth &\cr\bs{1.5ex}
\s| | | | | | | \quad /cm$^2$ & \cr
\m| Potential |$V,\phi$ \hidewidth |{ml^2}\over{t^2q} |{m^{1/2}l^{1/2}}
  \over t |volt |$\displaystyle{1\over 3}\times10^{-2}$ |statvolt &\cr\tska{7}{3pt}
\m| Power | $P$ | {ml^2}\over{t^3} | {ml^2}\over{t^3} | watt | $10^7$ |
  erg/sec & \cr
\m| Power | | m\over{l t^3} | m\over{l t^3} | watt/m$^3$ | 10 |
  erg/cm$^3$--sec \hidewidth & \cr \bs{1.75ex}
\s| \quad density | | | | | | & \cr
\m| Pressure | $p, P$ \hidewidth | m\over{l t^2} | m\over{l t^2} |
  pascal | 10 | dyne/cm$^2$ & \cr \tska{7}{2pt}
\s| Reluctance | {\cs R} | {q^2}\over{ml^2} | 1\over l |
  ampere--turn \hidewidth |$4\pi\times 10^{-9}$ \hidewidth | cm$^{-1}$ &\cr\bs{1.5ex}
\s| | | | | \quad /weber | | & \cr
\m| Resistance | $R$ | {ml^2}\over{tq^2} | t\over l | ohm |
  $\displaystyle{1\over 9}\times 10^{-11}$ \hidewidth | sec/cm & \cr \tska{7}{3pt}
\m| Resistivity | $\eta,\rho$ | {ml^3}\over{tq^2} | t | ohm--m |
  $\displaystyle{1\over 9}\times 10^{-9}$ | sec & \cr \tska{7}{3pt}
\s| Thermal con- \hidewidth | $\kappa, k$ | {ml}\over{t^3} |
  {ml}\over{t^3} | watt/m-- | $10^5$ | erg/cm--sec-- \hidewidth & \cr \bs{1.5ex}
\s| \quad ductivity | | | | \quad deg (K) $\ph{0}$ | | \quad
  deg (K) $\ph{0}$ & \cr \tska{7}{2pt}
\s| Time | $t$ | t | t | second (s) | 1 | second (sec)& \cr
\m| Vector | {\bf A} | {ml}\over{tq} | {m^{1/2} l^{1/2}}\over t |
  weber/m | $10^6$ | gauss--cm & \cr \bs{2.0ex}
\s| \quad potential | | | | | | & \cr
\m| Velocity | {\bf v} | l\over t | l\over t | m/s | $10^2$ | cm/sec & \cr
\m| Viscosity | $\eta,\mu$ | m\over{l t} | m\over{l t} | kg/m--s | 10 |
  poise & \cr \tska{7}{2pt}
\m| Vorticity | $\zeta$ | 1\over t | 1\over t | s$^{-1}$ | 1 | sec$^{-1}$ & \cr
\m| Work | $W$ | {ml^2}\over{t^2} | {ml^2}\over{t^2} | joule | $10^7$ |
  erg & \cr \tska{7}{2pt} \trule}}$$ \vfil\eject\end
```

Physical Quantity	Symbol	Dimensions SI	Dimensions Gaussian	SI Units	Conversion Factor	Gaussian Units
Permittivity	ϵ	$\frac{t^2q^2}{ml^3}$	1	farad/m	$36\pi \times 10^9$	
Polarization	$\mathbf{P}$	$\frac{q}{l^2}$	$\frac{m^{1/2}}{l^{1/2}t}$	coulomb/m^2	3×10^5	statcoulomb /cm^2
Potential	V, ϕ	$\frac{ml^2}{t^2q}$	$\frac{m^{1/2}l^{1/2}}{t}$	volt	$\frac{1}{3} \times 10^{-2}$	statvolt
Power	P	$\frac{ml^2}{t^3}$	$\frac{ml^2}{t^3}$	watt	10^7	erg/sec
Power density		$\frac{m}{lt^3}$	$\frac{m}{lt^3}$	watt/m^3	10	erg/cm^3 sec
Pressure	p, P	$\frac{m}{lt^2}$	$\frac{m}{lt^2}$	pascal	10	dyne/cm^2
Reluctance	$\mathcal{R}$	$\frac{q^2}{ml^2}$	$\frac{1}{l}$	ampere turn /weber	$4\pi \times 10^{-9}$	cm^{-1}
Resistance	R	$\frac{ml^2}{tq^2}$	$\frac{t}{l}$	ohm	$\frac{1}{9} \times 10^{-11}$	sec/cm
Resistivity	η, ρ	$\frac{ml^3}{tq^2}$	t	ohm m	$\frac{1}{9} \times 10^{-9}$	sec
Thermal conductivity	κ, k	$\frac{ml}{t^3}$	$\frac{ml}{t^3}$	watt/m deg (K)	10^5	erg/cm sec deg (K)
Time	t	t	t	second (s)	1	second (sec)
Vector potential	$\mathbf{A}$	$\frac{ml}{tq}$	$\frac{m^{1/2}l^{1/2}}{t}$	weber/m	10^6	gauss cm
Velocity	$\mathbf{v}$	$\frac{l}{t}$	$\frac{l}{t}$	m/s	10^2	cm/sec
Viscosity	η, μ	$\frac{m}{lt}$	$\frac{m}{lt}$	kg/m s	10	poise
Vorticity	ζ	$\frac{1}{t}$	$\frac{1}{t}$	s^{-1}	1	sec^{-1}
Work	W	$\frac{ml^2}{t^2}$	$\frac{ml^2}{t^2}$	joule	10^7	erg

The listing for page 13 begins on the next page.

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=13
\centerline{\headfont INTERNATIONAL SYSTEM (SI) NOMENCLATURE$^6$}
\nointerlineskip
$$\vbox{\tabskip=0pt \offinterlineskip \def\S{{$\ph{*}$}}
   %  \S is used to leave the space of an asterisk, so that text lines up.
\halign to \hsize{\vrule# \tabskip=0.5em plus2em minus0.5em&#\hfil\strut&\vrule#
&#\hfil&\vrule#&\hfil#\hfil&\vrule#\tabskip=0pt\hskip1.0pt&\vrule#
\tabskip=0.5em plus2em minus1em&#\hfil\strut&\vrule#&#\hfil&\vrule#
&\hfil#\hfil&\vrule#\tabskip=0pt\cr \trule \tskb{2pt} \bs{1pt}
&\hfil Physical|\hfil Name|Symbol|
&\hfil Physical|\hfil Name|Symbol&\cr \bs{1pt}
&\hfil Quantity|\hfil of Unit|\hfil for Unit|
&\hfil Quantity|\hfil of Unit|\hfil for Unit&\cr \tskb{2pt} \bs{1pt} \trule
\tskb{1.0pt} \trule \tskb{2pt} \bs{1pt}
&*length|meter|m|&electric|volt|V&\cr \bs{2pt}
&\om|\om|\om|&potential||&\cr \bs{2pt}
&*mass|kilogram|kg|&\om|\om|\om&\cr \bs{2pt}
&\om|\om|\om|&electric|ohm|$\Omega$&\cr \bs{2pt}
&*time|second|s|&resistance|\om|\om&\cr \tskb{2pt}
&*current|ampere|A|&electric|siemens|S&\cr \bs{2pt}
&\om|\om|\om|&conductance|\om|\om&\cr \bs{2pt}
&*temperature|kelvin|K|&\om|\om|\om&\cr \bs{2pt}
&\om|\om|\om|&electric|farad|F&\cr \bs{2pt}
&*amount of|mole|mol|&capacitance|\om|\om&\cr \bs{2pt}
&\S substance|\om|\om|&\om|\om|\om&\cr \bs{2pt} \bs{2pt}
&\om|\om|\om|&magnetic flux|weber|Wb&\cr \bs{2pt}
&*luminous|candela|cd|&\om|\om|\om&\cr \bs{2pt}
&\S intensity|\om|\om|&magnetic|henry|H&\cr \bs{2pt}
&\om|\om|\om|&inductance|\om|\om&\cr \bs{2pt}
&\dag plane angle|radian|rad|&\om|\om|\om&\cr \bs{2pt}
&\om|\om|\om|&magnetic|tesla|T&\cr \bs{2pt}
&\dag solid angle|steradian|sr|&intensity|\om|\om&\cr \tskb{2pt}
&\S frequency|hertz|Hz|&luminous flux|lumen|lm&\cr \tskb{2pt}
&\S energy|joule|J|&illuminance|lux|lx&\cr \tskb{2pt}
&\S force|newton|N|&activity (of a|becquerel|Bq&\cr \bs{2pt}
&\om|\om|\om|&radioactive|\om|\om&\cr \bs{2pt}
&\S pressure|pascal|Pa|&source)|\om|\om&\cr \tskb{2pt}
&\S power|watt|W|&absorbed dose|gray|Gy&\cr \bs{2pt}
&\om|\om|\om|&(of ionizing|\om|\om&\cr \bs{2pt}
&\S electric charge|coulomb|C|&radiation)|\om|\om&\cr \bs{1pt}
height2pt&\om|\om|\om|&\om|\om|\om&\cr \trule}}$$
\vskip 4pt
*SI base unit \qquad \dag Supplementary Unit
  %
  %  Second table on page 13
  %
\bsk \centerline{\headfont METRIC PREFIXES}
\nointerlineskip
$$\vbox{\tabskip=0pt\offinterlineskip \def\hw{\hidewidth}
   %  THE MACRO \hw IS JUST USED TO SAVE SPACE AND TYPING.
\halign to \hsize{\vrule#\tabskip=1em plus2em minus1em&\quad#\hfil&\vrule#
&#\hfil&\vrule#&\hfil#\hfil\strut&\vrule#\tabskip=0pt\hskip1.0pt&\vrule#
\tabskip=1em plus2em minus1em&\quad#\hfil&\vrule#&#\hfil&\vrule#
&\hfil#\hfil&\vrule#\tabskip=0pt\cr \trule \tskb{2pt}
&\om\hfil\hw Multiple\hw\hfil|\hfil\hw Prefix\hw|\hw Symbol\hw|
```

```
&\om\hfil\hw Multiple\hw\hfil|\hfil\hw Prefix\hw|\hw Symbol\hw&\cr
\tskb{2pt} \trule \tskb{1.0pt} \trule \tskb{2pt}
&$10^{-1}$|deci|d|&$10$|deca|da&\cr \tskb{1pt}
&$10^{-2}$|centi|c|&$10^2$|hecto|h&\cr \tskb{1pt}
&$10^{-3}$|milli|m|&$10^3$|kilo|k&\cr \tskb{1pt}
&$10^{-6}$|micro|$\mu$|&$10^6$|mega|M&\cr \tskb{1pt}
&$10^{-9}$|nano|n|&$10^9$|giga|G&\cr \tskb{1pt}
&$10^{-12}$|pico|p|&$10^{12}$|tera|T&\cr \tskb{1pt}
&$10^{-15}$|femto|f|&$10^{15}$|peta|P&\cr \tskb{1pt}
&$10^{-18}$|atto|a|&$10^{18}$|exa|E&\cr \tskb{2pt} \trule}}$$
\vfill\eject\end
```

INTERNATIONAL SYSTEM (SI) NOMENCLATURE[6]

Physical Quantity	Name of Unit	Symbol for Unit	Physical Quantity	Name of Unit	Symbol for Unit
*length	meter	m	electric potential	volt	V
*mass	kilogram	kg	electric resistance	ohm	Ω
*time	second	s	electric conductance	siemens	S
*current	ampere	A	electric capacitance	farad	F
*temperature	kelvin	K	magnetic flux	weber	Wb
*amount of substance	mole	mol	magnetic inductance	henry	H
*luminous intensity	candela	cd	magnetic intensity	tesla	T
†plane angle	radian	rad	luminous flux	lumen	lm
†solid angle	steradian	sr	illuminance	lux	lx
frequency	hertz	Hz	activity (of a radioactive source)	becquerel	Bq
energy	joule	J	absorbed dose (of ionizing radiation)	gray	Gy
force	newton	N			
pressure	pascal	Pa			
power	watt	W			
electric charge	coulomb	C			

*SI base unit †Supplementary Unit

METRIC PREFIXES

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E

The listing for page 14 begins on the next page.

```
%
% See prolog.tex for macro definitions.
%
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=[illegible]truein
\pageno=14
\centerline{\headfont PHYSICAL CONSTANTS (SI)$[illegible]$}
%
% This is another example of a ruled table. This one is implemented
% slightly differently from that on page [illegible] [illegible] tables in the formulary
% follow one of the examples we have [illegible] [illegible], instead of adding a
% strut before each line, we have put a strut in the [illegible] [illegible]
% the lines simpler, but requires that they [illegible] be the [illegible] height
%
[illegible] def quad{ [illegible] }
[illegible] & \vrule# & [illegible] # \hfil [illegible] quad & \vrule# & [illegible] # [illegible] quad
& [illegible] rule# & \quad # [illegible] quad & \vrule# & \quad # [illegible] & [illegible]
%
% [illegible] ines that all vrules [illegible]
% through the table, with no gaps. We have [illegible] defined [illegible], and defined
% a new measurement called a [illegible] quid. By adjusting the [illegible] of [illegible] quad
% and [illegible] quid, we [illegible] adjust the spacing of the table [illegible]
% [illegible] ideal as closely as possible [illegible]
% [illegible] the next page and in [illegible]
% different [illegible] As opposed to the table on page 1[illegible], this table
% [illegible] quids and [illegible] quads, rather than with [illegible]
% [illegible] of the [illegible] alternate containing [illegible] vrules and containing
% [illegible] entries
%
[illegible] \hfil Symbol [illegible] \hfil Value [illegible] \hfil Units & \cr
[illegible] \trule [illegible] {4}{1.0pt} \trule \tsk {4}{2pt}
%
% [illegible] heading of the table is followed by a double rule [illegible]
% [illegible] again a [illegible] skip, leaving a blank line of variable height,
% [illegible] uninterrupted. The table entries follow:
[illegible]
```

```
   & \quad\quad free space | | | & \cr \tskc{4}{2pt}
& Proton/electron mass | ${m_p}/{m_e}$ | $1.8362\times10^3$ | & \cr
   & \quad\quad ratio | | | & \cr \tskc{4}{2pt}
& Electron charge/mass | $e/{m_e}$ |$1.7588\times10^{11}$ | C$\,$kg$^{-1}$ & \cr
   & \quad\quad ratio | | | & \cr \tskc{4}{2pt}
& Rydberg constant | $R_\infty=\displaystyle {me^4 \over 8{\epsilon_0}^2
   ch^3}$ | $1.0974\times10^7$ | m$^{-1}$ & \cr \tskc{4}{2pt}
& Bohr radius | $a_0=\epsilon_0 h^2/\pi me^2$ | $5.2918\times10^{-11}$ | m & \cr
   \tskc{4}{2pt}
& Atomic cross section | $\pi {a_0}^2$ | $8.7974\times10^{-21}$ | m$^2$ & \cr
   \tskc{4}{2pt}
& Classical electron radius | $r_e=e^2/4\pi \epsilon_0 mc^2$ |$2.8179
   \times10^{-15}$ | m & \cr \tskc{4}{2pt}
& Thomson cross section |$(8\pi/3){r_e}^2$ |$6.6524\times10^{-29}$ |m$^2$ & \cr
   \tskc{4}{2pt}
& Compton wavelength of | $h/{m_ec}$ | $2.4263\times10^{-12}$ | m & \cr
   & \quad\quad electron |$\hbar /{m_ec}$ |$3.8616\times10^{-13}$ |m
   & \cr \tskc{4}{2pt}
& Fine-structure constant | $\alpha=e^2/2 \epsilon_0 h c$ |
   $7.2974\times10^{-3}$ | & \cr
   & | $\alpha^{-1}$ | \hfil $137.04$ | & \cr \tskc{4}{2pt}
& First radiation constant | $c_1=2\pi hc^2$ | $3.7418\times10^{-2}$ |
   W$\,$m$^2$ & \cr \tskc{4}{2pt}
& Second radiation | $c_2=hc/k$ | $1.4388\times10^{-2}$ | m$\,$K &\cr
   & \quad\quad constant | | | & \cr \tskc{4}{2pt}
& Stefan-Boltzmann | $\sigma$ | $5.6703\times10^{-8}$ |
   W$\,$m$^{-2}$K$^{-4}$ & \cr
& \quad\quad constant | | | & \cr \tskc{4}{2pt} \trule}}$$
   % END OF RULED TABLE.
\vfil\eject\end
```

PHYSICAL CONSTANTS (SI)[7]

Physical Quantity	Symbol	Value	Units
Boltzmann constant	k	1.3807×10^{-23}	$\mathrm{J\,K^{-1}}$
Elementary charge	e	1.6022×10^{-19}	C
Electron mass	m_e	9.1095×10^{-31}	kg
Proton mass	m_p	1.6726×10^{-27}	kg
Gravitational constant	G	6.6720×10^{-11}	$\mathrm{m^3 s^{-2} kg^{-1}}$
Planck constant	h $\hbar = h/2\pi$	6.6262×10^{-34} 1.0546×10^{-34}	$\mathrm{J\,s}$ $\mathrm{J\,s}$
Speed of light in vacuum	c	2.9979×10^{8}	$\mathrm{m\,s^{-1}}$
Permittivity of free space	ϵ_0	8.8542×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Proton/electron mass ratio	m_p/m_e	1.8362×10^{3}	
Electron charge/mass ratio	e/m_e	1.7588×10^{11}	$\mathrm{C\,kg^{-1}}$
Rydberg constant	$R_\infty = \dfrac{me^4}{8\epsilon_0{}^2 ch^3}$	1.0974×10^{7}	$\mathrm{m^{-1}}$
Bohr radius	$a_0 = \epsilon_0 h^2/\pi m e^2$	5.2918×10^{-11}	m
Atomic cross section	$\pi a_0{}^2$	8.7974×10^{-21}	$\mathrm{m^2}$
Classical electron radius	$r_e = e^2/4\pi\epsilon_0 mc^2$	2.8179×10^{-15}	m
Thomson cross section	$(8\pi/3) r_e{}^2$	6.6524×10^{-29}	$\mathrm{m^2}$
Compton wavelength of electron	$h/m_e c$ $\hbar/m_e c$	2.4263×10^{-12} 3.8616×10^{-13}	m m
Fine-structure constant	$\alpha = e^2/2\epsilon_0 hc$ α^{-1}	7.2974×10^{-3} 137.04	
First radiation constant	$c_1 = 2\pi hc^2$	3.7418×10^{-2}	$\mathrm{W\,m^2}$
Second radiation constant	$c_2 = hc/k$	1.4388×10^{-2}	$\mathrm{m\,K}$
Stefan-Boltzmann constant	σ	5.6703×10^{-8}	$\mathrm{W\,m^{-2}K^{-4}}$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=15
$$\vbox{\offinterlineskip \def\quad{\hskip 4pt} \def\quid{\hskip 1pt} \hrule
   %
   %  The definitions of \quad and \quid change from table to table in order
   %  to keep overall table widths the same.
   %
\halign {&\vrule# &\quad #\hfil \quid &\vrule# &\strut \quad #\hfil \quid
&\vrule# &\quad #\hfil \quid &\vrule# &\quad #\hfil \quid &\vrule# \cr \tskc{4}{2pt}
&\hfil Physical Quantity | \hfil Symbol | \hfil Value | \hfil Units & \cr
\tskc{4}{2pt} \trule \tskc{4}{1.0pt} \trule \tskc{4}{2pt}
& Wavelength associated | $\lambda_0 = hc/e$ | $1.2399\times10^{-6}$ | m &
\cr & \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Frequency associated | $\nu_0 = e/h$ | $2.4180\times10^{14}$ | Hz & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Wave number associated | $k_0 = e/hc$ | $8.0655\times10^5$ | m$^{-1}$ & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Energy associated with | $h\nu_0$ | $1.6022\times10^{-19}$ | J & \cr
& \quad\quad 1 eV | | | & \cr \tskc{4}{2pt}
& Energy associated with | $hc$ | $1.9865\times10^{-25}$ | J & \cr
& \quad\quad 1 m$^{-1}$ | | | & \cr \tskc{4}{2pt}
& Energy associated with | $me^3/8{\epsilon_0}^2 h^2$ |\hfil 13.606 | eV &\cr
& \quad\quad 1 Rydberg | | | & \cr \tskc{4}{2pt}
& Energy associated with | $k/e$ | $8.6173\times10^{-5}$ | eV & \cr
& \quad\quad 1 Kelvin | | | & \cr \tskc{4}{2pt}
& Temperature associated | $e/k$ | $1.1605\times10^4$ | K & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Avogadro number | $N_A$ | $6.0220\times10^{23}$ | mol$^{-1}$ & \cr \tskc{4}{2pt}
& Faraday constant |$F=N_Ae$ |$9.6485\times10^4$ | C$\,$mol$^{-1}$ &\cr\tskc{4}{2pt}
& Gas constant | $R=N_Ak$ | \hfil 8.3144 | J$\,$K$^{-1}$mol$^{-1}$ & \cr
    \tskc{4}{2pt}
& Loschmidt's number | $n_0$ | $2.6868\times10^{25}$ | m$^{-3}$ & \cr
& \quad\quad (no. density at STP) | | | & \cr \tskc{4}{2pt}
& Atomic mass unit | $m_u$ | $1.6606\times10^{-27}$ | kg & \cr \tskc{4}{2pt}
& Standard temperature | $T_C$ | \hfil 273.16 | K & \cr \tskc{4}{2pt}
& Atmospheric pressure | $p_0=n_0kT_0$ | $1.0133\times10^5$ | Pa & \cr \tskc{4}{2pt}
& Pressure of 1 mm Hg | | $1.3332\times10^2$ | Pa & \cr
& \quad\quad (1 torr) | | | & \cr \tskc{4}{2pt}
& Molar volume at STP | $V_0=RT_0/p_0$ | $2.2415\times10^{-2}$ | m$^3$ & \cr
    \tskc{4}{2pt}
& Molar weight of air | $M_{\rm air}$ |$2.8971\times10^{-2}$ | kg &\cr\tskc{4}{2pt}
& calorie (cal) | | \hfil 4.1868 | J & \cr \tskc{4}{2pt}
& Gravitational | $g$ | \hfil 9.8067 | m$\,$s$^{-2}$ & \cr
& \quad\quad acceleration | | | & \cr \tskc{4}{2pt}} \hrule}$$
\vfil\eject\end
```

Physical Quantity	Symbol	Value	Units
Wavelength associated with 1 eV	$\lambda_0 = hc/e$	1.2399×10^{-6}	m
Frequency associated with 1 eV	$\nu_0 = e/h$	2.4180×10^{14}	Hz
Wave number associated with 1 eV	$k_0 = e/hc$	8.0655×10^{5}	m^{-1}
Energy associated with 1 eV	$h\nu_0$	1.6022×10^{-19}	J
Energy associated with 1 m^{-1}	hc	1.9865×10^{-25}	J
Energy associated with 1 Rydberg	$me^3/8\epsilon_0{}^2h^2$	13.606	eV
Energy associated with 1 Kelvin	k/e	8.6173×10^{-5}	eV
Temperature associated with 1 eV	e/k	1.1605×10^{4}	K
Avogadro number	N_A	6.0220×10^{23}	mol^{-1}
Faraday constant	$F = N_A e$	9.6485×10^{4}	$\mathrm{C\,mol}^{-1}$
Gas constant	$R = N_A k$	8.3144	$\mathrm{J\,K^{-1}mol^{-1}}$
Loschmidt's number (no. density at STP)	n_0	2.6868×10^{25}	m^{-3}
Atomic mass unit	m_u	1.6606×10^{-27}	kg
Standard temperature	T_0	273.16	K
Atmospheric pressure	$p_0 = n_0 k T_0$	1.0133×10^{5}	Pa
Pressure of 1 mm Hg (1 torr)		1.3332×10^{2}	Pa
Molar volume at STP	$V_0 = RT_0/p_0$	2.2415×10^{-2}	m^3
Molar weight of air	M_{air}	2.8971×10^{-2}	kg
calorie (cal)		4.1868	J
Gravitational acceleration	g	9.8067	$\mathrm{m\,s}^{-2}$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=16
\centerline{\headfont PHYSICAL CONSTANTS (cgs)$^7$}
$$\vbox{\offinterlineskip \def\quad{\hskip 3pt} \def\quid{\hskip 0pt} \hrule
   %
   % The definitions of \quad and \quid change from table to table in order
   % to keep overall table widths the same.
   %
\halign {&\vrule# &\quad #\hfil \quid &\vrule# &\strut \quad #\hfil \quid
&\vrule# &\quad #\hfil \quid &\vrule# &\quad #\hfil \quid &\vrule# \cr \tskc{4}{2pt}
&\hfil Physical Quantity | \hfil Symbol | \hfil Value |\hfil Units &\cr\tskc{4}{2pt}
\trule
height1.0pt &\om | \om | \om | \cm & \cr
\trule \tskc{4}{2pt}
& Boltzmann constant | $k$ |$1.3807\times10^{-16}$ |erg/deg$\,$(K) &\cr\tskc{4}{2pt}
& Elementary charge | $e$ | $4.8032\times10^{-10}$ | statcoulomb & \cr
& | | | \quad (statcoul) & \cr \tskc{4}{2pt}
& Electron mass | $m_e$ | $9.1095\times10^{-28}$ | g & \cr \tskc{4}{2pt}
& Proton mass | $m_p$ | $1.6726\times10^{-24}$ | g & \cr \tskc{4}{2pt}
& Gravitational constant | $G$ | $6.6720\times10^{-8}$ |
     dyne-cm$^2/$g$^2$ & \cr \tskc{4}{2pt}
& Planck constant | $h$ | $6.6262\times10^{-27}$ | erg-sec & \cr
& | $\hbar=h/2\pi$ | $1.0546\times10^{-27}$ | erg-sec & \cr \tskc{4}{2pt}
& Speed of light in vacuum | $c$ | $2.9979\times10^{10}$ | cm/sec &\cr\tskc{4}{2pt}
& Proton/electron mass | ${m_p}/{m_e}$ | $1.8362\times10^3$ | & \cr
& \quad\quad ratio | | | & \cr \tskc{4}{2pt}
& Electron charge/mass | $e/{m_e}$ | $5.2728\times10^{17}$ |
  statcoul/g & \cr
& \quad\quad ratio | | | & \cr \tskc{4}{2pt}
& Rydberg constant | $R_\infty=\displaystyle{{2\pi^2me^4}\over{ch^3}}$ |
    $1.0974\times10^5$ | cm$^{-1}$ & \cr \tskc{4}{2pt}
& Bohr radius | $a_0=\hbar^2/{me^2}$ | $5.2918\times10^{-9}$ | cm &\cr\tskc{4}{2pt}
& Atomic cross section | $\pi {a_0}^2$ | $8.7974\times10^{-17}$ |
    cm$^2$ & \cr \tskc{4}{2pt}
& Classical electron radius | $r_e=e^2/{mc^2}$ | $2.8179\times10^{-13}$ |
    cm & \cr \tskc{4}{2pt}
& Thomson cross section | $(8\pi/3){r_e}^2$ | $6.6524\times10^{-25}$ |
    cm$^2$ & \cr \tskc{4}{2pt}
& Compton wavelength of | $h/{m_ec}$ | $2.4263\times10^{-10}$ | cm & \cr
& \quad\quad electron | $\hbar/{m_ec}$ |$3.8616\times10^{-11}$ |cm &\cr\tskc{4}{2pt}
& Fine-structure constant | $\alpha=e^2/\hbar c$ | $7.2974\times10^{-3}$ | & \cr
& | $\alpha^{-1}$ | \hfil $137.04$ | & \cr \tskc{4}{2pt}
& First radiation constant | $c_1=2\pi hc^2$ | $3.7418\times10^{-5}$ |
    erg-cm$^2/$sec & \cr \tskc{4}{2pt}
& Second radiation | $c_2=hc/k$ | \hfil $1.4388$ | cm-deg$\,$(K) & \cr
& \quad\quad constant | | | & \cr \tskc{4}{2pt}
& Stefan-Boltzmann | $\sigma$ | $5.6703\times10^{-5}$ | erg/cm$^2$- & \cr
& \quad\quad constant | | | \quad sec-deg$^4$ & \cr \tskc{4}{2pt}
& Wavelength associated | $\lambda_0$ | $1.2399\times10^{-4}$ | cm & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt} \trule}}$$
\vfil\eject\end
```

PHYSICAL CONSTANTS (cgs)[7]

Physical Quantity	Symbol	Value	Units
Boltzmann constant	k	1.3807×10^{-16}	erg/deg(K)
Elementary charge	e	4.8032×10^{-10}	statcoulomb (statcoul)
Electron mass	m_e	9.1095×10^{-28}	g
Proton mass	m_p	1.6726×10^{-24}	g
Gravitational constant	G	6.6720×10^{-8}	dyne-cm^2/g^2
Planck constant	h $\hbar = h/2\pi$	6.6262×10^{-27} 1.0546×10^{-27}	erg-sec erg-sec
Speed of light in vacuum	c	2.9979×10^{10}	cm/sec
Proton/electron mass ratio	m_p/m_e	1.8362×10^{3}	
Electron charge/mass ratio	e/m_e	5.2728×10^{17}	statcoul/g
Rydberg constant	$R_\infty = \dfrac{2\pi^2 me^4}{ch^3}$	1.0974×10^{5}	cm^{-1}
Bohr radius	$a_0 = \hbar^2/me^2$	5.2918×10^{-9}	cm
Atomic cross section	$\pi {a_0}^2$	8.7974×10^{-17}	cm^2
Classical electron radius	$r_e = e^2/mc^2$	2.8179×10^{-13}	cm
Thomson cross section	$(8\pi/3){r_e}^2$	6.6524×10^{-25}	cm^2
Compton wavelength of electron	$h/m_e c$ $\hbar/m_e c$	2.4263×10^{-10} 3.8616×10^{-11}	cm cm
Fine-structure constant	$\alpha = e^2/\hbar c$ α^{-1}	7.2974×10^{-3} 137.04	
First radiation constant	$c_1 = 2\pi hc^2$	3.7418×10^{-5}	erg-cm^2/sec
Second radiation constant	$c_2 = hc/k$	1.4388	cm-deg(K)
Stefan-Boltzmann constant	σ	5.6703×10^{-5}	erg/cm^2-sec-deg^4
Wavelength associated with 1 eV	λ_0	1.2399×10^{-4}	cm

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=17
$$\vbox{\offinterlineskip \def\quad{\hskip 4pt} \def\quid{\hskip 1pt} \hrule
   %
   %  The definitions of \quad and \quid change from table to table in order to
   %  keep overall table widths the same.
   %
\halign {&\vrule# &\quad #\hfil \quid &\vrule# &\strut \quad #\hfil \quid
&\vrule# &\quad #\hfil \quid &\vrule# &\quad #\hfil \quid &\vrule# \cr \tskc{4}{2pt}
&\hfil Physical Quantity | \hfil Symbol |\hfil Value |\hfil Units &\cr\tskc{4}{2pt}
\trule
height1.0pt &\om | \om | \om | \om & \cr
\trule \tskc{4}{2pt}
& Frequency associated | $\nu_0$ | $2.4180\times10^{14}$ | Hz & \cr
&\quad\quad  with 1 eV | | | & \cr \tskc{4}{2pt}
& Wave number associated | $k_0$ | $8.0655\times10^3$ | cm$^{-1}$ & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Energy associated with | | $1.6022\times10^{-12}$ | erg & \cr
& \quad\quad 1 eV | | | & \cr \tskc{4}{2pt}
& Energy associated with | | $1.9865\times10^{-16}$ | erg & \cr
& \quad\quad 1 cm$^{-1}$ | | | & \cr \tskc{4}{2pt}
& Energy associated with | | \hfil 13.606 | eV & \cr
& \quad\quad 1 Rydberg | | | & \cr \tskc{4}{2pt}
& Energy associated with | | $8.6173\times10^{-5}$ | eV & \cr
& \quad\quad 1 deg Kelvin | | | & \cr \tskc{4}{2pt}
& Temperature associated | | $1.1605\times10^4$ | deg$\,$(K) & \cr
& \quad\quad with 1 eV | | | & \cr \tskc{4}{2pt}
& Avogadro number | $N_A$ | $6.0220\times10^{23}$ | mol$^{-1}$ & \cr \tskc{4}{2pt}
& Faraday constant | $F=N_Ae$ | $2.8925\times10^{14}$ |
    statcoul/mol & \cr \tskc{4}{2pt}
& Gas constant | $R=N_Ak$ | $8.3144\times10^7$ | erg/deg-mol & \cr \tskc{4}{2pt}
& Loschmidt's number | $n_0$ | $2.6868\times10^{19}$ | cm$^{-3}$ & \cr
& \quad\quad (no. density at STP) | | | & \cr \tskc{4}{2pt}
& Atomic mass unit | $m_u$ | $1.6606\times10^{-24}$ | g & \cr \tskc{4}{2pt}
& Standard temperature | $T_0$ | \hfil 273.16 | deg$\,$(K) & \cr \tskc{4}{2pt}
& Atmospheric pressure | $p_0=n_0kT_0$ | $1.0133\times10^6$ |
    dyne/cm$^2$ & \cr \tskc{4}{2pt}
& Pressure of 1 mm Hg | | $1.3332\times10^3$ | dyne/cm$^2$ & \cr
& \quad\quad (1 torr) | | | & \cr \tskc{4}{2pt}
& Molar volume at STP | $V_0=RT_0/p_0$ |$2.2415\times10^4$ |cm$^3$ &\cr\tskc{4}{2pt}
& Molar weight of air | $M_{\rm air}$ | \hfil 28.971 | g & \cr \tskc{4}{2pt}
& calorie (cal) | | $4.1868\times10^7$ | erg & \cr \tskc{4}{2pt}
& Gravitational | $g$ | \hfil 980.67 | cm/sec$^2$ & \cr
& \quad\quad acceleration | | | & \cr \tskc{4}{2pt}} \hrule}$$
\vfil\eject\end
```

Physical Quantity	Symbol	Value	Units
Frequency associated with 1 eV	ν_0	2.4180×10^{14}	Hz
Wave number associated with 1 eV	k_0	8.0655×10^{3}	cm^{-1}
Energy associated with 1 eV		1.6022×10^{-12}	erg
Energy associated with 1 cm^{-1}		1.9865×10^{-16}	erg
Energy associated with 1 Rydberg		13.606	eV
Energy associated with 1 deg Kelvin		8.6173×10^{-5}	eV
Temperature associated with 1 eV		1.1605×10^{4}	deg (K)
Avogadro number	N_A	6.0220×10^{23}	mol^{-1}
Faraday constant	$F = N_A e$	2.8925×10^{14}	statcoul/mol
Gas constant	$R = N_A k$	8.3144×10^{7}	erg/deg-mol
Loschmidt's number (no. density at STP)	n_0	2.6868×10^{19}	cm^{-3}
Atomic mass unit	m_u	1.6606×10^{-24}	g
Standard temperature	T_0	273.16	deg (K)
Atmospheric pressure	$p_0 = n_0 k T_0$	1.0133×10^{6}	$dyne/cm^2$
Pressure of 1 mm Hg (1 torr)		1.3332×10^{3}	$dyne/cm^2$
Molar volume at STP	$V_0 = RT_0/p_0$	2.2415×10^{4}	cm^3
Molar weight of air	M_{air}	28.971	g
calorie (cal)		4.1868×10^{7}	erg
Gravitational acceleration	g	980.67	cm/sec^2

```
\input prolog \pageno=18
\hoffset=1.25truein \voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\centerline{\headfont FORMULA CONVERSION$^8$}\medskip\indent
Here $\alpha=10^2\,$m$\,$m$^{-1}$, $\beta=10^7\,$erg$\,$J$^{-1}$, $\epsilon_0
=8.8542\times10^{-12}\,$F$\,$m$^{-1}$, $\mu_0=4\pi\times10^{-7}\,$H$\,$m$^{-1}$,
$c=(\epsilon_0\mu_0)^{-1/2}=2.9979\times10^8\,$m$\,$s$^{-1}$, and $\hbar=1.0546
\times10^{-34}\,$J$\,$s. To derive a dimensionally correct SI formula from one
expressed in Gaussian units, substitute for each quantity according to
$\ov{Q}=\ov{k}Q$, where $\ov{k}$ is the coefficient in the second column of the
table corresponding to $Q$ (overbars denote variables expressed in Gaussian
units). Thus, the formula $\ov{a}_0=\ov{\hbar}^2/\ov{m} \hbox{\kern0.5pt}
\ov{e}^2$ for the Bohr radius becomes $\alpha a_0 = (\hbar\beta)^2/ [(m\beta
/\alpha^2) (e^2\alpha\beta/4\pi\epsilon_0)]$, or $a_0 = \epsilon_0 h^2/ \pi m
e^2$.  To go from SI to natural units in which $\hbar=c=1$ (distinguished by a
circumflex), use $Q=\un{k}^{-1} \un{Q}$, where $\un{k}$ is the coefficient
corresponding to $Q$ in the third column.  Thus $\un{a}_0 = 4\pi\epsilon_0
\hbar^2/[(\un{m}\hbar/c)(\un{e}^2 \epsilon_0\hbar c)] = 4\pi /\un{m}
\hbox{\kern0.5pt} \un{e}^2.$  (In transforming {\it from} SI units, do not
substitute for $\epsilon_0$, $\mu_0$, or $c$.)
$$\vbox{\vskip-3pt \offinterlineskip\hrule \def\quad{\hskip6pt}\def\quid{\hskip2pt}
 \halign{&\vrule# & \quad # \hfil \quid & \vrule# \quad&\strut#\hfil&\quad
\vrule#\quad&# \hfil& \quad \vrule# \cr \tskc{3}{1pt}
& \hfil Physical Quantity \hfil Gaussian Units \to SI|\hfil
   Natural Units to SI& \cr \tskc{3}{2pt}\trule \tskc{3}{1.0pt}\trule \tskc{3}{2pt}
& Capacitance | $\alpha/4\pi\epsilon_0$ | ${\epsilon_0}^{-1}$ & \cr \tskc{3}{1pt}
& Charge | $(\alpha\beta/4\pi\epsilon_0)^{1/2}$ |
   $(\epsilon_0 \hbar c)^{-1/2}$ & \cr \tskc{3}{0.5pt}
& Charge density | $(\beta/4\pi\alpha^5\epsilon_0)^{1/2}$ |
   $(\epsilon_0 \hbar c)^{-1/2}$ & \cr \tskc{3}{0.5pt}
& Current | $(\alpha\beta/4\pi\epsilon_0)^{1/2}$ |
   $(\mu_0/\hbar c)^{1/2}$ & \cr \tskc{3}{0.5pt}
& Current density | $(\beta/4\pi\alpha^3\epsilon_0)^{1/2}$ |
   $(\mu_0/\hbar c)^{1/2}$ & \cr \tskc{3}{0.5pt}
& Electric field | $(4\pi\beta\epsilon_0/\alpha^3)^{1/2}$ |
   $(\epsilon_0/\hbar c)^{1/2}$ & \cr \tskc{3}{0.5pt}
& Electric potential | $(4\pi\beta\epsilon_0/\alpha)^{1/2}$ |
   $(\epsilon_0/\hbar c)^{1/2}$ & \cr \tskc{3}{0.5pt}
& Electric conductivity |$(4\pi\epsilon_0)^{-1}$|${\epsilon_0}^{-1}$&\cr\tskc3{0.5pt}
& Energy | $\beta$ | $(\hbar c)^{-1}$ & \cr \tskc{3}{0.5pt}
& Energy density | $\beta/\alpha^3$ | $(\hbar c)^{-1}$ & \cr \tskc{3}{0.5pt}
& Force | $\beta/\alpha$ | $(\hbar c)^{-1}$ & \cr \tskc{3}{0.5pt}
& Frequency | 1 | $c^{-1}$ & \cr \tskc{3}{0.5pt}
& Inductance | $4\pi\epsilon_0/\alpha$ | ${\mu_0}^{-1}$ & \cr \tskc{3}{0.5pt}
& Length | $\alpha$ | 1 & \cr \tskc{3}{0.5pt}
& Magnetic induction | $(4\pi\beta/\alpha^3\mu_0)^{1/2}$ |
   $(\mu_0\hbar c)^{-1/2}$ & \cr \tskc{3}{0.5pt}
& Magnetic intensity | $(4\pi\mu_0\beta/\alpha^3)^{1/2}$ |
   $(\mu_0/\hbar c)^{1/2}$ & \cr \tskc{3}{0.5pt}
& Mass | $\beta/\alpha^2$ | $c/\hbar$ & \cr \tskc{3}{0.5pt}
& Momentum | [illegible] | $\hbar^{-1}$ & \cr \tskc{3}{0.5pt}
& Power | [illegible] | [illegible] & \cr \tskc{3}{0.5pt}
& Pressure | [illegible] | $(\hbar [illegible] & \cr \tskc{3}{0.5pt}
& Resistance | [illegible] |  [illegible]^{1/2}$ & \cr \tskc{3}{0.5pt}
& Time | 1 | [illegible] & \cr \tskc{3}{0.5pt}
& Velocity | [illegible] \trule}}$$ \vfil\eject\end
```

FORMULA CONVERSION[8]

Here $\alpha = 10^2\,\mathrm{cm\,m^{-1}}$, $\beta = 10^7\,\mathrm{erg\,J^{-1}}$, $\epsilon_0 = 8.8542\times10^{-12}\,\mathrm{F\,m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$, $c = (\epsilon_0\mu_0)^{-1/2} = 2.9979 \times 10^8\,\mathrm{m\,s^{-1}}$, and $\hbar = 1.0546 \times 10^{-34}\,\mathrm{J\,s}$. To derive a dimensionally correct SI formula from one expressed in Gaussian units, substitute for each quantity according to $\bar{Q} = \bar{k}Q$, where $\bar{k}$ is the coefficient in the second column of the table corresponding to Q (overbars denote variables expressed in Gaussian units). Thus, the formula $\bar{a}_0 = \bar{\hbar}^2/\bar{m}\bar{e}^2$ for the Bohr radius becomes $\alpha a_0 = (\hbar\beta)^2/[(m\beta/\alpha^2)(e^2\alpha\beta/4\pi\epsilon_0)]$, or $a_0 = \epsilon_0 h^2/\pi m e^2$. To go from SI to natural units in which $\hbar = c = 1$ (distinguished by a circumflex), use $Q = \hat{k}^{-1}\hat{Q}$, where $\hat{k}$ is the coefficient corresponding to Q in the third column. Thus $\hat{a}_0 = 4\pi\epsilon_0\hbar^2/[(\hat{m}\hbar/c)(\hat{e}^2\epsilon_0\hbar c)] = 4\pi/\hat{m}\hat{e}^2$. (In transforming *from* SI units, do not substitute for ϵ_0, μ_0, or c.)

Physical Quantity	Gaussian Units to SI	Natural Units to SI
Capacitance	$\alpha/4\pi\epsilon_0$	$\epsilon_0{}^{-1}$
Charge	$(\alpha\beta/4\pi\epsilon_0)^{1/2}$	$(\epsilon_0\hbar c)^{-1/2}$
Charge density	$(\beta/4\pi\alpha^5\epsilon_0)^{1/2}$	$(\epsilon_0\hbar c)^{-1/2}$
Current	$(\alpha\beta/4\pi\epsilon_0)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Current density	$(\beta/4\pi\alpha^3\epsilon_0)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Electric field	$(4\pi\beta\epsilon_0/\alpha^3)^{1/2}$	$(\epsilon_0/\hbar c)^{1/2}$
Electric potential	$(4\pi\beta\epsilon_0/\alpha)^{1/2}$	$(\epsilon_0/\hbar c)^{1/2}$
Electric conductivity	$(4\pi\epsilon_0)^{-1}$	$\epsilon_0{}^{-1}$
Energy	β	$(\hbar c)^{-1}$
Energy density	β/α^3	$(\hbar c)^{-1}$
Force	β/α	$(\hbar c)^{-1}$
Frequency	1	c^{-1}
Inductance	$4\pi\epsilon_0/\alpha$	$\mu_0{}^{-1}$
Length	α	1
Magnetic induction	$(4\pi\beta/\alpha^3\mu_0)^{1/2}$	$(\mu_0\hbar c)^{-1/2}$
Magnetic intensity	$(4\pi\mu_0\beta/\alpha^3)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Mass	β/α^2	$c/\hbar$
Momentum	β/α	$\hbar^{-1}$
Power	β	$(\hbar c^2)^{-1}$
Pressure	β/α^3	$(\hbar c)^{-1}$
Resistance	$4\pi\epsilon_0/\alpha$	$(\epsilon_0/\mu_0)^{1/2}$
Time	1	c
Velocity	α	c^{-1}

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=19
\centerline{\headfont MAXWELL'S EQUATIONS}
$$\vbox{\tabskip=0pt \offinterlineskip
\halign to \hsize{\vrule# \tabskip=1.0em plus2em minus0.5em&#\hfil\strut&\vrule#
&$\displaystyle#$\hfil&\vrule#&$\displaystyle#$\hfil&\vrule#\tabskip=0pt\cr
\trule \tskc{3}{2pt}
&\hfil Name or Description|\om\hfil SI\hfil|\om\hfil Gaussian\hfil&\cr
\tskc{3}{2pt} \trule \tskc{3}{1pt} \trule \tskc{3}{2.5pt}
&Faraday's law|\del\times\E=-{\partial\B\over\partial t}|\del\times\E
=-{1\over c}{\partial\B\over\partial t}&\cr \tskc{3}{2pt} \tskc{3}{2pt}
&Ampere's law|\del\times{\bf H}={\partial\D\over\partial t}+{\bf J}|\del\times
{\bf H}={1\over c}{\partial\D\over\partial t}+{4\pi\over c}{\bf J}&\cr\tskc{3}{4pt}
&Poisson equation|\del\cdot\D=\rho|\del\cdot\D=4\pi\rho&\cr \tskc{3}{2pt}
&[Absence of magnetic|\del\cdot\B=0|\del\cdot\B=0&\cr \bs{2pt}
&\quad monopoles]|\om|\om&\cr \tskc{3}{2pt} \bs{2pt}
&Lorentz force on|q\left(\E+{\bf v}\times\B\right)| q\left(\E+{1\over c}{\bf
v}\times\B\right)&\cr \bs{2pt} \bs{2pt} \bs{2pt}
&\quad charge $q$|\om|\om&\cr \tskc{3}{2pt}
&Constitutive|\D=\epsilon\E|\D=\epsilon\E&\cr \bs{1pt}
&\quad relations|\B=\mu{\bf H}|\B=\mu{\bf H}&\cr \tskc{3}{2pt} \trule}}$$
\vskip-6pt
In a plasma, $\mu\approx\mu_0=4\pi\times 10^{-7}\,{\rm H}\,{\rm m}^{-1}$
(Gaussian units: $\mu\approx 1$).  The permittivity satisfies
$\epsilon\approx\epsilon_0 = 8.8542\times 10^{-12}\,{\rm F}\,{\rm m}^{-1}$
(Gaussian: $\epsilon \approx 1)$ provided that all charge is regarded as free.
Using the drift approximation ${\bf v}_\perp= \E\times\B/B^2$ to calculate
polarization charge density gives rise to a dielectric constant
$K\equiv\epsilon/\epsilon_0 = 1+36\pi\times10^9\rho/B^2\>$ (SI) $=1+4\pi\rho
c^2/B^2\>$ (Gaussian), where $\rho$ is the mass density.

\indent
The electromagnetic energy in volume $V$ is given by
$$\eqalignno{W &= {1\over 2}\int_V\,dV({\bf H}\cdot\B+\E\cdot\D)&
(\rm SI)\ph{\rm Gaussian.}\cr
&={1\over 8\pi}\int_V\,dV({\bf H}\cdot\B+\E\cdot\D)&(\rm Gaussian).
\ph{\rm SI}\cr}$$

Poynting's theorem is
$${\partial W \over partial t}+\int_S\,{\bf H}\cdot d{\bf S}=-\int_V\,dV{\bf J}
\cdot\E,$$
where $S$ is the closed surface bounding $V$ and the Poynting vector (energy
flux across $S$) is given by ${\bf N}=\E\times{\bf H}\>$ (SI) or ${\bf
N}=c\E\times {\bf H}/4\pi\>$ (Gaussian).
\vfill\eject\end
```

MAXWELL'S EQUATIONS

Name or Description	SI	Gaussian
Faraday's law	$\nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$	$\nabla\times\mathbf{E} = -\dfrac{1}{c}\dfrac{\partial\mathbf{B}}{\partial t}$
Ampere's law	$\nabla\times\mathbf{H} = \dfrac{\partial\mathbf{D}}{\partial t}+\mathbf{J}$	$\nabla\times\mathbf{H} = \dfrac{1}{c}\dfrac{\partial\mathbf{D}}{\partial t}+\dfrac{4\pi}{c}\mathbf{J}$
Poisson equation	$\nabla\cdot\mathbf{D} = \rho$	$\nabla\cdot\mathbf{D} = 4\pi\rho$
[Absence of magnetic monopoles]	$\nabla\cdot\mathbf{B} = 0$	$\nabla\cdot\mathbf{B} = 0$
Lorentz force on charge q	$q\,(\mathbf{E}+\mathbf{v}\times\mathbf{B})$	$q\left(\mathbf{E}+\dfrac{1}{c}\mathbf{v}\times\mathbf{B}\right)$
Constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$

In a plasma, $\mu \approx \mu_0 = 4\pi\times 10^{-7}\,\mathrm{H\,m^{-1}}$ (Gaussian units: $\mu \approx 1$). The permittivity satisfies $\epsilon \approx \epsilon_0 = 8.8542\times 10^{-12}\,\mathrm{F\,m^{-1}}$ (Gaussian: $\epsilon \approx 1$) provided that all charge is regarded as free. Using the drift approximation $\mathbf{v}_\perp = \mathbf{E}\times\mathbf{B}/B^2$ to calculate polarization charge density gives rise to a dielectric constant $K \equiv \epsilon/\epsilon_0 = 1 + 36\pi\times 10^9\rho/B^2$ (SI) $= 1 + 4\pi\rho c^2/B^2$ (Gaussian), where ρ is the mass density.

The electromagnetic energy in volume V is given by

$$W = \frac{1}{2}\int_V dV(\mathbf{H}\cdot\mathbf{B}+\mathbf{E}\cdot\mathbf{D}) \qquad \text{(SI)}$$

$$= \frac{1}{8\pi}\int_V dV(\mathbf{H}\cdot\mathbf{B}+\mathbf{E}\cdot\mathbf{D}) \qquad \text{(Gaussian)}.$$

Poynting's theorem is

$$\frac{\partial W}{\partial t} + \int_S \mathbf{N}\cdot d\mathbf{S} = -\int_V dV\,\mathbf{J}\cdot\mathbf{E},$$

where S is the closed surface bounding V and the Poynting vector (energy flux across S) is given by $\mathbf{N} = \mathbf{E}\times\mathbf{H}$ (SI) or $\mathbf{N} = c\mathbf{E}\times\mathbf{H}/4\pi$ (Gaussian).

```
\input prolog.tex
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=20
\centerline{\headfont ELECTRICITY AND MAGNETISM}
\medskip\indent
In the following, $\epsilon=$ dielectric permittivity, $\mu=$ permeability
of conductor, $\mu^\prime=$ permeability of surrounding medium,
$\sigma=$ conductivity, $f=\omega/2\pi=$ radiation frequency,
$\kappa_m=\mu/\mu_0$ and $\kappa_e=\epsilon/\epsilon_0$.  Where subscripts are
used, '1' denotes a conducting medium and '2' a propagating (lossless
dielectric) medium.  All units are SI unless otherwise specified.

\msk \halign{#\hfil&\quad#& =\ #\hfil\cr
Permittivity of free space & $\epsilon_0$ & $8.8542
\times10^{-12}\,$F$\,$m$^{-1}$ \cr \sk
Permeability of free space & $\mu_0$ & $4\pi\times10^{-7}\,$H$\,$m$^{-1}$ \cr
|$1.2566\times10^{-6}\,$H$\,$m$^{-1}$ \cr \sk
Resistance of free space & $R_0$\hidewidth& $(\mu_0/\epsilon_0)^{1/2}=376.73\,
    \Omega$ \cr \sk
Capacity of parallel plates of area & $C$ & $\epsilon A/d$ \cr
    \noalign{\quad$A$, separated by distance $d$} \sk
Capacity of concentric cylinders & $C$ & $2\pi\epsilon l\ln(b/a)$ \cr
    \noalign{\quad of length $l$, radii $a,b$} \sk
Capacity of concentric spheres of & $C$ & $4\pi\epsilon ab/(b-a)$ \cr
    \noalign{\quad radii $a,b$} \sk
Self-inductance of wire of length & $L$ & $\mu l$ \cr
    \noalign{\quad $l$, carrying uniform current} \sk
Mutual inductance of parallel wires & $L$ & $(\mu^\prime l/4 \pi)
    \left[1+4\ln(d/a)\right]$ \cr
    \noalign{\quad of length $l$, radius $a$, separated}
    \noalign{\quad by distance $d$} \sk
Inductance of circular loop of radius & $L$ & $b\left\{\mu^\prime\left[\ln(8b/
    a)-2\right]+\mu/4\right\}$ \cr
    \bs{0.65ex}
    \noalign{\quad$b$, made of wire of radius $a$,}
    \noalign{\quad carrying uniform current} \sk
Relaxation time in a lossy medium & $\tau$ & $\epsilon/\sigma$ \cr \sk
Skin depth in a lossy medium & $\delta$ & $(2/\omega\mu\sigma)^{1/2}
    = (\pi f\mu\sigma)^{-1/2}$ \cr \sk
Wave impedance in a lossy medium & $Z$ & $\left[\mu/(\epsilon+i\sigma/\omega)
    \right]^{1/2}$ \cr \sk
Transmission coefficient at & $T$ & $4.22\times10^{-4}(f\kappa_{m1}
    \kappa_{e2}/\sigma)^{1/2}$ \cr
    \noalign{\quad conducting surface}
    \noalign{\quad(good only for $T \ll 1)^9$} \sk
Field at distance r from straight wire & $B_\theta$\hidewidth&
    $\mu I/2\pi r\,$tesla \cr
\quad carrying current $I$ (amperes) | $0.2I/r\,$gauss ($r$ in cm) \cr \sk
Field at distance $z$ along axis from & $B_z\hidewidth$&
    $\mu a^2I/[2(a^2+z^2)^{3/2}]$ \cr
    \noalign{\quad circular loop of radius $a$}
    \noalign{\quad carrying current $I$}}
\vfil\eject\end
```

ELECTRICITY AND MAGNETISM

In the following, ϵ = dielectric permittivity, μ = permeability of conductor, μ' = permeability of surrounding medium, σ = conductivity, $f = \omega/2\pi$ = radiation frequency, $\kappa_m = \mu/\mu_0$ and $\kappa_e = \epsilon/\epsilon_0$. Where subscripts are used, '1' denotes a conducting medium and '2' a propagating (lossless dielectric) medium. All units are SI unless otherwise specified.

Permittivity of free space	$\epsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$ $= 1.2566 \times 10^{-6}\ \mathrm{H\,m^{-1}}$
Resistance of free space	$R_0 = (\mu_0/\epsilon_0)^{1/2} = 376.73\,\Omega$
Capacity of parallel plates of area A, separated by distance d	$C = \epsilon A/d$
Capacity of concentric cylinders of length l, radii a, b	$C = 2\pi\epsilon l \ln(b/a)$
Capacity of concentric spheres of radii a, b	$C = 4\pi\epsilon ab/(b-a)$
Self-inductance of wire of length l, carrying uniform current	$L = \mu l$
Mutual inductance of parallel wires of length l, radius a, separated by distance d	$L = (\mu' l/4\pi)\left[1 + 4\ln(d/a)\right]$
Inductance of circular loop of radius b, made of wire of radius a, carrying uniform current	$L = b\left\{\mu'\left[\ln(8b/a) - 2\right] + \mu/4\right\}$
Relaxation time in a lossy medium	$\tau = \epsilon/\sigma$
Skin depth in a lossy medium	$\delta = (2/\omega\mu\sigma)^{1/2} = (\pi f \mu\sigma)^{-1/2}$
Wave impedance in a lossy medium	$Z = \left[\mu/(\epsilon + i\sigma/\omega)\right]^{1/2}$
Transmission coefficient at conducting surface (good only for $T \ll 1$)[9]	$T = 4.22 \times 10^{-4}(f\kappa_{m1}\kappa_{e2}/\sigma)^{1/2}$
Field at distance r from straight wire carrying current I (amperes)	$B_\theta = \mu I/2\pi r$ tesla $= 0.2I/r$ gauss (r in cm)
Field at distance z along axis from circular loop of radius a carrying current I	$B_z = \mu a^2 I/\left[2(a^2 + z^2)^{3/2}\right]$

```
\input prolog \pageno=21
\hoffset=1.0truein\voffset=1.0truein\hsize=6.25truein\vsize=9.0truein
\centerline{\headfont ELECTROMAGNETIC FREQUENCY/}
\centerline{\headfont WAVELENGTH BANDS$^{10}$} \nointerlineskip
$$\vbox{\tabskip=0pt \offinterlineskip
\def\ds{\ \ }\newdimen\digitwidth \setbox0=\hbox{\rm0}
\digitwidth=\wd0 \catcode`?= \active \def?{\kern\digitwidth}
  %  `?' HAS BEEN MADE ACTIVE AND DEFINED TO BE A SPACE EQUAL IN SIZE TO ONE
  %  DIGIT, SO THE COLUMNS OF NUMBERS ALL LINE UP CORRECLY.
\halign to\hsize{\kern-5pt \strut# &\vrule# \tabskip=1em plus1em
&\qquad#\hfil &\vrule# &#\hfil &\vrule# &#\hfil &\vrule# &#\hfil &\vrule#
    &#\hfil &\vrule# \tabskip=0pt \cr \trule
\om &height2pt &\om |\multispan3 | \om &\om &\om & \cr
|\om |\multispan3\hfil Frequency Range \hfil |\multispan3\hfil
Wavelength Range \hfil & \cr
|\om\hfil Designation \hfil |\multispan3 |\multispan3 & \cr
\noalign{\vskip-1.5ex \m veright1.64truein \vbox{\hrule width4.61truein}}
|\om |\om\hidewidth\hfil Lower \hidewidth |\om\hidewidth\hfil
Upper \hidewidth &height3ex & \om\hidewidth \hfil Lower
\hidewidth | \om\hidewidth \hfil Upper \hidewidth & \cr
\tska{5}{2pt} \trule \tska{5}{1.0pt} \trule \tska{5}{2pt}
| ULF\rlap* | | ?10\ts Hz | ??3\ts Mm | & \cr \tska{5}{2pt}
| ELF\rlap* | ?10\ts Hz | ??3\ts kHz | 100\ts km | ??3\ts Mm & \cr \tska{5}{2pt}
| VLF\  | ??3\ts kHz | ?30\ts kHz | ?10\ts km | 100\ts km & \cr \tska{5}{2pt}
| LF\ds | ?30\ts kHz | 300\ts kHz | ??1\ts km | ?10\ts km & \cr \tska{5}{2pt}
| MF\ds | 300\ts kHz | ??3\ts MHz | 100\ts m | ??1\ts km & \cr \tska{5}{2pt}
| HF\ds | ??3\ts MHz | ?30\ts MHz | ?10\ts m | 100\ts m & \cr \tska{5}{2pt}
| VHF\  | ?30\ts MHz | 300\ts MHz | ??1\ts m | ?10\ts m & \cr \tska{5}{2pt}
| UHF\  | 300\ts MHz | ??3\ts GHz | ?10\ts cm | ??1\ts m & \cr \tska{5}{2pt}
| SHF\rlap\dag | ??3\ts GHz | ?30\ts GHz | ??1\ts cm | ?10\ts cm &\cr\tska{5}{2pt}
| ??S | ??2.6 | ??3.95 | ??7.6 | ?11.5 & \cr \tska{5}{2pt}
| ??G | ??3.95 | ??5.85 | ??5.1 | ??7.6 & \cr \tska{5}{2pt}
| ??J | ??5.3 | ??8.2 | ??3.7 | ??5.7 & \cr \tska{5}{2pt}
| ??H | ??7.05 | ?10.0 | ??3.0 | ??4.25 & \cr \tska{5}{2pt}
| ??X | ??8.2 | ?12.4 | ??2.4 | ??3.7 & \cr \tska{5}{2pt}
| ??M | ?10.0 | ?15.0 | ??2.0 | ??3.0 & \cr \tska{5}{2pt}
| ??P | ?12.4 | ?18.0 | ??1.67 | ??2.4 & \cr \tska{5}{2pt}
| ??K | ?18.0 | ?26.5 | ??1.1 | ??1.67 & \cr \tska{5}{2pt}
| ??R | ?26.5 | ?40.0 | ??0.75 | ??1.1 & \cr \tska{5}{2pt}
| EHF | ?30\ts GHz | 300\ts GHz | ??1\ts mm | ??1\ts cm & \cr \tska{5}{2pt}
| \om \hfil Submillimeter \hfil | 300\ts GHz | ??3\ts THz | 100\ts$\mu$m |
    ??1\ts mm & \cr \tska{5}{2pt}
| \om \hfil Infrared \hfil | ??3\ts THz | 430\ts THz | 700\ts nm |
    [illegible] & \cr \tska{5}{2pt}
    \om \hfil Visible \hfil | 430\ts THz | 750\ts THz | 400\ts nm | 700\ts nm&\cr\tska{5}{2pt}
    \om \hfil Ultraviolet \hfil | 750\ts THz | [illegible] PHz | ?10\ts nm |
    [illegible] & \cr \tska{5}{2pt}
    \om \hfil X-Ray [illegible] PHz | [illegible] PHz | 100\ts pm |
    [illegible] & \cr \tska{5}{2pt}
| \om \hfil Gamma [illegible] PHz | [illegible] \ts pm & \cr \tska{5}{2pt}
[illegible]
Note: In spectroscopy [illegible] ( AA) [illegible] used (1\thinspace AA
[illegible]
*The boundary between [illegible] \smallskip\noindent\hbox [illegible]
[illegible] The UHF [illegible] further subdivided
approximately [illegible]
```

ELECTROMAGNETIC FREQUENCY/ WAVELENGTH BANDS[10]

Designation	Frequency Range		Wavelength Range	
	Lower	Upper	Lower	Upper
ULF*		10 Hz	3 Mm	
ELF*	10 Hz	3 kHz	100 km	3 Mm
VLF	3 kHz	30 kHz	10 km	100 km
LF	30 kHz	300 kHz	1 km	10 km
MF	300 kHz	3 MHz	100 m	1 km
HF	3 MHz	30 MHz	10 m	100 m
VHF	30 MHz	300 MHz	1 m	10 m
UHF	300 MHz	3 GHz	10 cm	1 m
SHF†	3 GHz	30 GHz	1 cm	10 cm
S	2.6	3.95	7.6	11.5
G	3.95	5.85	5.1	7.6
J	5.3	8.2	3.7	5.7
H	7.05	10.0	3.0	4.25
X	8.2	12.4	2.4	3.7
M	10.0	15.0	2.0	3.0
P	12.4	18.0	1.67	2.4
K	18.0	26.5	1.1	1.67
R	26.5	40.0	0.75	1.1
EHF	30 GHz	300 GHz	1 mm	1 cm
Submillimeter	300 GHz	3 THz	100 μm	1 mm
Infrared	3 THz	430 THz	700 nm	100 μm
Visible	430 THz	750 THz	400 nm	700 nm
Ultraviolet	750 THz	30 PHz	10 nm	400 nm
X Ray	30 PHz	3 EHz	100 pm	10 nm
Gamma Ray	3 EHz			100 pm

Note: In spectroscopy the angstrom (Å) is sometimes used (1 Å $= 10^{-8}$ cm $= 0.1$ nm).

*The boundary between ULF and ELF is variously defined.

†The SHF (microwave) band is further subdivided approximately as shown.[11]

```
\input prolog
\hoffset=1.25truein
\voffset=1.0truein
\hsize=6.0truein
\vsize=9.0truein
\pageno=22
\centerline{\headfont AC CIRCUITS}
\medskip\indent
For a resistance $R$, inductance $L$, and capacitance $C$ in series with  a
voltage source $V=V_0\exp(i\omega t)$ (here $i=\sqrt{-1}$), the current is given
by $I=dq/dt$, where $q$ satisfies
   $$ L{{d^2\!q}\over{dt^2}}+R{{dq}\over{dt}}+{q \over C} = V.$$
Solutions are $q(t)=q_s+q_t,\;I(t)=I_s+I_t$, where the steady state is
$I_s = i\omega q_s = V/Z$ in terms of the impedance $Z = R + i(\omega L -
1/\omega C)$ and $I_t = dq_t/dt.$  For initial conditions $q(0)\equiv q_0 =
\bar q_0+q_s$, $\;I(0)\equiv I_0,$ the transients can be of three types,
depending on $\Delta=R^2-4L/C$:
\medskip\noindent
(a) Overdamped, $\Delta>0$
$$\eqalign {q_t &= {I_0+\gamma_+ \bar q_0 \over \gamma_+-\gamma_-}
\exp(-\gamma_-t) - {I_0+\gamma_- \bar q_0 \over \gamma_+-\gamma_-}
\exp(-\gamma_+t), \cr
I_t &= {\gamma_+(I_0+\gamma_- \bar q_0) \over \gamma_+-\gamma_-} \exp(-\gamma_
+t)- {\gamma_-(I_0 + \gamma_+ \bar q_0) \over \gamma_+-\gamma_-}
\exp(-\gamma_-t), \cr}$$
where $\gamma_\pm=(R\pm\Delta^{1/2})/2L;$
\medskip\noindent
(b) Critically damped, $\Delta=0$
$$\eqalign{q_t &= \left[\bar q_0 + (I_0+\gamma_R \bar q_0)t\right]
\exp(-\gamma_Rt), \cr
I_t &= \left[I_0 - (I_0 + \gamma_R \bar q_0)\gamma_Rt\right]\exp(-\gamma_Rt),
\cr}$$
where $\gamma_R=R/2L;$
\medskip\noindent
(c) Underdamped, $\Delta < 0$
$$\eqalign{q_t &= \left[{\gamma_R \bar q_0 + I_0 \over \omega_1} \sin\omega_1t +
   \bar q_0 \cos\omega_1t\right]\exp(-\gamma_Rt), \cr
I_t &= \left[I_0\cos\omega_1t - {({\omega_1}^2+{\gamma_R}^2)\bar q_0 +
   \gamma_R I_0 \over \omega_1} \sin(\omega_1t)\right]\exp(-\gamma_Rt),\cr}$$
where $\omega_1=\omega_0(1-R^2C/4L)^{1/2}$ and $\omega_0=(LC)^{-1/2}\!$ is
the resonant frequency.  At $\omega=\omega_0$, $\;Z=R$.  The quality of the
circuit is $Q=\omega_0L/R$.  In\-stab\-il\-ity results when $L$, $R$, $C$ are
not all of the same sign.
\vskip0.5truein
\vfil\eject\end
```

AC CIRCUITS

For a resistance R, inductance L, and capacitance C in series with a voltage source $V = V_0 \exp(i\omega t)$ (here $i = \sqrt{-1}$), the current is given by $I = dq/dt$, where q satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V.$$

Solutions are $q(t) = q_s + q_t$, $I(t) = I_s + I_t$, where the steady state is $I_s = i\omega q_s = V/Z$ in terms of the impedance $Z = R + i(\omega L - 1/\omega C)$ and $I_t = dq_t/dt$. For initial conditions $q(0) \equiv q_0 = \bar{q}_0 + q_s$, $I(0) \equiv I_0$, the transients can be of three types, depending on $\Delta = R^2 - 4L/C$:

(a) Overdamped, $\Delta > 0$

$$q_t = \frac{I_0 + \gamma_+ \bar{q}_0}{\gamma_+ - \gamma_-}\exp(-\gamma_- t) - \frac{I_0 + \gamma_- \bar{q}_0}{\gamma_+ - \gamma_-}\exp(-\gamma_+ t),$$

$$I_t = \frac{\gamma_+(I_0 + \gamma_- \bar{q}_0)}{\gamma_+ - \gamma_-}\exp(-\gamma_+ t) - \frac{\gamma_-(I_0 + \gamma_+ \bar{q}_0)}{\gamma_+ - \gamma_-}\exp(-\gamma_- t),$$

where $\gamma_\pm = (R \pm \Delta^{1/2})/2L$;

(b) Critically damped, $\Delta = 0$

$$q_t = [\bar{q}_0 + (I_0 + \gamma_R \bar{q}_0)t]\exp(-\gamma_R t),$$

$$I_t = [I_0 - (I_0 + \gamma_R \bar{q}_0)\gamma_R t]\exp(-\gamma_R t),$$

where $\gamma_R = R/2L$;

(c) Underdamped, $\Delta < 0$

$$q_t = \left[\frac{\gamma_R \bar{q}_0 + I_0}{\omega_1}\sin\omega_1 t + \bar{q}_0\cos\omega_1 t\right]\exp(-\gamma_R t),$$

$$I_t = \left[I_0\cos\omega_1 t - \frac{(\omega_1{}^2 + \gamma_R{}^2)\bar{q}_0 + \gamma_R I_0}{\omega_1}\sin(\omega_1 t)\right]\exp(-\gamma_R t),$$

where $\omega_1 = \omega_0(1 - R^2C/4L)^{1/2}$ and $\omega_0 = (LC)^{-1/2}$ is the resonant frequency. At $\omega = \omega_0$, $Z = R$. The quality of the circuit is $Q = \omega_0 L/R$. Instability results when L, R, C are not all of the same sign.

```
\input prolog
\hoffset=1truein\voffset=1truein\hsize=6.5truein\vsize=9truein
\pageno=23
\centerline{\headfont DIMENSIONLESS NUMBERS OF FLUID MECHANICS$^{12}$}
\bsk  % BEGINNING OF TABLE.
\vbox{\tabskip=0pt \offinterlineskip \def\quid{\hskip0.5em\relax}
\halign to \hsize{\vrule#\tabskip=0.5em plus2em&\strut#\hfil&\vrule#&#
\hfil&\vrule#&#\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr \trule
&\hfil Name(s)|\om\hidewidth Symbol\hidewidth|\hfil Definition|\hfil
Significance&\cr \tskc{4}{2pt} \trule \tskc{4}{1.0pt} \trule \tskc{4}{2pt}
&Alfv\'en,|Al, Ka|$V_A/V$|\kern-0.5em*(Magnetic force/&\cr
  &\om\quid K\'arm\'an\hfil|\om|\om|\quad inertial force)$^{1/2}$&\cr\tskc{4}{2pt}
&Bond|Bd|$(\rho^\prime-\rho)L^2g/\Sigma$|Gravitational force/&\cr
&\om|\om|\om|\quad surface tension&\cr\tskc{4}{2pt}
&Boussinesq|B|$V/(2gR)^{1/2}$|(Inertial force/&\cr
  &\om|\om|\om|\quad gravitational force)$^{1/2}$&\cr\tskc{4}{2pt}
&Brinkman|Br|$\mu V^2/k\Delta T$|Viscous heat/conducted heat&\cr\tskc{4}{2pt}
&Capillary|Cp|$\mu V/\Sigma$|Viscous force/surface tension&\cr\tskc{4}{2pt}
&Carnot|Ca|$(T_2-T_1)/T_2$|Theoretical Carnot cycle&\cr
  &\om|\om|\om|\quad efficiency&\cr\tskc{4}{2pt}
&Cauchy,|Cy, Hk|$\rho V^2/\Gamma=\rm M^2$|Inertial force/&\cr
&\om\quid Hooke\hfil|\om|\om|\quad compressibility force&\cr\tskc{4}{2pt}
&Clausius|Cl|$LV^3\rho/k\Delta T$|Kinetic energy flow rate/heat&\cr
  &\om|\om|\om|\quad conduction rate&\cr\tskc{4}{2pt}
&Cowling|C|$(V_A/V)^2=\rm Al^2$|Magnetic force/inertial force&\cr\tskc{4}{2pt}
&Crispation|Cr|$\mu \kappa/\Sigma L$|Effect of diffusion/effect of&\cr
  &\om|\om|\om|\quad surface tension&\cr\tskc{4}{2pt}
&Dean|D|$D^{3/2}V/\nu(2r)^{1/2}$|Transverse flow due to&\cr
  &\om|\om|\om|\quad curvature/longitudinal flow&\cr\tskc{4}{2pt}
&[Drag|$C_D$|$(\rho^\prime-\rho)Lg/$|Drag force/inertial force&\cr
  &\om\quid coefficient]\hfil|\om|\quad$\rho^\prime V^2$|\om&\cr\tskc{4}{2pt}
&Eckert|E|$V^2/c_p\Delta T$|Kinetic energy/change in&\cr
  &\om|\om|\om|\quad thermal energy&\cr\tskc{4}{2pt}
&Ekman|Ek|$(\nu/2\Omega L^2)^{1/2}=$|(Viscous force/Coriolis force)$^{1/2}$
  \hidewidth&\cr
  &\om|\om|\quad(Ro/Re)$^{1/2}$|\om&\cr\tskc{4}{2pt}
&Euler|Eu|$\Delta p/\rho V^2$|Pressure drop due to friction/&\cr
  &\om|\om|\om|\quad dynamic pressure&\cr\tskc{4}{2pt}
&Froude|Fr|$V/(gL)^{1/2}$|\kern-0.5em\dag(Inertial force/gravitational or
  \kern0.25em&\cr
  &\om|\om|$V/NL$|\quad buoyancy force)$^{1/2}$&\cr\tskc{4}{2pt}
&Gay-Lussac|Ga|$1/\beta\Delta T$|Inverse of relative change in&\cr
  &\om|\om|\om|\quad volume during heating&\cr\tskc{4}{2pt}
&Grashof|Gr|$gL^3\beta\Delta T/\nu^2$|Buoyancy force/viscous force&\cr\tskc{4}{2pt}
&[Hall|$C_H$|$\lambda/r_L$|Gyrofrequency/&\cr
  &\om\quid coefficient]\hfil|\om|\om|\quad collision frequency&\cr\tskc{4}{2pt}
\tskc{4}{2pt} \trule}}
*(\dag) Also defined as the inverse (square) of the quantity shown.
\vfil\eject\end
```

DIMENSIONLESS NUMBERS OF FLUID MECHANICS[12]

Name(s)	Symbol	Definition	Significance
Alfvén, Kármán	Al, Ka	V_A/V	*(Magnetic force/ inertial force)$^{1/2}$
Bond	Bd	$(\rho' - \rho)L^2 g/\Sigma$	Gravitational force/ surface tension
Boussinesq	B	$V/(2gR)^{1/2}$	(Inertial force/ gravitational force)$^{1/2}$
Brinkman	Br	$\mu V^2/k\Delta T$	Viscous heat/conducted heat
Capillary	Cp	$\mu V/\Sigma$	Viscous force/surface tension
Carnot	Ca	$(T_2 - T_1)/T_2$	Theoretical Carnot cycle efficiency
Cauchy, Hooke	Cy, Hk	$\rho V^2/\Gamma = \mathrm{M}^2$	Inertial force/ compressibility force
Clausius	Cl	$LV^3\rho/k\Delta T$	Kinetic energy flow rate/heat conduction rate
Cowling	C	$(V_A/V)^2 = \mathrm{Al}^2$	Magnetic force/inertial force
Crispation	Cr	$\mu\kappa/\Sigma L$	Effect of diffusion/effect of surface tension
Dean	D	$D^{3/2}V/\nu(2r)^{1/2}$	Transverse flow due to curvature/longitudinal flow
[Drag coefficient]	C_D	$(\rho' - \rho)Lg/\rho' V^2$	Drag force/inertial force
Eckert	E	$V^2/c_p\Delta T$	Kinetic energy/change in thermal energy
Ekman	Ek	$(\nu/2\Omega L^2)^{1/2} = (\mathrm{Ro}/\mathrm{Re})^{1/2}$	(Viscous force/Coriolis force)$^{1/2}$
Euler	Eu	$\Delta p/\rho V^2$	Pressure drop due to friction/ dynamic pressure
Froude	Fr	$V/(gL)^{1/2}$ V/NL	†(Inertial force/gravitational or buoyancy force)$^{1/2}$
Gay-Lussac	Ga	$1/\beta\Delta T$	Inverse of relative change in volume during heating
Grashof	Gr	$gL^3\beta\Delta T/\nu^2$	Buoyancy force/viscous force
[Hall coefficient]	C_H	λ/r_L	Gyrofrequency/ collision frequency

*(†) Also defined as the inverse (square) of the quantity shown.

```
\input prolog \pageno=24  % BEGINNING OF TABLE.
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\halign to \hsize{\vrule#\tabskip=0.5em plus2em&\strut#\hfil&\vrule#&#
\hfil&\vrule#&#\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr \trule
&\hfil Name(s)|\om\hidewidth Symbol\hidewidth|\hfil Definition|\hfil
    Significance&\cr
  % DRAW DOUBLE RULE BENEATH HEADING.
\tskc{4}{2pt} \trule \tskc{4}{1.0pt} \trule \tskc{4}{2pt}
&Hartmann|H|$BL/(\mu\eta)^{1/2}=$|Magnetic force/&\cr
    &\om|\om|\quad (Rm\ts Re\ts C)$^{1/2}$\hidewidth|\quad dissipative
    force&\cr\tskc{4}{2pt}
&Knudsen|Kn|$\lambda/L$|Hydrodynamic time/&\cr
    &\om|\om|\om|\quad collision time&\cr\tskc{4}{2pt}
&Lorentz|Lc|$V/c$|Magnitude of relativistic effects&\cr\tskc{4}{2pt}
&Lundquist|Lu|$\mu_OLV_A/\eta=$|
    ${\bf J}\times{\bf B}$force/resistive magnetic&\cr
    &\om|\om|\quad Al\ts Rm|\quad diffusion force &\cr\tskc{4}{2pt}
&Mach|M|$V/C_S$|Magnitude of compressibility&\cr
    &\om|\om|\om|\quad effects&\cr\tskc{4}{2pt}
&Magnetic|Mm|$V/V_A=$ Al$^{-1}$|(Inertial force/magnetic force)$^{1/2}$
    \hidewidth&\cr
    &\om\quid Mach\hfil|\om|\om|\om&\cr\tskc{4}{2pt}
&Magnetic|Rm|$\mu_OLV/\eta$|Flow velocity/magnetic diffusion&\cr
    &\om\quid Reynolds\hfil|\om|\om|\quad velocity&\cr\tskc{4}{2pt}
&Newton|Nt|$F/\rho L^2V^2$|Imposed force/inertial force&\cr\tskc{4}{2pt}
&Nusselt|N|$\alpha L/k$|Total heat transfer/thermal&\cr
    &\om|\om|\om|\quad conduction&\cr\tskc{4}{2pt}
&P\'eclet|Pe|$LV/\kappa$|Heat convection/heat conduction\hidewidth&\cr\tskc{4}{2pt}
&Poisseuille|Po|$D^2\Delta p/\mu LV$|Pressure force/viscous force&\cr\tskc{4}{2pt}
&Prandtl,|Pr, Sc|$\nu/\kappa$|Momentum diffusion/&\cr
    &\om\quid Schmidt\hfil|\om|\om|\quad heat diffusion&\cr\tskc{4}{2pt}
&Rayleigh|Ra|$gH^3\beta\Delta T/\nu\kappa$|Buoyancy force/diffusion force&\cr
    \tskc{4}{2pt}
&Reynolds|Re|$LV/\nu$|Inertial force/viscous force&\cr\tskc{4}{2pt}
&Richardson|Ri|$(NH/\Delta V)^2$|Buoyancy effects/&\cr
    &\om|\om|\om|\quad vertical shear effects&\cr\tskc{4}{2pt}
&Rossby|Ro|$V/2\Omega L\sin\Lambda$|Inertial force/Coriolis force&\cr \tskc{4}{2pt}
&Stanton|St|$\alpha/\rho c_p V$|Thermal conduction loss/&\cr
    &\om|\om|\om|\quad heat capacity&\cr\tskc{4}{2pt}
&Stefan|Sf|$\sigma LT^3/k$|Radiated heat/conducted heat&\cr\tskc{4}{2pt}
&Stokes|S|$\nu/L^2f$|Viscous damping rate/&\cr
    &\om|\om|\om|\quad vibration frequency&\cr\tskc{4}{2pt}
&Strouhal|Sr|$fL/V$|Vibration speed/flow velocity&\cr\tskc{4}{2pt}
&Taylor|Ta|$(2\Omega L^2/\nu)^2$|Centrifugal force/viscous force&\cr
    &\om|\om|$R [illegible] 2}$|(Centrifugal force/&\cr
    &\om|\om|\quid [illegible]|\quad viscous force)$^{1/2}$&
    \cr\tskc{4}{2pt}
&Thring,|Th, [illegible] Convective heat transport/&\cr
&\om\quid [illegible] \quad radiative heat transport&\cr\tskc{4}{2pt}
&Weber|[illegible]|Inertial force/surface tension&\cr
    [illegible]
[illegible]
```

Name(s)	Symbol	Definition	Significance
Hartmann	H	$BL/(\mu\eta)^{1/2} = (\mathrm{Rm\,Re\,C})^{1/2}$	Magnetic force/ dissipative force
Knudsen	Kn	λ/L	Hydrodynamic time/ collision time
Lorentz	Lo	V/c	Magnitude of relativistic effects
Lundquist	Lu	$\mu_0 L V_A/\eta = \mathrm{Al\,Rm}$	$\mathbf{J} \times \mathbf{B}$force/resistive magnetic diffusion force
Mach	M	V/C_S	Magnitude of compressibility effects
Magnetic Mach	Mm	$V/V_A = \mathrm{Al}^{-1}$	(Inertial force/magnetic force)$^{1/2}$
Magnetic Reynolds	Rm	$\mu_0 LV/\eta$	Flow velocity/magnetic diffusion velocity
Newton	Nt	$F/\rho L^2 V^2$	Imposed force/inertial force
Nusselt	N	$\alpha L/k$	Total heat transfer/thermal conduction
Péclet	Pe	LV/κ	Heat convection/heat conduction
Poisseuille	Po	$D^2 \Delta p/\mu LV$	Pressure force/viscous force
Prandtl, Schmidt	Pr, Sc	ν/κ	Momentum diffusion/ heat diffusion
Rayleigh	Ra	$gH^3 \beta \Delta T/\nu\kappa$	Buoyancy force/diffusion force
Reynolds	Re	LV/ν	Inertial force/viscous force
Richardson	Ri	$(NH/\Delta V)^2$	Buoyancy effects/ vertical shear effects
Rossby	Ro	$V/2\Omega L \sin\Lambda$	Inertial force/Coriolis force
Stanton	St	$\alpha/\rho c_p V$	Thermal conduction loss/ heat capacity
Stefan	Sf	$\sigma LT^3/k$	Radiated heat/conducted heat
Stokes	S	$\nu/L^2 f$	Viscous damping rate/ vibration frequency
Strouhal	Sr	fL/V	Vibration speed/flow velocity
Taylor	Ta	$(2\Omega L^2/\nu)^2$ $R^{1/2}(\Delta R)^{3/2} \cdot (\Omega/\nu)$	Centrifugal force/viscous force (Centrifugal force/ viscous force)$^{1/2}$
Thring, Boltzmann	Th, Bo	$\rho c_p V/\epsilon\sigma T^3$	Convective heat transport/ radiative heat transport
Weber	W	$\rho LV^2/\Sigma$	Inertial force/surface tension

```
\input prolog
\hoffset=1truein\voffset=1truein\hsize=6.5truein\vsize=9truein
\pageno=25
{\headfont Nomenclature:}
\msk \def\Cr{\cr \noalign{\vskip 2.5pt}}
   %  \Cr HAS BEEN DEFINED TO LEAVE AN EXTRA SPACE AFTER EACH ALIGNED LINE.
\halign{#\hfil&\qquad#\hfil\cr
$B$&Magnetic induction\Cr
$C_s,c$&Speeds of sound, light\Cr
$c_p$&Specific heat at constant pressure (units $\rm m^2\ts s^{-2}\ts
    K^{-1}$)\Cr
$D=2R$&Pipe diameter\Cr
$F$&Imposed force\Cr
$f$&Vibration frequency\Cr
$g$&Gravitational acceleration\Cr
$H, L$&Vertical, horizontal length scales\Cr
$k=\rho c_p \kappa$&Thermal conductivity (units $\rm kg\ts m^{-1}\ts s^{-2}$)\Cr
$N=(g/H)^{1/2}$&Brunt--V\"ais\"al\"a frequency\Cr
$R$&Radius of pipe or channel\Cr
$r$&Radius of curvature of pipe or channel\Cr
$r_L$&Larmor radius\Cr
$T$&Temperature\Cr
$V$&Characteristic flow velocity\Cr
$V_A=B/(\mu_0\rho)^{1/2}$&Alfv\'en speed\cr
$\alpha$&Newton's-law heat coefficient, $\displaystyle k{\part T \over
    \part x}=\alpha\Delta T$\Cr
$\beta$&Volumetric expansion coefficient, $dV/V = \beta dT$\Cr
$\Gamma$&Bulk modulus (units $\rm kg\ts m^{-1}\ts s^{-2}$)\Cr
$\Delta R, \Delta V, \Delta p, \Delta T$&Imposed difference in two radii,
    velocities,\cr
    &pressures, or temperatures\Cr
$\epsilon$&Surface emissivity\Cr
$\eta$&Electrical resistivity\Cr
$\kappa$&Thermal diffusivity (units $\rm m^2\ts s^{-1}$)\Cr
$\Lambda$&Latitude of point on earth's surface\Cr
$\lambda$&Collisional mean free path\Cr
$\mu=\rho\nu$&Bulk viscosity\Cr
$\mu_0$&Permeability of free space\Cr
$\nu$&Kinematic viscosity (units $\rm m^2\ts s^{-1}$)\Cr
$\rho$&Mass density of fluid medium\Cr
$\rho [illegible]$&Mass density of bubble, droplet, or moving object\Cr
$\sigma$&Surface tension (units $\rm kg\ts s^{-2}$)\Cr
$\sigma$&Stefan--Boltzmann constant\Cr
$\Omega$&[illegible] angular velocity\Cr}
\vfill [illegible]
```

Nomenclature:

B	Magnetic induction
C_s, c	Speeds of sound, light
c_p	Specific heat at constant pressure (units $\mathrm{m^2\,s^{-2}\,K^{-1}}$)
$D = 2R$	Pipe diameter
F	Imposed force
f	Vibration frequency
g	Gravitational acceleration
H, L	Vertical, horizontal length scales
$k = \rho c_p \kappa$	Thermal conductivity (units $\mathrm{kg\,m^{-1}\,s^{-2}}$)
$N = (g/H)^{1/2}$	Brunt–Väisälä frequency
R	Radius of pipe or channel
r	Radius of curvature of pipe or channel
r_L	Larmor radius
T	Temperature
V	Characteristic flow velocity
$V_A = B/(\mu_0 \rho)^{1/2}$	Alfvén speed
α	Newton's-law heat coefficient, $k\dfrac{\partial T}{\partial x} = \alpha \Delta T$
β	Volumetric expansion coefficient, $dV/V = \beta dT$
Γ	Bulk modulus (units $\mathrm{kg\,m^{-1}\,s^{-2}}$)
$\Delta R, \Delta V, \Delta p, \Delta T$	Imposed difference in two radii, velocities, pressures, or temperatures
ϵ	Surface emissivity
η	Electrical resistivity
κ	Thermal diffusivity (units $\mathrm{m^2\,s^{-1}}$)
Λ	Latitude of point on earth's surface
λ	Collisional mean free path
$\mu = \rho\nu$	Bulk viscosity
μ_0	Permeability of free space
ν	Kinematic viscosity (units $\mathrm{m^2\,s^{-1}}$)
ρ	Mass density of fluid medium
ρ'	Mass density of bubble, droplet, or moving object
Σ	Surface tension (units $\mathrm{kg\,s^{-2}}$)
σ	Stefan–Boltzmann constant
Ω	Solid-body rotational angular velocity

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=26
{\def\ups{\upsilon} \def\NQ{\medskip\N\quad} \def\PH{$\phantom{1}$}
\def\sq{^{\phantom{1}2}} \def\medskip{\vskip4.0pt}
  % \PH SKIPS THE SPACE OF THE DIGIT '1' SO THAT EQUATIONS NUMBERS LINE UP.
  % \medskip HAS BEEN REDEFINED TO ADJUST THE SPACING OF THE TABLE.
  % \NQ SKIPS SPACE DOWN AND INDENTS ONE \quad TO ALIGN EQUATIONS.
\centerline{\headfont SHOCKS} \bsk\indent
At a shock front propagating in a magnetized fluid at an angle $\theta$ with
respect to the magnetic induction {\bf B}, the jump conditions are $^{13, 14}$
\NQ\PH(1) $\rho U = \ov{\rho}\ov{U} \equiv q$;
\NQ\PH(2) $\rho U^2+p+B_\perp\sq/2\mu=\ov{\rho}\ov{U}^2+
    \ov{p}+\ov{B}_\perp\sq/2\mu$;
\NQ\PH(3) $\rho UV - B_\parallel B_\perp/\mu =
    \ov{\rho}\ov{U}\ov{V} - \ov{B}_\parallel \ov{B}_\perp/\mu$;
\NQ\PH(4) $B_\parallel = \ov{B}_\parallel$;
\NQ\PH(5) $UB_\perp - VB_\parallel = \ov{U}\ov{B}_\perp -
    \ov{V}\ov{B}_\parallel$;
\NQ\PH(6) ${1\over2}(U^2+V^2)+w+(UB_\perp\sq-VB_\parallel B_\perp)/\mu\rho U$
\smallskip
\qquad\qquad\qquad$={1\over2}(\ov{U}^2+\ov{V}^2)+\ov{w}+(\ov{U}\ov{B}_\perp\sq
-\ov{V}\ov{B}_\parallel\ov{B}_\perp)/\mu\ov{\rho}\ov{U}$.
\medskip\N
Here $U$ and $V$ are components of the fluid velocity normal and tangential to
the front in the shock frame; $\rho = 1/\ups$ is the mass density; $p$ is the
pressure; $B_\perp = B \sin \theta$, $B_\parallel = B \cos \theta$; $\mu$ is the
magnetic permeability ($\mu = 4 \pi$ in cgs units); and the specific enthalpy is
$w = e + p\ups$, where the specific internal energy $e$ satisfies $de = Tds
- pd\ups$ in terms of the temperature $T$ and the specific entropy $s$.
Quantities in the region behind (downstream from) the front are distinguished by
a bar. If ${\bf B} = 0$, then$^{15}$
\NQ\PH(7) $U - \ov{U} = \left[(\ov{p} - p)(\ups - \ov{\ups})\right]^{1/2}$;
\NQ\PH(8) $(\ov{p} - p)(\ups -  \ov{\ups})^{-1} = q^2$;
\NQ\PH(9) $\ov{w} - w = {1\over2}(\ov{p} - p)(\ups + \ov{\ups})$;
\NQ(10) $\ov{e} - e = {1\over2}(\ov{p} + p)(\ups - \ov{\ups})$.
\medskip\N
In what follows we assume that the fluid is a perfect gas with adiabatic index
$\gamma = 1 + 2/n$, where $n$ is the number of degrees of freedom.  Then $p =
\rho RT/m$, where $R$ is the universal gas constant and $m$ is the molar weight;
the sound speed is given by ${C_s}^2 = (\partial p/\partial\rho)_s = \gamma
p\ups$; and $w = \gamma e = \gamma p \ups/(\gamma + 1)$.  For a general oblique
shock in a perfect gas the quantity $X = r^{-1}(U/V_A)^2$ satisfies$^{14}$
\NQ(11) $(X - \beta/\alpha)(X - \cos^2\theta)^2 =
    X\sin^2\theta\left\{\left[1 + (r-1)/2\alpha\right]X - \cos^2\theta\right\}$,
where $r=\ov{\rho}/\rho$, $\alpha={1\over2}\left[\gamma+1-(\gamma-1)r\right]$,
and $\beta={C_s}^2/{V_A}^2=4\pi\gamma p/B^2$.
\medskip\N
The density ratio is bounded by
\NQ(12) $1 < r < (\gamma+1)/(\gamma-1)$. \medskip\N
If the shock is normal to {\bf B} (i.e., if $\theta = \pi/2$), then
\NQ(13) $U^2=(r/\alpha)\left
    \{{C_s}^2+{V_A}^2\left[1+(1-\gamma/2)(r-1)\right]\right\}$;}
\vfil\eject\end
```

SHOCKS

At a shock front propagating in a magnetized fluid at an angle θ with respect to the magnetic induction $\mathbf{B}$, the jump conditions are [13,14]

(1) $\rho U = \bar{\rho}\bar{U} \equiv q$;

(2) $\rho U^2 + p + B_\perp{}^2/2\mu = \bar{\rho}\bar{U}^2 + \bar{p} + \bar{B}_\perp{}^2/2\mu$;

(3) $\rho UV - B_\| B_\perp/\mu = \bar{\rho}\bar{U}\bar{V} - \bar{B}_\| \bar{B}_\perp/\mu$;

(4) $B_\| = \bar{B}_\|$;

(5) $UB_\perp - VB_\| = \bar{U}\bar{B}_\perp - \bar{V}\bar{B}_\|$;

(6) $\frac{1}{2}(U^2 + V^2) + w + (UB_\perp{}^2 - VB_\| B_\perp)/\mu\rho U$
$$= \tfrac{1}{2}(\bar{U}^2 + \bar{V}^2) + \bar{w} + (\bar{U}\bar{B}_\perp{}^2 - \bar{V}\bar{B}_\| \bar{B}_\perp)/\mu\bar{\rho}\bar{U}.$$

Here U and V are components of the fluid velocity normal and tangential to the front in the shock frame; $\rho = 1/v$ is the mass density; p is the pressure; $B_\perp = B\sin\theta$, $B_\| = B\cos\theta$; μ is the magnetic permeability ($\mu = 4\pi$ in cgs units); and the specific enthalpy is $w = e + pv$, where the specific internal energy e satisfies $de = Tds - pdv$ in terms of the temperature T and the specific entropy s. Quantities in the region behind (downstream from) the front are distinguished by a bar. If $\mathbf{B} = 0$, then[15]

(7) $U - \bar{U} = [(\bar{p} - p)(v - \bar{v})]^{1/2}$;

(8) $(\bar{p} - p)(v - \bar{v})^{-1} = q^2$;

(9) $\bar{w} - w = \frac{1}{2}(\bar{p} - p)(v + \bar{v})$;

(10) $\bar{e} - e = \frac{1}{2}(\bar{p} + p)(v - \bar{v})$.

In what follows we assume that the fluid is a perfect gas with adiabatic index $\gamma = 1 + 2/n$, where n is the number of degrees of freedom. Then $p = \rho RT/m$, where R is the universal gas constant and m is the molar weight; the sound speed is given by $C_s{}^2 = (\partial p/\partial\rho)_s = \gamma pv$; and $w = \gamma e = \gamma pv/(\gamma + 1)$. For a general oblique shock in a perfect gas the quantity $X = r^{-1}(U/V_A)^2$ satisfies[14]

(11) $(X - \beta/\alpha)(X - \cos^2\theta)^2 = X\sin^2\theta \left\{[1 + (r - 1)/2\alpha]\, X - \cos^2\theta\right\}$,

where $r = \bar{\rho}/\rho$, $\alpha = \frac{1}{2}[\gamma + 1 - (\gamma - 1)r]$, and $\beta = C_s{}^2/V_A{}^2 = 4\pi\gamma p/B^2$.

The density ratio is bounded by

(12) $1 < r < (\gamma + 1)/(\gamma - 1)$.

If the shock is normal to $\mathbf{B}$ (i.e., if $\theta = \pi/2$), then

(13) $U^2 = (r/\alpha)\left\{C_s{}^2 + V_A{}^2[1 + (1 - \gamma/2)(r - 1)]\right\}$;

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=27
{\def\ups{\upsilon} \def\NQ{\medskip\N\quad}
\def\sq{^{\ph{1}2}} \def\medskip{\vskip4.0pt}
  % \PH SKIPS THE SPACE OF THE DIGIT '1', SO THAT EQUATION NUMBERS LINE UP.
  % \medskip HAS BEEN REDEFINED, TO ADJUST THE SPACING OF THE TABLE.
  % \NQ SKIPS SPACE DOWN AND INDENTS ONE \quad, TO ALIGN EQUATIONS.
\NQ(14) $U/\ov{U} = \ov{B}/B = r$;
\NQ(15) $\ov{V} = V$;
\NQ(16) $\ov{p} = p + (1 - r^{-1})\rho U^2 + (1 - r^2)B^2/2\mu$. \medskip\N
If $\theta = 0$, there are two possibilities: switch-on shocks, which require
$\beta < 1$ and for which
\NQ(17) $U^2 = r{V_A}^2$;
\NQ(18) $\ov{U} = {V_A}^2/U$;
\NQ(19) $\ov{B}_\perp\sq=2B_\parallel\sq(r-1)(\alpha-\beta)$;
\NQ(20) $\ov{V} = \ov{U}\ov{B}_\perp/B_\parallel$;
\NQ(21) $\ov{p} = p + \rho U^2(1 - \alpha + \beta)(1 - r^{-1})$, \medskip\N
and acoustic (hydrodynamic) shocks, for which
\NQ(22) $U^2 = (r/\alpha){C_s}^2$;
\NQ(23) $\ov{U} = U/r$;
\NQ(24) $\ov{V} = \ov{B}_\perp = 0$;
\NQ(25) $\ov{p} = p + \rho U^2(1 - r^{-1})$. \medskip\N
For acoustic shocks the specific volume and pressure are related by
\NQ(26)
$\ov{\ups}/\ups=\left[(\gamma+1)p+
    (\gamma-1)\ov{p}\right]/\left[(\gamma-1)p+(\gamma+1)\ov{p}\right]$.
\medskip\N
In terms of the upstream Mach number $M = U/C_s$,
\NQ(27) $\ov{\rho}/\rho=\ups/\ov{\ups}=
    U/\ov{U}=(\gamma+1)M^2/[(\gamma-1)M^2+2]$;
\NQ(28) $\ov{p}/p=(2\gamma M^2-\gamma+1)/(\gamma+1)$;
\NQ(29) $\ov{T}/T=[(\gamma-1)M^2+2](2\gamma M^2-\gamma+1)/(\gamma+1)^2M^2$;
\NQ(30) $\ov{M}^2=[(\gamma-1)M^2+2]/[2\gamma M^2-\gamma+1]$. \medskip\N
The entropy change across the shock is
\NQ(31) $\Delta s\equiv\ov{s}-s=c_\ups\ln[(\ov{p}/p)(\rho/\ov{\rho})^\gamma]$,
\medskip\N
where $c_\ups=R/(\gamma-1)m$ is the specific heat at constant volume; here $R$
is the gas constant.  In the weak-shock limit ($M\rightarrow1$),
\NQ(32) $\displaystyle \Delta s\rightarrow
    c_\ups{2\gamma(\gamma-1)\over3(\gamma+1)}
(M^2-1)^3\approx{16\gamma R\over3(\gamma+1)m}(M-1)^3$.
\medskip\N
The radius at time $t$ of a strong spherical blast wave resulting from the
explosive release of energy $E$ in a medium with uniform density $\rho$ is
\NQ(33) $R_S = C_0(Et^2/\rho) {1/5}$,
\medskip\N
where $C_0$ is a constant depending on $\gamma$. For $\gamma=7/5$, $C_0=1.033$.}
\vfil\eject\end
```

(14) $U/\bar{U} = \bar{B}/B = r$;

(15) $\bar{V} = V$;

(16) $\bar{p} = p + (1 - r^{-1})\rho U^2 + (1 - r^2)B^2/2\mu$.

If $\theta = 0$, there are two possibilities: switch-on shocks, which require $\beta < 1$ and for which

(17) $U^2 = r{V_A}^2$;

(18) $\bar{U} = {V_A}^2/U$;

(19) ${\bar{B}_\perp}^2 = 2{B_\parallel}^2(r - 1)(\alpha - \beta)$;

(20) $\bar{V} = \bar{U}\bar{B}_\perp/B_\parallel$;

(21) $\bar{p} = p + \rho U^2(1 - \alpha + \beta)(1 - r^{-1})$,

and acoustic (hydrodynamic) shocks, for which

(22) $U^2 = (r/\alpha){C_s}^2$;

(23) $\bar{U} = U/r$;

(24) $\bar{V} = \bar{B}_\perp = 0$;

(25) $\bar{p} = p + \rho U^2(1 - r^{-1})$.

For acoustic shocks the specific volume and pressure are related by

(26) $\bar{v}/v = [(\gamma + 1)p + (\gamma - 1)\bar{p}] \,/\, [(\gamma - 1)p + (\gamma + 1)\bar{p}]$.

In terms of the upstream Mach number $M = U/C_s$,

(27) $\bar{\rho}/\rho = v/\bar{v} = U/\bar{U} = (\gamma + 1)M^2/[(\gamma - 1)M^2 + 2]$;

(28) $\bar{p}/p = (2\gamma M^2 - \gamma + 1)/(\gamma + 1)$;

(29) $\bar{T}/T = [(\gamma - 1)M^2 + 2](2\gamma M^2 - \gamma + 1)/(\gamma + 1)^2 M^2$;

(30) $\bar{M}^2 = [(\gamma - 1)M^2 + 2]/[2\gamma M^2 - \gamma + 1]$.

The entropy change across the shock is

(31) $\Delta s \equiv \bar{s} - s = c_v \ln[(\bar{p}/p)(\rho/\bar{\rho})^\gamma]$,

where $c_v = R/(\gamma - 1)m$ is the specific heat at constant volume; here R is the gas constant. In the weak-shock limit ($M \to 1$),

(32) $$\Delta s \to c_v \frac{2\gamma(\gamma - 1)}{3(\gamma + 1)}(M^2 - 1)^3 \approx \frac{16\gamma R}{3(\gamma + 1)m}(M - 1)^3.$$

The radius at time t of a strong spherical blast wave resulting from the explosive release of energy E in a medium with uniform density ρ is

(33) $R_S = C_0(Et^2/\rho)^{1/5}$,

where C_0 is a constant depending on γ. For $\gamma = 7/5$, $C_0 = 1.033$.

```
\input prolog
\hoffset=1truein\hoffset=1truein\vsize=6.5truein\vsize=9truein
\pageno=28
\centerline{\headfont FUNDAMENTAL PLASMA PARAMETERS}
\bigskip indent
All quantities are in Gaussian cgs units except temperature ($T$, $T_e$, $T_i$)
expressed in eV and ion mass ($m_i$) expressed in units of the proton mass,
$\mu=m_i/m_p$; $\,Z$ is charge state; $\,k$ is Boltzmann's constant; $\,K$ is
wavelength; $\,\gamma$ is the adiabatic index; $\,\ln\Lambda$ is the Coulomb
logarithm.
\medskip\noindent {\headfont Frequencies} \smallskip
\def\CR{\cr\noalign{\vskip2.5pt}}
% \CR IS DEFINED TO SKIP SPACE AFTER EACH LINE.
\halign{\quad#\hfil&\qquad$#$\hfil\cr
electron gyrofrequency&f_{ce}=\omega_{ce}/2\pi=2.80\times10^6B\,\rm Hz\CR
    &\omega_{ce}=eB/m_ec=1.76\times10^7B\,\rm rad/sec\CR
ion gyrofrequency&f_{ci}=\omega_{ci}/2\pi=1.52\times10^3Z\mu^{-1}B\,\rm Hz\CR
    &\omega_{ci}=eB/m_ic=9.58\times10^3Z\mu^{-1}B\,\rm rad/sec\CR
electron plasma frequency&
    f_{pe}=\omega_{pe}/2\pi=8.98\times10^3{n_e}^{1/2}\,\rm Hz\CR
    &\omega_{pe}=(4\pi n_ee^2/m_e)^{1/2}\CR
    &\ph{\omega_{pe}}=5.64\times10^4{n_e}^{1/2}\,\rm rad/sec\CR
ion plasma frequency&f_{pi}=\omega_{pi}/2\pi\CR
    &\ph{f_{pi}}=2.10\times10^2Z\mu^{-1/2}{n_i}^{1/2}\,\rm Hz\CR
    &\omega_{pi}=(4\pi n_iZ^2e^2/m_i)^{1/2}\CR
    &\ph{\omega_{pi}}=1.32\times10^3Z\mu^{-1/2}{n_i}^{1/2}\rm rad/sec\CR
electron trapping rate&\nu_{Te}=(eKE/m_e)^{1/2}\CR
    &\ph{\nu_{Te}}=7.26\times10^8K^{1/2}E^{1/2}\,\rm sec^{-1}\CR
ion trapping rate&\nu_{Ti}=(eKE/m_i)^{1/2}\CR
    &\ph{\nu_{Ti}}=1.69\times10^7K^{1/2}E^{1/2}\mu^{-1/2}\,\rm sec^{-1}\CR
electron collision rate&
    \nu_e=2.91\times10^{-6}n_e\ln\Lambda{T_e}^{-3/2}\,\rm sec^{-1}\CR
ion collision rate&
    \nu_i=4.78\times10^{-8}n_iZ^2\ln\Lambda{T_i}^{-3/2}\,\rm sec^{-1}\CR \sk
\noalign{\headfont Lengths}
    \sk
electron deBroglie length&\lambdabar=\hbar/(m_ekT_e)^{1/2}=2.76\times10^{-8}
    {T_e}^{-1/2}\,\rm cm\CR
classical distance of&e^2/kT=1.44\times10^{-7}T^{-1}\,\rm cm\cr
    \quad minimum approach\CR
electron gyroradius&r_e=v_{Te}/\omega_{ce}=2.38{T_e}^{1/2}B^{-1}\,\rm cm\CR
ion gyroradius&r_i=v_{Ti}/\omega_{ci}\CR
    &\ph{r_i}=1.02\times10^2\mu^{1/2}Z^{-1}{T_i}^{1/2}B^{-1}\,\rm cm\CR
plasma skin depth&c/\omega_{pe}=5.31\times10^5{n_e}^{-1/2}\,\rm cm\CR
Debye length&\lambda_D=(kT/4\pi ne^2)^{1/2}\CR
    &\ph{\lambda_D}=7.43\times10^2T^{1/2}n^{-1/2}\,\rm cm\CR}}
\vfill\eject\end
```

FUNDAMENTAL PLASMA PARAMETERS

All quantities are in Gaussian cgs units except temperature (T, T_e, T_i) expressed in eV and ion mass (m_i) expressed in units of the proton mass, $\mu = m_i/m_p$; Z is charge state; k is Boltzmann's constant; K is wavelength; γ is the adiabatic index; $\ln\Lambda$ is the Coulomb logarithm.

Frequencies

electron gyrofrequency	$f_{ce} = \omega_{ce}/2\pi = 2.80 \times 10^6 B$ Hz
	$\omega_{ce} = eB/m_e c = 1.76 \times 10^7 B$ rad/sec
ion gyrofrequency	$f_{ci} = \omega_{ci}/2\pi = 1.52 \times 10^3 Z\mu^{-1} B$ Hz
	$\omega_{ci} = eB/m_i c = 9.58 \times 10^3 Z\mu^{-1} B$ rad/sec
electron plasma frequency	$f_{pe} = \omega_{pe}/2\pi = 8.98 \times 10^3 n_e{}^{1/2}$ Hz
	$\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$
	$= 5.64 \times 10^4 n_e{}^{1/2}$ rad/sec
ion plasma frequency	$f_{pi} = \omega_{pi}/2\pi$
	$= 2.10 \times 10^2 Z\mu^{-1/2} n_i{}^{1/2}$ Hz
	$\omega_{pi} = (4\pi n_i Z^2 e^2/m_i)^{1/2}$
	$= 1.32 \times 10^3 Z\mu^{-1/2} n_i{}^{1/2}$ rad/sec
electron trapping rate	$\nu_{Te} = (eKE/m_e)^{1/2}$
	$= 7.26 \times 10^8 K^{1/2} E^{1/2}$ sec^{-1}
ion trapping rate	$\nu_{Ti} = (eKE/m_i)^{1/2}$
	$= 1.69 \times 10^7 K^{1/2} E^{1/2} \mu^{-1/2}$ sec^{-1}
electron collision rate	$\nu_e = 2.91 \times 10^{-6} n_e \ln\Lambda T_e{}^{-3/2}$ sec^{-1}
ion collision rate	$\nu_i = 4.78 \times 10^{-8} n_i Z^2 \ln\Lambda T_i{}^{-3/2}$ sec^{-1}

Lengths

electron deBroglie length	$\lambda = \hbar/(m_e k T_e)^{1/2} = 2.76 \times 10^{-8} T_e{}^{-1/2}$ cm
classical distance of minimum approach	$e^2/kT = 1.44 \times 10^{-7} T^{-1}$ cm
electron gyroradius	$r_e = v_{Te}/\omega_{ce} = 2.38 T_e{}^{1/2} B^{-1}$ cm
ion gyroradius	$r_i = v_{Ti}/\omega_{ci}$
	$= 1.02 \times 10^2 \mu^{1/2} Z^{-1} T_i{}^{1/2} B^{-1}$ cm
plasma skin depth	$c/\omega_{pe} = 5.31 \times 10^5 n_e{}^{-1/2}$ cm
Debye length	$\lambda_D = (kT/4\pi n e^2)^{1/2}$
	$= 7.43 \times 10^2 T^{1/2} n^{-1/2}$ cm

```
\input prolog
\hoffset=1truein\hoffset=1truein\vsize=6.5truein\vsize=9truein
\pageno=29
{\headfont Velocities}
\smallskip{\def\CR{\cr\noalign{\vskip2.5pt}}
  %  \CR SKIPS A SPACE AFTER EACH LINE.
\halign{\quad#\hfil&\qquad$#$\hfil\cr
electron thermal velocity&v_{Te}=(kT_e/m_e)^{1/2}\CR
    &\ph{v_{Te}}=4.19\times10^7{T_e}^{1/2}\,\rm cm/sec\CR
ion thermal velocity&v_{Ti}=(kT_i/m_i)^{1/2}\CR
    &\ph{v_{Ti}}=9.79\times10^5\mu^{-1/2}{T_i}^{1/2}\,\rm cm/sec\CR
ion sound velocity&C_s=(\gamma ZkT_e/m_i)^{1/2}\CR
    &\ph{C_s}=9.79\times10^5(\gamma ZT_e/\mu)^{1/2}\,\rm cm/sec\CR
Alfv\'en velocity&v_A=B/(4\pi n_im_i)^{1/2}\CR
    &\ph{v_A}=2.18\times10^{11}\mu^{-1/2}{n_i}^{-1/2}B\,\rm cm/sec\CR
\noalign{\headfont Dimensionless}
    \sk
(electron/proton mass ratio)$^{1/2}$&(m_e/m_p)^{1/2}=2.33\times10^{-2}=1/42.9\CR
    number of particles in&(4\pi/3)n{\lambda_D}^3=
    1.72\times10^9T^{3/2}n^{-1/2}\cr
    \quad Debye sphere\CR
Alfv\'en velocity/speed of light&v_A/c=7.28\mu^{-1/2}{n_i}^{-1/2}B\CR
electron plasma/gyrofrequency&
    \omega_{pe}/\omega_{ce}=3.21\times10^{-3}{n_e}^{1/2}B^{-1}\cr
    \quad ratio\CR
ion plasma/gyrofrequency ratio&
    \omega_{pi}/\omega_{ci}=0.137\mu^{1/2}{n_i}^{1/2}B^{-1}\CR
thermal/magnetic energy ratio&\beta=8\pi nkT/B^2=4.03\times10^{-11}nTB^{-2}\CR
magnetic/ion rest energy ratio&B^2/8\pi n_im_ic^2=26.5\mu^{-1}{n_i}^{-1}B^2\CR
    \sk
\noalign{\headfont Miscellaneous}
    \sk
Bohm diffusion coefficient&D_B=(ckT/16eB)\CR
    &\ph{D_B}=6.25\times10^6TB^{-1}\,\rm cm^2/sec\CR
transverse Spitzer resistivity&
    \eta_\perp=1.15\times10^{-14}Z\ln\Lambda T^{-3/2}\,\rm sec\CR
    &\ph{\eta_\perp}=
    1.03\times10^{-2}Z\ln\Lambda T^{-3/2}\,\Omega\rm\,cm\CR}}
\smallskip\noindent
The anomalous collision rate due to low-frequency ion-sound turbulence is
$$\nu\hbox{*}\approx\omega_{pe}\widetilde W/kT=5.64\times10^4{n_e}^{1/2}
\widetilde W/kT\,\rm sec^{-1},$$
where $\widetilde W$ is the total energy of waves with $\omega/K < v_{Ti}$.
\smallskip\noindent
Magnetic pressure is given by
$$P_{\rm mag}=B^2/8\pi=3.98\times10^6B^2\,{\rm dynes/cm}^2=3.93(B/B_0)^2\,\rm
atm,$$
where $B_0=10\,\rm kG=1\,T$.
\smallskip\noindent
Detonation energy of 1 kiloton of high explosive is
$$W_{\rm kT}=10^{12}\,\rm cal=4.2\times10^{19}\,erg.$$
\vfill\eject\end
```

Velocities

electron thermal velocity	$v_{Te} = (kT_e/m_e)^{1/2}$ $= 4.19 \times 10^7 T_e{}^{1/2}$ cm/sec
ion thermal velocity	$v_{Ti} = (kT_i/m_i)^{1/2}$ $= 9.79 \times 10^5 \mu^{-1/2} T_i{}^{1/2}$ cm/sec
ion sound velocity	$C_s = (\gamma Z k T_e/m_i)^{1/2}$ $= 9.79 \times 10^5 (\gamma Z T_e/\mu)^{1/2}$ cm/sec
Alfvén velocity	$v_A = B/(4\pi n_i m_i)^{1/2}$ $= 2.18 \times 10^{11} \mu^{-1/2} n_i{}^{-1/2} B$ cm/sec

Dimensionless

(electron/proton mass ratio)$^{1/2}$	$(m_e/m_p)^{1/2} = 2.33 \times 10^{-2} = 1/42.9$
number of particles in Debye sphere	$(4\pi/3) n \lambda_D{}^3 = 1.72 \times 10^9 T^{3/2} n^{-1/2}$
Alfvén velocity/speed of light	$v_A/c = 7.28 \mu^{-1/2} n_i{}^{-1/2} B$
electron plasma/gyrofrequency ratio	$\omega_{pe}/\omega_{ce} = 3.21 \times 10^{-3} n_e{}^{1/2} B^{-1}$
ion plasma/gyrofrequency ratio	$\omega_{pi}/\omega_{ci} = 0.137 \mu^{1/2} n_i{}^{1/2} B^{-1}$
thermal/magnetic energy ratio	$\beta = 8\pi n k T/B^2 = 4.03 \times 10^{-11} n T B^{-2}$
magnetic/ion rest energy ratio	$B^2/8\pi n_i m_i c^2 = 26.5 \mu^{-1} n_i{}^{-1} B^2$

Miscellaneous

Bohm diffusion coefficient	$D_B = (ckT/16eB)$ $= 6.25 \times 10^6 T B^{-1}$ cm^2/sec
transverse Spitzer resistivity	$\eta_\perp = 1.15 \times 10^{-14} Z \ln \Lambda T^{-3/2}$ sec $= 1.03 \times 10^{-2} Z \ln \Lambda T^{-3/2}$ Ω cm

The anomalous collision rate due to low-frequency ion-sound turbulence is

$$\nu^* \approx \omega_{pe} \widetilde{W}/kT = 5.64 \times 10^4 n_e{}^{1/2} \widetilde{W}/kT \text{ sec}^{-1},$$

where $\widetilde{W}$ is the total energy of waves with $\omega/K < v_{Ti}$.

Magnetic pressure is given by

$$P_{\text{mag}} = B^2/8\pi = 3.98 \times 10^6 B^2 \text{ dynes/cm}^2 = 3.93 (B/B_0)^2 \text{ atm},$$

where $B_0 = 10$ kG $= 1$ T.

Detonation energy of 1 kiloton of high explosive is

$$W_{\text{kT}} = 10^{12} \text{ cal} = 4.2 \times 10^{19} \text{ erg}.$$

```
\input prolog
\hoffset=1.25truein
\voffset=1.0truein
\hsize=6.0truein
\vsize=9.0truein
\pageno=30
\centerline{\headfont PLASMA DISPERSION FUNCTION}
\bigskip\N Definition$^{16}$ (first form valid only for Im$\,\zeta>0$):
$$Z(\zeta)=\pi^{-1/2}\int_{-\infty}^{+\infty}{dt\,\exp\left(-t^2\right)\over
t-\zeta}=2i\exp\left(-\zeta^2\right)\int_{-\infty}^{i\zeta}dt\,\exp\left(-t^2
\right).$$
\N Physically, $\zeta=x+iy$ is the ratio of wave phase velocity to thermal
velocity.
\medskip\N Differential equation:
$${dZ\over d\zeta}=-2\left(1+\zeta Z\right),\>Z(0)=i\pi^{1/2};\quad{d^2Z\over d
\zeta^2}+2\zeta{dZ\over d\zeta}+2Z=0.$$
\N Real argument ($y=0$):
$$Z(x)=\exp\left(-x^2\right)\left(i\pi^{1/2}-2\int_0^x dt\,\exp\left(t^2\right)
\right).$$
\N Imaginary argument ($x=0$):
$$Z(iy)=i\pi^{1/2}\exp\left(y^2\right)\left[1-{\rm erf}(y)\right].$$
\N Power series (small argument):
$$Z(\zeta)=i\pi^{1/2}\exp\left(-\zeta^2\right)-2\zeta\left(1-2\zeta^2/3+4\zeta^
4/15-8\zeta^6/105+\cdots\right).$$
\N Asymptotic series, $\mid\zeta\mid\gg1$ (Ref. 17):
$$Z(\zeta)=i\pi^{1/2}\sigma\exp\left(-\zeta^2\right)-\zeta^{-1}\left(1+1/2\zeta
^2+3/4\zeta^4+15/8\zeta^6+\cdots\right),$$
where
$$\sigma=\left\{\,\vcenter{\halign{$#\;$&\hfil$#$\hfil&\hfil$\;#\;$\hfil&$#$
\hfil\cr
0\;&y&>&\mid x\mid^{-1}\cr
1\;&\mid y\mid&<&\mid x\mid^{-1}\cr
2\;&y&<&-\mid x\mid^{-1}\cr}}\right.$$
\N Symmetry properties (the asterisk denotes complex conjugation):
$$Z(\zeta\hbox{*})=-\left[Z(-\zeta)\right]\hbox{\kern-1pt*};$$
$$Z(\zeta\hbox{*})=\left[Z(\zeta)\right]\hbox{*}+2i\pi^{1/2}\exp[-(\zeta
\hbox{*})^2]\quad(y>0).$$
\N Two-pole approximations$^{18}$ (good for $\zeta$ in upper half plane except
when $y<\pi^{1/2}x^2\exp(-x^2),\;x\gg1$):
$$\eqalign{Z(\zeta)&\approx{0.50+0.81i\over a-\zeta}-{0.50-0.81i\over a\hbox{*}+
\zeta},\;\;a=0.51-0.81i;\cr
Z'(\zeta)&\approx{0.50+0.96i\over(b-\zeta)^2}+{0.50-0.96i\over(b\hbox{*}+\zeta)^
2},\;\;b=0.48-0.91i.\cr}$$
\vfil\eject\end
```

PLASMA DISPERSION FUNCTION

Definition[16] (first form valid only for $\operatorname{Im}\zeta > 0$):

$$Z(\zeta) = \pi^{-1/2} \int_{-\infty}^{+\infty} \frac{dt \exp\left(-t^2\right)}{t - \zeta} = 2i \exp\left(-\zeta^2\right) \int_{-\infty}^{i\zeta} dt \exp\left(-t^2\right).$$

Physically, $\zeta = x + iy$ is the ratio of wave phase velocity to thermal velocity.

Differential equation:

$$\frac{dZ}{d\zeta} = -2\left(1 + \zeta Z\right), \quad Z(0) = i\pi^{1/2}; \qquad \frac{d^2 Z}{d\zeta^2} + 2\zeta\frac{dZ}{d\zeta} + 2Z = 0.$$

Real argument ($y = 0$):

$$Z(x) = \exp\left(-x^2\right)\left(i\pi^{1/2} - 2\int_0^x dt \exp\left(t^2\right)\right).$$

Imaginary argument ($x = 0$):

$$Z(iy) = i\pi^{1/2} \exp\left(y^2\right)\left[1 - \operatorname{erf}(y)\right].$$

Power series (small argument):

$$Z(\zeta) = i\pi^{1/2} \exp\left(-\zeta^2\right) - 2\zeta\left(1 - 2\zeta^2/3 + 4\zeta^4/15 - 8\zeta^6/105 + \cdots\right).$$

Asymptotic series, $|\zeta| \gg 1$ (Ref. 17):

$$Z(\zeta) = i\pi^{1/2}\sigma \exp\left(-\zeta^2\right) - \zeta^{-1}\left(1 + 1/2\zeta^2 + 3/4\zeta^4 + 15/8\zeta^6 + \cdots\right),$$

where

$$\sigma = \begin{cases} 0 & y > |x|^{-1} \\ 1 & |y| < |x|^{-1} \\ 2 & y < -|x|^{-1} \end{cases}$$

Symmetry properties (the asterisk denotes complex conjugation):

$$Z(\zeta^*) = -\left[Z(-\zeta)\right]^*;$$

$$Z(\zeta^*) = \left[Z(\zeta)\right]^* + 2i\pi^{1/2}\exp[-(\zeta^*)^2] \quad (y > 0).$$

Two-pole approximations[18] (good for ζ in upper half plane except when $y < \pi^{1/2}x^2\exp(-x^2)$, $x \gg 1$):

$$Z(\zeta) \approx \frac{0.50 + 0.81i}{a - \zeta} - \frac{0.50 - 0.81i}{a^* + \zeta}, \quad a = 0.51 - 0.81i;$$

$$Z'(\zeta) \approx \frac{0.50 + 0.96i}{(b - \zeta)^2} + \frac{0.50 - 0.96i}{(b^* + \zeta)^2}, \quad b = 0.48 - 0.91i.$$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=31
\centerline{\headfont COLLISIONS AND TRANSPORT} \bsk\indent
Temperatures are in eV; the corresponding value of Boltzmann's constant
is $k = 1.60\times 10^{-12}\,$erg/eV; masses $\mu$, $\mu'$ are in units of
the proton mass; $e_\alpha = Z_\alpha e$ is the charge of species $\alpha$.
All other units are cgs except where noted. \msk
{\headfont Relaxation Rates} \ssk\indent
Rates are associated with four relaxation processes arising from the
interaction of test particles (labeled $\alpha$) streaming with velocity {\bf
v}$_\alpha$ through a background of field particles (labeled $\beta$):
$$ \eqalign{ \hbox to 108pt{slowing down\hfil} &{d{\bf v}_\alpha\over dt} =
-\nu_s^\AOB {\bf v}_\alpha \cr
\hbox to 108pt{transverse diffusion\hfil} &{d\over dt}({\bf v}_\alpha
- {\bf \bar v}_\alpha)^2_\perp = \nu_\perp^\AOB {v_\alpha}^2 \cr
\hbox to 108pt{parallel diffusion\hfil} &{d\over dt}({\bf v}_\alpha -
{\bf \bar v}_\alpha)^2_\parallel = \nu_\parallel^\AOB {v_\alpha}^2 \cr
\hbox to 108pt{energy loss\hfil} &{d\over dt}{v_\alpha}^2 = -\nu_
\epsilon^\AOB {v_\alpha}^2, \cr}$$
where the averages are performed over an ensemble of test particles and
a Maxwellian field particle distribution.  The exact formulas may be
written$^{19}$
$$\eqalign{\nu_s^\AOB &= (1+m_\alpha/m_\beta)\psi(x^\AOB )
\nu_0^\AOB ; \cr
\nu_\perp^\AOB &= 2\left[ (1-1/2x^\AOB) \psi(x^\AOB)+ \psi'(x^\AOB)\right]
\nu_0^\AOB ; \cr
\nu_\parallel^\AOB &= \left[ \psi(x^\AOB)/x^\AOB\right] \nu_0^\AOB ; \cr
\nu_\epsilon^\AOB &= 2\left[ (m_\alpha/m_\beta)\psi(x^\AOB) -
\psi'(x^\AOB)\right]\nu_0^\AOB ,}$$
where
$$\nu_0^\AOB = 4\pi {e_\alpha}^2{e_\beta}^2\lambda_{\alpha\beta}n_\beta/
{m_\alpha}^2{v_\alpha}^3 ; \qquad x^\AOB = m_\beta {v_\alpha}^2/
2kT_\beta;$$
$$\psi(x)={2\over\sqrt{\pi}} \int_0^x \!dt\, t^{1/2} e^{-t}; \quad \psi'(x)
= {d\psi\over dx},$$
and $\lambda_{\alpha\beta} = \ln \Lambda_{\alpha\beta}$ is the Coulomb
logarithm (see below).  Limiting forms of $\nu_s$, $\nu_\perp$ and
$\nu_\parallel$ are given in the following table.  All the expressions shown
\vfil\eject\end
```

COLLISIONS AND TRANSPORT

Temperatures are in eV; the corresponding value of Boltzmann's constant is $k = 1.60 \times 10^{-12}$ erg/eV; masses μ, μ' are in units of the proton mass; $e_\alpha = Z_\alpha e$ is the charge of species α. All other units are cgs except where noted.

Relaxation Rates

Rates are associated with four relaxation processes arising from the interaction of test particles (labeled α) streaming with velocity $\mathbf{v}_\alpha$ through a background of field particles (labeled β):

slowing down	$\dfrac{d\mathbf{v}_\alpha}{dt} = -\nu_s^{\alpha\backslash\beta}\mathbf{v}_\alpha$
transverse diffusion	$\dfrac{d}{dt}(\mathbf{v}_\alpha - \bar{\mathbf{v}}_\alpha)_\perp^2 = \nu_\perp^{\alpha\backslash\beta}{v_\alpha}^2$
parallel diffusion	$\dfrac{d}{dt}(\mathbf{v}_\alpha - \bar{\mathbf{v}}_\alpha)_\parallel^2 = \nu_\parallel^{\alpha\backslash\beta}{v_\alpha}^2$
energy loss	$\dfrac{d}{dt}{v_\alpha}^2 = -\nu_\epsilon^{\alpha\backslash\beta}{v_\alpha}^2,$

where the averages are performed over an ensemble of test particles and a Maxwellian field particle distribution. The exact formulas may be written[19]

$$\nu_s^{\alpha\backslash\beta} = (1 + m_\alpha/m_\beta)\psi(x^{\alpha\backslash\beta})\nu_0^{\alpha\backslash\beta};$$

$$\nu_\perp^{\alpha\backslash\beta} = 2\left[(1 - 1/2x^{\alpha\backslash\beta})\psi(x^{\alpha\backslash\beta}) + \psi'(x^{\alpha\backslash\beta})\right]\nu_0^{\alpha\backslash\beta};$$

$$\nu_\parallel^{\alpha\backslash\beta} = \left[\psi(x^{\alpha\backslash\beta})/x^{\alpha\backslash\beta}\right]\nu_0^{\alpha\backslash\beta};$$

$$\nu_\epsilon^{\alpha\backslash\beta} = 2\left[(m_\alpha/m_\beta)\psi(x^{\alpha\backslash\beta}) - \psi'(x^{\alpha\backslash\beta})\right]\nu_0^{\alpha\backslash\beta}.$$

where

$$\nu_0^{\alpha\backslash\beta} = 4\pi{e_\alpha}^2{e_\beta}^2\lambda_{\alpha\beta}n_\beta/{m_\alpha}^2{v_\alpha}^3; \qquad x^{\alpha\backslash\beta} = m_\beta{v_\alpha}^2/2kT_\beta;$$

$$\psi(x) = \frac{2}{\sqrt{\pi}}\int_0^x dt\, t^{1/2}e^{-t}; \quad \psi'(x) = \frac{d\psi}{dx},$$

and $\lambda_{\alpha\beta} = \ln\Lambda_{\alpha\beta}$ is the Coulomb logarithm (see below). Limiting forms of ν_s, $\nu_\perp$ and $\nu_\parallel$ are given in the following table. All the expressions shown

```
\input prolog \pageno=32
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
have units cm$^3\,$sec$^{-1}$.  Test particle energy $\epsilon$ and field
particle temperature $T$ are both in eV; $\mu=m_i/m_p$ where $m_p$ is the
proton mass; $Z$ is ion charge state; in electron--electron and ion--ion
encounters, field particle quantities are distinguished by a prime.  The two
expressions given below for each rate hold for very slow $(x^\AOB \ll 1)$ and
very fast $(x^\AOB \gg 1)$ test particles, respectively. \msk
  %  \app AND \lra ARE BOTH DEFINED IN PROLOG.TEX
\halign{\quad#\hfil &#\hfil &#\hfil \cr
&\hfil\undertext{Slow} &\hfil\undertext{Fast} \cr
\kern-1em Electron--electron \hidewidth \cr
$\displaystyle \nu_s^{e/e'}/n_{e'}\lambda_{ee'}$&$\app 5.8\times 10^{-6}
  T^{-3/2}$&$\lra 7.7\times 10^{-6} \epsilon^{-3/2}$\cr
$\displaystyle \nu_\perp^{e/e'}/n_{e'}\lambda_{ee'}$&$\app 5.8\times 10^{-6}
  T^{-1/2}\epsilon^{-1}$&$\lra 7.7\times 10^{-6} \epsilon^{-3/2}$\cr
$\displaystyle \nu_\parallel^{e/e'}/n_{e'}\lambda_{ee'}$&$\app 2.9\times
  10^{-6} T^{-1/2}\epsilon^{-1}$&$\lra 3.9\times 10^{-6}T\epsilon^{-5/2}$\cr
\noalign{\smallskip Electron--ion \smallskip}
$\displaystyle \nu_s^{e/i}/n_iZ^2\lambda_{ei}$&$\app 0.23 \mu^{3/2}T^{-3/2}$
  &$\lra 3.9\times 10^{-6}\epsilon^{-3/2}$ \cr
$\displaystyle \nu_\perp^{e/i}/n_iZ^2\lambda_{ei}$&$\app 2.5 \times 10^{-4}
  \mu^{1/2}T^{-1/2}\epsilon^{-1}$&$\lra 7.7\times 10^{-6}\epsilon^{-3/2}$ \cr
$\displaystyle \nu_\parallel^{e/i}/n_iZ^2\lambda_{ei}$&$\app 1.2
  \times 10^{-4}\mu^{1/2}T^{-1/2}\epsilon^{-1}$&$\lra 2.1\times
  10^{-9}\mu^{-1}T\epsilon^{-5/2}$ \cr
\noalign{\smallskip Ion--electron \smallskip}
$\displaystyle \nu_s^{i/e}/n_eZ^2\lambda_{ie}$&$\app 1.6 \times 10^{-9}\mu^{-1}
  T^{-3/2}$&$\lra 1.7\times 10^{-4}\mu^{1/2}\epsilon^{-3/2}$ \cr
$\displaystyle \nu_\perp^{i/e}/n_eZ^2\lambda_{ie}$&$\app 3.2
  \times 10^{-9}\mu^{-1}T^{-1/2}\epsilon^{-1}$&$\lra
  1.8\times 10^{-7}\mu^{-1/2}\epsilon^{-3/2}$ \cr
$\displaystyle \nu_\parallel^{i/e}/n_eZ^2\lambda_{ie}$&$\app 1.6
  \times 10^{-9}\mu^{-1}T^{-1/2}\epsilon^{-1}$&$\lra
  1.7\times 10^{-4}\mu^{1/2}T\epsilon^{-5/2}$ \cr
\noalign{\smallskip Ion--ion \smallskip}
$\displaystyle {\nu_s^{i/i'} \over n_{i'}Z^2Z'^2\lambda_{ii'}}$ &
  $\app 6.8\times 10^{-8}{\mu'^{1/2}\over\mu}
  \left(1+{\mu'\over\mu}\right)T^{-3/2}$ \hidewidth \cr \bs{0.5ex}
&&$\lra 9.0\times 10^{-8} \left( {1 \over \mu} + {1 \over \mu'} \right)
  {\mu^{1/2} \over \epsilon^{3/2}}$ \cr
$\displaystyle {\nu_\perp^{i/i'} \over n_{i'}Z^2Z'^2\lambda_{ii'}}$
  &$\app 1.4\times 10^{-7} \mu'^{1/2} \mu^{-1} T^{-1/2}
  \epsilon^{-1}$ \hidewidth \cr \bs{0.5ex}
&&$\lra 1.8\times 10^{-7}\mu^{-1/2}\epsilon^{-3/2}$\cr
$\displaystyle {\nu_\parallel^{i/i'} \over n_{i'}Z^2Z'^2\lambda_{ii'}}$
  &$\app 6.8\times 10^{-8} \mu'^{1/2} \mu^{-1}T^{-1/2}
   \epsilon^{-1}$ \hidewidth \cr \bs{0.5ex}
&&$\lra 9.0\times 10^{-8}\mu^{1/2} \mu'^{-1}T\epsilon^{-5/2}$ \cr} \msk\noindent
In the same limits, the energy transfer rate follows from the identity
$$\nu_\epsilon = 2\nu_s - \nu_\perp - \nu_\parallel ,$$
except for the case of fast electrons or fast ions scattered by ions, where
the leading terms cancel.  Then the appropriate forms are
$$\eqalign{\nu_\epsilon^{e/i} \longrightarrow 4.2 &\times 10^{-9}n_iZ^2
\lambda_{ei}\cr &\left[ \epsilon^{-3/2}\mu^{-1} - 8.9 \times 10^4 (\mu/T)^{1/2}
\epsilon^{-1} \exp (-1836 \mu \epsilon/T) \right]\,{\rm sec}^{-1} \cr}$$
\vfil\eject\end
```

have units $\mathrm{cm}^3\,\mathrm{sec}^{-1}$. Test particle energy ϵ and field particle temperature T are both in eV; $\mu = m_i/m_p$ where m_p is the proton mass; Z is ion charge state; in electron–electron and ion–ion encounters, field particle quantities are distinguished by a prime. The two expressions given below for each rate hold for very slow ($x^{\alpha/\beta} \ll 1$) and very fast ($x^{\alpha/\beta} \gg 1$) test particles, respectively.

	Slow	Fast
Electron–electron		
$\nu_s^{e/e'}/n_{e'}\lambda_{ee'}$	$\approx 5.8 \times 10^{-6} T^{-3/2}$	$\longrightarrow 7.7 \times 10^{-6} \epsilon^{-3/2}$
$\nu_\perp^{e/e'}/n_{e'}\lambda_{ee'}$	$\approx 5.8 \times 10^{-6} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 7.7 \times 10^{-6} \epsilon^{-3/2}$
$\nu_\parallel^{e/e'}/n_{e'}\lambda_{ee'}$	$\approx 2.9 \times 10^{-6} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 3.9 \times 10^{-6} T \epsilon^{-5/2}$
Electron–ion		
$\nu_s^{e/i}/n_i Z^2 \lambda_{ei}$	$\approx 0.23 \mu^{3/2} T^{-3/2}$	$\longrightarrow 3.9 \times 10^{-6} \epsilon^{-3/2}$
$\nu_\perp^{e/i}/n_i Z^2 \lambda_{ei}$	$\approx 2.5 \times 10^{-4} \mu^{1/2} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 7.7 \times 10^{-6} \epsilon^{-3/2}$
$\nu_\parallel^{e/i}/n_i Z^2 \lambda_{ei}$	$\approx 1.2 \times 10^{-4} \mu^{1/2} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 2.1 \times 10^{-9} \mu^{-1} T \epsilon^{-5/2}$
Ion–electron		
$\nu_s^{i/e}/n_e Z^2 \lambda_{ie}$	$\approx 1.6 \times 10^{-9} \mu^{-1} T^{-3/2}$	$\longrightarrow 1.7 \times 10^{-4} \mu^{1/2} \epsilon^{-3/2}$
$\nu_\perp^{i/e}/n_e Z^2 \lambda_{ie}$	$\approx 3.2 \times 10^{-9} \mu^{-1} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 1.8 \times 10^{-7} \mu^{-1/2} \epsilon^{-3/2}$
$\nu_\parallel^{i/e}/n_e Z^2 \lambda_{ie}$	$\approx 1.6 \times 10^{-9} \mu^{-1} T^{-1/2} \epsilon^{-1}$	$\longrightarrow 1.7 \times 10^{-4} \mu^{1/2} T \epsilon^{-5/2}$

Ion–ion

$$\frac{\nu_s^{i/i'}}{n_{i'} Z^2 Z'^2 \lambda_{ii'}} \approx 6.8 \times 10^{-8} \frac{\mu'^{1/2}}{\mu}\left(1 + \frac{\mu'}{\mu}\right) T^{-3/2} \longrightarrow 9.0 \times 10^{-8} \left(\frac{1}{\mu} + \frac{1}{\mu'}\right) \frac{\mu^{1/2}}{\epsilon^{3/2}}$$

$$\frac{\nu_\perp^{i/i'}}{n_{i'} Z^2 Z'^2 \lambda_{ii'}} \approx 1.4 \times 10^{-7} \mu'^{1/2} \mu^{-1} T^{-1/2} \epsilon^{-1} \longrightarrow 1.8 \times 10^{-7} \mu^{-1/2} \epsilon^{-3/2}$$

$$\frac{\nu_\parallel^{i/i'}}{n_{i'} Z^2 Z'^2 \lambda_{ii'}} \approx 6.8 \times 10^{-8} \mu'^{1/2} \mu^{-1} T^{-1/2} \epsilon^{-1} \longrightarrow 9.0 \times 10^{-8} \mu^{1/2} \mu'^{-1} T \epsilon^{-5/2}$$

In the same limits, the energy transfer rate follows from the identity

$$\nu_\epsilon = 2\nu_s - \nu_\perp - \nu_\parallel,$$

except for the case of fast electrons or fast ions scattered by ions, where the leading terms cancel. Then the appropriate forms are

$$\nu_\epsilon^{e/i} \longrightarrow 4.2 \times 10^{-9} n_i Z^2 \lambda_{ei} \left[\epsilon^{-3/2}\mu^{-1} - 8.9 \times 10^4 (\mu/T)^{1/2} \epsilon^{-1} \exp(-1836\mu\epsilon/T)\right] \mathrm{sec}^{-1}$$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=33
and
$$\eqalign{\nu_\epsilon^{i/i'} \longrightarrow 1.8 &\times 10^{-7} n_{i'} Z^2
Z'^2 \lambda_{ii'} \cr &\left[ \epsilon^{-3/2}\mu^{1/2}/\mu' - 1.1(\mu'/T)
^{1/2} \epsilon^{-1} \exp(-\mu'\epsilon/T)\right]\,{\rm sec}^{-1}.\cr}$$
In general, the energy transfer rate $\nu_\epsilon^\AOB$ is positive for
$\epsilon>\epsilon_\alpha\hbox{*}$ and negative for $\epsilon<\epsilon_\alpha
\hbox{*}$,
where $x\hbox{*} = (m_\beta/m_\alpha) \epsilon_\alpha\hbox{*}/T_\beta$ is the
solution of $\psi'(x\hbox{*})=(m_\alpha/m_\beta)\psi(x\hbox{*})$.  The ratio
$\epsilon_\alpha\hbox{*}/T_\beta$ is given for a number of specific $\alpha$,
$\beta$ in the following table:
  %  BEGINNING OF TABLE
$$\vbox{\offinterlineskip \def\quad{\hskip0.6em\relax} \hrule \halign{
  %  \quad IS REDEFINED TO ADJUST SPACING.
&\vrule# &\strut\quad\hfil#\hfil\quad &\vrule# &\quad\hfil#\hfil\quad
&\hfil#\hfil\quad &\hfil#\hfil\quad &\hfil#\hfil\quad &\hfil#\hfil &\hfil#\hfil
\quad \cr
  height2pt&\om|\om&\om&\om&\om&\om&\om&\cr
&$\AOB$ |$i/e$ &$e/e$, $i/i$ &$e/p$ &$e/$D &$e/$T, $e/$He$^3$ &$e/$He$^4$ &\cr
  height2pt&\om|\om&\om&\om&\om&\om&\om&\cr
\noalign{\hrule}
  height2pt&\om|\om&\om&\om&\om&\om&\om&\cr
&$\displaystyle {\epsilon_\alpha\hbox{*}\over
  T_\beta}$ |$1.5$ &$0.98$ &$4.8\times 10^{-3}$ & $2.6\times
  10^{-3}$ &$1.8\times 10^{-3}$ &\quad$1.4\times 10^{-3}$ &\cr
  height2pt&\om|\om&\om&\om&\om&\om&\om&\cr
\noalign{\hrule}}}$$
\indent
When both species are near Maxwellian, with $T_i \approxlt T_e$, there are
just two characteristic collision rates.  For $Z = 1$,
$$\eqalign{\nu_e&=2.9\times 10^{-6}n\lambda{T_e}^{-3/2}\,{\rm sec}^{-1};\cr
\nu_i&=4.8\times 10^{-8}n\lambda{T_i}^{-3/2}\mu^{-1/2}\,{\rm sec}^{-1}.\cr}$$
\smallskip
{\headfont Temperature Isotropization}
\smallskip\indent
Isotropization is described by
$${dT_\perp \over dt} = - {1 \over 2} {dT_\parallel \over dt} = -\nu_T^\alpha
(T_\perp - T_\parallel),$$
where, if $A \equiv T_\perp/T_\parallel - 1 > 0$,
$$\nu_T^\alpha = {2\sqrt{\pi}{e_\alpha}^2{e_\beta}^2n_\alpha\lambda_{\alpha
  \beta} \over {m_\alpha}^{1/2} (kT_\parallel)^{3/2}} A^{-2}\left[-3 + (A+3)
  {{\tan}^{-1} (A^{1/2}) \over A^{1/2}}\right] .$$
If $A < 0$, $\tan^{-1}(A^{1/2})/A^{1/2}$ is replaced by $\tanh^{-1}
  (-A)^{1/2}/(-A)^{1/2}$.
For $T_\perp \approx T_\parallel \equiv T$,
$$\eqalign{\nu_T^e &= 8.2\times 10^{-7}n\lambda T^{-3/2}\,{\rm sec}^{-1}; \cr
\noalign{\smallskip}
\nu_T^i &= 1.9\times 10^{-8}n\lambda Z^2\mu^{-1/2} T^{-3/2}\,{\rm sec}^{-1}.
\cr}$$
\vfil\eject\end
```

and

$$\nu_\epsilon^{i/i'} \longrightarrow 1.8 \times 10^{-7} n_{i'} Z^2 Z'^2 \lambda_{ii'}$$
$$\left[\epsilon^{-3/2} \mu^{1/2}/\mu' - 1.1(\mu'/T)^{1/2} \epsilon^{-1} \exp(-\mu'\epsilon/T)\right] \sec^{-1}.$$

In general, the energy transfer rate $\nu_\epsilon^{\alpha/\beta}$ is positive for $\epsilon > \epsilon_\alpha{}^*$ and negative for $\epsilon < \epsilon_\alpha{}^*$, where $x^* = (m_\beta/m_\alpha)\epsilon_\alpha{}^*/T_\beta$ is the solution of $\psi'(x^*) = (m_\alpha/m_\beta)\psi(x^*)$. The ratio $\epsilon_\alpha{}^*/T_\beta$ is given for a number of specific α, β in the following table:

α/β	i/e	e/e, i/i	e/p	e/D	e/T, e/He^3	e/He^4
$\dfrac{\epsilon_\alpha{}^*}{T_\beta}$	1.5	0.98	4.8×10^{-3}	2.6×10^{-3}	1.8×10^{-3}	1.4×10^{-3}

When both species are near Maxwellian, with $T_i \lesssim T_e$, there are just two characteristic collision rates. For $Z = 1$,

$$\nu_e = 2.9 \times 10^{-6} n\lambda T_e{}^{-3/2} \sec^{-1};$$
$$\nu_i = 4.8 \times 10^{-8} n\lambda T_i{}^{-3/2} \mu^{-1/2} \sec^{-1}.$$

Temperature Isotropization

Isotropization is described by

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = -\nu_T^\alpha (T_\perp - T_\parallel),$$

where, if $A \equiv T_\perp/T_\parallel - 1 > 0$,

$$\nu_T^\alpha = \frac{2\sqrt{\pi} e_\alpha{}^2 e_\beta{}^2 n_\alpha \lambda_{\alpha\beta}}{m_\alpha{}^{1/2}(kT_\parallel)^{3/2}} A^{-2} \left[-3 + (A+3)\frac{\tan^{-1}(A^{1/2})}{A^{1/2}}\right].$$

If $A < 0$, $\tan^{-1}(A^{1/2})/A^{1/2}$ is replaced by $\tanh^{-1}(-A)^{1/2}/(-A)^{1/2}$. For $T_\perp \approx T_\parallel \equiv T$,

$$\nu_T^e = 8.2 \times 10^{-7} n\lambda T^{-3/2} \sec^{-1};$$
$$\nu_T^i = 1.9 \times 10^{-8} n\lambda Z^2 \mu^{-1/2} T^{-3/2} \sec^{-1}.$$

```
\input prolog \pageno=34
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\smallskip {\headfont Thermal Equilibration} \smallskip\indent
If the components of a plasma have different temperatures, but no relative
drift, equilibration is described by
$${dT_\alpha \over dt} = \sum_\beta \bar \nu_\epsilon^\AOB(T_\beta -
  T_\alpha),$$
where
$$\bar \nu_\epsilon^\AOB=1.8\times 10^{-19}{(m_\alpha m_\beta)^{1/2}{Z_\alpha}^2
  {Z_\beta}^2n_\beta\lambda_{\alpha\beta}\over(m_\alpha T_\beta +
  m_\beta T_\alpha)^{3/2}}\,{\rm sec}^{-1}.$$
For electrons and ions with $T_e \approx T_i \equiv T$, this implies
$$\bar \nu_\epsilon^{e/i}/n_i=\bar \nu_\epsilon^{i/e}/n_e=3.2\times 10^{-9}
  Z^2\lambda/\mu T^{3/2} {\rm cm}^3\,{\rm sec}^{-1}.$$
\smallskip {\headfont Coulomb Logarithm}
\smallskip\indent
For test particles of mass $m_\alpha$ and charge $e_\alpha=Z_\alpha e$
scattering off field particles of mass $m_\beta$ and charge $e_\beta = Z_\beta
e$, the Coulomb logarithm is defined as $\lambda = \ln\Lambda \equiv \ln(r_{\rm
max} /r_{\rm min})$.  Here $r_{\rm min}$ is the larger of $e_\alpha
e_\beta/m_{\alpha\beta} \bar u^2$ and $\hbar/2 m_{\alpha\beta}\bar u$,
averaged over both particle velocity distributions, where
$m_{\alpha\beta}=m_\alpha m_\beta/(m_\alpha + m_\beta)$ and ${\bf u} = {\bf
v}_\alpha - {\bf v}_\beta$; $\,r_{\rm max} = (4 \pi \sum n_\gamma {e_\gamma}^2
/kT_\gamma)^{-1/2}$, where the summation extends over all species $\gamma$ for
which $\bar u^2 < {v_{T\gamma}}^2 = kT_\gamma/m_\gamma$. If this inequality
cannot be satisfied, or if either $\bar u {\omega_{c\alpha}}^{-1}<r_{\rm max}$
or $\bar u{\omega_{c\beta}}^{-1}<r_{\rm max}$, the theory breaks down.
Typically $\lambda \approx$ 10--20. Corrections to the transport coefficients
are $O(\lambda^{-1})$; hence the theory is good only to $\sim 10\%$ and fails
when $\lambda \sim 1$.
\indent
The following cases are of particular interest:
\smallskip
(a) Thermal electron--electron collisions
$$\vbox{\halign{\hfil# &#\hfil\qquad &#\hfil\cr
$\displaystyle \lambda_{ee}$&$\displaystyle = 23-\ln({n_e}^{1/2}{T_e}^{-3/2})$,
&$\displaystyle T_e \approxlt 10\,{\rm eV}$; \cr
\noalign{\smallskip}
&$\displaystyle = 24 - \ln({n_e}^{1/2}{T_e}^{-1})$,
&$\displaystyle T_e \approxgt 10\,{\rm eV}$. \cr}}$$
(b) Electron--ion collisions
$$\vbox{\halign{\hfil# &#\hfil\quad &#\hfil\cr
$\displaystyle \lambda_{ei} = \lambda_{ie}$ &$\displaystyle = 23-\ln\left({n_e}
^{1/2}ZT_e^{-3/2}\right)$,&$\displaystyle T_im_e/m_i<T_e<10Z^2\,{\rm eV}$;\cr
\noalign{\smallskip}
$\displaystyle \ph{\lambda_{ei} = \lambda_{ie}}$ &$\displaystyle = 24 - \ln
\left({n_e}^{1/2} T_e^{-1} \right)$, &$\displaystyle T_i m_e/m_i < 10 Z^2\,{\rm
eV} < T_e$ \cr \noalign{\smallskip}
$\displaystyle \ph{\lambda_{ei} = \lambda_{ie}}$ &$\displaystyle = 30 -
\ln \left( {n_i}^{1/2}{T_i}^{-3/2} Z^2 \mu^{-1} \right)$,
&$\displaystyle T_e < T_i Zm_e/m_i$. \cr}}$$
\vfill\eject\end
```

Thermal Equilibration

If the components of a plasma have different temperatures, but no relative drift, equilibration is described by

$$\frac{dT_\alpha}{dt} = \sum_\beta \bar{\nu}_\epsilon^{\alpha/\beta}(T_\beta - T_\alpha),$$

where

$$\bar{\nu}_\epsilon^{\alpha/\beta} = 1.8 \times 10^{-19} \frac{(m_\alpha m_\beta)^{1/2} Z_\alpha{}^2 Z_\beta{}^2 n_\beta \lambda_{\alpha\beta}}{(m_\alpha T_\beta + m_\beta T_\alpha)^{3/2}} \sec^{-1}.$$

For electrons and ions with $T_e \approx T_i \equiv T$, this implies

$$\bar{\nu}_\epsilon^{e/i}/n_i = \bar{\nu}_\epsilon^{i/e}/n_e = 3.2 \times 10^{-9} Z^2 \lambda/\mu T^{3/2} \mathrm{cm}^3 \sec^{-1}.$$

Coulomb Logarithm

For test particles of mass m_α and charge $e_\alpha = Z_\alpha e$ scattering off field particles of mass m_β and charge $e_\beta = Z_\beta e$, the Coulomb logarithm is defined as $\lambda = \ln \Lambda \equiv \ln(r_{\max}/r_{\min})$. Here $r_{\min}$ is the larger of $e_\alpha e_\beta / m_{\alpha\beta}\bar{u}^2$ and $\hbar/2m_{\alpha\beta}\bar{u}$, averaged over both particle velocity distributions, where $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ and $\mathbf{u} = \mathbf{v}_\alpha - \mathbf{v}_\beta$; $r_{\max} = (4\pi \sum n_\gamma e_\gamma{}^2/kT_\gamma)^{-1/2}$, where the summation extends over all species γ for which $\bar{u}^2 < v_{T_\gamma}{}^2 = kT_\gamma/m_\gamma$. If this inequality cannot be satisfied, or if either $\bar{u}\omega_{c\alpha}{}^{-1} < r_{\max}$ or $\bar{u}\omega_{c\beta}{}^{-1} < r_{\max}$, the theory breaks down. Typically $\lambda \approx 10$–20. Corrections to the transport coefficients are $O(\lambda^{-1})$; hence the theory is good only to $\sim 10\%$ and fails when $\lambda \sim 1$.

The following cases are of particular interest:

(a) Thermal electron-electron collisions

$$\begin{aligned} \lambda_{ee} &= 23 - \ln(n_e{}^{1/2} T_e{}^{-3/2}), & T_e &\lesssim 10\,\mathrm{eV}; \\ &= 24 - \ln(n_e{}^{1/2} T_e{}^{-1}), & T_e &\gtrsim 10\,\mathrm{eV}. \end{aligned}$$

(b) Electron-ion collisions

$$\begin{aligned} \lambda_{ei} = \lambda_{ie} &= 23 - \ln\left(n_e{}^{1/2} Z T_e{}^{-3/2}\right), & & T_i m_e/m_i < T_e < 10Z^2\,\mathrm{eV}; \\ &= 24 - \ln\left(n_e{}^{1/2} T_e^{-1}\right), & & T_i m_e/m_i < 10Z^2\,\mathrm{eV} < T_e \\ &= 30 - \ln\left(n_i{}^{1/2} T_i{}^{-3/2} Z^2 \mu^{-1}\right), & & T_e < T_i Z m_e/m_i. \end{aligned}$$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=35
(c) Mixed ion--ion collisions
$$\lambda_{ii'} = \lambda_{i'i} = 23 - \ln \left[ {ZZ'(\mu + \mu') \over \mu
  T_{i'} + \mu' T_i} \left( {n_i Z^2 \over T_i} + {n_{i'} {Z'}^2 \over
  T_{i'}} \right)^{1/2} \right].$$
(d) Counterstreaming ions (relative velocity $v_D = \beta_D c$) in the presence
of warm electrons, $kT_i/m_i, kT_{i'}/m_{i'} < {v_D}^2 < kT_e/m_e $
$$\lambda_{ii'} = \lambda_{i'i} = 35 - \ln\left[ {ZZ'(\mu + \mu') \over
  \mu\mu'{\beta_D}^2} \left({n_e \over T_e} \right)^{1/2} \right].$$
\smallskip
{\headfont Fokker-Planck Equation}
\smallskip
$${Df^\alpha\over Dt}\equiv{\partial f^\alpha\over\partial t}+{\bf
  v}\cdot\nabla f^\alpha + {\bf F}\cdot\nabla_{\bf v}f^\alpha
  = \left({\partial f^\alpha\over\partial t}\right)_{\rm coll},$$
where {\bf F} is an external force field.  The general form of the collision
integral is $(\partial f^\alpha/\partial t)_{\rm
coll} = -\sum_\beta\nabla_{\bf v}\cdot{\bf J}^\AOB$, with
$$\eqalign{{\bf J}^\AOB = 2\pi\lambda_{\alpha\beta}{{e_\alpha}^2{e_\beta}^2 \over
m_\alpha}\int &d^3\!v'(u^2\hbox{\tf I} - {\bf uu})u^{-3} \cr &\cdot \left\{{1\over
m_\beta}f^\alpha({\bf v})\nabla_{\bf v'}f^\beta({\bf v'})-{1\over
m_\alpha}f^\beta({\bf v'})\nabla_{\bf v}f^\alpha({\bf v})\right\} \cr}$$
(Landau form) where ${\bf u = v'-v}$ and \hbox{\tf I} is the unit dyad, or
alternatively,
$${\bf J}^\AOB = 4\pi\lambda_{\alpha\beta}{{e_\alpha}^2{e_\beta}^2\over
  {m_\alpha}^2}\left\{f^\alpha({\bf v})\nabla_{\bf v}H({\bf v})-
  {1\over2}\nabla_{\bf v}\cdot\left[ f^\alpha({\bf v})
  \nabla_{\bf v}\nabla_{\bf v}G({\bf v})\right] \right\},$$
where the Rosenbluth potentials are
$$G({\bf v})=\int f^\beta({\bf v'})ud^3\!v'$$
$$H({\bf v})=\left(1+ {m_\alpha\over m_\beta} \right)\int f^\beta({\bf v'})
  u^{-1}d^3\!v'.$$
\vfil\eject\end
```

(c) Mixed ion–ion collisions

$$\lambda_{ii'} = \lambda_{i'i} = 23 - \ln \left[\frac{ZZ'(\mu + \mu')}{\mu T_{i'} + \mu' T_i} \left(\frac{n_i Z^2}{T_i} + \frac{n_{i'} Z'^2}{T_{i'}} \right)^{1/2} \right].$$

(d) Counterstreaming ions (relative velocity $v_D = \beta_D c$) in the presence of warm electrons, $kT_i/m_i, kT_{i'}/m_{i'} < v_D{}^2 < kT_e/m_e$

$$\lambda_{ii'} = \lambda_{i'i} = 35 - \ln \left[\frac{ZZ'(\mu + \mu')}{\mu\mu'\beta_D{}^2} \left(\frac{n_e}{T_e} \right)^{1/2} \right].$$

Fokker-Planck Equation

$$\frac{Df^\alpha}{Dt} \equiv \frac{\partial f^\alpha}{\partial t} + \mathbf{v} \cdot \nabla f^\alpha + \mathbf{F} \cdot \nabla_{\mathbf{v}} f^\alpha = \left(\frac{\partial f^\alpha}{\partial t} \right)_{\mathrm{coll}}.$$

where $\mathbf{F}$ is an external force field. The general form of the collision integral is $(\partial f^\alpha/\partial t)_{\mathrm{coll}} = -\sum_\beta \nabla_{\mathbf{v}} \cdot \mathbf{J}^{\alpha/\beta}$, with

$$\mathbf{J}^{\alpha/\beta} = 2\pi\lambda_{\alpha\beta} \frac{e_\alpha{}^2 e_\beta{}^2}{m_\alpha} \int d^3v' (u^2 I - \mathbf{uu}) u^{-3}$$
$$\cdot \left\{ \frac{1}{m_\beta} f^\alpha(\mathbf{v}) \nabla_{\mathbf{v}'} f^\beta(\mathbf{v}') - \frac{1}{m_\alpha} f^\beta(\mathbf{v}') \nabla_{\mathbf{v}} f^\alpha(\mathbf{v}) \right\}$$

(Landau form) where $\mathbf{u} = \mathbf{v}' - \mathbf{v}$ and I is the unit dyad, or alternatively,

$$\mathbf{J}^{\alpha/\beta} = 4\pi\lambda_{\alpha\beta} \frac{e_\alpha{}^2 e_\beta{}^2}{m_\alpha{}^2} \left\{ f^\alpha(\mathbf{v}) \nabla_{\mathbf{v}} H(\mathbf{v}) - \frac{1}{2} \nabla_{\mathbf{v}} \cdot \left[f^\alpha(\mathbf{v}) \nabla_{\mathbf{v}} \nabla_{\mathbf{v}} G(\mathbf{v}) \right] \right\},$$

where the Rosenbluth potentials are

$$G(\mathbf{v}) = \int f^\beta(\mathbf{v}') u \, d^3v'$$

$$H(\mathbf{v}) = \left(1 + \frac{m_\alpha}{m_\beta} \right) \int f^\beta(\mathbf{v}') u^{-1} d^3v'.$$

```
\input prolog
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\pageno=36
If species $\alpha$ is a weak beam (number and energy density small compared
with background) streaming through a Maxwellian plasma, then
$${\bf J}^\AOB = -\nu_s^\AOB{\bf v}f^\alpha -
  {1\over2} \nu_\perp^\AOB v^2\nabla_{\bf v}f^\alpha +
  {1\over2}(\nu_\perp^\AOB - \nu_\parallel^\AOB){\bf vv}\cdot\nabla_
  {\bf v}f^\alpha.$$
\smallskip
{\headfont B-G-K Collision Operator}
\smallskip\indent
For distribution functions with no large gradients in velocity space, [illegible]
Fokker-Planck collision terms can be approximated according to
$${Df_e\over Dt} = \nu_{ee}(F_e-f_e)+ \nu_{ei}(\bar F_e - f_e),$$
$${Df_i\over Dt} = \nu_{ie}(\bar F_i-f_i) + \nu_{ii}(F_i-f_i).$$
The respective slowing-down rates $\nu_s^\AOB$ given in the Relaxation Rate
section above can be used for $\nu_{\alpha\beta}$, assuming slow ions and fast
electrons, with $\epsilon$ replaced by $T_\alpha$. (For $\nu_{ee}$ and
$\nu_{ii}$, one can equally well use $\nu_\perp$, and the result [illegible]
to whether the slow- or fast-test particle limit is employed.) The Maxwellians
$F_\alpha$ and $\bar F_\alpha$ are given by
$$F_\alpha=n_\alpha \left(m_\alpha\over 2\pi kT_\alpha\right)^{3/2} [illegible]
   \left[ {m_\alpha({\bf v} - {\bf v}_\alpha)^2 \over 2kT_\alpha} \right] [illegible]$$
$$\bar F_\alpha=n_\alpha \left(m_\alpha\over 2\pi k\bar T_\alpha\right) [illegible]
   \exp \left\{- \left[{m_\alpha({\bf v} - {\bf \bar v}_\alpha)^2\over 2k\bar T [illegible]
   \right] \right\},$$
where $n_\alpha$, [illegible] and $T_\alpha$ are the number density, [illegible]
drift velocity, and effective temperature obtained by taking moments of
[illegible]
```

If species α is a weak beam (number and energy density small compared with background) streaming through a Maxwellian plasma, then

$$\mathbf{J}^{\alpha/\beta} = -\nu_s^{\alpha/\beta}\mathbf{v}f^\alpha - \frac{1}{2}\nu_\perp^{\alpha/\beta}v^2\nabla_{\mathbf{v}}f^\alpha + \frac{1}{2}(\nu_\perp^{\alpha/\beta} - \nu_\parallel^{\alpha/\beta})\mathbf{v}\mathbf{v}\cdot\nabla_{\mathbf{v}}f^\alpha .$$

B-G-K Collision Operator

For distribution functions with no large gradients in velocity space, the Fokker-Planck collision terms can be approximated according to

$$\frac{Df_e}{Dt} = \nu_{ee}(F_e - f_e) + \nu_{ei}(\bar{F}_e - f_e);$$

$$\frac{Df_i}{Dt} = \nu_{ie}(\bar{F}_i - f_i) + \nu_{ii}(F_i - f_i).$$

Here the slowing-down rates $\nu_s^{\alpha/\beta}$ given in the Relaxation Rate section above can be used for ν_{ie}, assuming slow ions and fast electrons, with ϵ replaced by T_i. (For ν_{ee} and ν_{ii}, one can equally well use $\nu_\perp$, and the result is insensitive to whether the slow- or fast-test-particle limit is employed.) The Maxwellians F_α and $\bar{F}_\alpha$ are given by

$$F_\alpha = n_\alpha\left(\frac{m_\alpha}{2\pi kT_\alpha}\right)^{3/2}\exp\left\{-\left[\frac{m_\alpha(\mathbf{v}-\mathbf{v}_\alpha)^2}{2kT_\alpha}\right]\right\};$$

$$\bar{F}_\alpha = n_\alpha\left(\frac{m_\alpha}{2\pi k\bar{T}_\alpha}\right)^{3/2}\exp\left\{-\left[\frac{m_\alpha(\mathbf{v}-\bar{\mathbf{v}}_\alpha)^2}{2k\bar{T}_\alpha}\right]\right\}.$$

where n_α, $\mathbf{v}_\alpha$ and T_α are the number density, mean drift velocity, and effective temperature obtained by taking moments of f_α. Some latitude in the definition of $\bar{T}_\alpha$ and $\bar{\mathbf{v}}_\alpha$ is possible;[20] one choice is $\bar{T}_e = T_i$, $\bar{T}_i = T_e$, $\bar{\mathbf{v}}_e = \mathbf{v}_i$, $\bar{\mathbf{v}}_i = \mathbf{v}_e$.

Transport Coefficients

Transport equations for a multispecies plasma:

$$\frac{d^\alpha n_\alpha}{dt} + n_\alpha\nabla\cdot\mathbf{v}_\alpha = 0;$$

$$m_\alpha n_\alpha\frac{d^\alpha\mathbf{v}_\alpha}{dt} = -\nabla p_\alpha - \nabla\cdot P_\alpha + Z_\alpha e n_\alpha\left[\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha\times\mathbf{B}\right] + \mathbf{R}_\alpha;$$

```
\input prolog \pageno=37
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
$${3\over 2}n_\alpha {d^\alpha kT_\alpha \over dt} + p_\alpha \nabla
  \cdot \,{\bf v}_\alpha = -\nabla \cdot \,{\bf q}_\alpha - \hbox{\tf
  P}_\alpha:\nabla {\bf v}_\alpha + Q_\alpha.$$
Here $d^\alpha/dt \equiv \partial/\partial t + {\bf v}_\alpha\cdot\nabla$;
$\> p_\alpha=n_\alpha kT_\alpha$, where $k$ is Boltzmann's constant; ${\bf
R}_\alpha = \sum_\beta{\bf R}_{\alpha\beta}$ and $Q_\alpha = \sum_\beta
Q_{\alpha\beta}$, where ${\bf R}_{\alpha\beta}$ and $Q_{\alpha\beta}$ are
respectively the momentum and energy gained by the $\alpha{\rm th}$ species
through collisions with the $\beta{\rm th}$;$\,\hbox{\tf P}_\alpha$ is the
stress tensor; and ${\bf q}_\alpha$ is the heat flow.
\indent
The transport coefficients in a simple two-component plasma (electrons and
singly charged ions) are tabulated below.  Here $\parallel$ and $\perp$ refer
to the direction of the magnetic field ${\bf B} = {\bf b}B$; $\>{\bf u} = {\bf
v}_e - {\bf v}_i$ is the relative streaming velocity; $n_e = n_i \equiv  n$;
$\> {\bf j} = - ne{\bf u}$ is the current; $\omega_{ce} = 1.76 \times 10^7
B\,$sec$^{-1}$ and $\omega_{ci}=(m_e/m_i)\omega_{ce}$ are the electron and ion
gyrofrequencies, respectively; and the basic collisional times are taken to be
$$\tau_e={3\sqrt{m_e}(kT_e)^{3/2} \over 4\sqrt{2\pi}\,n\lambda e^4}=3.44 \times
10^5 {{T_e}^{3/2} \over n\lambda}\,{\rm sec},$$
where $\lambda$ is the Coulomb logarithm, and
$$\tau_i={3\sqrt{m_i}(kT_i)^{3/2} \over 4\sqrt{\pi}n\,\lambda e^4}=2.09 \times
10^7 {{T_i}^{3/2} \over n\lambda}\mu^{1/2}\,{\rm sec}.$$
In the limit of large fields $(\omega_{c\alpha}\tau_\alpha \gg 1, \>\alpha = i,
e)$ the transport processes may be summarized as follows:$^{21}$
\smallskip \def\quid{\hskip0.75em\relax}
\halign{\quad#\hfil\quad &$\displaystyle #$\hfil &$\displaystyle #$\hfil \cr
momentum transfer &\R_{ei} &= -\R_{ie} \equiv \R = \R_{\bf u} + \R_T; \cr
  \noalign{\smallskip}
frictional force &\R_{\bf u} &= ne({\bf j}_\parallel/\sigma_\parallel +
  {\bf j}_\perp/\sigma_\perp); \cr \noalign{\smallskip}
electrical &\sigma_\parallel &= 2.0\sigma_\perp = 2.0{ne^2\tau_e \over m_e};\cr
  \bs{5pt} conductivities \cr \noalign{\smallskip}
thermal force &\R_T &= -0.71n\nabla_\parallel (kT_e) - {3n \over 2
  \omega_{ce}\tau_e}{\bf b}\times\nabla_\perp (kT_e); \cr \noalign{\smallskip}
ion heating &Q_i &={3m_e\over m_i}{nk\over \tau_e}(T_e-T_i); \cr
  \noalign{\smallskip}
electron heating &Q_e &= -Q_i  R\cdot{\bf u}; \cr \noalign{\smallskip}
ion heat flux &{\bf q}_i &=-\kappa_\parallel^i\nabla_\parallel (kT_i)
  - \kappa_\perp^i\nabla_\perp (kT_i) + \kappa_\wedge^i{\bf b}\times
  \nabla_\perp (kT_i); \cr \noalign{\smallskip}
ion thermal &\kappa_\parallel^i &= 3.9{nkT_i\tau_i \over m_i}; \quid
  \kappa_\perp^i = {2nkT_i \over m_i\omega_{ci}^{\ph{1}2} \tau_i}, \quid
  \kappa_\wedge^i = {5nkT_i \over 2m_i\omega_{ci}}; \cr
  \bs{6pt} conductivities \cr \noalign{\smallskip}
electron heat flux &{\bf q}_e &={\bf q}_{\bf u}^e + {\bf q}_T^e; \cr
  \noalign{\smallskip}
frictional heat flux &{\bf q}_{\bf u}^e &= 0.71nkT_e{\bf u}_\parallel +
  {3nkT_e\over 2\omega_{ce}\tau_e}{\bf b}\times{\bf u}_\perp; \cr}
\vfil\eject\end
```

$$\frac{3}{2} n_\alpha \frac{d^\alpha kT_\alpha}{dt} + p_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - P_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha .$$

Here $d^\alpha/dt \equiv \partial/\partial t + \mathbf{v}_\alpha \cdot \nabla$; $p_\alpha = n_\alpha kT_\alpha$, where k is Boltzmann's constant; $\mathbf{R}_\alpha = \sum_\beta \mathbf{R}_{\alpha\beta}$ and $Q_\alpha = \sum_\beta Q_{\alpha\beta}$, where $\mathbf{R}_{\alpha\beta}$ and $Q_{\alpha\beta}$ are respectively the momentum and energy gained by the αth species through collisions with the βth; P_α is the stress tensor; and $\mathbf{q}_\alpha$ is the heat flow.

The transport coefficients in a simple two-component plasma (electrons and singly charged ions) are tabulated below. Here $\|$ and $\perp$ refer to the direction of the magnetic field $\mathbf{B} = \mathbf{b}B$; $\mathbf{u} = \mathbf{v}_e - \mathbf{v}_i$ is the relative streaming velocity; $n_e = n_i \equiv n$; $\mathbf{j} = -ne\mathbf{u}$ is the current; $\omega_{ce} = 1.76 \times 10^7 B\,\mathrm{sec}^{-1}$ and $\omega_{ci} = (m_e/m_i)\omega_{ce}$ are the electron and ion gyrofrequencies, respectively; and the basic collisional times are taken to be

$$\tau_e = \frac{3\sqrt{m_e}(kT_e)^{3/2}}{4\sqrt{2\pi}\, n\lambda e^4} = 3.44 \times 10^5 \frac{T_e^{\;3/2}}{n\lambda}\ \mathrm{sec},$$

where λ is the Coulomb logarithm, and

$$\tau_i = \frac{3\sqrt{m_i}(kT_i)^{3/2}}{4\sqrt{\pi}\, n\lambda e^4} = 2.09 \times 10^7 \frac{T_i^{\;3/2}}{n\lambda}\mu^{1/2}\ \mathrm{sec}.$$

In the limit of large fields ($\omega_{c\alpha}\tau_\alpha \gg 1$, $\alpha = i, e$) the transport processes may be summarized as follows:[21]

momentum transfer	$\mathbf{R}_{ei} = -\mathbf{R}_{ie} \equiv \mathbf{R} = \mathbf{R_u} + \mathbf{R}_T$;
frictional force	$\mathbf{R_u} = ne(\mathbf{j}_\| / \sigma_\| + \mathbf{j}_\perp / \sigma_\perp)$;
electrical conductivities	$\sigma_\| = 2.0\sigma_\perp = 2.0 \dfrac{ne^2\tau_e}{m_e}$;
thermal force	$\mathbf{R}_T = -0.71 n \nabla_\|(kT_e) - \dfrac{3n}{2\omega_{ce}\tau_e}\mathbf{b} \times \nabla_\perp(kT_e)$;
ion heating	$Q_i = \dfrac{3m_e}{m_i}\dfrac{nk}{\tau_e}(T_e - T_i)$;
electron heating	$Q_e = -Q_i - \mathbf{R} \cdot \mathbf{u}$;
ion heat flux	$\mathbf{q}_i = -\kappa_\|^i \nabla_\|(kT_i) - \kappa_\perp^i \nabla_\perp(kT_i) + \kappa_\wedge^i \mathbf{b} \times \nabla_\perp(kT_i)$;
ion thermal conductivities	$\kappa_\|^i = 3.9 \dfrac{nkT_i\tau_i}{m_i}$; $\quad \kappa_\perp^i = \dfrac{2nkT_i}{m_i\omega_{ci}^2\tau_i}$; $\quad \kappa_\wedge^i = \dfrac{5nkT_i}{2m_i\omega_{ci}}$;
electron heat flux	$\mathbf{q}_e = \mathbf{q}_\mathbf{u}^e + \mathbf{q}_T^e$;
frictional heat flux	$\mathbf{q}_\mathbf{u}^e = 0.71 nkT_e \mathbf{u}_\| + \dfrac{3nkT_e}{2\omega_{ce}\tau_e}\mathbf{b} \times \mathbf{u}_\perp$;

```
\input prolog \pageno=38
\hoffset=1.0truein\voffset=1.0truein\hsize=6.5truein\vsize=9.0truein
\def\quid{\hskip0.75em\relax}
\halign{\quad#\hfil\quad &$\displaystyle #$\hfil &$\displaystyle #$\hfil \cr
\noalign{\ssk}
thermal gradient &{\bf q}_T^e &=-\kappa_\parallel^e\nabla_\parallel
  (kT_e) - \kappa_\perp^e\nabla_\perp (kT_e) - \kappa_\wedge^e{\bf b}\times
  \nabla_\perp (kT_e); \cr
  \bs{1.5pt} heat flux \cr \noalign{\ssk}
electron thermal &\kappa_\parallel^e &=3.2{nkT_e\tau_e \over m_e}; \quid
  \kappa_\perp^e = 4.7{nkT_e \over m_e\omega_{ce}^{\ph{1}2}\tau_e}; \quid
  \kappa_\wedge^e = {5nkT_e \over 2m_e\omega_{ce}}; \cr
  \bs{6pt} conductivities \cr \noalign{\ssk}
stress tensor (both &P_{xx} &= -{\eta_0 \over 2}(W_{xx}+W_{yy})-{\eta_1 [illegible]
  (W_{xx}-W_{yy})-\eta_3W_{xy}; \cr \bs{4.5pt}
  species) \cr \noalign{\ssk}
&P_{yy} &= -{\eta_0 \over 2}(W_{xx}+W_{yy})+{\eta_1 \over 2}
  (W_{xx}-W_{yy})+\eta_3W_{xy}; \cr \noalign{\ssk}
&P_{xy} &= P_{yx} = -\eta_1W_{xy}+{\eta_3 \over 2}(W_{xx}-W_{yy}); \cr
  \noalign{\ssk}
&P_{xz} &= P_{zx} = -\eta_2W_{xz}-\eta_4W_{yz}; \cr \noalign{\ssk}
&P_{yz} &= P_{zy} = -\eta_2W_{yz}+\eta_4W_{xz}; \cr \noalign{\ssk}
&P_{zz} &= -\eta_0W_{zz} \cr
(here the $z$ axis is defined parallel to [illegible] \hidewidth \cr \noalign{\ssk}
ion viscosity&\eta_0^i &= 0.96nkT_i\tau_i; \quid \eta_1^i = {3nkT_i \over
  10\omega_{ci}^{\ph{2}2}\tau_i}; \quid \eta_2^i = {6nkT_i \over
  5\omega_{ci}^{\ph{1}2}\tau_i}; \cr
&\eta_3^i &={nkT_i \over 2\omega_{ci}}; \quid \eta_4^i={nkT_i \over
  \omega_{ci}}; \cr \noalign{\ssk}
electron viscosity&\eta_0^e &= 0.73nkT_e\tau_e; \quid \eta_1^e = [illegible]
  {nkT_e \over \omega_{ce}^{\ph{1}2}\tau_e}; \quid \eta_2^e = [illegible]nkT_e
  \over \omega_{ce}^{[illegible]}\tau_e}; \cr
&\eta_3^e &= -{nkT_e \over 2\omega_{ce}}; \quid \eta_4^e = -{nkT_e \over
  \omega_{ce}}; \cr [illegible]
[illegible] species the rate-of-strain tensor is defined as
$$W_{jk}={\partial v_j \over \partial x_k}+{\partial v_k \over \partial x_j}
  -{2\over3}\delta_{jk}\nabla\cdot{\bf v}.$$
[illegible] $B = 0$ the following simplifications occur:
$$ [illegible] {\bf u} = ne{\bf j}/\sigma_\parallel; \quad [illegible] \nabla (kT_e);
  \quad {\bf q}_i = -\kappa_\parallel^i\nabla (kT_i); $$
$$ [illegible]
  -\kappa_\parallel^e\nabla (kT_e); \quad [illegible] $$
[illegible] $\omega_{ce}\tau_e [illegible] \tau_i$, the electrons obey the
high-field expressions and the ions obey the zero-field expressions.
\indent
Collisional transport theory is applicable when (1) macroscopic time rates
of change satisfy $d/dt \ll 1/\tau$, where $\tau$ is the longest collisional
time scale, and (in the absence of a magnetic field) (2) macroscopic length
scales $L$ satisfy $L \gg l$, where $l = \bar v\tau$ is the mean
free path. In a strong field, $\omega_{ce}\tau \gg 1$, condition (2) is
replaced by $L_\parallel \gg l$ and $L_\perp \gg \sqrt{lr_e}$
($L_\perp \gg r_e$ in a uniform field).
\vfill\eject\end
```

thermal gradient heat flux
$$\mathbf{q}'_T = -\kappa'_{\parallel}\nabla_{\parallel}(kT_e) - \kappa'_{\perp}\nabla_{\perp}(kT_e) - \kappa'_{\wedge}\mathbf{b}\times\nabla_{\perp}(kT_e);$$

electron thermal conductivities
$$\kappa'_{\parallel} = 3.2\frac{nkT_e\tau_e}{m_e}; \quad \kappa'_{\perp} = 4.7\frac{nkT_e}{m_e\omega_{ce}^{\ 2}\tau_e}; \quad \kappa'_{\wedge} = \frac{5nkT_e}{2m_e\omega_{ce}};$$

stress tensor (both species)
$$P_{xx} = -\frac{\eta_0}{2}(W_{xx} + W_{yy}) - \frac{\eta_1}{2}(W_{xx} - W_{yy}) - \eta_3 W_{xy};$$
$$P_{yy} = -\frac{\eta_0}{2}(W_{xx} + W_{yy}) + \frac{\eta_1}{2}(W_{xx} - W_{yy}) + \eta_3 W_{xy};$$
$$P_{xy} = P_{yx} = -\eta_1 W_{xy} + \frac{\eta_3}{2}(W_{xx} - W_{yy});$$
$$P_{xz} = P_{zx} = -\eta_2 W_{xz} - \eta_4 W_{yz};$$
$$P_{yz} = P_{zy} = -\eta_2 W_{yz} + \eta_4 W_{xz};$$
$$P_{zz} = -\eta_0 W_{zz};$$

here the z axis is defined parallel to $\mathbf{B}$;

ion viscosity
$$\eta_0 = 0.96nkT_i\tau_i; \quad \eta_1 = \frac{3nkT_i}{10\omega_{ci}^{\ 2}\tau_i}; \quad \eta_2 = \frac{6nkT_i}{5\omega_{ci}^{\ 2}\tau_i};$$
$$\eta_3 = \frac{nkT_i}{2\omega_{ci}}; \quad \eta_4 = \frac{nkT_i}{\omega_{ci}};$$

electron viscosity
$$\eta'_0 = 0.73nkT_e\tau_e; \quad \eta'_1 = 0.51\frac{nkT_e}{\omega_{ce}^{\ 2}\tau_e}; \quad \eta'_2 = 2.0\frac{nkT_e}{\omega_{ce}^{\ 2}\tau_e};$$
$$\eta'_3 = -\frac{nkT_e}{2\omega_{ce}}; \quad \eta'_4 = -\frac{nkT_e}{\omega_{ce}}.$$

For both species the rate-of-strain tensor is defined as

$$W_{jk} = \frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\nabla\cdot\mathbf{v}.$$

When $\mathbf{B} = 0$ the following simplifications occur:

$$\mathbf{R}_{\mathbf{u}} = ne\mathbf{j}/\sigma_{\parallel}; \quad \mathbf{R}_T = -0.71n\nabla(kT_e); \quad \mathbf{q}_i = -\kappa_{\parallel}^i\nabla(kT_i);$$

$$\mathbf{q}'_{\mathbf{u}} = 0.71nkT_e\mathbf{u}; \quad \mathbf{q}'_T = -\kappa'_{\parallel}\nabla(kT_e); \quad P_{jk} = -\eta_0 W_{jk}.$$

For $\omega_{ce}\tau_e \gg 1 \gg \omega_{ci}\tau_i$, the electrons obey the high-field expressions and the ions obey the zero field expressions. Collisional transport theory is applicable when (1) macroscopic time rates of change satisfy $d/dt \ll 1/\tau$, where τ is the longest collisional time scale, and (in the absence of a magnetic field) (2) macroscopic length scales L satisfy $L \gg l$, where $l = v\tau$ is the mean free path. In a strong field, $\omega_{ce}\tau \gg 1$, condition (2) is replaced by $L_{\parallel} \gg l$ and $L_{\perp} \gg \sqrt{lr_e}$ ($L_{\perp} \gg r_e$ in a uniform field).

```
[illegible]put prolog
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[illegible]39
[illegible]ere $L_\parallel$ is a ma[illegible]scale parallel
[illegible] field [illegible] and $L[illegible] is the smaller of
[illegible]nabla_\perp B [illegible] the transverse plasma dimension.  In
[illegible], the standard [illegible]efficients are valid only when (3) the
[illegible] logarithm satisfie[illegible] \gg 1$, (4) the electron gyroradius
[illegible], (5) relative drift
[illegible] between [illegible] are small [illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]alpha [illegible]parallel
[illegible]alpha}^2/({[illegible]alpha}^2 + [illegible]mega_{c alpha}[illegible]ph{1}2}), [illegible]
[illegible]alpha wedge [illegible]alpha,\parallel \nu_ alpha [illegible]mega_{c\alpha}
[illegible]alpha}^2 + [illegible]alpha}^{[illegible]ph{1}2}). \cr}}}
[illegible]$ [illegible]perp$ and $ [illegible]wedge$ are the Pedersen and Hall
[illegible], respectively.
[illegible]ject\end
```

where $L_{\|}$ is a macroscopic scale parallel to the field $\mathbf{B}$ and $L_{\perp}$ is the smaller of $B/\mid \nabla_{\perp} B \mid$ and the transverse plasma dimension. In addition, the standard transport coefficients are valid only when (3) the Coulomb logarithm satisfies $\lambda \gg 1$; (4) the electron gyroradius satisfies $r_e \gg \lambda_D$, or $8\pi n_e m_e c^2 \gg B^2$; (5) relative drifts $\mathbf{u} = \mathbf{v}_\alpha - \mathbf{v}_\beta$ between two species are small compared with the thermal velocities, i.e., $u^2 \ll kT_\alpha/m_\alpha$, kT_β/m_β; and (6) anomalous transport processes owing to microinstabilities are negligible.

Weakly Ionized Plasmas

Collision frequency for scattering of charged particles of species α by neutrals is

$$\nu_\alpha = n_0 \sigma_s^{\alpha/0} (kT_\alpha/m_\alpha)^{1/2},$$

where n_0 is the neutral density and $\sigma_s^{\alpha/0}$ is the cross section, typically $\sim 5 \times 10^{-15}$ cm^2 and weakly dependent on temperature.

When the system is small compared with a Debye length, $L \ll \lambda_D$, the charged particle diffusion coefficients are

$$D_\alpha = kT_\alpha/m_\alpha \nu_\alpha.$$

In the opposite limit, both species diffuse at the ambipolar rate

$$D_A = \frac{\mu_i D_e - \mu_e D_i}{\mu_i - \mu_e} = \frac{(T_e + T_i) D_i D_e}{T_e D_i + T_i D_e},$$

where $\mu_\alpha = e_\alpha/m_\alpha \nu_\alpha$ is the mobility. The conductivity σ_α satisfies $\sigma_\alpha = n_\alpha e_\alpha \mu_\alpha$.

In the presence of a magnetic field $\mathbf{B}$ the scalars μ and σ become tensors,

$$\mathbf{J}^\alpha = \boldsymbol{\sigma}^\alpha \cdot \mathbf{E} = \sigma_{\|}^\alpha \mathbf{E}_{\|} + \sigma_{\perp}^\alpha \mathbf{E}_{\perp} + \sigma_{\wedge}^\alpha \mathbf{E} \times \mathbf{b},$$

where $\mathbf{b} = \mathbf{B}/B$ and

$$\sigma_{\|}^\alpha = n_\alpha e_\alpha{}^2/m_\alpha \nu_\alpha;$$

$$\sigma_{\perp}^\alpha = \sigma_{\|}^\alpha \nu_\alpha{}^2/(\nu_\alpha{}^2 + \omega_{c\alpha}^2);$$

$$\sigma_{\wedge}^\alpha = \sigma_{\|}^\alpha \nu_\alpha \omega_{c\alpha}/(\nu_\alpha{}^2 + \omega_{c\alpha}^2).$$

Here $\sigma_{\perp}$ and $\sigma_{\wedge}$ are the Pedersen and Hall conductivities, respectively.

```
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\centerline{\headfont APPROXIMATE MAGNITUDES}
\vskip1pt
\centerline{\headfont IN SOME TYPICAL PLASMAS}
\bsk  %  BEGINNING OF TABLE.
$$\vbox{\offinterlineskip \def\trule{\noalign{\hrule}}
\tabskip=0pt \halign to \hsize{
\vrule# \tabskip=3pt minus2pt&\strut #\hfil &\vrule #&\hfil$#$\hfil
      &\vrule #&\hfil$#$\hfil &\vrule #&\hfil$#$\hfil
      &\vrule #&\hfil$#$\hfil &\vrule #&\hfil$#$\hfil
      &\vrule #&\hfil$#$\hfil &\vrule #\tabskip=0pt\cr\trule
\tskc{7}{4pt}
&\hfil Plasma Type|n\ {\rm cm}^{-3}|T\ {\rm eV}|\omega_{pe}\ {\rm sec}^{-1}
|\lambda_D\ {\rm cm}|n{\lambda_D}^3|v_{ei}\ {\rm sec}^{-1}&\cr
\tskc{7}{4pt} \trule \tskc{7}{1pt} \trule \tskc{7}{4pt}

&Interstellar gas |1 |1 |6\times10^{4\ph{0}} |\om$7\times10^{2}$\hfil |
  4\times10^{8}|7\times10^{-5} &\cr \tskc{7}{3pt}
&Gaseous nebula   |10^3 |1 |2\times10^{6\ph{0}} |20 |10^7 |
  6\times10^{-2}& \cr \tskc{7}{3pt}
&Solar Corona |10^6 |10^2 |6\times10^{7\ph{0}} |7 |4\times10^{8} |
  6\times 10^{-2} &\cr \tskc{7}{3pt}
&Diffuse hot plasma |10^{12} |10^2 |6\times10^{10} |7\times10^{-3} |
  4\times10^{5} |40 &\cr \tskc{7}{3pt}
&Solar atmosphere, |10^{14} |1 |6\times10^{11} |7\times10^{-5} |40 |
  2\times10^{9\ph{-}} &\cr
  &\hskip 1em gas discharge ||||||&\cr \tskc{7}{3pt}
&Warm plasma |10^{14} |[illegible] |6\times10^{11} |2\times10^{-4} |10^3 |10^7&\cr
  \tskc{7}{3pt}
&Hot plasma |10^{14} |10^2 |6\times10^{11} |7\times10^{-4} |
  4\times10^{4}|4\times10^{6\ph{-}} &\cr \tskc{7}{3pt}
&Thermonuclear |10^{15} |10^4 |2\times10^{12} |2\times10^{-3} |10^7 |
  \times10^{4\ph{-}} &\cr
  &\hskip 1em plasma ||||||&\cr \tskc{7}{3pt}
&Theta pinch |10^{16} |10^2 |6\times10^{12} |7\times10^{-5} |
  4\times10^{3}|2\times10^{[illegible]\ph{-}} &\cr \tskc{7}{3pt}
&Dense hot plasma |10^{18} |10^2 |6\times10^{13} |7\times10^{-6} |
  4\times10^{2} |2\times10^{10\ph{.}} &\cr \tskc{7}{3pt}
&Laser Plasma |10^{20} |10^2 |6\times10^{14} |7\times10^{-7} |40 |
  2\times10^{12\ph{.}} &\cr \tskc{7}{4pt}\trule}}$$
\bsk  %  END OF TABLE.
The diagram (facing) gives comparable information in graphical form.$^{22}$
\vfill\eject\end
```
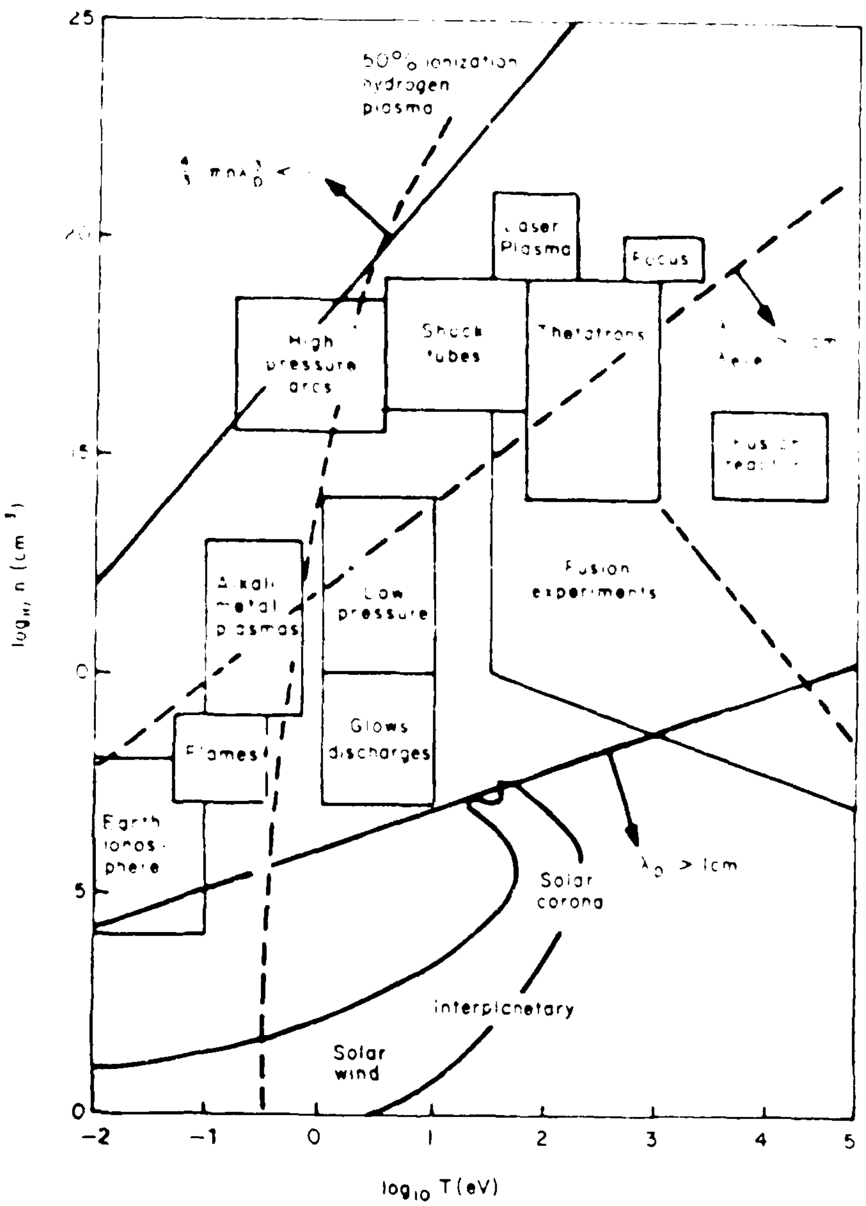

APPROXIMATE MAGNITUDES IN SOME TYPICAL PLASMAS

Plasma Type	n cm^{-3}	T eV	ω_{pe} sec^{-1}	λ_D cm	$n\lambda_D{}^3$	ν_{ei} sec^{-1}
Interstellar gas	1	1	6×10^4	7×10^2	4×10^8	7×10^{-5}
Gaseous nebula	10^3	1	2×10^6	20	10^7	6×10^{-2}
Solar Corona	10^6	10^2	6×10^7	7	4×10^8	6×10^{-2}
Diffuse hot plasma	10^{12}	10^2	6×10^{10}	7×10^{-3}	4×10^5	40
Solar atmosphere, gas discharge	10^{14}	1	6×10^{11}	7×10^{-5}	40	2×10^9
Warm plasma	10^{14}	10	6×10^{11}	2×10^{-4}	10^3	10^7
Hot plasma	10^{14}	10^2	6×10^{11}	7×10^{-4}	4×10^4	4×10^6
Thermonuclear plasma	10^{15}	10^4	2×10^{12}	2×10^{-3}	10^7	5×10^4
Theta pinch	10^{16}	10^2	6×10^{12}	7×10^{-5}	4×10^3	3×10^8
Dense hot plasma	10^{18}	10^2	6×10^{13}	7×10^{-6}	4×10^2	2×10^{10}
Laser Plasma	10^{20}	10^2	6×10^{14}	7×10^{-7}	40	2×10^{12}

The diagram (facing) gives comparable information in graphical form.[22]

The next page is an inserted figure



50% ionization
hydrogen
plasma
Laser
Plasma
Focus
High
pressure
arcs
Shock
tubes
Thetatrons
Alkali
metal
plasmas
Low
pressure
Fusion
experiments
Flames
Glows
discharges
Earth
ionos-
phere
Solar
corona
$\lambda_D > 1cm$
interplanetary
Solar
wind
$\log_{10} T(eV)$

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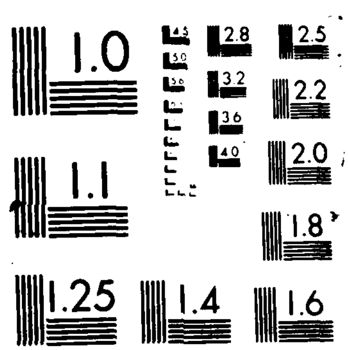
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

IONOSPHERIC PARAMETERS[23]

The following tables give average nighttime values. Where two numbers are entered, the first refers to the lower and the second to the upper portion of the layer.

Quantity	E Region	F Region
Altitude (km)	90–160	160–500
Number density (m^{-3})	1.5×10^{10}–3.0×10^{10}	5×10^{10}–2×10^{11}
Height-integrated number density (m^{-2})	9×10^{14}	4.5×10^{15}
Ion-neutral collision frequency (sec^{-1})	2×10^{3}–10^{2}	0.5–0.05
Ion gyro-/collision frequency ratio κ_i	0.09–2.0	4.6×10^{2}–5.0×10^{3}
Ion Pederson factor $\kappa_i/(1+\kappa_i{}^2)$	0.09–0.5	2.2×10^{-3}–2×10^{-4}
Ion Hall factor $\kappa_i{}^2/(1+\kappa_i{}^2)$	8×10^{-4}–0.8	1.0
Electron-neutral collision frequency	1.5×10^{4}–9.0×10^{2}	80–10
Electron gyro-/collision frequency ratio κ_e	4.1×10^{2}–6.9×10^{3}	7.8×10^{4}–6.2×10^{5}
Electron Pedersen factor $\kappa_e/(1+\kappa_e{}^2)$	2.7×10^{-3}–1.5×10^{-4}	10^{-5}–1.5×10^{-6}
Electron Hall factor $\kappa_e{}^2/(1+\kappa_e{}^2)$	1.0	1.0
Mean molecular weight	28–26	22–16
Ion gyrofrequency (sec^{-1})	180–190	230–300
Neutral diffusion coefficient ($m^2\ sec^{-1}$)	30–5×10^{3}	10^{5}

The terrestrial magnetic field in the lower ionosphere at equatorial lattitudes is approximately $B_0 = 0.35 \times 10^{-4}$ tesla. The earth's radius is $R_E = 6371$ km.

```
\input prolog \pageno=43
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\centerline{{\headfont SOLAR PHYSICS PARAMETERS}$^{24}$} \msk
\vbox{\offinterlineskip \tabskip=0pt \halign to\hsize{
\vrule#\tabskip=3pt minus2pt&\strut#\hfil&\vrule#&\hfil$#$\hfil&\vrule#
&$#$\hfil&\vrule#&\hfil#\hfil&\vrule#\tabskip=0pt\cr\trule \tskc{4}{2pt}
&\hfil Parameter|\hbox{Symbol}|\hfil\hbox{Value}|Units&\cr
\tskc{4}{2pt} \trule \tskc{4}{1pt} \trule \tskc{4}{2pt}
&Total mass|M_\odot|1.99\times10^{33}|g&\cr \tskc{4}{1.25pt}
&Radius|R_\odot|6.96\times10^{10}|cm&\cr \tskc{4}{1.25pt}
&Surface gravity|g_\odot|2.74\times10^4|cm\ts s$^{-2}$&\cr \tskc{4}{1.25pt}
&Escape speed|v_\infty|6.18\times10^7|cm\ts s$^{-1}$&\cr \tskc{4}{1.25pt}
&Upward mass flux in spicules|\hbox{---}|1.6\times10^{-9}|g\ts cm$^{-2}$
  \ts s$^{-1}$&\cr \tskc{4}{1.25pt}
&Vertically integrated atmospheric density|\hbox{---}|\om\hfil4.28\hfil|
  g\ts cm$^{-2}$&\cr \tskc{4}{1.25pt}
&Sunspot magnetic field strength|B_{\rm max}|\om\hfil2500\hbox{--}3500\hfil|
  G&\cr \tskc{4}{1.25pt}
&Surface temperature|T_0|\om\hfil6420\hfil|K&\cr \tskc{4}{1.25pt}
&Radiant power|\cal L_\odot|3.90\times10^{33}|erg\ts s$^{-1}$&\cr \tskc{4}{1.25pt}
&Radiant flux density|\cal F|6.41\times10^{10}|erg\ts cm$^{-2}$s$^{-1}$&\cr
  \tskc{4}{1.25pt}
&Optical depth at 500\ts nm, measured|\tau_{500}|\om\hfil0.99\hfil| \hbox{---}&\cr
  &\hskip 1em from photosphere|||&\cr \tskc{4}{1.25pt}
&Astronomical unit (radius of earth's orbit)|\rm AU|1.50\times10^{13}|cm&\cr
  \tskc{4}{1.25pt}
&Solar constant (intensity at 1\ts AU)|f|1.39\times10^6|
  erg\ts cm$^{-2}$\ts s$^{-1}$&\cr \tskc{4}{2pt}\trule}} \msk
{\headfont Chromosphere and Corona}$^{25}$ \ssk
\vbox{\offinterlineskip\tabskip=0pt\halign to\hsize{
\vrule#\tabskip=3pt minus2pt&\strut#\hfil&\vrule#&\hfil$#$\hfil&\vrule#
&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#\tabskip=0pt\cr\trule \tskc{4}{2pt}
&\om|\hbox{Quiet}|\hbox{Coronal}|\hbox{Active}&\cr \bs{4pt}
&\hfil Parameter (Units)|\om|\om|\om&\cr \bs{4pt}
&\om|\hbox{Sun}|\hbox{Hole}|\hbox{Region}&\cr
\tskc{4}{2pt} \trule \tskc{4}{1pt} \trule \tskc{4}{2pt}
&Chromospheric radiation losses|||&\cr
&\hskip 1em (erg\ts cm$^{-2}$\ts s$^{-1}$)|||&\cr \tskc{4}{1pt}
&\hskip 2em Low chromosphere|2\times10^6|2\times10^6|\approxgt10^7&\cr \tskc{4}{1pt}
&\hskip 2em Middle chromosphere|2\times10^6|2\times10^6|10^7&\cr \tskc{4}{1pt}
&\hskip 2em Upper chromosphere|3\times10^5|3\times10^5|2\times10^6&\cr \tskc{4}{1pt}
&\hskip 2em Total|4\times10^6|4\times10^6|\approxgt2\times10^7&\cr \tskc{4}{1.5pt}
&Transition layer pressure (dyne\ts cm$^{-2}$)|0.2|0.07|2&\cr \tskc{4}{1.5pt}
&Coronal temperature (K) at 1.1\ts R$_\odot$|1.1\hbox{--}1.6\times10^6|10^6|
  2.5\times10^6&\cr \tskc{4}{1.5pt}
&Coronal energy losses (erg\ts cm$^{-2}$\ts s$^{-1}$)|||&\cr \tskc{4}{1pt}
&\hskip 2em Conduction|2\times10^5|6\times10^4|10^5\hbox{--}10^7&\cr \tskc{4}{1pt}
&\hskip 2em Radiation|10^5|10^4|5\times10^6&\cr \tskc{4}{1pt}
&\hskip 2em Solar Wind|\approxlt 5\times10^4|7\times10^5|<10^5&\cr \tskc{4}{1pt}
&\hskip 2em Total|3\times10^5|8\times10^5|10^7&\cr \tskc{4}{1.5pt}
&Solar wind mass loss (g\ts cm$^{-2}$\ts s$^{-1}$)|\approxlt2\times10^{-11}|
  2\times10^{-10}|<4\times10^{-11}&\cr \tskc{4}{2pt}\trule}}
\vfill\eject\end
```

SOLAR PHYSICS PARAMETERS[24]

Parameter	Symbol	Value	Units
Total mass	$M_{\odot}$	1.99×10^{33}	g
Radius	$R_{\odot}$	6.96×10^{10}	cm
Surface gravity	$g_{\odot}$	2.74×10^{4}	$\mathrm{cm\,s^{-2}}$
Escape speed	v_{∞}	6.18×10^{7}	$\mathrm{cm\,s^{-1}}$
Upward mass flux in spicules	—	1.6×10^{-9}	$\mathrm{g\,cm^{-2}\,s^{-1}}$
Vertically integrated atmospheric density	—	4.28	$\mathrm{g\,cm^{-2}}$
Sunspot magnetic field strength	$B_{\max}$	2500–3500	G
Surface temperature	T_0	6420	K
Radiant power	$\mathcal{L}_{\odot}$	3.90×10^{33}	$\mathrm{erg\,s^{-1}}$
Radiant flux density	$\mathcal{F}$	6.41×10^{10}	$\mathrm{erg\,cm^{-2}s^{-1}}$
Optical depth at 500 nm, measured from photosphere	τ_{500}	0.99	-
Astronomical unit (radius of earth's orbit)	AU	1.50×10^{13}	cm
Solar constant (intensity at 1 AU)	f	1.39×10^{6}	$\mathrm{erg\,cm^{-2}\,s^{-1}}$

Chromosphere and Corona[25]

Parameter (Units)	Quiet Sun	Coronal Hole	Active Region
Chromospheric radiation losses ($\mathrm{erg\,cm^{-2}\,s^{-1}}$)			
Low chromosphere	2×10^{6}	2×10^{6}	$\gtrsim 10^{7}$
Middle chromosphere	2×10^{6}	2×10^{6}	10^{7}
Upper chromosphere	3×10^{5}	3×10^{5}	2×10^{6}
Total	4×10^{6}	4×10^{6}	$\gtrsim 2 \times 10^{7}$
Transition layer pressure ($\mathrm{dyne\,cm^{-2}}$)	0.2	0.07	2
Coronal temperature (K) at 1.1 R .	1.1 1.6×10^{6}	10^{6}	2.5×10^{6}
Coronal energy losses ($\mathrm{erg\,cm^{-2}\,s^{-1}}$)			
Conduction	2×10^{5}	6×10^{4}	10^{5} 10^{7}
Radiation	10^{5}	10^{4}	5×10^{6}
Solar Wind	$\lesssim 5 \times 10^{4}$	7×10^{5}	$< 10^{5}$
Total	3×10^{5}	8×10^{5}	10^{7}
Solar wind mass loss ($\mathrm{g\,cm^{-2}\,s^{-1}}$)	$\lesssim 2 \times 10^{-11}$	2×10^{-10}	$< 4 \times 10^{-11}$

```
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\vsize=9.0truein
\pageno=44
\centerline{{\headfont THERMONUCLEAR FUSION}$^{26}$}
\bigskip
Natural abundance of isotopes:
\smallskip\nointerlineskip
$$\vbox{\halign{#\qquad&$#$\hfil\cr
hydrogen&n_D^{\vphantom{3}}/n_H^{\vphantom{3}}=1.5\times10^{-4}\cr
helium&n_{\rm He^3}/n_{\rm He^4}=1.3\times10^{-6}\cr
\noalign{\vskip1.5pt}
lithium&n_{\rm Li^6}/n_{\rm Li^7}=0.08\cr}}$$
\halign{#\qquad\qquad&$#$\hfil&$#$\hfil \cr
Mass ratios: &m_e/m_D           &=2.72\times10^{-4} = 1/3670 \cr
             &(m_e/m_D)^{1/2}  &=1.65\times10^{-2} = 1/60.6 \cr
             &m_e/m_T           &=1.82\times10^{-4} = 1/5496 \cr
             &(m_e/m_T)^{1/2}  &=1.35\times10^{-2} = 1/74.1 \cr}
\medskip
Absorbed radiation dose is measured in rads: 1 rad = 10$^2$\ts erg\ts g$^{-1}$.
The curie (abbreviated Ci) is a measure of radioactivity:  1 curie =
$3.7\times10^{10}$\ts counts\ts sec$^{-1}$.
\medskip
Fusion reactions (branching ratios are correct for energies near the cross
section peaks; a negative yield means the reaction is endothermic):$^{27}$
\smallskip{\def\yields#1{\lower4.5pt\hbox{$\overrightarrow{\ph{0}#1\ph{0}}$}}
   %  THE \yields MACRO IS USED TO DRAW THE 'YIELDS' SYMBOL '--->' WITH
   %  A NUMBER UNDER IT.
\halign{\indent#\quad&#\quad&#\hfil&#\hfil&#\hfil\cr
&(1a)&D + D&\yields{50\%}&T(1.01\ts MeV) + p(3.02\ts MeV)\cr
&(1b)&     &\yields{50\%}&He$^3$(0.82\ts MeV) + n(2.45\ts MeV)\cr
&(2) &D + T&\yields{\ph{50\%}}&He$^4$(3.5\ts MeV) + n(14.1\ts MeV)\cr
&(3) &D + He$^3$&\yields{\ph{50\%}}&He$^4$(3.6\ts MeV) + p(14.7\ts MeV)\cr
&(4) &T + T &\yields{\ph{50\%}}&He$^4$ + 2n + 11.3\ts MeV\cr
&(5a)&He$^3$ + T&\yields{51\%}&He$^4$ + p + n + 12.1\ts MeV\cr
&(5b)&          &\yields{43\%}&He$^4$(4.8\ts MeV) + D(9.5\ts MeV)\cr
&(5c)&          &\yields{\ph{5}6\%}&He$^5$(2.4\ts MeV) + p(11.9\ts MeV)\cr
&(6) &p + Li$^6$&\yields{\ph{50\%}}&He$^4$(1.7\ts MeV) + He$^3$(2.3\ts MeV)\cr
&(7a)&p + Li$^7$&\yields{20\%}&2 He$^4$ + 17.3\ts MeV\cr
&(7b)&          &\yields{80\%}&Be$^7$ + n $-$ 1.6\ts MeV\cr
&(8) &D + Li$^6$&\yields{\ph{50\%}}&$2$He$^4$ + 22.4\ts MeV\cr
&(9) &p + B$^{11}$&\yields{\ph{50\%}}&3 He$^4$ + 8.7\ts MeV\cr
&(10)&n + Li$^6$&\yields{\ph{50\%}}&He$^4$(2.1\ts MeV) + T(2.7\ts MeV)\cr}}
\medskip
The total cross section in barns as a function of $E$, the energy in keV
of the incident particle [the first ion on the left side of Eqs. (1)--(5)],
assuming the target ion at rest, can be fitted by$^{28}$
$$\sigma_T(E)={A_5+\left[(A_4-A_3E)^2+1\right]^{-1}A_2\over E\left[\exp(A_1
E^{-1/2})-1\right]}$$
\vfill\eject\end
```

THERMONUCLEAR FUSION[26]

Natural abundance of isotopes:

hydrogen	$n_D/n_H = 1.5 \times 10^{-4}$
helium	$n_{\mathrm{He}^3}/n_{\mathrm{He}^4} = 1.3 \times 10^{-6}$
lithium	$n_{\mathrm{Li}^6}/n_{\mathrm{Li}^7} = 0.08$

Mass ratios:

$$m_e/m_D = 2.72 \times 10^{-4} = 1/3670$$
$$(m_e/m_D)^{1/2} = 1.65 \times 10^{-2} = 1/60.6$$
$$m_e/m_T = 1.82 \times 10^{-4} = 1/5496$$
$$(m_e/m_T)^{1/2} = 1.35 \times 10^{-2} = 1/74.1$$

Absorbed radiation dose is measured in rads: 1 rad $= 10^2$ erg g^{-1}. The curie (abbreviated Ci) is a measure of radioactivity: 1 curie $= 3.7 \times 10^{10}$ counts sec^{-1}.

Fusion reactions (branching ratios are correct for energies near the cross section peaks; a negative yield means the reaction is endothermic):[27]

(1a)	D + D	$\xrightarrow[50\%]{}$ T(1.01 MeV) + p(3.02 MeV)
(1b)		$\xrightarrow[50\%]{}$ He^3(0.82 MeV) + n(2.45 MeV)
(2)	D + T	$\longrightarrow$ He^4(3.5 MeV) + n(14.1 MeV)
(3)	D + He^3	$\longrightarrow$ He^4(3.6 MeV) + p(14.7 MeV)
(4)	T + T	$\longrightarrow$ He^4 + 2n + 11.3 MeV
(5a)	He^3 + T	$\xrightarrow[51\%]{}$ He^4 + p + n + 12.1 MeV
(5b)		$\xrightarrow[43\%]{}$ He^4(4.8 MeV) + D(9.5 MeV)
(5c)		$\xrightarrow[6\%]{}$ He^5(2.4 MeV) + p(11.9 MeV)
(6)	p + Li^6	$\longrightarrow$ He^4(1.7 MeV) + He^3(2.3 MeV)
(7a)	p + Li^7	$\xrightarrow[20\%]{}$ 2 He^4 + 17.3 MeV
(7b)		$\xrightarrow[80\%]{}$ Be^7 + n − 1.6 MeV
(8)	D + Li^6	$\longrightarrow$ 2He^4 + 22.4 MeV
(9)	p + B^{11}	$\longrightarrow$ 3 He^4 + 8.7 MeV
(10)	n + Li^6	$\longrightarrow$ He^4(2.1 MeV) + T(2.7 MeV)

The total cross section in barns as a function of E, the energy in keV of the incident particle [the first ion on the left side of Eqs. (1)–(5)], assuming the target ion at rest, can be fitted by[28]

$$\sigma_T(E) = \frac{A_5 + \left[(A_4 - A_3 E)^2 + 1\right]^{-1} A_2}{E\left[\exp(A_1 E^{-1/2}) - 1\right]}$$

The listing for page 45 begins on the next page.

```
\input prolog
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\pageno=45
\nointerlineskip
where the Duane coefficients $A_j$ for the principle fusion reactions are
as follows:
\ssk  % BEGINNING OF FIRST TABLE.
$$\vbox{\offinterlineskip\tabskip=0pt\halign to\hsize{\vrule#\tabskip=3pt plus
2pt minus2pt&\strut\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil
&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil
&\vrule#&\hfil$#$\hfil&\vrule#\tabskip=0pt\cr \trule \tskc{7}{2pt}
|&{}$D--D${}|{}$D--D${}|{}$D--T${}|{}$D--He$^3|{}$T--T${}|{}$T--He$^3&\cr
|&(1{\rm a})|(1{\rm b})|(2)|(3)|(4)|\null$(5a--c)$\null&\cr
\tskc{7}{2pt} \trule \tskc{7}{1pt} \trule \tskc{7}{2pt}
&A_1|46.097|47.88|45.95|89.27|38.39|123.1&\cr \tskc{7}{1pt}
&A_2|372|482|50200|25900|448|11250&\cr \tskc{7}{1pt}
&A_3|4.36\times10^{-4}|3.08\times10^{-4}|1.368\times10^{-2}
  |3.98\times10^{-3}|1.02\times10^{-3}|0&\cr \tskc{7}{1pt}
&A_4|1.220|1.177|1.076|1.297|2.09|0&\cr \tskc{7}{1pt}
&A_5|0|0|409|647|0|0&\cr \tskc{7}{2pt} \trule}}$$
\msk

Reaction rates $\overline{\sigma v}$ (in cm$^3\,$sec$^{-1}$), averaged over
Maxwellian distributions:
\ssk  % BEGINNING OF SECOND TABLE.
$$\vbox{\offinterlineskip\tabskip=0pt\halign to\hsize{\vrule#\tabskip=3pt plus2pt
minus2pt&\strut\hfil$#$&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil
&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil
&\vrule#\tabskip=0pt\cr \trule \tskc{6}{2pt}
&{\rm Temperature}\hfil|{}$D--D${}|{}$D--T${}|{}$D--He$^3
|{}$T--T${}|{}$T--He$^3&\cr
&{\rm (keV)}\hfil|{\rm (1a+1b)}|(2)|(3)|(4)|{}$(5a--c)${}&\cr
\tskc{6}{2pt} \trule \tskc{6}{1pt} \trule \tskc{6}{2pt}
&1.0\qquad|1.5\times10^{-22}|5.5\times10^{-21}|10^{-26}
  |3.3\times10^{-22}|10^{-28}&\cr \tskc{6}{1pt}
&2.0\qquad|5.4\times10^{-21}|2.6\times10^{-19}|1.4\times10^{-23}
  |7.1\times10^{-21}|10^{-25}&\cr \tskc{6}{1pt}
&5.0\qquad|1.8\times10^{-19}|1.3\times10^{-17}|6.7\times10^{-21}
  |1.4\times10^{-19}|2.1\times10^{-22}&\cr \tskc{6}{1pt}
&10.0\qquad|1.2\times10^{-18}|1.1\times10^{-16}|2.3\times10^{-19}
  |7.2\times10^{-19}|1.2\times10^{-20}  &\cr \tskc{6}{1pt}
&20.0\qquad|5.2\times10^{-18}|4.2\times10^{-16}|3.8\times10^{-18}
  |2.5\times10^{-18}|2.6\times10^{-19}&\cr \tskc{6}{1pt}
&50.0\qquad|2.1\times10^{-17}|8.7\times10^{-16}|5.4\times10^{-17}
  |8.7\times10^{-18}|5.3\times10^{-18}&\cr \tskc{6}{1pt}
&100.0\qquad|4.5\times10^{-17}|8.5\times10^{-16}|1.6\times10^{-16}
  |1.9\times10^{-17}|2.7\times10^{-17}&\cr \tskc{6}{1pt}
&200.0\qquad|8.8\times10^{-17}|6.3\times10^{-16}|2.4\times10^{-16}
  |4.2\times10^{-17}|9.2\times10^{-17}&\cr \tskc{6}{1pt}
&500.0\qquad|1.8\times10^{-16}|3.7\times10^{-16}|2.3\times10^{-16}
  |8.4\times10^{-17}|2.9\times10^{-16}&\cr \tskc{6}{1pt}
&1000.0\qquad|2.2\times10^{-16}|2.7\times10^{-16}|1.8\times10^{-16}
  |[illegible]\times10^{-17}|5.2\times10^{-16}&\cr \tskc{6}{2pt}\trule}}$$
\smallskip
For low energies ($T \approx 25\,$keV) the data may be represented by
\smallskip \nointerlineskip
```

```
$$(\overline{\sigma v})_{DD}=2.33\times 10^{-14} T^{-2/3} \exp(-18.76 T^{-1/3})
\,{\rm cm}^3\,{\rm sec}^{-1};$$
$$(\overline{\sigma v})_{DT}=3.68\times 10^{-12} T^{-2/3} \exp(-19.94 T^{-1/3})
\,{\rm cm}^3\,{\rm sec}^{-1},$$
where $T$ is measured in keV.
\medskip
The power density released in the form of charged particles is
\smallskip
\indent $P_{DD}=3.3\times10^{-13}{n_D}^2(\overline{\sigma v})_{DD}$\ts
watt\ts cm$^{-3}$ (including the subsequent

\hskip4.625truein D--T reaction);

\indent $P_{DT}=5.6\times10^{-13}n_D n_T(\overline{\sigma v})_{DT}$\ts
watt\ts cm$^{-3}$;
\medskip
\indent $P_{{D\rm He}^3}=2.9\times10^{-12}n_{D^{\vphantom{3}}}n_{\rm He^3}
(\overline{\sigma v})_{D{\rm He}^3}$\ts watt\ts cm$^{-3}$.
\vfill\eject\end
```

where the Duane coefficients A_j for the principle fusion reactions are as follows:

	D–D (1a)	D–D (1b)	D–T (2)	D–He^3 (3)	T–T (4)	T–He^3 (5a–c)
A_1	46.097	47.88	45.95	89.27	38.39	123.1
A_2	372	482	50200	25900	448	11250
A_3	4.36×10^{-4}	3.08×10^{-4}	1.368×10^{-2}	3.98×10^{-3}	1.02×10^{-3}	0
A_4	1.220	1.177	1.076	1.297	2.09	0
A_5	0	0	409	647	0	0

Reaction rates $\overline{\sigma v}$ (in cm^3 sec^{-1}), averaged over Maxwellian distributions:

Temperature (keV)	D–D (1a + 1b)	D–T (2)	D–He^3 (3)	T–T (4)	T–He^3 (5a–c)
1.0	1.5×10^{-22}	5.5×10^{-21}	10^{-26}	3.3×10^{-22}	10^{-28}
2.0	5.4×10^{-21}	2.6×10^{-19}	1.4×10^{-23}	7.1×10^{-21}	10^{-25}
5.0	1.8×10^{-19}	1.3×10^{-17}	6.7×10^{-21}	1.4×10^{-19}	2.1×10^{-22}
10.0	1.2×10^{-18}	1.1×10^{-16}	2.3×10^{-19}	7.2×10^{-19}	1.2×10^{-20}
20.0	5.2×10^{-18}	4.2×10^{-16}	3.8×10^{-18}	2.5×10^{-18}	2.6×10^{-19}
50.0	2.1×10^{-17}	8.7×10^{-16}	5.4×10^{-17}	8.7×10^{-18}	5.3×10^{-18}
100.0	4.5×10^{-17}	8.5×10^{-16}	1.6×10^{-16}	1.9×10^{-17}	2.7×10^{-17}
200.0	8.8×10^{-17}	6.3×10^{-16}	2.4×10^{-16}	4.2×10^{-17}	9.2×10^{-17}
500.0	1.8×10^{-16}	3.7×10^{-16}	2.3×10^{-16}	8.4×10^{-17}	2.9×10^{-16}
1000.0	2.2×10^{-16}	2.7×10^{-16}	1.8×10^{-16}	8.0×10^{-17}	5.2×10^{-16}

For low energies ($T \lesssim 25$ keV) the data may be represented by

$$(\overline{\sigma v})_{DD} = 2.33 \times 10^{-14} T^{-2/3} \exp(-18.76 T^{-1/3}) \text{ cm}^3 \text{ sec}^{-1};$$

$$(\overline{\sigma v})_{DT} = 3.68 \times 10^{-12} T^{-2/3} \exp(-19.94 T^{-1/3}) \text{ cm}^3 \text{ sec}^{-1},$$

where T is measured in keV.

The power density released in the form of charged particles is

$$P_{DD} = 3.3 \times 10^{-13} n_D{}^2 (\overline{\sigma v})_{DD} \text{ watt cm}^{-3} \text{ (including the subsequent D–T reaction);}$$

$$P_{DT} = 5.6 \times 10^{-13} n_D n_T (\overline{\sigma v})_{DT} \text{ watt cm}^{-3};$$

$$P_{DHe^3} = 2.9 \times 10^{-12} n_D n_{He^3} (\overline{\sigma v})_{DHe^3} \text{ watt cm}^{-3}.$$

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=46
\centerline{\headfont RELATIVISTIC ELECTRON BEAMS}
\medskip
\indent
Here $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic scaling factor;
quantities in analytic formulas are expressed in SI or cgs units, as indicated;
in numerical formulas, $I$ is in amperes (A), $B$ is in gauss (G), electron
linear density $N$ is in cm$^{-1}$, and temperature, voltage and energy are
in MeV; $\beta_z = v_z/c$; $k$ is Boltzmann's constant.
\medskip
Relativistic electron gyroradius:
$$r_e={mc^2\over eB}(\gamma^2 - 1)^{1/2}\ ({\rm cgs})=
1.70\times10^3(\gamma^2-1)^{1/2}B^{-1}\,{\rm cm}.$$

Relativistic electron energy:
$$W=mc^2\gamma=0.511\gamma\>{\rm MeV}.$$

Bennett pinch condition:
$$I^2=2Nk(T_e+T_i)c^2\ ({\rm cgs})=3.20\times10^{-4}N(T_e+T_i)\,\rm A^2.$$

Alfv\'en-Lawson limit:
$$I_A=(mc^3/e)\beta_z\gamma\ {\rm (cgs)}=(4\pi mc/\mu_0e)\beta_z\gamma\ {\rm
(SI)}=1.70\times10^4\beta_z\gamma\,\rm A.$$

The ratio of net current to $I_A$ is
$${I\over I_A} = {\nu\over\gamma}.$$
Here $\nu=Nr_e$ is the Budker number, where $r_e=e^2/mc^2=2.82\times10^
{-13}\,$cm is the classical electron radius.  Beam electron number density is
$$n_b = 2.08\times 10^8 J\beta^{-1}\,{\rm cm}^{-3},$$
where $J$ is the current density in A\ts cm$^{-2}$.  For a uniform beam of
radius $a$ (in cm),
$$n_b=6.63\times10^7 I a^{-2}\beta^{-1}\,{\rm cm}^{-3},$$
and
$${2r_e\over a}={\nu\over\gamma}.$$
\vfil\eject\end
```

RELATIVISTIC ELECTRON BEAMS

Here $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic scaling factor; quantities in analytic formulas are expressed in SI or cgs units, as indicated; in numerical formulas, I is in amperes (A), B is in gauss (G), electron linear density N is in cm^{-1}, and temperature, voltage and energy are in MeV; $\beta_z = v_z/c$; k is Boltzmann's constant.

Relativistic electron gyroradius:

$$r_e = \frac{mc^2}{eB}(\gamma^2 - 1)^{1/2} \text{ (cgs)} = 1.70 \times 10^3(\gamma^2 - 1)^{1/2} B^{-1} \text{ cm}.$$

Relativistic electron energy:

$$W = mc^2\gamma = 0.511\gamma \text{ MeV}.$$

Bennett pinch condition:

$$I^2 = 2Nk(T_e + T_i)c^2 \text{ (cgs)} = 3.20 \times 10^{-4} N(T_e + T_i) \text{ A}^2.$$

Alfvén-Lawson limit:

$$I_A = (mc^3/e)\beta_z\gamma \text{ (cgs)} = (4\pi mc/\mu_0 e)\beta_z\gamma \text{ (SI)} = 1.70 \times 10^4 \beta_z\gamma \text{ A}.$$

The ratio of net current to I_A is

$$\frac{I}{I_A} = \frac{\nu}{\gamma}.$$

Here $\nu = Nr_e$ is the Budker number, where $r_e = e^2/mc^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. Beam electron number density is

$$n_b = 2.08 \times 10^8 J\beta^{-1} \text{ cm}^{-3},$$

where J is the current density in $\mathrm{A\,cm}^{-2}$. For a uniform beam of radius a (in cm),

$$n_b = 6.63 \times 10^7 Ia^{-2}\beta^{-1} \text{ cm}^{-3},$$

and

$$\frac{2r_e}{a} = \frac{\nu}{\gamma}.$$

```
\input prolog
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein
\pageno=47
Child's law:  (non-relativistic) space-charge-limited current density between
parallel plates with voltage drop $V$ and separation $d$ (in cm) is
$$J=2.34\times 10^3 V^{3/2}d^{-2}\,\rm A\,cm^{-2}.$$
\smallskip
The saturated parapotential current (magnetically self-limited flow along
equi\-potentials in pinched diodes and transmission lines) is$^{29}$
$$I_p=8.5\times10^3G\gamma \ln\left[\gamma+(\gamma^2-1)^{1/2}\right]\,\rm A,$$
where $G$ is a geometrical factor depending on the diode structure:
\smallskip
\halign{\quad#\hfil\qquad&#\hfil \cr
 \lower5pt\hbox{$\displaystyle G={w \over 2\pi d}$}
   &for parallel plane cathode and anode \cr
 \hbox{7pt}
   &of width $w$, separation $d$; \cr

$\displaystyle G=\left( \ln {R_2 \over R_1} \right)^{-1}$ &for cylinders of
radii $R_1$ (inner) and $R_2$ (outer); \cr
\noalign{\vskip2pt}
\lower5pt\hbox{$\displaystyle G={R_c \over d_0}$}
   &for conical cathode of radius $R_c$, maximum \cr
\hbox{8pt}
   &separation $d_0$ (at $r=R_c$) from plane
anode. \cr}
\smallskip
For $\beta\to0\;(\gamma\to1)$, both $I_A$ and $I_p$ vanish.
\medskip
The condition for a longitudinal magnetic field $B_z$ to suppress
filamentation in a beam of current density $J$ (in A\ts cm$^{-2}$) is
\smallskip\nointerlineskip
$$B_z > 47 \beta_z(\gamma J)^{1/2}\,\rm G.$$

Voltage registered by Rogowski coil of minor cross-sectional area $A$, $n$
turns, major radius $a$, inductance $L$, external resistance $R$ and
capacitance $C$ (all in SI):
$$\vbox{\halign{\quad#\hfil\qquad\qquad&$#$\hfil \cr
externally integrated  &V=(1/RC)(nA\mu_0I/2\pi a); \cr
\noalign{\smallskip}
self-integrating       &V=(R/L)(nA\mu_0I/2\pi a) = RI/n. \cr}}$$

X-ray production, target with average atomic number $Z\;$ ($V \approxlt
[illegible] MeV):
\smallskip\nointerlineskip
$$\eta \equiv\hbox{x-ray power/beam power}=[illegible]\times10^{-4} ZV.$$

[illegible] at 1 meter generated by an e-beam depositing total charge $Q$
[illegible] while $V \ge 0.84V_{\rm max}$ in material with charge state $Z$:
$$[illegible]_{\rm max} [illegible] QZ^{1/2}\,\rm rads.$$
[illegible] end
```

Child's law: (non-relativistic) space-charge-limited current density between parallel plates with voltage drop V and separation d (in cm) is

$$J = 2.34 \times 10^3 V^{3/2} d^{-2}\ \mathrm{A\,cm^{-2}}.$$

The saturated parapotential current (magnetically self-limited flow along equipotentials in pinched diodes and transmission lines) is[29]

$$I_p = 8.5 \times 10^3 G\gamma \ln\left[\gamma + (\gamma^2 - 1)^{1/2}\right]\ \mathrm{A}.$$

where G is a geometrical factor depending on the diode structure:

$G = \dfrac{w}{2\pi d}$	for parallel plane cathode and anode of width w, separation d;
$G = \left(\ln \dfrac{R_2}{R_1}\right)^{-1}$	for cylinders of radii R_1 (inner) and R_2 (outer);
$G = \dfrac{R_c}{d_0}$	for conical cathode of radius R_c, maximum separation d_0 (at $r = R_c$) from plane anode.

For $\beta \to 0$ ($\gamma \to 1$), both I_A and I_p vanish.

The condition for a longitudinal magnetic field B_z to suppress filamentation in a beam of current density J (in A cm^{-2}) is

$$B_z > 47\beta_z(\gamma J)^{1/2}\ \mathrm{G}.$$

Voltage registered by Rogowski coil of minor cross-sectional area A, n turns, major radius a, inductance L, external resistance R and capacitance C (all in SI):

externally integrated	$V = (1/RC)(nA\mu_0 I/2\pi a)$;
self-integrating	$V = (R/L)(nA\mu_0 I/2\pi a) = RI/n$.

X-ray production, target with average atomic number Z ($V \lesssim 5$ MeV):

$$\eta \equiv \text{x-ray power/beam power} = 7 \times 10^{-4} ZV.$$

X-ray dose at 1 meter generated by an e-beam depositing total charge Q coulombs while $V \geq 0.84 V_{\max}$ in material with charge state Z:

$$D = 150 V_{\max}^{2.8} Q Z^{1/2}\ \text{rads}.$$

```
\input prolog
\voffset=1.0truein\hoffset=0.25truein\vsize=9.0truein\hsize=6.5truein
\pageno=48
\centerline{\headfont{BEAM INSTABILITIES}$^{30}$}
\bsk  %  BEGINNING OF TABLE.
\vbox{\tabskip=0pt \offinterlineskip \halign to\hsize
{\strut#&\vrule#\tabskip=3pt plus3pt minus2pt&#\hfil&\vrule#
&$#$\hfil&\vrule#&#\hfil&\vrule#\tabskip=0pt\cr \trule \tska{3}{3pt}
|\hfil Name|\om\hfil Conditions\hfil|\hfil Saturation Mechanism&\cr
\tska{3}{3pt} \trule \tska{3}{1pt} \trule \tska{3}{3pt}
|Electron-|V_d>\ov{V}_{ej},\;j=1,2|Electron trapping until &\cr
  |\quad electron|\om|\quad$\ov{V}_{ej}\sim V_d$&\cr \tska{3}{6pt}
|Buneman|V_d>(M/m)^{1/3}\ov{V}_i,|Electron trapping until&\cr
  |\om|V_d>\ov{V}_e|\quad$\ov{V}_e\sim V_d$&\cr \tska{3}{6pt}
|Beam-plasma|V_b>(n_p/n_b)^{1/3}\ov{V}_b|Trapping of beam electrons&\cr
    \tska{3}{6pt}
|Weak beam-|V_b<(n_p/n_b)^{1/3}\ov{V}_b|Quasilinear or nonlinear&\cr
  |\quad plasma|\om|\quad (mode coupling)&\cr \tska{3}{6pt}
|Beam-plasma|\ov{V}_e>V_b>\ov{V}_b|Quasilinear or nonlinear&\cr
  |\quad (hot-electron)|\om|\om&\cr \tska{3}{6pt}
|Ion acoustic|T_e\gg T_i,\;V_d\gg C_s|Quasilinear, ion tail form-&\cr
  |\om|\om|\quad ation, nonlinear scattering,&\cr
  |\om|\om|\quad or resonance broadening.&\cr \tska{3}{6pt}
|Anisotropic|T_{e\perp}>2T_{e\parallel}|Isotropization&\cr \bs{1pt}
  |\quad temperature|\om|\om&\cr
  |\quad (hydro)|\om|\om&\cr \tska{3}{6pt}
|Ion cyclotron|V_d>20\ov{V}_i\ ({\rm for}|Ion heating&\cr
  |\om|\om\hfil$T_e\approx T_i)$|\om&\cr \tska{3}{6pt}
|Beam-cyclotron|V_d>C_s|Resonance broadening&\cr
  |\quad (hydro)|\om|\om&\cr \tska{3}{6pt}
|Modified two-|V_d<(1+\beta)^{1/2}V_A,|Trapping&\cr
  |\quad stream (hydro)|V_d>C_s|\om&\cr \tska{3}{6pt}
|Ion-ion (equal|U<2(1+\beta)^{1/2}V_A|Ion trapping&\cr
  |\quad beams)|\om|\om&\cr
  \tska{3}{6pt}
|Ion-ion (equal|U<2C_s|Ion trapping&\cr
  |\quad beams)|\om|\om&\cr \tska{3}{3pt} \trule}}

For nomenclature, see p. 50.
\vfil\eject\end
```

BEAM INSTABILITIES[30]

Name	Conditions	Saturation Mechanism
Electron-electron	$V_d > \bar{V}_{ej},\ j = 1,2$	Electron trapping until $\bar{V}_{ej} \sim V_d$
Buneman	$V_d > (M/m)^{1/3}\bar{V}_i$, $V_d > \bar{V}_e$	Electron trapping until $\bar{V}_e \sim V_d$
Beam-plasma	$V_b > (n_p/n_b)^{1/3}\bar{V}_b$	Trapping of beam electrons
Weak beam-plasma	$V_b < (n_p/n_b)^{1/3}\bar{V}_b$	Quasilinear or nonlinear (mode coupling)
Beam-plasma (hot-electron)	$\bar{V}_e > V_b > \bar{V}_b$	Quasilinear or nonlinear
Ion acoustic	$T_e \gg T_i,\ V_d \gg C_s$	Quasilinear, ion tail formation, nonlinear scattering, or resonance broadening.
Anisotropic temperature (hydro)	$T_{e\perp} > 2T_{e\parallel}$	Isotropization
Ion cyclotron	$V_d > 20\bar{V}_i$ (for $T_e \approx T_i$)	Ion heating
Beam-cyclotron (hydro)	$V_d > C_s$	Resonance broadening
Modified two-stream (hydro)	$V_d < (1+\beta)^{1/2}V_A$, $V_d > C_s$	Trapping
Ion-ion (equal beams)	$U < 2(1+\beta)^{1/2}V_A$	Ion trapping
Ion-ion (equal beams)	$U < 2C_s$	Ion trapping

For nomenclature, see p. 50.

```
\input prolog \pageno=49
\voffset=1.0truein\hoffset=0.25truein\vsize=9.0truein\hsize=6.5truein
\vbox{\tabskip=0pt \offinterlineskip \def\ds{\displaystyle}
   % HERE, \ds IS USED FOR 'DISPLAYSTYLE'.
\halign to\hsize{\strut#&\vrule#\tabskip=3pt plus3pt minus2pt&#\hfil&\vrule#
&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#&\hfil$#$\hfil&\vrule#
&\hfil$#$\hfil&\vrule#\tabskip=0pt\cr\trule
\om&height3pt&\om|\multispan7&\cr
|\om|\multispan7\hfil Parameters of Most Unstable Mode\hfil&\cr
\om&height3pt&\om|\multispan7&\cr
\om|\om&\multispan9\hrulefill\cr \tska{5}{2pt} \bs{0.75ex}
  %  THIS USE OF \hrulefill IS MORE GENERAL AND ELEGANT IN MANY CASES
  %  THAN THE 'HARD-WIRED' \hrule USED UNDER 'Dimension' ON PAGE 10.
|\hfil\raise1ex\hbox{Name}|\om|\om|{\rm Wave}|{\rm Group}&\cr
|\om|{\rm Growth\ Rate}|{\rm Frequency}|{\rm Number}|{\rm Velocity}&\cr
\tska{5}{2pt} \trule \tska{5}{1pt} \trule \tska{5}{2pt}
|Electron-|\ds{1\over2}\omega_e|0|\ds0.9{\omega_e\over V_d}|0&\cr \bs{5pt}
  |\quad electron|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Buneman|\ds0.7\left({m\over M}\right)^{1/3}\omega_e|
  \ds0.4\left({m\over M}\right)^{1/3}\omega_e|\ds{\omega_e
  \over V_d}|\ds{2\over3}V_d&\cr \tska{5}{2pt}
|Beam-plasma|\ds0.7\left({n_b\over n_p}\right)^{1/3}\omega_e|\om$\omega_e-$
  \hfil|\ds{\omega_e\over V_b}|\ds{2\over3}V_b&\cr
  |\om|\om|\om\hfil$\ds0.4\left({n_b\over n_p}\right)^{1/3}\omega_e$|
  \om|\om&\cr \tska{5}{2pt}
|Weak beam-|\ds{n_b\over2n_p}\left({V_b\over\ov{V}_b}\right)^2\omega_e
  |\omega_e|\ds{\omega_e\over V_b}|\ds{3{\ov{V}_e}^2\over V_b}&\cr \bs{6pt}
  |\quad plasma|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Beam-plasma|\ds\left({n_b\over n_p}\right)^{1/2}{\ov{V}_e\over V_b}\omega_e
  |\ds{V_b\over\ov{V}_e}\omega_e|\lambda_D^{-1}|V_b&\cr \bs{5pt}
  |\quad (hot-electron)|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Ion acoustic|\ds\left({m\over M}\right)^{1/2}\omega_i|\ds\omega_i
  |\lambda_D^{-1}|C_s&\cr \tska{5}{2pt}
|Anisotropic|\Omega_e|\omega_e\cos\theta\sim\Omega_e|r_e^{-1}|
  \ov{V}_{e\perp}&\cr \bs{1pt}
  |\quad temperature|\om|\om|\om|\om&\cr
  |\quad (hydro)|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Ion cyclotron|0.1\Omega_i|1.2\Omega_i|r_i^{-1}|\ds{1\over3}\ov{V}_i&\cr
  \tska{5}{2pt}
|Beam-cyclotron|\ds0.7\Omega_e|\ds n\Omega_e|0.7
  \lambda_D^{-1}|\approxgt V_d;&\cr \bs{1pt}
  |\quad (hydro)|\om|\om|\om|\approxlt C_s\ph{;}&\cr \tska{5}{2pt}
|Modified two-|\ds{1\over2}\Omega_H|\ds0.9\Omega_H|\ds1.7
  {\Omega_H\over V_d}|\ds{1\over2}V_d&\cr \bs{5pt}
  |\quad stream|\om|\om|\om|\om&\cr
  |\quad (hydro)|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Ion-ion (equal|0.4\Omega_H|0|\ds1.2{\Omega_H\over U}|0&\cr \bs{4pt}
  |\quad beams)|\om|\om|\om|\om&\cr \tska{5}{2pt}
|Ion-ion (equal|\ds0.4\omega_i|0|\ds1.2{\omega_i\over U}|0&\cr \bs{4pt}
  |\quad beams)|\om|\om|\om|\om&\cr \tska{5}{2pt}\trule}}
For nomenclature, see p. 50.
\vfil\eject\end
```

Name	Parameters of Most Unstable Mode			
	Growth Rate	Frequency	Wave Number	Group Velocity
Electron-electron	$\frac{1}{2}\omega_e$	0	$0.9\frac{\omega_e}{V_d}$	0
Buneman	$0.7\left(\frac{m}{M}\right)^{1/3}\omega_e$	$0.4\left(\frac{m}{M}\right)^{1/3}\omega_e$	$\frac{\omega_e}{V_d}$	$\frac{2}{3}V_d$
Beam-plasma	$0.7\left(\frac{n_b}{n_p}\right)^{1/3}\omega_e$	$\omega_e - 0.4\left(\frac{n_b}{n_p}\right)^{1/3}\omega_e$	$\frac{\omega_e}{V_b}$	$\frac{2}{3}V_b$
Weak beam-plasma	$\frac{n_b}{2n_p}\left(\frac{V_b}{\bar{V}_b}\right)^2\omega_e$	ω_e	$\frac{\omega_e}{V_b}$	$\frac{3\bar{V}_e^2}{V_b}$
Beam-plasma (hot-electron)	$\left(\frac{n_b}{n_p}\right)^{1/2}\frac{\bar{V}_e}{V_b}\omega_e$	$\frac{V_b}{\bar{V}_e}\omega_e$	λ_D^{-1}	V_b
Ion acoustic	$\left(\frac{m}{M}\right)^{1/2}\omega_i$	ω_i	λ_D^{-1}	C_s
Anisotropic temperature (hydro)	Ω_e	$\omega_e \cos\theta \sim \Omega_e$	r_e^{-1}	$\bar{V}_{e\perp}$
Ion cyclotron	$0.1\Omega_i$	$1.2\Omega_i$	r_i^{-1}	$\frac{1}{3}\bar{V}_i$
Beam-cyclotron (hydro)	$0.7\Omega_e$	$n\Omega_e$	$0.7\lambda_D^{-1}$	$\gtrsim V_d$; $\lesssim C_s$
Modified two-stream (hydro)	$\frac{1}{2}\Omega_H$	$0.9\Omega_H$	$1.7\frac{\Omega_H}{V_d}$	$\frac{1}{2}V_d$
Ion-ion (equal beams)	$0.4\Omega_H$	0	$1.2\frac{\Omega_H}{U}$	0
Ion-ion (equal beams)	$0.4\omega_i$	0	$1.2\frac{\omega_i}{U}$	0

For nomenclature, see p. 50.

```
\input prolog \pageno=50
\hoffset=1.25truein\voffset=1.0truein\hsize=6.0truein\vsize=9.0truein \indent
In the preceding tables, subscripts $e$, $i$, $d$, $b$, $p$ stand for
''elec\-tron,'' ''ion,'' ''drift,'' ''beam,'' and ''plasma,'' respectively.
Thermal velocities are denoted by a bar.  In addition, the following are used: \ssk
\halign {$##$\hfil\quad&#\hfil\quad&$#$\hfil\quad&#\hfil \cr
m &electron mass &r_e, r_i &gyroradius \cr
M &ion mass &\beta &plasma/magnetic energy \cr
V &velocity |\quad density ratio \cr
T &temperature &V_A &Alfv\'en speed \cr
n_e, n_i &number density &\Omega_e,\Omega_i &gyrofrequency \cr
n &harmonic number &\Omega_H &hybrid gyrofrequency,\cr
C_s=(T_e/M)^{1/2} &ion sound speed |\quad ${\Omega_H}^2=\Omega_e\Omega_i$ \cr
\omega_e, \omega_i &plasma frequency &U &relative drift velocity of \cr
\lambda_D &Debye length |\quad two ion species \cr} \bsk
\centerline{\headfont LASERS} \msk
{\headfont System Parameters} \ssk
Efficiencies and power levels are approximately state-of-the-art (1987).$^{31}$\ssk
\vbox{\offinterlineskip \tabskip=0pt \halign to \hsize{
\vrule# \tabskip=3pt plus 2pt minus2pt&\strut #\hfil
&\vrule #&\hfil#\hfil &\vrule #&\hfil#\hfil
&\vrule #&\hfil#\hfil &\vrule #&\hfil#\hfil
&\vrule #\tabskip=0pt\cr\trule
height 2pt&\om|\om|\om|\multispan3 &\cr
&\om|\om|\om|\multispan3 Power levels available (W) &\cr \bs{6.5pt}
&\hfil Type|$\displaystyle{\rm Wavelength\atop(\mu m)}$|Efficiency|\om
&\om&\om&\cr \bs{8pt}
&\om|\om|\om\tabskip=0pt|\multispan3&\cr
\noalign{\vskip-0.4ex \moveright3.215truein \vbox{\hrule width2.79truein}}
\tskc{5}{3pt} &\om|\om|\om|Pulsed|CW&\cr
\tskc{5}{3pt} \trule \tskc{5}{1pt} \trule \tskc{5}{2pt}
&CO$_2$|10.6|0.01--0.02|$>2\times10^{13}$|$>10^5$&\cr
  &\om|\om|(pulsed)|\om|\om&\cr \tskc{5}{1pt}
&CO|5|0.4|$>10^9$|$>100$&\cr \tskc{5}{1pt}
&Holmium|2.06|0.03|$>10^7$|30&\cr \tskc{5}{1pt}
&Iodine|1.315|0.003|$>10^{12}$|--&\cr \tskc{5}{1pt}
&Nd-glass,|1.06|0.001--0.06|$\sim 10^{14}$|1--10$^3$&\cr
  &\quad YAG|\om|\om|(10-beam system)|\om&\cr \tskc{5}{1pt}
&\llap{*}Color|1--4|10$^{-3}$|$>10^6$|1&\cr
  &\quad center|\om|\om|\om|\om&\cr \tskc{5}{1pt}
&\llap{*}OPO|0.7--0.9|10$^{-3}$|$10^6$|1&\cr \tskc{5}{1pt}
&Ruby|0.6943|$<10^{-3}$|$10^{10}$|1&\cr \tskc{5}{1pt}
&He-Ne|0.6328|10$^{-4}$|--|1--50$\times10^{-3}$ &\cr \tskc{5}{1pt}
&\llap{*}Argon ion|0.45--0.60|10$^{-3}$|$5\times 10^4$|1--10&\cr \tskc{5}{1pt}
&N$_2$|0.3371|$$0.001--0.05|10$^5$--10$^6$|--&\cr \tskc{5}{1pt}
&\llap{*}Dye|0.3--1.1|10$^{-3}$|$>10^6$|140&\cr \tskc{5}{1pt}
&Kr-F|0.26|0.08|$>10^9$|--&\cr \tskc{5}{1pt}
&Xenon|0.175|0.02|$>10^8$|--&\cr \tskc{5}{2pt} \trule \noalign{\ssk}
\om\hbox{*Tunable sources}\hidewidth\cr}} \ssk  % END OF TABLE.
YAG stands for Yttrium--Aluminum Garnet and OPO for Optical Parametric
Oscillator.
\vfil\eject\end
```

In the preceding tables, subscripts e, i, d, b, p stand for "electron," "ion," "drift," "beam," and "plasma," respectively. Thermal velocities are denoted by a bar. In addition, the following are used:

m	electron mass	r_e, r_i	gyroradius
M	ion mass	β	plasma/magnetic energy density ratio
V	velocity		
T	temperature	V_A	Alfvén speed
n_e, n_i	number density	Ω_e, Ω_i	gyrofrequency
n	harmonic number	Ω_H	hybrid gyrofrequency, $\Omega_H{}^2 = \Omega_e \Omega_i$
$C_s = (T_e/M)^{1/2}$	ion sound speed		
ω_e, ω_i	plasma frequency	U	relative drift velocity of two ion species
λ_D	Debye length		

LASERS

System Parameters

Efficiencies and power levels are approximately state-of-the-art (1987).[31]

Type	Wavelength (μm)	Efficiency	Power levels available (W)	
			Pulsed	CW
CO_2	10.6	0.01–0.02 (pulsed)	$> 2 \times 10^{13}$	$> 10^5$
CO	5	0.4	$> 10^9$	> 100
Holmium	2.06	0.03	$> 10^7$	30
Iodine	1.315	0.003	$> 10^{12}$	
Nd-glass, YAG	1.06	0.001–0.06	$\sim 10^{14}$ (10-beam system)	1–10^3
*Color center	1–4	10^{-3}	$> 10^6$	1
*OPO	0.7–0.9	10^{-3}	10^6	1
Ruby	0.6943	$< 10^{-3}$	10^{10}	1
He-Ne	0.6328	10^{-4}		1–50×10^{-3}
*Argon ion	0.45–0.60	10^{-3}	5×10^4	1–10
N_2	0.3371	0.001–0.05	10^5–10^6	
*Dye	0.3–1.1	10^{-3}	$> 10^6$	140
Kr-F	0.26	0.08	$> 10^9$	
Xenon	0.175	0.02	$> 10^8$	

*Tunable sources

YAG stands for Yttrium Aluminum Garnet and OPO for Optical Parametric Oscillator.

```
\input prolog
\pageno=51
\hoffset=1.25truein
\voffset=1.0truein
\hsize=6.0truein
\vsize=9.0truein
{\headfont Formulas}
\medskip\indent
An e-m wave with {\bf k} $\parallel$ {\bf B} has an index of refraction
given by
$$n_\pm = [1-\omega_{pe}^{\ph{1}2}/\omega(\omega\mp\omega_{ce})]^{1/2},$$
where $\pm$ refers to the helicity.  The rate of change of polarization
angle $\theta$ as a function of displacement $s$ (Faraday rotation) is given
by
$$d\theta/ds = (k/2)(n_{-}-n_{+})=2.36\times 10^4 NBf^{-2}\,{\rm cm}^{-1},$$
where $N$ is the electron number density, $B$ is the field strength, and
$f$ is the wave frequency, all in cgs.
\smallskip\indent
The quiver velocity of an electron in an e-m field of angular frequency $\omega$
is
$$v_0=eE_{\rm max}/m\omega=25.6 I^{1/2}\lambda_0\,\rm cm\,sec^{-1}$$
in terms of the laser flux $I=cE_{\rm max}^{\ph{\mu}2}/8\pi$, with $I$ in
watt/cm$^2$, laser wavelength $\lambda_0$ in $\mu$m.  The ratio of quiver
energy to thermal energy is
$$W_{\rm qu}/W_{\rm th}=m_e{v_0}^2/2kT=1.81\times10^{-13} {\lambda_0}^2I/T,$$
where $T$ is given in eV.  For example, if $I=10^{15}\,$W\ts cm$^{-2}$,
$\>\lambda_0 = 1\, \mu$m, $T=2$\ts keV, then $W_{\rm qu}/W_{\rm th}\approx0.1$.
\smallskip\indent
Pondermotive force:
$$\hbox{{$\cal F$}\kern-0.74em{$\cal F$}}=N\nabla\langle E^2 \rangle/8\pi N_c,$$
where
$$N_c=1.1\times 10^{21} {\lambda_0}^{-2} {\rm cm}^{-3}.$$
\indent
For uniform illumination of a lens with $f$-number $F$, the diameter $d$
at focus (85\% of the energy) and the depth of focus $l$ (distance to first
zero in intensity) are given by
$$d\approx 2.44 F\lambda \theta/\theta_{DL}{\rm\quad and \quad}l\approx\pm2
F^2\lambda\theta/\theta_{DL}.$$
Here $\theta$ is the beam divergence containing 85\% of energy and
$\theta_{DL}$ is the diffraction-limited divergence:

$$\theta_{DL}=2.44\lambda/b,$$

where $b$ is the aperture.  These formulas are modified for nonuniform (such
as Gaussian) illumination of the lens or for pathological laser profiles.
\vfil\eject\end
```

Formulas

An e-m wave with $\mathbf{k} \parallel \mathbf{B}$ has an index of refraction given by

$$n_{\pm} = [1 - \omega_{pe}^2/\omega(\omega \mp \omega_{ce})]^{1/2},$$

where $\pm$ refers to the helicity. The rate of change of polarization angle θ as a function of displacement s (Faraday rotation) is given by

$$d\theta/ds = (k/2)(n_- - n_+) = 2.36 \times 10^4 NBf^{-2}\,\mathrm{cm}^{-1},$$

where N is the electron number density, B is the field strength, and f is the wave frequency, all in cgs.

The quiver velocity of an electron in an e-m field of angular frequency ω is

$$v_0 = eE_{\max}/m\omega = 25.6 I^{1/2}\lambda_0\,\mathrm{cm\,sec}^{-1}$$

in terms of the laser flux $I = cE_{\max}^2/8\pi$, with I in watt/cm^2, laser wavelength λ_0 in μm. The ratio of quiver energy to thermal energy is

$$W_{\mathrm{qu}}/W_{\mathrm{th}} = m_e v_0{}^2/2kT = 1.81 \times 10^{-13}\lambda_0{}^2 I/T,$$

where T is given in eV. For example, if $I = 10^{15}\,\mathrm{W\,cm}^{-2}$, $\lambda_0 = 1\,\mu$m, $T = 2\,$keV, then $W_{\mathrm{qu}}/W_{\mathrm{th}} \approx 0.1$.

Pondermotive force:

$$\mathcal{F} = N\nabla\langle E^2\rangle/8\pi N_c,$$

where

$$N_c = 1.1 \times 10^{21}\lambda_0{}^{-2}\mathrm{cm}^{-3}.$$

For uniform illumination of a lens with f-number F, the diameter d at focus (85% of the energy) and the depth of focus l (distance to first zero in intensity) are given by

$$d \approx 2.44F\lambda\theta/\theta_{DL} \quad \text{and} \quad l \approx \pm 2F^2\lambda\theta/\theta_{DL}.$$

Here θ is the beam divergence containing 85% of energy and θ_{DL} is the diffraction-limited divergence:

$$\theta_{DL} = 2.44\lambda/b,$$

where b is the aperture. These formulas are modified for nonuniform (such as Gaussian) illumination of the lens or for pathological laser profiles.

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\centerline{\headfont ATOMIC PHYSICS AND RADIATION}
\bigskip\inndent
Energies and temperatures are in eV; all other units are cgs except where
noted.  $Z$ is the charge state ($Z=0$ refers to a neutral atom); the subscript
$e$ labels electrons.  $N$ refers to number density, $n$ to principal quantum
number.  Asterisk superscripts on level population densities denote local
thermodynamic equilibrium (LTE) values.  Thus $N_n\hbox{*}$ is the LTE number
density of atoms (or ions) in level $n$.

Characteristic atomic collision cross section:
$$\pi {a_0}^2 = 8.80\times 10^{-17}\,{\rm cm}^2. \leqno(1)$$
Binding energy of outer electron in level labelled by quantum numbers $n, 1$:
$$E_\infty^Z(n,l) = -{Z^2E_\infty^H \over(n-\Delta_l)^2}, \leqno{(2)}$$
where $E_\infty^H = 13.6\,{\rm eV}$ is the hydrogen ionization energy and
$\Delta_l = 0.75l^{-5}$, $l \approxgt 5$, is the quantum defect.
\medskip
{\headfont Excitation and Decay}

\inndent
Cross section (Bethe approximation) for electron excitation by dipole allowed
transition $m \rightarrow n$ (Refs. 32, 33):
$$\sigma_{mn} = 2.36 \times 10^{-13}{f_{nm}g(n,m) \over \epsilon \Delta
E_{nm}}\,{\rm cm}^2, \leqno{(3)}$$
where $f_{nm}$ is the oscillator strength, $g(n,m)$ is the Gaunt factor,
$\epsilon$ is the incident electron energy, and $\Delta E_{nm} = E_n - E_m$.

Electron excitation rate averaged over Maxwellian velocity distribution,
$X_{mn} = N_e \langle \sigma_{mn}v \rangle$ (Refs. 34, 35):
$$X_{mn} = 1.6\times10^{-5}{f_{nm}\langle g(n,m)\rangle N_e
\over \Delta E_{nm} T_e^{1/2}}\exp\left( -{\Delta E_{nm} \over T_e}
\right)\,{\rm sec}^{-1}, \leqno{(4)}$$
where $\langle g(n,m)\rangle$ denotes the thermal averaged Gaunt factor
(generally $\sim 1$ for atoms, $\sim 0.2$ for ions).
\vfill\eject\end
```

ATOMIC PHYSICS AND RADIATION

Energies and temperatures are in eV; all other units are cgs except where noted. Z is the charge state ($Z = 0$ refers to a neutral atom); the subscript e labels electrons. N refers to number density, n to principal quantum number. Asterisk superscripts on level population densities denote local thermodynamic equilibrium (LTE) values. Thus $N_n{}^*$ is the LTE number density of atoms (or ions) in level n.

Characteristic atomic collision cross section:

$$\pi a_0{}^2 = 8.80 \times 10^{-17}\,\mathrm{cm}^2. \tag{1}$$

Binding energy of outer electron in level labelled by quantum numbers n, l:

$$E_\infty^Z(n,l) = -\frac{Z^2 E_\infty^H}{(n-\Delta_l)^2}, \tag{2}$$

where $E_\infty^H = 13.6\,\mathrm{eV}$ is the hydrogen ionization energy and $\Delta_l = 0.75 l^{-5}$, $l \gtrsim 5$, is the quantum defect.

Excitation and Decay

Cross section (Bethe approximation) for electron excitation by dipole allowed transition $m \to n$ (Refs. 32, 33):

$$\sigma_{mn} = 2.36 \times 10^{-13} \frac{f_{nm}\, g(n,m)}{\epsilon \Delta E_{nm}}\,\mathrm{cm}^2, \tag{3}$$

where f_{nm} is the oscillator strength, $g(n,m)$ is the Gaunt factor, ϵ is the incident electron energy, and $\Delta E_{nm} = E_n - E_m$.

Electron excitation rate averaged over Maxwellian velocity distribution, $X_{mn} = N_e \langle \sigma_{mn} v \rangle$ (Refs. 34, 35):

$$X_{mn} = 1.6 \times 10^{-5} \frac{f_{nm} \langle g(n,m) \rangle N_e}{\Delta E_{nm} T_e^{1/2}} \exp\left(-\frac{\Delta E_{nm}}{T_e}\right) \mathrm{sec}^{-1}, \tag{4}$$

where $\langle g(n,m) \rangle$ denotes the thermal averaged Gaunt factor (generally ~ 1 for atoms, ~ 0.2 for ions).

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Rate for electron collisional deexcitation:
$$Y_{nm}=({N_m}\hbox{*}/{N_n}\hbox{*})X_{mn}. \leqno{(5)}$$
Here ${N_m}\hbox{*}/{N_n}\hbox{*}=(g_m/g_n)\exp(\Delta E_{nm}/T_e)$ is the
Boltzmann relation for level population densities, where $g_n$ is the
statistical weight of level $n$.

Rate for spontaneous decay $n \rightarrow m$ (Einstein $A$ coefficient)$^{34}$
$$A_{nm}=4.3\times10^7 (g_n/g_m)f_{nm}(\Delta E_{nm})^2\,{\rm sec}^{-1}.
\leqno{(6)}$$

Intensity emitted per unit volume from the transition $n \rightarrow m$ in an
op\-tic\-ally thin plasma:
$$I_{nm}=1.6\times 10^{-19}A_{nm}N_n\Delta E_{nm}\, {\rm watt/cm}^3. \leqno{(7)}$$

Condition for steady state in a corona model:
$$N_0N_e \langle \sigma_{0n}v \rangle =N_nA_{n0},\leqno{(8)}$$
where the ground state is labelled by a zero subscript.

Hence for a transition $n \rightarrow m$ in ions, where $\langle g(n,0)\rangle
\approx 0.2$,
$$I_{nm} = 5.1\times 10^{-25}{f_{nm}g_0 N_eN_0 \over g_m T_e^{1/2}}
\left({\Delta E_{nm} \over \Delta E_{n0}} \right)^3 \exp \left (-{\Delta
E_{n0} \over T_e} \right )\,{{\rm watt}\over{\rm cm}^3}. \leqno(9)$$
\medskip
{\headfont Ionization and Recombination}

\inndent
In a general time-dependent situation the number density of the
charge state $Z$ satisfies
$${dN(Z)\over dt} = N_e \big[ -S(Z)N(Z) - \alpha(Z)N(Z) \leqno(10)$$
\vskip-0.3truein
$$\qquad\qquad\qquad + S(Z-1)N(Z-1) + \alpha(Z+1)N(Z+1)\big].$$
Here $S(Z)$ is the ionization rate.  The recombination rate $\alpha(Z)$
has the form $\alpha(Z)=\alpha_r(Z)+N_e\alpha_3(Z)$, where $\alpha_r$ and
$\alpha_3$ are the radiative and three-body recombination rates, respectively.
\vfill\eject\end
```

Rate for electron collisional deexcitation:

$$(5) \qquad Y_{nm} = (N_m{}^*/N_n{}^*)X_{mn}.$$

Here $N_m{}^*/N_n{}^* = (g_m/g_n)\exp(\Delta E_{nm}/T_e)$ is the Boltzmann relation for level population densities, where g_n is the statistical weight of level n.

Rate for spontaneous decay $n \longrightarrow m$ (Einstein A coefficient)[34]

$$(6) \qquad A_{nm} = 4.3 \times 10^7 (g_n/g_m) f_{nm} (\Delta E_{nm})^2 \ \mathrm{sec}^{-1}.$$

Intensity emitted per unit volume from the transition $n \longrightarrow m$ in an optically thin plasma:

$$(7) \qquad I_{nm} = 1.6 \times 10^{-19} A_{nm} N_n \Delta E_{nm} \ \mathrm{watt/cm}^3.$$

Condition for steady state in a corona model:

$$(8) \qquad N_0 N_e \langle \sigma_{0n} v \rangle = N_n A_{n0},$$

where the ground state is labelled by a zero subscript.

Hence for a transition $n \longrightarrow m$ in ions, where $\langle g(n,0) \rangle \approx 0.2$,

$$(9) \qquad I_{nm} = 5.1 \times 10^{-25} \frac{f_{nm} g_0 N_e N_0}{g_m T_e^{1/2}} \left(\frac{\Delta E_{nm}}{\Delta E_{n0}} \right)^3 \exp\left(- \frac{\Delta E_{n0}}{T_e} \right) \frac{\mathrm{watt}}{\mathrm{cm}^3}.$$

Ionization and Recombination

In a general time-dependent situation the number density of the charge state Z satisfies

$$(10) \qquad \frac{dN(Z)}{dt} = N_e \Big[- S(Z)N(Z) - \alpha(Z)N(Z) + S(Z-1)N(Z-1) + \alpha(Z+1)N(Z+1) \Big].$$

Here $S(Z)$ is the ionization rate. The recombination rate $\alpha(Z)$ has the form $\alpha(Z) = \alpha_r(Z) + N_e\alpha_3(Z)$, where α_r and α_3 are the radiative and three-body recombination rates, respectively.

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Classical ionization cross-section$^{36}$ for any atomic shell $j$
$$\sigma_i=6\times10^{-14}  b_jg_j(x)/{U_j}^2\,{\rm cm}^2. \leqno{(11)}$$
Here $b_j$ is the number of shell electrons; $U_j$ is the binding energy
of the ejected electron; $x=\epsilon/U_j$, where $\epsilon$ is the incident
electron energy; and $g$ is a universal function with a minimum value $g_{\min}
\approx 0.2$ at $x\approx4$.

Ionization from ion ground state, averaged over Maxwellian electron
distribution, for $0.02 \approxlt T_e/E_\infty^Z \approxlt 100$ (Ref. 35):
$$S(Z) = 10^{-5}{(T_e/E_\infty^Z)^{1/2} \over (E_\infty^Z)^{3/2} (6.0 + T_e/
E_\infty^Z)}\exp\left (-{E_\infty^Z\over T_e}\right )\,\rm cm^3/sec,
\leqno{(12)}$$
where $E_\infty^Z$ is the ionization energy.

Electron-ion radiative recombination rate $\left ( e + N(Z) \rightarrow
N(Z-1)+h\nu \right )$ for $T_e/Z^2 \approxlt 400\,{\rm eV}$ (Ref. 37):
$$\alpha_r(Z) = 5.2 \times 10^{-14}Z \left ({E_\infty^Z \over T_e}
\right )^{1/2} \big[ 0.43 + {1 \over 2} \ln(E_\infty^Z/T_e) \leqno(13)$$
$$\qquad\qquad\qquad + 0.469(E_\infty^Z/T_e)^{-1/3} \big] \,{\rm cm^3/sec}.$$

For $1\,{\rm eV} < T_e/Z^2 < 15\,{\rm eV}$, this becomes approximately$^{35}$
$$\alpha_r(Z)=2.7\times 10^{-13} Z^2{T_e}^{-1/2}\,{\rm cm^3/sec}.\leqno{(14)}$$

Collisional (three-body) recombination rate for singly ionized plasma:$^{38}$
$$\alpha_3=8.75\times 10^{-27}{T_e}^{-4.5}\,{\rm cm^6/sec}.\leqno{(15)}$$

Photoionization cross section for ions in level $n, l$ (short-wavelength limit):
$$\sigma_{\rm ph}(n,l) = 1.64 \times 10^{-16}Z^5/n^3 K^{7+2l}\,\rm cm^2,
\leqno{(16)}$$
where $K$ is the wavenumber in Rydbergs (1~Rydberg $=1.0974\times
10^5\,{\rm cm}^{-1})$.
\vfill eject\end
```

Classical ionization cross-section[36] for any atomic shell j

$$\sigma_i = 6 \times 10^{-14} b_j g_j(x)/U_j{}^2 \text{ cm}^2. \quad (11)$$

Here b_j is the number of shell electrons; U_j is the binding energy of the ejected electron; $x = \epsilon/U_j$, where ϵ is the incident electron energy; and g is a universal function with a minimum value $g_{\min} \approx 0.2$ at $x \approx 4$.

Ionization from ion ground state, averaged over Maxwellian electron distribution, for $0.02 \lesssim T_e/E_\infty^Z \lesssim 100$ (Ref. 35):

$$S(Z) = 10^{-5} \frac{(T_e/E_\infty^Z)^{1/2}}{(E_\infty^Z)^{3/2}(6.0 + T_e/E_\infty^Z)} \exp\left(-\frac{E_\infty^Z}{T_e}\right) \text{ cm}^3/\text{sec}, \quad (12)$$

where E_∞^Z is the ionization energy.

Electron-ion radiative recombination rate $(e + N(Z) \rightarrow N(Z-1) + h\nu)$ for $T_e/Z^2 \lesssim 400\,\text{eV}$ (Ref. 37):

$$\alpha_r(Z) = 5.2 \times 10^{-14} Z \left(\frac{E_\infty^Z}{T_e}\right)^{1/2} \left[0.43 + \frac{1}{2}\ln(E_\infty^Z/T_e) + 0.469(E_\infty^Z/T_e)^{-1/3}\right] \text{ cm}^3/\text{sec}. \quad (13)$$

For $1\,\text{eV} < T_e/Z^2 < 15\,\text{eV}$, this becomes approximately[35]

$$\alpha_r(Z) = 2.7 \times 10^{-13} Z^2 T_e{}^{-1/2} \text{ cm}^3/\text{sec}. \quad (14)$$

Collisional (three-body) recombination rate for singly ionized plasma:[38]

$$\alpha_3 = 8.75 \times 10^{-27} T_e{}^{-4.5} \text{ cm}^6/\text{sec}. \quad (15)$$

Photoionization cross section for ions in level n, l (short-wavelength limit):

$$\sigma_{\text{ph}}(n,l) = 1.64 \times 10^{-16} Z^5/n^3 K^{7+2l} \text{ cm}^2. \quad (16)$$

where K is the wavenumber in Rydbergs (1 Rydberg $= 1.0974 \times 10^5\,\text{cm}^{-1}$).

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{\headfont Ionization Equilibrium Models}

\inndent
Saha equilibrium:$^{39}$
$${N_eN_1\hbox{*}(Z)\over N_n\hbox{*}(Z-1)}=6.0\times 10^{21}{g_1^Z {T_e}^{3/2}
\over g_n^{Z-1}}\exp\left(-{E_\infty^Z(n,1)\over T_e}\right)\,{\rm cm}^{-3},
\leqno{(17)}$$
where $g_n^Z$ is the statistical weight for level $n$ of charge state
$Z$ and $E_\infty^Z(n,1)$ is the ionization energy of the neutral atom
initially in level $(n, 1)$, given by Eq.~(2).

In a steady state at high electron density,
$${N_eN\hbox{*}(Z)\over N\hbox{*}(Z-1)}={S(Z-1)\over\alpha_3}, \leqno{(18)}$$
a function only of $T$.

Conditions for LTE:$^{39}$

(a) Collisional and radiative excitation rates for a level $n$ must satisfy
$$Y_{nm}\approxgt10A_{nm}.\leqno{(19)}$$

(b) Electron density must satisfy
$$N_e\approxgt 7\times 10^{18}Z^7n^{-17/2} (T/E_\infty^Z)^{1/2} {\rm cm}^{-3}.
\leqno{(20)}$$

Steady state condition in corona model:
$$ {N(Z-1) \over N(Z)} = {\alpha_r \over S(Z-1)}. \leqno{(21)}$$

Corona model is applicable if$^{40}$
$$10^{12}{t_I}^{-1}<N_e<10^{16}{T_e}^{7/2}\,{\rm cm}^{-3},\leqno{(22)}$$
where $t_I$ is the ionization time.
\vfil\eject\end
```

Ionization Equilibrium Models

Saha equilibrium:[39]

$$\frac{N_e N_1{}^*(Z)}{N_n{}^*(Z-1)} = 6.0 \times 10^{21} \frac{g_1^Z T_e{}^{3/2}}{g_n^{Z-1}} \exp\left(-\frac{E_\infty^Z(n,l)}{T_e}\right) \text{cm}^{-3}. \tag{17}$$

where g_n^Z is the statistical weight for level n of charge state Z and $E_\infty^Z(n,l)$ is the ionization energy of the neutral atom initially in level (n,l), given by Eq. (2).

In a steady state at high electron density,

$$\frac{N_e N^*(Z)}{N^*(Z-1)} = \frac{S(Z-1)}{\alpha_3}, \tag{18}$$

a function only of T.

Conditions for LTE:[39]

(a) Collisional and radiative excitation rates for a level n must satisfy

$$Y_{nm} \gtrsim 10 A_{nm}. \tag{19}$$

(b) Electron density must satisfy

$$N_e \gtrsim 7 \times 10^{18} Z^7 n^{-17/2} (T/E_\infty^Z)^{1/2} \text{cm}^{-3}. \tag{20}$$

Steady state condition in corona model:

$$\frac{N(Z-1)}{N(Z)} = \frac{\alpha_r}{S(Z-1)}. \tag{21}$$

Corona model is applicable if[40]

$$10^{12} t_I{}^{-1} < N_e < 10^{16} T_e{}^{7/2} \text{cm}^{-3}. \tag{22}$$

where t_I is the ionization time.

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{\headfont Radiation}

\inndent
{\it N.\ B.}  Energies and temperatures are in eV; all other quantities are in
cgs units except where noted.  $Z$ is the charge state ($Z=0$ refers to a
neutral atom); the subscript $e$ labels electrons.  $N$ is number density.

Average radiative decay rate of a state with principal quantum number $n$
is
$$A_n=\sum_{m<n}A_{nm}=1.6\times 10^{10}Z^4n^{-9/2}\,\rm  sec. \leqno{(23)}$$

Natural linewidth $(\Delta E$ in eV):
$$\Delta E \,\Delta t=h=4.14\times 10^{-15}\,{\rm eV\,sec}, \leqno{(24)}$$
where $\Delta t$ is the lifetime of the line.

Doppler width:
$$\Delta \lambda/\lambda=7.7\times 10^{-5} (T/\mu)^{1/2},\leqno{(25)}$$
where $\mu$ is the mass of the emitting atom or ion scaled by the proton mass.

Optical depth for a Doppler-broadened line:$^{39}$
$$\tau=1.76\times 10^{-13}\lambda(Mc^2/kT)^{1/2}NL=5.4\times
10^{-9}\lambda(\mu/T)^{1/2}NL,\leqno{(26)}$$

where $\lambda$ is the wavelength and $L$ is the physical depth of the plasma;
$M$, $N$, and $T$ are the mass, number density, and temperature of the absorber;
$\mu$ is $M$ divided by the proton mass.  Optically thin means $\tau < 1$.

Resonance absorption cross section at center of line:
$$\sigma_{\lambda=\lambda_c}=5.6\times 10^{-13} \lambda^2/\Delta
\lambda\,{\rm cm}^2. \leqno{(27)}$$

Wien displacement law (wavelength of maximum black-body emission):
$$\lambda_{\max}=2.50\times 10^{-5}T^{-1}\,{\rm cm}.\leqno{(28)}$$

Radiation from the surface of a black body at temperature $T$:
$$W=1.03\times10^5T^4\,{\rm watt/cm}^2. \leqno{(29)}$$
\vfil\eject\end
```

Radiation

N. B. Energies and temperatures are in eV; all other quantities are in cgs units except where noted. Z is the charge state ($Z = 0$ refers to a neutral atom); the subscript e labels electrons. N is number density.

Average radiative decay rate of a state with principal quantum number n is

(23) $$A_n = \sum_{m<n} A_{nm} = 1.6 \times 10^{10} Z^4 n^{-9/2} \text{ sec.}$$

Natural linewidth (ΔE in eV):

(24) $$\Delta E \, \Delta t = h = 4.14 \times 10^{-15} \text{ eV sec},$$

where Δt is the lifetime of the line.

Doppler width:

(25) $$\Delta\lambda/\lambda = 7.7 \times 10^{-5} (T/\mu)^{1/2},$$

where μ is the mass of the emitting atom or ion scaled by the proton mass.

Optical depth for a Doppler-broadened line:[39]

(26) $$\tau = 1.76 \times 10^{-13} \lambda (Mc^2/kT)^{1/2} NL = 5.4 \times 10^{-9} \lambda (\mu/T)^{1/2} NL,$$

where λ is the wavelength and L is the physical depth of the plasma; M, N, and T are the mass, number density, and temperature of the absorber; μ is M divided by the proton mass. Optically thin means $\tau < 1$.

Resonance absorption cross section at center of line:

(27) $$\sigma_{\lambda=\lambda_c} = 5.6 \times 10^{-13} \lambda^2/\Delta\lambda \text{ cm}^2.$$

Wien displacement law (wavelength of maximum black-body emission):

(28) $$\lambda_{\max} = 2.50 \times 10^{-5} T^{-1} \text{ cm}.$$

Radiation from the surface of a black body at temperature T:

(29) $$W = 1.03 \times 10^5 T^4 \text{ watt/cm}^2.$$

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Bremsstrahlung from hydrogen-like plasma:$^{26}$
$$P_{\rm Br}=1.69\times 10^{-32}N_e{T_e}^{1/2}\sum\left[Z^2N(Z)\right]\,{\rm
watt/cm}^3,\leqno{(30)}$$
where the sum is over all ionization states $Z$.

Bremsstrahlung optical depth:$^{41}$
$$\tau=5.0\times 10^{-38}N_eN_iZ^2 \overline{g}LT^{-7/2},\leqno{(31)}$$
where $\overline{g} \approx 1.2$ is an average Gaunt factor and $L$ is the
physical path length.

Inverse bremsstrahlung absorption coefficient$^{42}$ for radiation of angular
frequency $\omega$:
$$\kappa = 3.1\times10^{-7}Z{n_e}^2 \ln \Lambda \,T^{-3/2} \omega^{-2}
(1-\omega_p^2/\omega^2)^{1/2}\,{\rm cm}^{-1};\qquad\leqno(32)$$
here $\Lambda$ is the electron thermal velocity divided by $V$, where $V$ is
the larger of $\omega$ and $\omega_p$ multiplied by the larger of $Ze^2/kT$ and
$\hbar /(mkT)^{1/2}$.

Recombination (free-bound) radiation:
$$P_r=1.69\times 10^{-32}N_eT_e{}^{1/2} \sum \left [ Z^2N(Z) \left (
{E_\infty^{Z-1}\over T_e} \right) \right]\,{\rm watt/cm}^3. \leqno{(33)}$$

Cyclotron radiation$^{26}$ in magnetic field {\bf B}:
$$P_c=6.21\times 10^{-28}B^2N_eT_e\,{\rm watt/cm}^3. \leqno{(34)}$$

For $N_ekT_e = N_ikT_i = B^2/16\pi$ ($\beta=1$, isothermal plasma),$^{26}$
$$P_c=5.00\times 10^{-38}N_e^2T_e^2\,{\rm watt/cm}^3. \leqno{(35)}$$

Cyclotron radiation energy loss $e$-folding time for a single electron:$^{41}$
$$t_c \approx {9.0\times 10^8B^{-2} \over 2.5 + \gamma}\,{\rm sec},
\leqno{(36)}$$
where $\gamma$ is the kinetic plus rest energy divided by the rest energy
$mc^2$.

Number of cyclotron harmonics$^{41}$ trapped in a medium of finite depth $L$:
$$m_{\rm tr}=(57\beta BL)^{1/6}, \leqno{(37)}$$
where $\beta=8\pi NkT/B^2$.

Line radiation is given by summing Eq. (9) over all species in the plasma.
\vfil\eject\end
```

Bremsstrahlung from hydrogen-like plasma:[26]

(30) $$P_{\mathrm{Br}} = 1.69 \times 10^{-32} N_e T_e{}^{1/2} \sum \left[Z^2 N(Z) \right] \text{ watt/cm}^3.$$

where the sum is over all ionization states Z.

Bremsstrahlung optical depth:[41]

(31) $$\tau = 5.0 \times 10^{-38} N_e N_i Z^2 \overline{g} L T^{-7/2}.$$

where $\overline{g} \approx 1.2$ is an average Gaunt factor and L is the physical path length.

Inverse bremsstrahlung absorption coefficient[42] for radiation of angular frequency ω:

(32) $$\kappa = 3.1 \times 10^{-7} Z n_e{}^2 \ln \Lambda\, T^{-3/2} \omega^{-2} (1 - \omega_p^2/\omega^2)^{1/2} \text{ cm}^{-1};$$

here Λ is the electron thermal velocity divided by V, where V is the larger of ω and ω_p multiplied by the larger of Ze^2/kT and $\hbar/(mkT)^{1/2}$.

Recombination (free-bound) radiation:

(33) $$P_r = 1.69 \times 10^{-32} N_e T_e{}^{1/2} \sum \left[Z^2 N(Z) \left(\frac{E_\infty^{Z-1}}{T_e} \right) \right] \text{ watt/cm}^3.$$

Cyclotron radiation[26] in magnetic field $\mathbf{B}$:

(34) $$P_c = 6.21 \times 10^{-28} B^2 N_e T_e \text{ watt/cm}^3.$$

For $N_e kT_e = N_i kT_i = B^2/16\pi$ ($\beta = 1$, isothermal plasma).[26]

(35) $$P_c = 5.00 \times 10^{-38} N_e^2 T_e^2 \text{ watt/cm}^3.$$

Cyclotron radiation energy loss e-folding time for a single electron:[41]

(36) $$t_c \approx \frac{9.0 \times 10^8 B^{-2}}{2.5 + \gamma} \text{ sec}.$$

where γ is the kinetic plus rest energy divided by the rest energy mc^2.

Number of cyclotron harmonics[41] trapped in a medium of finite depth L:

(37) $$m_{\mathrm{tr}} = (57\beta BL)^{1/6},$$

where $\beta = 8\pi NkT/B^2$.

Line radiation is given by summing Eq. (9) over all species in the plasma.

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Most of the formulas and data in this collection are well known and for
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ii) to indicate where to find derivations, explanations,
examples, etc., which have been omitted from this compilation.
Additional material can also be found in D.~L. Book, NRL Memorandum Report No.~
3332 (1977).
\smallskip\frenchspacing
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