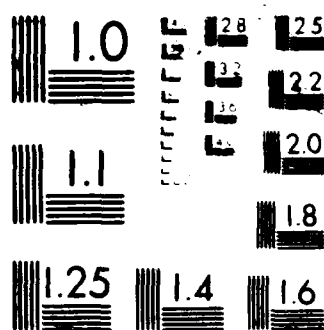


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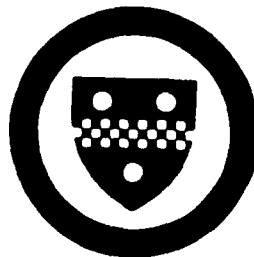
ON THE LEAST SQUARES ESTIMATOR IN  
MOVING AVERAGE MODELS OF ORDER ONE

H. A. Niroomand Chapah & M. Bhaskara Rao  
Center for Multivariate Analysis  
University of Pittsburgh

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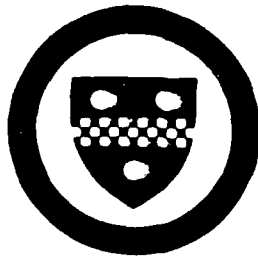
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>A simple expression is derived in this paper for the error sum of squares in the context of moving average models of order one. A computer program is developed to estimate the parameter of a moving average model of order one based on the method of least squares. (Keywords: time series analysis; consistency; algorithms; inference; multivariate analysis.) |                       |                                                                                   |

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MOVING AVERAGE MODELS OF ORDER ONE**

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# ON THE LEAST SQUARES ESTIMATOR IN MOVING AVERAGE MODELS OF ORDER ONE

By H.A. Niroomand Chapah and M. Bhaskara Rao  
University of Sheffield, UK

Keywords : Moving average model; Least squares estimator; Consistency

## Language

Fortran 77

## Description and purpose

Given a time series data, estimate  $\beta$  in the moving average model  
 $Y_t = e_t + \beta e_{t-1}$ ,  $t = 1, 2, 3, \dots$  of order one using the method of least squares.

## Theory

Let  $Y_t = e_t + \beta e_{t-1}$ ,  $t = 1, 2, 3, \dots$  be a moving average process of order one, where  $e_0 = 0$  and  $e_1, e_2, \dots$  is a sequence of independent identically distributed random variables with mean zero and variance  $\sigma^2$ . There are several methods of estimation are available in the literature for estimating  $\beta$  for a given time series data  $y_1, y_2, \dots, y_N$ . McClave (1974) compared the performance of some five estimators of  $\beta$  for moderate sample sizes by simulating the process  $Y_t$ ,  $t = 1, 2, \dots, 100$ . All these methods use approximations of one kind or the other and involve selection of some indices for a good degree of approximation. Consequently, the execution of these methods looks complex and it is natural to search for a simple method which performs competitively well with these methods. The method of least square is intuitively very appealing, and recently, under some conditions, Macpherson and Fuller (1983) showed that the least squares

estimator is consistent for  $\beta$  in  $[-1,1]$ . We have compared the performance of least squares estimator for moderate sample sizes vis-a-vis with the five estimators examined by McClave (1974). See Niroomand Chapeh and Bhaskara Rao (1987). The least squares estimator comes out better than these five estimators. Additionally, as the present article demonstrates, the execution of the least squares method is much simpler than these five methods.

Following Macpherson and Fuller (1983), the least squares estimator of  $\beta$  is that value of  $\theta$  in  $[-1,1]$  that minimizes

$$Q_N(\theta) = \sum_{t=1}^N [e_t(Y;\theta)]^2,$$

where  $e_t(Y;\theta) = Y_t - \theta e_{t-1}(Y;\theta)$ ,  $t = 1, 2, \dots, N$  and  $e_0(Y;\theta) = 0$ . It works out that

$$e_k(Y;\theta) = Y_k - \theta Y_{k-1} + \theta^2 Y_{k-2} - \dots + (-1)^{k-1} \theta^{k-1} Y_1,$$

$k = 1, 2, \dots, N$ . Consequently,  $Q_N(\theta)$  is a polynomial in  $\theta$  of degree  $2N-2$ .

More explicitly, for  $N$  even,

$$\begin{aligned} Q_N(\theta) = & \sum_{i=1}^N Y_i^2 + (\sum_{i=1}^{N-1} Y_i^2 + 2 \sum_{i=1}^{N-2} Y_i Y_{i+2}) \theta^2 + \\ & (\sum_{i=1}^{N-2} Y_i^2 + 2 \sum_{i=1}^{N-3} Y_i Y_{i+2} + 2 \sum_{i=1}^{N-4} Y_i Y_{i+4}) \theta^4 + \dots + \\ & (\sum_{i=1}^{N/2+1} Y_i^2 + 2 \sum_{i=1}^{N/2} Y_i Y_{i+2} + \dots + 2 \sum_{i=1}^2 Y_i Y_{i+N-2}) \theta^{N-2} + \\ & (\sum_{i=1}^{N/2} Y_i^2 + 2 \sum_{i=1}^{N/2-1} Y_i Y_{i+2} + \dots + 2 \sum_{i=1}^1 Y_i Y_{i+N-2}) \theta^N + \\ & (\sum_{i=1}^{N/2-1} Y_i^2 + 2 \sum_{i=1}^{N/2-2} Y_i Y_{i+2} + \dots + 2 \sum_{i=1}^1 Y_i Y_{i+N-4}) \theta^{N+2} + \\ & \dots \dots + Y_1^2 \theta^{2N-2} - 2(\sum_{i=1}^{N-1} Y_i Y_{i+1}) \theta \end{aligned}$$

$$\begin{aligned}
 & - 2(\sum_{i=1}^{N-2} Y_i Y_{i+1} + \sum_{i=1}^{N-3} Y_i Y_{i+3}) \theta^3 - \dots \\
 & - 2(\sum_{i=1}^{N/2} Y_i Y_{i+1} + \sum_{i=1}^{N/2-1} Y_i Y_{i+3} + \dots + \sum_{i=1}^1 Y_i Y_{i+N-1}) \theta^{N-1} \\
 & - 2(\sum_{i=1}^{N/2-1} Y_i Y_{i+1} + \sum_{i=1}^{N/2-2} Y_i Y_{i+3} + \dots + \sum_{i=1}^1 Y_i Y_{i+N-3}) \theta^{N+1} \\
 & - \dots - 2Y_1 Y_2 \theta^{2N-3}.
 \end{aligned}$$

The above expression looks formidable to include in a computer program. However, the above can be simplified as follows. Let  $A$  be the matrix of order  $N \times N$  defined by

$$A = \begin{bmatrix}
 Y_1 & 0 & 0 & 0 & \dots & 0 & 0 \\
 Y_2 & -Y_1 & 0 & 0 & \dots & 0 & 0 \\
 Y_3 & -Y_2 & Y_1 & 0 & \dots & 0 & 0 \\
 Y_4 & -Y_3 & Y_2 & -Y_1 & \dots & 0 & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 Y_{N-1} & -Y_{N-2} & Y_{N-3} & -Y_{N-4} & \dots & (-1)^{N-2} Y_1 & 0 \\
 Y_N & -Y_{N-1} & Y_{N-2} & -Y_{N-3} & \dots & (-1)^{N-2} Y_2 & (-1)^{N-1} Y_1
 \end{bmatrix}$$

Let  $\theta$  be the column vector of order  $N \times 1$  defined by

$$\theta^T = [1, \theta, \theta^2, \theta^3, \dots, \theta^{N-1}],$$

where  $T$  stands for operation transpose. It can be verified that

$$Q_N(\theta) = \theta^T (A^T A) \theta.$$

We use this simple form of  $Q_N(\theta)$  in the computer program to estimate  $\theta$ .

### A case study

We used the method of least squares to fit a first order moving average model to the first order differences of the time series data "IBM Common Stock Closing Prices - Daily, 17th May, 1961 to 2nd November, 1962". See Box and Jenkins (1976, p.239 and p.526). The fitted model works out to be

$$\nabla Y_t = e_t + 0.089e_{t-1},$$

$\nabla Y_t = Y_t - Y_{t-1}$  and  $Y_t$ 's are the original series. Box and Jenkins obtained the estimate of  $\beta$  to be 0.09.

### Structure

SUBROUTINE ZXMD(FCN,M,NSIG,A1,B1,NSRCH,X,F,WORK,IWORK,IER)

Formal parameters

|        |               |                                                                                                          |
|--------|---------------|----------------------------------------------------------------------------------------------------------|
| M      | Integer       | Input: number of unknowns parameters                                                                     |
| NSIG   | Integer       | Input: number of digits of accuracy required in the parameter estimates                                  |
| A1, B1 | Real arrays   | Constraint vectors of length of M<br>Input: $X(I)$ is required to satisfy - $A1(I) \leq X(I) \leq B1(I)$ |
| NSRCH  | Integer       | Number of starting points to be generated<br>Input: suggested value = $\min(2**M+5, 100)$                |
| X      | Real array    | Vector of length M containing the final parameter estimates (output)                                     |
| F      | Real          | Value of the function at the final parameter estimates (output)                                          |
| WORK   | Real array    | Work vector of length $M*(M+1)/2+11*M$                                                                   |
| IWORK  | Integer array | Work vector of length M                                                                                  |
| IER    | Real          | Error parameter (output)                                                                                 |

Terminal error

IER=129 indicates that the algorithm has converged to a point which may only be a saddle point

IER=130 indicates that it was not possible to calculate the solution to NSIG digits (See remarks)

IER=131 indicates that iteration was terminated after  $200*(N+1)$  function evaluations (See remarks)

IER=132 indicates that  $Al(I).GE.B1(I)$  for some  $I = 1,2,...,N$ . No attempt is made to find the minimum in this case.

Remarks

When IER is returned as 130 or 131, the parameter estimates in X may not be reliable. Further checking should be performed. Use of a larger NSRCH may produce more reliable parameter estimates.

Auxiliary algorithms

SUBROUTINE ZXMWDR will use the FCN(M,X,F) to minimize F

M Integer Input: number of unknown parameters

X Real array Vector of length M containing the final parameter estimates (output)

F Real Output: Value of the function at the final parameter estimates

SUBROUTINE VMULFF(AMATT,AMAT,L,KA,KB,IA,IB,C,IC,IER) will find the multiplication of two matrices AMATT and AMAT

Formal parameters

AMATT Real array Input: N by N matrix stored in full storage mode

AMAT Real array Input: N by N matrix stored in full storage mode

L Integer Input: number of rows in AMATT

KA Integer Input: number of columns in AMATT

KB Integer Input: number of columns in AMAT

IA Integer Input: row dimension of matrix AMATT

IB Integer Input: row dimension of matrix AMAT

C        Real array    Output: N by N matrix containing the product  $C = AMATT * AMAT$   
IC       Integer        Input: row dimension of matrix C  
IER      Integer        Output: Error indicator  
                         IER=129 indicates that AMATT or AMAT or C was dimensioned  
                         incorrectly

#### References

- Box, G.E.P. and Jenkins, G.M. (1976) Time Series Analysis : Forecasting and Control.  
Holden-Day, Oakland, California
- Macpherson, B.D. and Fuller, W.A. (1983) - Consistency of the least squares estimator  
of the first order moving average parameter, Ann.Stat., 11,326-329.
- McClave, J.T. (1974) - A comparison of moving average estimation procedures,  
Comm.Stat., 3, 865-883.
- Niroomand Chapeh, H.A. and Bhaskara Rao, M. (1987) - A comparison of various  
methods of estimation in moving average models (in preparation).

C THIS PROGRAM WILL FIT THE FIRST ORDER MOVING AVERAGE MODEL TO DATA

INTEGER IER,N,L,KA,KB,IA,IB,IC,NSIG,NSRCH,IWORK(1),M,IV,N=

REAL X(1),A1,B1,WORK(12),F,AMAT(N,N),AMATT(N,N),C(N,N)=

EXTERNAL FCN

COMMON N,C

F1=0.0

F2=0.0

OPEN(UNIT=100, FILE='DATA.DAT',STATUS='OLD')

OPEN(UNIT=99, FILE='RES.DAT', STATUS='NEW')

READ(100,\*) IV

M=1

NSIG=4

NSRCH=7

A1=1.0

E1=1.0

N=

L=N

KA=N

KB=N

IA=N

IB=N

IC=N

C THE GIVEN DATA IS PUT INTO THE MATRIX A AS EXPLAINED IN THE THEORY

DO 2 J=1,N

DO 2 I=J,N

```

      AMATT(1,1) = -1.0**J-1 *IV/I-J+1
1      CONTINUE
      DO 100      I=1,N
      DO 100      J=1,N
      AMATT(1,1) = AMAT(1,1)
      CONTINUE
      CALL  WCHIFF(AMATT,AMAT,1,KA,KB,IA,IB,C,IC,IER)
      CALL  EXCHWD(FCN,M,NSIG,A1,B1,NSRCH,X,F,WORK,IWORK,IER)
      WRITE (4,100)      X 1.
100     FORMAT ('FINAL RES IS 1- AMAT', F10.4)
      STOP
      END
      SUBROUTINE 1.FCN M,X,F
      INTEGER      M,I,J,N
      REAL          X(1),F,C N=N,N=1
      COMMON        N,C
      FI=0.0
      DO 100      J=0,N-1
      SUM1=0.0
      DO 100      I=1,J+1
20      SUM1=SUM1+0.1*(J+1-I)
200     FI=FI+X 1.0**J*SUM1
      FD=0.0
      DO 100      J=1*N-1,N,-1

```

```
      SUM2=0.0
      DO      30      I=J+2-N,N
30      SUM2=SUM2+C(I,J+2-I)
300      F2=F2+(X(1)**J)*SUM2
      F=F1+F2
      RETURN
      END
```

Concluding remarks : This work is useful for drawing inferences for signal process models when the observations form a moving average model of order one. A manuscript is under preparation detailing applications of the results of this paper to signal processing.

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