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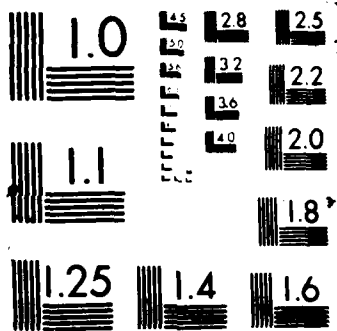
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I - RESEARCH SUMMARY

During the last year significant results have been obtained in a variety of problems. Here we give a brief description of these results and, under separate cover, we remit some preprints wherein details may be found.

FINITE ELEMENT METHODS FOR THE LADYZHENSKAYA MODEL OF VISCOUS FLOW

The Navier-Stokes equations are the most commonly used mathematical model describing the motion of viscous incompressible fluids. Although this model is generally accepted and is even thought by many to be valid for turbulent flows, it does have some shortcomings. Of the latter, the most important one is that it is not known and not believed that solutions are globally, in time, unique.

Among the other models proposed for viscous incompressible flows is the one of Ladyzhenskaya. Her model differs from the Navier-Stokes model only in the

constitutive equation used. Whereas for the Navier-Stokes equations one uses a linear constitutive model, Ladyzhenskaya introduces a nonlinear one. One advantage of her model is that one can show, in both 2-D and 3-D, that solutions are globally unique in time.

For the stationary case, the Ladyzhenskaya equations are given by

$$\operatorname{div} \mathbf{u} = 0$$

and

$$-\operatorname{div} \left[A(\mathbf{u}) \left(\operatorname{grad} \mathbf{u} + (\operatorname{grad} \mathbf{u})^T \right) \right] + \mathbf{u} \cdot \operatorname{grad} \mathbf{u} + \operatorname{grad} p = \mathbf{f}$$

on some domain Ω and where $\mathbf{u}$ is the velocity, p the pressure, $\mathbf{f}$ the given body force and

$$A(\mathbf{v}) = \gamma_0 + \gamma_1 |\mathbf{v}|^{p-2}, \quad p \geq 2.$$

The Navier-Stokes model corresponds to $\gamma_1 = 0$ (or $p=2$). Ladyzhenskaya herself has shown that these equations, with suitable boundary conditions, always have a solution. She did not explore the uniqueness question with respect to the stationary equations. It is well known that one can show that the Navier-Stokes equations have a unique solution whenever

$$N \|\mathbf{f}\|_{-1} / \gamma_0^2 \leq 1.$$

where N is a constant such that

$$\int_{\Omega} \mathbf{w} \cdot \operatorname{grad} \mathbf{u} \cdot \mathbf{v} \leq N \|\mathbf{u}\|_1 \|\mathbf{v}\|_1 \|\mathbf{w}\|_1 \quad \text{for all } \mathbf{u}, \mathbf{v}, \mathbf{w} \in H_0^1(\Omega).$$

For the Ladyzhenskaya equations we have obtained an improved uniqueness condition in the sense that her equations can be shown to have a unique solution for "larger" values of the data f .

We have also studied the finite element approximation of the Ladyzhenskaya model. We have shown that one may use the same finite element spaces for the velocity and pressure as are used for Navier-Stokes calculations. Furthermore, we have obtained error estimates for the finite element solution which are identical, with respect to the rate of convergence, as those available for finite element approximations of the Navier-Stokes equations.

At this time we are writing a code to implement some finite element methods for the Ladyzhenskaya model. The main goals of these computations are to see what effect her model has on actual solutions, as well as to study how one may implement finite element methods and solution algorithms for that model. Once the computations have been carried out, a paper will be written concerning both the theoretical and computational results.

SURVEY OF FINITE ELEMENT METHODS FOR INCOMPRESSIBLE VISCOUS FLOWS

Over the past few years, mainly with support from AFOSR, we have been involved in various aspects of research in finite element methods for incompressible viscous flows. As a result of an invitation to give a survey lecture on this subject at a Finite Element Workshop held at ICASE, we prepared a survey paper. This paper collects many important algorithmic and mathematical results obtained by us, our colleagues, and the finite element community as a whole. A copy of this paper is enclosed and will appear in a Workshop Proceedings which is to be published by Springer.

We feel that this paper is not of sufficient length to adequately cover the many aspects of the subject considered. Therefore we are presently rewriting and expanding the paper so that it will result in a valuable survey of algorithms and the analysis of algorithms for the Navier-Stokes equations. Our aim is to have it published by *SIAM Review*, or, if its length and scope warrant it, as a book, perhaps in a SIAM or Springer series.

FINITE ELEMENT METHODS FOR HYPERBOLIC EQUATIONS

We have studied various finite element methods for first order hyperbolic systems of differential equations. First we consider equations of the positive symmetric type, i.e., of the Friedrichs type. Such equations arise in most applications, e.g., fluid mechanics and electromagnetics. Standard finite element methods for these type equations are not optimally accurate in the sense that the error in the finite element approximation does not converge at the same rate as the error in the best approximation to the solution of the differential equation in the finite element spaces employed. We study a nonstandard finite element method which yields optimal accuracy in the H^1 norm and, for sufficiently regular solutions, in the L^2 norm as well. The method may be viewed as a combined Galerkin and least squares method.

If we denote the hyperbolic system by

$$L(u) = 0$$

for some unknown function u , the method is based on the weak formulation

$$\left(L(u) , v + \delta L_0(v) \right) = 0$$

where $L_0(\cdot)$ is operator related to the principal part of $L(\cdot)$, δ is a parameter related to the grid size, and the test functions v can be chosen in the same space in which the solution u is sought. We show that with the proper choice of the parameter δ that the optimal estimates mentioned above are obtained.

Another aspect of this research is based on our work and those of others on first order elliptic systems of differential equations. These methods do not apply to general hyperbolic systems, but fortunately are useful in many applications, e.g., again, fluid mechanics and electromagnetics. Using the work on elliptic differential equations, a suitable projection, onto the finite element space, of the solution of the hyperbolic system is defined, and the error in this projection estimated. This projection is then used to obtain analogous estimates for the error in the finite element approximation of the solution of the hyperbolic equations.

WORK OF WILLIAM LAYTON SUPPORTED IN PART BY THE GRANT

During the past year the Grant has provided partial support for Professor William Layton of the Georgia Institute of Technology during his visit to Carnegie-Mellon University. We now give a brief account of some of his work which was partially supported by the grant.

One subject of interest to Layton is the approximation of singularly perturbed problems. For example he has studied nonstandard defect correction methods applied to convection-diffusion problems with dominant convection

terms. It is shown that these methods converge uniformly in the viscosity parameter and with optimal rates, outside of boundary layers. Numerical codes have been written implementing the method for 1-D and 2-D problems. Another aspect of this research is the approximation of the derivatives of the solution of singularly perturbed two-point boundary problems. It is shown that outside boundary layers certain difference quotients of a computed approximation to the solution give optimal approximations to the derivative of the solution, uniformly in the viscosity parameter. By looking at stretched variables, an accurate formula for approximating the derivative within the boundary layer is also provided.

Another research project was concerned with the approximation of solutions of second order elliptic differential equations. For example, it is shown that standard central difference discretizations of linear second order elliptic equations without lower order terms are of monotone type, although not necessarily positive. Consequently, a maximum principle holds. Another aspect of this research concerns itself with Galerkin approximations to the solution of semilinear elliptic equations of the type

$$Lu = f(u)$$

where L is a linear second order elliptic operator. Optimal error estimates are obtained whenever df/du is bounded inside the resolvent set of L .

II - PERSONNEL SUPPORTED BY THE GRANT

1. Max Gunzburger, Professor of Mathematics; 3 summer months; Principal investigator.
2. William Layton, Visiting Associate Professor; partial academic year support.
3. Qiang Du and Amnon Meir, Graduate Students; summer support and partial academic year support.

III - PUBLICATIONS RELEVANT TO THE GRANT

1. Finite element approximations for first order elliptic systems in three dimensions; preprint enclosed; (C. Chang and M. Gunzburger).
2. Finite element methods for the Ladyzhenskaya model of incompressible viscous flows; in preparation; (Q. Du and M. Gunzburger).
3. Nonstandard finite element methods for hyperbolic systems; in preparation; (Q. Du, M. Gunzburger and W. Layton).
4. Mathematical aspects of finite element methods for incompressible viscous flow problems; to appear in the Proceedings of the ICASE Workshop on Finite Element Methods, to be published by Springer; (M. Gunzburger). Preprint enclosed.

5. *A study of defect-correction, finite difference methods for convection diffusion equations; preprint enclosed; (V. Ervin and W. Layton).*
6. *On the approximation of the derivatives of singularly perturbed boundary value problems; preprint enclosed; (V. Ervin and W. Layton).*
7. *On central difference approximations to general second order elliptic equations; preprint enclosed; (W. Layton).*
8. *L^2 -estimates for Galerkin methods for semilinear elliptic equations; preprint enclosed; (E. Harrell and W. Layton).*
9. *A guide to finite element methods for incompressible viscous flows; in preparation; and expanded version of 3 which will be submitted as a review article to a major journal or be published as monograph; (M. Gunzburger).*

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