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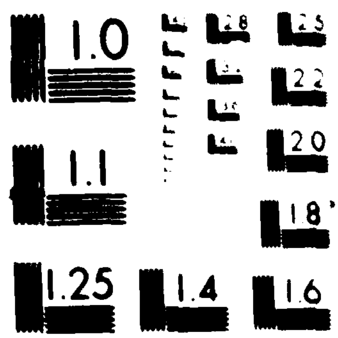
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Interim Scientific Report

July 1, 1985 - September 30, 1986

AFOSR-TR- 87-1188

Estimation and Control of Distributed Models for Certain Elastic Systems Arising in Large Space Structures

AFOSR Grant No. AFOSR-84-0271

Principal Investigator:

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1. Summary.

During the reporting period progress was made toward the goal of developing efficient and accurate estimation and control algorithms for elastic structures composed of beams and plates. Specificially, theory and algorithms were developed for the estimation of elastic coefficients and damping terms for static and dynamic models of beams and plates, the control of deformation of dynamic models of beams and plates by means of actuators of small support, and the optimal placement of actuators to control the deformation of beams and plates. These efforts have produced computer codes on which experimentation is currently being conducted to test methods and algorithms.

2. Research Objectives.

The research objective of this project is to study the estimation and control of elastic systems composed of beams and plates in order to develop efficient and accurate control and estimation algorithms. In the case of estimation basic to this goal is the development of an understanding of properties of the parameter to state mapping, an approximation theory associated with the particular models and minimization problems, and the suitability of different minimization algorithms for efficient codes for various problems. In control of prime importance is to determine properties of optimal controls and feedback, best location based on design of the actuators and the geometry and elastic properties of the body, and suitable algorithms and codes for control. Toward these objectives the work during the past year has centered primarily on the estimation and control of both static and dynamic linear models.

Our considerations for estimation problems have centered around the problems of the form

(1) $$Au = \Delta(a_0 \Delta u) = f \quad in \quad \Omega$$

and

(2) $$u_{tt} + Bu_t + Au = f \quad in \quad \Omega \times (0,T)$$

where B is of the form

$$B = \Delta(b_0 \Delta),$$

Ω is a bounded domain in $\Re^2$ with a suitable boundary, and appropriate boundary and initial conditions are given. The estimation problem may be stated as follows. Given an observation z taken in an observation space Z of the deformation, find a parameter $\tilde{a}_0$ from within an admissible set Qad in a parameter space Q that minimizes a fit-to-data functional

(3) $$J(a_0) \; ||Cu(a_0) - z||_Z^2 + \epsilon ||a_0||_Q^2$$

over the set Qad. Here C denotes an operator that takes the solutions $u = u(a) \; in \; X$ to the states $Cu(a_0) \; in \; Z$.

To solve this problem numerically, it is important to analyze such properties of the optimal estimator as regularity and stability with respect to data and constraints. Regularity results of the optimal estimator have enabled us to develop a suitable approximation theory. Moreover, stability with respect to the observation z is indespensible in producing reliable methods.

The objective in control over the last year was to develop algorithms to control plates and beams to a desired deformation. The motivating problem is that of controlling a large mirror (modeled as a plate) to a sepcified deformation. Hence, given a bounded domain $\Omega \subset \Re^2$ with equation (2) in which

$$f = \sum_{i=1}^{w} \beta_i(t)\Phi_i \tag{4}$$

for Φ_i in $H^{-2}(\Omega)$, of importance is to find $\hat{\beta}(t) = (\beta_1(t), ..., \beta_w(t))$ in U minimizing

$$J(\beta) = \int_0^T \int_\Omega (u(x,t;\beta) - z(x,t))^2 dxdt + \epsilon\|\hat{\beta}\|_U^2 \tag{5}$$

where U for example may be $(L^2(0,T))^w$. The functions Φ_i determine the control mechanism and may represent control at a point, distributed along a curve, or over a subset of Ω of positive measure. Furthermore, in the case of point controls a problem of interest is to determine optimal locations of the actuators.

3. Status of Research.

Over the last year we have sought to extend results obtained in [2] and [3] to more general cases for plates. In [2] estimation for static plates and beams is considered in which the theory is basically one dimensional. The plates are rectangular and the flexural rigidity coefficient is assumed to be expressible as a tensor product of H^1 functions. For these special problems we consider H^1-regularization and develop an approximation theory using the resulting regularity properties and tensor products of linear splines to approximate a_0. In the case in which no such assumption is made on the admissible class of flexural rigidities or the geometry of the domain (other than smoothness), H^2-regularization is used. Regularity properties arising from this regularization enable us to relate the admissible sets from approximating problems to that of the original Qad. Numerical studies of test problems have produced very accurate results. This work is at the submission stage [9].

For dynamic plates we have written codes for the estimation of flexural rigidity and damping coefficients. These codes use H^2-regularization and in numerical test problems have proved to be accurate and stable. Currently, we are developing the theory for these problems. We are seeking to determine specific regularity assumptions on the function f, the initial conditions u_0 and u_1, the smoothness of the domain boundary Γ, and the observation z to obtain an approximation theory. In addition for the regularized problems, we have determined sufficient conditions for stability with respect to data.

In related work on parabolic equations. We are considering regularity properties for estimation problems to find diffusion coefficients that are dependent on both space and time variables. This work [7] is with Professor K. Kunisch and is approximately at the submission stage. Also, for parabolic problems we are considering generalized problems with Professor S. Gutman [8] to estimate discontinuous coefficients. In these considerations the existence is studied by means of the notion of G-convergent operators. These ideas carryover to elliptic problems and so will have application to static plate problems.

Concerning the control of plates by means of point controls and controls of small support, our work on dynamic models of plates extends that for static in [4]. In the dynamic case the formulation of the problem is determined by the presence of damping. In the case of little or no damping [5] the space U is taken to be $(H^1(0,T))^w$. On the other hand for damped cases [6], U may be taken to be $(L^2(0,T))^w$.

The controllers Φ_i in (4) may be a function in $L^2(\Omega)$ or may be a Dirac delta measure δ_{x_i} with mass at x_i. We have coded the optimization problem (5) in both cases with good results for test cases.

For the case that $\Phi_i = \Phi(\cdot, x_i)$ the optimal vector β is in fact a function of the actuator location $X = (x_1, ..., x_w)$. Hence, mappings $\Omega^w \mapsto U$ and $\Omega^w \mapsto \Re$ may be defined by $X \mapsto \beta(X)$ where $\beta(X)$ represents the solution of (5) for the actuator configuration X and by $X \mapsto j(X) = J(\beta(X))$, respectively. The question may be considered: Given a set F of Ω and a desired motion $z = z(x,t)$ of the plate is there an optimal set of actuator locations to control the plate to this motion. This problem is analyzed in [5] and [6] by considering the continuity and differentiability properties of the mappings above. Furthermore, in [5] numerical results are given for the case of locating one actuator on a rectangular plate.

4. Publications. July 1, 1985 - July, 1986

1. Regularity properties in the estimation of elliptic diffusion, with K. Kunisch, *J. of Applicable Analysis.*
2. Estimation of elastic parameters in beams and certain plates: H^1 regularization, to appear *J. Optimization Theory and Application.*
3. Identification of elastic coefficients in beams: penalization, submitted.
4. Control of the deformation of static beams and Kirchhoff plates, submitted.
5. Control of dynamic models of beams and plates with small damping: Location of actuators, submitted.
6. Control of certain dynamic models of beams and plates: Location of actuators, Proc. VII Internation Conference of Nonlinear Analysis and Appl. V. Lakshmikantham, Ed., Marcel Dekker, 1986.
7. Regularity properties for estimated diffusion coefficients for parabolic boundary value problems, with K. Kunisch, in preparation.
8. Estimation of discontinuous diffusion coefficients in parabolic problems, with S. Gutman, in preparation.
9. Estimation of an elastic coefficient in a Kirchhoff plate, in preparation.

5. List of Professional Personnel.

(i) Professor Luther W. White, Principal Investigator.

(ii) Kuppusany Ravindran, Research Assistant, Master of Science, 1986.

6. Interactions.

(i) **Talks:**

(a) Control of certain dynamic models of Plates: Location of Actuators, VII International Conference on Nonlinear Analysis and Applications, July 28-August 1, 1986.

(b) Regularity of regularized least square optimal estimators of elastic coefficients in static plates, Math. Research Center, University of Wisconsin, August, 1986.

(c) Estimation of elastic coefficients in plate problems. O.U. School of Electrical Engineering and Computer Science, December, 1985.

(ii) **Consultations:**

(a) Consulted with Prof. D.L. Russell at the MIPAC facility at the University of Wisconsin-Madison, August 14-20, 1986.

(b) Consulted with Prof. J.T. Oden, University of Texas-Austin, Department of Engineering Mechanics.

7. New Discoveries.

Research in the second year of this project has produced codes for the following.

(i) Estimation of flexural rigidity in Kirchhoff plates using cubic splines.
(ii) Estimation of damping and flexural rigidity in dynamic models for plates using cubic splines.
(iii) Control of the deformation of a dynamic plate with point actuators.
(iv) Optimal location of a point actuator for the control of a dynamic model of a plate.

New results include the following.

(i) Regularity properties for optimal estimators of regularized output least squares estimators of flexural rigidity in static plate models.
(ii) Development of an approximation theory for estimators considered in (i).
(iii) Differentiability and continuity properties of the optimal control with respect to actuator locations for dynamic plate and beam models.
(iv) Sufficient conditions for stability of regularized output least squares estimators of flexural rigidity with respect to data.

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