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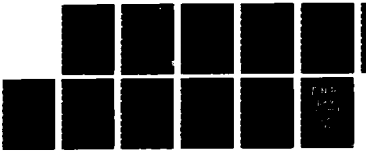

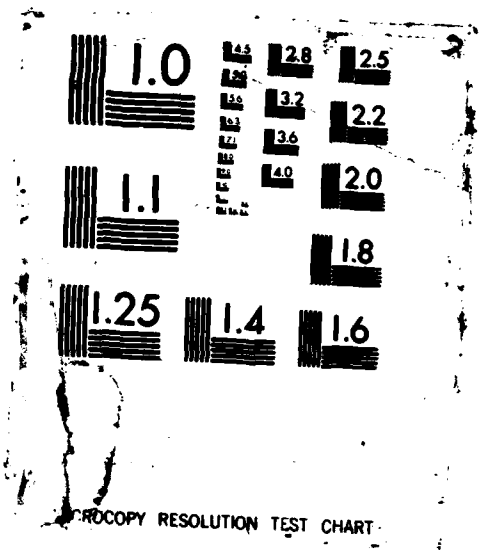
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ON A NEW GRAPHICAL METHOD OF DETERMINING THE CONNECTEDNESS IN THREE DIMENSIONAL DESIGNS*

BY

AFOSR-TR- 87-1284

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Summary

In this paper we study the connectedness of 3 dimensional designs by reducing the dimension of designs from three to two. A new graphical method of determining the connectedness of designs is presented. The method is easier and simpler than the earlier known methods of Birkes, Dodge and Seely (1976) and Srivastava and Anderson (1970). A generalization of this method for 4 or higher dimensional designs is also discussed.

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Keywords and Phrases: Additive models, Connectedness, Contrast, Equivalence Class, Graph.

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1. Introduction

A 3 dimensional (i.e., 3-factor) design involves 3 factors treatment, row and column. The sets of treatments, rows and columns are the sets of levels of 3 factors. The determination of connectedness of 3 and higher dimensional designs is a difficult task. The method of Srivastava and Anderson (1970) involves the use of λ-operator. The method of Birkes, Dodge and Seely (1976) known as BDS algorithm, uses the incidence matrix of a design to find the estimable parametric functions by R-and Q-processes. The λ-operator is indeed a powerful mathematical tool and R- and Q-processes are indeed good mathematical algorithms. When it comes to actual doing and finding, we find those two earlier known methods are difficult to implement and there is something conspicuously missing. Our search leads to a new concept of connected path for three dimensional designs, a generalization of a powerful idea of Bose (1947) for two dimensional designs. The proposed method takes into account the idea of reduction in dimensionality, discussed in Section 2, and the concept of connected path, discussed in Section 3, and some graph theoretic tools. The method consists of drawing graphs, i.e., joining points by lines directly from the design and does not require to write down the incidence matrix and construct another matrix from it to perform R- and Q-algorithmic processes like in BDS algorithm.

2. Reduction in Dimensionality

Consider a 3 dimensional design with a rows, b columns and v treatments. The standard additive model for the observations is

$$E(y_{ijk}) = \mu + \alpha_i + \beta_j + \tau_k,\ i=1,\ldots,a,\ j=1,\ldots,b,\ k=1,\ldots,v, \quad (1)$$

where the observations y_{ijk}'s are correlated or uncorrelated random variables with equal or unequal variances. Consider a pair of columns (j,j'), the row i and the pair of treatments (k,k') occurring in the design corresponding to i and (j,j'). Define $\tau_{(k,k')} = \tau_{k'} - \tau_{k'}$ $\beta_{(j,j')} = \beta_{j'} - \beta_j$. Then we have from (1),

$$E(y_{ij'k'} - y_{ijk}) = \beta_{(j,j')} + \tau_{(k,k')},\ j \neq j'. \quad (2)$$

Similarly for a pair of rows (i,i'), the column j and the pair of treatments (k,k'), we have

$$E(y_{i'jk'} - y_{ijk}) = \alpha_{(i,i')} + \tau_{(k,k')},\ i \neq i', \quad (3)$$

where $\alpha_{(i,i')} = \alpha_{i'} - \alpha_i$. The pair of columns (j,j') (or. the pair of rows (i,i')) can be treated as a one-dimensional block. We thus reduce the dimension of a 3 dimensional design to two by considering all pairs of columns (or rows) and all pairs of treatments. The equations (2) and (3) are essentially less than full rank reparametrization of (1).

Denote the sets whose members are the pairs of rows, columns and treatments, respectively by

$$I = \{(i,i'),\ i \neq i',\ i,\ i' \in (1,\ldots,a)\},$$

$$J = \{(j,j'),\ j \neq j',\ j,\ j' \in (1,\ldots,b)\},$$

$$K = \{(k,k'),\ k,k' \in (1,\ldots,v)\}. \quad (4)$$

The cardinalities of the three sets are $|I| = a(a-1)$, $|J| = b(b-1)$ and $|K| = v^2$. In case, the treatments are not replicated more than once in a row or a column of the design, we assume $k \neq k'$ in K. Then $|K| = v(v-1)$.

We write $k \sim k'$ if two treatments k and k' are connected, i.e., $\tau_{(k,k')}$ is estimable. Similarly for rows and columns.

Definition 1. Two members (k_1,k_2) and (k_3,k_4) in K are said to be connected, $(k_1,k_2) \sim (k_3,k_4)$, if $\tau_{(k_3,k_4)} - \tau_{(k_1,k_2)}$ is estimable under (1).

Note in Definition 1, the cases $k_1=k_2$, $k_3=k_4$ and $k_1=k_3$, $k_2=k_4$ are trivially true. Clearly, $\sim$ is an equivalence relation on the set K. Let $K_1, K_2, \ldots, K_q$, $q(\geq 1)$ is an integer, be the distinct equivalence classes of $\sim$ on K.

Lemma 1. If $(k_1,k_2) \sim (k_3,k_4)$ then $(k_{u_1},k_{u_2}) \sim (k_{u_3},k_{u_4})$, where (u_1,u_2,u_3,u_4) is any permutation of (1,2,3,4) in the dihedral group D_4 of permutations of order 8.

Lemma 2. The following are true.

a. $(k_1,k_2) \sim (k_2,k_3) \sim \ldots \sim (k_{w-1},k_w) \sim (k_w,k_1) \Rightarrow k_1 \sim k_2 \sim k_3 \sim \ldots \sim k_{w-1} \sim k_w$, where $w(\geq 2)$ is an integer,

b. $k_1 \sim k_2 \Rightarrow (k_1,k) \sim (k_2,k)$ for any k,

c. $(k_1,k_2) \sim (k_3,k_4)$ and $k_3 \sim k_4 \Rightarrow k_1 \sim k_2$,

d. $k_1 \sim k_2$, $k_3 \sim k_4 \Rightarrow (k_1,k_2) \sim (k_3,k_4)$.

The proofs of Lemmas 1 and 2 are easy. Lemmas 1 and 2 are very useful in finding the equivalence classes. To illustrate this, suppose $(k_1,k_2) \sim (k_3,k_4)$, $(k_2,k_5) \sim (k_4,k_6)$ and $(k_5,k_6) \sim (k_6,k_5)$. Then,

by Lemma 1, we have $(k_1,k_3) \sim (k_2,k_4)$ and $(k_2,k_4) \sim (k_5,k_6)$. Thus, $(k_1,k_3) \sim (k_5,k_6)$. It now follows from Lemma 2 that $k_1 \sim k_3$ and $k_2 \sim k_4$. Note that $c_1\tau_{(k_3,k_4)} - c_2\tau_{(k_1,k_2)}$, c_1 and c_2 are real constants, is a constrasts of τ's. However, in Definition 1, we treat (k_1,k_2) and (k_3,k_4) as two treatments in the reduced 2 dimensional design and consider a simple contrast $\tau_{(k_3,k_4)} - \tau_{(k_1,k_2)}$ (i.e., $c_1 = c_2$). The estimability or nonestimability of $c_1\tau_{(k_3,k_4)} - c_2\tau_{(k_1,k_2)}$ can easily be determined from the equivalence classes $K_1,K_2,\ldots,K_q$.

Theorem 1. A 3 dimensional design is connected w.r.t. treatment if and only if (iff) any two members in K are connected.

Proof. Suppose any two members in K are connected. Then, for an two treatments k_1 and k_2, we have $(k_1,k_2) \sim (k_2,k_1)$. Thus $k_1 \sim k_2$ by Lemma 2. If the design is connected, then for any two pairs (k_1,k_2) and (k_3,k_4) in K we have $k_1 \sim k_2$ and $k_3 \sim k_4$. Therefore, $(k_1,k_2) \sim (k_3,k_4)$ by Lemma 2. This completes the proof.

Theorem 2. A 3 dimensional design is completely connected (i.e., connected w.r.t. treatment, row and column) iff any two members in each of I, J, and K are connected.

Proof. Similar to the proof of Theorem 1.

If the design is connected w.r.t. treatment then $q = 1$, i.e., there is one and only one equivalence class. In case the design is not connected w.r.t. treatment we have $q \geq 2$ and any two members in an equivalence class are connected and any member in an equivalence class is not connected to any member in a different equivalence class. The

equivalence classes $K_1, K_2, \ldots, K_q$ give all the estimable and non-estimable row and column contrasts from the equivalence classes $I_1, \ldots, I_r$ and $J_1, \ldots, J_p$, $p, r(\geq 1)$ integers, of $\sim$ on I and J.

It is easy to see that any two members in K occuring in two distinct rows (or, columns) of (j,j') (or, (i,i')) are connectd. Two pairs of columns (j_1, j_2) and (j_3, j_4) are connected if a pair of treatments in (j_1, j_2) is connected to a pair of treatments in (j_3, j_4). If (j_1, j_2) and (j_3, j_4) have a common pair of treatments then $(j_1, j_2) \sim (j_3, j_4)$ or, in other words, every pair of treatments in (j_1, j_2) is connected to any pair of treatments in (j_3, j_4). Similarly, two pairs of rows (i_1, i_2) and (i_3, i_4) can be connected. If there is a pair of treatments (k,k) appearing in any row (or, column) of (j,j') (or, (i,i')) than $j \sim j'$ (or, $i \sim i'$).

3. A Simple Graphical Method

Suppose the pairs of treatments (k_1, k_2) and (k_3, k_4) are occuring in a block (j_1, j_2) and the rows i_1 and i_2 of a 3 dimensional design. It follows from (1) that

$$E(y_{i_1 j_1 k_1} - y_{i_1 j_2 k_2} - y_{i_2 j_1 k_3} + y_{i_2 j_2 k_4}) = \tau(k_1, k_2) - \tau(k_3, k_4). \quad (5)$$

We therefore say that (k_1, k_2) and (k_3, k_4) are connected in the analytical sense of Definition 1 if they occur in a block (j_1, j_2), or, in other words (k_1, k_2) and (k_3, k_4) are connected by the block (j_1, j_2) which contains them. From Lemmas 1 and 2, it follows that (k_1, k_2) and (k_3, k_4) with $k_1 = k_3$, occuring in two different blocks may be connected because some other pairs of treatments, e.g. (k_5, k_2) and

(k_5,k_4), are connected by some other block containing them. In this case, we say that (k_1,k_2) and (k_3,k_4) are connected by a third block.

Suppose (k_1,k_2) is occuring in a block (j_1,j_2) and a row i_1 and (k_3,k_4) is occuring in a block (j_3,j_4) and a row i_2 of a 3 dimensional design. It follows from (1) that

$$E(y_{i_1j_1k_1} - y_{i_1j_2k_2} - y_{i_2j_3k_3} + y_{i_2j_4k_4}) = \tau_{(k_1,k_2)} - \tau_{(k_3,k_4)} + \beta_{(j_1,j_2)} - \beta_{(j_3,j_4)}. \tag{6}$$

If (k_1,k_2) and (k_3,k_4) are connected by a block, then it follows from (6) that (j_1,j_2) and (j_3,j_4) are also connected and vice versa. We say combining (5) and (6) that two blocks (j_1,j_2) and (j_3,j_4) are connected if every treatment (k_1,k_2) in (j_1,j_2) is connected to every treatment (k_3,k_4) in (j_3,j_4).

We now introduce a new concept of connected path for three dimensional designs. Consider a graph whose points are the pairs of treatments and any two points are joined by an edge if they are connected, in the sense of Definition 1, by a block. When two points (k_1,k_2) and (k_3,k_4) are joined by a block (j_1,j_2), we write

$$(k_1,k_2)\ \underline{(j_1,j_2)}\ (k_3,k_4).$$

The adjacency of any two points in the graph can be determined from the fact that all points occuring in the same block of the design are adjacent and applying the Lemmas 1 and 2.

Definition 2. Two points (k_1,k_2) and (k_1',k_2') in the graph are said to be connected if there is a path

$$(k_1,k_2) \overset{(j_1,j_2)}{\sim} (k_3,k_4) \sim \cdots \sim (k_1',k_2'),$$

joining (k_1,k_2) and (k_1',k_2').

It follows from Theorem 1 that the design is connected w.r.t. treatment if any two points of the graph are connected (i.e., the graph is connected). Thus for a design connected w.r.t. treatment there is a single component in the graph (or, equivalently, there is one and only one equivalence in K as discussed in Section 2).

We now discuss a simple procedure of determining whether a design is connected or not. We assume without any loss of generality that $b \leq a$ and consider the column blocks (1,2), (1,3),...,(1,b) and (2,3). Note that the points in any block form a completely connected subgraph. Two subgraphs are connected when any point of one subgraph can be joined by a connected path to any point of the other subgraph. Lemmas 1 and 2 are the only tools needed in verifying connectedness or disconnectedness of two subgraphs. The main step in our procedure is to check whether b subgraphs are part of a single component of the graph or, in other words, whether these b subgraphs are connected or not. If these b subgraphs belong to different components of the graph (i.e., any of them is disconnected from some others), then the design is not connected w.r.t. treatment and block (column). If b subgraphs belong to a single component of the graph, then the design is connected w.r.t. block (column). This in fact means the connectedness of b(b-1) subgraphs corresponding to b(b-1) column blocks. The

connectedness of the subgraphs (j_1,j_2) and (j_2,j_1) implies the connectedness of treatments (k_1,k_2) in (j_1,j_2) and (k_2,k_1) in (j_2,j_1) occuring in the same row. This, in turn, implies the connectedness of two treatments k_1 and k_2, or, in other words, all treatments in every row of the design are connected.

4. Examples

We now present two examples to illustrate the procedure discussed in the earlier Section. Our first example of a 3 dimensional design is taken from Shah and Khatri (1973).

1	2	5	6
3	4	7	8
8	6	1	3
7	5	2	4

The column blocks (1,2), (1,3), (1,4) and (2,3) are shown below.

(1,2)	(1,3)	(1,4)	(2,3)
1 2	1 5	1 6	2 5
3 4	3 7	3 8	4 7
8 6	8 1	8 3	6 1
7 5	7 2	7 4	5 2

From the block (1,4) we have

$$(1,6) \overset{(1,4)}{\sim} (3,8) \overset{(1,4)}{\sim} (8,3).$$

It follows from Lemma 2 that $1 \sim 6$ and therefore $(8,6) \sim (8,1)$. Note that (8,6) is in the block (1,2) and (8,1) is in the block

(1,3). Thus the third block (1,4) is connecting the treatments (8,6) and (8,1). This fact is expressed by

$$(8,6) \overset{(1,4)}{\sim} (8,1).$$

Similarly from the block (2,3),

$$(2,5) \overset{(2,3)}{\sim} (5,2),$$

and this implies $2 \sim 5$. Thus (1,2) in the block (1,2) and (1,5) in the block (1,3) is connected by a third block (2,3) and we have

$$(1,2) \overset{(2,3)}{\sim} (1,5).$$

It is interesting to observe

$$(1,6) \overset{(1,4) \text{ or } (2,3)}{\sim} (6,1).$$

The subgraphs corresponding to the blocks (1,2) and (1,3) form one connected component and the subgraphs corresponding to the blocks (1,4) and (2,3) form another connected component in the graph. Therefore the design is not connected w.r.t. treatment and block (column).

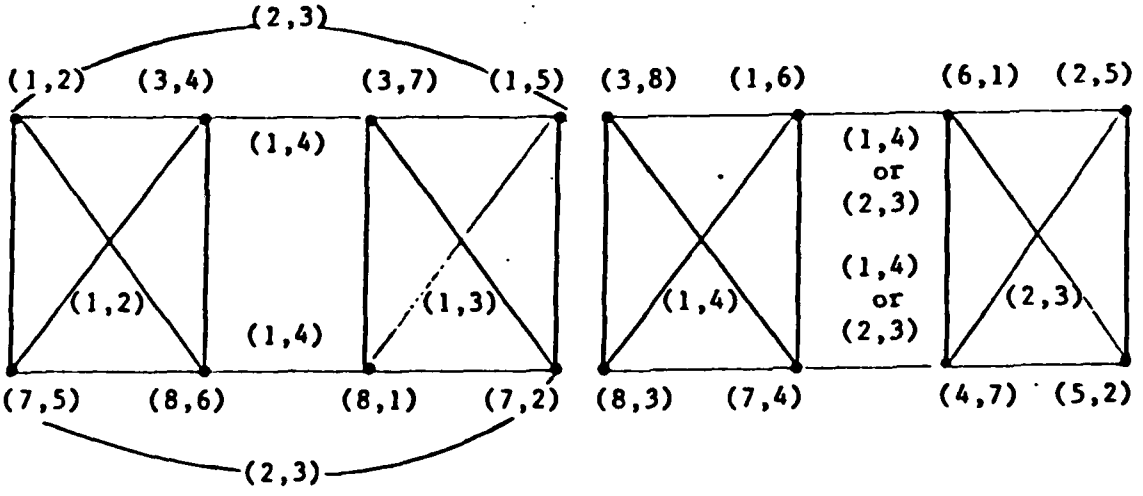

Figure 1. Connected and disconnected subgraphs corresponding to the blocks (1,2), (1,3), (1,4) and (2,3).

The second example of a 3 dimensional design is given below.

1	2	3	
2	3	4	1
3	4	1	2
	1		3

From the column blocks (1,2) and (2,3) we have

$$(1,2) \overset{(1,2)}{\sim} (2,3) \overset{\substack{(1,2)\\ \text{or}\\ (2,3)}}{\sim} (3,4) \overset{(2,3)}{\sim} (4,1).$$

From Lemma 2, it follows that the treatments 1 2 3 4 and the design is completely connected.

5. Interrelation

The graphical method, BDS algorithm and λ operator in Srivastava and Anderson are techniques in solving the same problem of determining connectedness. There is a common ground to all these methods and this can be seen from the equations (5) and (6). The R-process and the Q-process in the BDS algorithm can be applied to find estimable constrasts involving only τ parameters. A loop L defined in the BDS algorithm can be obtained from the observation $y_{i_1j_1k_1}$, $y_{i_1j_2k_2}$, $y_{i_2j_2k_4}$ and $y_{i_2j_1k_3}$ and the corresponding τ-functional is $\tau(L) = \tau_{(k_1,k_2)} - \tau_{(k_3,k_4)}$, which appears in (5). Thus the observations corresponding to the treatments (k_1,k_2) and (k_3,k_4) in the rows i_1 and

i_2 of the block (j_1,j_2) gives a loop. A loop can also be a combination of several such loops. A quasi-loop defined in the BDS algorithm is in fact obtained this way. From the connected paths discussed in Section 3, one can write down the loops and the quasi-loops in the BDS algorithm. There is obviously an even number of pairs, in the sense of BDS algorithm, in a loop or a quasi-loop. The graphical method discussed in this paper is lot simpler to implement than the BDS algorithm. To illustrate this let us consider Example 1 on page 103 in Birkes et. al. (1976). Assuming $\gamma = \tau$ and considering the blocks (2,3) and the rows 2 and 3, we get $(2,4) \overset{(2,3)}{\sim} (2,1)$ and $(4,4) \overset{(2,3)}{\sim} (2,1)$. It follows from Lemma 2 that $1 \sim 2$ and $1 \sim 4$, i.e., $\tau_1 - \tau_2$ and $\tau_1 - \tau_4$ are estimable. Considering the block (1,2) and the rows 2 and 4, we find the treatments $(3,2) \overset{(1,2)}{\sim} (1,3)$. Now $1 \sim 2$ implies $(1,3) \overset{(2,3)}{\sim} (2,3)$. Thus we have the connected path $(3,2) \overset{(1,2)}{\sim} (1,3) \overset{(2,3)}{\sim} (2,3)$ and this, in turn, implies $2 \sim 3$ i.e., $\tau_2 - \tau_3$ is estimable. We do not need to write the matrices N and $\bar{M}$ and to do complicated operations as in the BDS algorithm.

The idea of a loop or a quasi-loop is present in Definiton 4.2 and in the condition (a) of Theorem 4.2 in Srivastava and Anderson (1970) when τ-functional is a simple contrast of τ's. To illutrate this, suppose (k_1,k_2) and (k_2,k_1) are occurring in (j_1,j_2) and the rows i_1 and i_2. Then $T_1 = (\underline{t}_0 = (i_1,j_1,k_1), \underline{t}_2 = (i_2,j_2,k_1))$,

$T_2 = \{\underline{t}_1 = (i_1, j_2, k_2), \underline{t}_3 = (i_2, j_1, k_2)\}$, $\delta = 2$, and $T = (\underline{t}_0, \underline{t}_1, \underline{t}_2, \underline{t}_3)$ is a chain connecting the levels k_1 and k_2 of treatment in the notation of Srivastava and Anderson. (Incidentally, there are some misprints in the equation (4.6) and the other places, although the results are all correct.)

6. Higher Dimensional Designs

The method discussed in Sections 2-5 for 3 dimensional designs can be generalized for designs of dimensions 4 or more. Consider a 4 dimensional design with a blocks of type I, b blocks of type II, c blocks of type III and v treatments. The standard additive model is

$$E(y_{ij\ell k}) = \mu + \alpha_i + \beta_j + \gamma_\ell + \tau_k, \quad i=1,\ldots,a, \quad j=1,\ldots,b, \quad \ell=1,\ldots,c, \quad k=1,\ldots,v. \tag{7}$$

Suppose the treatments k_1, k_2, k_3 and k_4 are occurring in the blocks i_1 and i_2 of type I, j_1 and j_2 of type II and ℓ_1, ℓ_2, ℓ_3, ℓ_4 of type III. Then

$$E(y_{i_1 j_1 \ell_1 k_1} - y_{i_1 j_2 \ell_2 k_2} - y_{i_2 j_1 \ell_3 k_3} - y_{i_2 j_2 \ell_4 k_4}) = \tau((k_4,k_3), (k_2,k_1)) + \gamma((\ell_4,\ell_3), (\ell_2,\ell_1)), \tag{8}$$

where $\tau((k_4,k_3), (k_2,k_1)) = \tau_{(k_2,k_1)} - \tau_{(k_4,k_3)}$, $\gamma((\ell_4,\ell_3), (\ell_2,\ell_1)) = \gamma_{(\ell_2,\ell_1)} - \gamma_{(\ell_4,\ell_3)}$ and $\gamma_{(\ell_2,\ell_1)} = \gamma_{\ell_1} - \gamma_{\ell_2}$. A 4 dimensional design is thus reduced to a 2 dimensional design. Any two treatments occurring in a block, say $((\ell_4,\ell_3), (\ell_2,\ell_1))$, are connected. Any two blocks are connected if treatment, $((k_4,k_3), (k_2,k_1))$, in one block,

say $((\ell_4,\ell_3), (\ell_2,\ell_1))$, is connected to a treatment, say $((k_4',k_3'), (k_2',k_1'))$, in the other block, say $((\ell_4',\ell_3'), (\ell_2',\ell_1'))$. The connectedness or disconectedness w.r.t. treatment and type III block can easily be determined by defining an equivalence relation on appropriate sets. The determination of connectedness w.r.t. type I block and type II block is then straightforward. The idea of reduction in dimensionality jutifies the partitioning of the parameters in the model (7) using the BDS algorithm, see Example 2 of Section 10 in Birkes, Dodge, and Seely (1976).

Remarks

If the model is nonadditive, i.e., interactions are present in the model, then the idea of reduction of dimensionality does not work. However, the idea in this paper helps in identifying the estimable parametric functions. To illustrate this, we consider a nonadditive model for a 3 dimensional design

$$E(y_{ijk}) = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \tau_k, \tag{8}$$

where $(\alpha\beta)_{ij}$ is the interaction effect between the i^{th} row and the j^{th} column. A loop can still be obtained from the observations $y_{i_1j_1k_1}$, $y_{i_1j_2k_2}$, $y_{i_2j_2k_4}$ and $y_{i_2j_1k_3}$ and the corresponding τ, $(\alpha\beta)$ - functional is $\tau_{k_1} - \tau_{k_2} - \tau_{k_3} + \tau_{k_4} + (\alpha\beta)_{i_1j_1} - (\alpha\beta)_{i_1j_2} - (\alpha\beta)_{i_2j_1} + (\alpha\beta)_{i_2j_2}$, the right hand side of (5) under (8).

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