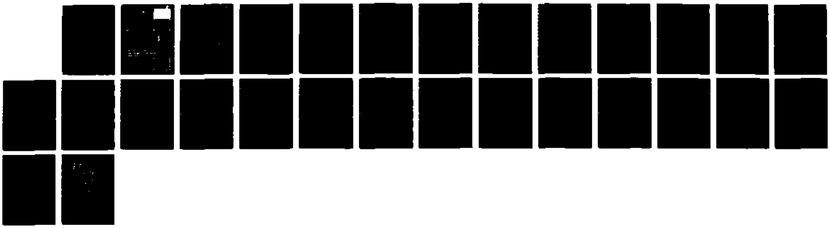
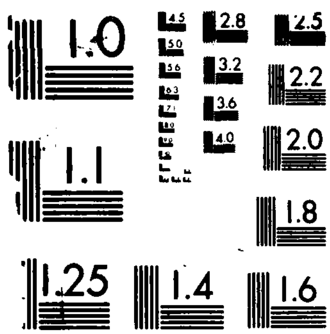
AD-A186 475 OPTIMIZING THE LOCATION OF INPUT/OUTPUT STATIONS WITHIN 1/1
FACILITIES LAYOUT..(U) GEORGIA INST OF TECH ATLANTA
PRODUCTION AND DISTRIBUTION RESE.. B MONTREUIL ET AL.
UNCLASSIFIED AUG 87 PDRC-87-05 N00014-86-K-0173 F/G 23/2 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

Georgia Institute of Technology

PRODUCTION and DISTRIBUTION RESEARCH CENTER

SCHOOL OF INDUSTRIAL
AND SYSTEMS
ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
A UNIT OF THE UNIVERSITY SYSTEM
OF GEORGIA
ATLANTA, GEORGIA 30332

(4)

PDRC Report Series 87-05
August 1987

OPTIMIZING THE LOCATION OF INPUT/OUTPUT STATIONS WITHIN FACILITIES LAYOUT

by

*Benoit Montreuil
**H. Donald Ratliff

PDRC 87-05

*Purdue University

**School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

This work was supported in part by the Office of Naval Research under Contract No N00014-86-K-0173. Reproduction in whole or in part is permitted for any purpose of the U. S. Government.

1. INTRODUCTION

This paper addresses the design task of characterizing and locating input/output stations within a facilities layout. Depending upon the particular case, an input/output station could be a door to an office or a department, a floor location where unit loads enter and leave a department, or an automated transfer station integrated to autonomous robotic vehicle systems. In all cases, the locations of input/output stations have a strong impact on both the cost of flow among departments and the internal configuration of the departments.

In the past, characterizing and locating input/output (I/O) stations has generally been restricted to a secondary consideration in the detailed phase of facilities design. However, practical relevancy has recently lead researchers to more fundamental consideration of input/output station locations when evaluating layouts.

O'Brien and Abdel Barr (Ref. 9) have proposed a layout improvement algorithm named S-ZAKY, similar to CRAFT (Ref. 1), which computes the expected flow distance savings at each iteration based on the location of the input/output (I/O) stations. They argue that this represents a more realistic improvement assessment than procedures which use intercentroid travel (e.g. CRAFT). They require an input and an output at fixed relative locations for each department. This has the impact of specifying much of the internal configuration of the department. Hence, the I/O station locations were prespecified, used for evaluation, and not subject to alteration for optimization sake. Warnecke et al. (Ref. 14) confirmed that interstation flow distance measurements are more representative of actual flows than centroid to centroid approximations.

Warnecke and Dangelmaier (Ref. 13) incorporated into the INTALA interactive layout software a layout construction algorithm permitting three internal configurations for each department. Each of these three configurations fixes an input station and an output station at given relative locations within the department. At each iteration the interactive heuristic selects a new department to enter the layout, then looks for the best combination location/configuration for the department based on the currently located departments. Hence, INTALA provides primitive capability for integrating I/O station location and facility layout.

2. STATION CHARACTERIZATION AND LOCATION FOR A GIVEN LAYOUT

This paper attempts to provide a systematic methodology for accomplishing the design task of characterizing and locating I/O stations for a given facility layout, and to demonstrate how it can potentially have a strong impact on the overall flow efficiency of a facility. The proposed methodology is summarized below.

The first task is to define the station set and characterize each station. Simply put, it means that for each department the designer must decide how many stations are to be used in its operation and what flow types will be assigned to each station. For example, the engineer may decide that a given assembly department should work with two stations, one global input station and one global output station.

The second task is to specify all interstation flows based on the station characterizations. This is similar to what is commonly done in specifying department flows for facilities layout studies, except that it is performed at the station level instead of the department level. The result is a set of interstations flows (e.g., 500 trips/period from the output station of the cell XYZ to the input station of storage zone ABC).

As a third task, the designer must specify the location boundary region for each station. This is usually based on considerations such as technological feasibility and ease of implementation. One extreme is to fix a station at a given location within the department. At the other extreme, a station may be left free to be located anywhere within the department.

The fourth task is to actually optimize the station locations given the flows and boundary regions. The objective is to obtain the set of locations which maximizes the overall flow efficiency of the layout. An efficient model and algorithm for this problem is presented in Section 8.

A final task is to perform sensitivity analysis on the optimization results. The purpose of the sensitivity analysis is to evaluate the robustness of the solution with respect to decisions made within the first three tasks.

3. STATION SET CHARACTERIZATION

The first task the designer has to perform is to determine the set of stations to be located, as well as a specific characterization of each station, which is logical and representative of the system operation. In order to facilitate this operation, it is proposed that each department be studied in order to categorize its stations according to the following terminology.

A station will first be categorized as either a FREE station or a DEPARTMENT station. A DEPARTMENT station belongs to a specific department and only handles flows associated with its department. A FREE station does not belong to any department and may handle flows associated with any departments or other stations. Instances where defining a FREE station is useful include representation of interfloor elevators or stairs, interbuilding conveyor entrances/exits, critical doors or interfaces within the facility, and intersection nodes within the flow structure. A DEPARTMENT station will further be categorized according to the types of flow it handles. In this development, a DEPARTMENT station will be designated as an INPUT, OUTPUT, INPUT/OUTPUT, or INTERNAL station.

An INPUT station handles flow coming into a department. An OUTPUT station handles flow going out of a department. An INPUT/OUTPUT (I/O) station handles both input and output flows. Finally an INTERNAL station has flows only with DEPARTMENT stations belonging to its own department. Defining INTERNAL stations is especially useful with process oriented departments. For example, if three different sets of equipment are located within a department and there are distinct flows associated with each group, it is frequently useful to define an INTERNAL station corresponding to each equipment set.

It is also convenient to define GLOBAL and FLOW GROUP stations. A GLOBAL station is a DEPARTMENT station through which are directed all flow of a given type (i.e., INPUT, OUTPUT, or INPUT/OUTPUT) for its department. For example, a GLOBAL OUTPUT station is a DEPARTMENT station which deals with *all* outgoing flows from its department. A FLOW GROUP station is a DEPARTMENT station dealing with flows identified as part of its associated group. A station which handles only flows of tools is an example of a FLOW GROUP station. Hence, GLOBAL and FLOW GROUP stations are identified by the set of flows they deal with within a given flow type.

Finally, it is convenient to designate stations in terms of logical assumptions made about their location. A station whose location is restricted to be within a boundary region to be subsequently established will be called a BOUNDARY REGION station. In subsequent discussions this is the default categorization. Stations which are assumed to be at the centroid of a department will be called CENTROID stations. FIXED stations will refer to stations proposed to be fixed to a specific location, different from the department centroid.

In order to illustrate the overall methodology, an example case will be discussed throughout the paper. This case is adapted from Ref. 2. Relevant data including department descriptions and areas as well as the interdepartment directed flows is given in Table 1. Figure 1 presents the block layout for the facility. Neither the building shape nor the department block layout have been optimized.

Table 2 summarizes the resulting station set characterization. The Receiving and Shipping departments have been attributed a single station, respectively GLOBAL OUTPUT and GLOBAL INPUT. The Milling, Presses, Lathes, Drills, Welding and Grinding departments are typical process departments. The multiplicity of equipments within each has led the engineer to assign a GLOBAL INPUT station and a GLOBAL OUTPUT station to each of these departments, coupled with a CENTROID INTERNAL station. The flow logic is as follows. Material arrives through the GLOBAL INPUT station, then it is directed toward one of the various equipment cells. Once the work is completed in a cell, the material is sent to the GLOBAL OUTPUT station for final departure. Flow toward and from the various cells is approximated by flow to and from the CENTROID INTERNAL station. The Plating and Assembly departments can be internally laid out to take advantage of station locations, hence they have been assigned GLOBAL INPUT and GLOBAL OUTPUT stations. Since the internal configuration of the Stores and Warehouse departments is extremely flexible, the engineer has decided to permit FLOW GROUP stations based on geographical locations for these two departments. For example, the Warehouse department has two FLOW GROUP INPUT stations. One deals with incoming flows from Welding and Plating while the other deals with flow from Assembly.

DEPARTMENT	CODE	AREA	RE	MI	PR	LA	DR	WE	PL	GR	AS	WR	SH	ST
			FROM/TO FLOWS											
RECEIVING	RE	1800		30										30
MILLING	MI	1200			40	10			10					
PRESSES	PR	2000				35	5							
LATHES	LA	3600					20	10	45					
DRILLS	DR	3200							5	20	10			
WELDING	WE	1000				10					15	10		
PLATING	PL	3500						5		45	10	20		
GRINDING	GR	2000						20			25		60	
ASSEMBLY	AS	2100										50		
WAREHOUSE	WR	2600												
SHIPPING	SH	2000												
STORES	ST	1500			20	30								

TABLE 1. Prespecified Data for the Example Case

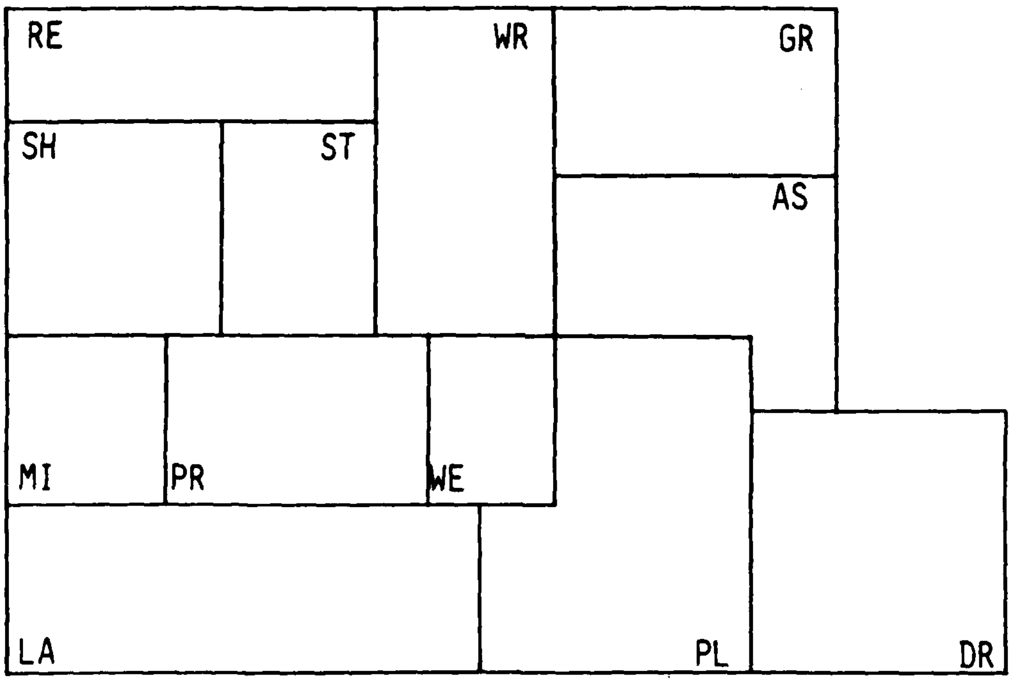


FIGURE 1. Prespecified Layout for the Example Case

TABLE 2

STATION CHARACTERIZATION AND BOUNDARY SPECIFICATION

STATION Department	#	CHARACTERISTICS Flow Type	Flow Set	Location Assumption	BOUNDARIES X-axis Min,Max	Y-axis Min,Max	FLOW GROUP Specification
Receiving	1	O	GL	B	0,78	117,140	
Milling	1	I	GL	B	0,33	36,61	
	2	INT	GL	C	16.5,16.5	54,54	
	3	O	GL	B	0,33	36,61	
Presses	1	I	GL	B	33,89	36,72	
	2	INT	GL	C	61,61	54,54	
	3	O	GL	B	33,89	36,72	
Lathes	1	I	GL	B	0,100	0,36	
	2	INT	GL	C	50,50	18,18	
	3	O	GL	B	0,100	0,36	
Drills	1	I	GL	B	156,209	0,61	
	2	INT	GL	C	182.5,182.5	30.5,30.5	
	3	O	GL	B	156,209	0,61	
Welding	1	I	GL	B	89,116	36,72	
	2	INT	GL	C	102.5,102.5	54,54	
	3	O	GL	B	89,116	36,72	
Plating	1	I	GL	B	100,156	0,33	
	2	O	GL	B	116,156	0,72	
Grinding	1	I	GL	B	116,174	105,140	
	2	INT	GL	C	145,145	122.5,122.5	
	3	O	GL	B	116,174	105,140	
Assembly	1	I	GL	B	116,174	72,105	
	2	O	GL	B	116,174	72,105	
Warehouse	1	I	FG	B	78,116	72,140	From Welding & Plating
	2	I	FG	B	78,116	72,140	From Assembly
Shipping	1	I	GL	B	0,45	72,117	
Stores	1	I	GL	B	45,78	72,117	
	2	O	FG	B	45,78	72,117	To Press
	3	O	FG	B	45,78	72,117	To Lathes

ABBREVIATIONS

FLOW TYPE: I=INPUT, O=OUTPUT, INT=INTERNAL

FLOW SET: GL=GLOBAL, FG=FLOW GROUP

LOCATION ASSUMPTION: B=BOUNDARY, C=CENTROID

4. FLOW SPECIFICATION

Once all stations to be located have been characterized, the designer must specify the magnitude of all interstation flows. This process is similar to specification of the standard interdepartment FROM-TO and FLOW-BETWEEN charts (for background reference, see Ref. 12). For example, rather than determining flow from a department A to a department B for a FROM-TO chart, flow is determined from each OUTPUT station of department A to each INPUT station of department B. If the designer limits himself to a single GLOBAL I/O station for every department, then the flow list will correspond directly to the standard interdepartment FLOW-BETWEEN chart. Furthermore, if he defines two stations for every department, one as a GLOBAL INPUT station and one as a GLOBAL OUTPUT station, then the flow list will correspond directly to the standard interdepartment FROM-TO chart.

Table 3 presents the resulting flow set for the example, based on the station characteristics expressed in Table 2 and the interdepartment FROM-TO flows from Table 1. The interstation flows are flows between stations of distinct departments. These flows are straightforward to determine. For example, Table 1 shows that there is a flow of 30 from Receiving to Milling. Receiving has a single GLOBAL OUTPUT station (RE, 1), and Milling has a GLOBAL INPUT station (MI, 1), hence there is a flow of 30 from station (RE, 1) to station (MI,1). As another example, there is a flow of 50 from Assembly to Warehouse. Warehouse has a FLOW GROUP INPUT station (WR, 2) specialized in dealing with flow incoming from Assembly. Assembly has a GLOBAL OUTPUT station (AS, 2). Therefore, there is a flow of 50 from (AS, 2) to (WR, 2).

The intradepartmental flows are flows between stations within a given department. Table 2 specifies that the Lathes department has GLOBAL INPUT, INTERNAL and

TABLE 3
INTERSTATION FLOW SET SPECIFICATION

INTERSTATION FLOWS					
INTERDEPARTMENT			INTRADEPARTMENT		
STATION	STATION	FLOW	STATION	STATION	FLOW
RE.1	MI.1	30	MI.1	MI.2	30
RE.1	ST.1	30	MI.2	MI.3	60
MI.3	PR.1	40	PR.1	PR.2	60
MI.3	LA.1	10	PR.2	PR.3	60
MI.3	PL.1	10	LA.1	LA.2	85
PR.3	LA.1	35	LA.2	LA.3	75
PR.3	DR.1	5	DR.1	DR.2	25
LA.3	DR.1	20	DR.2	DR.3	35
LA.3	WE.1	10	WE.1	WE.2	35
LA.3	PL.1	45	WE.2	WE.3	35
DR.3	PL.1	5	GR.1	GR.2	65
DR.3	GR.1	20	GR.2	GR.3	105
Dr.3	AS.1	10			
WE.3	LA.1	10			
WE.3	AS.1	15			
WE.3	WR.1	10			
PL.2	WE.1	5			
PL.2	GR.1	45			
PL.2	AS.1	10			
PL.2	WR.1	20			
GR.3	WE.1	20			
GR.3	AS.1	25			
GR.3	SH.1	60			
AS.2	WR.2	50			
ST.2	PR.1	20			
ST.3	LA.1	30			

OUTPUT stations. Given the flow logic expressed earlier, all flow coming into the Lathes department through (LA, 1) is approximated by flow to the INTERNAL station (LA, 2). By summing all flows to (LA, 1), the flow from (LA, 1) to (LA, 2) can be computed as 85. The same logic leads to a flow of 75 from (LA, 2) to the GLOBAL OUTPUT station (LA, 3). Material transformation, scrap, unit load, lot sizing, handling methods may all justify the difference between the total input and output flows.

5. BOUNDARY REGION SPECIFICATION

It is generally desirable to specify a boundary region within which a station can be located. An *infinite* boundary region for a station means that the station can be located anywhere within the facility. A *point* boundary region indicates that the station is to be fixed at a prespecified location. For example, a CENTROID station is assumed to be fixed at a point which is the department centroid. Departments where the input and output are technologically predetermined, such as product line assembly departments when the layout is difficult to change, may also have a point boundary region.

Department boundaries are typically used as station boundaries when the designer does not conceive major restrictions on station locations. Also in very aggregate studies, the station boundaries can usually be taken as the department boundaries. In less aggregate studies, the designer may compensate for technological constraints on stations by restricting them to be a given distance within the department boundary region.

For convenience, the resulting station boundaries for the example case have been included in Table 2. All boundary regions are presented as the extreme X-axis and Y-axis coordinates of a rectangle. The limitation to rectangular boundary regions will be justified in a latter section. The CENTROID INTERNAL stations have *point* boundary regions at their department centroid. The designer, not conceiving a priori any major restriction on location for other stations, has used the department boundaries as boundary regions. This can be visualized by referring to the layout in Figure 1.

6. STATION LOCATION OPTIMIZATION

Given a set of stations, each restricted to be within a specified boundary region, the location optimization task is to find the set of station locations that permit the most effective interstation flows.

The best measure of *efficiency* to be used for station location is subject to debate. Extremely detailed measures of efficiency can be obtained by simulation studies of a given set of station locations. However, in order to *determine* the locations, it is necessary to use a more aggregate measure of efficiency.

The optimization criterion proposed here is to minimize the sum of rectilinear (L1) distance travelled by all interstation flows. It provides a reasonable approximation to the actual objective and still yields a tractable model. Under the assumptions of a convex piece-wise linear boundary region for each station, the location problem can be solved using a linear programming model. Under the additional assumption of a rectangular boundary region for each station, a much more efficient network theory based solution methodology can be applied. The impact of these assumptions on model realism will be discussed subsequently.

The linear programming model for optimizing the station locations under L1 criterion and with rectangular boundary regions is presented below. First let

S	:	set of stations to locate
f_{ij}	:	flow between stations i and j
F	:	set of positive flow $\{(i,j) : i \in S , j \in S , i < j \text{ and } f_{ij} > 0\}$
(x_i , y_i)	:	location of station i on X and Y axes
$(\underline{x}_i , \overline{x}_i)$	:	lower and upper bounds on X_i
$(\underline{y}_i , \overline{y}_i)$	:	lower and upper bounds on Y_i

The station location problem can then be stated as:

Minimize

(1) $$\sum_F f_{ij} (| x_i - x_j | + | y_i - y_j |)$$

Subject to

(2) $$\underline{x}_k \leq x_i \leq \overline{x}_i \quad \forall \{i \in S\}$$

(3) $$\underline{y}_i \leq y_i \leq \overline{y}_i \quad \forall \{i \in S\}$$

Minimizing expression 1 corresponds to minimizing the sum, over all stations having a positive flow, of the product of the rectilinear (L1) distance between the stations and their flow value. It will be referred to as the L1 flow score. Expressions 2 and 3 guarantee that the stations are located within their boundary rectangular grid region. It is a simple matter (Ref. 3) to reformulate expression 1 as a linear function with additional variables and linear constraints.

Expressions 2 and 3 can be altered to obtain a linear programming formulation as long as the station locations are restricted to be within any specified convex polyhedron. As a simple numerical example, station i can be restricted to be located within the triangle formed by points (5,6) (10,12) and (15,4) by using the following expressions defining the three line equations forming the triangle internal region:

$$y_i - (6/5)\, x_i \leq 0$$

$$y_i + (1/5)\, x_i \geq 7$$

$$y_i + (8/5)\, x_i \leq 28$$

7. RELATIONSHIP WITH MULTIFACILITY LOCATION MODEL

The model expressed by (1) to (3) is equivalent to the well-known and very efficiently solved rectilinear multifacility location problem (Ref. 11). The rectilinear multifacility location problem is defined as follows. A set of *new facilities* , with *no location constraints*, are to be located in order to minimize the sum of the rectilinear distance travelled by all flows between the new facilities, and between the new facilities and a set of *existing facilities* specified by given locations.

It is easy to see that the rectilinear multifacility model is a special case of the station location model. The set of new facilities corresponds to a set of FREE stations with *infinite* boundary regions. The set of existing facilities corresponds to a set of FREE stations with *point* boundary regions.

To see that the models are in fact equivalent let any station with a *point* boundary region correspond to an *existing* facility, and any station with an *infinite* boundary region correspond to a *new* facility. The complication occurs when stations have a rectangular boundary region. To enforce the boundary region an *existing* facility can be created *at each of the four corners* of the rectangular boundary region with a large (BIG-M) flow defined between the *new* facility and each of the four created existing facilities. Using the properties derived by Picard and Ratliff (Ref. 11), locating the new facility anywhere within the rectangular region between the four existing facilities leads to *equivalent* optimal solutions for the BIG-M flows (score related to these is identical). Furthermore, the new facility *cannot* be located outside the 4-facility region in a global optimal solution since the BIG-M flows *dominate* the original flows. Hence the stations are restricted to their rectangular boundaries within the framework required by the rectilinear multifacility location model.

8. OPTIMIZATION METHODOLOGY WITH RECTANGULAR BOUNDARIES

The transformation process described in section 7 allows development of a specialized algorithm for solving station location optimization under L1 criterion with rectangular boundaries. The algorithm is a variation of the algorithm designed by Picard and Ratliff (Ref. 11) for rectilinear multifacility location. The proposed algorithm for station location is presented in figure 2.

The location optimizations for the horizontal and vertical dimensions are performed independently. For each dimension, the algorithm moves iteratively from the lowest boundary coordinate to the highest boundary coordinate. At each boundary coordinate C, it divides the stations among three sets. The set ← includes all stations whose upper boundary is smaller than C, these are *already located due to the increasing order of treatment of C. The* set → includes all stations whose lower bound is larger than C, these *cannot* be located at C. The set ⊕ includes all other stations, in fact all those which *may* be located at C.

Given these three station sets, the algorithm creates a graph G as follows. A node 1 is defined to include *all* stations within set ← and a node 2 is defined to include *all* stations within set →. Then a node is defined to represent *each* station within set ⊕. A link is then defined between each pair of nodes (m and n). The capacity of a link, C_{mn} , is set equal to the sum of the flows (f_{ij}) between all stations included in node m and all stations included in node n.

The algorithm then finds the MINIMUM CUT (Ref. 10) between nodes 1 and 2 in graph G, based on the link capacities (C_{mn}) . The cut is denoted as $\overline{L/L}$, with node $1 \in L$. The cut divides the nodes in two sets L and $\bar{L}$. Hence the stations are divided in two sets L and $\bar{L}$. The set L includes all stations in set ← and stations in set ⊕ which have more flow with the other stations in L than with all those in $\bar{L}$. A station

```
BEGIN
 FOR EACH DIMENSION INDEPENDENTLY (hor. and vert.) DO
  BEGIN
   REPEAT
    FOR EACH BOUNDARY COORDINATE C, IN INCREASING ORDER, DO
     BEGIN
       1. LOCATE AT C ALL UNLOCATED STATIONS WHOSE UPPER BOUNDARY = C
       2. DIVIDE ALL S STATIONS AMONG 3 SETS ←, ⊕, → AS FOLLOWS:
            ←: {stations whose upper boundary ≤ C}
            ⊕: {stations whose lower boundary ≤ C
                    and whose upper boundary > C}
            →: {stations whose lower boundary > C}
       3. DEFINE A GRAPH G AS FOLLOWS:
            • node 1: {Stations ∈ ←}
            • node 2: {Stations ∈ → }
            • node (3,...,||⊕|| +2) for each station ∈⊕
            • a link between each pair of nodes (m and n),
            with capacity (c_mn) equal to the sum of the
            flows (f_ij) between all stations i∈m
            and j ∈ n.
       4. FIND THE MINIMUM CUT (REF. 10) BETWEEN NODES 1 AND 2 IN
          GRAPH G, BASED ON THE LINK CAPACITIES (c_mn);
          DENOTE THE CUT BY L/L̄ , WITH NODE 1 ∈ L
       5. LOCATE AT C ALL STATIONS i ∈⊕ WHOSE NODE (i+2) ∈L
          AND FIX THEIR BOUNDARIES AT C
     END
    UNTIL ALL S STATIONS ARE LOCATED
  END
END
```

FIGURE 2. Proposed algorithm for the rectilinear station location model with grid rectangular boundary regions

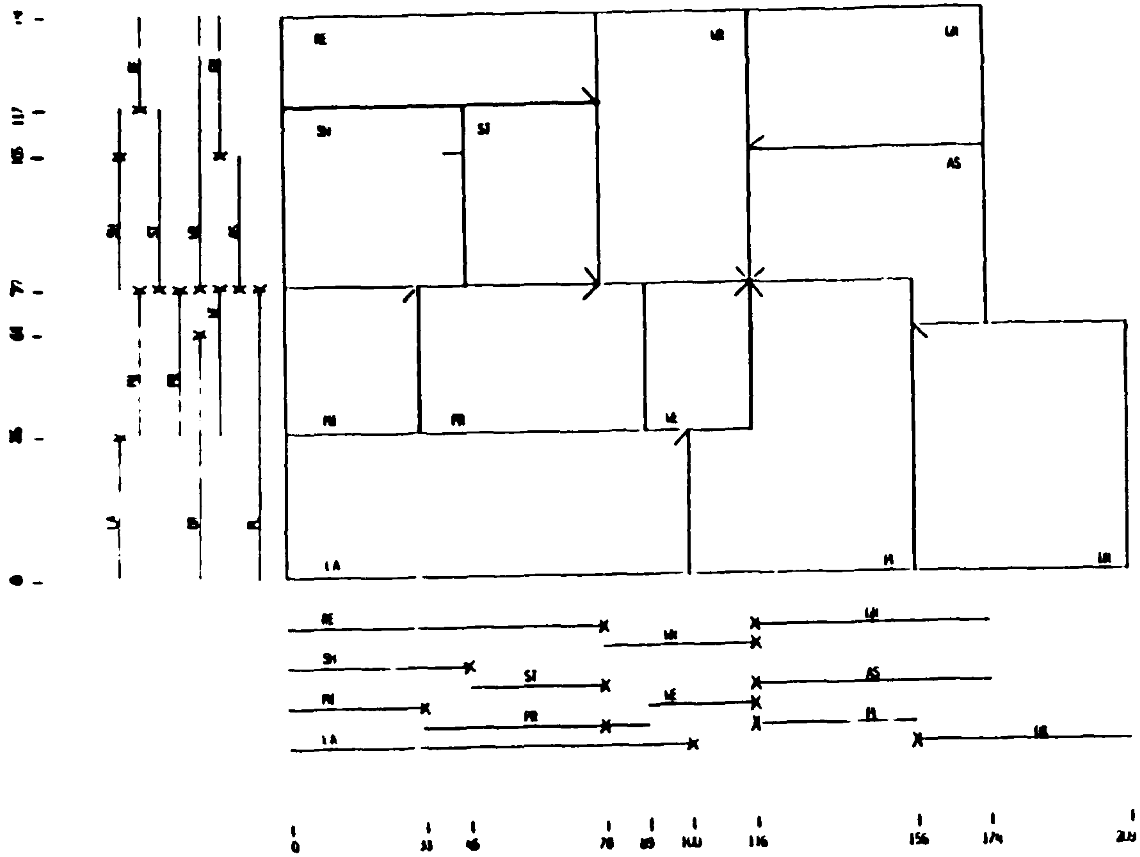
in set $\oplus$ which ends up in L is more attracted to the lower coordinate locations than higher-than-C coordinate locations. However, this was not true at the previous C investigated, otherwise it would already be located. Hence this indicates that such a station should be located exactly at C. For algorithmic purposes its boundaries are then fixed at C. The algorithm then moves to the following C and repeats the process until all stations are located.

Figure 3 presents an application of the methodology for the example case under the simpler assumption of a single GLOBAL I/O station for each department. The interstation flows f_{ij} are non-directional (between), so flow from i to j and from j to i are added to get f_{ij} . The boundary region defined for each department's station is depicted graphically by an horizontal boundary line and a vertical boundary line. Both horizontal and vertical optimizations are detailed. For each iteration, the three sets $\leftarrow$, $\oplus$ and $\rightarrow$ are listed, then the minimum cut computed is included, finally the resulting location decisions are shown. The final optimal locations are graphically represented on the layout and the associated L1 flow score is tabulated.

Cases with over 100 stations have been solved within 100 seconds on a IBM AT personal computer with a mathematical co-processor, and within a few seconds on a CDC CYBER 174. Hence, the methodology can be applied to most industrial design cases without bothering about optimization solution time within the overall design process.

9. SCORE BOUNDING

As part of the suggested sensitivity analysis phase, it may be worthwhile to compute lower and upper bounds on the flow score given the flows and boundary regions in order to assess how restrictive are the imposed constraints.



VERTICAL STATION LOCATION OPTIMIZATION

0 : ← : none ⊕ : LA1, DR1 → : rest
cut : none/all none located at 0

30 : ← : LA1 ⊕ : DR1, MI1, PR1, WE1, PL1 → : rest
cut : LA1/rest LA1 located at 30

61 : ← : LA1, DR1 ⊕ : MI1, PR1, WE1, PL1, → : rest
cut : LA1, DR1/rest DR1 located at 61

72 : ← : rest ⊕ : AS1, SH1, ST1, WR1 → : RE1, GR1
cut : rest/RE1, GR1, SH1
MI1, PR1, AS1, WR1, WE1, PL1, ST1 located at 72

105 : ← : rest ⊕ : GR1, SH1 → : RE1
cut : rest/RE1 GR1, SH1 located at 105

117 : ← : rest ⊕ : RE1 → : none
cut : none/all RE1 located at 117

HORIZONTAL STATION LOCATION OPTIMIZATION

0 : ← : none ⊕ : MI1, RE1, SH1, LA1 →: rest .
cut : none/all none located at 0

33: ← : MI1 ⊕ : RE1, SH1, LA1, PR1 →:
cut : MI1/rest MI1 located at 33

45: ← : MI1, SH1 ⊕ : RE1, LA1, PR1, ST1 → : rest
cut: MI1, SH1/rest SH1 located at 45

78 : ← : MI1, SH1, RE1, ST1 ⊕ : PR1, WR1, LA1 → : rest
cut : MI1, SH1, RE1, ST1, PR1/rest
RE1, ST1, PR1 located at 78

89 : ← : rest ⊕ : WE1, WR1, LA1 → : PL1, GR1, AS1, DR1
cut : MI1, SH1, RE1, ST1, DR1/rest
none located at 89

100 : ← : rest ⊕ : WE1, WR1 → : PL1, GR1, AS1, DR1
cut : MI1, SH1, RE1, ST1, DR1, LA1/rest
LA1 located at 100

116 : ← : rest ⊕: PL1, GR1, AS1 → : DR1
cut :rest/DR1
WE1, WR1, PL1, GR1, PS1 located at 116

156 : ← : rest ⊕ : DR1 → : none
cut : all/none DR1 located at 156

INTERSTATION FLOW EXCHANGE

Stations	flow	Stations	flow
RE1, MI1	30	WE1, AS1	15
RE1, ST1	30	WE1, WR1	10
MI1, PR1	40	PL1, WE1	5
MI1, LA1	10	PL1, GR1	45
MI1, PL1	10	PL1, AS1	10
PR1, LA1	35	PL1, WR1	20
PR1, DR1	5	GR1, WE1	20
LA1, DR1	20	GR1, AS1	25
LA1, WE1	20	GR1, SH1	60
LA1, PL1	45	AS1, WR1	50
DR1, PL1	5	ST1, PR1	20
DR1, GR1	20	ST1, LA1	30
DR1, AS1	10		

Operating Hypothesis: A single global I/O station for each department

RECTILINEAR SCORE

CENTROID SCORE	:	51930
OPTIMIZED STATION LOCATION SCORE	:	28600
Δ	:	22530
%	:	48.8

Figure 3 . Example of application of the proposed station location optimization algorithm for a given layout.

A lower bound on the L1 flow score can be obtained by assuming for each pair of stations having a positive flow that they are located so that the distance between them is as small as possible. Summing the obtained distance times the flow over all pairs independently gives a lower bound $\underline{S}_{L1}$ on the L1 score. Taking advantage of the rectangular boundaries, $\underline{S}_{L1}$ can be computed efficiently as follows:

$$(4) \qquad \underline{S}_{L1} = \sum_{F} f_{ij} \left[\max\left((\underline{x}_i - \overline{x}_j), (\underline{x}_j - \overline{x}_i), 0\right) + \max\left((\underline{y}_i - \overline{y}_j), (\underline{y}_i - \overline{y}_j), 0\right) \right]$$

By setting the $(\underline{x}_k, \overline{x}_k)$ and $(\underline{y}_k, \overline{y}_k)$ equal to their largest possible region, which usually corresponds to the department boundaries, $\underline{S}_{L1}$ permits to extend the boundary to the lowest L1 score attainable with full degree of freedom in station design given the actual layout.

Inversely, by assuming for each pair of stations having a positive flow that they are located as far from each other as possible, and then summing those distances times the flow over all pairs independently gives an upper bound $\overline{S}_{L1}$. Taking again advantage of the rectangular boundaries, $\overline{S}_{L1}$ can be computed as follows:

$$(5) \qquad \overline{S}_{L1} = \sum_{F} f_{ij} \left[\max\left((\overline{x}_i - \underline{x}_j), (\overline{x}_j - \underline{x}_i)\right) + \max\left((\overline{y}_i - \underline{y}_j), (\overline{y}_j - \underline{y}_i)\right) \right]$$

$\overline{S}_{L1}$ corresponds to the worst possible station design strategy by locating stations always as far as possible from each other. Determining this upper bound is useful to find out if the L1 flow score is sensitive upon the set of station locations. If $\underline{S}_{L1}$ and $\overline{S}_{L1}$ are not significantly different, then optimizing the station location will not have a significant impact on the flow score. Then the actual station locations may be based on other considerations than flow.

For the example of figure 3, $\underline{S}_{L1}$ is 11,950, and $\overline{S}_{L1}$ is 105,505. The L1 flow score obtained for the optimal solution is 26,600, which is 25% of the upper bound and 223% of the lower bound. The score being therefore more than twice the lower bound, it indicates that the single-station operating constraint is strongly affecting the expected distance travelled.

10. RECTANGULAR BOUNDARY REGION ASSUMPTION

On first observation, restricting the station location boundary region to be rectangles may appear very limiting. However, when analyzing actual layouts, one rarely observes elaborate department shapes. In fact, it is good layout practice to keep the department shapes as simple as possible. The majority of non-rectangular departments are composites of at most three rectangular areas. For example, department AS in Figure 3 can be viewed as a composite at two rectangular areas.

There are several approaches to modeling the station location constraint for these nonrectangular departments as rectangles. Sometimes it is straightforward to define a rectangular region inside the department within which the optimal location is to be located with very high probability.

If it is not obvious how to define the boundary region within an interior rectangle, an enclosing rectangle can be defined and used in the optimization. If the optimum location turns out to be inside the department then the problem is solved. If not, then it is often possible to use knowledge of the optimum to define an interior rectangle or at most a couple of possibilities which need to be enumerated.

It therefore appears that a rectangular boundary assumption can be used in practice without significant loss of applicability.

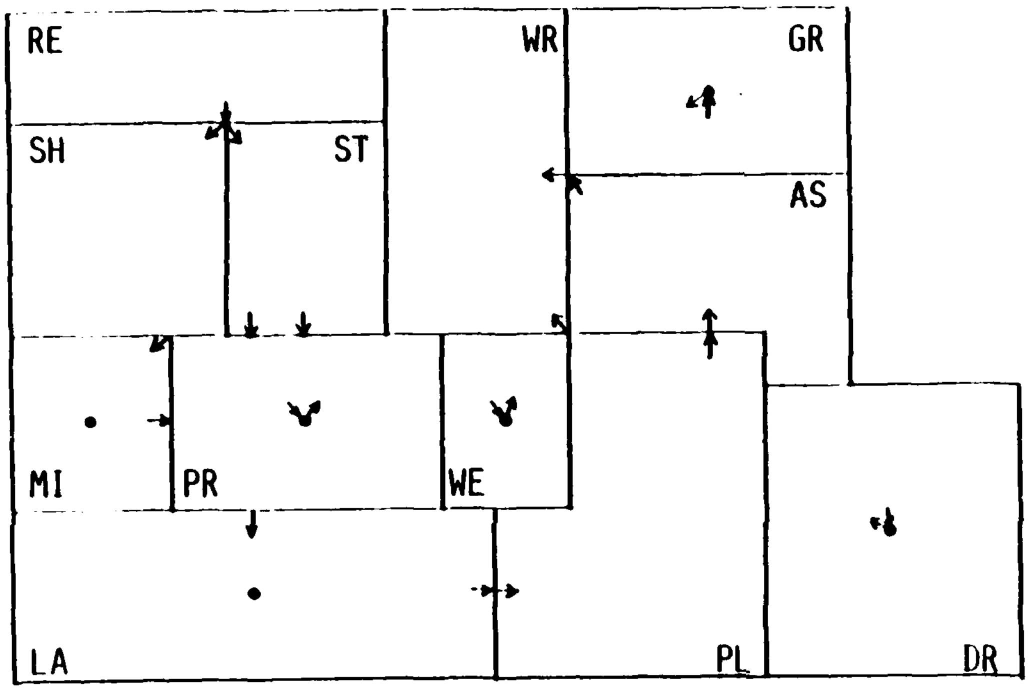
11. RESULTING DESIGN FOR THE EXAMPLE CASE

Given the station characterization and boundary specification expressed in Table 2, and the interstation flow set specification as stated in Table 3, the optimization model results in the design pictured in figure 4 after a computing time of 52 seconds on an IBM PC AT with mathematical co-processor.

The resulting flow score is 35730, this is higher than the score of 26600 obtained for the simple single station assumption. This increase is directly related to the use of CENTROID INTERNAL stations to represent adequately internal movement through the pertinent process departments, as decided by the designer.

It is interesting to note that the algorithm has suggested that, for 4 out of 6 of the process departments with a CENTROID INTERNAL station, their stations should all be clustered as a centralized input/output location within the department. This was not suggested, however, for the Milling and Lathes departments. The design provides illustration of typical clusterings of stations from different departments along their common boundary, hence providing minimal travel for their associated flows. An example is the clustering of the OUTPUT station of Lathes and the INPUT station of Plating.

One can note in Table 2 that the boundary regions for both stations of Plating had to be studied carefully, even if a full freedom of location within the department was suggested. This is due to the L shape of the department. The stated boundaries were decided upon by studying the boundary region for the stations having flow with the stations of the Plating department. As a sensitivity test, the engineer solved the case with (PL,1) restricted to X : [116,156], Y : [0,72]. The suggested location for (PL,1) was (116,18) which confirms its attraction to a lower horizontal coordinate; as expected, the flow score increased to 36530.



DEPT	STATION	LOCATION X-AXIS	LOCATION Y-AXIS
RE	1	45.00	117.00
MI	1	33.00	61.00
MI	2	16.50	54.00
MI	3	33.00	54.00
PR	1	61.00	54.00
PR	2	61.00	54.00
PR	3	61.00	54.00
LA	1	50.00	36.00
LA	2	50.00	18.00
LA	3	100.00	18.00
DR	1	182.50	30.50
DR	2	182.50	30.50
DR	3	182.50	30.50
WE	1	102.50	54.00
WE	2	102.50	54.00
WE	3	102.50	54.00
PL	1	100.00	18.00
PL	2	145.00	72.00
GR	1	145.00	122.50
GR	2	145.00	122.50
GR	3	145.00	123.50
AS	1	145.00	72.00
AS	2	116.00	105.00
WR	1	116.00	72.00
WR	2	116.00	105.00
SH	1	45.00	117.00
ST	1	45.00	117.00
ST	2	61.00	72.00
ST	3	50.00	72.00

FIGURE 4. Optimal Station Locations for the Example Case Given the Specifications from Tables 2 and 3.

Finally, the algorithm has taken advantage of the added freedom allowed by the designer when he defined distinct FLOW GROUP stations for the Stores and Warehouse departments. These FLOW GROUP stations ended up in distinct locations in the suggested design.

12. CONCLUSION

This paper has proposed a methodology for characterizing and locating input/output stations within a facility layout. The methodology is to first define the station set and characterize each station; second, specify all interstation flows based on the station characterizations; third, specify the location boundary region for each station; fourth, optimize the station locations given the flows and boundary regions; and, finally, perform sensitivity analysis on the optimization results. A specialized algorithm was proposed to solve the optimization problem of minimizing the rectilinear flow score given rectangular boundary regions.

The material presented in this paper has been used through various empirical research studies with industrial firms (Ref. 5, 6,7,8) and has led in all cases to significant and representative designs. However, there are a number of areas which seem productive for further research.

At the optimization model level, further research is necessary in addressing other optimization criteria, more complex boundary regions, and the cost of relocating stations in an actual layout. Furthermore, certain station set definition and station characterization decisions could be subject to optimization modeling.

The location methodology presented here minimizes the expected distance travelled. Impacts of this are resulting station clusterings and high flow links between stations. These have some potentially very interesting impacts on factory handling automation which could be studied in more depth. As a negative impact in some settings, it may create congestion. Simulation based approaches to deal with this phenomenon could be designed. Empirical studies could also be performed to assess typical station characterizations in industry, impacts of design constraints to overall efficiency achieved, and so on. These would require extensive industry collaboration to insure validity and representativity of the results.

Finally, as suggested in Ref. 4 and 13, a promising avenue for further research is the development of methodologies and models to sustain the integrated design of facility layout and station location. The methodology presented in this paper could be used as a building block for this research.

13. REFERENCES

1. Armour, G. C. and E. S. Buffa, "A Heuristic Algorithm and Simulation Approach to Relative Allocation of Facilities," *Management Science*, Vol. 9, No. 1, January 1963, pp. 294-309.

2. Barwikowski, S. S., "Computerized Layout Technique-A Successful Marriage," 1980 ORSA/TIMS Joint National Conference, Baltimore, MD., USA, 1980.

3. Hillier, F.S. and G.J. Lieberman, "Introduction to Operations Research, 3rd ed.", Holden-Day, San Francisco, USA, 1980.

4. Montreuil, B., "Interactive Optimization Based Facilities Layout," Ph.D. Dissertation, Georgia Institute of Technology, Atlanta, GA., USA, 1982.

5. Montreuil, B., "Experimental Investigation of the Computer-Aided Tree Facilities Layout Approach," Research Report to Tompkins Associates, Inc., Raleigh, NC, USA, June 1984.

6. Montreuil, B., "Adapting to Added Working Constraints to a Layout Design Study with the Tree Layout Approach," Research Report to Tompkins Associates, Inc., Raleigh, NC, USA., January 1985.

7. Montreuil, B., "Investigation Experimentale D'Amenagement De PME Industrielles Par L'Approche Arbre," Research Report to Quebec Industrial Research Center (C.R.I.Q.), Quebec City, Quebec, Canada, May 1985.

8. Montreuil, B. and A. Ramudhin, "Hierarchical Facilities Layout Optimization an Experimental Investigation," 1986 ORSA/TIMS Conference, Los Angeles, CA, USA, April 14, 1986.

9. O'Brien C. and S. E. Z. Abdel Barr, "An Interactive Approach to Computer Aided Facility Layout," *International Journal of Production Research*, Vol. 18, No. 2, 1980, pp. 201-211.

10. Phillips, D. T. and A. Garcia-Diaz, *FUNDAMENTALS OF NETWORK ANALYSIS*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1981.

11. Picard, J. C. and H. D. Ratliff, "A Cut Approach to the Rectilinear Distance Facility Location Problem," *Operations Research*, Vol. 26, No. 3, May-June 1978.

12. Tompkins, J. A. and J. A. White, *FACILITIES PLANNING*, John Wiley and Sons, New York, USA, 1984.

13. Warnecke, H. J. and W. Dangelmaier, "Progress in Computer Aided Plant Layout," Technical Report, Institute for Manufacturing Engineering and Automation-Fraunhofer, Stuttgart, Fed. Rep. of Germany, 1983.

14. Warnecke H. J., W. Dangelmaier and H. Kuhnle, "Computer-Aided Layout Planning," *Material Flow*, Vol. 1, 1982, pp. 35-48.

14. ACKNOWLEDGEMENTS

This work was partly supported by the Canadian Research Council in Natural Sciences and Engineering under grant No. A5482, by the U.S. Office of Naval Research under Contract No. N00014-83-h-0147, and by the U.S. National Science Foundation under Grant No. 501 1287-0040-0F103.

END
DATE
FILMED
JAN
1988