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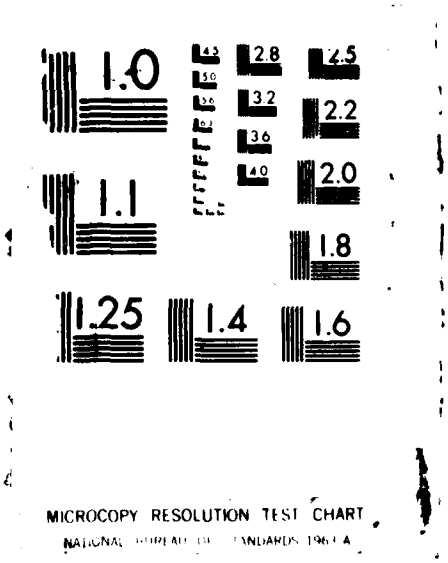
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REPORT DOCUMENTATION PAGE

1b RESTRICTIVE MARKINGS	
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release: distribution unlimited
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Cornell University	6b OFFICE SYMBOL (If applicable):
6c ADDRESS (City, State, and ZIP Code): Cornell University, Ithaca, NY 14853	7a NAME OF MONITORING ORGANIZATION: AFOSR/NM
	7b ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bolling AFB DC 20332-6448
8a NAME OF FUNDING / SPONSORING ORGANIZATION: AFOSR	8b OFFICE SYMBOL (If applicable): NM
8c ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bolling AFB DC 20332-6448	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-85-0157
	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO 61102F; PROJECT NO 2304; TASK NO A4; WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

Dynamical Characteristics of Weak Turbulence

12 PERSONAL AUTHOR(S)

John Guckenheimer

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Final	FROM _____ TO _____	8/87	3

16 SUPPLEMENTARY NOTATION

17 COSATI CODES		
FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

The P.I. studied global bifurcations in planar vector fields. In particular, codimension two bifurcations involving a simple saddle point was constructed together with related results applied to Hilbert's 16th problem. Also investigated were dynamical systems with symmetry groups. Notable was the discovery of heteroclinic cycles that are structurally stable within the class of symmetric systems. This has implications for the behavior of the K-S eg and turbulence modelling. Finally some work on ID maps was initiated. Preliminary results on the measure of an attracting set has implications for the famous Henon map. Nine papers were written.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	
22a NAME OF RESPONSIBLE INDIVIDUAL: Dr. Arje Nachman	22b TELEPHONE (Include Area Code): (202) 767-5027 / 22c OFFICE SYMBOL: NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Final Technical Report
AFOSR Grant 85-0157
John Guckenheimer
Cornell University

This is the final technical report for AFOSR Grant 85-0157. The general area of the research has been the investigation of nonlinear dynamical systems and their bifurcations. A number of different investigations of bifurcation in multiparameter systems of differential equations have been undertaken. A brief description of the research completed during the past year and still in progress follows, separated into three distinct areas of inquiry.

(1) The investigation of global bifurcations in planar vector fields:

Earlier work had classified the types of codimension one and two bifurcations involving behavior near an equilibrium point in planar vector fields. In studying higher codimension bifurcations in models of chemical reactors, we found that it was necessary to study codimension two bifurcations involving the presence of homoclinic orbits for these systems. A classification of codimension two bifurcations involving a single saddle point was constructed and applied to chemical reactor problems. Work this year with Rand and Schlomiuk applies these ideas to study the number of limit cycles near homoclinic orbits in perturbations of quadratic planar vector fields. This project is a small step in towards the solution of "Hilbert's 16th Problem," an old and famous outstanding problem in mathematics.

Our principal results are a combination of a stronger numerical conclusion with a weaker theorem. The question we examine is the number of limit cycles that can appear near a homoclinic orbit that appears as the termination of a continuous family of periodic orbits when the integrable system having a family of periodic orbits is perturbed. The solution to this problem is sought through a singular perturbation calculation that computes the asymptotic expansion of the return map for the perturbed system at the original homoclinic orbit. We are able to prove rigorously that no more than five limit cycles can appear, but extensive symbolic and numerical computations provide strong evidence for a maximum of two.

Another manuscript which is nearing completion gives a review of classical results that give necessary and sufficient conditions for a quadratic planar vector field to have a

continuous family of periodic orbits. The literature on this subject is confusing, with contradictory results stated in the literature. In addition, there are a number of important papers on the subject that have appeared only in Russian or Dutch, making it difficult to obtain a comprehensive understanding of the status of this problem. Our intent is to make these results accessible to the general mathematical public in the West.

(2) The investigation of dynamical systems with symmetry groups:

A significant discovery in this work with Armbruster and Holmes is the occurrence of heteroclinic cycles that are structurally stable within the class of symmetric systems. This discovery was motivated by studies of models for turbulent boundary layers and may be closely related to the phenomenon of intermittency in fluid flows. Further work has shown that the bifurcation behavior we have described for O(2) symmetric systems also explains the initial several bifurcations in the "Kuramoto-Sivashinsky" equation, a fourth order partial differential equation that has been derived to explain a variety of pattern formation problems. Structurally stable heteroclinic solutions are found for this equation. The results of our calculations have been compared with much more extensive calculations of solutiions of the PDE. Our model with a fourth order system of ordinary differential equations provides bifurcation values that are within 2% of those of the more extensive calculations and gives the correct qualitiative picture of the flows in these parameter regimes.

We are optimistic that our techniques will enable us to give a rigorous proof of these results for the Kuramoto-Sivashinsky equation. We are somewhat limited in this effort by the lack of theory which applies to the boundary conditions that we study. Most numerical calculations (including ours) work with periodic boundary conditions, while theoretical results concerning the boundedness of solutions, the existence of inertial manifolds, etc. require more restrictive Neumann boundary conditions.

(3) The investigation of one dimensional mappings:

The theory of iterations of one dimensional mappings is a subject that has matured in the last decade. We have continued our work in this area. The latest results establish that attracting Cantor sets that occur at the limit of period doubling sequences of bifurcations have Lebesgue measure zero.

Stewart Johnson and I are currently pursuing generalizations of this result and also hope to use it in achieving a deeper understanding of the scaling behavior associated with period doubling. A result that appears to be close at hand is that any attracting Cantor set for a unimodal map with negative Schwarzian derivative has Lebesgue measure zero. A key ingredient of this work are estimates of the "distortion" of iterates from being a quadratic mapping. It appears that appropriate iterates of a unimodal map always have bounded quadratic distortion. This theorem has the potential for leading to effective estimates for the behavior of the Henon mapping and similar two dimensional diffeomorphisms.

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