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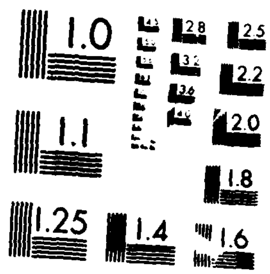

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AERONAUTICAL RESEARCH LABORATORIES

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Aerodynamics Technical Memorandum 386

HOVERING HELICOPTER FLIGHT DYNAMICS -
A STUDY OF VERTICAL MOTION (U)

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C. R. GUY

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SUMMARY

This document examines the flight dynamic characteristics of a hovering helicopter in response to collective stick inputs. Only vertical motion is considered. A mathematical model of the aerodynamics/kinematics and rotor system dynamics of a representative aircraft, linearized about the hovering flight condition, is used as a basis for the study. The dynamic behaviour of this model is compared with flight test results for similar inputs and the coefficients are adjusted to improve model performance. The model structure is then modified by the incorporation of additional terms to improve its accuracy. Although no attempt is made to optimize the coefficients of the model using parameter identification techniques, the approach used here provides acceptable results.

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CONTENTS

1. INTRODUCTION

The complex aerodynamics and flight mechanics associated with helicopters makes accurate mathematical modelling of flight dynamic characteristics difficult. The behaviour of such models when compared with flight test results for similar inputs often only indicates the general trend of response. Some examples of such models and comparisons of their behaviour with flight test results are given in references 1 to 4. The models used are usually relatively large, non-linear and complicated, thus making them unsuited to techniques such as parameter identification which might be used to obtain improved representation.

In order to understand helicopter flight dynamic characteristics more precisely and obtain improved mathematical models, it is necessary to simplify the task by subdividing it. As the hovering flight case is perhaps the most straightforward to consider, only this part of the flight envelope is addressed here. In addition, this document considers only vertical motion response to collective stick inputs without interactions in other motions. However, similar techniques to those developed here can be used to determine the model structure for yawing, pitching and rolling motions as well as interactions, as demonstrated in reference 5. Flight test results illustrating dynamic response to stick and pedal inputs for a Sea King helicopter in this condition are available (reference 6), as is a flight dynamic mathematical model linearized about the hover condition (reference 7). Although the flight test results and model relate to different versions of this aircraft, the basic model structure should be applicable to both versions.

A linear model rather than a more complex non-linear one (e.g. as described in reference 4) was selected both to simplify the problem and understand its fundamental nature. In addition, a linear model lends itself to the application of parameter identification techniques at a later stage to obtain optimum fitting of the model to flight data. The linear hover model would be obtained from the six degree of freedom non-linear equations of motion using small perturbation theory. As the manoeuvres considered are essentially small perturbations from the steady state hover condition, the linearized model approach is justified.

The exercise is essentially one entailing fitting of coefficients within an existing model structure followed by modification of structure through the addition of terms. A comparison of model behaviour with flight test results for the same input is made at each stage to ensure reasonable agreement is achieved. In adopting this approach it was found necessary to add terms to the model which were not included in the basic model of reference 7.

In determining vertical motion of the aircraft about the hover, two separate 'sub-systems' influence overall behaviour. These are the rotor system dynamics and the aircraft dynamics. The mathematical modelling of the various elements of each 'sub-system' is described together with simulation and some analytical studies. Comparisons of model behaviour with flight test results are also presented, along with some notes on stabilizer systems.

2. ROTOR SYSTEM DYNAMICS

2.1. General

Flight test results from a Sea King helicopter (reference 6) indicate that variations occur in engine torque and rotor speed when a collective stick input is applied to the aircraft in steady hovering flight (e.g. Figure 1). It also appears that the normal acceleration of the aircraft contains an oscillatory component related to engine torque variations. For these reasons, and the need to consider torque variations when analysing yaw response, the rotor system dynamics need to be included in the flight dynamic mathematical model of the aircraft. Earlier helicopter mathematical models (e.g. reference 8) assume constant rotor speed during manoeuvres whereas more recent studies (e.g. references 3 and 4) have taken rotor system dynamics into account.

The main purpose of this work is to determine a realistic model structure that gives representative dynamic behaviour of the system being considered. This can then be used for stability analysis and simulation purposes. In line with the configuration of the aircraft vertical motion dynamics (section 3) which is based on reference 7, a linear model of the form:

$$\underline{\dot{x}} = A\underline{x} + B\underline{u} \qquad (1)$$

is used for the rotor system where

$\underline{x}$ is the state vector

$\underline{u}$ is the control vector

A is the state matrix and

B is the control matrix.

As stated previously, the approach used derives the model and its coefficients from an amalgamation of established theory and flight test results.

Instrumented variables available from the flight test results of reference 6 which are involved in the rotor system dynamics model are collective stick position, collective blade angle, engine torque and rotor speed (RPM). Parameters have been determined so that a reasonable approximation to dynamic response is obtained, but no attempt has been made at this stage to optimize the coefficients.

2.2. Mathematical Modelling

Based on information from a number of sources, a block diagram of the elements forming the simplified rotor system dynamics for a Sea King helicopter can be constructed as shown in Figure 2. A breakdown of the elements shown on this diagram, including mathematical representations, is given below.

It should be noted that the model developed here is linearized about a specific flight condition, in this case hovering flight at a given aircraft weight. As shown, transfer functions and equations relate to changes in variables about this flight condition. Hence to get actual values, constants and integrator initial conditions need to be added where appropriate. These are considered in Section 2.3.

2.2.1. Flying controls

A detailed, non-linear mathematical model of the flying controls for the Sea King helicopter has been described in reference 9. However, for this purpose it is sufficient to use a linear representation of the form adopted by Blakelock (reference 10), with time constant and gain determined through ground tests on a Sea King aircraft. In transfer function form this is:

$$\frac{\theta_C}{\theta_{CST}} = \frac{K_1}{1 + \tau_1 s} \qquad (2)$$

or, in equation form (omitting initial conditions):

$$\dot{\theta}_C = \left(\frac{K_1}{\tau_1} \right) \theta_{CST} - \left(\frac{1}{\tau_1} \right) \theta_C \qquad (3)$$

where θ_C = collective blade angle ; change from trim

θ_{CST} = collective stick angle; change from trim

K_1 = collective flying controls gearing

τ_1 = collective flying controls time constant

s = Laplace operator and

$\dot{\theta}_C = \frac{d}{dt}(\theta_C)$

2.2.2. Collective pitch anticipator

Reference 11 states that the output signal from the collective pitch anticipator comprises two terms thus:

i. A rate component which is used as feedforward to enhance the engine response to changes in collective pitch; i.e. it helps reduce transient droop, and

ii. A position component which moves the droop law thereby producing overcancelled static droop.

These characteristics can be modelled by a transfer function of the form:

$$\frac{\Omega_{REF}}{\theta_{CST}} = \frac{K_2(1 + \tau_2 s)}{(1 + \tau_3 s)} \qquad (4)$$

i.e. $$\dot{\Omega}_{REF} = \left(\frac{K_2}{\tau_3}\right) \theta_{CST} + \left(\frac{K_2 \tau_2}{\tau_3}\right) \dot{\theta}_{CST} - \left(\frac{1}{\tau_3}\right) \Omega_{REF} \qquad (5)$$

where Ω_{REF} = rotor reference RPM; change from trim

K_2 = anticipator gain and

τ_2, τ_3 = anticipator time constants

Values for K_2, τ_2 and τ_3 were found from the simulation by trial and error, flight trial and model results being compared.

2.2.3. Governor

The governor compares the rotor RPM relative to the fuselage (Ω) with the rotor reference RPM (Ω_{REF}) to produce the RPM error signal (Ω_E). The demand signal to the fuel flow computer is proportional to Ω_E. Consequently:

$$\Omega_E = \Omega_{REF} - \Omega \tag{6}$$

2.2.4. Fuel flow computer

The fuel flow computer produces a fuel flow change related to the demand (RPM error) signal Ω_E. Padfield (reference 3) has indicated that the transfer function for this is:

$$\frac{\omega_f}{\Omega_E} = \frac{K_3}{1 + \tau_4 s} \tag{7}$$

i.e. $$\dot{\omega}_f = \left(\frac{K_3}{\tau_4} \right) \Omega_E - \left(\frac{1}{\tau_4} \right) \omega_f \tag{8}$$

where ω_f = fuel flow rate; change from trim

K_3 = computer gain and

τ_4 = computer time constant

The gain and time constant were determined from a comparison of model runs with flight test results.

2.2.5. Engine

Reference 3 also describes the relationship between engine torque change from trim (Q_E) and fuel flow rate change (ω_f) to be:

$$\frac{Q_E}{\omega_f} = \frac{K_4(1 + \tau_5 s)}{(1 + \tau_6 s)} \qquad (9)$$

i.e. $$\dot{Q}_E = \left(\frac{K_4}{\tau_6}\right) \omega_f + \left(\frac{K_4 \tau_5}{\tau_6}\right) \dot{\omega}_f - \left(\frac{1}{\tau_6}\right) Q_E \qquad (10)$$

where K_4, τ_5 and τ_6 are gains and time constants. The reference states that τ_5 and τ_6 vary with Q_E (or, more correctly, high speed turbine RPM) as illustrated in Figure 3. However, from the flight results of reference 6, the overall system response did not appear sensitive to variations in τ_5 and τ_6. Consequently, in order to retain linearity, τ_5 and τ_6 were chosen to be constant and were set to correspond approximately to the hover torque value at a specified aircraft weight.

2.2.6. Rotor

The rotor torque change from trim (Q_R) is assumed to be proportional to the change in collective blade angle (θ_C). This follows because blade angle changes will produce load changes which will result in torque variations. The gearing K_6 is calculated from flight test results so that Q_R is given as a percentage of 'nominal maximum' torque; i.e.

$$Q_R = K_6 \theta_C \qquad (11)$$

The difference between the engine torque Q_E and rotor torque Q_R is Q, such that:

$$Q = Q_E - Q_R \tag{12}$$

Then, relating torque difference to rotor speed change ($\dot{\Omega}_R$):

$$\dot{\Omega}_R = \left(\frac{1}{I_R} \right) Q = K_5 Q \tag{13}$$

where I_R is the rotor moment of inertia about its shaft (i.e. z-axis). In addition, the speed change of the rotor (Ω_R) is:

$$\Omega_R = \int \dot{\Omega}_R \, dt \tag{14}$$

It should be noted that rotor speed (angular velocity) in flight is measured relative to the fuselage. However, any change in rotor angular velocity produces a reaction on the fuselage which causes the fuselage to have an angular velocity change relative to a point fixed in space but in a direction opposite to that of the rotor. Hence the fuselage angular velocity must be taken into account when considering the feedback of rotor RPM to the governor. However:

$$\Omega_F = r \tag{15}$$

where r is the aircraft yaw rate change from trim

$$\therefore \Omega = \Omega_R + r \tag{16}$$

where Ω is the rotor RPM change from trim sensed by the governor and Ω_R and r are, by definition, of opposite sign.

Flight results indicate that for a collective step input, the change in r is roughly four times less than the change in Ω. This follows from the fuselage moment of inertia about its z-axis being approximately four times greater than the rotor moment of inertia. In the present exercise r is neglected as only pure vertical motion is being considered, but the statement above should be noted. Nevertheless, the effect on overall vehicle response should be small. Variable r is, of course, required to be included when the yaw channel and collective - yaw interaction are considered (reference 5).

2.3. Simulation

The specialized language CSMP-10 (ARL) (reference 12) was used for simulation of the above equations. However, in order to obtain actual values of variables rather than simply changes in value about the (hover) trim flight condition, constants and integrator initial conditions need to be included in the equations where outputs of actual values are required for comparison with flight test results. These are detailed below. A block diagram for simulation of the rotor system is shown in Figure 4. Notation used on this diagram is standard for the CSMP-10(ARL) language and corresponds closely with the notation used previously.

2.3.1. Flying controls

The change in collective pitch angle (THCST) is generated as a function of time and converted to collective stick angle measured relative to the aircraft horizontal datum (COLL STK) by the addition of a constant. Thus:

$$\text{COLL STK} = \text{THCST} + \text{CONSTANT} \tag{17}$$

Similarly, actual collective blade angle (THETA C) is computed from blade angle change (THEC) thus:

$$\text{THETA C} = \text{THEC} + \text{CONSTANT} \tag{18}$$

In addition, if an autopilot term is included, then:

$$\text{THCAP} = (0.86)\ \text{THCST} + \text{AUTOC} \tag{19}$$

where AUTOC is the autopilot signal and THCAP is an intermediate variable within the flying controls simulation which is generated as shown on Figure 4.

2.3.2. Engine

To obtain actual engine torque (TORQUE) from torque change (QE) a constant equal to the hover torque needs to be included. Hence:

$$\text{TORQUE} = \text{QE} + \text{CONSTANT} \tag{20}$$

2.3.3. Rotor

Because Q (the difference between engine torque and rotor torque) is generated as a percentage of 'nominal maximum' torque, the units of rotor angular velocity sensed by tne governor (OMEGA) are non-standard. To obtain rotor RPM (ROTOR RPM) from OMEGA, a gain and constant need to be incorporated thus:

$$\text{ROTOR RPM} = (2600)\ \text{OMEGA} + \text{CONSTANT} \tag{21}$$

2.4. Transfer Function Analysis

Using the system structure shown in Figure 2, the transfer functions and algebraic equations developed in section 2.2 can be integrated into the block diagram illustrated in Figure 5. Note that because aircraft yaw rate is not considered in this exercise, then

$$\Omega_R = \Omega. \tag{22}$$

Suitable values for constants are:

K_1	=	0.86	τ_1	=	0.1
K_2	=	$9.09.10^{-6}$	τ_2	=	280.0
K_3	=		τ_3	=	10.0
K_4	=	$K_3.K_4 = 7.0 \times 10^4$	τ_4	=	0.275
K_5	=	$9.26.10^{-5}$	τ_5	=	0.075
K_6	=	10.57	τ_6	=	0.26

For the purpose of analysis, it is convenient to calculate the transfer functions Ω_R/θ_{CST} and Q/θ_C, the latter being of use in computing the transfer function of the complete aircraft vertical dynamics (see section 3.4). By combining blocks and using the relationships:

$$
\begin{aligned}
K_7 &= K_1.K_6 \\
K_8 &= K_3.K_4 \\
K_9 &= K_5.K_8 \\
K_{10} &= K_2.K_9 \text{ and} \\
K_{11} &= K_5.K_7
\end{aligned}
\qquad (23)
$$

it can be shown that:

$$
\frac{\Omega_R}{\theta_{CST}} = \frac{s^3[K_{10}\tau_1\tau_2\tau_5 - K_{11}\tau_3\tau_4\tau_6] + s^2[K_{10}(\tau_1\tau_2+\tau_1\tau_5+\tau_2\tau_5) - K_{11}(\tau_3\tau_4+\tau_3\tau_6+\tau_4\tau_6)] + s[K_{10}(\tau_1+\tau_2+\tau_5) - K_{11}(\tau_3+\tau_4+\tau_6)] + [K_{10} - K_{11}]}{\tau_1\tau_3(s+1/\tau_1)(s+1/\tau_3)[s^3(\tau_4\tau_6) + s^2(\tau_4+\tau_6) + s(1+K_9\tau_5)+K_9]} \qquad (24)
$$

On substituting the constants above:

$$
\frac{\Omega_R}{\theta_{CST}} = \frac{-(0.006685)(s^3 + 3.5s^2 - 16.025s + 1.638)}{(s + 0.1)(s + 10)(s^3 + 7.4285s^2 + 20.785s + 90.657)} \qquad (25)
$$

and factoring the cubic polynomials gives:

$$
\frac{\Omega_R}{\theta_{CST}} = \frac{-(0.006685)(s + 6.15)(s - 2.54)(s - 0.1)}{(s + 0.1)(s + 10)(s + 6.44)(s + 0.52 \pm j3.72)} \qquad (26)
$$

Similarly, it can be shown that:

$$\frac{Q}{\theta_{CST}} = \frac{-(72.19)(s)(s+6.15)(s-2.54)(s-0.1)}{(s+0.1)(s+10)(s+6.44)(s+0.52 \pm j3.72)} \tag{27}$$

and

$$\frac{Q}{\theta_C} = \frac{-(8.39)(s)(s+6.15)(s-2.54)(s-0.1)}{(s+0.1)(s+6.44)(s+0.52 \pm j3.72)} \tag{28}$$

The above transfer functions are useful for stability analysis.

3. AIRCRAFT VERTICAL MOTION DYNAMICS

3.1. Basic Mathematical Model

From reference 7, the linearized equation of motion for the vertical dynamics of an SH-3D (Sea King) Helicopter is given as:

$$\dot{w} = -(0.00011)u + (0.000036)v - (0.3242)w + (0.0000057)\dot{\phi} + (0.00018)\dot{\theta} - (0.424)\theta_C \tag{29}$$

where w = normalized vertical velocity ; change from trim

u = normalized longitudinal velocity ; change from trim

v = normalized lateral velocity ; change from trim

$\dot{\phi}$ = roll rate ; change from trim

$\dot{\theta}$ = pitch rate ; change from trim and

θ_C = collective blade angle ; change from trim

In addition, from section 2.2.1, the flying controls equation including the autopilot term is:

$$\dot{\theta}_C = (8.6)\theta_{CST} - (10.0)\theta_C + (K_{CA})\theta_{CA} \quad (30)$$

where θ_{CA} = collective autopilot demand signal; change from trim and K_{CA} = collective autopilot gain.

A block diagram representation of these equations, which uses transfer function notation and includes cross-couplings, is shown in Figure 6.

If the cross-couplings are removed from equation (29), the relationship between θ_C and w has the form:

$$\frac{w}{\theta_C} = \frac{z\delta_C}{s - z_w} \quad (31)$$

where $z\delta_C$ is the gain between θ_C and w. The form of this equation is the same as that specified by Heffley (reference 13). The term $(s-z_w)$ is attributed to the heave damping mode and the value of z_w in hover is inversely proportional to the square root of the disc loading. From Wilson (reference 14):

$$z_w \approx - g \sqrt{\frac{\rho}{2W/A}} \quad (32)$$

where g = gravitational acceleration
ρ = air density
W = aircraft weight and
A = disc area

and substituting data for Sea King at the hover weight used:

$$z_w = -0.44$$

$$\therefore \quad \frac{w}{\theta_C} = \frac{z\delta_C}{s + 0.44} \tag{33}$$

This compares with z_w = -0.3242 for the SH-3D given by Murphy and Narendra in reference 7.

Initially a computer model using Murphy and Narendra's equation together with a representation of the flying controls was constructed. In order to compare this with the flight test results of reference 6, a common collective stick step input was used. Comparison of time histories of the variables θ_{CST}, θ_C, $\dot{w}$ and w are shown in Figure 7. As can be seen, the quantities w and $\dot{w}$ for the model bear no resemblance to those for the aircraft.

3.2. Model Development

In order to overcome some shortcomings of the basic model, the gain $z\delta_c$ was increased from 0.424 to 3.25 (Figure 8). However, there is no clear reason why such a large change in $z\delta_c$ is required. Nevertheless, when incorporated this gives rise to reasonable agreement in w, although comparison of the behaviour for $\dot{w}$ is poor for about 1.5s after application of the input.

To achieve improved agreement between model and flight results for $\dot{w}$, a 'blade angle derivative term' was added to the model. If cross-coupling terms are omitted, the equation relating vertical velocity to blade angle then becomes:

$$\dot{w} = -(0.3242)w - (3.25)\,\theta_C - (0.92)\theta_D \qquad (34)$$

where $$\theta_D = \dot{\theta}_C - 0.1\,\dot{\theta}_D \qquad (35)$$

Time histories for the variables θ_{CST}, θ_C, $\dot{w}$ and w are shown in Figure 9. It will be seen that the initial response in vertical acceleration $\dot{w}$ is now approximated, although behaviour of the model after this initial pulse still requires improvement. The reason for the high initial acceleration is not understood, although a similar phenomenon, albeit less pronounced, occurs in other flight results (e.g. references 1 and 3). As rotor speed remains reasonably constant, it is possible that the initial pulse in $\dot{w}$ arises from a change of pitch. It is postulated that when the blade angle is changed rapidly as in the step case used here, an incidence angle change exists for a short time until steady inflow is re-established. During this transient a differing level of thrust may be produced which gives rise to the change in $\dot{w}$. However, the reasons for the high initial acceleration require further investigation, including the incorporation of improved flight test instrumentation (e.g. direct blade and coning angles), so that a better data base is established.

As a final step in determining the structure of the model, it was noted that flight results for $\dot{w}$ exhibited a periodic variation following the initial impulse. Observations showed that the magnitude of this periodicity was proportional to the difference between the rotor and engine torques, and that it was also in phase with this torque difference. A similar periodic variation in normal acceleration is illustrated in reference 3, and it would not appear unreasonable that acceleration and torque difference were related. On the other hand, both torque difference and $\dot{w}$ variations may be due to blade lag combined with pitch/lag coupling. Again, improved flight test information on rotor behaviour is required. However, for expediency it has been decided to use the empirical approach here rather than attempt to incorporate such terms. Nevertheless, this is another area where further work is required.

Figure 10 shows time histories of variables for a collective step down from the hover when a torque difference term is added to the vertical motion equation, while Figure 11 is similar but for a collective step up. In both cases time histories for engine torque (Q_E) and rotor RPM are included, the rotor system dynamic model being that described in section 2. It will be noted that the time histories for $\dot{w}$ are improved from previous results. In neither case have the model parameters been optimized to obtain the results, as this would best be done using parameter estimation techniques.

In its final form, the vertical equation of motion for the model is:

$$\dot{w} = -(0.00011)u + (0.000036)v - (0.3242)w + (0.0000057)\dot{\phi} + (0.00018)\dot{\theta} - (3.25)\theta_C - (0.92)\theta_D + (0.15)Q \qquad (36)$$

where Q is defined in equation (12). In addition, the constants associated with the cross-coupling terms have not yet been verified against flight results.

3.3. Simulation

The simulation block diagram, in CSMP-10(ARL) format, for vertical motion with the model in its final form is shown in Figure 12. This includes representation of autopilot and cross-coupling terms together with means for obtaining actual values of variables as described in section 2.3. WKN (vertical velocity in knots) is obtained from W(vertical velocity in ft/s) and altitude HT is obtained by integrating W. The model time histories shown in Figures 10 and 11 use this simulation combined with that shown in Figure 4. Reasonable comparisons of the time histories of variables have also been obtained for pulse and doublet collective stick inputs.

3.4. Transfer Function Analysis

When the autopilot input and cross-coupling terms are omitted from the final version of the model, the transfer function block diagram is as shown in Figure 13. The transfer function for the flying controls (θ_C/θ_{CST}) is as specified in equation (2) but the vertical motion dynamics (w/θ_C) now comprise the following:

i. The blade angle derivative term

$$\frac{\theta_{CD}}{\theta_C} = \frac{9.2s}{s + 10} \qquad (37)$$

which can be obtained from equations (34) and (35).

ii. The proportional term

$$\frac{\theta_{CP}}{\theta_C} = 3.25 \qquad (38)$$

which is simply the increased gain $z\delta_C$ and

iii. The torque difference term

$$\frac{\theta_{CQ}}{\theta_C} = \frac{1.26(s)(s + 6.15)(s - 2.54)(s - 0.1)}{(s + 0.1)(s + 6.44)(s + 0.52 \pm j3.72)} \qquad (39)$$

which is derived from equation (28) (the Q/θ_{CST} transfer function multiplied by the appropriate gain).

The vertical velocity can then be computed based on equation (31) thus:

$$\frac{w}{\theta_C'} = \frac{1}{s + 0.3242} \qquad (40)$$

where $\theta_C' = \theta_{CD} + \theta_{CP} + \theta_{CQ}$ (41)

It should be observed that both w and h are measured positive downwards in standard notation and that height h is obtained from w thus:

$$\frac{h}{w} = \frac{1}{s} \qquad (42)$$

If all terms are included, the overall transfer function is:

$$\frac{h}{\theta_{CST}} = \frac{-117.9(s + 3.2)(s + 0.36 \pm j3.2)}{(s)(s + 0.3242)(s + 10)(s + 10)(s + 0.52 \pm j3.72)} \qquad (43)$$

However, if some simplification is required for analysis purposes where higher order terms are not important, the torque difference term can be omitted to give:

$$\frac{h}{\theta_{CST}} = \frac{-107(s + 2.61)}{(s)(s + 10)(s + 10)(s + 0.3242)} \qquad (44)$$

or, with the derivative term also omitted:

$$\frac{h}{\theta_{CST}} = \frac{-27.95}{(s)(s + 10)(s + 0.3242)} \qquad (45)$$

3.5. Stabilization

This section serves to illustrate how the transfer functions generated in section 3.4 can be used when height stabilization systems are incorporated in the helicopter. The Sea King Mk 50 helicopter is fitted with two forms of height stabilization, namely Radio Altitude Hold and Barometric Height Hold. The former provides accurate height stabilization up to 500 ft and also enables automatic transition and hover manoeuvres to be performed. Barometric height hold provides stabilization in height up to ceiling altitude. Full details of both systems are given in references 15 - 17.

3.5.1. Barometric height hold

In order to configure the system in a manner suitable for stability analysis, it is necessary to make a number of assumptions and simplifications to the actual aircraft flight control system. These include:

i. Ignoring 'open-loop spring operation' (see reference 15) as this serves merely to re-datum the system when relatively large autostabilizer signals are generated and is non-linear in its operation.

ii. Ignoring the (non-linear) authority limits; i.e. only small perturbations about a given flight condition are considered.

When these simplifications are made, the control law governing Barometric Height Hold is (from reference 15):

$$\theta_{CA} = \left(\frac{0.0622}{s + 2.22}\right)(h - h_{ref}) \qquad (46)$$

where h_{ref} is the reference altitude and the smoothing term (lag) is a gust filter. A block diagram incorporating this law and the aircraft vertical motion dynamics is shown in Figure 14. This forms the basis of analysis to determine barometric height hold system stability.

3.5.2. Radio altitude hold

As with Barometric Height Hold, it is necessary to configure the system in a manner suitable for stability analysis. The Radio Altitude Hold circuit used in the actual aircraft flight control system includes an inertial height smoother and is quite complex. Both the system and its mathematical model are described in reference 17. A simplified version of this is presented in reference 15, although even this contains *non-linear elements*. Consequently *simplifications* and assumptions similar to those used in section 3.5.1 need to be [illegible] obtain a representation suitable for stability analysis.

When simplifications are incorporated in the model given in reference 15, the control law governing Radio Altitude Hold becomes:

$$\theta_{CA} = (h' - h_{ref})\left\{0.37\left[\frac{s^2 + 0.22s + 0.036}{s[s + 0.22]}\right]\right\} + h'\left\{\frac{0.062}{s + 0.22}\right\} + 0.46w \quad (47)$$

and

$$\frac{h'}{h} = \frac{K_{HS}}{s + \tau_{HS}} \quad (45)$$

A block diagram representing this law is shown in Figure 15 and again this forms the basis for system stability analysis. It should be noted that an inertial height smoother modifies the height signal h via, basically, a simple lag circuit, to give h′. Provision for this is indicated on Figure 15, where K_{HS} and τ_{HS} are both estimated to be 1.0.

4. CONCLUDING REMARKS

A mathematical model representing the dynamic flight behaviour of a hovering helicopter has been constructed. Only vertical motion about the hover point has been considered. The model developed comprises two parts, the rotor system dynamics and the aircraft vertical motion dynamics. To obtain an accurate representation of vertical flight behaviour, it has been found necessary to incorporate a rotor system term in the flight dynamic model for vertical motion.

The rotor system model consists of a number of components which are linked together. A dynamic representation of each element has been made and parameters adjusted so that the dynamics of the model are similar to those of the physical system. Simulation of the model is discussed and a transfer function analysis carried out.

The aircraft vertical motion dynamic model is based on previously published work which includes cross-coupling terms. However, in order to obtain an accurate representation of aircraft vertical motion, a number of additions to the basic model have been found necessary. In conjunction with adjusting the gain relating vertical velocity change to collective blade angle change, extra terms have been added. These are a blade angle derivative term and a torque difference term which are required so that the model behaviour can be adequately fitted to the flight test results for similar manoeuvres. Whilst some bases for these additional terms have been postulated, further work is required to establish a better understanding of flight behaviour. This includes instrumentation of additional rotor variables during flight test.

After these modifications are made, the flight dynamic characteristics of the model are shown to be similar to those of the flight test results. However, no attempt has been made to optimize parameters as this would best be done using identification methods. The linear model developed is ideally suited to this approach. Again, simulation and transfer function analyses have been conducted; the latter is of use in determining both the stability of the bare aircraft and system stability when height hold loops are added.

The technique used to develop the model structure has been demonstrated to give a realistic representation of vertical flight behaviour for small perturbations from the hover. The derived transfer functions should be of use for handling quality and flight control system studies and the linearised model should be useful for gust response analyses. It is proposed to use the technique to observe flight behaviour in other axes, both in hovering and forward flight, so that a greater understanding of helicopter dynamics can be obtained.

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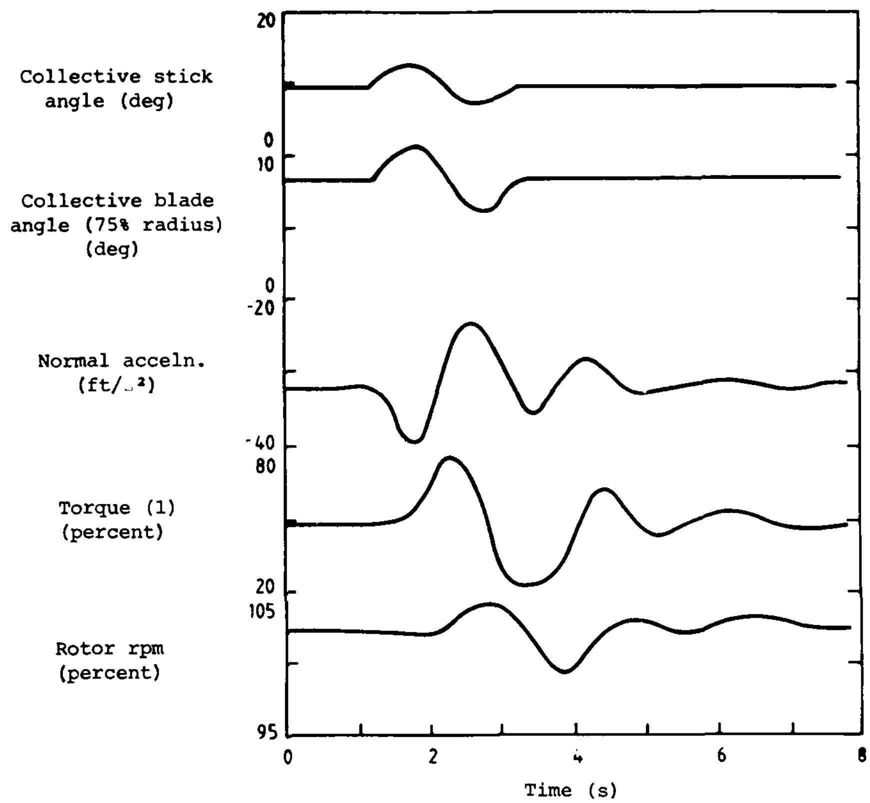


FIG. 1. COLLECTIVE DOUBLET INPUT - 80 KN - AFCS OFF
(FLIGHT TEST RESULTS)

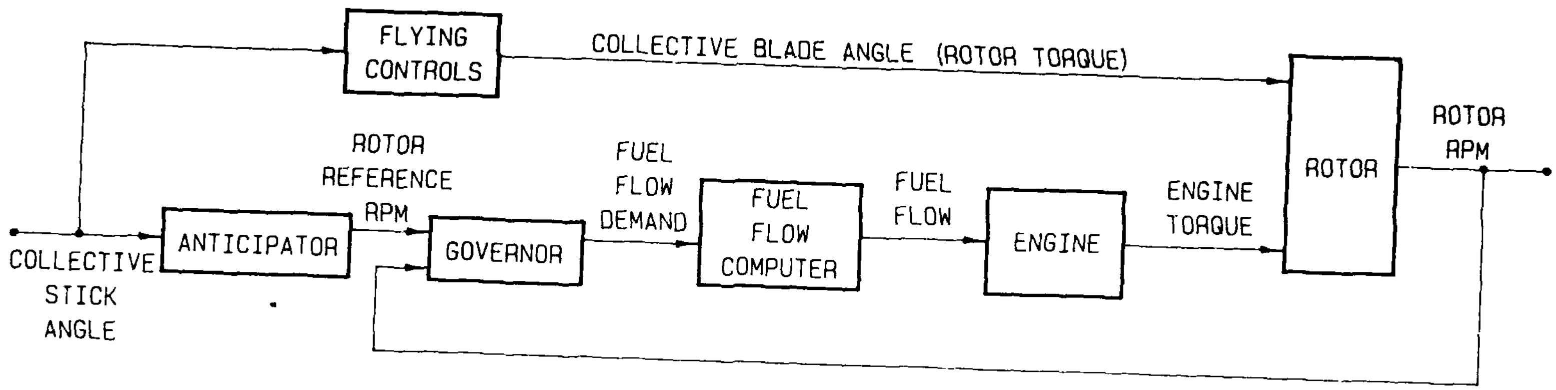


FIG. 2. ROTOR SYSTEM ELEMENTS BLOCK DIAGRAM

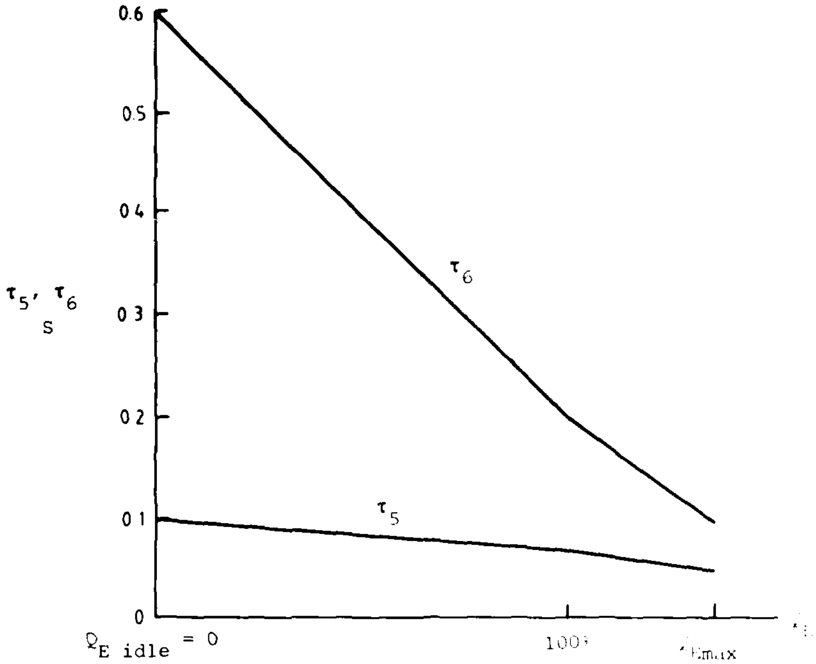


FIG. 3. ENGINE TIME CONSTANTS (reference 3)

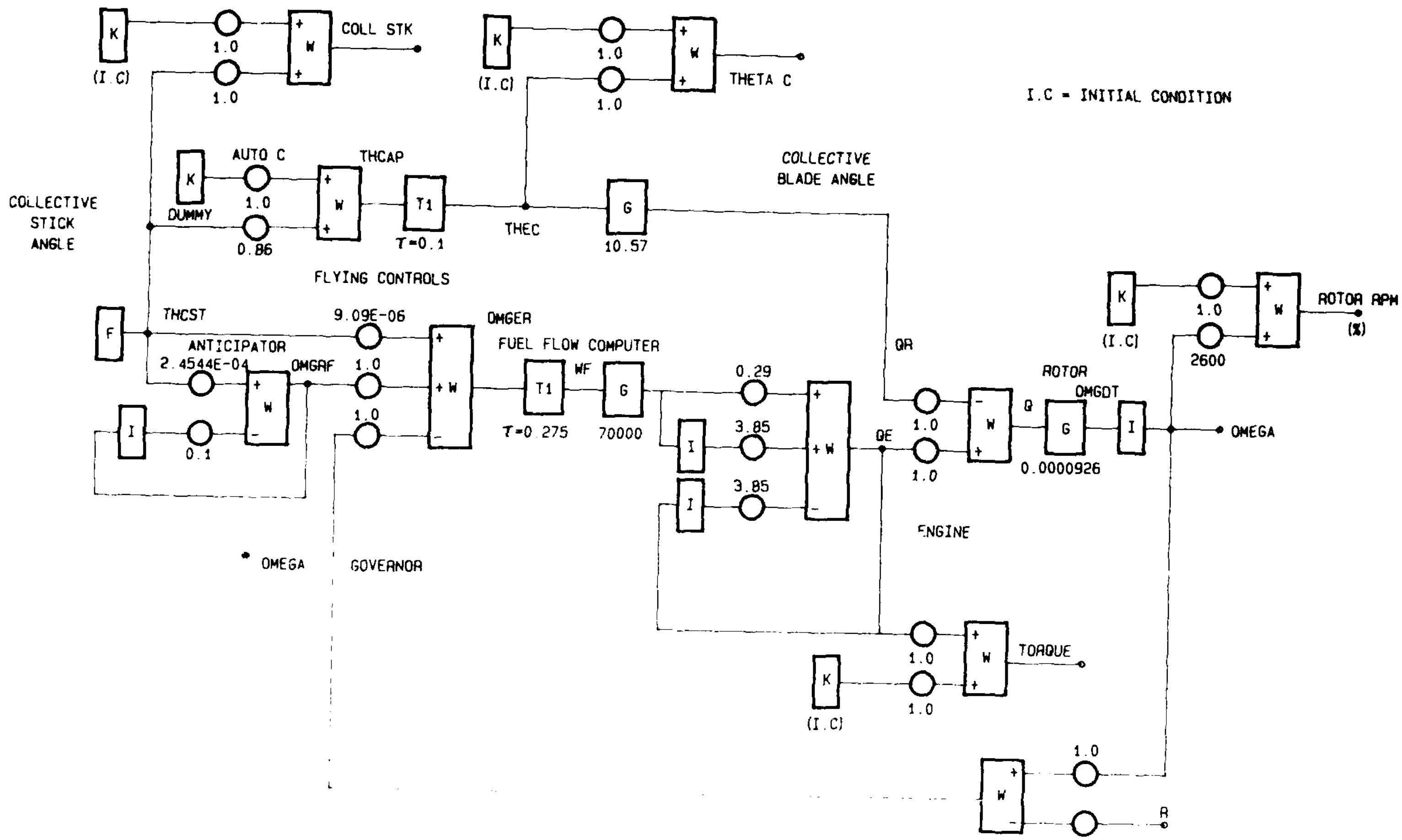


FIG. 4. [illegible] SIMULATION BLOCK DIAGRAM

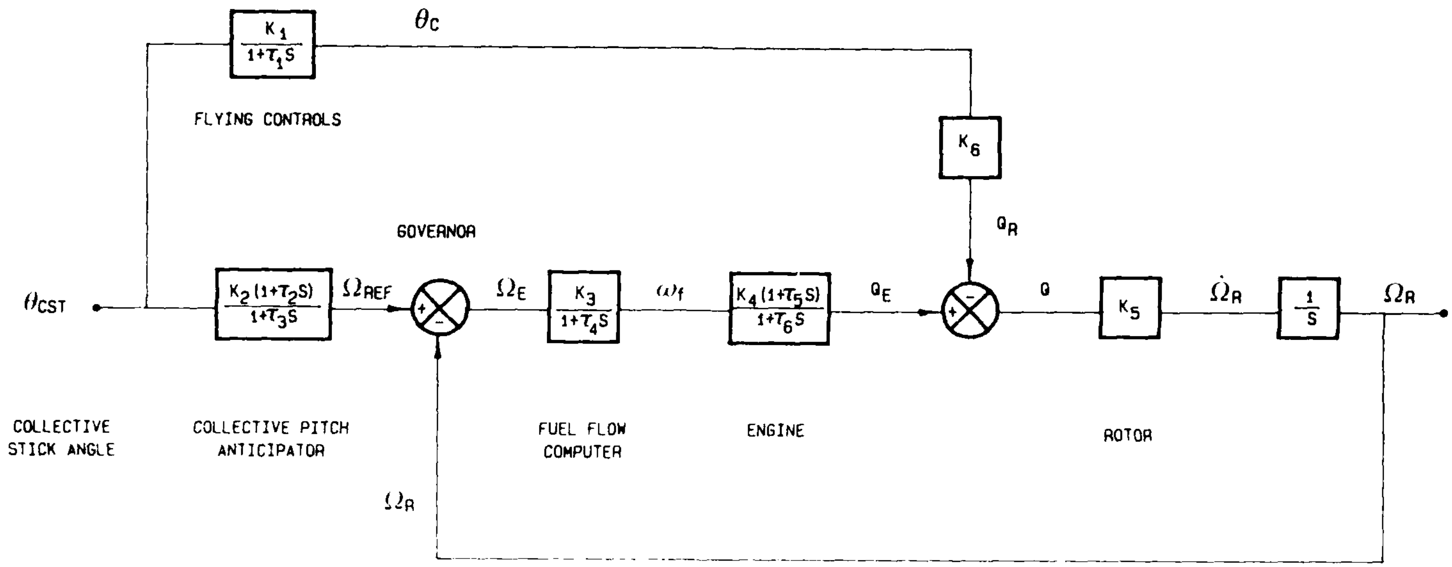


FIG. 5. ROTOR SYSTEM TRANSFER FUNCTION BLOCK DIAGRAM

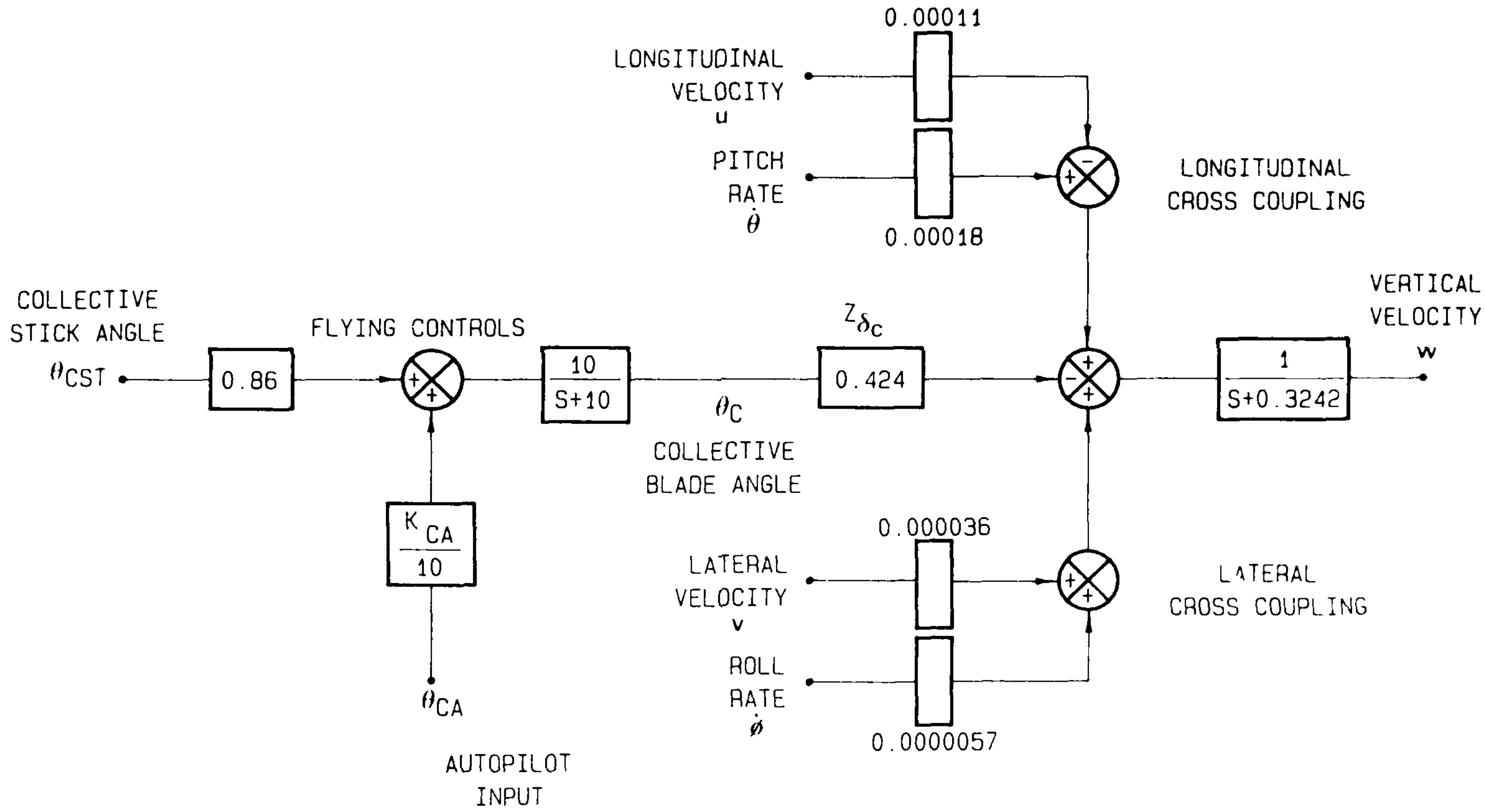
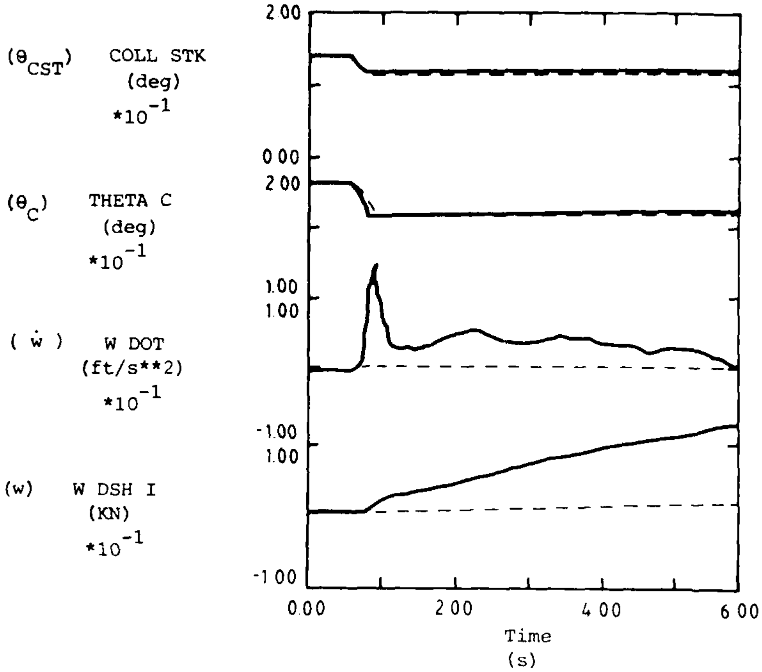


FIG. 6. BLOCK DIAGRAM OF BASIC AIRCRAFT VERTICAL MOTION DYNAMICS

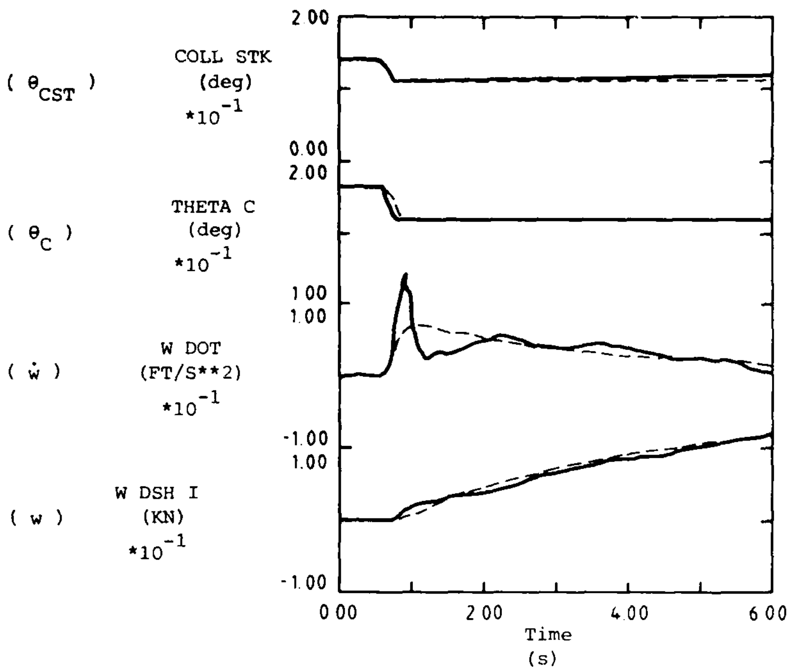


Collective step down, hover, stab off

Key: ———— Flight test

-------- Model

FIG. 7. COMPARISON OF MODEL AND FLIGHT TEST RESULTS (BASIC MODEL)

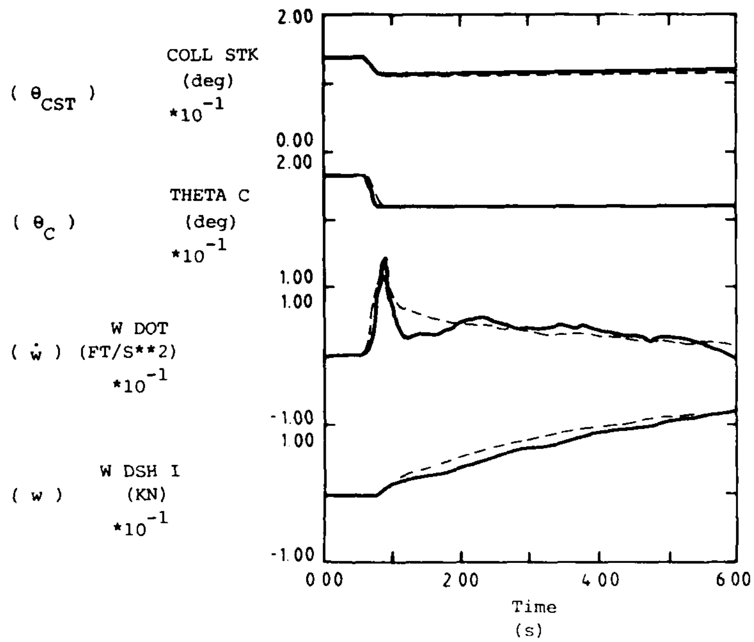


Collective step down, hover, stab off

Key: ——— Flight test

- - - - - - - Model

FIG. 8. COMPARISON OF MODEL AND FLIGHT TEST RESULTS (INCREASED GAIN Z_{δ_C})

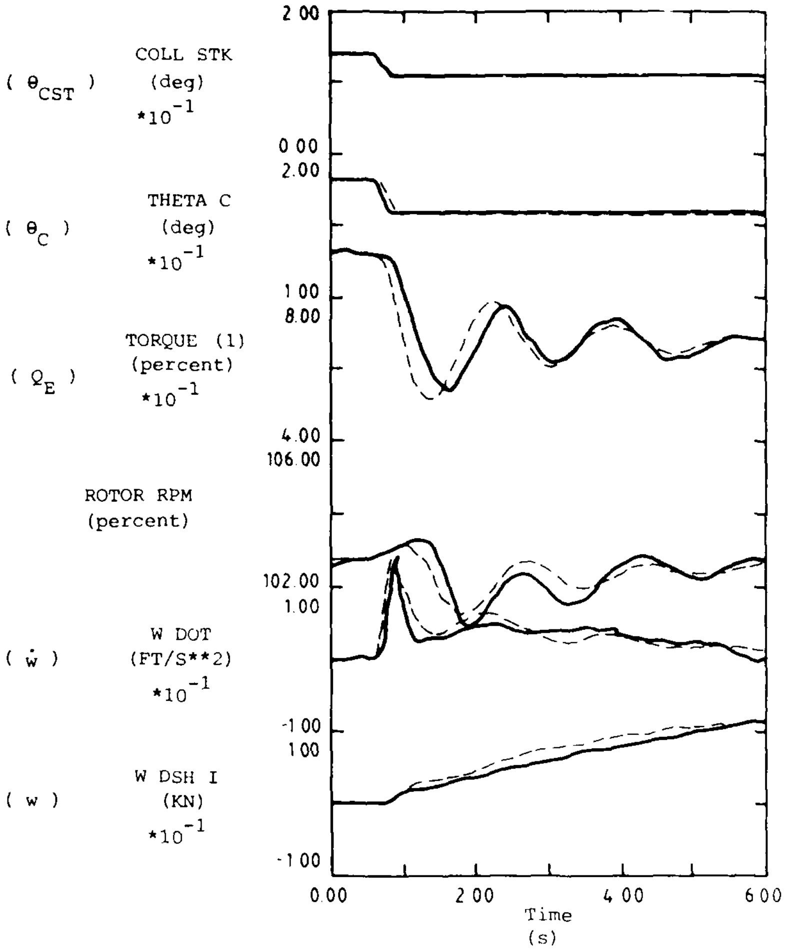


Collective step down, hover, stab off

Key: ——— Flight test

- - - - - - - Model

FIG. 9. COMPARISON OF MODEL AND FLIGHT TEST RESULTS (DERIVATIVE TERM ADDED)

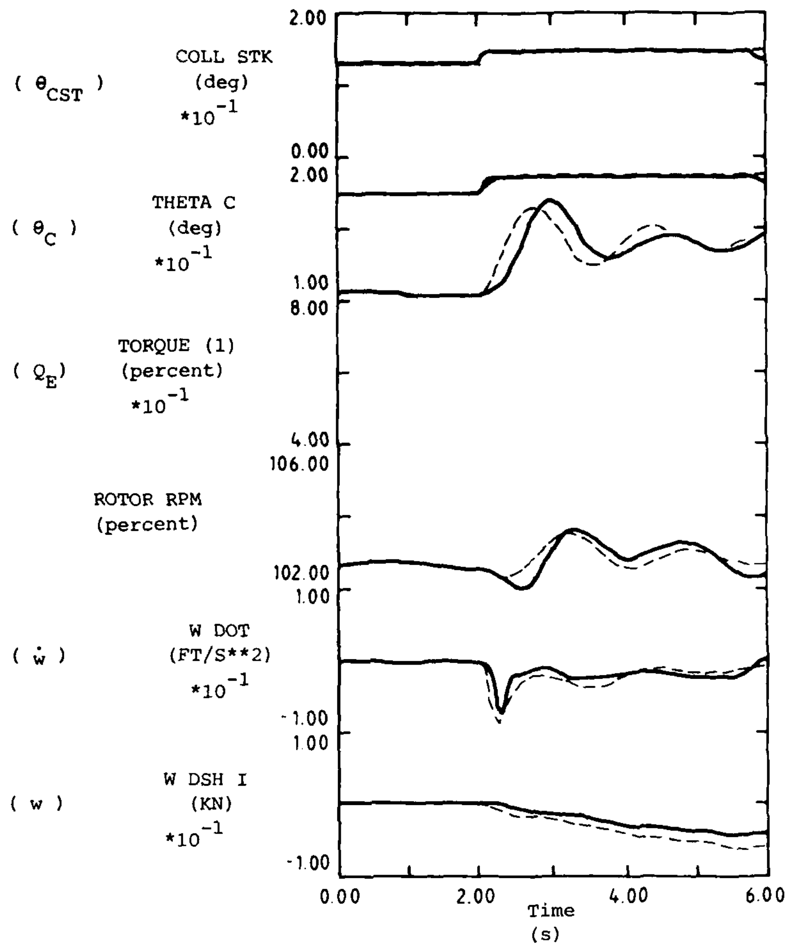


Collective step down, hover, stab off

Key: ——— Flight Test

- - - - - - - Model

FIG. 10. COMPARISON OF MODEL AND FLIGHT TEST RESULTS (TORQUE TERM ADDED) - COLLECTIVE STEP DOWN



Collective step up, hover, stab off

Key: ———— Flight test

-------- Model

FIG. 11. COMPARISON OF MODEL AND FLIGHT TEST RESULTS (TORQUE TERM ADDED) - COLLECTIVE STEP UP

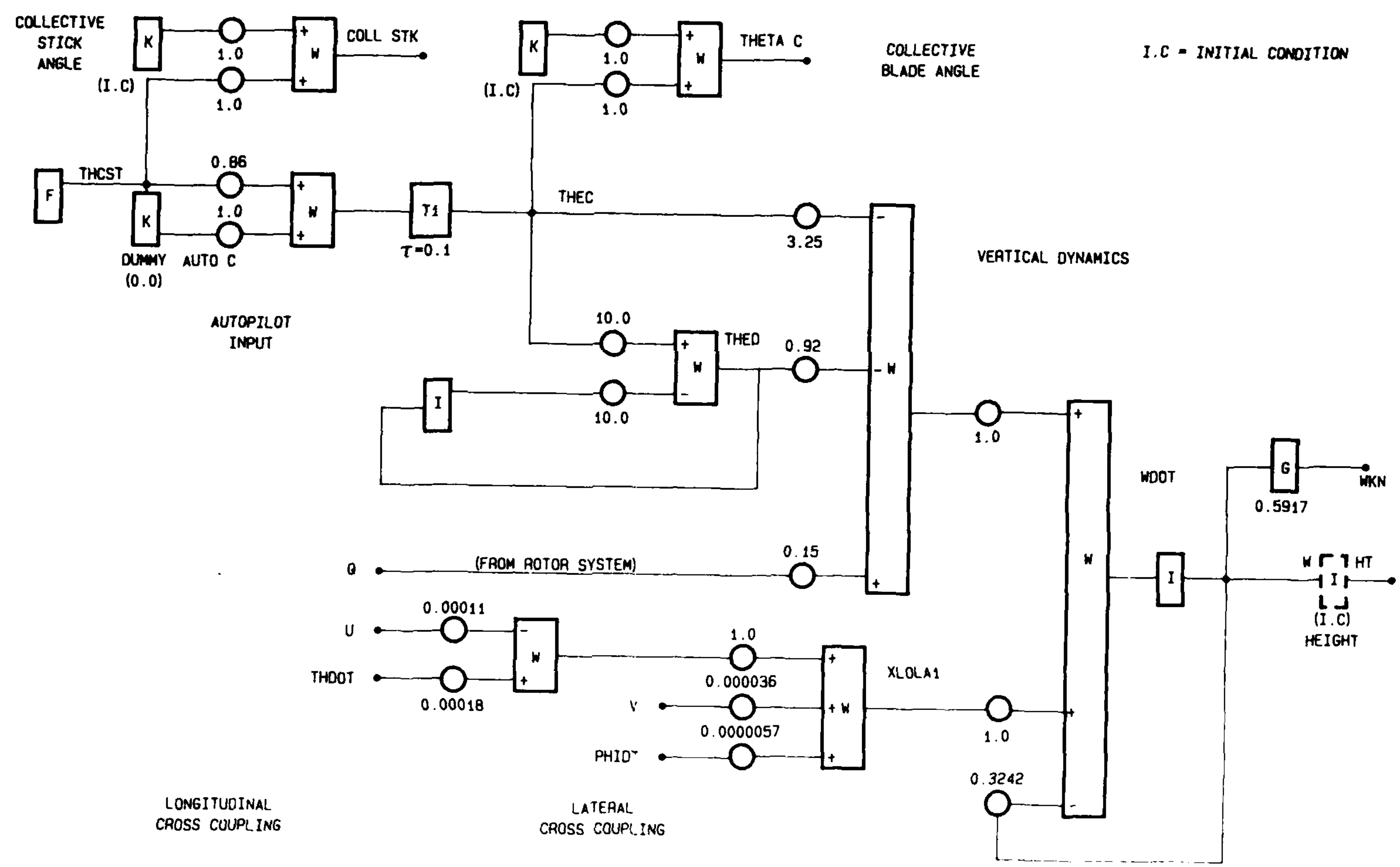


FIG. 12. AIRCRAFT VERTICAL MOTION SIMULATION BLOCK DIAGRAM

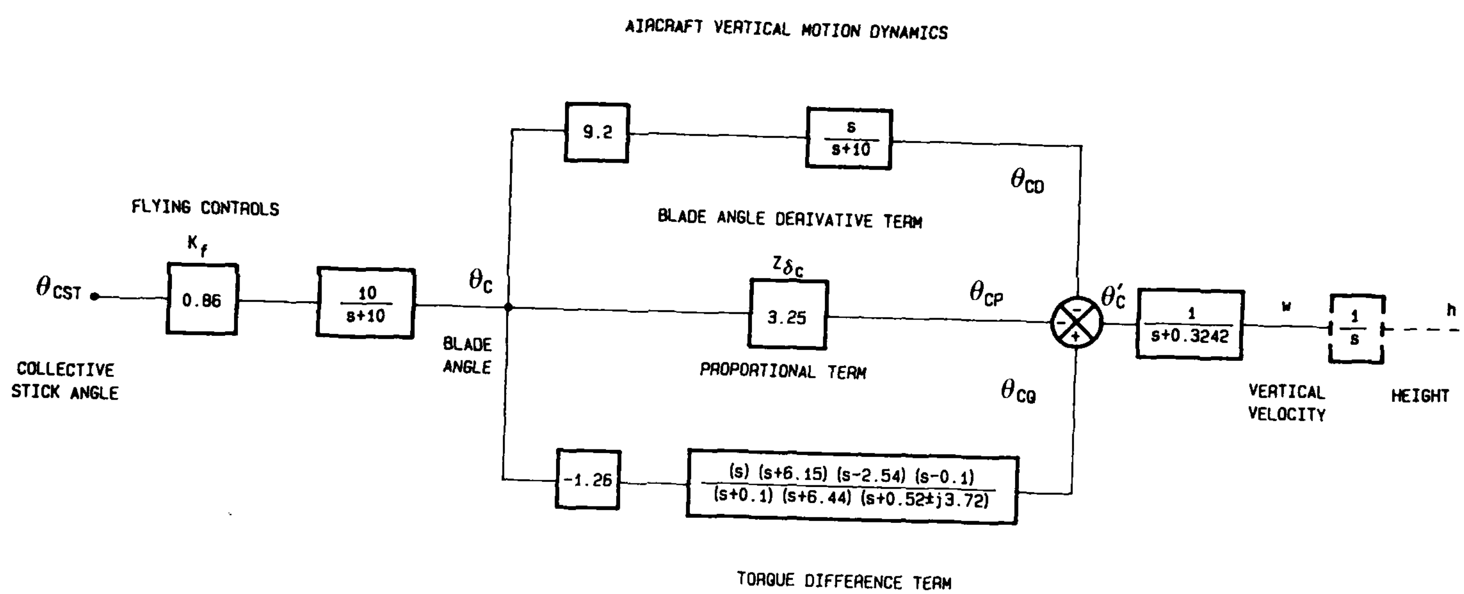


FIG. 13. AIRCRAFT VERTICAL MOTION TRANSFER FUNCTION BLOCK DIAGRAM

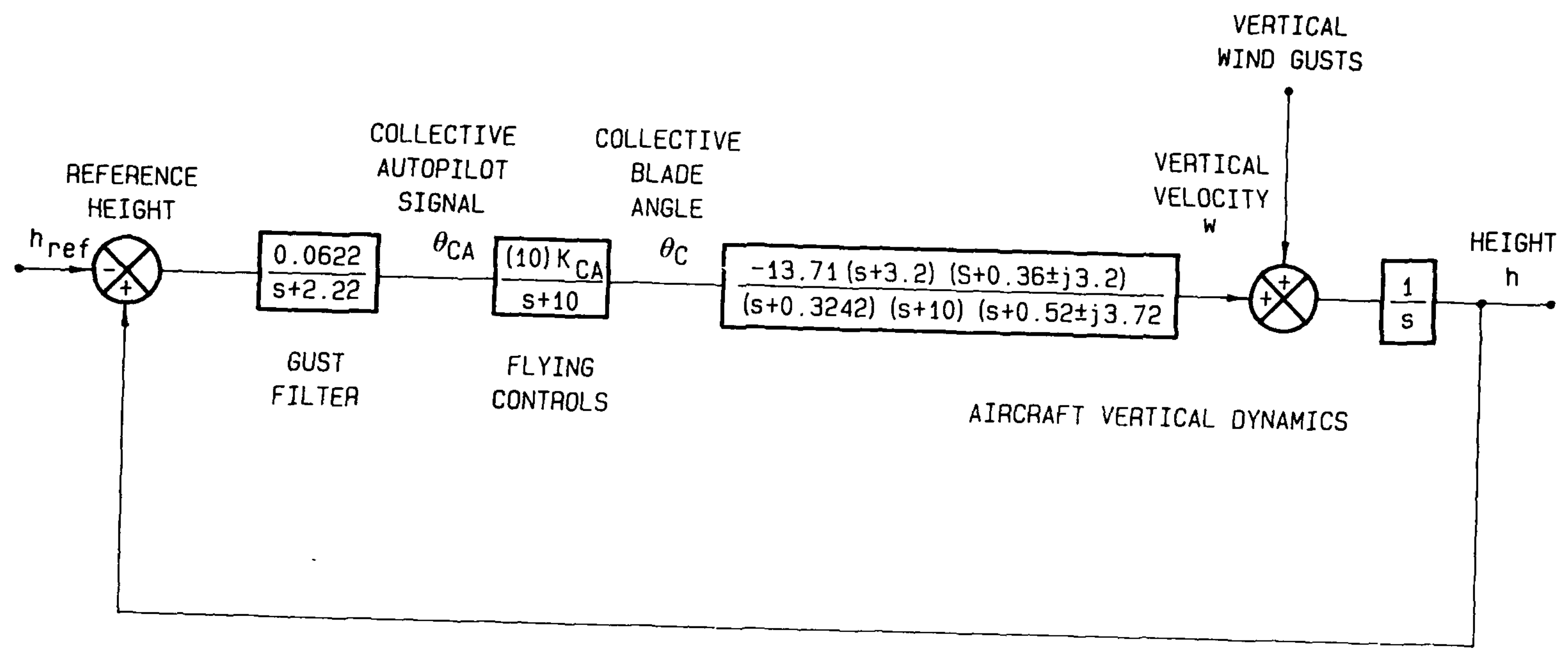


FIG. 14. BAROMETRIC HEIGHT HOLD BLOCK DIAGRAM

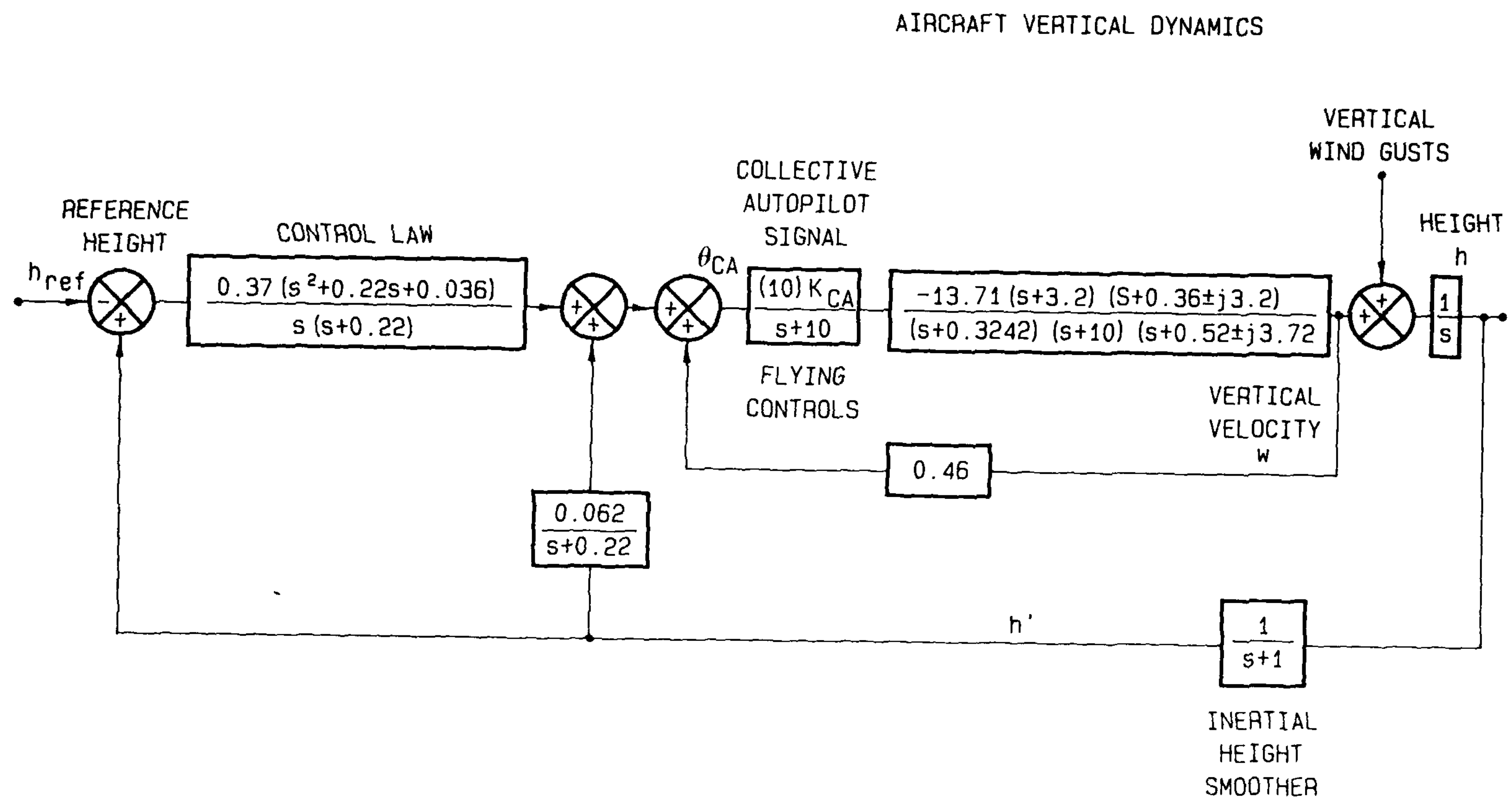


FIG. 15. RADIO ALTITUDE HOLD BLOCK DIAGRAM

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16. Abstract

This document examines the flight dynamic characteristics of a hovering helicopter in response to collective stick inputs. Only vertical motion is considered. A mathematical model of the aerodynamics/kinematics and rotor system dynamics of a representative aircraft, linearized about the hovering flight condition, is used as a basis for the study. The dynamic behaviour of this model is compared with flight test results for similar inputs and the coefficients are adjusted to improve model performance. The model structure is then modified by the incorporation of additional terms to improve its accuracy. Although no attempt is made to optimize the coefficients of the model using parameter identification techniques, the approach used here provides acceptable results.

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