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INVESTIGATION ON IMPROVED ITERATIVE METHODS FOR SOLVING
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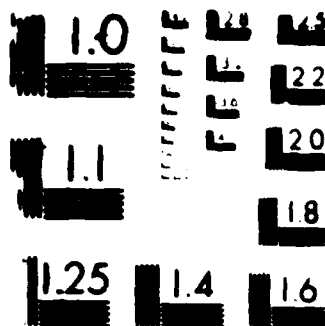
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**Investigations on Improved Iterative Methods for Solving
Sparse Systems of Linear Equations**

Submitted by

Kent State University
Kent, Ohio 44242

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) This effort involved research investigating improved iterative methods for solving sparse systems of linear equations. Research papers appearing in print during this period included such titles as "On the minimum module of normalized polynomials", "Extended numerical computations on the "1/9" conjecture in rational approximation Theory", and "A study of semiterative methods for nonsymmetric systems of linear equations". A major accomplishment during this period of effort was the use of summability methods and conformal mapping techniques in the study of iterative methods, thus enhancing the theoretical foundations of such methods.					
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1. Summary of Research in the Period July, 1984 - June 30, 1985. Broadly speaking, the research supported by the Air Force Office of Scientific Research during this period has centered about general matrix methods, and applications of matrix theory in solving large systems of linear equations.

Listed below are those research papers, appearing in print in this period (July, 1984 - June, 1985) or pending publication, which were outgrowths of the research supported by the Air Force Office of Scientific Research. (All carry, or will carry, an acknowledgement of AFOSR support.)

1. S. Ruscheweyh and R.S. Varga, "On the minimum moduli of normalized polynomials", Rational Approximation and Interpolation, Proceedings, Tampa, Florida 1983, (P.R. Graves-Morris, E.B. Saff, and R.S. Varga, eds.), Lecture Notes in Mathematics 1105, pp. 150-159, Springer-Verlag, Heidelberg, 1984. MR86e:3007.
2. A.J. Carpenter, A. Ruttan, and R.S. Varga, "Extended numerical computations on the "1/9" conjecture in rational approximation theory", Rational Approximation and Interpolation, Proceedings, Tampa, Florida 1983, (P.R. Graves-Morris, E. B. Saff, and R.S. Varga, eds.), Lecture Notes in Mathematics 1105, pp. 383-411, Springer-Verlag, Heidelberg, 1984.
3. A.S. Cavaretta, Jr., A. Sharma, and R.S. Varga, "A Theorem of J.L. Walsh", revisited, Pacific J. Math. 118 (1985), 313-322. MR86m:30039.
4. R.S. Varga and Wu Wen-Da, "On the rate of overconvergence of the generalized Eneström-Kakeya functional for polynomials", J. Comp. Math. 3 (1985), 275-288. MR87d:65034.
5. M. Eiermann, W. Niethammer, and R.S. Varga, "A study of semiiterative methods for nonsymmetric systems of linear equations, Numer. Math. 47 (1985), 505-533.
6. A.S. Cavaretta, Jr., A. Sharma, and R.S. Varga, "Converse results in the Walsh theory of equiconvergence", RAIRO Modél Math. Anal. Numér. 19 (1985), 601-609. MR87g:30001.

The above research papers can be roughly grouped into the following areas:

- A. Applications of function theory,
- B. The use of summability methods and approximate conformal mapping techniques in the study of iterative methods.

We now focus on area B, and the associated paper by M. Eiermann, W. Niethammer, and R.S. Varga, listed as #5 above.

The Use of Summability Methods and Approximate Conformal Mapping Techniques in the Study of Iterative Methods.

To iteratively solve the matrix equation (in fixed point form)

(1)
$$\mathbf{x} = T\mathbf{x} + \mathbf{c},$$



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(where 1 is not an eigenvalue of T), one takes as the basic iteration

$$(2) \quad \mathbf{x}_{m+1} = T\mathbf{x}_m + \mathbf{c} \quad (\mathbf{x}_0 = \mathbf{a}, m \geq 0),$$

which is well-known to be convergent, for arbitrary $\mathbf{a}$, iff the spectral radius, denoted by $\rho(T)$, satisfies $\rho(T) < 1$. Associated with (2) is a *semiiterative* method (SIM), defined by

$$(3) \quad \mathbf{y}_m = \sum_{i=0}^m \pi_{m,i} \mathbf{x}_i$$

where the (complex) numbers $\{\pi_{m,i}\}_{i=0, m=0}^{m, \infty}$ define an infinite triangular matrix

$$(4) \quad P = \begin{bmatrix} \pi_{0,0} & & & \\ \pi_{1,0} & \pi_{1,1} & & \\ \pi_{2,0} & \pi_{2,1} & \pi_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where we assume

$$(5) \quad \sum_{i=0}^m \pi_{m,i} = 1 \quad (m \geq 0).$$

On setting

$$(6) \quad p_m(z) := \sum_{i=0}^m \pi_{m,i} z^i \quad (m \geq 0),$$

then $p_m(z)$ is a (complex) polynomial of degree at most m , with $p_m(1) = 1$ ($m \geq 0$).

If $\mathbf{x}$ denotes the unique solution of (1) (since 1 is not an eigenvalue of T), then the error vector ε_m associated with the iterates $\mathbf{x}_m$ of (2), i.e., $\varepsilon_m := \mathbf{x} - \mathbf{x}_m$, satisfies

$$(7) \quad \varepsilon_m = T\varepsilon_{m-1} = \dots = T^m \varepsilon_0 \quad (m \geq 0).$$

Analogously, if $\tilde{\varepsilon}_m := \mathbf{x} - \mathbf{y}_m$, then $\tilde{\varepsilon}_m$ satisfies

$$(8) \quad \tilde{\varepsilon}_m = p_m(T)\varepsilon_0.$$

We now assume that the set of eigenvalues of T (denoted by $\sigma(T)$) is contained in a compact set Ω (where $1 \notin \Omega$). The assumption of such a set Ω is typical in applications of iterative methods applied to matrix equations arising from physical problems. Then, given Ω , choose any infinite matrix P of (4), (which then defines the semiiterative method of (3)). Then, the *asymptotic convergence factor*, with respect to P , is defined by

$$(9) \quad K(\Omega, P) := \overline{\lim}_{m \rightarrow \infty} \left\{ \max_{z \in \Omega} |p_m(z)| \right\}^{1/m},$$

and we set

$$(10) \quad K(\Omega) := \inf \{ K(\Omega, P) : P \text{ induces a SIM of the form (3)} \}.$$

Then, $K(\Omega)$ is called the *convergence factor* for Ω , and any $\hat{P}$ for which

$$(11) \quad K(\Omega) = K(\Omega, \hat{P})$$

is called an *asymptotically optimal SIM* (an AOSIM) with respect to Ω . The whole object is to determine AOSIM's for a given compact set Ω (containing the eigenvalues of T), for such SIM's give the *fastest* asymptotic convergence rates, and these are thus preferred for actual computation!

To describe our main results, let $\bar{C} := C \cup \{\infty\}$ denote the extended plane, and let

$$(12) \quad \mathbf{M} := \{ \Omega \subset \mathbb{C} : \Omega \text{ is compact; } 1 \in C \setminus \Omega; C \setminus \Omega \text{ is simply connected, and } \Omega \text{ contains more than one point} \}.$$

Then, by the Riemann Mapping Theorem, there is, for each $\Omega \in \mathbf{M}$, a conformal mapping

$$(13) \quad \begin{cases} \Psi : \bar{C} \setminus \{ \omega : |\omega| \leq 1 \} \rightarrow \bar{C} \setminus \Omega, \text{ and} \\ \Psi(\infty) = \infty : \Psi'(\infty) =: \gamma(\Omega) > 0. \end{cases}$$

Now, there is a unique $\hat{\omega}$ with $\Psi(\hat{\omega}) = 1$, and if $\hat{\eta} := |\hat{\omega}| > 1$, then this number $\hat{\eta}$ is the key to finding an AOSIM with respect to $\Omega \in \mathbf{M}$.

More precisely, we have (cf. [5, Corollary [2]]) the

Theorem. *Given $\Omega \in \mathbf{M}$, then any SIM generated by a P satisfying (5) is an AOSIM with respect to Ω iff*

$$(14) \quad K(\Omega, P) = \frac{1}{\hat{\eta}}.$$

Thus, for any $\Omega \in \mathbf{M}$, an AOSIM *always* exists for Ω , and such a SIM can be obtained from the conformal mapping of (13). What is very interesting, in our opinion, is that there is a strong connection with the theory of Faber polynomials and the construction of nearly optimal semiiterative methods, for each $\Omega \in \mathbf{M}$. To be precise, let us normalize to $\gamma(\Omega) = 1$ in (13), so that

$$(15) \quad \Psi(\omega) = \omega + \sum_{k=0}^{\infty} \alpha_k \omega^{-k} \quad (|\omega| > 1).$$

If $\phi(z)$ denotes the inverse mapping of $\Psi(\omega)$, then we can write the Laurent expansion of $(\phi(z))^n$ as

$$(16) \quad (\phi(z))^n = \gamma_0 z^n + \sum_{k=0}^{n-1} \beta_{n,k} z^k + \sum_{k=-\infty}^{n-2} \beta_{n,k} z^k.$$

The principal part (or polynomial part) of $(\phi(z))^n$ is defined to be the *Faber polynomial* of degree n for the set Ω . It turns out that, on assuming some additional smoothness on the boundary $\partial\Omega$ of Ω , these Faber polynomials can be used to generate a SIM for the region Ω which is *asymptotically optimal* (i.e., an AOSIM). This is given in Theorem 22 of [5].

In our opinion, these results from complex function theory form the *foundation* of a wide-reaching theory of iterative methods.

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