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RESONANT LANDAU LEVEL-OPTICAL PHONON INTERACTION IN
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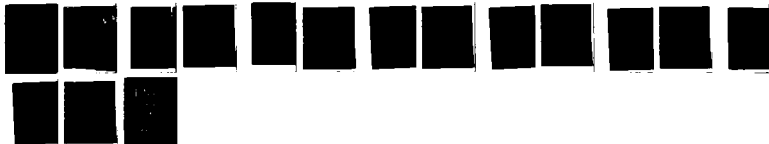
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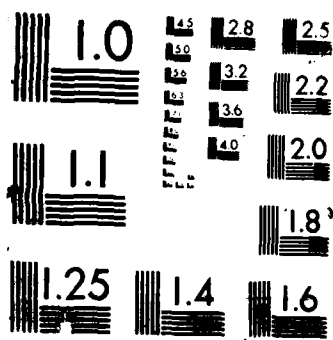
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RESONANT LANDAU LEVEL-OPTICAL PHONON INTERACTION IN
TWO-DIMENSIONALLY CONFINED CHARGE CARRIER SYSTEMS*

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I. INTRODUCTION

The study of the phenomenon of resonant coupling of Landau level with the longitudinal optical phonon in a bulk semiconductor received considerable impetus with the observation¹ of the anomalous magneto-optical interband absorption behavior of InSb, reported a little over a decade ago. Subsequent studies clarified various aspects of this phenomenon as it affects impurity absorption,² cyclotron resonance³ and combined resonance.⁴ A systematic theoretical study⁵ of the phenomenon in free carrier systems has led to a classification of the relative importance of self-energy and electron-light vertex corrections. In this lecture I shall discuss certain aspects of this phenomenon in the context of the new and novel two-dimensionally confined charge carrier systems that can be made to occur in semiconductor quantum wells,⁶ superlattices,^{7,8} heterojunctions^{9,10} and the metal-oxide-semiconductor systems.¹¹ The fabrication and study¹² of these systems within the recent past have opened a rather unique opportunity for the exploration of new phenomena arising from the reduced dimensionality, as well as from the special conditions that can be realized in these systems. In addition, new ramifications of established phenomena can be fruitfully employed to gain information regarding fundamental interactions unique to the system.

The theoretical results I shall present cover work done¹³⁻¹⁵ in collaboration with my post-doctoral associates, Drs. S. Das Sarma and R. N. Nucho. I shall develop the subject in four stages. In Section II I shall schematically illustrate the different kinds of new systems in which two-dimensionally confined carriers have been created. Here I shall lay greater emphasis on the heterojunction and superlattice systems since the metal-oxide-semiconductor system is

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discussed at some length in the lectures by Dr. B. D. McCombe. This should also serve to contrast the situation with that in the bulk, particularly in the presence of a magnetic field. In Section III I shall discuss the nature of possible electron-phonon couplings that these new systems can generate, either by virtue of changes in selection rules, zone folding effects or because of a new kind of phonon branch. In Section IV I shall provide results for the resonant splitting of combined Landau Level-LO phonon system, obtained in the simplest approximation capable of revealing the phenomenon - namely, that which retains only the resonant self energy term for the electron. The splitting will be found to be proportional to the electron-phonon coupling strength. In Section V the magneto-optical-absorption will be calculated, allowing for a finite width of the electronic levels in a phenomenological way. Finally, I shall conclude with some remarks about the electron-light vertex corrections.

II. SYSTEMS

A schematic representation of the energy diagram of quantum wells, heterojunctions and superlattices is provided in Figures 1 and 2 for the two most extensively studied systems; GaAs/Al_xGa_{1-x}As and InAs/GaSb. The formation of two-dimensionally confined bands in these systems arises from the presence of band edge discontinuities and band bending which act as barriers to motion normal to the interface between the two materials involved. In the Si MOS inversion layers an external electric field is needed to achieve sufficient band bending so as to lead to the formation of quantized energy states for motion normal to the interface. In the GaAs/Al_xGa_{1-x}As and InAs/GaSb heterojunctions such a field is not needed. A comparison of Figures 1 and 2 also shows schematically the difference in the band line up of the GaAs/AlAs and InAs/GaSb systems. The unusual band line up of the latter system leads to the formation^{13,16} of two-dimensionally confined electron and hole bands with electrons in InAs and holes in GaSb. It also leads to a semiconductor to semimetal change with increasing superlattice thickness^{16,17}.

Carriers in these systems are thus in two-dimensional subbands where their motion parallel to interfaces is "free" whereas their motion in Z-direction (normal to the interface) is confined. It is important, however, to clearly recognize certain differences between a single two-dimensionally confined layer (such as inversion layers, accumulation layers in heterostructures or a few quantum wells) and a repeated structure of these layers such as the superlattice. In the former case the motion of a wave vector k_z in the normal direction does not exist, whereas in the latter case it does. The range of values taken by k_z however decreases as the size of the unit cell of the superlattice (comprising of one slab each of the two materials involved) increases. In Figure 3 (panel b) is shown the band structure¹³, in the (001) direction, of the InAs/GaSb (001) superlattice consisting of 16 atomic layers each of InAs and GaSb. Only

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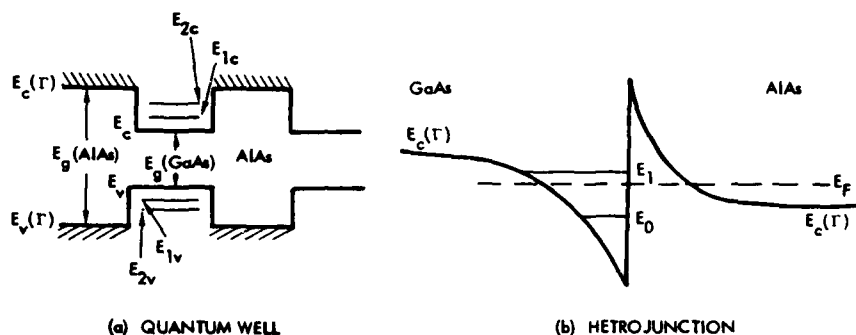


Fig. 1. Schematic representation of two-dimensionally confined electronic states in (a) GaAs/Al_xGa_{1-x}As quantum well and (b) GaAs/Al_xGa_{1-x}As heterojunction. This is also similar to Inversion/Accumulation layers in MOS systems.

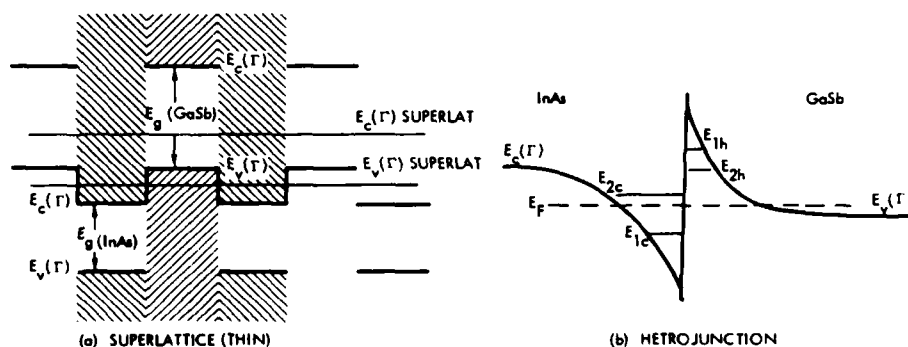


Fig. 2. (a) Schematic representation of thin InAs/GaSb superlattice reflecting the unusual starting band line up at the Γ -point. (b) The situation for the InAs/GaSb heterojunction giving rise to two-dimensional electron and hole bands (in InAs and GaSb respectively) as a consequence of the conduction (InAs) to valence (GaSb) band edge discontinuity.

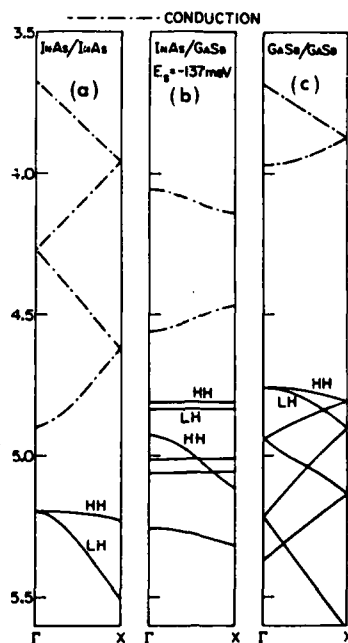


Fig. 3. Shows the zone folding effect in InAs/GaSb(001) superlattice. Panels (a) and (c) show the bulk band structure of InAs and GaSb plotted in the zone folded scheme in the (001) direction. Panel (b) shows the band structure of a superlattice with 16 atomic layers each of InAs and GaSb in the unit cell. E_g denotes the band edge discontinuity employed in the calculations (see Ref. 13).

a limited energy region covering the fundamental band gaps, the first conduction band of bulk InAs (panel a), and the topmost valence band of bulk GaSb (panel c) is shown for comparison with the bulk band structures of these materials plotted in the zone folded scheme (panels a and c).

The confinement of the electronic motion in the Z-direction leads to a unique situation when a magnetic field with a component in the Z-direction is imposed on the system - the parallel motion is quantized into Landau orbits, thus leading to complete quantization of the spatial motion of the charge carriers. It is the lack of motion in the Z-direction which, as we will see in Section IV, will give rise to sharp resonance splitting in the magneto-optical absorption. This feature makes such measurements in two-dimensionally confined carrier systems of potential significance in providing information regarding electron-optical phonon coupling.

III. ELECTRON-OPTICAL PHONON INTERACTION

Recent experiments¹⁸⁻²⁰ on the GaAs/Al_xGa_{1-x}As system reveal strong evidence for enhanced electron-LO phonon interaction in GaAs. Resonance enhanced normal¹⁸ and umklapp¹⁹ Raman processes have been observed in superlattices and ascribed to zone folding effects and the wave vector dependent Fröhlich type²¹ electron-LO phonon interaction. Phonon assisted recombination in quantum well structures has been ascribed²⁰ to the breakdown of translational invariance and momentum selection rules²². Of equal practical significance is the possible importance of the full Bloch wave form of the electron wave function demanded by the superlattice when considering certain physical phenomena. In particular, for the electron-optical phonon coupling, we note that the usual continuum approximation implicit in the use of Fröhlich's model for bulk semiconductors may not be valid in the present context. For the inversion layers, the description of the Z-dependence of the electron charge distribution has been sought in terms of an envelope function characteristic only of a particular electric subband but independent of $\vec{k}_{11}$, the wave vector in the two-dimensional plane. However, for superlattices the charge distribution within the unit cell may depend strongly upon the particular $\vec{k}_{11}$ value via the Bloch form of the wave function. That such a dependence of the charge variation in the Z-direction may be important is reflected in calculations done in our group on InAs/GaSb superlattices up to 50 Å thick in unit cell. This suggests the possible inadequacy of the effective mass approximation in phenomena involving electron states in a reasonable portion of the $\vec{k}_{11}$ space. In particular, the electron-optical phonon coupling in such a situation needs to be very carefully investigated. For simplicity here we proceed with a discussion of the electron-optical phonon coupling within the approximation of neglecting the $\vec{k}_{11}$ dependence.

The electronic charge density at any point $\vec{R} \equiv (\vec{r}, z)$ may be written as,

$$\rho_{el}(\vec{r}, z) = \frac{-1}{V} \sum_{i,j} \sum_{\vec{q}, \vec{q}'} \xi_{i\vec{q}}^*(z) \xi_{j\vec{q}'}(z') e^{-i(\vec{q}-\vec{q}') \cdot \vec{r}} C_{\vec{q}i}^{\dagger} C_{\vec{q}'j} \quad (1)$$

where $\vec{r}$ and $\vec{q}$ are the two-dimensional position and wavevectors, respectively. V is the total volume, and $C_{\vec{q}i}^{\dagger}$ ($C_{\vec{q}i}$) is the creation (annihilation) operator for an electron (or a hole) with two-dimensional momentum $\vec{q}$ in the i th subband. (We take $e = \hbar = 1$.) For a finite structure (in the Z -direction) such as inversion layers, i stands only for the quantized band, whereas for a repeated structure like the superlattice it may be considered to incorporate the wave vector k_z . $\xi_{i\vec{q}}(z)$ gives the Z -dependence of the wave functions of the quantized bands. We set $\xi_{i\vec{q}}(z) \equiv \xi_i(z)$, independent of $\vec{q}$. Equation (1) for the electronic charge distribution suggests

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that its coupling with the ionic charge may also be dependent upon the particular distribution that the latter may take. Consequently in the following we consider the form of the electron-phonon coupling arising from phonon modes in two broad categories, (i) coupling of the confined electrons with bulk LO phonon modes and (ii) coupling with quasi-two-dimensional phonon modes, i.e., phonon modes which reflect peaking of the ionic charge displacement somewhere within the confined region in which the electrons reside.

Consider first the coupling of these confined electrons to bulk LO phonons. We assume that the bulk phonons remain unaffected by the formation of the superlattice. Then within the Fröhlich model, the electron-phonon coupling with a two-dimensional momentum exchange $\vec{k}$ is readily found to be

$$M_{ij}(\vec{k}, k_z) = \left\{ \frac{2\pi}{V} \omega_{LO} (\epsilon_\infty^{-1} - \epsilon_0^{-1}) \right\}^{\frac{1}{2}} \cdot f_{ij}(k_z) (k^2 + k_z^2)^{-1} \quad (2)$$

where,

$$f_{ij}(k_z) = \int dz \xi_i^*(z) \xi_j(z) e^{ik_z \cdot z}, \quad (3)$$

k_z is the z -component of phonon momentum, ω_{LO} is the longitudinal optical phonon frequency and $\epsilon_\infty, \epsilon_0$ are the high frequency and low frequency dielectric constants, respectively. Equation (2) gives the matrix elements for coupling between a three-dimensional phonon of momentum $(\vec{k}, k_z)$ and a two-dimensionally confined electron which scatters from subband i to subband j . The leading order coupled electron-phonon processes involve the square of the matrix element in Eq. (2). Such an effective coupling strength, say $V_{ijlm}(\vec{k}, k_z)$, is thus given by

$$V_{ijlm}(\vec{k}, k_z) = M_{ij}(\vec{k}, k_z) M_{lm}^*(\vec{k}, k_z) \quad (4)$$

It signifies scattering of an electron from i to j with emission of a LO phonon of momentum $(\vec{k}, k_z)$ and the subsequent absorption of the phonon by another electron which gets scattered from l to m .

For systems without translational invariance in the z -direction, k_z in Eqs. (2) through (4) becomes completely arbitrary. In that case, one sums over k_z in Eq. (4) to obtain the effective electron-phonon coupling with the exchange of two-dimensional momentum $\vec{k}$. Neglecting LO phonon dispersion and summing over all k_z , we find

$$V_{ijlm}(\vec{k}) = \left\{ \frac{2\pi}{A} \cdot \omega_{LO} (\epsilon_\infty^{-1} - \epsilon_0^{-1}) \right\} f_{ijlm}(\vec{k}) \cdot \frac{1}{2k} \quad (5)$$

where A is the area and,

$$f_{ijlm}(\vec{k}) = \int dz \int dz' \xi_i^*(z) \xi_j(z) e^{-k|z-z'|} \xi_l^*(z') \xi_m(z') \quad (6)$$

The integrals over z, z' extend over a small region of space due to the two-dimensional confinement in these systems. For comparison, we recall that the Fröhlich interaction between electrons and LO phonons in three dimensions, (continuum solid model) is given by²¹,

$$V(k) = \left\{ \frac{2\pi}{V} \cdot \omega_{LO} (\epsilon_\infty^{-1} - \epsilon_0^{-1}) \right\} \cdot \frac{1}{k^2} \quad (7)$$

where k in Eq. (7) is the three-dimensional momentum exchange.

For the case of the superlattice, in addition to the modifications in the e-ph. coupling strength arising from charge confinement effects, we also have the formation of mini Brillouin zones in the k_z direction. However, noting the expected^{19,23} insignificant influence of this minizone formation on the bulk optical phonon branch we may, in considering virtual processes requiring summation over all k_z choose to consider it to lie within the original BZ, thus allowing for umklapp processes for electron-phonon scattering. Alternatively, one could incorporate the entire bulk optical phonon branch via a summation over the appropriate number of phonon modes within the minizones. This is referred to as the zone folding effect. In general, frequencies of these modes will be different due to the dispersion (wave-vector dependence) of the LO phonon mode which we assume to be small. It is important to note that since the effective coupling strength given by Eq. (5) sums over all k_z (and not just the unit minizone), it contains both the normal and the umklapp processes involving phonons with arbitrary k_z have the zone folding effects already built into it. It is only for the processes involving fixed momenta (e.g., $k_z = 0$ for processes involving only zone center phonons) that the zone folding effects will give rise to an explicit enhancement of the electron-phonon coupling strength via an increase in the number of phonon branches alone.

We turn now to the electron-phonon interactions belonging to category (ii) in which the longitudinal optical phonons are assumed to be quasi-two-dimensional just as the carriers themselves. The effective coupling strength for this model can be written as

$$V_{ijlm}(\vec{k}) = \frac{2\pi}{A} \omega_{LO} (\epsilon_\infty^{-1} - \epsilon_0^{-1}) g_{ijlm}(\vec{k}) \cdot \frac{1}{2k} \quad (8)$$

where

$$g_{ijlm}(\vec{k}) = \int dz \int dz' \xi_i^*(z) \xi_j(z) \xi_l^*(z') \xi_m(z') \left\{ \int dz_1 \int dz_2 \eta(z_1) \eta(z_2) e^{-k(|z-z_1|+|z'-z_2|)} \right\} \quad (9)$$

$\eta(z)$ in Eq. (9) gives the envelope function describing the decay of the phonons in the normal direction. The position of the peak in $\eta(z)$ will naturally be governed by the specific nature of quasi-two-dimensional vibrational modes which may exist in these systems. For instance, an interface phonon is characterized by an $\eta(z)$ which peaks at the interface. From Eq. (9) it is clearly seen that maximum electron-phonon coupling is obtained when the quasi-two-dimensional phonons are peaked in the same place where the electronic charge density (given by $|\xi_i(z)|^2$) reaches a maximum. The influence of the electron-phonon interaction manifested in the measurements on these systems is then via an effective coupling strength which is an average over various phonon modes and their couplings with the spatial distribution of the electrons.

Above considerations based on a Fröhlich model clearly bring out certain special features of the electron-LO phonon interaction in these novel quasi-two-dimensionally confined systems. The form factor arises only because the carriers in these systems are quantized into two-dimensional subbands. Zone-folding effects may give rise to a strong enhancement in the coupling strength in superlattices via the formation of new phonon branches. Lack of a wave vector conserving selection rule in the Z-direction makes it possible for phonons with all k_z to take part in the interaction process. This enhances the coupling strength even further. Significant effects may arise from the coupling of the confined carriers to quasi-two-dimensional phonons (possibly interface modes) which may exist in these systems.

IV. THE RESONANT SPLITTING PHENOMENON

Let us consider two Landau levels n and $(n-1)$ belonging to the same subband, j . We denote them by quantum numbers $N \equiv (j, n)$ and $(N-1) \equiv (j, n-1)$. (See Figure 4.) They differ by cyclotron energy, $\omega_{cj} = B_z(m_j^*c)^{-1}$, where B_z is the component of the magnetic field in the normal direction and m_j^* the effective mass in the j th subband. By varying the external magnetic field it is possible to make $\omega_{cj} = \omega_{LO}$. In that situation, the following two states will be degenerate in the absence of electron-phonon interaction: (1) An electron in the N th Landau level and (2) an electron in the $(N-1)$ Landau level + 1 LO phonon of energy ω_{LO} . Presence of electron-phonon interaction will, however, strongly mix these two states to lift the degeneracy at the point $\omega_{cj} = \omega_{LO}$ giving rise to two split energy levels (Figure 4). Such a splitting will be present even in the presence of a weak electron-phonon interaction since it

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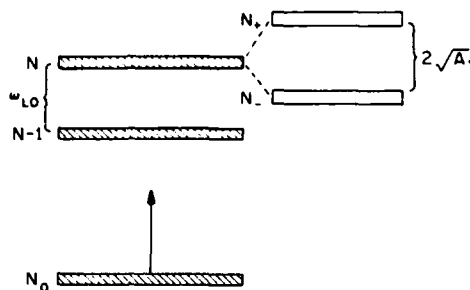


Fig. 4. Schematic representation of the resonant magneto-phonon splitting of the N th Landau level when it is degenerate in energy with the $(N-1)$ level plus 1 LO phonon. N_0 is the level from which optical transition may originate.

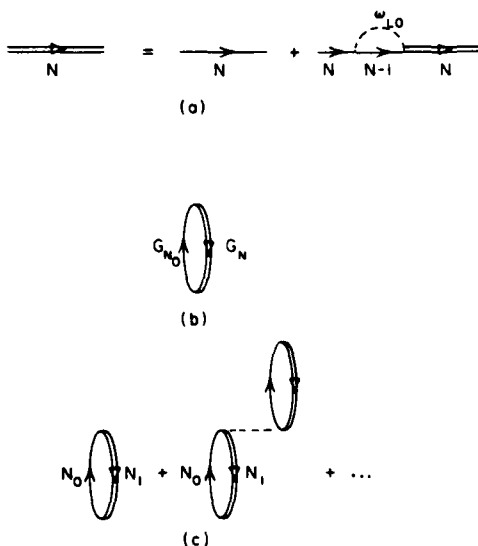


Fig. 5. (a) Dyson's equation for the resonant self-energy coupling between the $(N-1)$ Landau level + 1 LO phonon with the N th Landau level (double and single lines denote dressed and free electron Green's functions, respectively, whereas the dashed line is the LO phonon propagator). (b) Lowest order diagram for absorption in the long wavelength limit. (c) Phonon-induced electron-light vertex correction.

is a resonance process. Such a resonant coupling between a pure electronic state and an electron-phonon composite state can manifest itself in any experiment involving cyclotron transitions to the N th Landau level. Such transitions may involve initial and final Landau levels belonging to different subbands (combined inter-subband cyclotron resonance) or it may originate from the $(N - 1)$ level in the same subband (ordinary cyclotron resonance). The relevant magneto-absorption peak should split into two peaks around $\omega_c = \omega_{LO}$ due to the resonant coupling, provided the broadening of the peaks is less than the energy separation of the split levels. From an experimental point of view, the complications of absorbing cyclotron resonance in the Reststrahlen region suggests the use of the combined inter-subband cyclotron resonance technique. Similar resonant effects have been observed in the bulk²⁴. However, the two-dimensional confinement in the systems we are considering here give rise to features quite different from the corresponding bulk problem.

To investigate the resonant effects quantitatively, consider the self-energy contribution, Σ_N , to the electronic state $N \equiv (j, n)$ from an intermediate composite state consisting of an electron in the state $(N - 1) \equiv (j, n - 1)$ and one LO phonon. Noting the virtual nature of the process, we sum over all the relevant intermediate states involving LO phonons and find,

$$\Sigma_N(\omega) = (\omega^+ - E_{j,n} - \omega_{LO} + \mu)^{-1} \cdot A_1 \quad (10)$$

where $\omega^+ = (\omega + i\eta)$ with $\eta \rightarrow 0^+$, μ is the chemical potential and A_1 is given by

$$A_1 = (c/4\pi\omega_{LO}) \int_0^\infty dq f_{jj}(q) \cdot |F_{n-1,n}(q)|^2 \quad (11)$$

$$c = (2\pi e^2/A) (\epsilon_\infty^{-1} - \epsilon_0^{-1}) \cdot \omega_{LO}^2 \quad (12)$$

$$f_{jj}(q) = \int_0^\infty dz \int_0^\infty dz' |\xi_j(z)|^2 \cdot |\xi_j(z')|^2 e^{-q|z-z'|} \quad (13)$$

$$F_{n_1, n_2}(q) = \frac{n_1!}{n_2!} \left(\frac{1}{2} q^2 \ell^2\right)^{n_2 - n_1} \left\{ \frac{n_1 - n_2}{n_1} \left(\frac{1}{2} q^2 \ell^2\right) \right\}^2 e^{-\frac{1}{2} q^2 \ell^2} \quad (14)$$

The factor $F_{n_1 n_2}(q)$ is a direct result of the Landau quantization in the problem with $\ell = (c\hbar/eB)^{1/2}$ as the radius of lowest cyclotron orbit. Finally, $\mathcal{L}$ is the Laguerre polynomial.

While the resonant splitting phenomenon is independent of the details of the form of A_1 , its magnitude and magnetic field dependence etc., do indeed depend upon the specific nature of the coupling. Equations (11) through (14) provide its explicit form and reveal the functional dependence on the magnetic field B as well as its simple proportionality to the effective electron-phonon coupling strength. It should be pointed out that the simple form of the self-energy (Eq. (10)) is due to the quasi-zero-dimensional nature of the problem in presence of a magnetic field. This is very different from the corresponding three-dimensional result and a reflection of the two-dimensional confinement.

Now, the dressed electron propagator is obtained from the Dyson equation (Figure 5a). Employing the result for Σ_N given by Eq. (10), we find,

$$G_N(\omega^+) = \left[\omega^+ - \epsilon_{j,n} - A_1 \cdot (\omega^+ - \epsilon_{j,n-1} - \omega_{LO})^{-1} \right]^{-1}, \quad (15)$$

where $\epsilon_{j,n} = (E_{j,n} - \mu)$. Rearranging terms, Eq. (15) gives,

$$G_N(\omega) = (\omega - \epsilon_{j,n-1} - \omega_{LO})(\omega_1 - \omega_2)^{-1} \left[(\omega - \omega_1)^{-1} - (\omega - \omega_2)^{-1} \right] \quad (16)$$

where

$$\omega_{1,2} = \frac{1}{2} \left[(\epsilon_{j,n} + \epsilon_{j,n-1} + \omega_{LO}) \pm \left[(\epsilon_{j,n} - \epsilon_{j,n-1} - \omega_{LO})^2 + 4A_1 \right]^{1/2} \right]. \quad (17)$$

At resonance, $\omega_{cj} \equiv (\epsilon_{j,n} - \epsilon_{j,n-1}) = \omega_{LO}$, and Eq. (17) provides the resonant splitting, δ , as,

$$\delta \equiv (\omega_1 - \omega_2) = 2 \sqrt{A_1}. \quad (18)$$

From Eq. (16), the spectral weight of the two poles at ω_1 and ω_2 can be readily obtained. The spectral function is given by,

$$\rho_N(\omega) = -2 \operatorname{Im} G_N(\omega + i\eta) \Big|_{\eta \rightarrow 0^+} \quad (19)$$

Introducing, $\Delta = (\omega_{cj} - \omega_{LO})$ and using Eqs. (16) and (19) we obtain,

$$\begin{aligned} \rho_N(\omega) &= \pi(\omega - \epsilon_{j,n} + \Delta) \cdot (\Delta^2 + 4A_1)^{-1/2} \\ &\times \left[\delta(\omega - \omega_1) - \delta(\omega - \omega_2) \right] \end{aligned} \quad (20)$$

From Eq. (20), the spectral weights of the peaks at ω_1 and ω_2 can be readily obtained. In the non-resonant limit, i.e., for $(\Delta^2/A_1) \gg 1$, one finds,

$$\rho_N(\omega) \approx \pi \left(1 + \frac{2A_1}{\Delta^2}\right)^{-1} \cdot \left\{ \left(1 + \frac{A_1}{\Delta^2}\right) \cdot \delta\left(\omega - \epsilon_{j,n} - \frac{A_1}{\Delta}\right) + \left(\frac{A_1}{\Delta^2}\right) \cdot \delta\left(\omega - \epsilon_{j,n} + \Delta + \frac{A_1}{\Delta}\right) \right\} \quad (21)$$

which shows that the second peak at $\omega \approx (\epsilon_{j,n} - \Delta - A_1/\Delta)$ carries very little spectral weight. However, near resonance, i.e., for $(\Delta^2/A_1) \ll 1$, one finds

$$\rho_{N_2}(\omega) \approx \frac{\pi}{2} \cdot \left(1 + \frac{\Delta^2}{8A_1}\right)^{-1} \cdot \left\{ \left(1 + \frac{\Delta}{2\sqrt{A_1}}\right) \delta(\omega - \omega_1) + \left(1 - \frac{\Delta}{2\sqrt{A_1}} + \frac{\Delta^2}{8A_1}\right) \cdot \delta(\omega - \omega_2) \right\} \quad (22)$$

where

$$\omega_{1,2} \approx \left(\epsilon_{j,n} - \frac{\Delta}{2} \pm \sqrt{A_1} \pm \frac{\Delta^2}{8\sqrt{A_1}}\right) \quad (23)$$

From Eq. (22) one readily notes that near resonance both the quasi-particle peaks carry nearly equal weight.

V. MAGNETO-OPTICAL ABSORPTION

The resonant splitting can be seen in optical absorption provided certain conditions relating widths of the Landau levels and the splitting itself are satisfied. The relevant cross-section for magneto-absorption experiment (or Raman scattering) is, in the long wave length limit, provided by the imaginary part of the lowest order diagram shown in Figure 5(b). It may be interpreted as the polarizability, $\pi(\omega)$, of the coupled electron-phonon system. As noted before, since the observation of resonant splitting is intimately connected to its magnitude compared to the widths of the absorption peaks, it behooves us to take explicit account of the Landau level widths at this stage. A first principles calculation of the Landau level widths arising from background impurity and acoustic phonon scattering, while feasible, is neither within the scope of this lecture nor, fortunately, essential for us to proceed with the calculations. It suffices to introduce widths $(\Gamma_{j,n})$ to the Landau levels in a phenomenological manner, thus interpreting

the Landau level energy $\epsilon_{j,n}$ as $(\epsilon_{j,n} + i\Gamma_{j,n})$. The polarisability function involving transition from the level N_0 to the level N (Figure 5(b)) is then given by,

$$\pi_{N_0,N}(\omega) \propto \int \frac{d\epsilon}{2\pi} G_{N_0}(\epsilon) G_N(\epsilon + \omega) \quad (24)$$

$$= (\omega - \omega_0)(\omega - \omega_1 - i\gamma)^{-1} (\omega - \omega_2 - i\gamma)^{-1} \quad (25)$$

where $\omega_0 = (\epsilon_{j,n} - \epsilon_{i,n_0})$, $\omega_{1,2} = (\omega_0 \pm \sqrt{A_1})$, and γ is determined by the widths Γ_{i,n_0} , $\Gamma_{j,n}$ and $\Gamma_{j,n-1}$ of the Landau levels (i, n_0) , (j, n) and $(j, n-1)$ respectively. It is approximately given by,

$$\gamma = \left\{ \Gamma_{i,n_0} + \frac{1}{2} (\Gamma_{j,n} + \Gamma_{j,n-1}) \right\} \quad (26)$$

Thus the behavior of the absorption spectrum, which is proportional to the imaginary part of $\pi(\omega)$, is reflected in,

$$\text{Im } \pi_{N_0,N}(\omega) \propto (\omega - \omega_0)(\omega_1 - \omega_2)^{-1} \cdot \left\{ \frac{\gamma}{(\omega - \omega_1)^2 + \gamma^2} - \frac{\gamma}{(\omega - \omega_2)^2 + \gamma^2} \right\} \quad (27)$$

Expression (27) shows that for the two resonant absorption peaks to be well resolved, the resonant splitting $\delta = 2\sqrt{A_1}$ must satisfy the inequality, $\delta > \gamma$. For sufficiently pure systems at low temperatures it should be possible to satisfy this condition.

VI. CONCLUSION

In the previous three sections we have seen that the phenomenon of the resonant magneto-absorption in two dimensionally confined charge carrier systems offers a unique way of seeking information regarding electron-optical phonon coupling, including features special to the particular system. Equations (18) and (27) show that at resonance, the peak to peak splitting in magneto-absorption is $2\sqrt{A_1}$. For a purely two-dimensional system, $f_{jj}(q) = 1$ so that from Eq. (14) we find that $A_1 \propto B^{3/2}$. Consequently, at resonance the splitting is proportional to $B_{\text{RES}}^{3/2}$, where $B_{\text{RES}} = (m_j^* \omega_{LO}/c)$. By contrast, the result for a three-dimensional system is $\delta \propto B_{\text{RES}}$. Thus other factors (such as impurity scattering, ω_{LO} etc.) being identical, the resonant splitting is enhanced in two dimensions. This, combined with the absence of any Z-motion, argues strongly in favor of a study of magneto-optical anomalies in these systems.

There are several directions in which the results of Sections II and IV can be refined, some of which I have already noted. However, one direction which is relatively straightforward and can be noted in short is the correction to the absorption cross-section due to

phonon induced electron-light vertex corrections, shown in Figure (5(c)). Formally, these corrections simply lead to the replacement of the irreducible $\pi(\omega)$, given by Eq. (24), by the reducible $\pi(\omega)$ given by,

$$\tilde{\pi}(\omega) = \pi(\omega) \left\{ 1 - V D_{LO}^{(\omega)} \pi(\omega) \right\}^{-1} \quad (28)$$

where $D_{LO}(\omega)$ is the phonon propagator given by

$$D_{LO}(\omega) = 2\omega_{LO} \cdot (\omega^2 - \omega_{LO}^2)^{-1} \quad (29)$$

and V is the effective electron-phonon coupling strength in the presence of the magnetic field. The absorption peaks are now given by $\text{Im } \tilde{\pi}(\omega)$ and, as is readily realized from Eq. (28), can be considerably shifted from those given by Eq. (27). It is however worthy of note that the nature of these corrections are in the present case expected to be considerably different, qualitatively and quantitatively, from their counterpart in normal three-dimensional systems. In particular, the kind of carrier density dependence⁵ found by Ngai et al., for free carrier systems in three dimensions does not appear to realize in the present systems. The interested reader can find a discussion of these effects in Ref. 15.

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