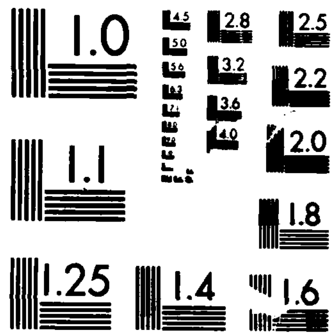
AD-A187 492 SCHEDULING JOB SHOPS WITH DELAYS(U) MASSACHUSETTS INST OF TECH CAMBRIDGE S X LOU ET AL MAR 87 N00014-85-K-0213 1/1

UNCLASSIFIED F/G 12/3 NL


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

MASSACHUSETTS INSTITUTE OF TECHNOLOGY — VLSI PUBLICATIONS

VLSI Memo No. 87-368
March 1987

SCHEDULING JOB SHOPS WITH DELAYS

Sheldon X. C. Lou, Garrett Van Ryzin, and Stanley B. Gershwin

Abstract

In this paper, the presence of delay in a job shop is addressed. We show that delay is an important consideration in many manufacturing systems that are modeled as continuous flow processes. A scheduling policy for a job shop with delays is then derived using theoretical arguments and heuristics.

Microsystems Research Center Room 39-321
Massachusetts Institute of Technology
Cambridge Massachusetts 02139
Telephone (617) 253-8138

Acknowledgements

Published in Proceedings, 1987 IEEE International Conference on Robotics and Automation, Raleigh, NC, March 31 - April 2, 1987. This work was supported in part by the Defense Advanced Research Projects Agency under contract no. N00014-85-K-0213.

Author Information

Laboratory for Information and Decision Systems, MIT, Cambridge, MA 02139, Lou: Room 35-431, (617) 253-3640; Van Ryzin, c/o Gershwin; Gershwin: Room 35-433, (617) 253-2149.

SCHEDULING JOB SHOPS WITH DELAYS

Sheldon X.C. Lou
Garrett Van Ryzin
Stanley B. Gershwin

Massachusetts Institute of Technology

Abstract

In this paper, the presence of delay in a job shop is addressed. We show that delay is an important consideration in many manufacturing systems that are modeled as continuous flow processes. A scheduling policy for a job shop with delays is then derived using theoretical arguments and heuristics.

1 INTRODUCTION

It is well known that the optimal solution of the job shop scheduling problem is, in general, NP-hard [2]. Except for a few problems under very specific conditions, no computationally tractable solution for optimization can be found. Due to this formidable computational complexity, which necessitates the use of static, oversimplified models, traditional job shop scheduling approaches have not proven satisfactory in practice.

The approach proposed in [1], which in turn is a natural extension of [3], makes use of a hierarchical control structure to remedy these problems. A high level controller, similar to what described in [3], works at long time scales and deals only with work stations (work centers). It treats the production process as a continuous material flow. Its objective is to control the flow over a long time horizon so that the demand is satisfied as closely as possible and inventories are kept low, while keeping the system within production rate capacity constraints.

The actual loading of individual parts into machines is left to low level controllers which work at shorter time scale. The low level deals only with single work stations which have far fewer machines than the whole job shop. The low level attempts to fulfill the production goal determined by the high level controller. In this way, the two level controller can avoid the formidable computation requirements encountered in traditional approaches. Further, it dynamically adjusts the production to cope with real-time events.

While the two-level, continuous flow model does simplify the job shop scheduling problem, it comes with a hidden cost, namely that the differential equations representing the system must often include delay. To see this, notice that any work station that typically processes many parts at a time (i.e. where the number of total parts in processes is much greater than 1) will have average interarrival times that are much less than the processing time for a single part. For such a system, the time parts spend in the system cannot be ignored and thus delay must be explicitly included in the formulation.

In this paper, which is a summary of the work to appear in [8], we analyze the high level controller for systems with delay. In Section 2 we look at some examples of manufacturing systems with delay and show that a rather large class of manufacturing systems require delay formulations. In Section 3 we then show that a delay system can be approximated by a system of first order differential equations without delay. We use the results of [6] and let our approximation

To appear in the Proceedings of the 1987 IEEE International Conference on Robotics and Automation, Raleigh, North Carolina, March 31 - April 2, 1987

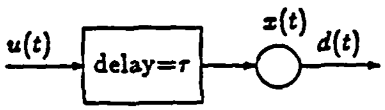


Figure 1: A single work station with delay

get arbitrarily good to arrive at a solution for the optimal control. Due to the difficulty of computing the optimal value function, we next explore a suboptimal strategy based on quadratic approximations to the value function. Finally, conclusions are presented in Section 4.

2 The Importance of Delay in Manufacturing Systems

We mentioned that delay arises in manufacturing systems that work on many parts at one time. We will now examine this phenomenon more closely and also try to indicate in what ways delay introduces difficulties into the scheduling problem.

Firstly, let us point out that introducing delay does not necessarily complicate the control problem. Consider, for example, a single work station with delay as shown in Fig. 1. where $x(t)$ is the inventory in the buffer, τ is the delay (processing time), $u(t)$ is the loading rate, which is bounded and the bound itself is a random variable ([3]), $d(t)$ is the demand rate, which is assumed to be deterministic and known. The dynamics of this system can be modeled as

$$\dot{x}(t) = u(t-\tau) - d(t) \tag{1}$$

By simply defining $\tilde{x}(t) = x(t+\tau)$ and $\tilde{d}(t) = d(t+\tau)$, both of which can be determined completely at time t, we can see this problem is no different than the non-delay problem. We simply use $x(t+\tau)$, rather than $x(t)$, as the current state and solve the problem as though there were no delay.

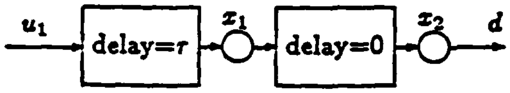


Figure 2: A two stage system

Unfortunately, delay cannot always be handled so simply. For example, consider the simple two stage system depicted in Fig. 2.

The system is described by

$$\begin{aligned} \dot{x}_1(t) &= u_1(t-\tau_1) - u_2(t) && (2) \\ \dot{x}_2(t) &= u_2(t) - d(t) && (3) \\ 0 &\leq x_1(t) && (4) \\ 0 &\leq u_1(t) \leq \alpha_1(t) && (5) \\ 0 &\leq u_2(t) \leq \alpha_2(t) && (6) \end{aligned}$$

Suppose the constraints for u_1 and u_2 depend on some random processes (e.g. the machine state). We must determine u_1 and u_2 based on the *present* constraints yet the value of the future inventory, $x_1(t)$, depends on both the present $u_1(t)$ *and* the future $u_2(t)$, the constraints on which we do not know.

Another example where delay makes the problem more complex is in the scheduling of a reentrant job shop. A reentrant job shop is one where parts visit the same work station several times [7]. A simple reentrant job shop is shown in Fig. 3.

New parts are processed by the work station then go back to the same work station for a second process. After the second process is finished, they leave the system. There are buffers after the first and second processes whose levels are denoted by x_1 and x_2 respectively. Suppose the processing time for the second process is negligible. We then get the same system equations, (2) and (3). The only difference now is that the constraints on u_1 and u_2 are also coupled, namely,

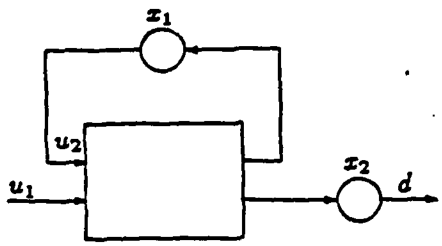


Figure 3: A reentrant job shop

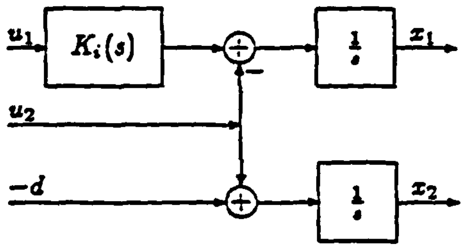


Figure 4: Diagram of two systems

$a_1u_1(t) + a_2u_2(t) \le \alpha(t)$ for some a_1, a_2 and α. This further complicates the control. Thus, a single reentrant work station with delay also cannot be trivially handled.

In the next section, we expand on the ideas suggested by these examples and define the control problem in exact terms.

3 Solution For Delay Systems

To demonstrate our solution technique more clearly, we first investigate the simple problem described in (2) to (6). The technique, however, is extendible to more complex systems.

The objective functional is

$$\min_{u \in \Omega(\alpha)} \int g(x_1, x_2)dt \tag{7}$$

here $\Omega(\alpha)$ is a polyhedron defined by (5) and (6) and $g(\cdot)$ is some function of x_1 and x_2. Without delay, this is the same formulation as in [3].

At time t, the parts in the first process that were loaded between $t-\tau_1$ and t will contribute to the future inventory and, therefore, should become part of the current state. Unfortunately, the problem then becomes an infinite dimensional one.

In order to overcome this difficulty we approximate the past u_1 by a finite dimensional first order system. We then let the approximation become better and better so that it approaches the original system.

Let us first define new variables $y_1(t)$ to $y_m(t)$ through the following equations.

$$\begin{aligned} \frac{\tau}{m}\dot{y}_1(t) &= u_1(t) - y_1(t) \\ \frac{\tau}{m}\dot{y}_2(t) &= y_1(t) - y_2(t) \\ &\vdots \\ \frac{\tau}{m}\dot{y}_m(t) &= y_{m-1}(t) - y_m(t) \end{aligned} \tag{8}$$

The initial conditions are set to zero at $-\infty$ and we assume that $u_1(-\infty) = 0$. Eq. (8) defines a cascade of m first order systems with time constant $\frac{\tau}{m}$. Its input and output are $u_1(t)$ and $y_m(t)$ respectively. As a motivation for using (8), note that its transfer function is $1/(1+s\tau/m)^m$ which yields the well known limit $e^{-s\tau}$, the transfer function of a delay τ, as $m \to \infty$.

Now define

$$\dot{x}_1(t) = y_m(t) - u_2(t) \tag{9}$$
$$\dot{x}_2(t) = u_2(t) - d(t) \tag{10}$$

Combine (8)-(10), we obtain a new system. The diagrams of this system or the original system defined by (2) and (3) can be drawn as in Fig. 4.

The only difference between the system defined by (2) and (3) and the system defined by (8)- (10) is the first box $K_i(s)$. For the first system, it is a delay element with delay τ. For the second system, it is a linear system defined by (8). If we can show that for the same u_1, u_2

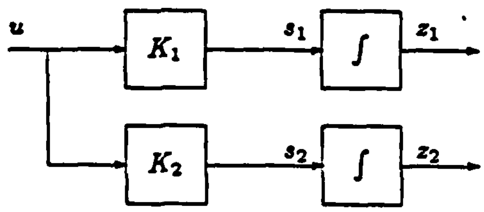


Figure 5: Compare two integrals

and d, the output of the first system approaches the output of the second one, then we can establish an equivalence between the two systems. By superposition, it is sufficient to show that the integral of $y_m(t)$ approaches the integral of $u_1(t-\tau)$ as m goes to infinity.

To show this result, we compare the two systems shown in Fig. 5. In Fig. 5, $K_1(s)$ is the system defined by (8) and $K_2(s)$ is a pure delay of τ. We will prove that $z_1(t)$ approaches $z_2(t)$ uniformly in t as $m \to \infty$. First we need the following lemma which is similar to [6].

Lemma 1 *If $u(t)$ is differentiable with $|\dot{u}(t)| < K$ for all $t \in (-\infty, +\infty)$, then*

$$\lim_{m\to\infty} \sup_{t\in(-\infty,+\infty)} \|s_1(t) - s_2(t)\| = 0$$

Proof: (see [8]).

The integrals in Fig. 5 start from $-\infty$. Since the initial conditions are zero at $-\infty$ and $u(-\infty)$ equals zero, we can switch the linear operators K_1, K_2 with the integrators, $1/s$. Because $u(t)$ is bounded, the integral of $u(t)$ has a bounded derivative. Combining these facts with Lemma 1, we obtain the following lemma.

Lemma 2 *If $u(t) \in L_1$ and is bounded, then*

$$\lim_{m\to\infty} \sup_{t\in(-\infty,+\infty)} \|z_1(t) - z_2(t)\| = 0$$

Proof: (see [8]).

Using Lemma 2, we see that the output of the system defined by (8)-(10) approaches the output of the system defined by (2) and (3). Therefore, if u_1 is optimal for the first system, it will also be optimal for the second one.

We will now consider the optimal control for the system defined by (8)-(10). Define $x = [x_1\ x_2\ y_1 \ldots y_m]'$, $u = [u_1\ u_2]'$. This system can be written in a compact form as

$$\dot{x} = Ax + Bu + Cd \tag{11}$$

Using the same approach as in [3], it can be shown that the optimal control u for the problem defined by (11) and the constraints (4)-(6) can be obtained by solving

$$\min_{u\in\Omega(\alpha)} \nabla_x J^*(x,\alpha)Bu \tag{12}$$

Eq. (12) can be rewritten as

$$\min_{u\in\Omega(\alpha)} [\frac{\partial J^*}{\partial x_1} - \frac{\partial J^*}{\partial x_2}]u_2 + \frac{\partial J^*}{\partial y_1}u_1 \tag{13}$$

Where $J^*(x,\alpha)$ is the *optimal* cost to go. Unfortunately, J^* remains unknown and is, even for simple problems, difficult to compute. Therefore, we seek an approximation for J^*. Experience ([4],[5]) shows that satisfactory results can be obtained with relatively crude approximations for J^*. The one we will use is in a quadratic form with coefficients that are functions of α, namely,

$$J^*(x,\alpha) \cong x'R(\alpha)x + S(\alpha)x \tag{14}$$

Then

$$\nabla_x J^*(x,\alpha) \cong R(\alpha)x + S(\alpha) \tag{15}$$

Using (15) we can rewrite (13) as

$$\min_{u\in\Omega(\alpha)} [\beta_1(x,\alpha)u_1 + \beta_2(x,\alpha)u_2] \tag{16}$$

where

$$\beta_i(x,\alpha) = \sum_{j=1}^{m} p_{ij}(\alpha)y_j + \sum_{j=1}^{2} q_{ij}(\alpha)x_j + \rho_j(\alpha) \tag{17}$$

Letting m go to infinity we get

$$\beta_i = \int_0^\tau f_i(\sigma,\alpha)u_1(t-\sigma)d\sigma + \sum_{j=1}^{2} q_{ij}(\alpha)x_j + \rho_j(\alpha) \tag{18}$$

Eq. (16) and (18) describe the sub-optimal control law for our system. Note that instead of a very complex dynamic program, the control (production rates) is determined by a linear program (16). The problem is further simplified due to the simple structure of $\Omega(\alpha)$. This calculation can easily be performed in real time.

The terms in β_i are easy to interpret. Note that β_i is a function of x_i, α and the past control u_1. The second and third terms in β_i are identical to the terms found in the quadratic approximation presented in [3] for a non-delay system. Added to this is a convolution of the control $u_1(t)$ with some weighting function $f_i(\sigma)$. If a constant weighting is applied, the first term would be the integral of $u_1(t)$ from $t-\tau$ to t, which is the number of the parts currently under processing. Other weighting functions are, of course, possible.

Further research will be conducted to obtain the correct form of β_i. Further, since a real manufacturing system is much more complex than the system described here, more complex system models will be investigated which will take into account factors such as reentrant process and time varying demand. Finally, numerical computations and simulation experiments will be used to help develop solution techniques.

4 Conclusion

In this paper, we examined the effects of delay in manufacturing systems. We proposed a general model for a network of work stations that includes delay. We then presented a technique for analyzing delay systems by augmenting the states to include an approximation of the past control. Using quadratic approximations to the optimal value function, we show that the control takes a particularly simple form.

REFERENCES

1. S.X.C. Lou, *Job Shop Scheduling Using Flow Rate Control*, M.I.T. Laboratory for Information and Decision Systems, LIDS-P-1618, September, 1986.

2. R.W.Conway, W.L.Maxwell, *Theory of Scheduling*, Addison-Wesley, Reading, Mass., 1967.

3. J.Kimemia, S.B.Gershwin, *An Algorithm for the Computer Control of a Flexible Manufacturing System*, IIE Transactions, Vol. 15, No.4, December 1983, pp. 353-362.

4. S.B. Gershwin, R.Akella, and Y.F.Choong (1985), *Short-Term Production Scheduling of and Automated Manufacturing Facility*, IBM Journal of Research and Development, Vol. 29, No. 4, pp 392-400, July, 1985.

5. R.Akella, Y.F.Choong, and S.B.Gershwin (1984), *Performance of Hierarchical Production Scheduling Policy*, IEEE Transactions on Components, Hybrids, and Manufacturing Technology, Vol. CHMT-7, No.3, September, 1984.

6. I.M.Repin, *On the Approximate Replacement of Systems with Lag by Ordinary Dynamical Systems*, Journal of Applied Mathematics and Mechanics, Vol. 29, 1965, pp. 254-264 (translation of PMM, Vol.29, No.2, 1965, pp.226-235).

7. S.C.Graves, H.C.Meal, D.Stefek, A.H.Zeghmi (1983), *Scheduling of Re-Entrant Flow Shops*, Journal of Operations Management, Vol.3, No.4, August 1983, pp 197-207.

8. G.Van Ryzin, *Control of Manufacturing Systems With Delays* M.S. Thesis under preparation, Laboratory for Information and Decision Systems, MIT.

END
Feb.
1988
DTIC