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AD-A187 670

# RESEARCH MEMORANDUM

## ALTERNATIVE MODELING APPROACHES FOR SETTING COST-EFFECTIVE QUALIFICATION STANDARDS

Laurie J. May

A Division of

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**REPORT DOCUMENTATION PAGE**

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION / AVAILABILITY OF REPORT	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE		Approved for public release; distribution unlimited.	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) CRM 87-54		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION Center for Naval Analyses	6b. OFFICE SYMBOL (if applicable) CNA	7a. NAME OF MONITORING ORGANIZATION Commandant of the Marine Corps (Code RDS)	
6c. ADDRESS (City, State, and ZIP Code) 4401 Ford Avenue Alexandria, Virginia 22302-0268		7b. ADDRESS (City, State, and ZIP Code) Headquarters, Marine Corps Washington, D.C. 20380	
8a. NAME OF FUNDING / ORGANIZATION Office of Naval Research	8b. OFFICE SYMBOL (if applicable) ONR	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-87-C-0001	
8c. ADDRESS (City, State, and ZIP Code) 800 North Quincy Street Arlington, Virginia 22217		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO. 65153M	PROJECT NO. C0031
		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Alternative Modeling Approaches for Setting Cost-Effective Qualification Standards			
12. PERSONAL AUTHOR(S) Laurie J. May			
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM TO	14. DATE OF REPORT (Year, Month, Day) May 1987	15. PAGE COUNT 44
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	Aptitude tests; Cost effectiveness; Cost models; Enlisted personnel;	
05	03	Mathematical models; Military requirements; Optimization; Performance	
05	09	(human); Personnel selection; Qualifications; Standards; Test scores	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
<p>The Rand Corporation has developed a model that determines cost-effective enlistment standards for military occupational specialties. Since this model may have an impact on enlistment policy, it is important that the modeling approach used to determine the enlistment standards be fully understood. This research memorandum describes the modeling technique used by the Rand Corporation and compares the Rand approach to alternative modeling methods in an attempt to determine the sensitivity of the model outcome to the methodology employed.</p>			
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL Lt.Col. Robert L. Earl		22b. TELEPHONE (Include Area Code) (202) 694-3491	22c. OFFICE SYMBOL RDS-40

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Subj: Center for Naval Analyses Research Memorandum 87-54

Encl: (1) CNA Research Memorandum 87-54, "Alternative Modeling Approaches for Setting Cost-Effective Qualification Standards," by Laurie J. May, May 1987

1. Enclosure (1) is forwarded as a matter of possible interest.
2. The Job Performance Measurement Project is a joint-service effort to develop good measures of job performance, to relate these measures to aptitude test scores, and to use this relationship along with cost data to determine enlistment standards. This Research Memorandum examines several alternative modeling approaches that may be used in this project.

William H. Sims  
Director, Manpower and Training Program  
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**ALTERNATIVE MODELING  
APPROACHES FOR SETTING  
COST-EFFECTIVE  
QUALIFICATION STANDARDS**

Laurie J. May

*Marine Corps Operations Analysis Group*

A Division of

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## ABSTRACT

The Rand Corporation has developed a model that determines cost-effective enlistment standards for military occupational specialties. Since this model may have an impact on enlistment policy, it is important that the modeling approach used to determine the enlistment standards be fully understood. This research memorandum describes the modeling technique used by the Rand Corporation and compares the Rand approach to alternative modeling methods in an attempt to determine the sensitivity of the model outcome to the methodology employed.

## EXECUTIVE SUMMARY

In recent years, the services have been under increasing pressure to justify their enlistment standards. In a pioneering effort, the Rand Corporation developed a model that addresses the difficult problem of determining cost-effective standards. The model has been applied by Rand to estimate the optimal cutoff score on the Combat Arms composite for assigning Army recruits to the Infantry specialty. The resulting qualification standards were similar to those established by the Army and were generally viewed as intuitively reasonable.

Since the Rand approach may affect enlistment policy, it is important that its modeling method be fully understood. This research memorandum examines the modeling approach used by the Rand Corporation and discusses the impact that different modeling approaches have on the determination of qualification standards. All approaches are evaluated using the same data set initially used by Rand.

The Rand model links qualification standards or, specifically, the Army Combat Arms aptitude composite cutoff score to personnel mixes. The Combat Arms cutoff score is the minimum score an individual can obtain on the Army Combat Arms ASVAB composite and still qualify for the infantry. The feasible personnel mixes are limited by fixing performance or the number of accessions at some predetermined level. Each Combat Arms cutoff score is assumed to generate a specific personnel mix. The optimal personnel mix is selected from the feasible choices by evaluating the costs associated with the various mixes.

The Rand model uses two measures of performance: one that reflects an individual's ability to remain in the service (retained man-months, or RMMs) and another that extends this retention measure by including an assessment of on-the-job performance (qualified man-months, or QMMs). One of these performance measures or the number of accessions must be fixed so that the solution of the cost-effective model can be determined.

The overall framework of the Rand model is general in the sense that specific model variants can be created by altering what factors are held fixed (the constraints) and the optimization criterion. By changing the performance constraint (that is, by fixing the level of retained man-months as opposed to qualified man-months or vice versa) and the corresponding optimization criterion, the Rand Corporation, in effect, developed two versions of the model—which will be denoted herein as RAND 1 and RAND 2. An alternative to the Rand approach is to use a net-benefit approach (NET

BENEFIT), which uses a different criterion for selecting the optimal cutoff score. The main features and the strengths and drawbacks of these three alternative approaches are outlined in table I. As shown in the table, all three approaches have both positive and negative features and no one approach stands out as clearly superior to the others.

Fixing one of the performance measures as opposed to the other has important implications regarding the model's criterion for selecting the optimal cutoff score and the real-world conditions within which the model operates. In the first version of the model, RAND 1, retained man-months, which are basically a proxy for end-strength, are constrained to a fixed level and optimal standards are determined by selecting the personnel mix that minimizes the total cost per qualified man-month. This method is equivalent to valuing an additional qualified man-month at average cost, which is a questionable assumption. In the second model variant, RAND 2, qualified man-months are held constant and total cost is minimized. In this version of the model, retained man-months are allowed to vary, which, as noted by the Rand authors, may be an unrealistic condition. An alternative to the Rand approach is a net-benefit approach. In this approach, retained man-months are fixed and the optimal standard is chosen by selecting the personnel mix that maximizes the difference between benefit (qualified man-months) and cost. The main drawback of the net-benefit approach is that to make the model operational, benefit and cost must be measured in the same metric. Thus, qualified man-months must be converted to a dollar value to be able to compare benefit and cost. These three approaches are illustrated in figure I.

Table II gives the optimal cutoff scores for the different models using the input data provided with the Rand model in [1].<sup>1</sup> The optimal cutoff scores are 85 and 95 for the two variants of the Rand model. Using the same input data, the net-benefit model yields different results depending on the assumption regarding the dollar value of a qualified man-month. If the total benefit curve is assumed to be linear (the dollar value of a qualified man-month is constant), then the optimal cutoff score under this approach is 85 also. However, if the total benefit curve is assumed to be concave (the dollar value of a qualified man-month falls as additional qualified man-months are obtained), the optimal cutoff score may be less than 85.

---

1. Reference [1] does not give all the information needed to execute the RAND 2 model. Therefore, the optimal cutoff score for RAND 2 is calculated using the November 1981 version of the Rand model. The optimal cutoff score varies depending on which version of the Rand model is applied. The version of RAND 2 presented in [1] generated a cutoff score of 90.

TABLE I  
**MAIN FEATURES OF THREE ALTERNATIVE COST-PERFORMANCE MODELS**

Model	Predetermined fixed factors	Criterion for selection of optimal mix	Weaknesses	Strengths
RAND 1	End-strength	Personnel mix with lowest average cost per unit of performance	Nonstandard economic selection criteria	End-strength fixed Easy to measure key inputs
RAND 2	Performance	Personnel mix with lowest total cost	Endstrength not held fixed Performance level predetermined	Standard economic selection criteria Easy to measure key inputs
NET BENEFIT	End-strength	Personnel mix with greatest difference between benefit and cost	Difficult to measure benefit	End-strength fixed Standard economic selection criteria

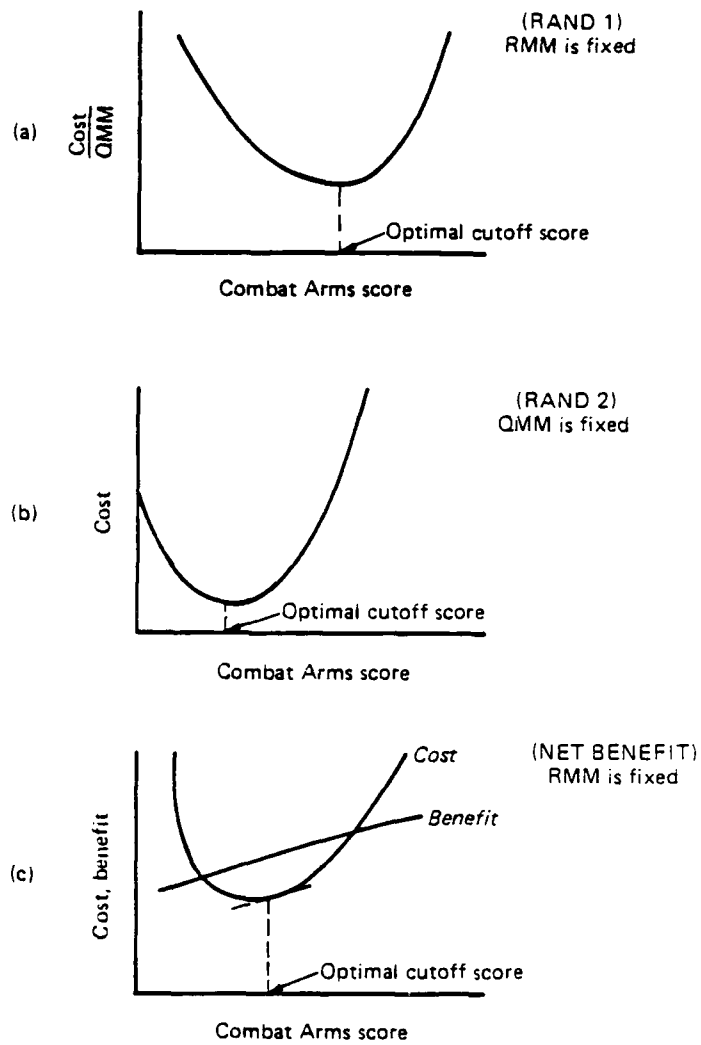


FIG. 1: ILLUSTRATION OF ALTERNATIVE COST-EFFECTIVE MODELS

TABLE II  
THE OPTIMAL COMBAT ARMS CUTOFF SCORE

Model	Constraints	Criteria for selection	Cutoff score
RAND 1 <sup>a</sup>	Fix RMMS	Minimize cost per QMM	85
RAND 2 <sup>b</sup>	Fix QMMs	Minimize total cost	95
Net benefit Linear benefit curve Nonlinear benefit curve Demand elasticity = 0.5-6.0	Fix RMMS	Maximize net benefit	85 70-85

a. Using version of Rand model detailed in [1].

b. November 1981 version of the Rand model.

## CONCLUSION

In conclusion, although the three models generate somewhat different answers, all of the models confirm that the main Rand result of a cutoff score of 85 is a reasonable estimate of the optimal cutoff score. The main version of the Rand model (RAND 1) and the linear net-benefit model yield an optimal cutoff score of 85. Similarly, RAND 2 generates a cutoff score of 95. However, the nonlinear net-benefit model demonstrates that if the value of a qualified man-month is not constant, then the optimal cutoff score depends on how the value of a QMM varies as additional QMMs are obtained, and the optimal score may differ significantly from 85. Nevertheless, for all three models, the difference between the cost and performance of the optimal personnel mix and the second best personnel mix is small. Thus, choosing a suboptimal cutoff score that is near the optimal cutoff score probably would not result in significant differences.

Because all three approaches have some negative features, no single model stands out as clearly superior to the others, making it extremely difficult to choose a "best" model. The choice of a "best" model is basically a policy decision. Depending on the importance assigned to the various strengths and weaknesses of the models, different models emerge as the "best." If holding end-strength fixed and ease of measurement are important but using a standard economic criteria for selecting the optimal cutoff score is not very important, then RAND 1 is the best model. In contrast, if using a standard economic criteria for cutoff-score selection is very important along with ease of measurement and predetermining performance and allowing end-strength to vary are not problematic, RAND 2 emerges as the best model. Finally, if fixing end-strength and using a standard criteria for cutoff-score selection are important and lower bound measures are acceptable then NET BENEFIT is the best model.

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## INTRODUCTION

Service enlistment standards are generally defined in terms of educational level and scores on the Armed Services Vocational Aptitude Battery (ASVAB). In the late 1970s, the appropriateness of these standards was questioned because problems were found with the ASVAB score scale. Congress has mandated that the services develop defensible enlistment standards that directly link ASVAB scores to job performance.

Historically, the services have set mental standards primarily on the basis of the probability of success in training courses. Little was known about the empirical relationship between the ASVAB and job performance because of the lack of adequate on-the-job performance measures.

In conjunction with the Office of the Secretary of Defense, the Rand Corporation developed a general model that satisfied the congressional intent to link enlistment standards to measures of job performance. But in addition to establishing the validity of the ASVAB, the services were tasked to justify the specific level of their enlistment standards. The Rand Corporation chose to analyze the problem of determining enlistment standards with a cost-performance model. The Rand model has been used to estimate the optimal entrance cutoff score for assigning Army recruits to the Infantry specialty.

The Rand model is a pioneer effort in this arena of linking enlistment standards to job performance measures. Since this model may have a significant impact on enlistment policy, it is important that it be fully understood. There are a variety of aspects of the Rand model that could benefit from further exploration as described in [2]. Building on the work set forth in [2], this research memorandum examines the modeling approach used in the Rand model and explores how alternative modeling approaches can affect the model outcome. A comprehensive description of the model and applications to selected Army specialties are presented in two Rand publications [1, 3].

## THE PROBLEM

To determine optimal qualification standards, a methodology for choosing among alternative cutoff scores must be developed. Different enlistment standards result in different mixes of personnel in terms of both education and aptitude levels. Determination of the optimal enlistment standards entails relating entrance cutoff scores to the personnel mix obtained. Once the link between cutoff score and the resulting

distribution of accessions is established, the cost and performance level of the various mixes must be evaluated. Given the cost and performance of the personnel mixes associated with the different cutoff scores, an optimization criterion must be applied to determine which mix, and thus which cutoff score, is the best. Figure 1 outlines the general procedure used for determining the optimal cutoff score. The optimal cutoff score will depend on the optimization criteria used and what factors are constrained (held fixed).

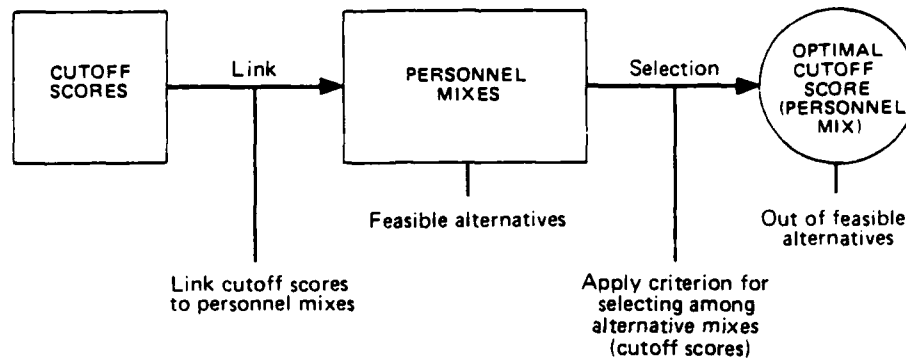


FIG. 1: GENERAL OPTIMIZATION PROCEDURE FOR DETERMINING OPTIMAL CUTOFF SCORE

### Criterion for Selection

In formulating an optimization problem, it is necessary to specify the criterion that will be used to choose among the alternatives available. In the case of determining optimal enlistment standards, a criterion for choosing among alternative cutoff scores must be specified. The most common criterion of choice among alternatives is the goal of maximizing something (such as a firm's profit) or minimizing something (such as a firm's costs). As an example, in determining optimal enlistment standards, the criterion for selection might be to minimize the cost of the force. The optimization process under this criterion for selection would entail finding the cutoff score that yielded the cheapest mix of personnel. The criterion of choice is selected by the model builder. Although for many problems the criterion of choice is obvious, several criteria could be used in the optimal enlistment-standards problem.

## Constraints

In building an optimization model, it is often necessary to constrain certain factors to a range of values or a specific value to make the model operational and reflect real-world conditions. Imposing constraints limits the number of alternative choices. The set of alternative choices is limited or constrained to some subset of choices from which the optimal choice is made. As with the criterion for selection, the model builder must decide which factors, if any, to constrain. As a guide, the model builder should only impose constraints that are consistent with real-world conditions. In determining the optimal cutoff score, constraints must be imposed on the force size to limit the choices to the realistic alternatives given the end-strength constraints placed on the military.

If no constraints are placed on force size or performance level, there will be an infinite number of personnel mixes associated with each cutoff score. For example, a different personnel mix associated with a cutoff score of 80 for each force size. However, the military cannot choose among all possible force sizes because they must comply with the congressionally mandated end-strength cap. Constraining the force size dramatically limits the alternative personnel mixes and results in a one-to-one correspondence between force mix and cutoff score. The constrained optimization procedure for determining the optimal cutoff score is described pictorially in figure 2.

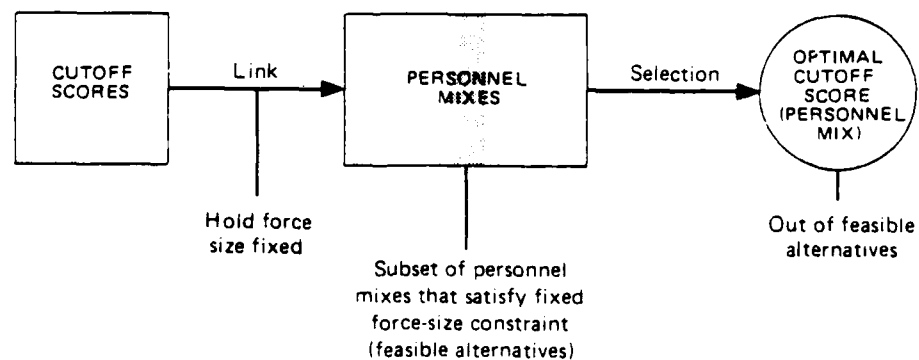


FIG. 2: CONSTRAINED OPTIMIZATION PROCEDURE FOR DETERMINING OPTIMAL CUTOFF SCORE

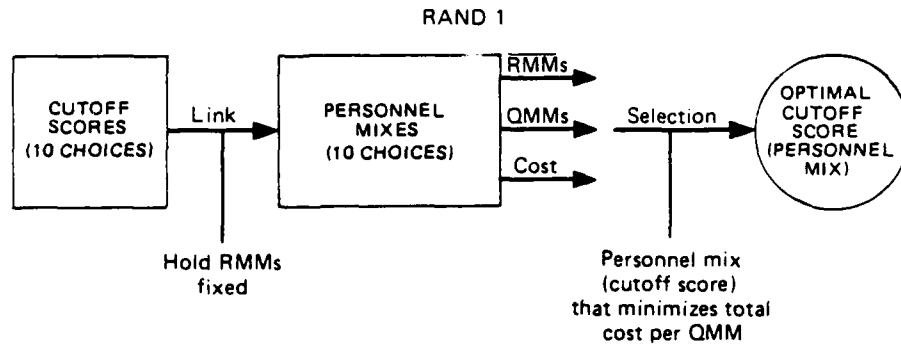
## THE RAND MODEL

The problem of determining cost-effective enlistment standards can be modeled in a variety of ways depending on the factors that are constrained and the criterion for selection that is used. The Rand Corporation uses a general optimization model to determine cost-effective enlistment standards. Two specific model variants (RAND 1 and RAND 2) are created by altering the constraints and the criteria for selection.

The first variant of the Rand model (RAND 1) relates cutoff score to personnel mix given that retained man-months are fixed. Retained man-months (RMMs) are defined as the number of months of service an individual contributes after completing basic and advance training. Holding RMMs fixed is analogous to fixing the force size. Ten cutoff scores are linked to corresponding personnel mixes. These ten personnel mixes all contribute the same number of retained man-months but have different associated costs and performance levels. In the Rand model, performance is measured in terms of qualified man-months (QMMs). Qualified man-months are defined as the number of months of service an individual contributes after completing basic and advanced training and passing the Army Skill Qualification Test. Each of the ten personnel mixes has a unique cost and yields a unique number of qualified man-months.

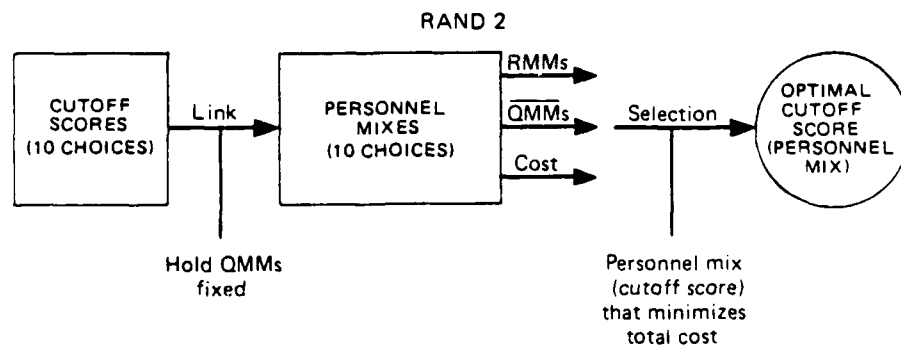
To choose the optimal personnel mix and thus the optimal cutoff score, RAND 1 uses the minimization of the total cost per qualified man-month as the criterion for selection. Thus, as shown in figure 3, RAND 1 seeks to find the enlistment standard that mimimizes the cost per QMM given the set of alternatives is constrained by the condition that RMMs are fixed at a predetermined level.

The second variant of the Rand model (RAND 2) differs significantly from RAND 1 in that it constrains different factors and uses a different criterion for selection. RAND 2 relates cutoff score to personnel mix while holding qualified man-months constant. RAND 2 fixes the on-the-job performance level of the force but allows retained man-months to vary. Because different factors are constrained, the relationship between cutoff score and personnel mix differs between RAND 1 and RAND 2. RAND 2 relates the ten cutoff scores being considered to ten personnel mixes that are different from those considered in RAND 1. By altering the constraints, the subset of alternative choices is different in the two models. In RAND 2, each of the ten alternative personnel mixes contribute the same number of QMMs but differ in terms of RMMs and cost.



**FIG. 3: RAND 1 PROCEDURE FOR DETERMINING OPTIMAL CUTOFF SCORE**

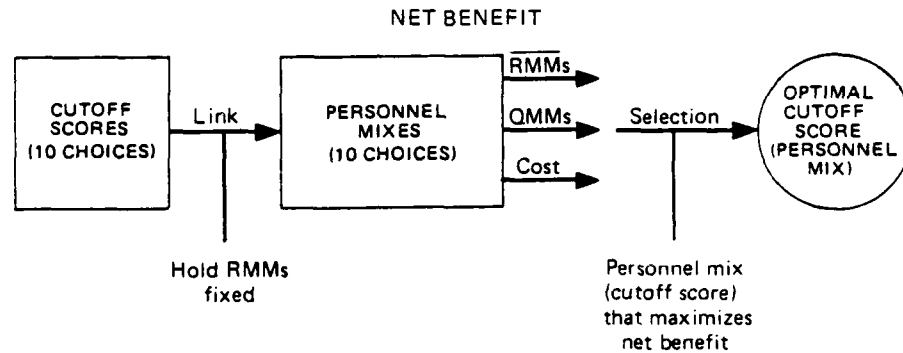
RAND 2 uses a different criterion than RAND 1 to choose the optimal cutoff score. In RAND 2 the minimization of the total cost of the force is used as the criterion for selection. Therefore, as shown in figure 4, RAND 2 seeks to find the standard that minimizes total cost given the alternatives are limited to those mixes that contribute a fixed number of QMMs.



**FIG. 4: RAND 2 PROCEDURE FOR DETERMINING OPTIMAL CUTOFF SCORE**

## ALTERNATIVE APPROACHES

In addition to the approaches used by the Rand Corporation, the optimal cutoff score could be determined by using other criteria for selection or by constraining different factors. NET BENEFIT is an alternative model that uses the maximization of net benefit as the criterion for choosing an optimal cutoff score. Net benefit is defined as the difference between performance (measured as QMMs) and cost. Under this approach, the cutoff scores can be linked to the personnel mixes as in RAND 1, in which retained man-months are held constant. The feasible personnel mixes would be the same as in RAND 1. NET BENEFIT differs from RAND 1 in that it uses a different criterion for selecting among the alternatives. Figure 5 describes the optimization procedure used in the net-benefit approach.



**FIG. 5: NET BENEFIT PROCEDURE FOR DETERMINING OPTIMAL CUTOFF SCORE**

## DIFFERENCES AMONG THE MODELS

All three models outlined above differ in their choice of selection criteria. RAND 1 and the alternative model, NET BENEFIT, differ from RAND 2 in terms of what factors are held fixed. Table 1 summarizes the main differences among the three models.

**TABLE 1**  
**MAIN FEATURES OF ALTERNATIVE COST-PERFORMANCE MODELS**  
**FOR DETERMINING ENLISTMENT STANDARDS**

Model	Criteria	Predetermined fixed factors
RAND 1	Minimize total cost per qualified man-month	Retained man-months
RAND 2	Minimize total cost	Qualified man-months
NET BENEFIT	Maximize difference between benefit and cost	Retained man-months

RAND 1 uses the minimization of total cost per qualified man-month as the criterion of choice. In addition, RAND 1 constrains retained man-months to a fixed value, which is consistent with the real-world constraints on force size. Under this scenario, the optimal cutoff score is the score that results in the personnel mix with the lowest average cost for qualified man-months.

The criterion of minimizing average cost is not a standard optimization criterion used in economics. The criterion of minimizing average cost has the positive feature of considering both cost and on-the-job performance factors in determining the optimal cutoff score. However, it does not take into account the value of additional performance (QMMs) in determining the optimal solution. This point is best illustrated by an example. Suppose an individual must choose between two alternative baskets of fruit. The first basket contains two apples and costs \$1. The second basket contains 50 apples and costs \$20. The average price of an apple is \$0.50 in the first basket and \$0.40 in the second basket. Thus, if the criterion for selection is to minimize the cost per apple, the second basket is optimal. However, this criterion for selection does not consider whether the individual wants 50 apples nor how much he or she is willing to pay for an additional apple. The individual may not want to spend an additional \$19 to get 48 additional apples. The criterion of minimizing average cost does not take into account how much the purchaser is willing to pay for one additional unit.

RAND 2 uses the minimization of total cost as the criterion of choice. Although this criterion is commonly used in optimization problems, under its use only cost factors are considered in choosing the best cutoff score among the alternatives. In

addition, RAND 2 limits the alternatives by constraining qualified man-months to a predetermined level. Therefore, on-the-job performance is predetermined by the model builder in this variant and is not an outcome of the model. In contrast, RAND 2 allows retained man-months to vary. Thus, the personnel-mix alternatives are limited to those mixes that yield a set number of QMMs, with each mix yielding a different number of RMMs. This set of alternatives does not reflect the real-world choices available to the military, because it does not account for the end-strength constraint. Given the set of personnel mixes that satisfy the predetermined on-the-job performance level, RAND 2 selects the cutoff score associated with cheapest mix as the optimal standard.

The alternative model (NET BENEFIT) uses the maximization of net benefit as the criterion of choice. Maximizing net benefit is also a commonly used criterion in optimization problems. Under its use, both cost and on-the-job performance factors are considered in choosing the best cutoff score among the alternatives. NET BENEFIT limits the alternatives by constraining retained man-months, which is consistent with real-world conditions. Given the set of alternatives, NET BENEFIT selects the cutoff score associated with the personnel mix with the highest net benefit as the optimal standard. Net benefit is calculated as the difference between qualified man-months (on-the-job performance) and cost. The major drawback of this approach is that to make it operational, benefit and cost must be measured in the same metric. Net benefit can only be calculated (and have any meaning) if QMMs and cost are converted to the same metric. Cost is expressed in dollars. Qualified man-months could be expressed in dollars if the value of a QMM were known. Since the value of a QMM is not directly known, it must be estimated.

## MODEL RESULTS

Since all three models differ in their choice of the selection criteria and constraints, the model outcomes (that is, the optimal cutoff scores) could differ significantly. Using the same input data, the three models are used to determine the optimal cutoff score for the Army Infantry specialty. The data set is the one used in the version of the Rand model presented in [1].<sup>1</sup> The results are compared and contrasted.

---

1. There have been several versions of the Rand model. Different versions of the model generate slightly different results. The results presented in this paper use the version of the Rand model reported in [1] for the RAND 1 and NET BENEFIT models. The necessary information to execute RAND 2 is not provided in [1]. Thus, the November 1981 version of the Rand model is used to run RAND 2, which generates slightly different results than those reported in [1].

## Operationalizing the Models

The link between the entrance cutoff score and the distribution of new accessions must be established. Raising or lowering the cutoff score has a direct effect on the ability distribution of accessions—that is, on the proportion of individuals in each AFQT (Armed Forces Qualification Test) category, cutoff-score grade interval, and educational level. Because cutoff scores have not been changed very often, there is virtually no observable information on how the mix of accessions changes as the cutoff score varies. Therefore, the effect that changing the cutoff score has on the distribution of accessions must be assumed.

The Rand model assumes that new persons enlisting after the new cutoff score is applied are distributed in proportion to their previous shares among the ability-mix categories. For example, suppose that the recruits in some baseline group are distributed evenly across aptitude categories I to IV, each category containing 25 percent of the accessions. In addition, assume that when a higher cutoff score is applied to this cohort, the proportion of individuals who qualify for category IV falls to zero. Under the proportionality assumption, this 25 percent is distributed to categories I to III in proportion to the percentages these categories represented before the new cutoff score was applied. That is, categories I to III accounted for 75 percent of the baseline group so that this 25 percent is distributed by adding  $25/75$  of 25 (or 8.3) to each of the remaining aptitude categories. Therefore, after the new cutoff score is applied, the accession mix is distributed evenly across aptitude categories I to III, with each category containing one-third (33.3 percent) of the accessions.

In applying the Rand model to the Army Infantry specialty, the cohort of 1977 accessions was used as the baseline group with a Combat Arms cutoff score of 76 [1] to determine the baseline mix of personnel. The 1981 Infantry accession goal of 12,168 men is used as the baseline number of accessions. This baseline group was stratified into categories defined by educational level, AFQT category, and 5-point intervals on the Combat Arms cutoff-score scale. For example, high school graduates in AFQT category II who have a Combat Arms composite score between 80 and 85 are grouped in one category, and high school graduates in AFQT category II who have a Combat Arms composite score between 85 and 90 are grouped into another category. Table 2 gives the categorization of the baseline group in terms of broad AFQT categories and education level.

**TABLE 2**  
**THE CATEGORIZATION OF THE**  
**BASELINE ACCESSIONS**

	<u>AFQT category</u>	
	<u>I-III A</u>	<u>III B-IV</u>
HS	2,574	3,923
NHS	1,495	4,176

For each potential cutoff score, the new distribution of personnel is calculated by determining which categories would be eliminated and distributing the percentage eliminated proportionally across the remaining categories. In addition, the number of category-IV non-high school graduates is restricted to zero. If retained man-months are held constant, the number of accessions from each category is adjusted to achieve the retention goal. Table 3 shows how the distribution of personnel is linked to the cutoff scores given retained man-months are held constant. These are the feasible personnel mixes considered by the RAND 1 and NET BENEFIT models. If qualified man-months are fixed, the number of accessions entering each category is adjusted so that this objective is obtained. Table 4 gives the distribution of personnel linked to the cutoff scores under the condition that QMMs rather RMMs are held constant. Because different factors are constrained, different personnel mixes are associated with the cutoff scores. Table 4 gives the personnel mixes considered by RAND 2.

For each feasible cutoff score and corresponding personnel mix, the number of QMMs and cost of the corresponding personnel mix is calculated. Table 5 gives the number of QMMs, RMMs, and costs associated with each cutoff score under the alternative constraints. Figures 6 and 7 show the relationship between QMMs, RMMs, and cost given the alternative constraints.

### **Measuring the Value of a Qualified Man-Month**

In contrast to the two Rand models, the net-benefit model requires additional assumptions regarding the value of a qualified man-month to make the model operational. A method for translating a QMM into its dollar equivalent must be developed. In comparing the two Rand models to the net-benefit model, three techniques for translating a QMM into its dollar value are used.

**TABLE 3**  
**THE CATEGORIZATION OF ACCESSIONS FOR**  
**ALTERNATIVE CUTOFF SCORES GIVEN RMMs ARE FIXED<sup>a</sup>**

Cutoff score	Total accessions	Education	AFQT category	
			I-III A	IIIB-IV
115+	11,242	HS	8,513	257
		NHS	2,309	163
110+	11,331	HS	7,796	495
		NHS	2,689	351
105+	11,510	HS	7,066	861
		NHS	2,917	666
100+	11,644	HS	6,434	1,364
		NHS	2,930	916
95+	11,757	HS	5,547	2,108
		NHS	2,822	1,280
90+	11,757	HS	4,733	2,965
		NHS	2,572	1,487
85+	11,729	HS	4,331	3,475
		NHS	2,404	1,519
80+	11,650	HS	3,747	4,252
		NHS	2,144	1,507
76+	11,580	HS	3,267	4,979
		NHS	1,897	1,437
70+	11,522	HS	2,902	5,528
		NHS	1,709	1,383

a. Using the version of the Rand model detailed in [1].

**TABLE 4**  
**THE CATEGORIZATION OF ACCESSIONS FOR**  
**ALTERNATIVE CUTOFF SCORES GIVEN QMMs ARE FIXED<sup>a</sup>**

Cutoff score	Total accessions	Education	AFQT category	
			I-III A	IIIB-IV
115+	9,108	HS	6,897	208
		NHS	1,871	132
110+	9,294	HS	6,394	406
		NHS	2,206	288
105+	9,575	HS	5,878	716
		NHS	2,427	554
100+	9,833	HS	5,433	1,152
		NHS	2,475	773
95+	10,156	HS	4,792	1,821
		NHS	2,438	1,105
90+	10,432	HS	4,199	2,631
		NHS	2,282	1,320
85+	10,592	HS	3,911	3,138
		NHS	2,171	1,372
80+	10,854	HS	3,491	3,961
		NHS	1,998	1,404
76+	11,158	HS	3,148	4,797
		NHS	1,828	1,385
70+	11,586	HS	2,918	5,559
		NHS	1,718	1,391

a. Using the November 1981 version of the Rand model.

**TABLE 5**  
**QMMs, RMMs, AND TOTAL COST OF ALTERNATIVE MIXES**

Cutoff score	RMMs are fixed <sup>a</sup>			QMMs are fixed <sup>b</sup>		
	RMMs	QMMs	Total cost <sup>c</sup>	RMMs	QMMs	Total cost <sup>c</sup>
115+	247,116	240,320	\$601,531.1	200,273	194,688	\$452,162.7
110+	247,116	237,387	569,038.5	202,812	194,688	437,768.8
105+	247,116	234,028	540,208.5	206,008	194,688	425,767.8
100+	247,116	230,531	518,072.4	209,719	194,688	418,029.4
95+	247,116	225,363	490,967.7	215,834	194,688	411,114.6
90+	247,116	219,389	470,001.6	223,607	194,688	411,656.3
85+	247,116	215,562	461,111.6	228,722	194,688	415,567.9
80+	247,116	208,968	449,867.7	237,767	194,688	425,410.8
76+	247,116	202,031	442,293.9	247,128	194,688	438,155.2
70+	247,116	193,619	437,529.6	259,396	194,688	457,293.2

- a. Using the version of Rand model detailed in [1].  
b. Using the November 1981 version of the Rand model.  
c. In thousands of 1981 dollars.

Since military services are not sold in a market, the economic value of a unit of service (a qualified man-month) is unknown. Although it is virtually impossible to estimate accurately the value of a QMM, it is possible to calculate lower-bound estimates. Using a lower-bound estimate for the value of a QMM biases the net-benefit model to favor personnel mixes that yield lower performance, thus resulting in a conservative lower-bound estimate of the optimal cutoff score.

The simplest method for estimating the value of a qualified man-month is to assume that the dollar value of the QMMs equals the cost at some base cutoff score. Under the assumption that the value of a QMM is constant, the percentage of change in QMMs and cost resulting from a change in the cutoff score can be directly compared because they are relative to the same base. This is the approach used by the Congressional Budget Office in [4]. Alternatively, using the same assumptions, the

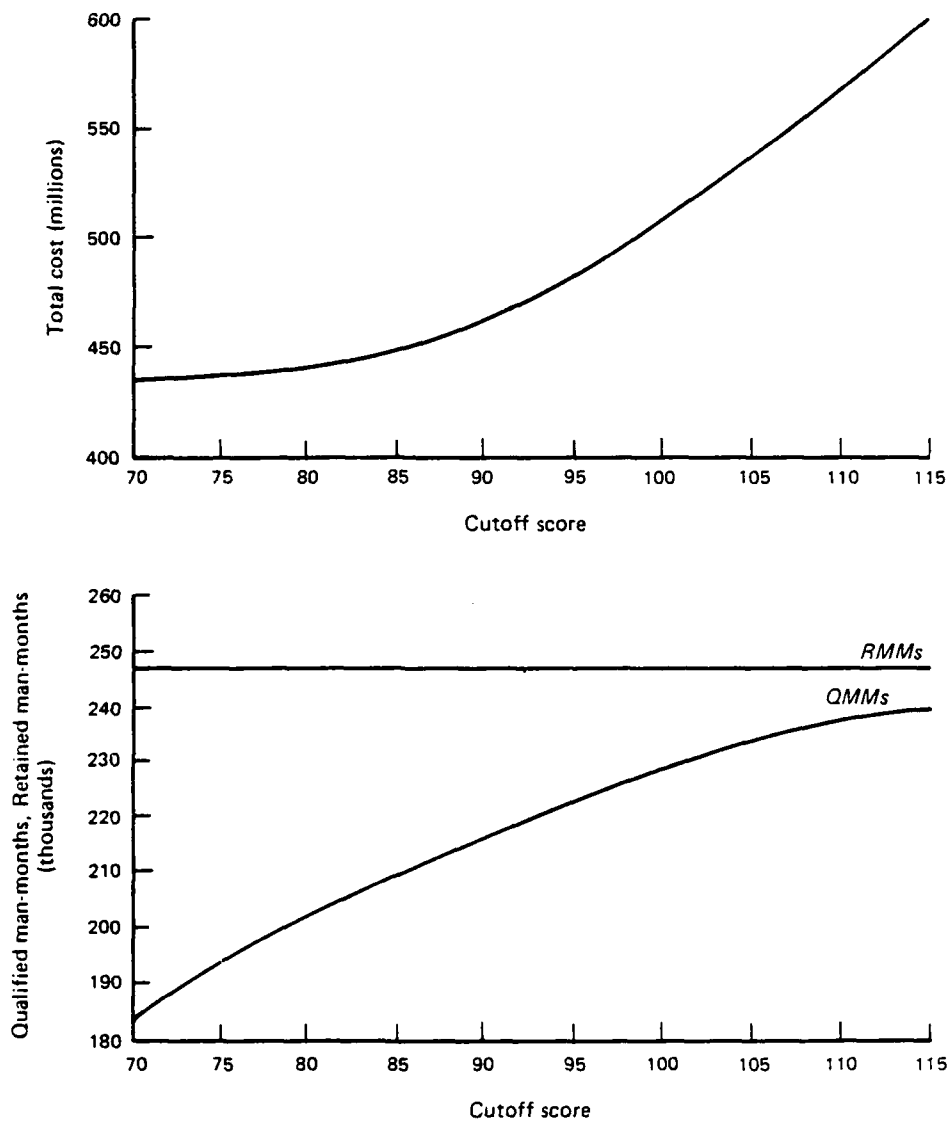


FIG. 6: ILLUSTRATION OF MAJOR VARIABLES: RAND 1

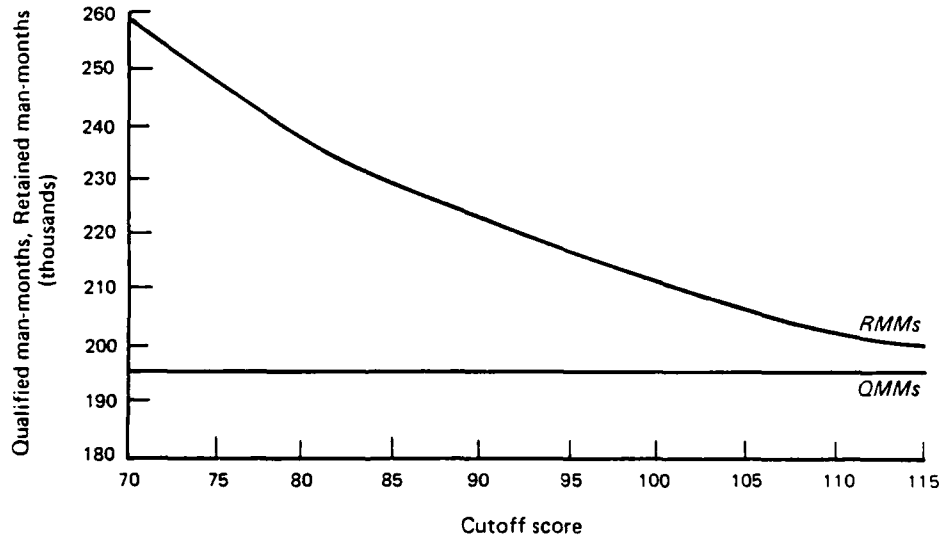
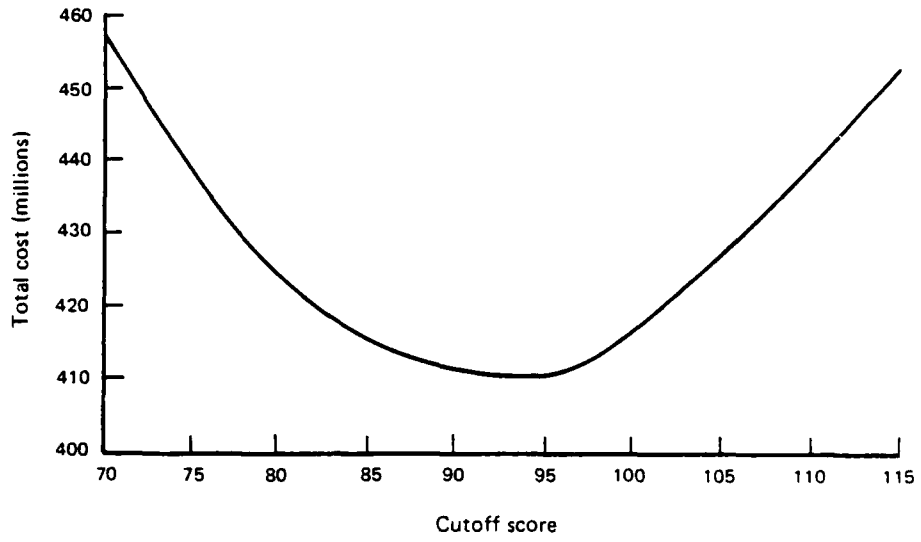


FIG. 7: ILLUSTRATION OF MAJOR VARIABLES: RAND 2

implicit value of a QMM can be determined using the condition that the value of the QMMs equals the cost at some base cutoff score (see figure 8). As shown in table 6, if it is assumed the QMM value (benefit) equals cost at a cutoff score of 70, then, using the data in table 5, the value of a QMM is \$2,260. If the value of the QMMs equals cost at a cutoff score of 76, the estimate for the value of a QMM is \$2,189.

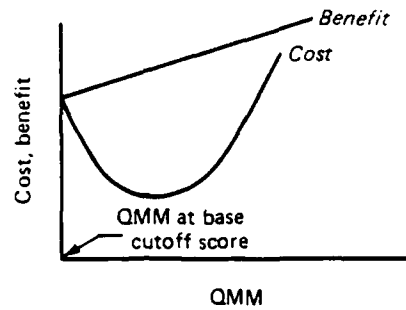
Alternatively, a lower-bound estimate for the value of a qualified man-month can be calculated by invoking two main assumptions. They are that, given a fixed number of retained man-months, the value of a QMM is constant over the range of personnel mixes considered, and there exists some personnel mix for which the value of the QMMs (benefit) is at least as great as the cost of this mix. The latter assumption is equivalent to assuming that there is some mix of personnel for which the military is able to at least cover its manpower costs. Graphically, the value of the QMMs curve (benefit curve) must be at least tangent to the cost curve, as shown in figure 9.

The minimum value of a qualified man-month under the assumption that the military covers its manpower costs is used as the dollar value of a QMM. That is, the value of a QMM that sets the benefit curve tangent to the cost curve is used as the estimate for the value of a QMM. Using this approach, the estimate of the value of a QMM is \$2,139, as shown in table 6. The details of the calculation are given in appendix A.

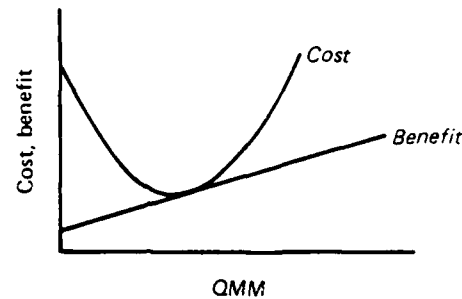
**TABLE 6**  
**THE ESTIMATED VALUE OF A QUALIFIED MAN-MONTH**

Estimation method	Value of a QMM
Linear benefit curve	
Benefit = cost at cutoff score = 70	\$2,260
Benefit = cost at cutoff score = 76	2,189
Benefit tangent to cost	2,139
Nonlinear benefit curve	
Elasticity = 0.5	2,260 <sup>a</sup>
Elasticity = 1.0	2,260 <sup>a</sup>
Elasticity = 2.0	2,238 <sup>a</sup>
Elasticity = 3.0	2,211 <sup>a</sup>
Elasticity = 4.0	2,196 <sup>a</sup>
Elasticity = 6.0	2,180 <sup>a</sup>

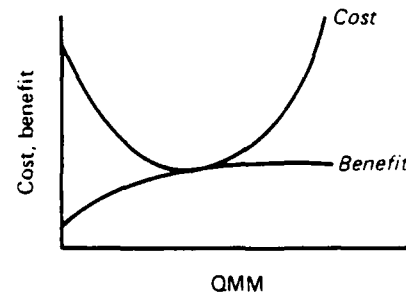
a. At a cutoff score of 70.



**FIG. 8: NET BENEFIT GIVEN BENEFIT EQUALS COST AT THE BASE CUTOFF SCORE**



**FIG. 9: NET BENEFIT GIVEN THERE IS SOME PERSONNEL MIX FOR WHICH BENEFIT IS AT LEAST AS GREAT AS COST (LINEAR CASE)**



**FIG. 10: NET BENEFIT GIVEN THERE IS SOME PERSONNEL MIX FOR WHICH BENEFIT IS AT LEAST AS GREAT AS COST (NONLINEAR CASE)**

As a variation of the above approach, a lower-bound estimate of the value of a qualified man-month is calculated using the same assumption regarding covering cost, but the assumption that the value of a QMM is constant is relaxed. It is possible that the value of the QMMs falls as additional QMMs are obtained. Rather than assuming that the value of the QMMs is constant, it is assumed the elasticity of demand is constant. The elasticity of demand is the percentage of change in the quantity of QMMs given a 1-percent change in the value (price) of a QMM. For example, if the elasticity were 2, the quantity of QMMs would rise by 2 percent for every 1-percent fall in the value (price) of QMMs. The total benefit curve (value of the QMMs curve) is non-linear in this case, as shown in figure 10, and its shape depends on the elasticity of demand. The estimates for the value of a QMM differ for each cutoff score. However, as shown in appendix A, the value of a QMM in general can be expressed as a function of the value of a QMM at the base cutoff score of 70 as follows:

$$V = V_0 - \left[ \frac{QMM - QMM_0}{QMM_0} \right] \frac{V_0}{\eta} \quad (1)$$

where  $\eta$  equals the elasticity of demand,  $V$  equals the value of a QMM associated with a specific cutoff score,  $QMM$  equals the number of QMMs associated with the cutoff score of consideration,  $QMM_0$  and  $V_0$  equal the number of QMMs and the value of a QMM at the cutoff score of 70, respectively. The estimated values of a QMM at the cutoff score of 70 assuming different elasticities, are given in table 6. The estimated value of a QMM at each cutoff score can be calculated by using the data given in table 5 and applying equation 1.

### Optimal Cutoff Score

RAND 1 holds RMMs constant and selects the personnel mix that minimizes the average cost of a QMM (see table 7). As shown in table 8, under this model variant, the optimal cutoff score is 85. RAND 2 holds QMMs constant and selects the personnel mix that minimizes total cost (see table 7). Although RAND 2 significantly differs from RAND 1, it yields an optimal cutoff score of 95. Similarly, the net-benefit model yields an optimal cutoff score of 85 given the assumption that the value of a QMM is constant (i.e., the benefit curve is linear). Given that the estimate for the value of a QMM is a lower-bound estimate, the outcome of the net-benefit model has the interpretation that the optimal cutoff score is at least 85. In contrast, if the value of a QMM falls as additional QMMs are obtained (i.e. the benefit curve is concave), the optimal cutoff score varies depending on the elasticity of demand (which determines

the curvature of the benefit curve). As shown in table 8, the optimal cutoff score ranges from 70 to 85 for the elasticity values are considered here. Given the underestimation of the value of a QMM, these results are interpreted as minimum values for the optimal cutoff score. Therefore, if the elasticity is greater than or equal to 3, the optimal cutoff is at least 80 for the nonlinear net-benefit model. The difference in both cost and performance between the optimal cutoff score and the second best cutoff score is relatively small for all three models. Therefore, although the optimal cutoff score is the "best" cutoff score, it is not much better than the next best score. In addition, because all three models have some negative features, none of the approaches stands out as being clearly superior, which makes it extremely difficult to choose a "best" model.

**TABLE 7**  
**THE OPTIMAL CUTOFF SCORE FOR RAND 1 AND RAND 2**

Cutoff score	RAND 1	RAND 2
	Average cost	Total cost <sup>a</sup>
115+	2503	\$452,162.7
110+	2397	437,768.8
105+	2308	425,767.8
100+	2247	418,029.4
95+	2179	411,114.6
90+	2142	411,656.3
85+	2139	415,567.9
80+	2153	425,410.8
76+	2189	438,155.2
70+	2260	457,293.2

a. In thousands of 1981 dollars.

## CONCLUSIONS

All three models confirm that, given the Rand input data, a cutoff score of 85 is a reasonable estimate of the optimal Arms Combat cutoff score. The main Rand variant and the linear net-benefit model yield an optimal cutoff score of 85. RAND 2 generates an optimal cutoff score of 95. The results of the nonlinear net-benefit model vary depending on the assumed level of the elasticity of demand. If the elasticity of demand is 6 or less, the minimum optimal cutoff score ranges from 85 to 70. Since the estimate for the value of a QMM is clearly a lower-bound estimate, these results are lower-bound cutoff scores and thus do not necessarily conflict with an optimal score

of 85. Therefore, although the main Rand approach (RAND 1) may not be the "best" approach, it appears to provide a reasonable methodology for determining optimal enlistment standards.

**TABLE 8**  
**THE OPTIMAL COMBAT ARMS CUTOFF SCORE**

Model	Constraints	Criteria for selection	Cutoff score
Rand variant 1	Fix RMMs	Minimize cost per QMM	85
Rand variant 2	Fix QMMs	Minimize total cost	95
Net benefit	Fix RMMs	Maximize net benefit	
Linear benefit curve			
	Benefit = cost at score = 70		85
	Benefit = cost at score = 76		85
	Benefit target to cost		85
Nonlinear benefit curve			
	Demand elasticity = 0.5		70
	Demand elasticity = 1.0		70
	Demand elasticity = 2.0		76
	Demand elasticity = 3.0		80
	Demand elasticity = 4.0		80
	Demand elasticity = 6.0		85

## REFERENCES

- [1] Rand Corporation, R-2874-MRAL, *Recruit Aptitudes and Army Job Performance*, by D. Armor, R. Fernandez, K. Bers, and D. Schwarzbach, Sep 1982
- [2] CNA, Research Memorandum 86-200, *The Rand Cost-Performance Model for Setting Qualification Standards: Preliminary Comments*, by Laurie J. May and Paul W. Mayberry, Unclassified, Sep 1986
- [3] Rand Corporation, R-3067-MIL, *Setting Enlistment Standards and Matching Recruits to Jobs Using Job Performance Criteria*, by R. Fernandez, Jan 1985
- [4] Congressional Budget Office, *Quality Soldiers: Cost of Manning the Active Army*, Jun 1985

APPENDIX A

ESTIMATING THE VALUE OF A QUALIFIED MAN-MONTH

## APPENDIX A

### ESTIMATING THE VALUE OF A QUALIFIED MAN-MONTH

The estimate for the value of a qualified man-month (QMM) that sets the benefit curve tangent to the cost curve is solved by an iterative process. An initial starting value for the dollar value of a QMM is selected, and alternatives are tried until the value that sets the benefit curve tangent to the cost curve is found. The benefit curve is tangent to the cost curve when net benefit equals zero for one cutoff score and is negative for all other cutoff scores.

In the linear benefit-curve case, net benefit is given by

$$NB = V(QMM) - TC, \quad (A-1)$$

where  $NB$  equals net benefit,  $TC$  equals total cost, and  $V$  is the unknown dollar value of a qualified man-month (QMM). Different values of  $V$  are tried until the value of  $V$  that sets the benefit curve tangent to the cost curve is found. This value of  $V$  is used as the estimate of the dollar value of a QMM. Table A-1 summarizes the results for the linear benefit-curve case in which  $V$  equals \$2,139.

**TABLE A-1**  
**NET BENEFIT FOR GIVEN VALUES**  
**OF  $V$  (LINEAR CASE)**

Cutoff score	Net benefit	
	$V = 2139.114$	$V = 2139.113$
115 +	-87,459,224	-87,459,464
110 +	-61,240,645	-61,240,882
105 +	-39,595,929	-39,596,163
100 +	-24,940,310	24,940,541
95 +	-8,879,856	-8,880,081
90 +	-703,519	-703,738
85 +	92	123
80 +	2,861,326	2,861,535
76 +	10,126,559	10,126,761
70 +	23,356,486	23,356,680

In the case of the nonlinear benefit curve, the expression for net benefit is more complicated than in the linear case. The shape of the benefit curve depends on how fast the value of a QMM declines over the feasible personnel mixes. A general expression for a nonlinear benefit curve can be derived from the relationship between the percentage of change in the number of QMMs and the percentage of change in the value of a QMM. This relationship can be expressed mathematically as follows:

$$\eta = \frac{\% \text{ change in the number of QMMs}}{\% \text{ change in the value of a QMM}} \quad (\text{A-2})$$

where  $\eta$  equals the elasticity of "demand."

If the elasticity is greater than 1, a 1-percent change in the number of QMMs corresponds to a less than 1-percent decline in the value of a QMM. Alternatively, if the elasticity is less than 1, the reverse holds true. Therefore, if the elasticity is very small (less than 1) a 1-percent change in the demand for QMMs corresponds to a greater than 1-percent decline in the value of an additional QMM.

The percentage of change in the number of QMMs that occurs as the cutoff score is raised from 70 can be expressed as

$$\% \text{ change in } QMM = \frac{QMM - QMM_0}{QMM_0} \quad (\text{A-3})$$

where  $QMM$  is the level of qualified man-months associated with a specific cutoff score, and  $QMM_0$  is the level of qualified man-months at the cutoff score of 70. The percentage of change in the value of a  $QMM$  that occurs as the cutoff score is raised from 70 can be expressed as

$$\% \text{ change in the value of a } QMM = \frac{V_0 - V}{V_0} \quad (\text{A-4})$$

where  $V$  is the value of a QMM associated with the specific cutoff score, and  $V_0$  is the value at the cutoff score of 70. Therefore, the elasticity can be expressed as

$$\eta = \frac{QMM - QMM_0}{QMM_0} \cdot \frac{V_0}{V_0 - V} \quad (\text{A-5})$$

Rearranging yields

$$V = V_0 - \left[ \frac{QMM - QMM_0}{QMM_0} \right] \frac{V_0}{\eta} \quad (\text{A-6})$$

where  $V$  equals the value of a QMM. Therefore, substituting for  $V$  in equation A-1, net benefit can be expressed as

$$NB = \left\{ V_0 - \left[ \frac{QMM - QMM_0}{QMM_0} \right] \frac{V_0}{\eta} \right\} QMM - TC \quad (A-7)$$

Different values of  $V_0$  are tried until the value of  $V_0$  that sets the benefit curve tangent to the cost curve is found. This value of  $V_0$  is used as the estimate of the dollar value of a QMM at the cutoff score of 70 and is in turn used to estimate the value of a qualified man-month ( $V$ ) at the other cutoff scores. Table A-2 summarizes the results for a specific case of the nonlinear net-benefit curve in which the elasticity equals 0.5 and  $V_0$  equals \$2,260. The value of  $V_0$ , and thus  $V$ , varies with the value of the elasticity.

**TABLE A-2**  
**NET BENEFIT FOR GIVEN VALUES**  
**OF  $V_0$  (NONLINEAR CASE)**

Cutoff score	Net benefit	
	$V_0 = 2259.74517$	$V_0 = 2259.74516$
115 +	-320,442,775	-320,442,776
110 +	-275,128,597	-275,128,599
105 +	-232,108,098	-232,108,099
100 +	-195,758,140	-195,758,142
95 +	-148,711,648	-148,711,649
90 +	-106,207,012	-106,207,013
85 +	-84,406,739	-84,406,741
80 +	-52,522,156	-52,522,157
76 +	-25,425,010	-25,425,012
70 +	0	-2

The above formulation for net benefit is equivalent to treating the military as if it were a monopoly producer. Total benefit equals the price of the last QMM obtained times the total number of QMMs obtained. Total benefit equals the shaded area in figure A-1(a).

## ALTERNATIVE MEASURES OF TOTAL BENEFIT

Alternatively, total benefit can be defined as including consumer surplus. That is, total benefit can be defined as the sum of the value (or price) of each QMM, which is the area under the demand curve. Under this scenario, the military is treated as both producer and consumer, and thus what is traditionally considered consumer surplus is included as part of the benefit. The benefit of an additional QMM equals the value of (or what the military is willing to pay for) an additional QMM. Under this formulation total benefit equals the shaded area in figure A-1(b), which is greater than the shaded area in figure A-1(a).

Although including consumer surplus as part of the total benefit associated with acquiring QMMs may have some theoretical appeal, it is extremely difficult to measure with any reasonable degree of accuracy. Measurement of consumer surplus requires knowledge of the entire demand curve for QMMs. Since the demand for QMMs is not observable, measuring consumer surplus as part of total benefit is virtually impossible.

It is possible, however, to estimate consumer surplus over a small range of the demand curve given the elasticity of demand is assumed to be constant over this range. The lowest cutoff score considered is 70, which yields 193,619 QMMs (see table 5 of main text). It might be argued that the value of the first 193,619 QMMs is basically constant and can be approximated by a horizontal demand curve. In contrast, additional QMMs beyond this quantity decline in value. Under this scenario, total benefit equals the shaded area in figure A-1(c).

Net benefit under this scenario equals the value of QMM at the cutoff score 70 ( $V_0$ ) times the quantity of QMMs at this cutoff score ( $QMM_0$ ) plus the area under the demand curve between  $QMM_0$  and the current level of QMM. Mathematically, net benefit equals

$$NB = V_0 QMM_0 + \int_{QMM_0}^{QMM} V_0 - \left[ \frac{QMM - QMM_0}{QMM_0} \right] \frac{V_0}{\eta} - TC, \quad (A-8)$$

recalling that,

$$V = V_0 - \left[ \frac{QMM - QMM_0}{QMM_0} \right] \frac{V_0}{\eta}. \quad (A-9)$$

Different values of  $V_0$  are tried until the value of  $V_0$  that sets the benefit curve tangent to the cost curve is found. This value of  $V_0$  is used as the estimate of the dollar value

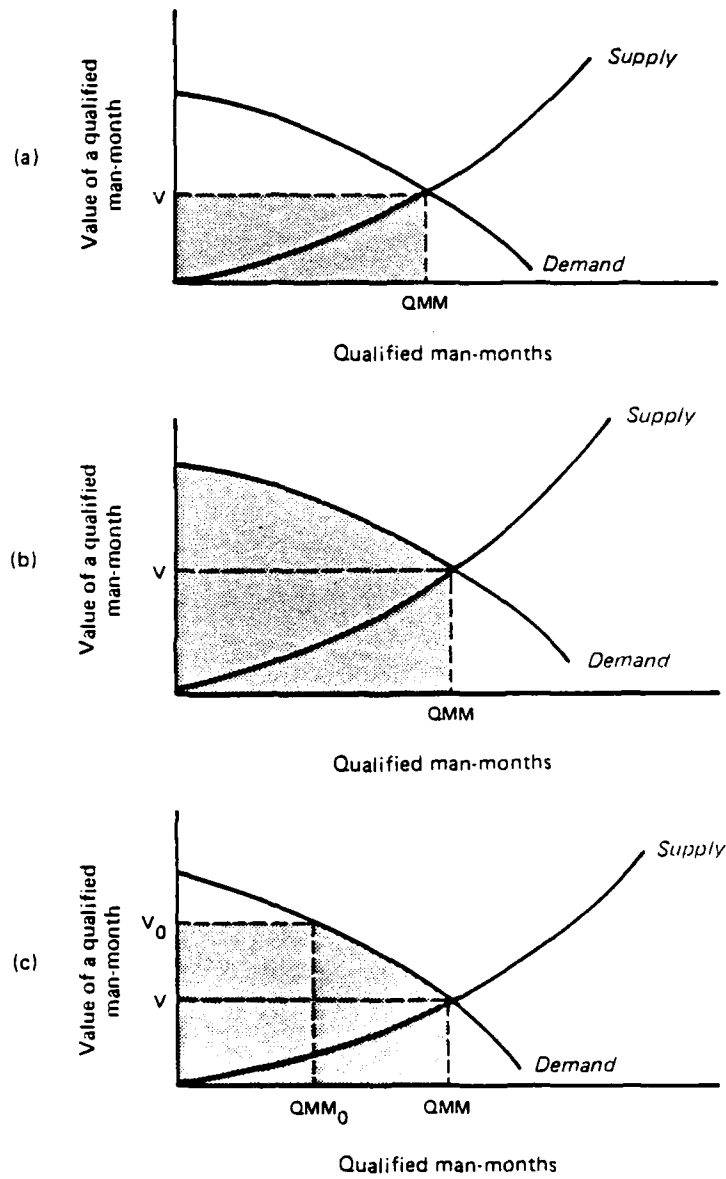


FIG. A-1: ALTERNATIVE APPROACHES TO MEASURING TOTAL BENEFIT

of a QMM at the cutoff score of 70 and is given for each scenario in table A-3. In turn, this estimate of  $V_0$  is used to estimate  $V$  at the other cutoff scores.

As shown in table A-3, this approach for estimating net benefit yields basically the same answers as the method that ignores consumer surplus which was used for the main results presented in the main text.

**TABLE A-3**

**THE ESTIMATED VALUE OF A QUALIFIED  
MAN-MONTH AND THE OPTIMAL COMBAT  
ARMS CUTOFF SCORE GIVEN TOTAL  
BENEFIT INCLUDES CONSUMER SURPLUS**

<b>Demand elasticity</b>	<b>Value of <math>QMM_0(V_0)</math></b>	<b>Cutoff score</b>
0.5	2,260	70
1.0	2,260	70
2.0	2,237	76
3.0	2,209	80
4.0	2,195	80
6.0	2,178	85

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