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ESTIMATION OF GRAVITY VECTOR COMPONENTS FROM BELL
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Estimation of Gravity Vector Components from Bell Gravity
Gradiometer and Auxiliary Data under Consideration of Topography
and Associated Analytical Upward Continuation Aspects.

H. Baussus von Luetzow

U.S. Army Engineer Topographic Laboratories
Fort Belvoir, VA 22060-5546

Abstract

Following an introduction, the paper briefly discusses gravity gradiometer applications. It outlines the estimation of first order derivatives of the anomalous gravity potential from Bell gravity gradiometer and auxiliary data in the context of a Wiener-Kolmogorov optimization scheme under consideration of computable "topographic noise," accomplished on the basis of the Pellinen-Moritz solution of the boundary value problem of physical geodesy. The paper also addresses four different methods of analytical upward continuation of first order derivatives of the anomalous gravity potential under identification of a finite difference method using Laplace's equation as the most economical and efficient one. Relevant conclusions are then presented.



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1. INTRODUCTION

The estimation of gravity vector components from Bell gravity gradiometer and auxiliary data has been discussed by a number of authors, including Jordan (1982), White and Goldstein (1984), Center, Jordan and Peacock (1985), and Baussus von Luetzow (1985). Apart from gravity gradiometer self noise power spectra or corresponding covariance functions, the collocation theory of physical geodesy was employed to derive spectra and cross-spectra or equivalent covariance functions involving first and second order derivatives of the anomalous gravity potential. The basic covariance functions, pertaining either to a randomized anomalous gravity potential or to a randomized gravity anomaly, accomplished by the subtraction of low frequency harmonic representations of anomalous potential or gravity anomaly data, presuppose the existence of homogeneity and isotropy. The application of collocation theory does not face difficulties in the case of quasi-flat terrain and is a prerequisite to arrive at small estimation errors for gravity vector components in the context of an airborne survey over a square area of $300 \times 300 \text{ km}^2$, covered with parallel traverses and cross traverses 5 km apart, conducted at an elevation of 600 m at a speed of about 360 km hr^{-1} , and employing only a limited amount of measured gravity vector component data. It is, however, well known that the assumption of homogeneity and isotropy does not satisfactorily hold in the presence of moderate to strong mountainous terrain. Some efforts have been made to apply heuristic topographic corrections to gravity gradiometer measurements. Still, the problem remains to provide for a Wiener-Kolmogorov-type estimation process free of computable topographic noise so that the assumption of homogeneity and isotropy is reasonably fulfilled with respect to modified, signal-type measurements disturbed by gradiometer self noise only. The solution to this problem is possible under application of approaches to the geodetic boundary value problem and the correlated interpolation of gravity anomalies and deflections of the vertical in mountainous terrain by Pellinen (1969), Moritz (1969), and Baussus von Luetzow (1971, 1981). Section 3 as the main section of this paper addresses the solution under consideration of the estimation process and the computation of "topographic noise". The preceding section addresses in a compact way gravity gradiometer applications to place the determination of gravity vector components over relatively large areas and the analytical upward continuation of vertical and horizontal derivatives of the anomalous gravity potential in the proper perspective. The relatively short sections 4-7 discuss different methods of analytical upward continuation, followed by several conclusions.

2. GRAVITY GRADIOMETER APPLICATIONS

Geodetic research and the development of associated technology has been considerably stimulated by military requirements and actual or potential military applications with critical funding provided by the U.S. Government. The result thereof was an increase of scientific knowledge, new instrumentation and techniques, and technology transfer to the civilian sector, particularly in navigation and surveying.

The main applications of gravity gradiometers follow:

- Establishment of the spatial gravity field in ICBM¹ launch areas (most important application). This includes gravity vector components on the ground for calibration, local to global coordinate transformations, geodetic azimuth determination, and gravity programmed inertial positioning (beneficial to Small ICBM or Midgetman). An accurate spatial gravity field, extending to

¹Intercontinental Ballistic Missile

a height of 200 km, provides also for comparisons/calibration for the Geopotential Research Mission (GRM) and a Superconducting Gravity Gradiometer Mission (SGGM). Instrument: Bell Gravity Gradiometer.

- ° Accuracy enhancement of SLBM's². Instrument: Bell Gravity Gradiometer.
- ° Improvement of global gravity field for geophysical applications (GRM). Instrument: Superconducting Gravity Gradiometer.
Byproducts: ICBM target error reduction (not critical), cruise missile navigation, improved satellite ephemeris.
- ° High accuracy space navigation. Instrument: Superconducting Gravity Gradiometer (six axis version).
- ° Gravity vector densification over large areas for gravity programmed inertial navigation and for geodetic network adjustments. Instrument: Bell Gravity Gradiometer.
- ° Subterranean mass detection (cavities, oil, water). Instrument: Superconducting Gravity Gradiometer (vertical axis version).
- ° Test of Newton's square law and tests of Einstein's theory of relativity. Instrument: Superconducting Gravity Gradiometer.

3. DETERMINATION OF DERIVATIVES OF THE ANOMALOUS GRAVITY POTENTIAL OVER MOUNTAINOUS TERRAIN USING BELL GRAVITY GRADIOMETER MEASUREMENTS AND AUXILIARY DATA

A. Notations and Relations Used

T anomalous gravity potential

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ derivatives taken along the local horizon in a northern and eastern direction

$$\bar{T}_x = \frac{\partial T}{\partial x}$$

$$\bar{T}_{xx} = \frac{\partial^2 T}{\partial x^2}$$

$n = n_1 + n_2$ Bell gravity gradiometer instrument noise with white noise component n_1 and red noise component n_2

$\wedge$ signal variable indicator

$\delta \bar{T}_x$ topographic noise component of $\bar{T}_x$

$\delta \bar{T}_{xx}$ topographic noise component of $\bar{T}_{xx}$

$\bar{T}_x$ lower order spherical harmonics representation of T_x used for subtraction

$\bar{T}_{xx}$ lower order spherical harmonics representation of T_{xx} used for subtraction

$\bar{\delta T}_x$ lower order spherical harmonics representation of δT_x used for subtraction

² Sea Launched Ballistic Missile

lower order spherical harmonics representation of
used for subtraction

$$\hat{T}_x = \bar{T}_x - \delta T_x$$

$$\hat{\bar{T}}_x = \hat{T}_x - \overline{\hat{T}_x} = \bar{T}_x - \delta T_x - (\bar{T}_x - \delta \bar{T}_x)$$

$$\hat{T}_{xx} = \bar{T}_{xx} - \delta T_{xx}$$

$$\hat{\bar{T}}_{xx} = \hat{T}_{xx} - \overline{\hat{T}_{xx}} = \bar{T}_{xx} - \delta T_{xx} - (\bar{T}_{xx} - \delta \bar{T}_{xx})$$

a_{ik} weight factor with measurement index i and computation index K , applicable to $\bar{T}_x$ in Wiener-Kolmogorov estimation

b_{jk} weight factor with measurement index j and computation index K , applicable to $\bar{T}_{xx}$ in Wiener-Kolmogorov estimation

R earth's mean radius

σ unit sphere (full solid angle)

r length of radius vector from earth's center to a moving point on earth's surface

r_p length of radius vector from earth's center to a fixed point on earth's surface

r_A length of radius vector from earth's center to a fixed point on a level surface above earth's surface

ψ angle between radius vectors $\vec{r}$ and $\vec{r}_p$ or $\vec{r}_A$

$$l = (r^2 + r_A^2 - 2rr_A \cos \psi)^{1/2}$$

α azimuth angle counted clockwise from north

$S_g = S(r_A, \psi, r)$ generalized Stokes Function

h elevation of terrain referring to a movable point on earth's surface

h_p elevation of terrain referring to a fixed point on earth's surface

h_A elevation of level surface above terrain

$$\ell_0 = 2 R \sin \frac{\psi}{2}$$

$$\ell_1 = [\ell_0^2 + (h - h_A)^2]^{1/2} \quad \text{planar approximation to } \ell$$

Δg	gravity anomaly on earth's surface
C	topographic correction to gravity anomaly
$\Delta g_F = \Delta g + C$	Faye anomaly
b	Bouguer gradient
k	gravitational constant
ρ	standard density

$$\delta T = 4\pi k \rho h_p R + k \rho R^2 \iint_{\sigma} \ln \frac{\ell_1 + h - h_p}{\ell_0} d\sigma$$

potential of the topographic masses

$$G_1(v) = \frac{R^2}{2\pi} \iint_{\sigma} \frac{v(h - h_p)}{\ell_0^3} d\sigma \quad \text{with } v \text{ as an arbitrary variable}$$

ϕ	geographic latitude
λ	geographic longitude

$$\Theta = \frac{\pi}{2} - \phi$$

B. Estimation Process

A Wiener-Kolmogorov estimation can, under consideration of $\bar{T}_x$ and T_{xx} only, be characterized by

$$T_{xK} = \sum a_{iK} T_{xi} + \sum b_{jK} (T_{xxj} + n_j) \quad (1)$$

The derivatives $\bar{T}_{xi}$, also called truth data, are, of course, also associated with measurement errors. With respect to deflections of the vertical, proportional to $\bar{T}_x$ and $\bar{T}_y$, efforts are being made to determine them astrogeodetically with an error of 0.1 arcsec rms. Gravity anomalies can be measured with a relatively greater accuracy. In order to compute the vertical derivative $\frac{\partial T}{\partial r} = T_r$, high degree spherical harmonic expansions presented by Rapp (1987) or the GRIM3-L1 Model described by Reigber, Balmino, Müller, Bosch and Moynot (1985) may be employed. These models are also useful for the subtraction of lower harmonics components from measured first and second order derivatives of T .

Regarding eq. (1), it is assumed that all measured quantities refer to a level surface. In the case of quasi-flat terrain, $\bar{T}_{xK}$ -estimates have been made from $\bar{T}_{xi}$ on the earth's surface and from airborne $\bar{T}_{xK}$ -measurements, employing spatial covariance functions. This is not possible in the presence of pronounced topography. In this case, it is

$$\bar{T}_{xK}(h_p) = \bar{T}_{xK}(h_A) + \frac{\partial \bar{T}_{xK}}{\partial z}(h_p - h_A) \quad (2)$$

where $\frac{\partial \bar{T}_{xK}}{\partial z}$ is a representative gradient obtained from gravity gradiometer measurements. There is no significant degradation in accuracy if eq. (2) is used.

For simplicity, eq. (1) presupposes the prior application of quasi-systematic corrections obtained from lower harmonic expansions. In the presence of pronounced mountainous terrain, eq. (1) is reformulated as

$$\bar{T}_{xK}^0 = \sum a_{iK}^{(2)} \bar{T}_{xi} + \sum b_{jK}^{(2)} (\bar{T}_{xj}^0 + n_j) \quad (3)$$

C. Computation of "Topographic Noise"

The essential task is the computation of topographic noise components of the derivatives $\bar{T}_x$, $\bar{T}_y$, $\bar{T}_z$. In this respect, the solution nomenclature employed by Moritz (1969)³ is applied. In order to obtain solutions at elevation h_A instead of solutions at elevation $h = h_p$, Stokes generalized function S_g replaces Stokes function $S(\psi)$ in the solutions where applicable.⁴

First, the solution for T at a point P_A is written as

$$T = \sum_i^3 \bar{T}_i \quad (4)$$

³ Moritz (1969), p. 31.

⁴ S_g is explicitly formulated by Moritz (1966). See p. 49.

The solution components are

$$T_1 = \frac{R}{4\pi} \iint_{\sigma} [\Delta g_F + G_1(\Delta g_F)] S_g d\sigma \quad (5)$$

$$T_2 = \frac{R}{4\pi} \iint_{\sigma} G_1(-bh_p + \frac{3}{2R} \delta T) S_g d\sigma \quad (6)$$

$$T_3 = k_s R^2 \iint_{\sigma} \left(\ln \frac{\ell_1 + h - h_A}{\ell_0} - \frac{h - h_A}{\ell_0} \right) d\sigma \quad (7)$$

T_1 represents the "signal" term of T since its structure is compatible with the existence of a reproducible kernel in the sense of Krarup, a prerequisite for the application of the collocation method of physical geodesy.

T_2 and T_3 are clearly "noise" terms which do not permit the establishment of a statistical estimation structure. T_2 appears to be a more slowly varying function. Moritz (1969) states that the term δT may be neglected in the context of a planar approximation.⁵ Accordingly, T_3 appears to be mainly responsible for fluctuations of a non-stationary nature.

An improvement of the above analysis is possible by the replacement of Δg_F by the isostatic anomaly Δg_i , provided that isostatic conditions prevail. Neglecting a second order improvement involving consideration of G_1 -terms, the additional "topographic noise" can be written as

$$T_4 = \frac{R}{4\pi} \iint_{\sigma} (\Delta g_F - \Delta g_i) S_g d\sigma \quad (8)$$

Without consideration of the term T_4 , the "topographic noise" terms involving vertical and horizontal derivatives of T_2 and T_3 can be formulated as

$$\frac{\partial T_2}{\partial z} = \frac{\partial T_2}{\partial r_A} = \frac{R}{4\pi} \iint_{\sigma} G_1(-bh_p + \frac{3}{2R} \delta T) \frac{\partial S_g}{\partial r_A} d\sigma \quad (9)$$

⁵ Moritz (1969), p. 32.

$$\frac{\partial T_3}{\partial z} = k_s R^2 \iint_{\sigma} \frac{\partial}{\partial h_A} \left(\ln \frac{\ell_1 + h - h_A}{\ell_0} - \frac{h - h_A}{\ell_0} \right) d\sigma \quad (10)$$

$$\left\{ \frac{\partial T_2}{\partial x} \right\} = \frac{R}{4\pi} \iint_{\sigma} G_1 (-b h_p + \frac{3}{2R} \delta T) \frac{\partial S_2}{\partial \psi} \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} d\sigma \quad (11)$$

$$\left\{ \frac{\partial T_2}{\partial x} \right\} = k_s R^3 \iint_{\sigma} \frac{h - h_A}{\ell_0^2} \left(\frac{1}{\ell_1} - \frac{1}{\ell_0} \right) \sin \psi \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} d\sigma \quad (12)$$

D. Averaged Second Order Derivatives

Because of the application of a moving average of gravity gradiometer measurements over a time interval Δt of about 10 seconds, it is not necessary to compute second order derivatives with respect to x, y, z of T_2 and T_3 . As an example, the averaged second order derivative of T_3 with respect to x is

$$\overline{\frac{\partial^2 T_3}{\partial x^2}} = \frac{1}{\Delta x} \left[\left(\frac{\partial T_3}{\partial x} \right)_{t_2} - \left(\frac{\partial T_3}{\partial x} \right)_{t_1} \right] \quad (13)$$

In eq. (13) it is $\Delta x = v_A (t_2 - t_1) = v_A \cdot \Delta t$, v_A being the horizontal aircraft speed.

4. STANDARD ANALYTICAL UPWARD CONTINUATION

Assuming that the three first order derivatives of the anomalous gravity potential have been estimated from gradiometric surveys at rectangular grid points of a level surface at elevation $h_A = \text{const.}$, Poisson's integral formula

$$\overline{T}_p = \frac{R(r^2 - R^2)}{4\pi} \iint_{\sigma} \frac{\overline{T}}{\ell^3} d\sigma \quad (14)$$

can be used for the analytical upward continuation of T or of its derivatives

$\frac{\partial T}{\partial r}$, $\frac{\partial T}{\partial \phi}$, $\frac{\partial T}{\partial \lambda}$. According to Heiskanen and Moritz (1967), for elevations smaller than 250 km, the planar form of eq. (14) is sufficiently accurate. For elevations up to 150 km, which apply in ICBM launch areas, the horizontal integration distance has to be ten times the elevation to assure good accuracy, i.e., a very large information base is required. Additionally, rectangular grid data has to be interpolated to conform with circular zone data to be employed in the computation.

5. IMMEDIATE SPATIAL WIENER KOLMOGOROV ESTIMATION

The three derivatives of the anomalous gravity potential may be estimated directly for grid points at higher elevations in the context of a Wiener-Kolmogorov scheme, using spatial covariance functions or equivalent power spectra. For higher elevations, data derived from a greater number of adjacent surveys would be required, and matrix inversions would become very complicated. Further complications would arise in the presence of pronounced topography because of the non-applicability of effective collocation methods.

6. SPATIAL COLLOCATION UPWARD CONTINUATION OF T_r , T_ϕ , T_λ

If T_r , T_ϕ , T_λ are first estimated at grid points of a level surface with elevation h_A , a collocation upward continuation would be a replacement of the Poisson integral formula approach by statistical methods. Again, a large information base would be required, inversions of matrices associated with higher elevations would become complicated, and complications would exist in the presence of pronounced topography. It would be possible though to apply the collocation extrapolation only to signal quantities T_r , T_ϕ , T_λ and to compute the "topographic noise" components separately for higher elevations by means of eqs. (9) - (12).

7. NUMERICAL UPWARD CONTINUATION USING LAPLACE'S DIFFERENTIAL EQUATION

The upward continuation methods discussed in sections 4-6 are characterized by the use of a specific derivative of the anomalous gravity potential, given at a level surface, by the requirement of a large information base for extrapolations to higher elevations, by associated large inversion matrices in collocation approaches, and by the inadequacy of these in the presence of mountainous terrain in the absence of a separation into signal estimation and "topographic noise" computation. Fortunately, the availability of three vector components at a level surface makes it possible to use information at two adjacent level surfaces as a prerequisite for applying Laplace's equation. Hereby, the information base can be reduced significantly, and inversions of matrices are not needed.

In Cartesian coordinates, the availability of $\frac{\partial T}{\partial x} = T_x$, $\frac{\partial T}{\partial y} = T_y$, $\frac{\partial T}{\partial z} = T_z$ at the level Z_0 makes it possible to compute T_x , T_y , T_z at the level $Z_1 = Z_0 + \Delta Z$ since

$$\frac{\partial}{\partial z} \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \frac{\partial T}{\partial z} \quad (15a)$$

$$\frac{\partial}{\partial z} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \frac{\partial T}{\partial z} \quad (15b)$$

$$\frac{\partial}{\partial z} \frac{\partial T}{\partial z} = - \left(\frac{\partial}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y} \right) \quad (15c)$$

The derivative $\frac{\partial^2 f}{\partial z^2}$ can, of course, be determined separately through use of gradiometer information, and a representative value can be employed for analytical upward continuation.

If a variable f satisfying Laplace's equation is given at two adjacent levels separated by ΔZ , Laplace's equation

$$\frac{\partial^2 f}{\partial z^2} = -\Delta^2 Z = -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) \quad (16)$$

permits application of the numerical algorithm

$$f_2 = 2f_1 - f_0 - (\Delta z)^2 (\Delta^2 f)_1 \quad (17)$$

In more appropriate spherical coordinates r , θ , λ , eq. (17) has to be replaced by

$$f_2 = 2f_1 - f_0 - \frac{(\Delta r)^2}{r^2} \left(2r \frac{\partial f}{\partial r} + \cot \theta \frac{\partial f}{\partial \theta} + \frac{\partial^2 f}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2 f}{\partial \lambda^2} \right)_1 \quad (18)$$

where the derivatives in the bracketed term are to be evaluated by finite differences. Higher order finite difference algorithms may be applied if considered advantageous.

It is possible to use eq. (18) only for "signal" components and to compute the "topographic noise" effects separately, using grid lengths of 1 km instead of 5 km.

8. CONCLUSIONS

Wiener-Kolmogorov-type estimation of gravity vector components or of related first order derivatives of the anomalous gravity potential from Bell gravity gradiometer and auxiliary data in the context of an airborne area survey can also be accomplished in the presence of pronounced mountainous terrain. A specific solution of the boundary value problem of physical geodesy permits the determination of "signal" and "topographic noise" solution terms. "Topographic noise" terms can be computed for each first derivative of the anomalous gravity potential, and their consideration permits the application of algorithms, including numerical weight factors, valid in the case of quasi-flat terrain. A slight degradation in accuracy may be expected. The standard analytical upward continuation method and two spatial collocation methods for the estimation of first order derivatives of the anomalous gravity potential can be replaced by numerical upward continuation using Laplace's equation because of the availability of three gravity vector components at the information base level surface. In this case, the standard information base can be reduced considerably, and no matrix inversions are required. Analytical upward continuation of "topographic noise" components may also be accomplished separately, using a higher resolution grid. The computation of "topographic noise" effects is mathematically laborious, can,

however, be achieved by means of high speed computers. The alternative in the presence of mountainous terrain is to reduce flight traverses from 300 km to about 100 km, to use highly accurate "truth" data at both ends of the traverse, and to repeat single traverse surveys.

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