

AD-A189 067

AMPLITUDE AND PHASE CONTROL WITH COARSELY QUANTIZED  
PHASE SHIFTERS IN PLANAR ARRAYS (U) ROME AIR DEVELOPMENT  
CENTER GRAIFFISS AFB NY P R FRANCHI ET AL. DEC 84

1/1

UNCLASSIFIED

RADC-TR-86-204

F/G 9/1

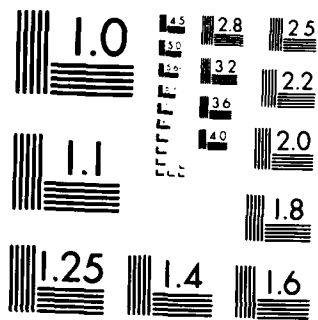
NL

END

PLATE

FIMEL

8-



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

DTIC FILE COPY

4

RADC-TR-86-204  
In-House Report  
December 1986



AD-A189 067

# AMPLITUDE AND PHASE CONTROL WITH COARSELY QUANTIZED PHASE SHIFTERS IN PLANAR ARRAYS

Peter R. Franchi and Robert A. Shore

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

DTIC  
ELECTE  
FEB 19 1988  
S D  
E

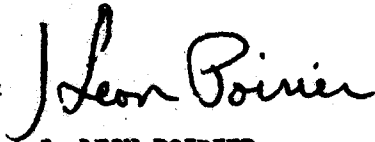
ROME AIR DEVELOPMENT CENTER  
Air Force Systems Command  
Griffiss Air Force Base, NY 13441-5700

88 2 16 00 9

This report has been reviewed by the RADC Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-86-204 has been reviewed and is approved for publication.

APPROVED:



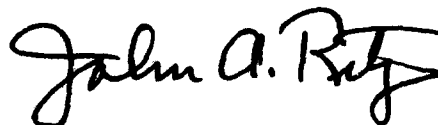
J. LEON POIRIER  
Chief, Applied Electromagnetics Division  
Electromagnetics Directorate

APPROVED:



ALLAN C. SCHELL  
Director of Electromagnetics

FOR THE COMMANDER:



JOHN A. RITZ  
Directorate of Plans & Programs

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (KECS) Hanscom AFB MA 01731-5000. This will assist us in maintaining a current mailing list.

Do not return copies of this report unless contractual obligations or notices on a specific document requires that it be returned.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>Unclassified</b>		1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT  <b>Approved for public release; distribution unlimited</b>	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		4. PERFORMING ORGANIZATION REPORT NUMBER(S)  <b>RADC-TR-86-204</b>	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S)	
6a. NAME OF PERFORMING ORGANIZATION <b>Rome Air Development Center</b>	6b. OFFICE SYMBOL (If applicable) <b>EECS</b>	7a. NAME OF MONITORING ORGANIZATION <b>Rome Air Development Center (EECS)</b>	
6c. ADDRESS (City, State and ZIP Code) <b>Hanscom AFB Massachusetts 01731</b>		7b. ADDRESS (City, State and ZIP Code) <b>Hanscom AFB Massachusetts 01731</b>	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION <b>Rome Air Development Center</b>	8b. OFFICE SYMBOL (If applicable) <b>EECS</b>	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State and ZIP Code) <b>Hanscom AFB Massachusetts 01731</b>		10. SOURCE OF FUNDING NOS	
11. TITLE (Include Security Classification) <b>(U) Amplitude and Phase Control (cont.)</b>		PROGRAM ELEMENT NO <b>61102F</b>	PROJECT NO <b>2305</b>
		TASK NO <b>J3</b>	WORK UNIT NO <b>04</b>
12. PERSONAL AUTHOR(S) <b>Franchi, Peter R., and Shore, Robert A.</b>			
13a. TYPE OF REPORT <b>In-house report for</b>	13b. TIME COVERED <b>FROM May 86 TO Aug 86</b>	14. DATE OF REPORT (Yr., Mo., Day) <b>1986 December</b>	15. PAGE COUNT <b>26</b>
16. SUPPLEMENTARY NOTATION			
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD <b>0903</b>	GROUP <b>1709</b>	SUB. GR.	
		<b>Planar array antennas</b>	
		<b>Phase quantization</b>	
		<b>Phase-only weight control</b>	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)			
<p>The purpose of this report is to show that deep nulls can be imposed at specified locations in the <math>\phi = 0^\circ</math> pattern cut of planar array antenna patterns using phase-only control of coarsely quantized phase shifters. This is done by using the number of phase shifters available for control in the columns of the array to compensate for the small number of bits of the individual phase shifters. A null synthesis algorithm is derived to calculate the phase shifts to impose nulls in the <math>\phi = 0^\circ</math> pattern cut of planar array antenna patterns by matching the pattern of a linear array with nulls imposed at the same locations using continuously variable, combined phase and amplitude control. Results of calculations are presented.</p>			
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <b>UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input checked="" type="checkbox"/> DTIC USERS <input type="checkbox"/></b>		21. ABSTRACT SECURITY CLASSIFICATION <b>Unclassified</b>	
22a. NAME OF RESPONSIBLE INDIVIDUAL <b>Robert A. Shore</b>		22b. TELEPHONE NUMBER (Include Area Code) <b>(617) 861-2058</b>	22c. OFFICE SYMBOL <b>RADC/EECS</b>

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

Block 11 (cont. ).

With Coarsely Quantized Phase Shifters in Planar Arrays

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	



## Contents

1. INTRODUCTION	1
2. ANALYSIS	2
3. RESULTS	9
REFERENCES	19
APPENDIX A: COMBINED PHASE AND AMPLITUDE NULLING IN LINEAR ARRAYS	21

## Illustrations

1. Comparison of Perturbed $v = 0$ Pattern (—) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers and 4-Bit Phase Shifters, With Linear Array Pattern (-----) With Nulls Imposed at 3, 6, 9, 12, and 15 Degrees Using Continuously Variable, Combined Phase and Amplitude Control	12
2. Unperturbed $u = 0$ Pattern (-----) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers, and Perturbed Pattern (—) Corresponding to Nulls Imposed in the $v = 0$ Pattern at 3, 6, 9, 12, and 15 Degrees. NBIT = 4	13

## Illustrations

3. Comparison of Perturbed $v = 0$ Pattern (——) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers and 2-Bit Phase Shifters, With Linear Array Pattern (-----) With Nulls Imposed at 3, 6, 9, 12, and 15 Degrees Using Continuously Variable, Combined Phase and Amplitude Control	14
4. Unperturbed $u = 0$ Pattern (-----) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers, and Perturbed Pattern (——) Corresponding to Nulls Imposed in the $v = 0$ Pattern at 3, 6, 9, 12, and 15 Degrees. NBIT = 2	15
5. Comparison of Perturbed $v = 0$ Pattern (——) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers and 2-Bit Phase Shifters, With Linear Array Pattern (-----) With Nulls Imposed at -12, -8, -4, 4, 8, and 12 Degrees Using Continuously Variable, Combined Phase and Amplitude Control	17
6. Unperturbed $u = 0$ Pattern (-----) of 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Amplitude Tapers, and Perturbed Pattern (——) Corresponding to Nulls Imposed in the $v = 0$ Pattern at -12, -8, -4, 4, 8, and 12 Degrees. NBIT = 2	18

## Tables

1. Summary of Results Obtained for a 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Row and Amplitude Tapers and Nulls Imposed in the $\phi = 0^\circ$ Pattern Cut at 3, 6, 9, 12, and 15 Degrees	10
2. Summary of Results Obtained for a 100 x 20 Element Array With 40 dB, $\bar{n} = 6$ Taylor Row and Amplitude Tapers and Nulls Imposed in the $\phi = 0^\circ$ Pattern Cut at 3, 6, 9, 12, and 15 Degrees	11
3. Summary of Results Obtained for a 100 x 100 Element Array With 40 dB, $\bar{n} = 6$ Taylor Row and Amplitude Tapers and Nulls Imposed in the $\phi = 0^\circ$ Pattern Cut at -12, -8, -4, 4, 8, and 12 Degrees	16
4. Summary of Results Obtained for a 100 x 20 Element Array With 40 dB, $\bar{n} = 6$ Taylor Row and Amplitude Tapers and Nulls Imposed in the $\phi = 0^\circ$ Pattern Cut at -12, -8, -4, 4, 8, and 12 Degrees	16

# Amplitude and Phase Control With Coarsely Quantized Phase Shifters in Planar Arrays

## I. INTRODUCTION

The growing importance of phased array antennas has resulted in considerable interest in phase-only control of array antenna patterns, since the required phase controls are already available as part of a beam steering system. Shore<sup>1</sup> provides an extensive bibliography. Even though it has been established theoretically that phase-only control can be used to impose nulls at desired locations in array antenna patterns, important questions remain to be answered about its practicality. One of these questions concerns the precision of the phase shifters required to achieve deep nulls. If phased arrays with low precision phase shifters can be used to give effective null control, this would, of course, imply correspondingly lower costs of the arrays than if high precision phase shifters are required.

This report is the second on the subject of a possible way of overcoming the limitations on depth of null and on other important pattern parameters that would otherwise result from using coarsely quantized phase shifters to control the pattern. The basic idea is to use the number of phase shifters available for control in the columns of a planar array to compensate for the small number of bits of the

---

(Received for publication 5 January 1987)

1. Shore, R. A. (1985) A Review of Phase-Only Sidelobe Nulling Investigations at RADC, RADC-TR-85-145, AD A166602.

individual phase shifters by averaging over the columns. In an earlier report,<sup>2</sup> we showed that deep nulls could be imposed in the  $\phi = 0^\circ$  pattern cut of a planar array pattern using phase shifters of even 2 bits by matching the pattern of a linear array with nulls imposed at the desired locations using phase-only control with continuously variable phase shifters. In this report, we extend those results to show that coarsely quantized phase shifters in a planar array can be used to give a  $\phi = 0^\circ$  pattern cut that closely matches the pattern of a linear array with nulls imposed using continuously variable phase and amplitude control. Thus, there is no special problem involved in imposing nulls at symmetric pattern locations using a planar array with phase-only control, the way there is if a linear array is used. In the following section of the report, we present the analysis and description of the nulling algorithm. Some typical results obtained are given in the concluding section.

## 2. ANALYSIS

We consider a rectangular planar array of elements with  $2N$  columns and  $2M$  rows. The inter-element spacing of elements in the rows is denoted by  $d_x$ , and that of the column elements is denoted by  $d_y$ . The field pattern of the array is given by the expression

$$P(u, v) = \sum_{n=1}^{2N} \sum_{m=1}^{2M} w_{nm} \exp [j (d_{x_n} u + d_{y_m} v)]$$

where

$$d_{x_n} = \frac{2N-1}{2} - (n-1), \quad n = 1, 2, \dots, 2N$$

$$d_{y_m} = \frac{2M-1}{2} - (m-1), \quad m = 1, 2, \dots, 2M$$

$$u = kd_x \sin(\theta) \cos(\phi),$$

$$v = kd_y \sin(\theta) \sin(\phi),$$

$$k = \text{wavenumber} = (2\pi)/\lambda,$$

---

2. Franchi, P. R., and Shore, R. A. (1986) Phase-Only Nulling With Coarsely Quantized Phase Shifters, RADC-TR-86-127, AD A175768.

and where  $w_{nm}$  is the complex weight of the  $nm$ th element. The pattern angles,  $\theta$  and  $\phi$ , are referred to a coordinate system with the  $z$ -axis normal to the plane of the array, and with the  $x$ - and  $y$ -axes parallel to the rows and columns respectively.

Let the unperturbed weights be denoted by  $w_{onm} = a_n b_m \exp(j\phi_{onm})$  where it is assumed that the amplitude taper can be factored into the product of a row taper and a column taper. The normalization

$$\sum_{n=1}^{2N} a_n = 1, \quad \sum_{m=1}^{2M} b_m = 1$$

is imposed on these tapers. The phase-only perturbed weights are then denoted by  $w_{nm} = w_{onm} \exp(j\Delta\phi_{nm})$ . The planar-array phase shifters are assumed to be quantized with  $N_{\text{BIT}}$  bits, so that the phase perturbations are given by

$$\Delta\phi_{nm} = i_{nm} B, \quad i_{nm} = 0, \pm 1, \pm 2, \dots, \quad B = \frac{2\pi}{2^{N_{\text{BIT}}}} \quad (1)$$

Throughout this report, we will work exclusively with real patterns, as a condition for which we assume that the amplitude tapers are even-symmetrical,

$$a_{2N-n+1} = a_n, \quad n = 1, 2, \dots, 2N$$

$$b_{2M-m+1} = b_m, \quad m = 1, 2, \dots, 2M,$$

and the initial phases,  $\{\phi_{onm}\}$ , and the phase perturbations,  $\{\Delta\phi_{nm}\}$ , are odd-symmetrical,

$$\phi_{o 2N-n-1, 2M-m-1} = -\phi_{onm}$$

$$\Delta\phi_{2N-n-1, 2M-m+1} = -\Delta\phi_{nm}$$

The objective of the nulling method to be described here is to find settings of the coarsely quantized phase shifters of the planar array that will impose nulls at the locations  $(u_j, 0)$ ,  $j = 1, 2, \dots, J$ , in the  $v = 0$  ( $\phi = 0^\circ$ ) pattern cut. This is done by matching the pattern of a linear array with the number of elements and inter-element spacing, equal respectively to the number of columns and inter-column spacing of the planar array; with initial amplitude taper equal to the row taper,  $\{a_n\}$ , of the planar array; and with nulls imposed at the location  $u_j$ ,  $j =$

1, 2, ..., J, using combined continuously variable, phase and amplitude control of the linear array pattern.

The unperturbed pattern,  $p_{0 \text{ lin}}(u)$ , of the linear array is given by

$$\begin{aligned} p_{0 \text{ lin}}(u) &= \sum_{n=1}^{2N} a_n e^{j\phi_{0n}} e^{jd_n u} \\ &= 2 \sum_{n=1}^{2N} a_n \cos(\phi_{0n} + d_n u) \end{aligned}$$

where  $\{\phi_{0n}\}$  are the initial phases required to scan the mainbeam in a specified direction,

$$d_n = d_{x_n} = \frac{2N-1}{2} - (n-1), \quad n = 1, 2, \dots, 2N$$

and

$$u = kd_x \sin(\theta).$$

Let  $\{\epsilon_n\}$  and  $\{\Delta\phi_n\}$  be the perturbations of the amplitudes and phases, respectively, of the linear array weights required to impose nulls at the locations  $u = u_j$ ,  $j = 1, 2, \dots, J$ . (See Appendix A for a convenient method of calculating the weight perturbations.) The perturbations satisfy the symmetry conditions

$$\epsilon_{2N-n+1} = \epsilon_n, \quad n = 1, 2, \dots, 2N$$

$$\Delta\phi_{2N-n+1} = -\Delta\phi_n, \quad n = 1, 2, \dots, 2N$$

to ensure that the perturbed pattern will be real. The perturbed pattern,  $p_{\text{lin}}(u)$ , of the linear array is then given by

$$\begin{aligned} p_{\text{lin}}(u) &= \sum_{n=1}^{2N} (a_n + \epsilon_n) e^{j(\phi_{0n} + \Delta\phi_n + d_n u)} \\ &= 2 \sum_{n=1}^N (a_n + \epsilon_n) \cos(\phi_{0n} + \Delta\phi_n + d_n u) \end{aligned}$$

Now let the  $v = 0$  pattern cut of the planar array pattern to match the perturbed linear array pattern be denoted  $p_{\text{pl}}(u, 0)$  with

$$\begin{aligned}
p_{pl}(u, 0) &= \sum_{n=1}^{2N} a_n \sum_{m=1}^{2M} b_m e^{j(\phi_{onm} + \Delta\phi_{nm} + d_n u)} \\
&= 2 \sum_{n=1}^N a_n \sum_{m=1}^{2M} b_m \cos(\phi_{onm} + \Delta\phi_{nm} + d_n u)
\end{aligned} \tag{2}$$

where the  $\{\Delta\phi_{nm}\}$  are the phase perturbations of the elements of the planar array to be determined. Equating  $p_{pl}(u, 0)$  with  $p_{lin}(u)$  and setting the initial phases equal to zero for simplicity (broadside mainbeams) then yields

$$\sum_{m=1}^{2M} b_m \cos(\Delta\phi_{nm} + d_n u) = \left(1 + \frac{\delta_n}{a_n}\right) \cos(\Delta\phi_n + d_n u)$$

or

$$\sum_{m=1}^{2M} b_m \cos \Delta\phi_{nm} = \left(1 + \frac{\delta_n}{a_n}\right) \cos \Delta\phi_n \tag{3a}$$

$$\sum_{m=1}^{2M} b_m \sin \Delta\phi_{nm} = \left(1 + \frac{\delta_n}{a_n}\right) \sin \Delta\phi_n \tag{3b}$$

In complex form, Eqs. (3a) and (3b) can be written as

$$\sum_{m=1}^{2M} b_m \exp(j \Delta\phi_{nm}) = \left(1 + \frac{\delta_n}{a_n}\right) \exp(j \Delta\phi_n) \tag{4}$$

One conclusion that can be drawn from Eq. (4) is that  $\sum_{m=1}^{2M} b_m$  must be greater than one for Eq. (4) to be satisfied for all  $n$ . For, in general, some of the  $\Delta\phi_{nm}$  will be greater than zero so that, for these values of  $n$ , the right-hand side (RHS) of Eq. (4) will have magnitude greater than unity. If  $\sum_{m=1}^{2M} b_m = 1$ , the left-hand side (LHS) of Eq. (4) can have at most unit magnitude only if all the  $\Delta\phi_{nm}$  are equal for a given value of  $n$ . Otherwise, the resulting vector in the complex plane will have magnitude less than one. Thus, for Eq. (4) to be satisfied, the  $\{b_m\}$ , initially normalized to sum to one, must be renormalized by multiplying them by a constant,  $\beta$ , greater than unity.

As a consequence of this renormalization of the column taper, the mainbeam loss of the planar array pattern that results from imposing nulls at a set of locations is greater than the loss incurred in nulling at the same locations in the corresponding linear array pattern. For, in the case of the linear array pattern, the

mainbeam loss incurred in imposing nulls is

$$\sum_{n=1}^{2N} a_n - \sum_{n=1}^{2N} (a_n + \delta_n) \cos \Delta\phi_n = 1 - \sum_{n=1}^{2N} a_n \left(1 + \frac{\delta_n}{a_n}\right) \cos \Delta\phi_n$$

while, for the planar array, the mainbeam loss is

$$\sum_{n=1}^{2N} a_n - \sum_{m=1}^{2M} b_m - \sum_{n=1}^{2N} a_n \sum_{m=1}^{2M} b_m \cos \Delta\phi_{nm}$$

$$\text{Eq. (3a)} \quad \beta - \sum_{n=1}^{2N} a_n \left(1 + \frac{\delta_n}{a_n}\right) \cos \Delta\phi_n > 1 - \sum_{n=1}^{2N} a_n \left(1 + \frac{\delta_n}{a_n}\right) \cos \Delta\phi_n$$

We now discuss the procedure used for determining the phase shifts to satisfy Eqs. (3a) and (3b), which, using Eq. (1), we write in the form

$$\sum_{m=1}^{2M} b_m \cos (i_{nm} B) = \left(1 + \frac{\delta_n}{a_n}\right) \cos \Delta\phi_n \quad (5a)$$

$$\sum_{m=1}^{2M} b_m \sin (i_{nm} B) = \left(1 + \frac{\delta_n}{a_n}\right) \sin \Delta\phi_n \quad (5b)$$

(We will here keep the renormalizing column taper constant,  $\beta$ , explicit, and use  $b_m$  to refer to the column taper normalized to sum to unity.) To choose  $\beta$ , we note that the LHS of Eq. (5a) cannot exceed  $\beta$ . Thus, if we let  $u_{\max}$  denote the maximum value of the RHS of Eq. (5a),

$$\beta \geq u_{\max}$$

Accordingly, we choose a trial value of  $\beta$  somewhat greater than  $u_{\max}$ . It is also desirable to keep the phase perturbations as small as possible so as to minimize the distortion of the initial array pattern. The LHS of Eq. (5a) must be capable of approximating the minimum value,  $u_{\min}$ , of the RHS of Eq. (5a), and, since the LHS of Eq. (5a) cannot be less than  $\beta \cos (KB)$  where  $KB$  is the largest perturbation we will allow in satisfying Eq. (5a),  $K$  must satisfy

$$\beta \cos (KB) \geq u_{\min}$$

$$K \geq \text{Int} \left\{ \frac{1}{B} \cos^{-1} \left( \frac{u_{\min}}{\beta} \right) \right\} + 1$$

Similarly, if LB is the magnitude of the largest perturbation we will allow in satisfying Eq. (5b), the magnitude of the LHS of Eq. (5b) cannot exceed  $\beta \sin(LB)$ . Hence, if the maximum magnitude of the RHS of Eq. (5b) is denoted  $v_{\max}$ ,

$$\beta \sin(LB) \geq v_{\max}$$

or

$$L \geq \text{Int} \left\{ \frac{1}{B} \sin^{-1} \left( \frac{v_{\max}}{\beta} \right) \right\} + 1$$

Having chosen values for  $\beta$ ,  $K$ , and  $L$ , we start with Eq. (5b), assume even column symmetry of the  $\{i_{nm}\}$ ,  $i_{n, 2m-m+1}$ , and let

$$S_{nm'} = \beta \sum_{m=M}^{m'} b_m \sin(i_{nm} B), \quad m' = M, M-1, \dots, 1$$

$$S_{n, M+1} = 0$$

Then, starting with  $m' = M$  (i. e., at the center of the column) and proceeding consecutively with  $m' = M - 1, M - 2, \dots$ , to  $m' = 1$ , we set  $i_{nm} = \pm L$  according to the agreement of the choice of sign with that of  $\Delta\phi_n$  and, if by so doing,

$$|S_{nm'}| = |S_{n, m'+1} + \beta b_{m'} \sin(i_{nm'} B)| \leq \frac{1 + \frac{\delta_n}{a_n}}{2} \sin \left[ \frac{\Delta\phi_n}{n} \right] \quad (6)$$

If, with  $|i_{nm'}| = L$ ,

$$|S_{nm'}| > \frac{1}{2} \left( 1 + \frac{\delta_n}{a_n} \right) \sin \left[ \frac{\Delta\phi_n}{n} \right]$$

we set  $i_{nm} = \pm(L - 1)$  and again check to see if (6) is satisfied. If not, we again decrease  $|i_{nm}|$  and so on until  $|i_{nm}| = 1$ . If  $|S_{nm'}|$  fails to satisfy the inequality (6) when  $i_{nm} = \pm 1$ , we leave  $i_{nm} = 0$  (that is, no phase perturbation) and proceed to the next value of  $m'$ . Thus, we attempt to satisfy Eq. (5b) as closely as possible by summing terms of the same sign as the RHS of Eq. (5b).

starting with the largest magnitude terms and working down to the smallest magnitude terms. The column amplitude taper is essential for the algorithm to work well since the small amplitude terms at the edge of the column are used to "fine tune" the fit of the LHS of Eq. (5b) to the RHS. The more pronounced the taper, the closer the fit that can be made.

Having satisfied Eq. (5b) as closely as possible for a given  $n$ , we then turn to Eq. (5a). As a result of the settings of the  $\{i_{nm}\}$  made in satisfying Eq. (5b), the LHS of Eq. (5a) has some value, in general not equal to the RHS. To adjust the  $\{i_{nm}\}$  so that the LHS of Eq. (5a) is made to agree with the RHS without disturbing the fit already made to Eq. (5b), we change only those values of the  $\{i_{nm}\}$  left equal to zero in satisfying Eq. (5b), and when the  $nm^{\text{th}}$  phase shifter is set equal to  $i_{nm}B$ , the symmetrically placed phase shifter in the column (index  $n, 2M - m + 1$ ) is set equal to  $-i_{nm}B$  instead of being set equal to  $i_{nm}B$  as when satisfying Eq. (5b). Thus,  $\sum_m b_m \sin(i_{nm}B)$  is left unchanged. Let

$$T_{n, M+1} = \sum_{m=1}^M b_m \cos(i_{nm}B)$$

after satisfying Eq. (5b), and let

$$T_{nm'} = T_{n, M-1} - \sum_{m=M}^{m'} b_m [\cos(i_{nm}B) - 1], \quad m' = M, M-1, \dots, 1,$$

where the summation is over only those indices for which  $i_{nm} = 0$  after satisfying Eq. (4b). Then, if  $i_{nm'}$  was left equal to zero after satisfying Eq. (5b), we now set  $i_{nm'} = \pm K$  according to the agreement of the choice of sign with that of  $\Delta\phi_n$  and, if by so doing,

$$T_{nm'} = T_{n, m'-1} - b_{m'} [\cos(i_{nm'}B) - 1] = \frac{1 + \frac{\delta_n}{a_n}}{2} \cos \Delta\phi_n \quad (7)$$

If, with  $i_{nm'} = K$ ,

$$T_{nm'} < \frac{1 + \frac{\delta_n}{a_n}}{2} \cos \Delta\phi_n$$

we set  $i_{nm'} = \pm(K - 1)$  and again check to see if Eq. (7) is satisfied. If not, we again decrease  $i_{nm'}$  and so on until  $i_{nm'} = 1$ . If  $T_{nm'}$  fails to satisfy the inequality (7) when  $i_{nm'} = \pm 1$ , we leave  $i_{nm'}$  unchanged at zero and proceed to

the next value of  $m'$ . In other words, we try to satisfy Eq. (5a) working downward from above the RHS in contrast to the procedure used to satisfy Eq. (5b), which works upward from below the RHS. When Eqs. (5a) and (5b) have been satisfied as closely as possible for all  $n$  given the initial choice of  $\beta$ ,  $K$ , and  $L$ , we calculate the depth of null at the locations  $(u_j, 0)$ ,  $j = 1, 2, \dots, J$ , using Eqs. (2) and (1) with the  $\{b_m\}$  renormalized to sum to  $\beta$ . The entire procedure can then be repeated with a different choice of  $\beta$ ,  $K$ , and  $L$  if desired to see if any improvement in nulling can thereby be obtained.

Although the nulling algorithm we have described here is designed to match a perturbed linear array pattern obtained using combined phase and amplitude perturbations, it can also be used when phase-only perturbations are used to solve the linear array nulling problem, the situation for which the multi-null algorithm described by Franchi and Shore<sup>2</sup> was designed. Since the algorithm given here allows planar array phase shifts greater than  $B$  in magnitude, it is more flexible than the algorithm for which phase shifts are restricted to be not greater than  $B$  in magnitude.<sup>2</sup> (When  $K$  and  $L$  are set equal to 1, the two algorithms are essentially the same.) Applied to the same problem as the algorithm of Franchi and Shore,<sup>2</sup> the algorithm given here yields superior results especially when the number of bits in the planar array phase shifters exceeds 4 or 5 and it becomes difficult to satisfy Eqs. (5a) and (5b) if  $K$  and  $L$  equal 1.

### 3. RESULTS

In this section, we present the results of calculations performed with the nulling algorithm described in Section 2. The first example is that of imposing five nulls in the  $\phi = 0^\circ$  pattern cut--at 3, 6, 9, 12, and 15 degrees--of a 100-column planar array. The inter-column and inter-row spacing of the array is taken to be a half wavelength, and a 40 dB,  $\bar{n} = 6$  Taylor distribution is used for the row and column amplitude tapers. Table 1 summarizes the results obtained when the planar array has 100 rows (i.e., a 100 x 100 element array). The columns of the table give, respectively, the number of bits in the planar-array phase shifters, the largest phase perturbations (in units of  $B = (2\pi)/2^{\text{NBIT}}$ ) allowed in satisfying Eqs. (5a) and (5b), the value of the multiplicative normalization constant  $\beta$ , the shallowest null depth among the five null locations, the average power at the null locations of the perturbed and unperturbed patterns, the average power reduction at the null locations, and the statistical ensemble average of the power at a null location of the pattern if phase errors are uniformly distributed in the interval

Table 1. Summary of Results Obtained for a 100 x 100 Element Array With 40dB  $\bar{n} = 6$  Taylor Row and Amplitude Tapers and Nulls Imposed in the  $\phi = 0^\circ$  Pattern Cut at 3, 6, 9, 12, and 15 Degrees

N BIT	K	L	$\beta$	Shallowest Null Depth (dB)	Average Perturbed Power (dB)	Average Unperturbed Power (dB)	Reduction (dB)	Statistical Average Null Depth (dB)
2	1	1	1.092	-71.8	-73.9	-44.4	29.5	-40.0
3	1	1	1.081	-74.3	-78.8	-44.5	34.3	-44.9
4	2	1	1.023	-76.5	-82.1	-45.0	37.1	-50.7
5	3	2	1.084	-87.2	-91.2	-44.5	46.7	-56.6
6	6	3	1.067	-94.6	-97.5	-44.6	52.9	-62.6

$[-B/2, +B/2]$ . This value is computed from the formula<sup>3</sup>

$$\frac{|F(u, v)|^2}{\left(\sum_n \sum_m |w_{nm}| \right)^2} = \left[ 1 - \left(\frac{\sin B}{B}\right)^2 \right] \frac{\sum_n \sum_m |w_{nm}|^2}{\left(\sum_n \sum_m |w_{nm}| \right)^2}$$

with  $|F(u, v)|^2$  the statistical ensemble average power at a location in the pattern for which the error-free pattern has a null. The significance of the statistical ensemble average power at a null location is that this is the depth of null that might be expected if nothing were done to correct for the quantization of the phase shifters. The tabulated values of K, L, and  $\beta$ , were those found empirically, after a thorough though not exhaustive search, to give the deepest shallowest null depth among the five imposed null locations.

It is apparent from the table that deep nulls can be formed with coarsely quantized phase shifters by our method of treating the columns of the planar array as single weights for a linear array and by compensating for the small number of bits with the number of phase shifters available for control in the columns. Thus, for example, with 2-bit phase shifters, the planar array is able to form nulls with an average depth of -74 dB as compared with the -40 dB nulls that would be expected if nothing were done to compensate for the small number of bits. Since the statistical average null depth decreases by approximately 6 dB for each extra phase

3. Shore, R.A. (1982) Statistical Analysis of the Effect of Phase Quantization on Array Antenna Patterns, RADC-TR-82-190, AD A123704.

shifter bit, the -74 dB null level obtained with 2-bit phase shifters is equivalent to that obtainable with 8-bit phase shifters if no compensating method is used.

To examine the effect of smaller column size on the performance of the nulling algorithm, calculations were performed with the same parameters as described above in generating Table 1 except that the column size was reduced from 100 to 20 elements. The results for these computations are summarized in Table 2. It is apparent that the smaller column size reduces the ability of the algorithm to produce deep nulls at the desired locations. This is, of course, only as expected, since the fewer the phase shifters that are available in the columns of the array, the less will be the ability of the algorithm to find appropriate choices of the  $\{i_{nm}\}$  to satisfy Eqs. (5a) and (5b).

Table 2. Summary of Results Obtained for a 100 x 20 Element Array With 40 dB,  $\bar{n} = 6$  Taylor Row and Amplitude Tapers and Nulls Imposed in the  $\phi = 0^\circ$  Pattern Cut at 3, 6, 9, 12, and 15 Degrees

N BIT	K	L	$\epsilon$	Shallowest Null Depth (dB)	Average Perturbed Power (dB)	Average Unperturbed Power (dB)	Reduction (dB)	Statistical Average Null Depth (dB)
2	1	1	1.072	-52.4	-55.4	-44.6	10.8	-33.0
3	1	2	1.054	-52.7	-56.8	-44.8	12.0	-37.9
4	3	1	1.052	-61.1	-65.0	-44.8	20.2	-43.7
5	4	1	1.080	-64.4	-67.4	-44.5	22.9	-49.6
6	6	2	1.053	-69.6	-73.9	-44.8	29.1	-55.6

As an example of the patterns obtained with the nulling method, we show in Figure 1 the  $v = 0$  ( $\phi = 0^\circ$ ) perturbed pattern cut of the 100 x 100 element planar array with 4-bit phase shifters and with nulls imposed at 3, 6, 9, 12, and 15 degrees. This pattern is superimposed on the perturbed linear array pattern (with nulls imposed at the same five locations) that the planar array nulling algorithm is designed to match. The two patterns are virtually indistinguishable from one another, and are very close to the unperturbed 100 element 40 dB,  $\bar{n} = 6$  Taylor pattern apart from the vicinity of the imposed null locations. In contrast, Figure 2 shows the unperturbed (40 dB,  $\bar{n} = 6$  Taylor) and perturbed planar array  $u = 0$  pattern cuts. The perturbed pattern has a sidelobe level considerably greater than that of the unperturbed pattern. The marked difference between the behavior of the  $v = 0$  and  $u = 0$  pattern cuts is the result of the fact that the algorithm we have used

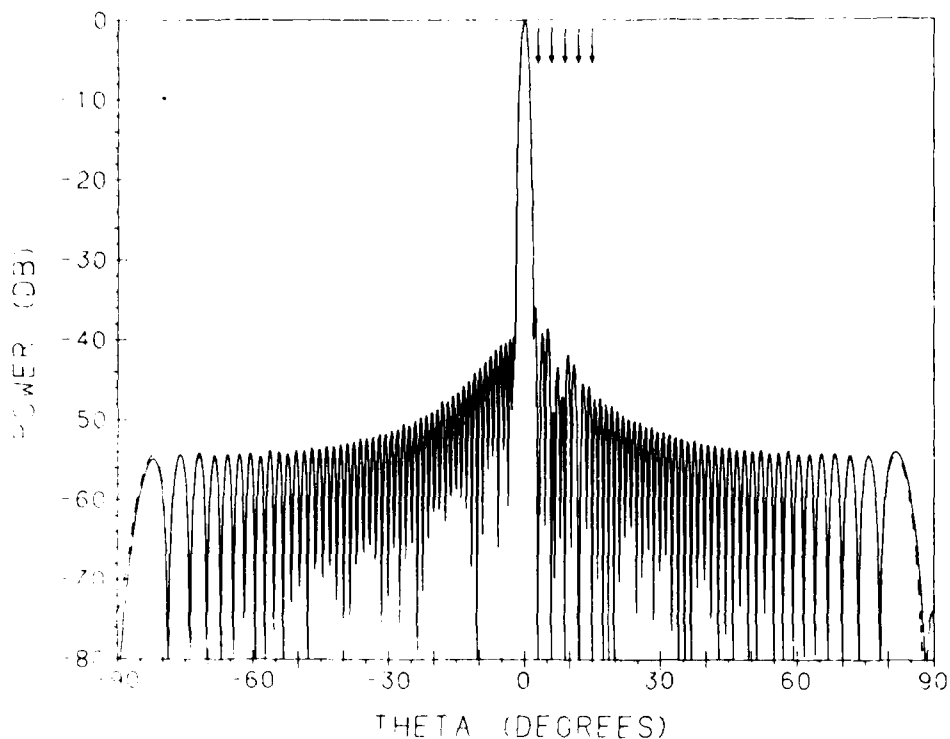


Figure 1. Comparison of Perturbed  $v = 0$  Pattern (—) of  $100 \times 100$  Element Array With 40 dB,  $\bar{n} = 6$  Taylor Amplitude Tapers and 4-bit Phase Shifters, With Linear Array Pattern (----) With Nulls Imposed at 3, 6, 9, 12, and 15 Degrees Using Continuously Variable, Combined Phase and Amplitude Control

is concerned solely with matching the  $v = 0$  pattern cut with the given pattern of a minimally perturbed linear array, and it contains no safeguards for the integrity of the  $u = 0$  pattern cut.

Figures 3 and 4 show the patterns corresponding to Figures 1 and 2 when the planar array contains 2-bit instead of 4-bit phase shifters. The  $v = 0$  perturbed planar array pattern cut is very close to the model perturbed linear array pattern, but the phase perturbations used to match the linear array pattern result in a  $u = 0$  pattern cut that is widely distorted as compared with the unperturbed pattern.

The second example for which computations were performed was that of imposing six nulls in three symmetrically located pairs— $\pm 4$ ,  $\pm 8$ , and  $\pm 12$  degrees—in the  $\phi = 0^\circ$  pattern cut of a 100-column planar array. As with the first example, half wavelength interelement spacing and a 40 dB,  $\bar{n} = 6$  Taylor distribution for the row and column amplitude tapers were used. Tables 3 and 4 summarize the results obtained when the columns of the planar array have 100 and 20 elements, respectively. It will be noticed that by comparing Table 3 with Table 1 or compar-

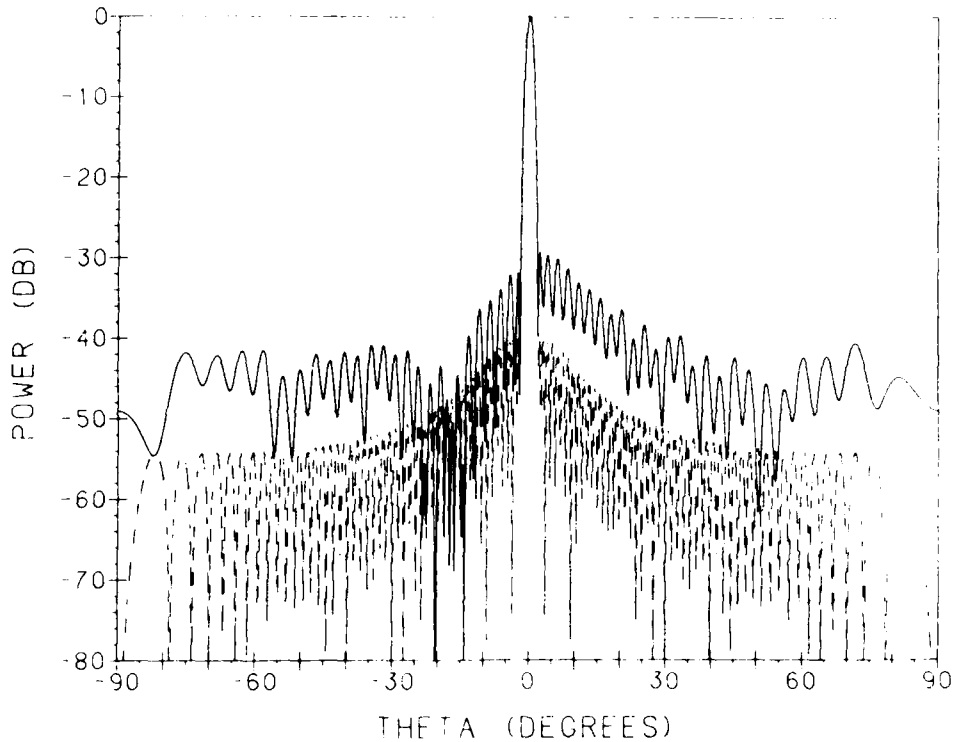


Figure 2. Unperturbed  $u = 0$  Pattern (-----) of 100 x 100 Element Array With 40dB,  $\bar{n} = 6$  Taylor Amplitude Tapers, and Perturbed Pattern (——) Corresponding to Nulls Imposed in the  $v = 0$  Pattern at 3, 6, 9, 12, and 15 Degrees. NBIT = 4

ing Table 4 with Table 2, the null depths obtained for this example are considerably deeper than those obtained for the five-null example. The explanation for this is that, in the linear array nulling problem, the solution to which the planar array  $\psi = 0^\circ$  pattern cut is matched, all the phase perturbations are identically zero because of the symmetry of the problem. This means that the RHS of Eq. (5b) is zero for all  $n$ , so that Eq. (5b) can be exactly satisfied initially by the algorithm by taking all the  $\{i_{nm}\}$  to be zero. This leaves all the phase shifters in the column available for satisfying Eq. (5a). Hence, in effect, the algorithm has only one of the two equations to satisfy, the result being the deeper null depths noted. It is thus interesting that imposing nulls at pattern locations that are symmetrically placed relative to the mainbeam is actually simpler to accomplish with phase-only control in planar arrays than is imposing nulls at not symmetrically placed locations. This is the opposite of the situation for linear arrays which re-

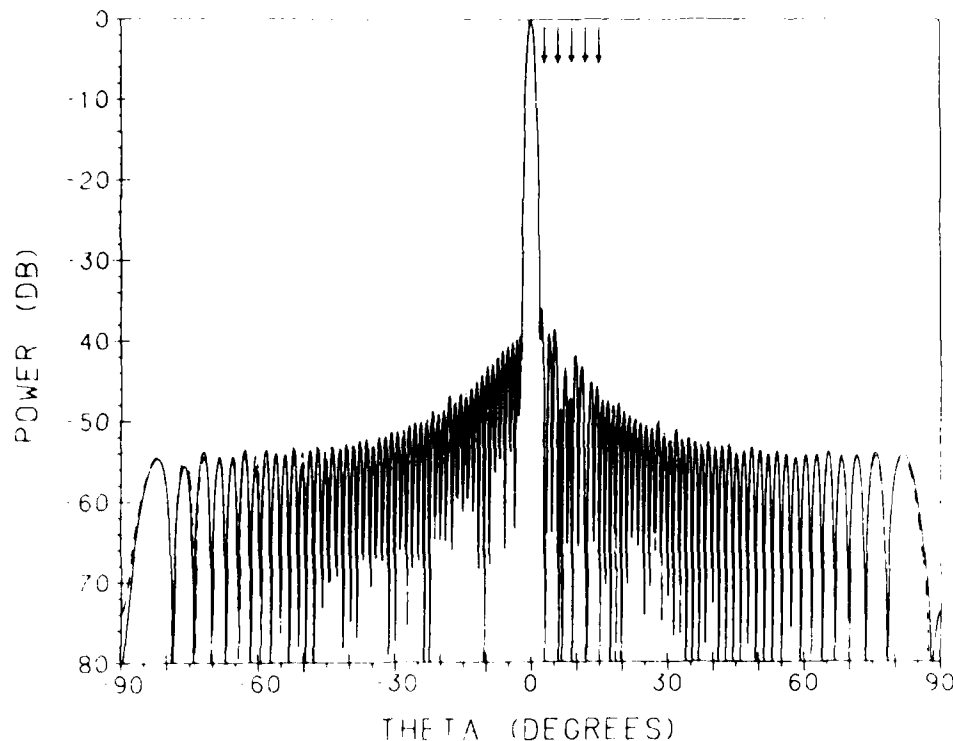


Figure 3. Comparison of Perturbed  $v = 0$  Pattern (—) of  $100 \times 100$  Element Array With 40dB,  $\bar{n} = 6$  Taylor Amplitude Tapers and 2-bit Phase Shifters, With Linear Array Pattern (-----) With Nulls Imposed at 3, 6, 9, 12, and 15 Degrees Using Continuously Variable, Combined Phase and Amplitude Control

quire large phase perturbations (that result in large pattern distortion) to impose nulls at symmetrically placed locations.<sup>4</sup>

Figure 5 shows a comparison of the model perturbed 100-element linear array pattern and the  $v = 0$  pattern cut of a  $100 \times 100$  element planar array with 2-bit phase shifters and nulls imposed at  $\pm 4$ ,  $\pm 8$ , and  $\pm 12$  degrees. The two patterns cannot be distinguished from one another. Figure 6 shows the corresponding  $u = 0$  pattern cut of the  $100 \times 100$  element, 2-bit phase shifter, planar array along with the unperturbed  $u = 0$  pattern. As with the five-null example, the perturbed pattern has, in general, a much higher sidelobe level than the unperturbed pattern.

4. Shore, R.A. (1984) Nulling at symmetric pattern location with phase-only weight control, IEEE Trans. Antennas Propag. AP-32:530-533.

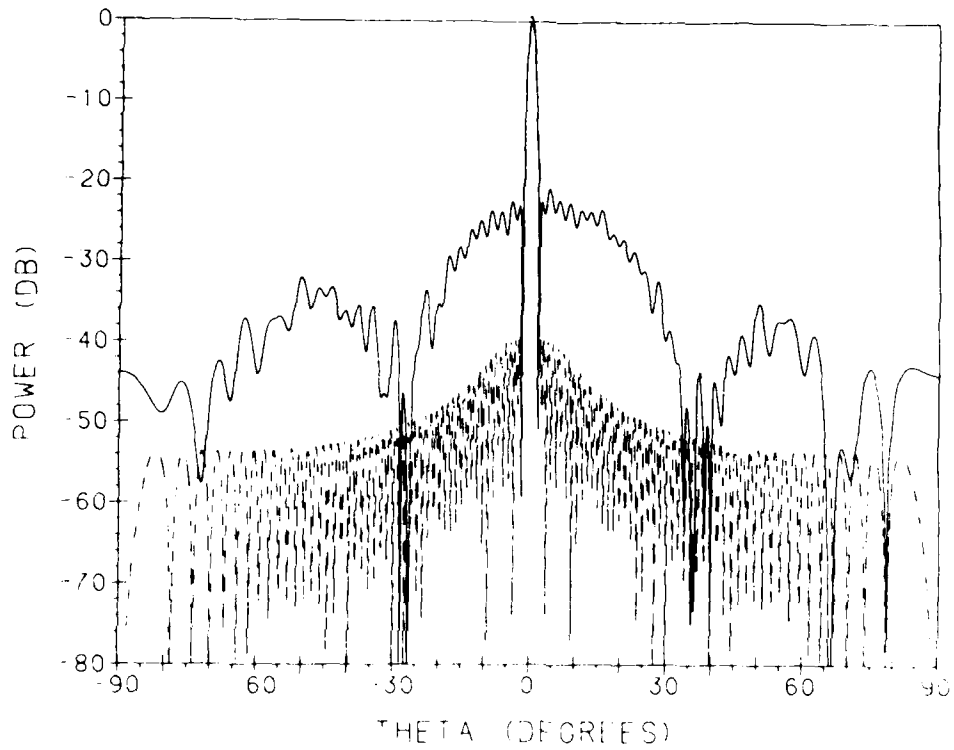


Figure 4. Unperturbed  $u = 0$  Pattern (-----) of  $100 \times 100$  Element Array With  $40\text{dB}$ ,  $\bar{\pi} = 6$  Taylor Amplitude Tapers, and Perturbed Pattern (——) Corresponding to Nulls Imposed in the  $v = 0$  Pattern at  $3, 6, 9, 12,$  and  $15$  Degrees.  $\text{NBIT} = 2$

Table 3. Summary of Results Obtained for a 100 x 100 Element Array With 40dB,  $\bar{n} = 6$  Taylor Row and Amplitude Tapers and Nulls Imposed in the  $\phi = 0^\circ$  Pattern Cut at -12, -8, -4, 4, 8, and 12 Degrees

N BIT	K	L	$\epsilon$	Shallowest Null Depth (dB)	Average Perturbed Power (dB)	Average Unperturbed Power (dB)	Reduction (dB)	Statistical Average Null Depth (dB)
2	1	1	1.054	-91.5	-96.1	-44.0	52.1	-40.0
3	1	1	1.044	-95.6	-99.8	-44.1	55.7	-44.9
4	2	1	1.034	-112.9	-115.5	-44.1	71.4	-50.7
5	4	1	1.035	-119.3	-123.5	-44.1	79.4	-56.6
6	8	1	1.035	-129.0	-131.7	-44.1	87.6	-62.6

Table 4. Summary of Results Obtained for a 100 x 20 Element Array With 40dB,  $\bar{n} = 6$  Taylor Row and Amplitude Tapers and Nulls Imposed in the  $\phi = 0^\circ$  Pattern Cut at -12, -8, -4, 4, 8, and 12 Degrees

N BIT	K	L	$\epsilon$	Shallowest Null Depth (dB)	Average Perturbed Power (dB)	Average Unperturbed Power (dB)	Reduction (dB)	Statistical Average Null Depth (dB)
2	1	1	1.023	-65.8	-67.0	-44.2	22.8	-33.0
3	1	1	1.010	-82.8	-85.3	-44.2	41.1	-37.9
4	2	1	1.039	-92.0	-96.0	-44.1	51.9	-43.7
5	6	1	1.038	-106.7	-107.7	-44.1	63.6	-49.6
6	8	1	1.033	-118.9	-122.1	-44.1	78.0	-55.7

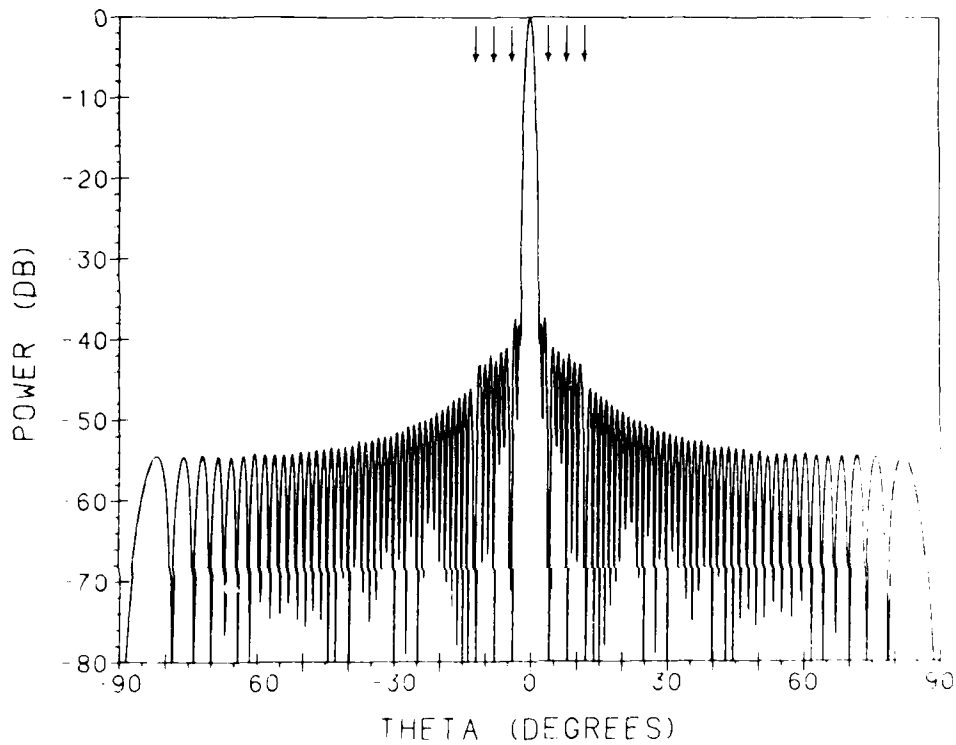


Figure 5. Comparison of Perturbed  $v = 0$  Pattern (—) of  $100 \times 100$  Element Array With 40 dB,  $\bar{n} = 6$  Taylor Amplitude Tapers and 2-Bit Phase Shifters, With Linear Array Pattern (-----) With Nulls Imposed at -12, -8, -4, 4, 8, and 12 Degrees Using Continuously Variable, Combined Phase and Amplitude Control

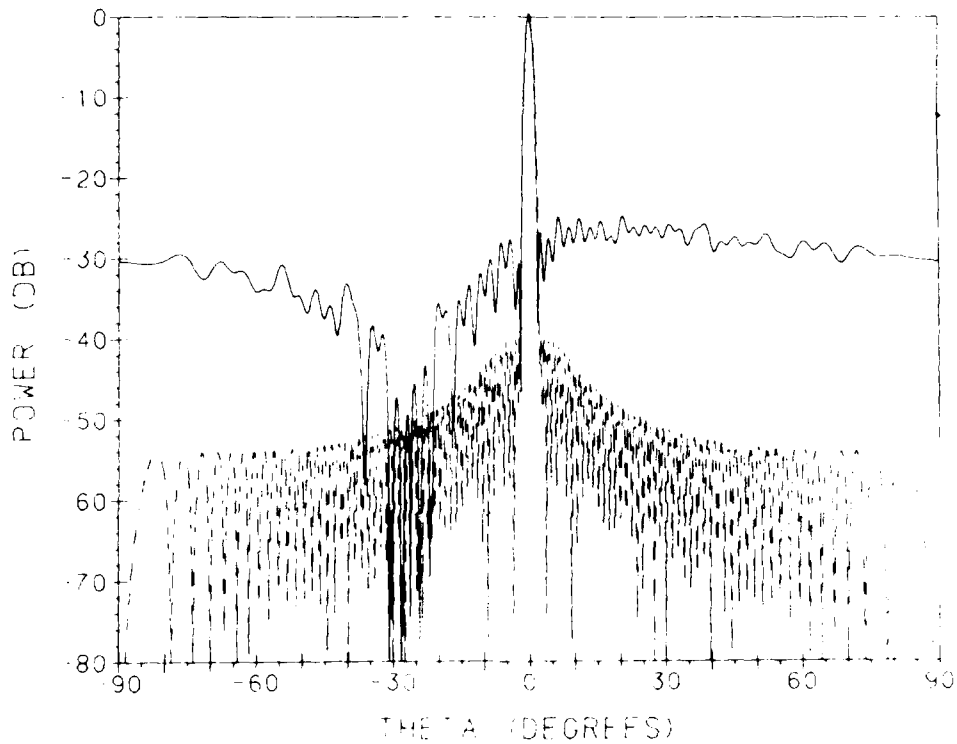


Figure 6. Unperturbed  $u = 0$  Pattern (-----) of 100 x 100 Element Array With 40dB,  $\bar{n} = 6$  Taylor Amplitude Tapers, and Perturbed Pattern (——) Corresponding to Nulls Imposed in the  $v = 0$  Pattern at -12, -8, -4, 4, 8, and 12 Degrees. NBIT = 2

## References

1. Shore, R. A. (1985) A Review of Phase-Only Sidelobe Nulling Investigations at RADC, RADC-TR-85-145, AD A166602.
2. Franchi, P. R., and Shore, R. A. (1986) Phase-Only Nulling With Coarsely Quantized Phase Shifters, RADC-TR-86-127, AD A175768.
3. Shore, R. A. (1982) Statistical Analysis of the Effect of Phase Quantization on Array Antenna Patterns, RADC-TR-82-190, AD A123704.
4. Shore, R. A. (1984) Nulling at symmetric pattern location with phase-only weight control, IEEE Trans. Antennas Propag. AP-32:530-533.
5. Rao, C. R., and Mitra, S. K. (1971) Generalized Inverse of Matrices and Its Applications, Wiley, New York.

## Appendix A

### Combined Phase and Amplitude Nulling in Linear Arrays

In this appendix, we obtain expressions for the perturbations, minimized in a least squares sense, of the weights of a linear array that will impose nulls in the pattern at prescribed locations. We consider an array of  $N$  isotropic, equispaced elements with inter-element spacing  $d$ . Let  $w_{on}$ ,  $n = 1, 2, \dots, N$ , be the initial weights and  $\Delta w_n$ ,  $n = 1, 2, \dots, N$  the complex weight perturbations required to impose nulls at the locations  $u_p$ ,  $p = 1, 2, \dots, J$ , subject to the condition that

$$\sum_{n=1}^N |\Delta w_n|^2 \tag{A1}$$

be minimized. The unperturbed field pattern,  $p_o(u)$ , is given by

$$p_o(u) = \sum_{n=1}^N w_{on} e^{jd_n u}$$

and the perturbed pattern,  $p(u)$ , by

$$p(u) = \sum_{n=1}^N (w_{on} + \Delta w_n) e^{jd_n u} = p_o(u) + \sum_{n=1}^N \Delta w_n e^{jd_n u}$$

with

$$\begin{aligned} d_n &= (N - 1)/2 - (n - 1), \quad n = 1, 2, \dots, N \\ u &= kd \sin(\theta) \\ k &= \text{wavenumber} = (2\pi)/\lambda \end{aligned}$$

and

$\theta$  = pattern angle measured from broadside to the array.  
For nulls to be imposed at  $\theta_j$ ,  $j = 1, 2, \dots, J$ , we have

$$p(u_j) = 0, \quad j = 1, 2, \dots, J$$

or

$$\sum_{n=1}^N \Delta w_n e^{jd_n u_j} = p_0(u_j), \quad j = 1, 2, \dots, J \quad (A2)$$

In matrix form, the system of Eqs. (A2) can be written

$$\underline{A} \underline{\Delta w} = \underline{p_0}$$

where

$$\underline{A} = \begin{bmatrix} e^{jd_1 u_1} & e^{jd_2 u_1} & \dots & e^{jd_N u_1} \\ \vdots & \vdots & & \vdots \\ e^{jd_1 u_J} & e^{jd_2 u_J} & \dots & e^{jd_N u_J} \end{bmatrix}$$

$$\underline{\Delta w} = [\Delta w_1, \Delta w_2, \dots, \Delta w_N]^T$$

and

$$\underline{p_0} = [p_0(u_1), p_0(u_2), \dots, p_0(u_J)]^T$$

The solution to Eq. (A3) that minimizes Eq. (A1) is then<sup>5</sup>

$$\underline{\Delta w} = -\underline{A}^\dagger (\underline{A}\underline{A}^\dagger)^{-1} \underline{p_0}$$

5. Rao, C.R., and Mitra, S.K. (1971) Generalized Inverse of Matrices and Its Applications, Wiley, New York.

*MISSION*  
*of*  
*Rome Air Development Center*

*RADC plans and executes research, development, test and selected acquisition programs in support of Command, Control, Communications and Intelligence (C<sup>3</sup>I) activities. Technical and engineering support within areas of competence is provided to ESD Program Offices (POs) and other ESD elements to perform effective acquisition of C<sup>3</sup>I systems. The areas of technical competence include communications, command and control, battle management, information processing, surveillance sensors, intelligence data collection and handling, solid state sciences, electromagnetics, and propagation, and electronic, maintainability, and compatibility.*

ATE  
LMED  
8