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MODEL SELECTION FOR THE MULTIPLE MODEL ADAPTIVE
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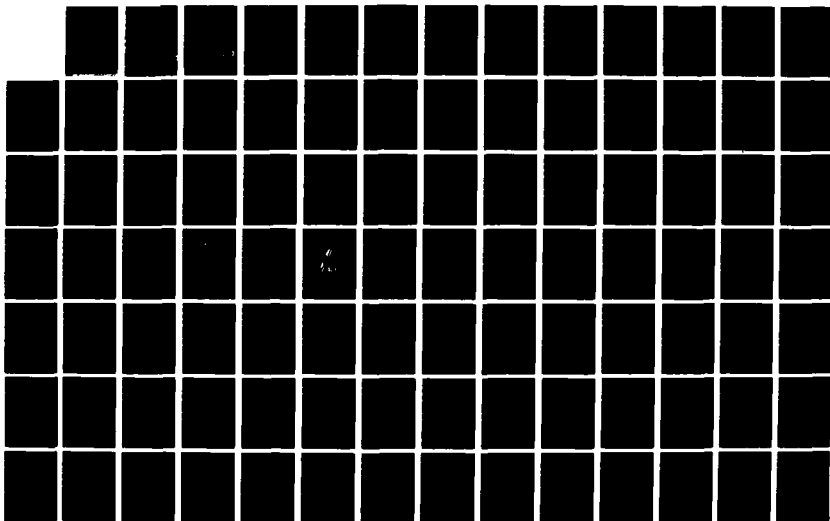
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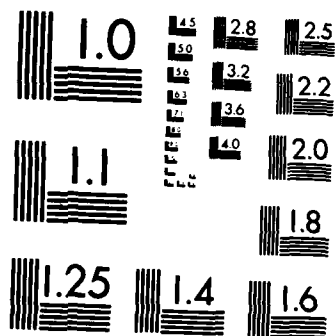
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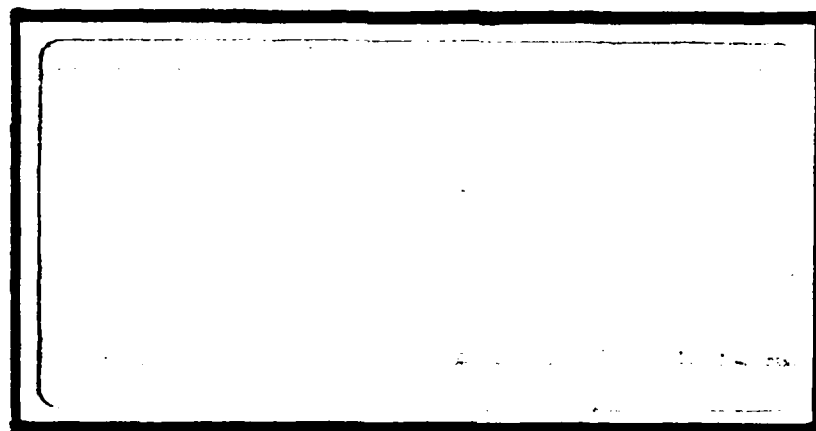
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THESIS

James R. Matthes, Jr.
Captain, USAF

AFIT/GE/ENG/87D-40

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Electrical Engineering

James R. Matthes, Jr., B.S.

Captain, USAF

December 1987

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Acknowledgments

The thesis represents the culmination of my AFIT experience. This effort was accomplished to a large degree by depriving myself of one of my favorite recreational pursuits -- sleep. Being cognizant of my limited ability, I realize that without the guidance and suggestions of my peers and professors this effort would have not been possible.

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Heartfelt thanks are also due to my wonderful wife, Helene, and my two precious children, Noelle and Aaron. Helene constantly provided me with the love, encouragement, and understanding that seemed to ease the pain associated with the AFIT experience while Noelle and Aaron were always there to keep my focus on the important matters of life.

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-- Captain James R. Matthes, Jr.

Table of Contents

	<u>Page</u>
Acknowledgments	ii
List of Figures	vii
List of Tables	xxii
Abstract	xxiv
1. Introduction	1
1.1 Overview	1
1.2 Background	2
1.3 Problem Statement	3
1.4 Summary of Current Knowledge	4
1.4.1 In-Flight Simulation	4
1.4.2 Adaptive Control	6
1.4.3 MMAE	11
1.5 Assumptions	12
1.6 Approach	13
1.7 Overview	15
2. Aircraft Description and Models	16
2.1 AFTI/F-16 Description	16
2.2 Aircraft Models	20
2.2.1 State Space Model	20
2.2.2 Autoregressive Difference Equation Model	26
2.3 Actuator Model	28
2.4 Summary	29
3. Control Law Algorithm	30
3.1 Introduction	30
3.2 Model Following Techniques	30
3.3 Porter's Control Law	34
3.3.1 Fixed Gain Controller Matrices	37
3.3.2 Adaptive Controller Matrices	43
3.4 Summary	45

4.	Parameter Estimation	47
4.1	Introduction	47
4.2	Multiple Model Algorithm	48
4.2.1	Prediction Error	53
4.2.2	Prediction Error Covariance	54
4.3	Multiple Model Parameter Estimator	55
4.4	Summary	59
5.	Model Selection Criteria	60
5.1	Introduction	60
5.2	Performance Boundaries	61
5.3	Sensor Noise Effects	67
5.4	Summary	71
6.	Simulation Setup and Results	73
6.1	Simulation Setup	73
6.2	Simulation Results	77
6.2.1	Single Model Analysis	77
6.2.1.1	Performance Boundary Evaluation	77
6.2.1.2	Control Surface Activity ...	101
6.2.2	Two-Model Configurations	106
6.2.3	Three-Model Configurations	123
6.2.4	Control Law Gain Adjustment	150
6.2.5	Sensor Noise Effects	175
6.3	Summary	215
7.	Conclusions and Recommendations	217
7.1	Introduction	217
7.2	Conclusions	217
7.3	Recommendations for Further Study	221
	Appendix A	223
	Appendix B	242
	Appendix C	247
	Appendix D	251

Bibliography	260
Vita	263

List of Figures

<u>Figure</u>		<u>Page</u>
1-1	Parameter Adaptive Control System	9
2-1	The AFTI/F-16	17
2-2	AFTI/F-16 Control Surfaces and Definition of Positive Deflections	19
2-3	Positive Directions of Forces, Moments, and Angles in the Body Reference Frame	22
2-4	Flight Envelope of Interest	23
3-1	Explicit Model Following System	32
3-2	Improved Explicit Model Following System	33
3-3	System Block Diagram - Discrete Case	38
4-1	Multiple Model Parameter Estimator	56
5-1	Region of Performance Robustness	63
5-2	Intersection of Performance Boundaries	67
5-3	Noise Effects on Model Selection	72
6-1	Top Level Description of Parameter Adaptive System Simulation Package	74
6-2	Flight Path Angle Reference Tracking Signal ...	76
6-3	Pitch Rate Reference Tracking Signal	76
6-4	Pitch Rate Response Nominal Model: 18K 0.45M Operating Point: 18K 0.45M	79

6-5	Pitch Rate Performance Criterion	
	Nominal Model: 10K 0.45M	
	Operating Point: 18K 0.45M	79
6-6	Flaperon Position	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.45M	80
6-7	Elevator Position	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.45M	80
6-8	Flaperon Rate	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.45M	81
6-9	Elevator Rate	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.45M	81
6-10	Pitch Rate Response	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	83
6-11	Pitch Rate Performance Criterion	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	83
6-12	Flaperon Position	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	84
6-13	Elevator Position	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	84
6-14	Flaperon Rate	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	85
6-15	Elevator Rate	
	Nominal Model: 18K 0.45M	
	Operating Point: 18K 0.8M	85

6-16	Performance Boundary for Nominal Model at 18K 0.45M	88
6-17	Performance Boundary for Nominal Model at 38K 0.7M	89
6-18	Pitch Rate Response Nominal Model: 18K 0.45M Operating Point: 26K 0.45M	93
6-19	Pitch Rate Performance Criterion Nominal Model: 18K 0.45M Operating Point: 26K 0.45M	93
6-20	Flaperon Position Nominal Model: 18K 0.45M Operating Point: 26K 0.8M	94
6-21	Elevator Position Nominal Model: 18K 0.45M Operating Point: 26K 0.45M	94
6-22	Flaperon Rate Nominal Model: 18K 0.45M Operating Point: 26K 0.45M	95
6-23	Elevator Rate Nominal Model: 18K 0.45M Operating Point: 26K 0.45M	95
6-24	Pitch Rate Response Nominal Model: 38K 0.7M Operating Point: 26K 0.45M	96
6-25	Pitch Rate Performance Criterion Nominal Model: 38K 0.7M Operating Point: 26K 0.45M	96
6-26	Flaperon Position Nominal Model: 38K 0.7M Operating Point: 26K 0.45M	97

6-27	Elevator Position	
	Nominal Model: 38K 0.7M	
	Operating Point: 26K 0.45M	97
6-28	Flaperon Rate	
	Nominal Model: 38K 0.7M	
	Operating Point: 26K 0.45M	98
6-29	Elevator Rate	
	Nominal Model: 38K 0.7M	
	Operating Point: 26K 0.45M	98
6-30	Lift Curve	101
6-31	Performance Boundaries for Configuration 1	108
6-32	Performance Boundaries for Configuration 2	109
6-33	Performance Boundaries for Configuration 3	110
6-34	Performance Boundaries for Configuration 4	111
6-35	Performance Boundaries for Configuration 5	112
6-36	Pitch Rate Response/Configuration 2	
	Operating Point: 14K 0.35M	114
6-37	Pitch Rate Performnace Criterion	
	Configuration 2	
	Operating Point: 14K 0.35M	114
6-38	Model Probability Weightings	
	Configuration 2	
	Operating Point: 14K 0.35M	115
6-39	Pitch Rate Response/Configuration 4	
	Operating Point: 14K 0.9M	116
6-40	Pitch Rate Performance Criterion	
	Configuration 4	
	Operating Point: 14K 0.9M	116

6-41	Model Probability Weightings Configuration 4 Operating Point: 14K 0.9M	117
6-42	Pitch Rate Response/Configuration 3 Operating Point: 10K 0.9M	118
6-43	Pitch Rate Performance Criterion Configuration 3 Operating Point: 10K 0.9M	118
6-44	Model Probability Weightings Configuration 3 Operating Point: 10K 0.9M	119
6-45	Performance Boundary Overlap	122
6-46	Three Model Configurations	125
6-47	Performance Boundaries for Configuration 6	126
6-48	Performance Boundaries for Configuration 7	127
6-49	Performance Boundaries for Configuration 8	128
6-50	Model Selection Data for Configuration 6	142
6-51	Model Selection Data for Configuration 7	143
6-52	Model Selection Data for Configuration 8	144
6-53	Model Probability Weightings Configuration 6 Operating Point: 26K 0.45M	145
6-54	Model Probability Weightings Configuration 8 Operating Point: 26K 0.65M	147
6-55	Pitch Rate Response Nominal Model: 22K 0.65M Operating Point: 22K 0.65M SIG1=0.4 SIG2=0.7 RHO=0.8	153

6-56	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.8	153
6-57	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.8	154
6-58	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.8	154
6-59	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.8	155
6-60	Elevator Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.8	155
6-61	Pitch Rate Response	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	158
6-62	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	158
6-63	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	159
6-64	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	159

6-65	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	160
6-66	Elevator Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.7 RHO=0.4	160
6-67	Pitch Rate Response	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	161
6-68	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	161
6-69	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	162
6-70	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	162
6-71	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	163
6-72	Elevator Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.8 SIG2=0.7 RHO=0.8	163

6-73	Pitch Rate Response	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	164
6-74	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	164
6-75	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	165
6-76	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	165
6-77	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	166
6-78	Elevator Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.1 SIG2=0.7 RHO=0.8	166
6-79	Pitch Rate Response	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	168
6-80	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	168
6-81	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	169

6-82	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	169
6-83	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	170
6-84	Elevator Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=1.5 RHO=0.8	170
6-85	Pitch Rate Response	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.1 RHO=0.8	171
6-86	Pitch Rate Performance Criterion	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.1 RHO=0.8	171
6-87	Flaperon Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.1 RHO=0.8	172
6-88	Elevator Position	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.1 RHO=0.8	172
6-89	Flaperon Rate	
	Nominal Model: 22K 0.65M	
	Operating Point: 22K 0.65M	
	SIG1=0.4 SIG2=0.1 RHO=0.8	173

6-90	Elevator Rate Nominal Model: 22K 0.65M Operating Point: 22K 0.65M SIG1=0.4 SIG2=0.1 RHO=0.8	173
6-91	Pitch Rate Response Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	176
6-92	Pitch Rate Performance Criterion Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	176
6-93	Flaperon Position Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	177
6-94	Elevator Position Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	177
6-95	Flaperon Rate Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	178
6-96	Elevator Rate Configuration 8 Operating Point: 10K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	178
6-97	Pitch Rate Response Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	179
6-98	Pitch Rate Performance Criterion Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	179

6-99	Flaperon Position Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	180
6-100	Elevator Position Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	180
6-101	Flaperon Rate Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	181
6-102	Elevator Rate Configuration 8 Operating Point: 38K 0.9M SIG1=0.4 SIG2=0.1 RHO=0.8	181
6-103	Model Probability Weightings Operating Point: 10K 0.35M	183
6-104	Prediction Error Variance (1,1) for Model 1 Operating Point: 10K 0.35M	184
6-105	Prediction Error Variance (2,2) for Model 1 Operating Point: 10K 0.35M	184
6-106	Prediction Error Variance (1,1) for Model 2 Operating Point: 10K 0.35M	185
6-107	Prediction Error Variance (2,2) for Model 2 Operating Point: 10K 0.35M	185
6-108	Model Probability Weightings Operating Point: 10K 0.35M Sensor Noise of Table 6-15	187
6-109	Prediction Error Variance (1,1) for Model 1 Operating Point: 10K 0.35M Sensor Noise of Table 6-15	188

6-110	Prediction Error Variance (2,2) for Model 1 Operating Point: 10K 0.35M Sensor Noise of Table 6-15	188
6-111	Prediction Error Variance (1,1) for Model 2 Operating Point: 10K 0.35M Sensor Noise of Table 6-15	189
6-112	Prediction Error Variance (2,2) for Model 2 Operating Point: 10K 0.35M Sensor Noise of Table 6-15	189
6-113	Model Probability Weightings Operating Point: 10K 0.35M Sensor Noise * 100	190
6-114	Prediction Error Variance (1,1) for Model 1 Operating Point: 10K 0.35M Sensor Noise * 100	191
6-115	Prediction Error Variance (2,2) for Model 1 Operating Point: 10K 0.35M Sensor Noise * 100	191
6-116	Prediction Error Variance (1,1) for Model 2 Operating Point: 10K 0.35M Sensor Noise * 100	192
6-117	Prediction Error Variance (2,2) for Model 2 Operating Point: 10K 0.35M Sensor Noise * 100	192
6-118	Model Probability Weightings Operating Point: 10K 0.35M	195
6-119	Prediction Error Variance (1,1) for Model 1 Operating Point: 10K 0.35M	196
6-120	Prediction Error Variance (2,2) for Model 1 Operating Point: 10K 0.35M	196
6-121	Prediction Error Variance (1,1) for Model 2 Operating Point: 10K 0.35M	197

6-122	Prediction Error Variance (2,2) for Model 2 Operating Point: 10K 0.35M	197
6-123	Model Probability Weightings Operating Point: 10K 0.35M Sensor Noise * 100	198
6-124	Prediction Error Variance (1,1) for Model 1 Operating Point: 10K 0.35M Sensor Noise * 100	199
6-125	Prediction Error Variance (2,2) for Model 1 Operating Point: 10K 0.35M Sensor Noise * 100	199
6-126	Prediction Error Variance (1,1) for Model 2 Operating Point: 10K 0.35M Sensor Noise * 100	200
6-127	Prediction Error Variance (2,2) for Model 2 Operating Point: 10K 0.35M Sensor Noise * 100	200
6-128	Model Probability Weightings Configuration 8 Operating Point: 26K 0.9M	203
6-129	Prediction Error Variance (1,1) for Model 1 Configuration 8 Operating Point: 26K 0.9M	204
6-130	Prediction Error Variance (2,2) for Model 1 Configuration 8 Operating Point: 26K 0.9M	204
6-131	Prediction Error Variance (1,1) for Model 2 Configuration 8 Operating Point: 26K 0.9M	205
6-132	Prediction Error Variance (2,2) for Model 2 Configuration 8 Operating Point: 26K 0.9M	205

6-133	Prediction Error Variance (1,1) for Model 3 Configuration 8 Operating Point: 26K 0.9M	206
6-134	Prediction Error Variance (2,2) for Model 3 Configuration 8 Operating Point: 26K 0.9M	206
6-135	Model Probability Weightings Configuration 8 Operating Point: 26K 0.9M Sensor Noise of Table 6-15	207
6-136	Model Probability Weightings Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	208
6-137	Prediction Error Variance (1,1) for Model 1 Configuration 8 Operating Point: 26K 0.9M Sensor Noise *100	209
6-138	Prediction Error Variance (2,2), for Model 1 Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	209
6-139	Prediction Error Variance (1,1) for Model 2 Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	210
6-140	Prediction Error Variance (2,2) for Model 2 Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	210
6-141	Prediction Error Variance (1,1) for Model 3 Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	211

6-142	Prediction Error Variance (2,2) for Model 3 Configuration 8 Operating Point: 26K 0.9M Sensor Noise * 100	211
6-143	Model Probability Weightings Configuration 8 Operating Point: 38K 0.6M	213
6-144	Model Probability Weightings Configuration 8 Operating Point: 38K 0.6M Sensor Noise of Table 6-15	213
6-145	Model Probability Weightings Configuration 8 Operating Point: 38K 0.6M Sensor Noise * 10	214
6-146	Model Probability Weightings Configuration 8 Operating Point: 38K 0.6M Sensor Noise * 100	214

List of Tables

<u>Table</u>	<u>Page</u>
2-1 Flight Conditions of Interest	24
2-2 Control Surface Position and Rate Limits	29
6-1 Flight Conditions Failing Performance Criteria Nominal Flight Condition=18,000 feet 0.45 Mach .	86
6-2 Flight Conditions Failing Performance Criteria Nominal Flight Condition=38,000 feet 0.7 Mach ..	87
6-3 Flight Conditions of Nominal Models with Similar Dynamic Pressures	90
6-4 Eigenvalues for the A matrix of State Space Model	99
6-5 Angle of Attack for Nominal Flight Conditions ..	100
6-6 Aerodynamic Gains for Transfer Functions of Equations (6-3) through (6-6)	104
6-7 Aerodynamic Gains for Transfer Functions of Equations (6-8) through (6-11)	105
6-8 Two Model Configurations	107
6-9 Three Model Configurations	124
6-10 Model Selection Data for Configuration 6	130
6-11 Model Selection Data for Configuration 7	134
6-12 Model Selection Data for Configuration 8	138
6-13 Performance Index Comparison of Selected Flight Conditions	149
6-14 Design Parameter Values for Simulations at 22,000 ft 0.65 Mach	156

6-15	Sensor Noise Data	182
6-16	Two Model Configuration for Noise Consideration	182
6-17	Two Model Configuration for Noise Consideration	194

ABSTRACT

This thesis extends the research accomplished by Capt Pineiro and Lt Berens in the area of adaptive algorithm implementation. Specifically, this thesis explores the performance characteristics of the multiple model estimation algorithm and how they influence the selection of aircraft models to allow the parameter adaptive control system to maintain tracking performance over a desired portion of the flight envelope. The aircraft dynamic equations used are those of the AFTI/F-16 and the control law design is based on the method developed by Professor Porter.

Numerous fixed gain simulations are presented in order to determine the performance robustness to plant variations of selected flight conditions. From these simulations, performance or robustness boundaries for each nominal flight condition are determined. Data is then generated to ascertain the effect on the control surfaces' responses of replacing the nominal model of a performance boundary with another model within the same boundary. That leads to the determination of the critical factors to be considered in

choosing an aircraft model to represent a specific set of performance boundaries.

Following the determination of what factors are important in establishing performance bounds for a fixed gain system, model selection for the multiple model algorithm is evaluated. In attempting to select a set of aircraft models that would ensure an acceptable level of tracking performance over the desired flight envelope, several two-, three, and four-model configurations are examined. Each configuration is evaluated to determine the amount of overlap of performance bounds required for proper tracking performance, the amount of the flight envelope that is covered, and control surface performance.

After selecting a set of aircraft models that results in the best overall system response, the effect of adjusting the control law gains on the performance of the multiple model estimation algorithm is evaluated. By assuming that all states are accessible, sensor noise is then added to each of the longitudinal states to study how noise impacts model selection. A set of models that produces acceptable tracking performance over the desired flight envelope and the most immunity to sensor noise is then selected.

MODEL SELECTION FOR THE MULTIPLE MODEL ADAPTIVE ALGORITHM FOR IN-FLIGHT SIMULATION

1. Introduction

1.1 Overview

There are a large number of estimation and control problems where algorithm design and implementation are dependent on the modeling and evaluation of uncertain parameters. One such problem was analyzed by Capt Luis Pineiro in his thesis, Parameter Adaptive Model Following For In Flight Simulation (1). Pineiro's thesis documented the fact that parameter adaptive control techniques can be implemented to allow a specific system to maintain a desired level of tracking performance in the presence of plant parameter changes. Lt Thomas Berens continued Pineiro's research by implementing a multiple model algorithm (2) as the primary parameter estimation technique. This estimation algorithm allows the system to maintain tracking performance when the plant has deviated considerably from the nominal condition. This thesis extends Beren's research by investigating the performance characteristics of the multiple model algorithm

and how they affect the selection of aircraft models that will yield a desired level of tracking performance over a specific portion of the flight envelope.

1.2 Background

Using the techniques developed by Professor Brian Porter (3;4;5;6;7;8), Pineiro developed an adaptive, fast sampling control law to compensate for changing aircraft parameters of the Advanced Fighter Technology Integration F-16 (AFTI/F-16) operating in a model following configuration. This technique involved the use of on-line, recursive, step response matrix identifiers to update the control law gains as needed to account for plant parameter variations. Pineiro's research was limited to the linearized, longitudinal, rigid body dynamics of the AFTI/F-16 using perturbation equations of motion at a nominal flight condition of 10,000 feet altitude and Mach 0.9 (1).

In order to keep the computational loading and convergence time to a minimum, Pineiro allowed only a small number of parameters to be estimated (only four parameters were adjustable out of 20) while retaining an accurate representation of the plant. The remaining parameters that were not among those to be estimated were fixed at values representa-

tive of the current flight condition. The results of Pineiro's research documented the fact that parameter-adaptive control techniques can be implemented to allow a specific system to maintain a desired level of tracking performance in the presence of plant parameter changes (produced by large changes in the stability derivatives)(1).

Capt Pineiro recommended that analysis of this research area continue with the parameter adaptive algorithm extended to include a multiple model estimator (1). In his thesis, Lt Berens implemented such an estimation algorithm. The multiple model algorithm implementation can be approached by assuming that the uncertain parameters take on discrete values within a possible range of parameter values. For each flight condition a complete system model is designed which produces an estimate of the selected parameters. Following the design of a bank of system models, each model is weighted by the probability of that particular model being correct and averaged with the outputs of all other models (9).

1.3 Problem Statement

The purpose of this thesis is to extend the research

accomplished by Capt Pineiro and Lt Berens in the area of adaptive algorithm implementation. Specifically, this thesis explores the performance characteristics of the multiple model estimation algorithm, how they influence the selection of aircraft models, and how well they allow the system to maintain tracking performance over a desired portion of the flight envelope.

1.4 Summary of Current Knowledge

This section summarizes the current knowledge that is pertinent to this thesis effort. The areas of in-flight simulators, adaptive control, and multiple model adaptive estimation are examined in the following paragraphs.

1.4.1 In-Flight Simulators. An in-flight simulator is an aircraft that is able to fly with the same characteristics as another aircraft. While flying the simulator, the pilot actually experiences the flying qualities and feel of the aircraft that is being simulated (10).

The in-flight simulator has become an extremely important tool in the aircraft research and development process. As the cost of developing aircraft has increased, the benefits of in-flight simulation have become more pronounced.

In-flight simulation provides the capability to fly aircraft systems prior to committing to the full scale or production phases of the acquisition process. Another area in which the simulator has been extremely helpful is the area of experimental testing of aircraft. Investigations into flight control systems, aircraft flying qualities, and system integration have been addressed in detail with the advent of the in-flight simulator (11).

An advanced in-flight simulator designated as the Variable-Stability In-Flight Simulator Test Aircraft (VISTA) will be developed by the USAF. VISTA will be a high performance, supersonic, six degree of freedom simulator based on a two place fighter aircraft. By having a variable stability system, VISTA will be able to support a diverse range of aeronautical research in the fields of flight controls, handling qualities, and flying characteristics. It will also have the ability to simulate current, modified, or new aircraft (11).

Because VISTA will be able to operate over a large range of flight conditions, wide variations in the aircraft stability derivatives will be possible. Jones and Porter have shown that degradation in closed loop-system behavior will

inevitably occur in the case of large plant parameter changes and a fixed control law (3). To ensure that the simulation is not degraded substantially due to inaccurate stability derivatives, a model-following mode of operation may be used. The term model-following means that the response characteristics of the host aircraft are tailored to 'follow' the responses of the simulated aircraft as generated by a computer model (12).

1.4.2 Adaptive Control. Hartman and Krebs (13) have shown that adaptive control is a method of maintaining desired tracking performance throughout the flight envelope. In his research, Pineiro's main objective was to implement a parameter adaptive control law that would compensate for the degraded simulation performance that occurs when the stability derivatives of the host aircraft take on values which have deviated considerably from the expected nominal values (1).

The area of adaptive control has become an area of great current interest in automatic control research. The reason for this is that adaptive control is a technique that proposes solutions when the system varies with time in a manner that is not totally specifiable a-priori. This is in

contrast to the manner in which most automatic control techniques afford solutions to time invariant systems (14). The main benefit of adaptive control is that the system parameters can be adjusted on-line as the system environment changes. Hagglund (14) stated in his doctoral thesis, New Estimation Techniques For Adaptive Control,

The idea of adaptive control is to automate the analysis and design. Hence, an adaptive controller does not only control the process, but it also collects information about the process behaviour from the control signals and the measured output signals. Based on this information, or analysis, the controller parameters are adjusted on-line.

There are numerous schemes reported in the literature for parameter adaptive control, three of which are gain scheduling, model reference control, and self tuning regulators (15).

The actual concept of gain scheduling started with the development of flight control systems. In fact, it is the predominant method used in current aircraft to handle variations of parameters in flight control systems. Mach number and dynamic pressure are measured and used as scheduling variables. When the scheduling variables have been chosen, the control parameters are evaluated at a number of opera-

ting conditions using an acceptable design method. The drawbacks of gain scheduling are that it is an open loop compensation method (no feedback is provided to correct for an incorrect schedule, resulting in gains being implemented which could degrade system performance or even result in system instability) and the design is time consuming (15).

Model reference adaptive controllers try to obtain a control behavior close to that of a reference model for a given input signal. The disadvantage of this type of adaptive control is that if a fixed reference model is used, the system will approach the behavior of the reference model as opposed to the behavior of a proper model at the current operating condition. The advantage of the model reference system is that, when the operating condition is near the fixed reference model and the input is one for which the system has been designed, quick adaptation is possible (16).

Another method of adaptively adjusting the parameters of a system is to use a self tuning regulator (15), also referred to as a parameter adaptive controller. It should be noted that self tuning regulators are one form of a parameter adaptive controller and are not equivalent to that

entire class. A block diagram of the self tuning regulator used by Pineiro is shown in Figure 1-1 (1). The adaptive controller has three different parts, the first of which (see Figure 1-1) is the ordinary feedback loop formed by the control algorithm (regulator) and the aircraft dynamics (process). The input and output of the process are fed into a parameter estimator which performs on-line estimation of

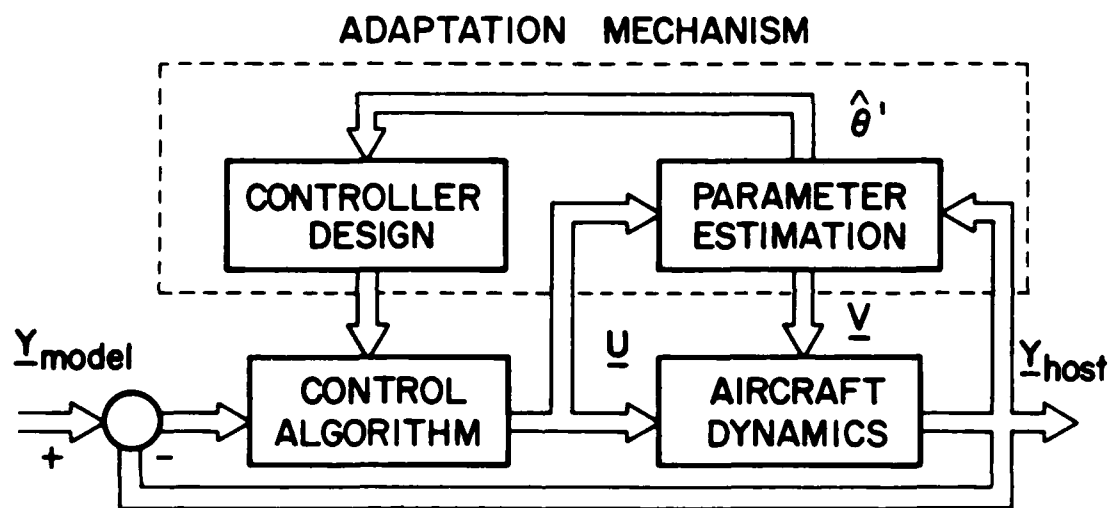


Figure 1-1. Parameter Adaptive Control System (1)

the parameters of the process. The estimated parameters are then used to modify the controller gains by a design calculation. The term adaptation mechanism refers to the parameter estimator and the design calculation (14). The host aircraft dynamics for the adaptive controller designed by Pineiro were the linearized, longitudinal, rigid body dyna-

mics of the Advanced Fighter Technology Integration F-16 (AFTI/F-16). The signals that were used as tracking references (Y model in Figure 1-1) were the elevators and flaperons and the adaptive system outputs (Y host in Figure 1-1) were flight path angle and pitch rate (1).

The control algorithm and controller design that were implemented by Pineiro were the techniques developed by Professor Brian Porter (3;4;5;6;7;8). Porter and Jones have shown that on-line recursive identifiers can be used to provide updated step response matrices. Inclusion of these updated step response matrices in a digital proportional plus integral control law, results in highly effective adaptive digital set point tracking controllers for multivariable plants (3). Using these design techniques, Pineiro developed an adaptive control law to compensate for changing aircraft parameters of the AFTI/F-16 operating in a model following configuration. This technique involved the use of on-line, recursive, step response matrix identifiers to update the control law gains as needed to account for plant parameter variations.

As shown in Figure 1, the parameter adaptive controller also contains a recursive parameter estimator. Many estima-

tion schemes have been used, including stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering, and the maximum likelihood method (15). A summary of the different estimation methods is given in Ljung and Soderstrom (17). The parameter identification algorithm used by Pineiro was a modified recursive least squares algorithm developed by Hagglund (1).

1.4.3 Multiple Model Adaptive Estimation (MMAE). In his research, Berens continued Pineiro's effort by extending the parameter adaptive algorithm to include a multiple model estimator (2). MMAE is especially beneficial when the application has parameters that can undergo large changes such as is the case with the stability derivatives of the host aircraft of an in-flight simulator (18). Multiple model adaptation has been successfully applied to numerous problems including flight control problems (19).

Multiple model implementation can be approached by assuming that the uncertain parameters can take on discrete values within a possible range of parameter values. A complete system model is designed for each flight condition,

resulting in a bank of n system models. Based upon the characteristics of the residuals generated for each of the n system models, the correctness of each of the estimated parameters is calculated iteratively from the conditional probability for each of the n system models being the best model to use. The conditional probability is determined using the past measurement history. The closer the value of the conditional probability is to one, the more weight is placed on that given model's parameter estimates being correct. The parameter estimate of each system model is weighted by the value of its probability, and the adaptive parameter estimate is determined by the probabilistically weighted average of all the system model outputs (9;18).

1.5 Assumptions

The assumptions in this thesis are those typically adopted when using linearized longitudinal perturbation equations. These assumptions are those used by Pineiro and Berens and are listed below.

- * The aircraft is a rigid body with constant mass.
- * The atmosphere is fixed with respect to the earth.

- * The earth's surface is an inertial reference frame.
- * Linearization about an operating condition is valid for preliminary aircraft models.
- * Aerodynamics are constant for Mach and altitude.

1.6 Approach

Using the control law design parameters and commanded inputs of flight path angle and pitch rate as given by Pineiro, numerous fixed gain simulations were accomplished in order to determine the performance robustness to plant variations, of selected flight conditions. From these simulations, performance or robustness boundaries for each nominal flight condition were determined. Data was then generated to ascertain the effect on the control surfaces' responses of replacing the nominal model of a performance boundary with another model within the same boundary on the control surfaces responses. That led to the determination of the critical factors to be considered in choosing an aircraft model to represent a specific set of performance bounds.

Following the determination as to what factors were important in establishing performance bounds for a fixed gain system, model selection for the multiple model algo-

rithm was evaluated. In attempting to select a set of aircraft models that would ensure an acceptable level of tracking performance over the desired flight envelope, several two-, three-, and four-model configurations were examined. (The term model configuration refers to the number of models that were explicitly implemented in the multiple model algorithms). Each configuration was evaluated to determine the amount of overlap of performance bounds required for proper tracking performance, the region of the flight envelope that was covered, and control surface performance.

After selecting a set of aircraft models that resulted in the best overall system response (discussed in a later section), the effect of adjusting the control law gains on the performance of the multiple model estimation algorithm was evaluated. By assuming that all states were accessible, sensor noise was then added to each of the longitudinal states to study how noise impacts model selection. A set of models that produced acceptable tracking performance over the desired flight envelope and the most immunity to sensor noise was then selected.

1.7 Overview

The material in this thesis begins in Chapter 2 with a description of the AFTI/F-16 and a presentation of the mathematical aircraft model. The actuator model is also discussed in Chapter 2. Chapter 3 contains a brief description of the multivariable design theory developed by Professor Brian Porter of the University of Salford, England. The multiple model estimation algorithm is discussed in Chapter 4 and model selection criteria are presented in Chapter 5. Chapter 6 presents the simulation results, and a discussion of conclusions and recommendations for future study is found in Chapter 7.

2. Aircraft Description and Models

2.1 AFTI/F-16 Description

Because the VISTA is still in the process of being developed and the aircraft dynamic equations are not available, the Advanced Fighter Technology Integration (AFTI)/F-16 was used as the host aircraft for the parameter adaptive control system. The reason for this choice is the anticipated similarity of the AFTI and proposed VISTA. The AFTI/F-16 aircraft, shown in Figure 2-1, is an F-16A air superiority fighter modified to be a testbed for evaluating new aircraft technologies (20). These modifications include the addition of two vertical canards which are mounted on the engine inlet, control surfaces that allow independent motion of the trailing edge flaps and independent motion of the horizontal tail halves, and a redundant, digital fly-by-wire flight control system. The AFTI/F-16's mission is very important in that the technologies tested on it may be incorporated in the Air Force's future fighter aircraft.

The unaugmented AFTI/F-16 aircraft is statically unstable in the longitudinal axis for subsonic flight. The reason this condition exists is that the center of gravity

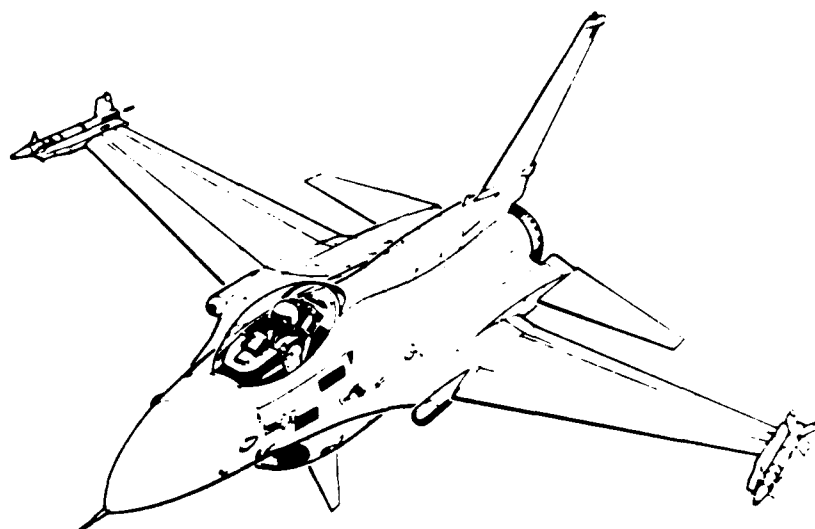


Figure 2-1. The AFTI/F-16

is located behind the aircraft's center of pressure. The instability, manifested by an unstable short period mode, allows the aircraft to withstand higher load factors and reduces drag, thereby enhancing maneuverability. Additionally, the aircraft has a lightly damped dutch roll mode. Therefore, the flight control system has a twofold purpose: to stabilize the aircraft longitudinally and to improve the dutch roll damping (20).

In addition to being able to perform conventional maneuvers such as pitching longitudinally, rolling laterally, and turning with zero sideslip, the AFTI/F-16 can also perform unconventional maneuvers that require decoupling of the aircraft's modes of responses. These unconventional maneuvers include pitch-pointing, yaw-pointing, and lateral

and longitudinal translation. Conventional and decoupled maneuvers can also be blended, which gives the AFTI/F-16 an even wider array of possible maneuvers (20).

The control surfaces that are used to maneuver the AFTI/F-16, as well as the definitions of positive surface deflections, are shown in Figure 2-2. The surfaces include left and right horizontal tail halves, left and right flaperons, left and right canards, and the rudder. The horizontal tail halves have a dual function in that they can be deflected symmetrically as elevators to pitch the aircraft, or they can be deflected asymmetrically to augment rolling. Likewise, the flaperons can be deflected symmetrically to function as conventional flaps or asymmetrically to function as ailerons to control roll. The canards can be used in a snow plow configuration to function as a speedbrake or they can be used to provide sideforces on the aircraft. As in a conventional aircraft, the rudder is used for yawing. The surfaces used in this thesis are shown in heavy black in Figure 2-2 and include the left and right tail halves and the left and right flaperons.

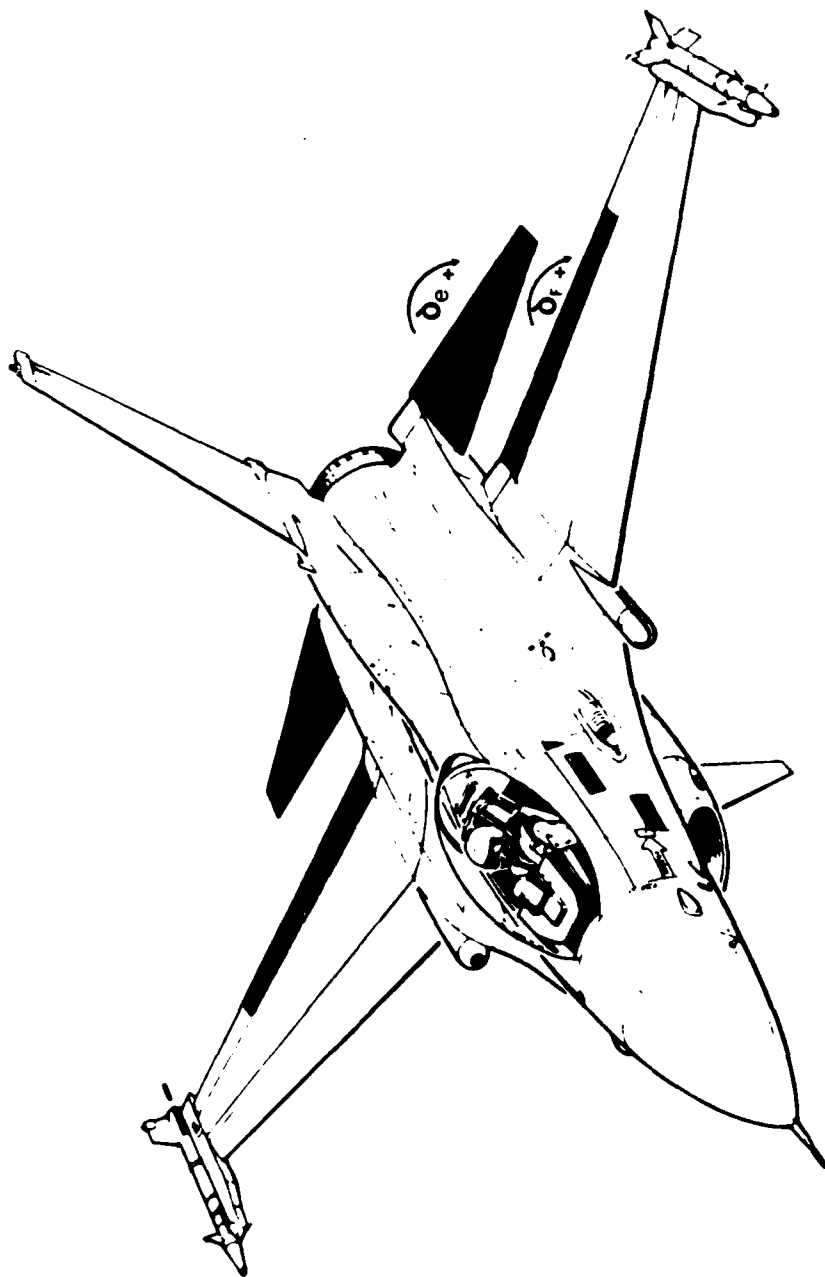


Figure 2-2. AFTI/F-16 Control Surfaces and Definition of Positive Deflections

2.2 Aircraft Models

2.2.1 State Space Model. The aircraft is modeled by a set of first order differential equations in the state space form

$$\dot{\underline{x}} = \underline{Ax} + \underline{Bu} \quad (2-1)$$

The state equations are perturbation equations about a trim condition and are derived from the aircraft equations of motion which consist of forces at the center of gravity and moments with reference to the body axis. Detailed derivations of the longitudinal state perturbation equations used in this thesis are presented in a thesis by Barfield (21).

Equation (2-2) shows the longitudinal axis state space model for the AFTI/F-16. The primed terms in Equation (2-2) are dimensionalized derivatives (Appendix A) in the body

$$\begin{bmatrix} \dot{\theta} \\ \dot{u} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ X'_{\theta} & X'_u & X'_{\alpha} & X'_q \\ Z'_{\theta} & Z'_u & Z'_{\alpha} & Z'_q \\ M'_{\theta} & M'_u & M'_{\alpha} & M'_q \end{bmatrix} \begin{bmatrix} \theta \\ u \\ \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ X'_{\delta e} & X'_{\delta f} \\ Z'_{\delta e} & Z'_{\delta f} \\ M'_{\delta e} & M'_{\delta f} \end{bmatrix} \begin{bmatrix} \delta e \\ \delta f \end{bmatrix} \quad (2-2)$$

where

θ is pitch angle

u is forward velocity

α is angle of attack

q is pitch rate

δ_e is the total elevator where both horizontal
tail halves deflect symmetrically

δ_f is the total flap where both flaperons deflect
symmetrically

axis. The definition of the body axis along with the positive directions of forces, moments, and angles are found in Figure 2-3.

A Flight Dynamics Laboratory aerodynamic data package for the AFTI/F-16 was used to obtain linearized aerodynamic coefficients and dimensionalized derivatives by trimming the aircraft at various flight conditions. Aircraft weight and inertias were common to all flight conditions and were also provided by the program (see Appendix A).

This thesis is concerned with that portion of the flight envelope which ranges in altitude from 10,000 to 38,000 feet and from 0.25 to 0.9 in Mach number. This portion of the flight envelope is shown graphically in Figure 2-4. The specific flight conditions considered in the envelope of interest are listed in Table 2-1. The

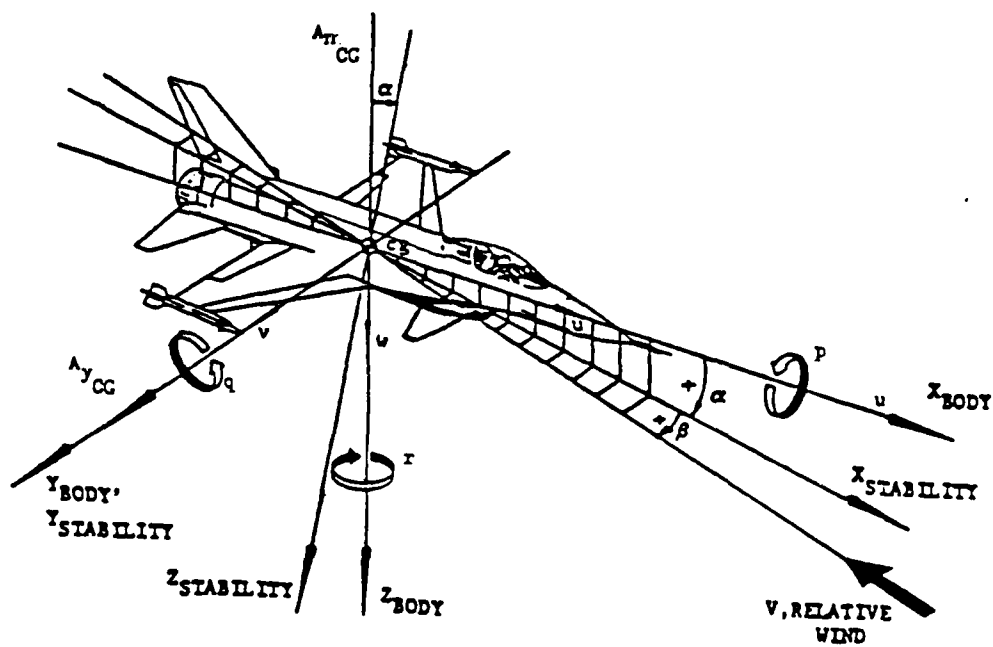


Figure 2-3. Positive Directions of Forces, Moments, and Angles in Body Reference Frame

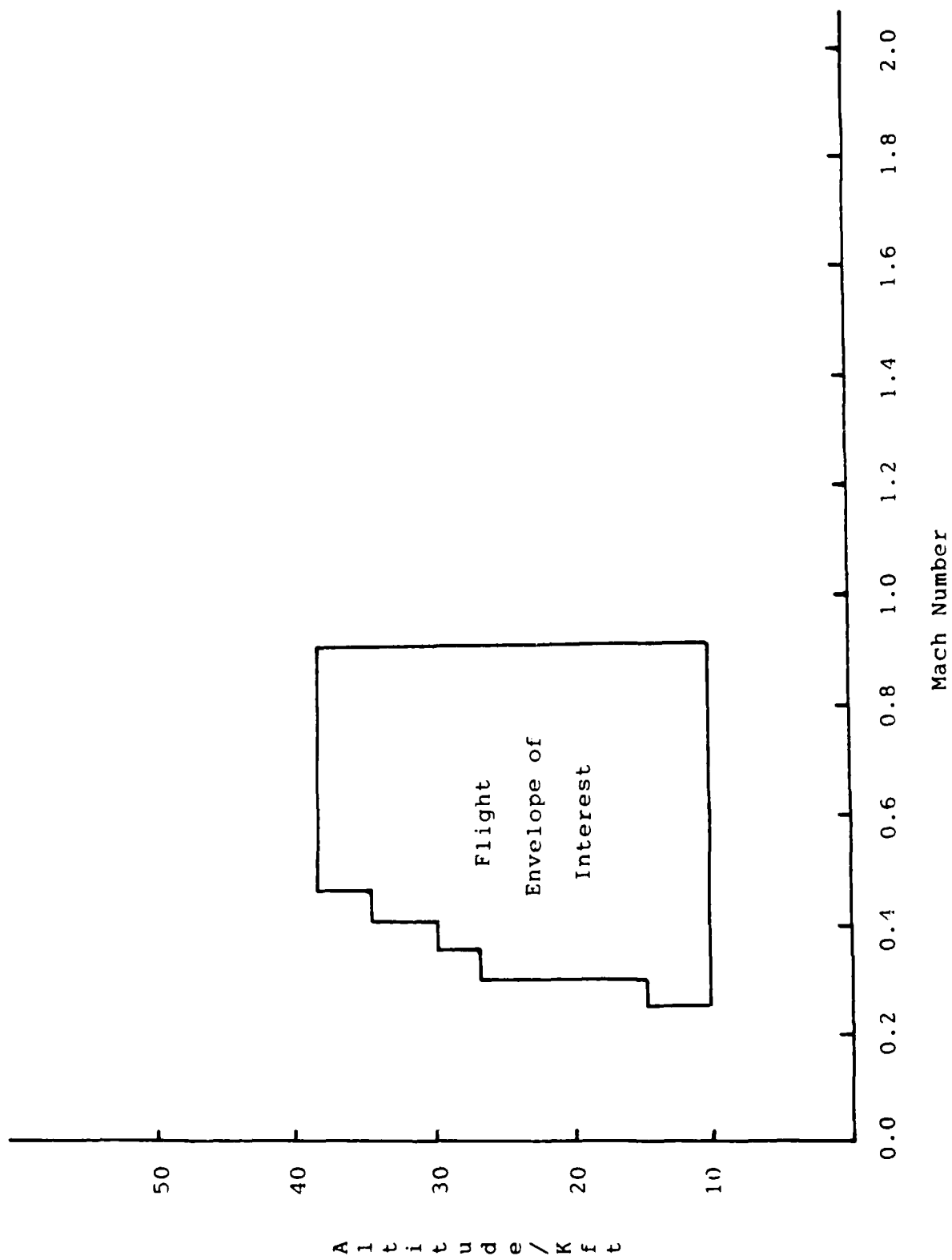


Figure 2-4. Flight Envelope of Interest

Table 2-1.

Flight Conditions of Interest

Alt (kft)/Mach No.		Alt (kft)/Mach No.		Alt (kft)/Mach No.	
38	0.45	26	0.40	18	0.85
38	0.50	26	0.45	18	0.90
38	0.55	26	0.50	14	0.25
38	0.60	26	0.55	14	0.30
38	0.65	26	0.60	14	0.35
38	0.70	26	0.65	14	0.40
38	0.75	26	0.70	14	0.45
38	0.80	26	0.75	14	0.50
38	0.85	26	0.80	14	0.55
38	0.90	26	0.85	14	0.60
34	0.40	26	0.90	14	0.65
34	0.45	22	0.30	14	0.70
34	0.50	22	0.35	14	0.75
34	0.55	22	0.40	14	0.80
34	0.60	22	0.45	14	0.85
34	0.65	22	0.50	14	0.90
34	0.70	22	0.55	10	0.25
34	0.75	22	0.60	10	0.30
34	0.80	22	0.65	10	0.35
34	0.85	22	0.70	10	0.40
34	0.90	22	0.75	10	0.45
30	0.35	22	0.80	10	0.50
30	0.40	22	0.85	10	0.55
30	0.45	22	0.90	10	0.60
30	0.50	18	0.30	10	0.65
30	0.55	18	0.35	10	0.70
30	0.60	18	0.40	10	0.75
30	0.65	18	0.45	10	0.80
30	0.70	18	0.50	10	0.85
30	0.75	18	0.55	10	0.90
30	0.80	18	0.60		
30	0.85	18	0.65		
30	0.90	18	0.70		
26	0.30	18	0.75		
26	0.35	18	0.80		

longitudinal dimensionalized derivatives for each of the flight conditions listed in Table 2-1 have been obtained using the Flight Dynamics Laboratory's AFTI/F-16 aerodynamic data package. A state space model for each flight condition is formed by inserting the proper dimensionalized derivatives into Equation (2-1). Appendix A presents the state space models for each of the flight conditions.

An output vector, y , is defined for the model in Equation (2-1) in order to command desired maneuvers. The output vector is added to the state space aircraft model with an output matrix, C . The aircraft model state and output equations then take the form of

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}\tag{2-3}$$

The output vector used in this study is the same vector as that used by Pineiro (1) and Berens (2) in which flight path angle and pitch rate are commanded. Flight path angle is given by

$$\gamma = \theta - \alpha\tag{2-4}$$

Because pitch angle, pitch rate, and angle of attack

are state variables, flight path angle and pitch rate are easily included in the model by placing unit elements with the proper signs in the appropriate positions in the $\underline{C}$ matrix. This results in an output equation

$$\begin{bmatrix} \gamma \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ u \\ \alpha \\ q \end{bmatrix} \quad (2-5)$$

2.2.2 Autoregressive Difference Equation Model. The AFTI/F-16 plant state and output equations of Equation (2-3) may be discretized for a sampling period T by using the relationships

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \exp \left[\begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} T \right] \quad (2-6)$$

and

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \int_0^T \exp \left[\begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} t \right] \begin{bmatrix} 0 \\ \underline{B}_2 \end{bmatrix} dt \quad (2-7)$$

The resulting sampled data state and output equations become

$$\begin{bmatrix} \underline{x}_1((k+1)T) \\ \underline{x}_2((k+1)T) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1(kT) \\ \underline{x}_2(kT) \end{bmatrix} + \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} \underline{u}(kT) \quad (2-8)$$

and

$$\underline{y}(kT) = [\underline{C}_1 \quad \underline{C}_2] \begin{bmatrix} \underline{x}_1(kT) \\ \underline{x}_2(kT) \end{bmatrix} \quad (2-9)$$

Pineiro showed (Appendix B) that the input-output relationship of Equations (2-8) and (2-9) can be expressed in terms of an Nth order autoregressive vector difference equation of the form

$$\begin{aligned} \underline{y}(kT) = & \underline{B}_1 \underline{u}((k-1)T) - \underline{A}_1 \underline{y}((k-1)T) + \dots \dots \dots (2-10) \\ & + \underline{B}_n \underline{u}((k-N)T) - \underline{A}_n \underline{y}((k-N)T) + \underline{e}(kT) \end{aligned}$$

or equivalently

$$\underline{y}(kT) = \underline{\Upsilon}^T \underline{\Theta}(kT) + \underline{e}(kT) \quad (2-11)$$

where the equation error vector $\underline{e}(kT)$ is assumed to be a zero-mean Gaussian white-noise vector with covariance $\underline{A}_e$, $\underline{\Upsilon}(kT) \in R^{m \times \ell}$ is a matrix of past values of $\underline{y}$ and $\underline{u}$, the matrices $\underline{A}_i \in R^{m \times m}$ ($i=1,2,\dots,N$), $\underline{B}_i \in R^{m \times m}$ ($i=1,2,\dots,N$) and the vector $\underline{\Theta} \in R^{\ell}$ are the parameters of the Nth order difference equation, m is equal to the number of inputs and

ℓ is equal to the number of parameters in the difference equation. For the case AFTI/F-16 longitudinal aircraft model of Equation (2-3), the parameter vector Θ consists of a column vector of 20 elements which are unique for a given flight condition.

2.3 Actuator Model

Consideration of control surface position and rate limits is important in the design of a realistic control system. The actuator transfer function model for both the elevator and the flaperon surfaces are first order models of the form

$$T_{\text{actuator}}(s) = \frac{44}{(s+44)} \quad (2-12)$$

The control surface position and rate limits used in this study are those given in Pineiro's thesis and are found in Table 2-2. The position limits assume that the elevator and flaperon have a trimmed position of zero degrees. Due to the fact that each flight condition has a different nonzero trim value of elevator and flaperon position, the surface position limits are adjusted accordingly for each point.

Table 2-2.

Control Surface Position and Rate Limits

Surface	Position Limit (deg)	Rate Limit (deg/sec)
Elevators	+25	90
Flaperons	+20 -23	78

The position limits for each flight condition are found in Appendix C.

2.4 Summary

This section presents the mathematical development of the equations describing the AFTI/F-16. The AFTI/F-16 is chosen as the plant due to the non-availability of similar equations for the VISTA. The autoregressive difference equation representing an equivalent state space representation of the aircraft is also presented, along with the flight conditions considered in this thesis.

3. Control Law Algorithm

3.1 Introduction

This chapter describes the control algorithm that is implemented in the parameter adaptive control system as shown in Figure 1-1. The control law should be such that the plant is able to follow the time response generated by a computer model of the vehicle being simulated. The multi-variable control algorithm used in this thesis is that developed by Professor Brian Porter of the University of Salford, England. Porter's method employs output feedback with high-gain error-actuated controllers. Output feedback may be advantageous since state variables may be difficult to measure.

3.2 Model Following Technique (1)

One method of achieving in-flight simulation is through the implementation of the model following concept (12). A model following in-flight simulation scheme uses the signals from the evaluation pilot's cockpit as inputs to an aircraft computer which simulates the equations of motion of the desired aircraft. The output of the same aircraft computer

is comprised of the time histories that describe the responses of the simulated aircraft to the inputs from the evaluation pilot. The variable stability system has the responsibility of maneuvering the control surfaces of the host aircraft so that the aircraft response at the pilot's station is the same as those of the emulated airplane. Obviously, achieving satisfactory model following performance depends on having an accurate knowledge of the host aircraft's stability derivatives.

The dynamics of the host vehicle can be represented by a differential equation of the form

$$\dot{\underline{x}}_h(t) = \underline{A}_h \underline{x}_h(t) + \underline{B}_h \underline{u}_h(t) \quad (3-1)$$

and the dynamics of the vehicle to be simulated can be represented by

$$\dot{\underline{x}}_m(t) = \underline{A}_m \underline{x}_m(t) + \underline{B}_m \underline{u}_m(t) \quad (3-2)$$

where $\underline{A}_h$, $\underline{A}_m$, $\underline{B}_h$, $\underline{B}_m$ are matrices that contain the stability derivatives, $\underline{x}_h$ and $\underline{x}_m$ are the states of the respective differential equations, $\underline{u}_h$ and $\underline{u}_m$ are the required control surface deflection, and the subscripts h and m signify the host and model aircraft, respectively.

By substituting the states and the rate of change of the states of the model into Equation (3-1), a control law for exact model following can be obtained by solving for the control input $\underline{u}_h(t)$:

$$\dot{\underline{x}}_m(t) = \underline{A}_h \underline{x}_m(t) + \underline{B}_h \underline{u}_h(t) \quad (3-3)$$

Assuming $\underline{B}_h$ is invertible and solving for $\underline{u}_h(t)$ leads to

$$\underline{u}_h(t) = \underline{B}_h^{-1} \dot{\underline{x}}_m(t) - \underline{B}_h^{-1} \underline{A}_h \underline{x}_m(t) \quad (3-4)$$

$$\underline{u}_h(t) = \underline{K}_1 \dot{\underline{x}}_m(t) - \underline{K}_2 \underline{x}_m(t) \quad (3-5)$$

From Equations (3-4) and (3-5) it can be seen that the control inputs to the host aircraft are defined in terms of the states of the model and the first derivative of those states. Also, Equations (3-4) and (3-5) show that accurate stability derivatives are required to have good model following performance. As shown in Figure 3-1, the states and

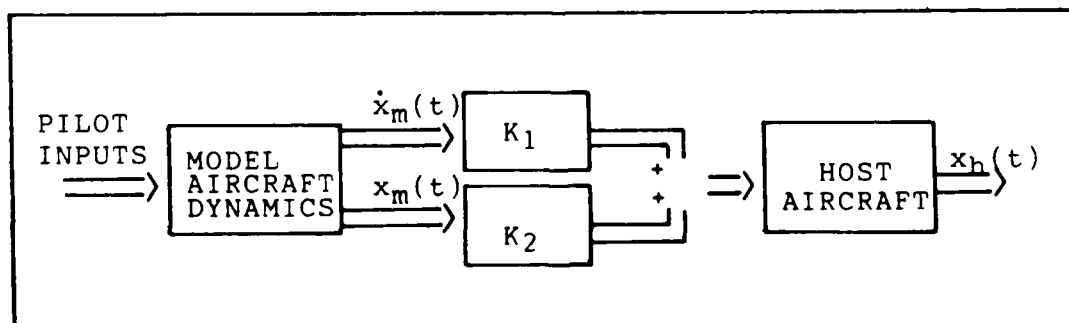


Figure 3-1. Explicit Model Following System (1)

rates of change of the states are available from the variable stability system's computer that contains the model aircraft dynamics.

Figure 3-2 shows an improved explicit model following system. Notice that there is a feedback loop containing a gain matrix $\underline{K}_h$ around the plant. The purpose of this feed-

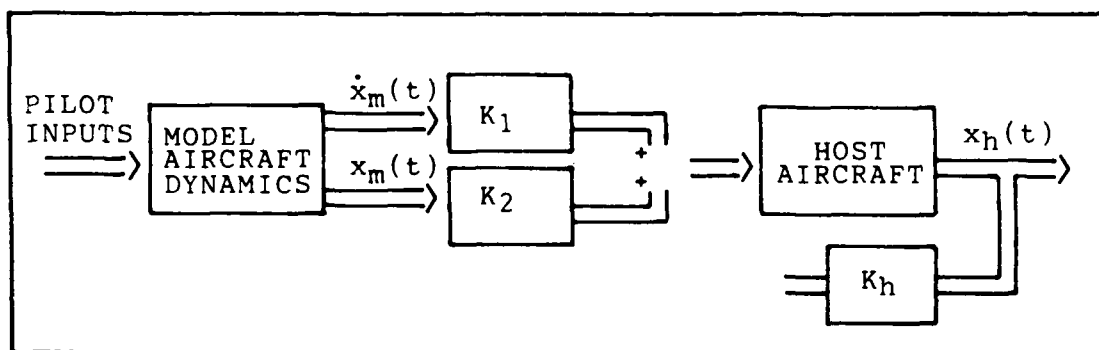


Figure 3-2. Improved Explicit Model Following System (1)

back loop is to reduce the sensitivity of the explicit model following system (see Figure 3-1) to plant parameter variations.

With the inclusion of the feedback loop the feedforward gains become

$$\underline{K}_1 = \underline{B}_h^{-1} \quad (3-6)$$

$$\underline{K}_2 = \underline{B}_h^{-1} \underline{A}_h - \underline{K}_h \quad (3-7)$$

If

$$\underline{K}_h \gg \underline{B}_h^{-1} \underline{A}_h \quad (3-8)$$

then the feedback loop becomes increasingly tighter which makes the system less dependent on $\underline{K}_1$ (less sensitive to plant parameter variations).

3.3 Porter's Control Law (1)

The multivariable control law used in this thesis to achieve model following was developed by Professor Brian Porter. Porter developed a control law that can be used to make the plant follow the responses generated by a computer model of the vehicle to be simulated. This section presents Porter's control law.

The host aircraft in this thesis is completely controllable and observable, and is described by the continuous time state space model

$$\dot{\underline{x}}(t) = \underline{A} \underline{x}(t) + \underline{B} \underline{u}(t) \quad (3-9)$$

$$\underline{y}(t) = \underline{C} \underline{x}(t) \quad (3-10)$$

where

$\underline{A}$ = continuous plant matrix (n x n)

$\underline{B}$ = continuous input control matrix ($n \times m$)
having a rank "m"

$\underline{C}$ = continuous output matrix ($l \times n$)

$\underline{y}$ = output vector with l outputs

$\underline{u}$ = input vector with m inputs

$\underline{x}$ = state variable vector with n states

For the linearized, longitudinal dynamics of the AFTI/F-16,
the state variable vector has 4 states as described in
Chapter 2:

θ = pitch

u = forward velocity

α = angle of attack

q = pitch rate

The input vector consists of 2 inputs

δ_e = elevator

δ_f = flaperons

The 2 elements of the output vector are

γ = flight path angle

q = pitch rate

In using Porter's design method, it is desirable to partition the state equations so that $\underline{B}_2$ and $\underline{C}_2$ are square ($m \times m$) and ($\ell \times \ell$) matrices, respectively. This process yields

$$\begin{bmatrix} \dot{\underline{x}}_1(t) \\ \dot{\underline{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \underline{B}_2 \end{bmatrix} \underline{u} \quad (3-11)$$

and

$$\underline{y}(t) = [\underline{C}_1 \quad \underline{C}_2] \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} \quad (3-12)$$

where it is required that the number of inputs to the system equal the number of outputs, $m=\ell$. This requirement ensures that the column dimension of $\underline{B}$ equals the row dimension of $\underline{C}$. In equation (3-12) the elements associated with $\underline{x}_1(t)$ and $\underline{x}_2(t)$ are

$$\underline{x}_1(t) = [\theta] \quad (3-13)$$

$$\underline{x}_2(t) = [u \quad \alpha \quad q] \quad (3-14)$$

The plant of Equations (3-11) and (3-12) is defined as regular if the matrix, $\underline{CB}$, has full rank " m ". The input output configuration of this thesis provides for a regular design. For regular plants with stable transmission zeros,

the control law is a discrete proportional-plus-integral (PI) output feedback control law (see Figure 3-3) expressed as

$$\underline{u}(kT) = (1/T) [\underline{K}_1 \underline{e}(kT) + \underline{K}_2 \underline{z}(kT)] \quad (3-15)$$

and

$$\underline{u}(t) = \underline{u}(kT) \text{ for } t \in [kT, (k+1)T] \quad (3-16)$$

where

$\underline{r}(kT)$ is the sampled reference tracking signal

k is an integer

$T = (1/f)$ is the sampling period

$\underline{K}_1$ and $\underline{K}_2$ are the $(m \times m)$ controller matrices

$\underline{e}(kT)$ is the error vector [$\underline{e}(kT) = \underline{r}(kT) - \underline{y}(kT)$]

$\underline{z}(kT)$ is the digital integral of the error vector
[$\underline{z}((k+1)T) = \underline{z}(kT) + T \underline{e}(kT)$]

When $\underline{CB}$ does not have full rank, the plant is labeled as irregular and a proportional-plus integral-plus derivative (PID) controller structure may be implemented (6;7).

3.3.1 Fixed Gain Controller Matrices.

The continuous time state equations given in Equations (3-11) and (3-12) can be discretized for the sampling period

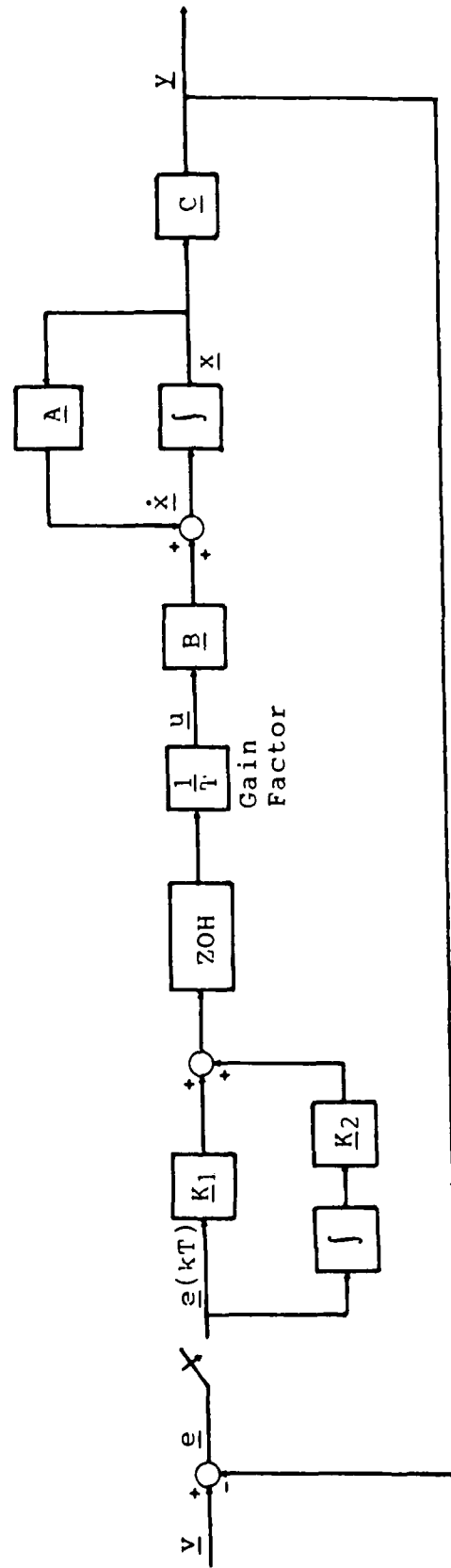


Figure 3-3. System Block Diagram - Discrete Design

T by using

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \exp \left\{ \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} T \right\} \quad (3-17)$$

and

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \int_0^T \exp \left\{ \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} t \right\} \begin{bmatrix} 0 \\ B_2 \end{bmatrix} dt \quad (3-18)$$

The resulting sampled data state and output equations for the plant are

$$\begin{bmatrix} x_1((k+1)T) \\ x_2((k+1)T) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} + \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} u(kT) \quad (3-19)$$

and

$$y(kT) = [C_1 \quad C_2] \begin{bmatrix} x_1(kT) \\ x_2(kT) \end{bmatrix} \quad (3-20)$$

The augmented closed-loop state and output equations for the control law of equation (3-15) are (8)

$$\begin{bmatrix} z((k+1)T) \\ x_1((k+1)T) \\ x_2((k+1)T) \end{bmatrix} = \begin{bmatrix} I & -TC_1 & -TC_2 \\ f\Psi_{1K_2} & \Phi_{11} - f\Psi_{1K_1}C_1 & \Phi_{12} - f\Psi_{1K_1}C_2 \\ f\Psi_{2K_2} & \Phi_{21} - f\Psi_{2K_1}C_1 & \Phi_{22} - f\Psi_{2K_1}C_2 \end{bmatrix} \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \end{bmatrix} + \begin{bmatrix} TI \\ f\Psi_{1K_1} \\ f\Psi_{2K_1} \end{bmatrix} r(kT) \quad (3-21)$$

and

$$\underline{y}(kT) = [\underline{0} \quad \underline{C}_1 \quad \underline{C}_2] \begin{bmatrix} \underline{z}(kT) \\ \underline{x}_1(kT) \\ \underline{x}_2(kT) \end{bmatrix} \quad (3-22)$$

A transformation block diagonalizes equations (3-21)

and (3-22) as

$$\begin{bmatrix} \bar{\underline{x}}_1\{(k+1)T\} \\ \bar{\underline{x}}_2\{(k+1)T\} \end{bmatrix} = \begin{bmatrix} \bar{\underline{A}}_1 & \underline{0} \\ \underline{0} & \bar{\underline{A}}_4 \end{bmatrix} \begin{bmatrix} \bar{\underline{x}}_1(kT) \\ \bar{\underline{x}}_2(kT) \end{bmatrix} + \begin{bmatrix} \bar{\underline{B}}_1 \\ \bar{\underline{B}}_2 \end{bmatrix} \underline{r}(kT) \quad (3-23)$$

$$\underline{y}(kT) = [\bar{\underline{C}}_1 \quad \bar{\underline{C}}_2] \begin{bmatrix} \bar{\underline{x}}_1(kT) \\ \bar{\underline{x}}_2(kT) \end{bmatrix} \quad (3-24)$$

where

$$\bar{\underline{x}}_1(kT) = \begin{bmatrix} \underline{z}(kT) \\ \underline{x}_1(kT) \end{bmatrix} \quad (3-25)$$

$$\bar{\underline{x}}_2(kT) = \underline{x}_2(kT) \quad (3-26)$$

$$\bar{\underline{C}}_1 = [\underline{K}_1^{-1} \underline{K}_2 \quad \underline{0}] \quad (3-27)$$

$$\bar{\underline{C}}_2 = \underline{C}_2 \quad (3-28)$$

$$\bar{\underline{A}}_1 = \begin{bmatrix} \underline{I}_m - T\underline{K}_1^{-1} \underline{K}_2 & \underline{0} \\ T\underline{A}_{12} \underline{C}_2^{-1} \underline{K}_1^{-1} \underline{K}_2 & \underline{I}_{n-m} + \underline{A}_{11} T - T\underline{A}_{12} \underline{C}_2^{-1} \underline{C}_1 \end{bmatrix} \quad (3-29)$$

$$\bar{\underline{B}}_1 = \begin{bmatrix} \underline{0} \\ T\underline{A}_{12} \underline{C}_2^{-1} \end{bmatrix} \quad (3-30)$$

$$\bar{\underline{A}}_4 = [\underline{I}_m - \underline{B}_2 \underline{K}_1 \underline{C}_2] \quad (3-31)$$

$$\bar{\underline{B}}_2 = [\underline{B}_2 \underline{K}_1] \quad (3-32)$$

As the sampling frequency is increased, the closed-loop transfer function takes on the asymptotic form (23)

$$\underline{G}(z) = \underline{G}_1(z) + \underline{G}_2(z) \quad (3-33)$$

where z is the discrete transform operator and

$$\underline{G}_1(z) = \bar{\underline{C}}_1 (z\underline{I}_n - \underline{I}_n - T\underline{A}_o)^{-1} T\underline{B}_o \quad (3-34)$$

$$\underline{G}_2(z) = \bar{\underline{C}}_2 (z\underline{I}_m - \underline{I}_m - \underline{A}_4)^{-1} \underline{B}_4 \quad (3-35)$$

with

$$\underline{A}_o = \begin{bmatrix} \underline{K}_1^{-1} \underline{K}_2 & 0 \\ \underline{A}_{12} \underline{C}_2^{-1} \underline{K}_1^{-1} \underline{K}_2 & \underline{A}_{11} - \underline{A}_{12} \underline{C}_2^{-1} \underline{C}_1 \end{bmatrix} \quad (3-36)$$

$$\underline{B}_o = \begin{bmatrix} 0 \\ \underline{A}_{12} \underline{C}_2^{-1} \end{bmatrix} \quad (3-37)$$

$$\underline{A}_4 = [-\underline{B}_2 \underline{K}_1 \underline{C}_2] \quad (3-38)$$

In Equations (3-33), (3-34), and (3-35), $\underline{G}_1(z)$ and $\underline{G}_2(z)$ are the slow and fast mode transfer functions respectively. The slow modes contain two sets of eigenvalues Z_1 and Z_2 which are given by the equations

$$Z_1 = \{z \in \mathbb{C} : \det(z\underline{I}_m - \underline{I}_m + T\underline{K}_1^{-1} \underline{K}_2) = 0\} \quad (3-39)$$

$$Z_2 = \{z \in \mathbb{C} : \det(z\underline{I}_{n-m} - \underline{I}_{n-m} - T\underline{A}_{11} + T\underline{A}_{12} \underline{C}_2^{-1} \underline{C}_1) = 0\} \quad (3-40)$$

The fast mode eigenvalues are given by

$$Z_3 = \{z \in \mathbb{C} : \det(z\mathbf{I}_m - \mathbf{I}_m + \mathbf{C}_2\mathbf{B}_2\mathbf{K}_1) = 0\} \quad (3-41)$$

Because of the form of $\mathbf{A}_0$, $\mathbf{B}_0$, and $\mathbf{C}_1$, the eigenvalues of $\mathbf{A}_0$ are uncontrollable or unobservable. Therefore, as the sampling frequency increases, the slow transfer function asymptotically approaches zero and the overall system transfer function contains only the fast modes, as given in $\mathbf{G}_2(z)$, which can be expressed in the equivalent form

$$\mathbf{G}(z) = \mathbf{G}_2(z) = (z\mathbf{I}_m - \mathbf{I}_m + \mathbf{C}_2\mathbf{B}_2\mathbf{K}_1)^{-1} \mathbf{C}_2\mathbf{B}_2\mathbf{K}_1 \quad (3-42)$$

The desired fixed gain controller matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ are then given by

$$\mathbf{K}_1 = [\mathbf{C}_2\mathbf{B}_2]^{-1} \boldsymbol{\xi} \quad (3-43)$$

$$\mathbf{K}_2 = \rho \mathbf{K}_1 \quad (3-44)$$

where ρ is a positive scalar greater than zero and assigns the ratio of proportional to integral control, and $\boldsymbol{\xi}$ is a diagonal tuning matrix. The diagonal elements of $\boldsymbol{\xi}$ determine the weighting effect of a particular error signal on the respective control input. Hence, selection of the di-

agonal weighting matrix elements can be altered to achieve desired tracking characteristics.

3.3.2 Adaptive Controller Matrices. To avoid performance degradation in the presence of large plant parameter variations, Porter has shown (3;4;5;6;7) that by incorporating fast on-line recursive identifiers which provide updated step-response matrices, satisfactory response characteristics can be achieved. The significance of using the step response matrix is that it can be obtained from real time input-output measurements to reflect the current characteristics of the plant, thus forming the basis for an adaptive system.

The discrete step response matrix is defined as (4)

$$\underline{H}(T) = \int_0^T \underline{C} \exp(\underline{A}t) \underline{B} dt \quad (3-45)$$

For small sampling periods $\underline{H}(T) \approx T\underline{CB}$, and the control law design equation becomes

$$\underline{u}(kT) = \bar{\underline{K}}_1 \underline{e}(kT) + \bar{\underline{K}}_2 \underline{z}(kT) \quad (3-46)$$

where

$$\bar{\underline{K}}_1 = \underline{H}^{-1}(T) \underline{\xi} \quad (3-47)$$

$$\bar{\underline{K}}_2 = \rho \bar{\underline{K}}_1 \quad (3-48)$$

Equations (3-47) and (3-48) show that for the case of plants with variable parameters, updated step response matrices need to be provided to the control-law design equations. This requirement can be achieved due to the fact that the step response matrix can be obtained from real time input-output data reflecting the current plant. Thus, by using the step response matrix, an adaptive control system can be designed to track plants with variable parameters. The elements of the step response matrix for the control law equation (3-47) can be obtained by expressing the state and output equations ((3-19) and (3-20)) as an Nth order autoregressive difference equation, as shown in Chapter 2, of the form

$$\begin{aligned} \underline{y}(kT) = & \underline{B}_1 \{ \underline{u}(k-1)T \} - \underline{A}_1 \underline{y}((k-1)T) + \dots \\ & + \underline{B}_n \underline{u}((k-N)T) - \underline{A}_n \underline{y}((k-N)T) + \underline{e}(kT) \end{aligned} \quad (3-49)$$

or

$$\underline{y}(kT) = \underline{\Upsilon}^T(kT) \underline{\Theta} + \underline{e}(kT) \quad (3-50)$$

where the equation error vector $\underline{e}(kT)$ is assumed to be a zero-mean Gaussian white-noise vector, $\underline{\Upsilon}^T(kT) \in R^{m \times 1}$ is the measurement matrix consisting of past values of $\{\underline{y}_k\}$ and

$\{u_k\}$, the matrices $A_i \in R^{m \times m}$ ($i=1,2,\dots,N$), $B_i \in R^{m \times m}$ ($i=1,2,\dots,N$), and the vector $\theta \in R^1$ are the parameters of the Nth order difference equation.

It can be shown that, by using the definition of the step response matrix, the following equation can be derived

$$\underline{H}(T) = \underline{B}_1 \cong TCB \quad (3-51)$$

Equation (3-51) clearly shows that by identifying $\underline{B}_1$ in real-time using input-output data, updated step-response matrix estimates can be provided for the control law design equation, (3-47). From Equation (3-49), it can be seen that $\underline{B}_1$ can be identified as a set of parameters in the parameter vector θ .

3.4 Summary

This chapter briefly presents the concept of model following in-flight simulation and discusses a control law design technique that can be used to accomplish model following. The design technique discussed is the Porter method, and both fixed gain and adaptive controller gain matrices are presented. Finally, the method of adaptively updating the controller gain matrices via the parameters of

an autoregressive difference equation are presented. The next chapter discusses in detail the parameter estimation algorithm used in this thesis.

4. Parameter Estimation

4.1 Introduction

This chapter details the algorithm, used in this thesis, that allows the process parameters to be identified. Due to the fact that the stability derivatives change as the aircraft operates through varying flight conditions, a parameter identification algorithm is required that estimates the time varying parameter vector $\Theta(t)$. In his thesis, Pineiro incorporated a modified recursive least squares algorithm developed by Hagglund (14) to account for time varying parameters. In order to minimize the number of parameters to be identified, Pineiro fixed some parameters that were not used in the control law and were assumed to be constant about a nominal flight condition. Thus, the number of parameters estimated by the estimation algorithm was reduced to four.

In an attempt to decrease the parameter estimation convergence time required to estimate the complete set of plant model parameters, Berens (2) implemented a multiple model algorithm parameter estimation technique in lieu of the modified recursive least squares algorithm used by Pineiro.

The algorithm discussed in this chapter and used in this thesis is the multiple model parameter estimation technique which was presented by Wittenmark (24) and implemented by Berens. The manner in which models are to be selected for the multiple model algorithm, to provide satisfactory tracking performance in the flight envelope of interest, is the primary emphasis of this thesis.

4.2 Multiple Model Algorithm

The multiple model adaptive algorithm is a parameter estimation technique which can yield satisfactory parameter estimates over a desired parameter space. By selecting a discrete number of parameter values that are dispersed throughout the region of expected parameter space and incorporating them as a-priori data, the multiple model estimation algorithm provides a method of discriminating which of the a-priori models is nearest to the true parameter value.

The multiple model algorithm defines the parameter vector as a random variable whose conditional probability density function can be approximated as a weighted sum of Dirac delta functions. The weights that are associated with

each of the a-priori models are dependent on the fit of a given model to the current real-world condition and are updated from modeling errors generated for each candidate model.

As is discussed earlier, the parameter vector, $\underline{\theta}(kT)$, is composed of the coefficients of an autoregressive difference equation. Each flight condition has a parameter vector associated with it and hence becomes the source of the a-priori information required by the multiple model algorithm. The candidate model parameter vectors are designated as

$$\underline{\theta}_i(kT) \quad i = 1, 2, \dots, N \quad (4-1)$$

where N is the number of candidate models being considered.

One method of estimating $\underline{\theta}(kT)$ is to form a weighted sum of a-priori models as given by the equation

$$\underline{\theta}(kT) = \sum_{i=1}^N a_i(kT) \underline{\theta}_i(kT) \quad (4-2)$$

where $a_i(kT)$ is a weighting factor associated with a particular candidate model. The conditions on the weighting factors are given by

$$0 \leq a_i(kT) \leq 1 \quad (4-3)$$

and

$$\sum_{i=1}^N a_i(kT) = 1 \quad (4-4)$$

From the above equations, estimating the parameter vector $\underline{\theta}(kT)$ becomes a matter of properly choosing the weighting factors, $a_i(kT)$, associated with each candidate model, which in turn are dependent on the fit of the associated model.

Iserman and Lachman (25) present a list of quantities that give information as to which candidate model parameter vector, $\underline{\theta}_i(kT)$, provides the best fit of the parameter vector associated with the actual flight condition. These quantities include

1) The parameter covariance matrices

$$\underline{P}_i(kT) = E\{(\underline{\theta}(kT) - \underline{\theta}_i(kT))(\underline{\theta}(kT) - \underline{\theta}_i(kT))^T\}, \underline{P}_i(kT) \in R^{m \times m} \text{ for } i=1,2,\dots$$

2) The covariance of the prediction error

$$\underline{A}_{ie}(kT) = E\{\underline{e}_i(kT)\underline{e}_i^T(kT)\}, \underline{A}_{ie}(kT) \in R^{m \times m}$$

3) The autocorrelation function

$$\underline{\psi}_{ee}(kT, \tau) = E\{\underline{e}_i(kT)\underline{e}_i^T(kT + \tau)\}, \underline{\psi}_{ee}(kT, \tau) \in R^{m \times m}$$

4) The crosscorrelation function

$$\underline{\psi}_{ue}(kT, \tau) = E\{\underline{u}(kT)\underline{e}_i^T(kT + \tau)\}, \underline{\psi}_{ue}(kT, \tau) \in R^{m \times m}$$

As shown in this thesis, the multiple model algorithm as given by Wittenmark (24) utilizes the variance of the prediction error as the method of determining the best model fit.

The multiple model algorithm (9,18,19,24) provides a method of blending a-priori information with parameter estimation to formulate a weighting factor for each candidate model. As stated earlier, this approach views $\underline{\theta}(kT)$ as a random vector that has a conditional probability density function approximated as a weighted sum of Dirac delta functions. The quantities that affect the shape of the approximated probability density function are the mean vector, covariance and the weighting factors associated with each of the individual models of the multiple model algorithm.

The mean of each individual distribution is the a-priori model parameter values themselves. The covariance of each distribution is the corresponding parameter covariance matrix, $\underline{P}_i(kT)$, which, as shown later, does not need to be directly estimated. The weighting factors, $a_i(kT)$, are selected via a Bayesian approach and are calculated based on the assumption that the residuals of the best fitting model

to the true model are zero mean white Gaussian. This observation can be used to test which model best fits the input/output data. Assuming an a-priori distribution, $p(\underline{\Theta}_i | \underline{y}(0))$, for each model (if there is no a-priori knowledge the assumption can be made that all models are equally probable, i.e. $p(\underline{\Theta}_i | \underline{y}(0)) = 1/N$, a conditional probability can be calculated and updated following a measurement by the equations as given by Wittenmark (24)

$$p(\underline{\Theta} | \underline{y}(kT)) = p(\underline{\Theta} | \underline{y}(kT), \underline{y}(k-1)T) \quad (4-5)$$

$$= \frac{p(\underline{y}(kT) | \underline{\Theta}, \underline{y}((k-1)T)) p(\underline{\Theta} | \underline{y}((k-1)T))}{p(\underline{y}(kT) | \underline{y}((k-1)T))} \quad (4-6)$$

where

$$p(\underline{y}(kT) | \underline{\Theta}, \underline{y}((k-1)T)) = (2\pi \underline{A}_{ie}(kT))^{-1/2} E \quad (4-7)$$

and

$$E = \exp\{-.5 \underline{e}_i^T(kT) (\underline{A}_{ie}(kT))^{-1} \underline{e}_i(kT)\}$$

Therefore the weighting coefficients, $a_i(kT)$, take the form of

$$a_i(kT) = C a_i((k-1)T) [\underline{A}_{ie}(kT)]^{-1/2} \exp\{-.5 \underline{e}_i^T(kT) (\underline{A}_{ie}(kT))^{-1} \underline{e}_i(kT)\} \quad (4-8)$$

where C is a normalization factor, such that

$$\sum_{i=1}^N \{a_i(kT)\} = 1 \quad (4-9)$$

$$\underline{\theta}(kT) = \sum_{i=1}^N \{a_i(kT) \underline{\theta}_i(kT)\} \quad (4-10)$$

and $\underline{e}_i$ is the residual generated by the i th model and $\underline{A}_{ie}$ is the prediction error covariance. Multiple model algorithm weighting coefficient update equations similar to those presented above are given by Maybeck (9) and Andersson (26).

4.2.1 Prediction Error. To define the error associated with a specific candidate model (the i th model), the prediction error or residual is defined from Equation (2-11) as

$$\underline{e}_i(kT) = \underline{y}(kT) - \underline{\Upsilon}^T(kT) \underline{\theta}_i(kT) \quad (4-11)$$

The parameter error is defined as

$$\Delta \underline{\theta}_i(kT) = \underline{\theta}_i(kT) - \underline{\theta}(kT) \quad (4-12)$$

where $\underline{\theta}(kT)$ is the best fitting model parameter vector. By incorporating Equation (4-12) into Equation (4-11), the prediction error becomes

$$\underline{e}_i(kT) = \underline{y}(kT) - \underline{\Upsilon}^T(kT) \underline{\theta}(kT) - \underline{\Upsilon}^T(kT) \Delta \underline{\theta}_i(kT) \quad (4-13)$$

The first two terms on the right hand side of Equation (4-13) represent the error introduced by actuator and sensor noise as well as the error due to linear approximation errors. The third term is an additional modeling error term induced by the model not being the best fitting model.

4.2.2 Prediction Error Covariance. As can be seen in Equation (4-8), the multiple model algorithm as given by Wittenmark (24) utilizes the covariance of the prediction error as the method of determining the best model fit. Berens (2) showed that an estimate of the prediction error covariance is given by

$$\underline{A}_{ie}(kT) \sim (1/k) \left(\sum_{ii=1}^k \underline{e}_i(ii) \underline{e}_i^T(ii) \right) \quad (4-14)$$

Equation (4-14) is an ergodic approximation to the true ensemble average and assumes $\underline{e}_i$ is zero-mean.

The prediction error covariance of systems with generalized inputs is slowly time varying. Old values of $\underline{A}_{ie}(kT)$ are discarded exponentially by replacing Equation (4-14) with a recursive fading memory filter (9),

$$\underline{A}_{ie}(kT) \sim j \underline{A}_{ie}((k-1)T) + (1-j) \underline{e}_i(kT) \underline{e}_i^T(kT) \quad (4-15)$$

where

$$0 < j < 1$$

Small j corresponds to short memory and j large corresponds to long memory.

The performance of Equation (4-15) is restricted by the fact that the prediction error covariance, $\underline{A}_{ie}(kT)$, is assumed to vary slowly with time. When the time varying assumption is violated, the performance of Equation (4-15) can be enhanced by low pass filtering of the input, $\underline{u}(kT)$, and the output, $\underline{y}(kT)$ (discussed in Section 4.3). From Equations (4-11) and (4-14), it can be seen that $\underline{A}_{ie}(kT)$ is a function of $\underline{\Upsilon}(kT)$. It is shown earlier that $\underline{\Upsilon}(kT)$ is formed entirely of past values of $\underline{u}(kT)$ and $\underline{y}(kT)$. Therefore, by reducing the high frequency variation in $\underline{u}(kT)$ and $\underline{y}(kT)$ via low pass filtering, high frequency variation in $\underline{A}_{ie}(kT)$ is also reduced.

4.3 Multiple Model Parameter Estimator

The multiple model parameter estimator which incorporates a-priori information for use in in-flight simulation is shown in Figure 4-1 (9). As discussed earlier, the a-

priori information is in the form of a set of autoregressive difference equation model parameters which describe the aircraft operating at a specific flight condition. Figure 4-1 shows that the a-priori information is stored in a bank of parallel secondary, or elemental, estimators. Each of the

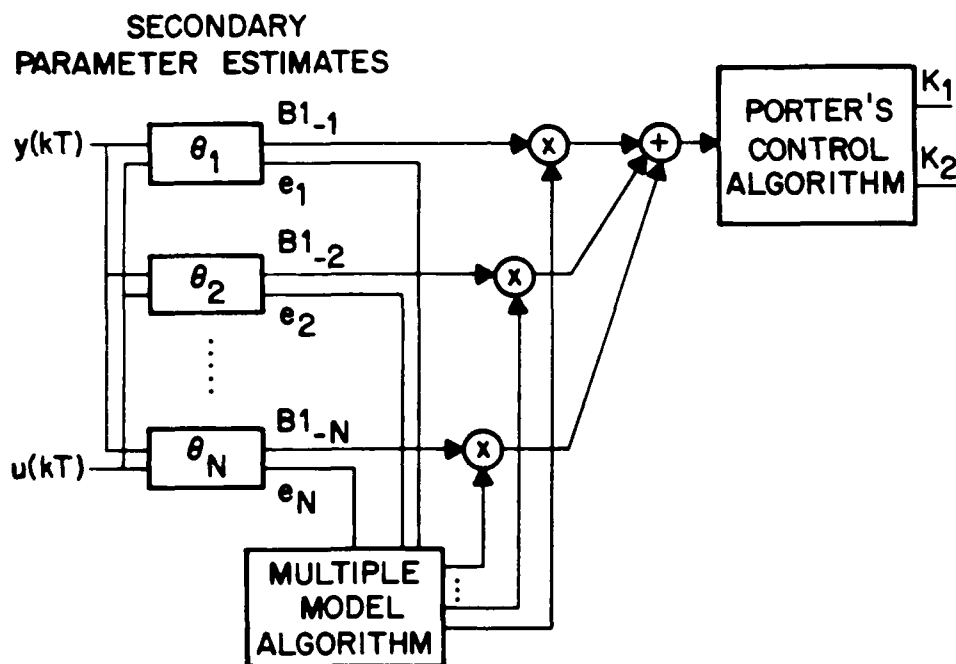


Figure 4-1. Multiple Model Parameter Estimator

secondary estimators is also referred to as a model in the bank and is designed for a specific flight condition. The prediction error from each of the secondary estimators is used by the multiple model algorithm to form a primary (also referred to as adaptive or composite) estimate of the best fitting model parameters. The primary estimates are then

used to update the control law gains.

The secondary parameter estimation task can be accomplished by incorporating one of several different recursive estimation techniques such as recursive least squares or Haggglund's algorithm. The number of parameters to be estimated by the recursive parameter estimation routine is also a design option. The secondary estimator may be chosen to estimate all, some (as in Pineiro's simulation), or none of the parameters.

A number of problems become apparent if the secondary parameter estimators are allowed to update all the parameters recursively at every sampling period. As the number of inputs, outputs, and the order of the autoregressive difference equation increases, the computational effort may become impractical due to long convergence and computation time. Also, as the number of parameters to be updated increases, the system requires more excitation for parameter convergence (17).

To reduce the computational effort afforded by full scale secondary parameter estimation and to decrease the convergence time, the secondary estimators may be allowed to estimate part of the parameter vector while the remaining

parameters are assumed to be fixed at a nominal flight condition. At times, this assumption may not be satisfactory and in fact it may preclude convergence to the proper model.

Secondary parameter estimation can be avoided altogether by leaving all of the parameters in each secondary estimator fixed at a nominal flight condition. This method requires relatively little computational effort and provides good estimates of the parameter vector, especially when a robust control law is being implemented. Fixed secondary parameter estimation is the method incorporated in this thesis.

Three filters are incorporated into the system shown in Figure 1-1 to smooth variations in the parameter estimates. The first is a digital band pass filter which filters $y(k)$ and $u(k)$. The low frequency components of the input-output signals must be removed to reduce parameter estimate bias, while high frequency signal components are removed to smooth input excitation (27). The filter is implemented as a sixth-order butterworth digital bandpass filter (2,28). A second non-linear filter is added to the weighting coefficients within the multiple model algorithm. The purpose of this filter is to smooth sudden changes in the model proba-

bilities that could result in destabilizing rapid changes in the primary parameter estimates. Therefore, the rate at which the weighting coefficients can change in a given sampling period is controlled by a rate limiting filter (2). The third filter is a low pass filter that filters the primary parameter estimates before they are implemented by the control law.

4.4 Summary

This chapter presents the multiple model estimation algorithm that was used in this thesis. The equations used to evaluate the weighting coefficients for the correctness of candidate models are presented as well as the overall implementation of the multiple model adaptive estimator. The next chapter presents important factors to be considered when selecting models for the multiple model estimation algorithm in the context of achieving desired tracking performance over a desired parameter space.

5. Model Selection Criteria

5.1 Introduction

Because the aircraft can maneuver through a continuum of different flight conditions, an infinite number of state space system models of the aircraft are possible. The large number of system models that would be required to characterize all sets of potential parameters could not be implemented in a realistic control system due to the computational burden that would be imposed as well as having a limited amount of computer memory available. To alleviate this situation, the multiple model algorithm with a discrete set of system models to represent the region of selected parameter space can be implemented. (The parameter space for this thesis is defined in Chapter 2 and is shown in Figure 2-4.) Selecting a discrete number of models to represent a continuous parameter space means that the actual values of the parameters do not correspond exactly to one of the models but are close to one. Therefore, the models must be 'close' enough to one another to ensure that the selected model is indeed representative of the actual flight condition.

However, the system models cannot be 'too close' or

difficulties occur with the multiple model algorithm. If the system models are such that the difference between measurement data and the model predicted values is not significant for more than one model, then selection of the 'proper' model cannot take place. As shown in Chapter 4, the multiple model algorithm generates a residual for each of the system models which is then passed on to be processed by the hypothesis conditional probability computation (see Figure 4-1). The algorithm performance depends on the fact that system models that are not representative of the current flight condition have large residuals, while those models that are near the true flight condition have small residuals. Therefore, it can be seen that a careful choice of system models must be made. The selected models must be close, but not too close. This chapter presents the criteria upon which model placement was based to allow the multiple model algorithm to select the model nearest the actual flight condition.

5.2 Performance Boundaries

The performance of an actual physical system can only be approximated by an appropriate mathematical model. The degree that a mathematical model represents an actual system

to depends on the model uncertainties that are present at a given instant in time. These model uncertainties can arise in several ways, including parameter variations or incorrectly modeled dynamics. Both stability and satisfactory performance are of prime concern in the face of these modeling uncertainties (29).

In designing a feedback control system, a model (referred to as the nominal model) must be selected from the set of available models to represent the plant's behavior. When a design has been performed with a specific nominal model, it is obvious that modelling errors are present when there is any deviation of the plant model from the nominal design model. A system is labeled as robust with respect to these modelling errors when the feedback control system remains stable when the nominal plant model is replaced by another plant without altering the controller (21).

Banda and Ridgely (29) point out that there are two types of robustness; stability robustness and performance robustness. Stability robustness occurs when the closed-loop system remains stable in the presence of plant variations. Performance robustness is present when the performance of the closed-loop system is acceptable in the face of

such variations. This chapter is primarily concerned with the issue of performance robustness.

Obviously, if a single control system possesses enough performance robustness, the entire flight envelope shown in Figure 2-4 could be approximated with a single nominal model. In any case, a given nominal model has a region of performance robustness that is associated with it as shown in Figure 5-1. This performance robustness region indicates that satisfactory system performance is achieved when the nominal plant is replaced by any plant model within the

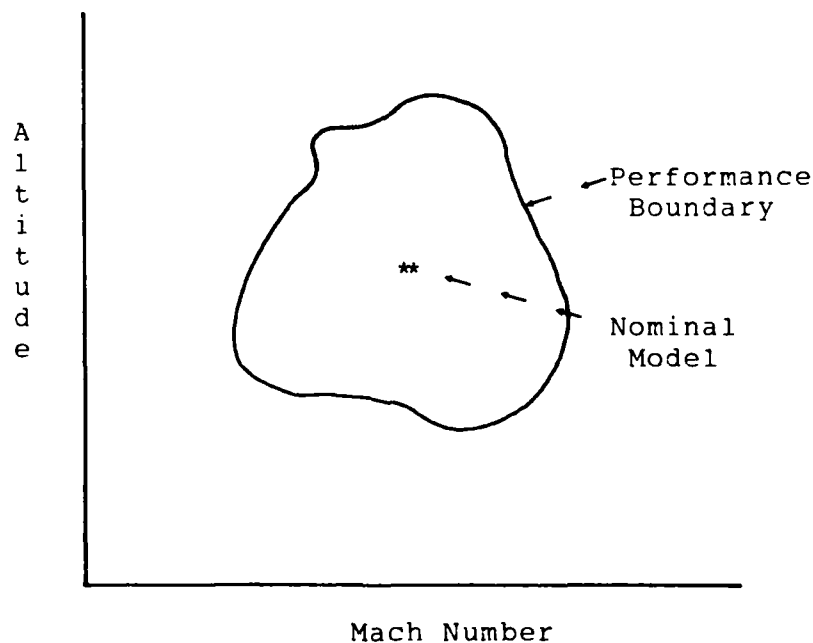


Figure 5-1. Region of Performance Robustness

region encompassed by the performance boundary (see Figure 5-1).

Recall that an aircraft state space model can be represented by a autoregressive difference equation

$$\underline{y}(kT) = \underline{\Upsilon}^T(kT)\underline{\Theta}(kT) + \underline{e}(kT) \quad (5-1)$$

where $\underline{\Theta}$ contains the parameters of the difference equation.

In Chapter 3, it is shown that the control law gains are given by

$$\underline{K}_1 = \underline{H}^{-1}(T)\underline{\xi} \quad (5-2)$$

$$\underline{K}_2 = \rho \underline{K}_1 \quad (5-3)$$

where the step response matrix is

$$\underline{H}(T) = \underline{B}_1 \quad (5-4)$$

and $\underline{B}_1$ can be identified as a subset of the parameter vector $\underline{\Theta}$. Each flight condition has a unique parameter vector associated with it and hence a unique step response matrix. Therefore, it can be, seen from Equations (5-2) and (5-3), that each model has a specific set of gains associated with it (assuming the tuning matrix $\underline{\xi}$ and the parameter ρ remain

constant).

In determining a performance boundary for a specific nominal model, the control law gains remain fixed at the values that are determined by the nominal model's parameter vector. By replacing the nominal plant model with other models and driving the closed-loop system with the commanded inputs outlined in Chapter 2, the performance boundary can be established for a given performance criteria. Hence, plant models that are outside the performance boundary fail to meet the desired performance criteria, while those models inside the boundary yield acceptable closed-loop performance.

The performance criteria used in this thesis are the criteria found in the VISTA statement-of-work and are also the criteria used by Pineiro and Berens. The criteria for tracking performance require that the average response error absolute value should not exceed the average absolute value of ten percent of the reference signal. The time-average response error absolute value is given by

$$\frac{1}{t} \int_{0.01}^t (\text{cmd input} - \text{output}) dt \quad (5-5)$$

where

t = time

cmd input = commanded input

The time-average value of ten percent of the reference signal is

$$\frac{1}{t} \int_{0.01}^t 0.1 * \text{cmd input } dt \quad (5-6)$$

For satisfactory closed-loop tracking response to be accomplished over the flight envelope of interest, the performance boundaries of the required number of models must overlap as shown in Figure 5-2. This requirement is due to the fact that if the operating point lies outside the performance boundaries, the multiple model algorithm assigns probabilities to the respective models that result in control law gains that cause the closed loop system to not meet the performance criteria. From Figure 5-2, it can be seen that there is a minimum number of models that will result in the performance boundaries totally covering the desired flight envelope. This thesis addresses the formation of performance boundaries, the number of models required to yield

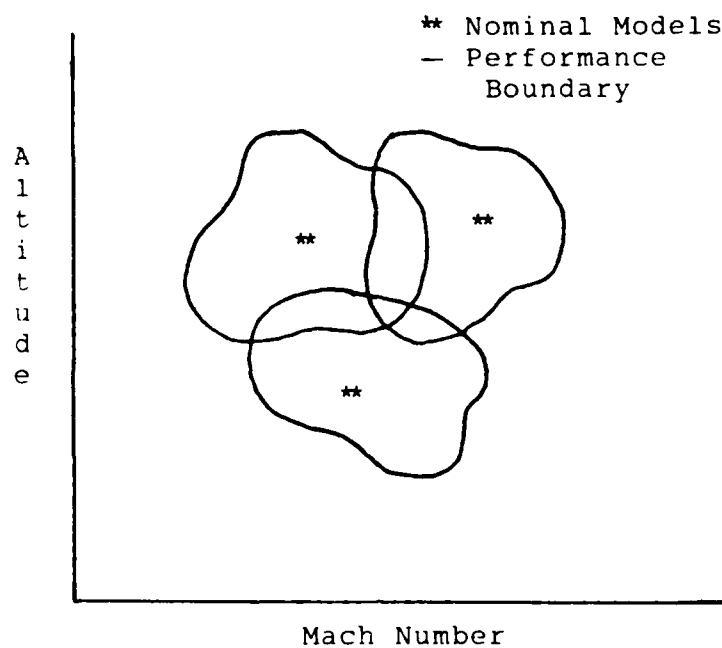


Figure 5-2. Overlapping of Performance Boundaries

acceptable tracking performance over the flight envelope, and the amount of overlapping of the performance boundaries that the multiple model algorithm requires.

5.3 Sensor Noise Effects

By simple algebraic manipulation of the autoregressive difference equation, it was shown in Chapter 4 that the prediction error is defined as

$$\underline{e}(kT) = y(kT) - \underline{\Upsilon}^T(kT)\underline{\Theta}(kT) \quad (5-7)$$

Therefore, the prediction error covariance becomes

$$E(\underline{e}(kT)\underline{e}^T(kT)) = \underline{A}_e \quad (5-8)$$

If the operating point of a fixed gain system (described above) is the nominal plant model, then the prediction error variance (assuming no sensor noise) is due to the modeling error introduced when representing a continuous system with an autoregressive difference equation. When sensor noise is present, then the prediction error variance is composed of the linear approximation errors just mentioned and the error introduced by addition of the sensor noise.

As shown in Chapter 4, the multiple model algorithm assigns a conditional probability to each candidate model based on the calculated value of each model's prediction error covariance. When selecting a model as the best model to represent the current plant dynamics from a set of candidate models, the prediction error covariance is composed of three error terms. These error terms are the errors introduced by sensor noise and linear approximation (as for the fixed gain case), as well as a modelling error term induced by the model not being the best fitting model. Therefore,

for the multiple model case, the prediction error covariance becomes

$$\underline{A}_{ie} = \underline{A}_e + \underline{A}_n + \underline{A}_{im} \quad (5-9)$$

where

$\underline{A}_{ie}$ is the total error covariance associated with the i th candidate model

$\underline{A}_e$ is the error covariance introduced by linear approximation

$\underline{A}_n$ is the error covariance introduced by the addition of sensor noise

$\underline{A}_{im}$ is the error covariance introduced by a model not being the best fitting model

It can be seen from Equation (5-9) that, in order for each candidate model to have a distinct prediction error covariance, the third term on the right hand side of the equal sign must be the dominant term. If the error associated with the sensor noise becomes the dominant term, it will mask the modelling error for each of the candidate models. In essence, a large sensor noise causes the residuals generated for each candidate model to be the same and not distinct. Therefore, proper model selection will not occur.

The above discussion results in the

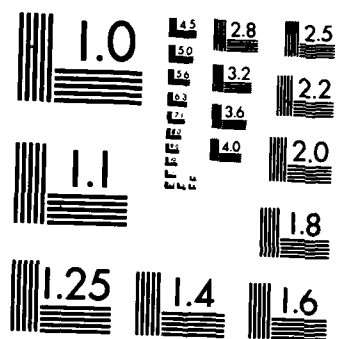
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MODEL SELECTION FOR THE MULTIPLE MODEL ADAPTIVE
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ty which must be satisfied in order for the multiple model algorithm to be able to distinguish one model from another.

$$\underline{A}_e + \underline{A}_{im} \gg \underline{A}_n \quad (5-10)$$

Assuming that the linear approximation error term is small, equation (5-10) becomes

$$\underline{A}_{im} \gg \underline{A}_n \quad (5-11)$$

Equation (5-11) shows that, for a given level of sensor noise, there is a minimum spacing between models that is necessary for the multiple model algorithm to determine the proper model. This is shown graphically in Figure 5-3. Figure 5-3 shows that if a second model is placed in the region where the sensor noise would mask the residuals, then proper model selection cannot occur. Therefore, there is a region that is not acceptable for placement of models subsequent to the placement of an initial model (determined by sensor noise level). On the other hand, when models are placed such that the modelling errors dominate the sensor noise, proper model selection can be made. Additionally, Equation (5-11) shows that the farther apart the candidate models are spaced, the more noise resistant the multiple

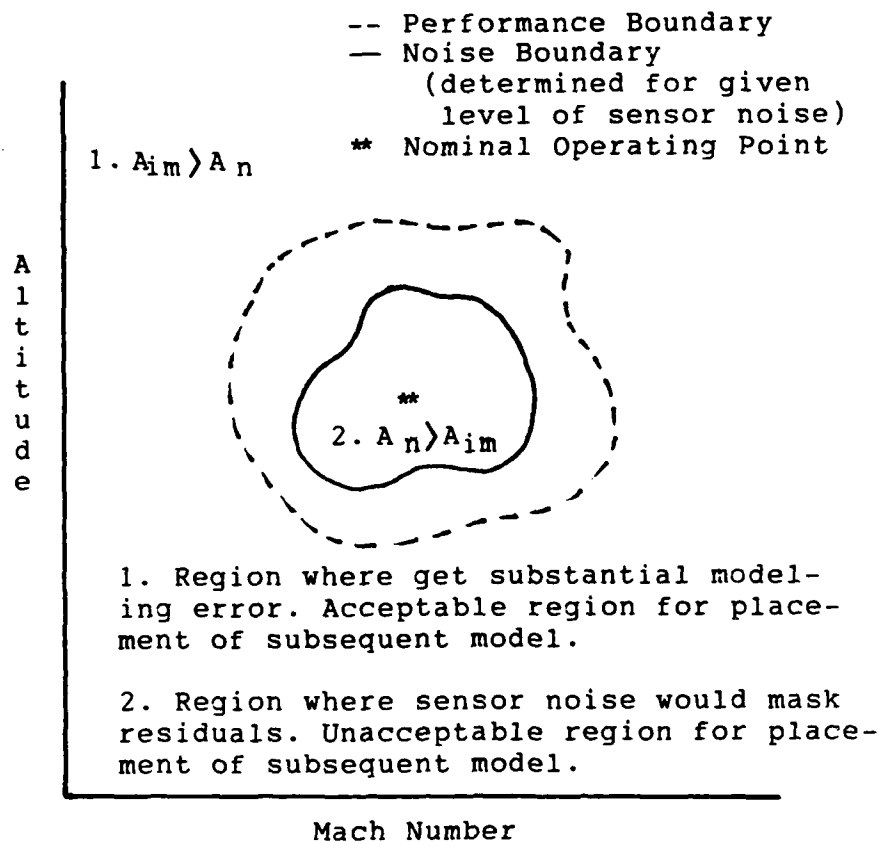


Figure 5-3. Noise Effects on Model Selection

model algorithm becomes. This thesis addresses the issues just discussed by investigating how model selection for the multiple model algorithm is affected by the addition of sensor noise.

5.4 Summary

This chapter presents the factors considered in this thesis for selecting models for the multiple model algorithm. Specifically, the concept of establishing perfor-

mance boundaries using fixed gain simulations is discussed, showing how overlapping boundaries can be used to establish desired tracking performance over a given set of flight conditions. Sensor noise effects are also shown to be a factor to be considered when selecting a discrete number of aircraft models for the multiple model algorithm.

6. Simulation Setup and Results

6.1 Simulation Setup

The parameter adaptive simulation software used in this thesis was developed by Pineiro and Berens. Figure 6-1 shows the top level description of the parameter adaptive system simulation package conducted with the aid of MATRIX_x control design software and its System Build simulation tool. The block in Figure 6-1 that is labeled ADAPT2 implements the multiple model parameter estimation algorithm and also performs the design calculations of updating the control law gains $\underline{K}_1$ and $\underline{K}_2$. The block labeled CTRL accepts the updated control law gains and implements the control law equations that are presented in Equations (3-47) and (3-48) with a sampling time, T , of 0.01 second. The block labeled A/C implements the longitudinal linearized dynamics of the AFTI/F-16 as presented in Chapter 2. The A/C block also implements actuator dynamics and includes nonlinearities such as rate and position limits. The block labeled SEN2 allows sensor noise to be added to the longitudinal states as desired. For a more detailed presentation of each of the blocks in Figure 6-1 consult Beren's thesis (2).

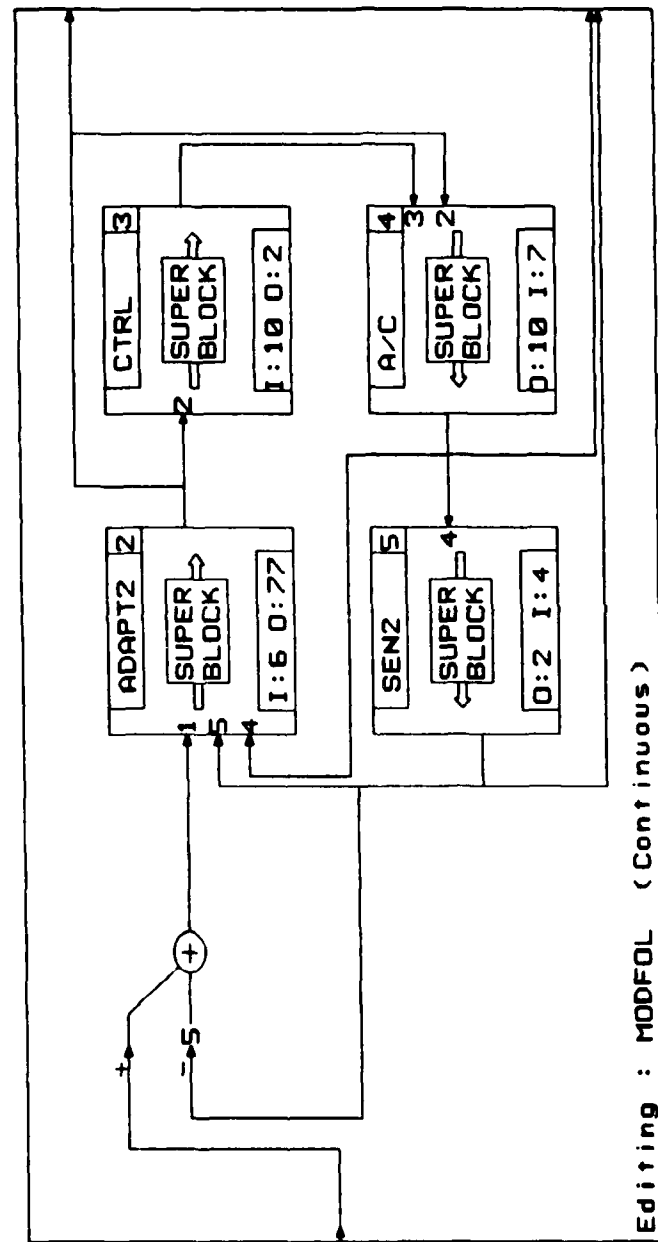


Figure 6-1. Top Level Description of Parameter Adaptive Simulation Package

The reference tracking signals used are those that were implemented by Pineiro and Berens. The inputs were obtained from real-time, nonlinear simulation with elevators and flaperons as control surfaces and with flight path angle and pitch rate as outputs. The reference tracking signals are shown on Figures 6-2 and 6-3.

In preparation to run the simulations, several data files were created to input specific flight condition data along with the necessary control surface deflection limits into the parameter adaptive simulation software. These data files consist of the A and B matrices of the state space representation of a flight condition as presented in Appendix A, along with the elevator and flaperon position limits as listed in Appendix C. This flight condition data was then input using the MATRIX_x System Build options into the block labeled A/C as shown on Figure 6-1. In addition, several parameter vectors were created from the state space models using MATRIX_x. These parameter vectors were input as needed into the ADAPT2 block (see Figure 6-1) as the fixed secondary parameter estimator models of the multiple model parameter estimator algorithm (see Figure 4-2).

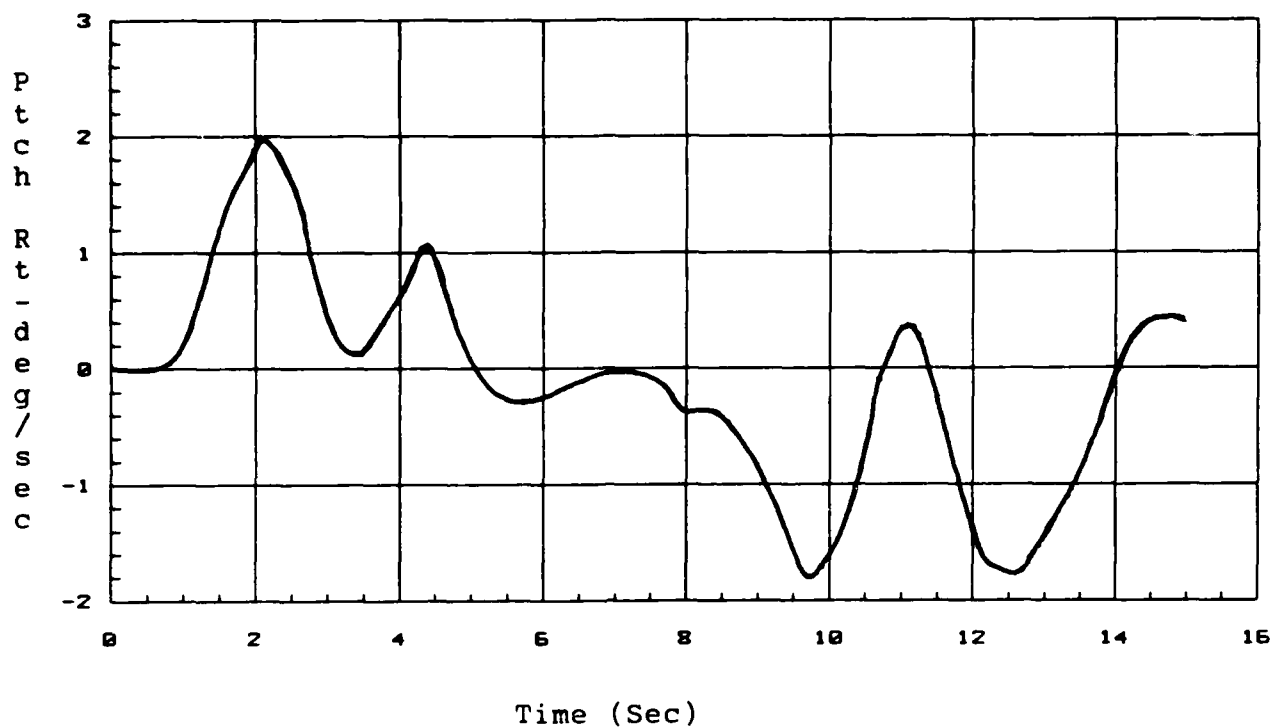


Figure 6-2. Pitch Rate Input

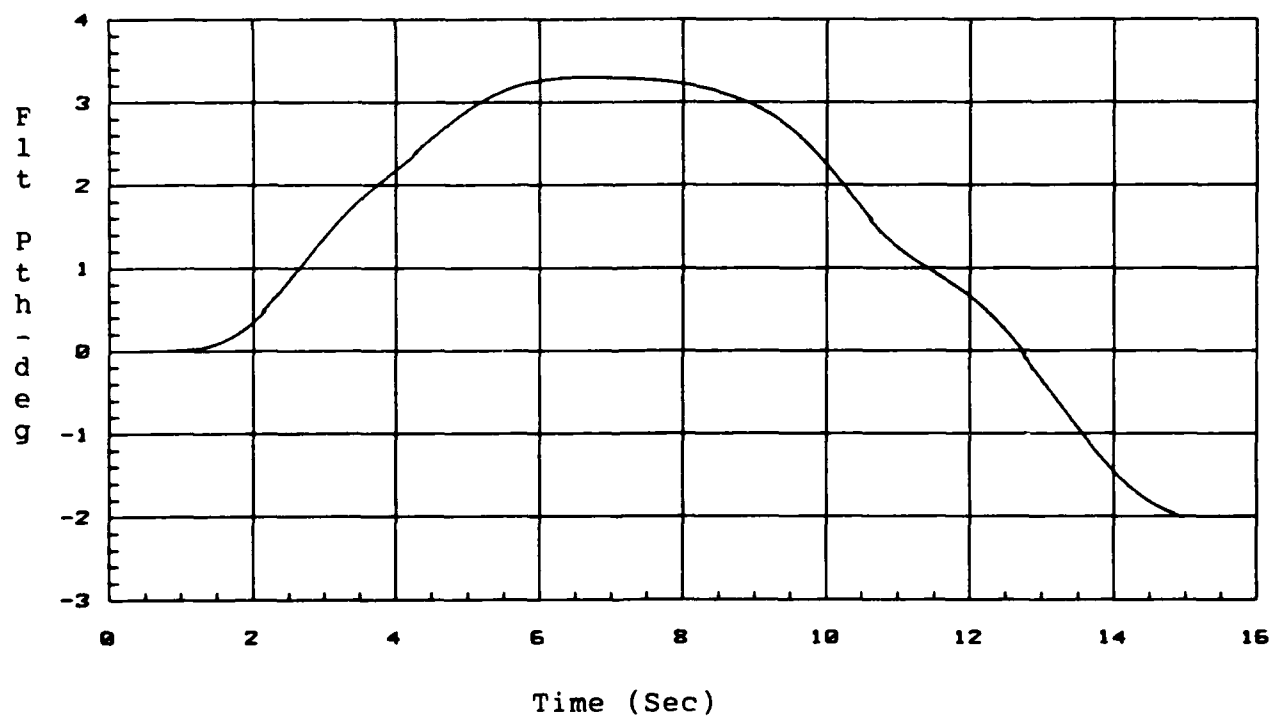


Figure 6-3. Flight Path Angle Input

6.2 Simulation Results

6.2.1 Single Model Analysis

6.2.1.1 Performance Boundary Evaluation. In preparation for determining the number of models and the spacing of those models to obtain satisfactory tracking performance over the desired flight envelope, fixed gain simulations were run to determine the performance boundaries of a single model. Recall that, as discussed in Chapter 5, by using the performance criteria of Equations (5-5) and (5-6) performance boundaries for a given model can be obtained by replacing the nominal state space aircraft model with another aircraft model and running subsequent simulations. If the performance criteria are satisfied, then the non-nominal aircraft model yields satisfactory tracking performance with the nominal models control law gains. This satisfactory performance dictates that the off-nominal model is within the performance boundary for the nominal model. Failure to meet the performance criteria indicates that the off-nominal model is outside of the nominal model performance boundary.

The first point chosen as a nominal aircraft model was at the flight condition of 18,000 ft altitude and 0.45 Mach. The parameter vector (Appendix D) was calculated and imple-

mented into parameter adaptive system for the purpose of defining the control law gains via the step response matrix for fixed gain simulations. The first simulation was run with the nominal aircraft plant and the nominal gains. The response of this simulation is shown on Figures 6-4 and 6-5. Figure 6-4 plots the pitch rate input versus the output response while Figure 6-5 shows a plot of the performance index for this simulation. Figures 6-6 through 6-9 present the elevator and flaperon positions and rates for this simulation. From the data presented for this simulation, it can be seen that the performance criteria are well satisfied and that tracking response of the reference signal is very good. It was noticed early in the simulations that the pitch rate response was the critical response for the commanded inputs. The pitch rate response failed the performance criteria well before the flight path angle response for all simulations run. For this reason, only the pitch rate responses are presented. Subsequent simulations were made by replacing the nominal aircraft model with aircraft models at the same altitude and decreased/increased Mach number. Mach number was decreased/increased from the nominal point in increments of 0.05 until the resulting data

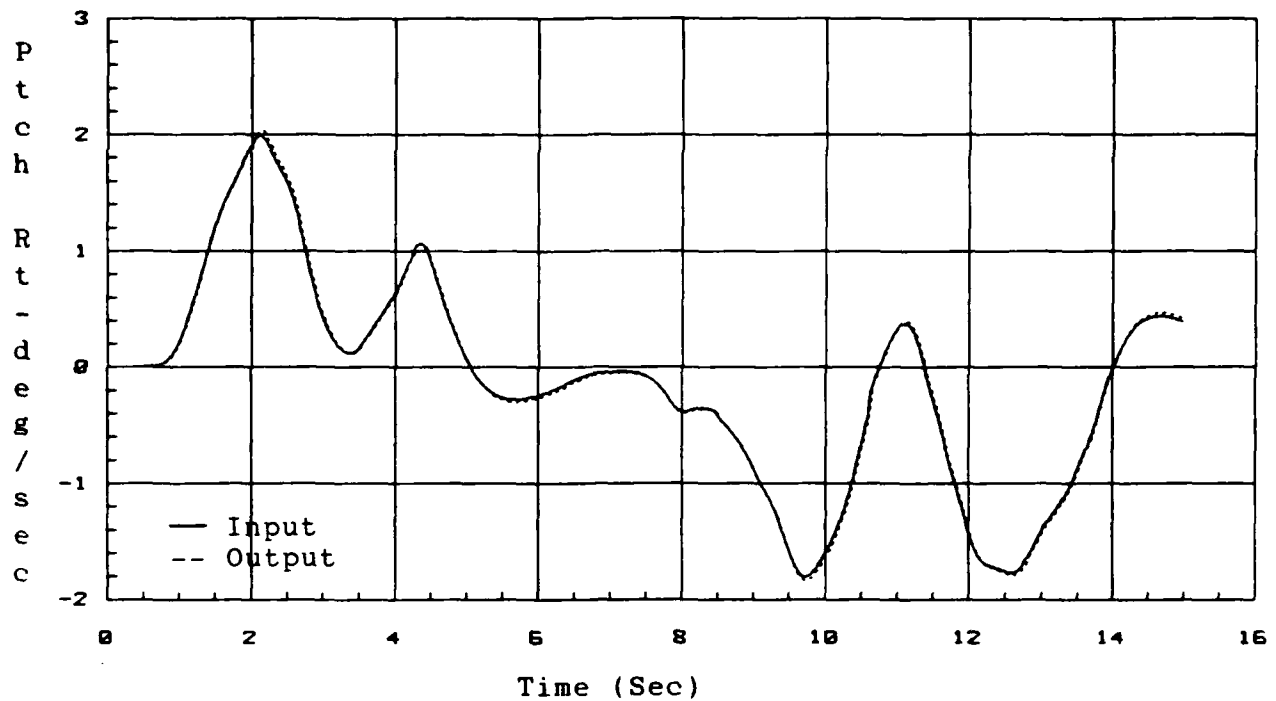


Figure 6-4. Pitch Rate Response
Nominal Model: 18K 0.45M/Operating Point: 18K 0.45M

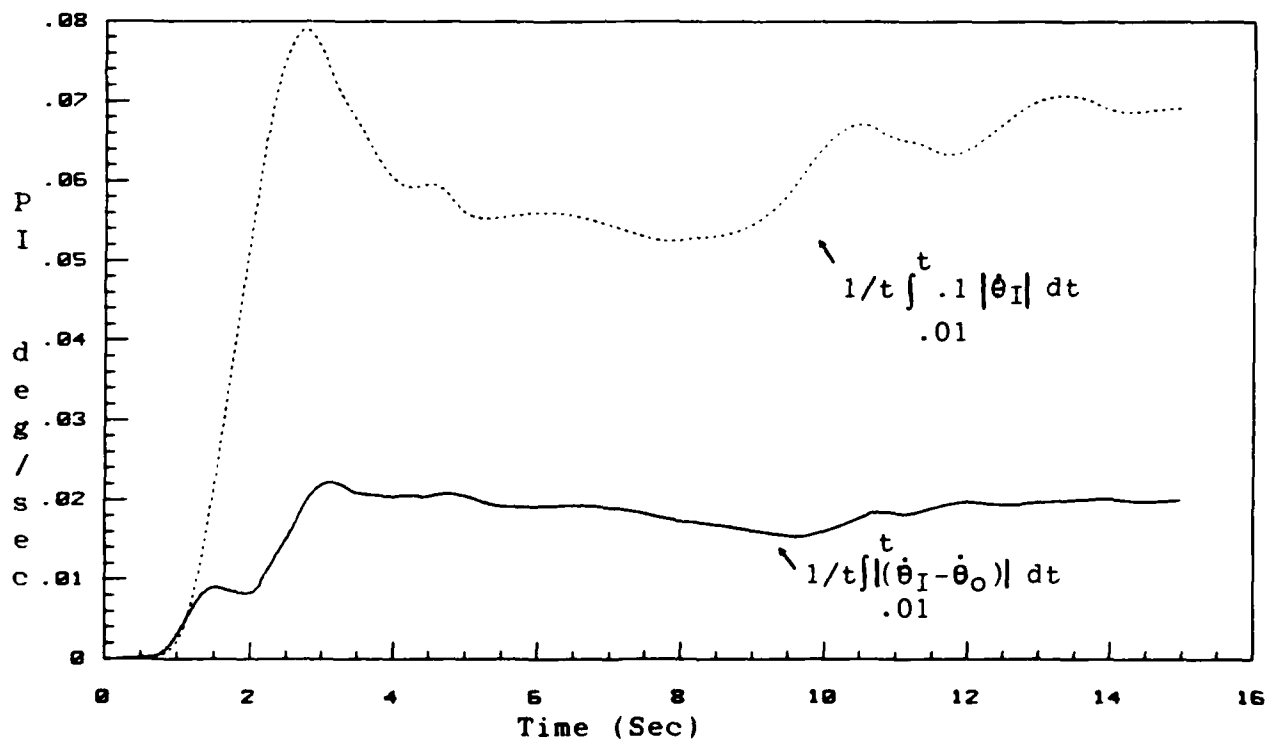


Figure 6-5. Pitch Rate Performance Criterion
Nominal Model 18K 0.45M/Operating Point: 18K 0.45M

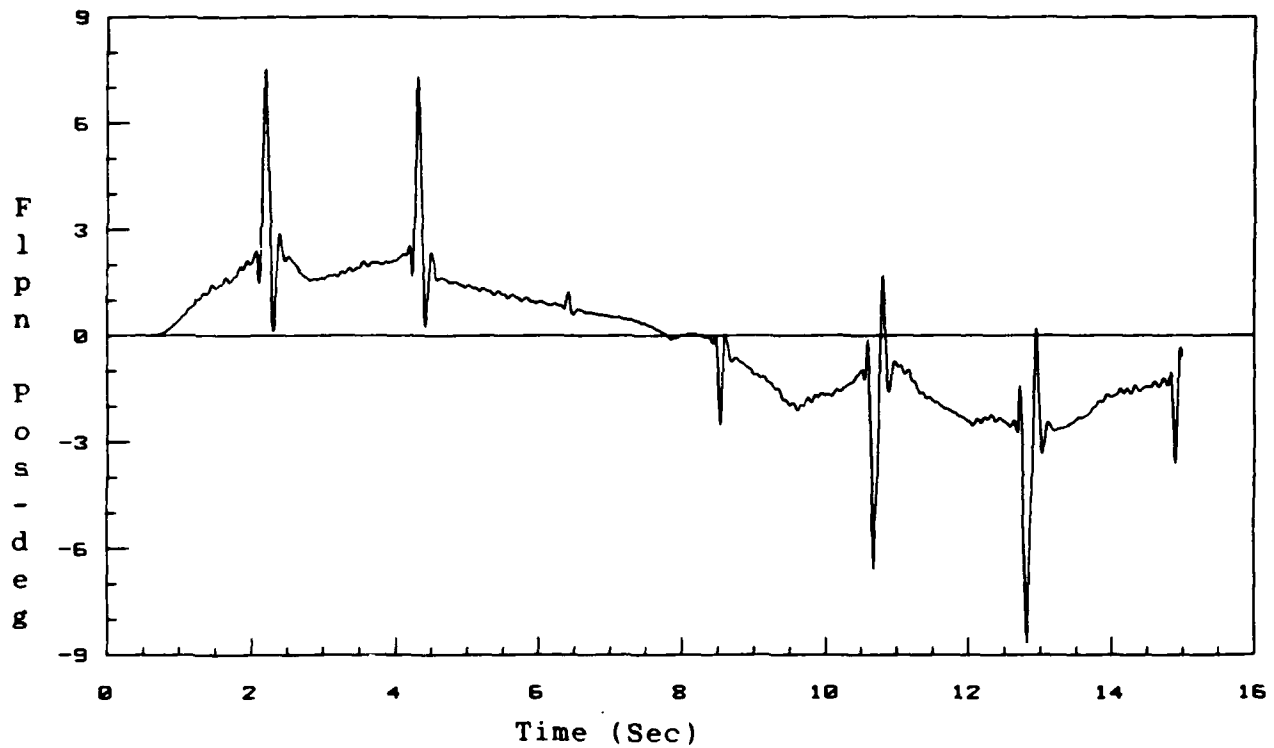


Figure 6-6. Flaperon Position
Nominal Model: 18K 0.45M/Operating Point: 18K 0.45M

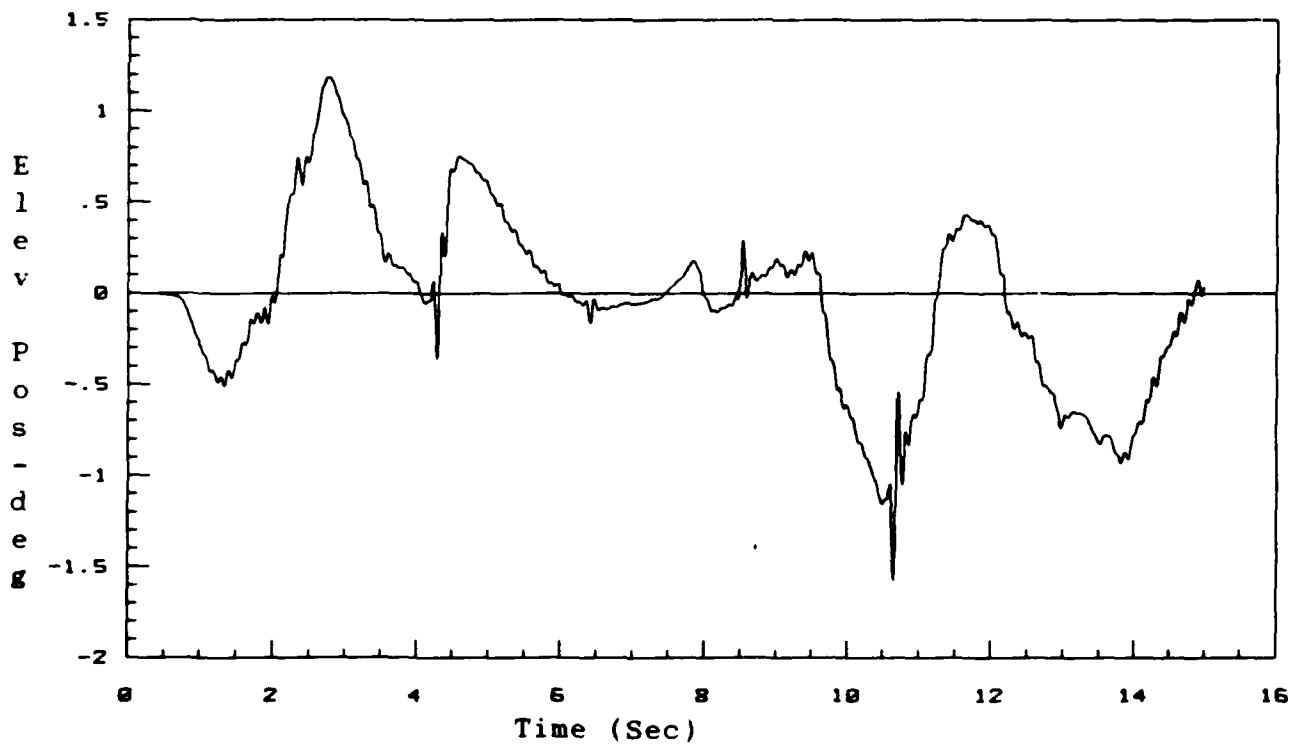


Figure 6-7. Elevator Position
Nominal Model: 18K 0.45M/Operating Point: 18K 0.45M

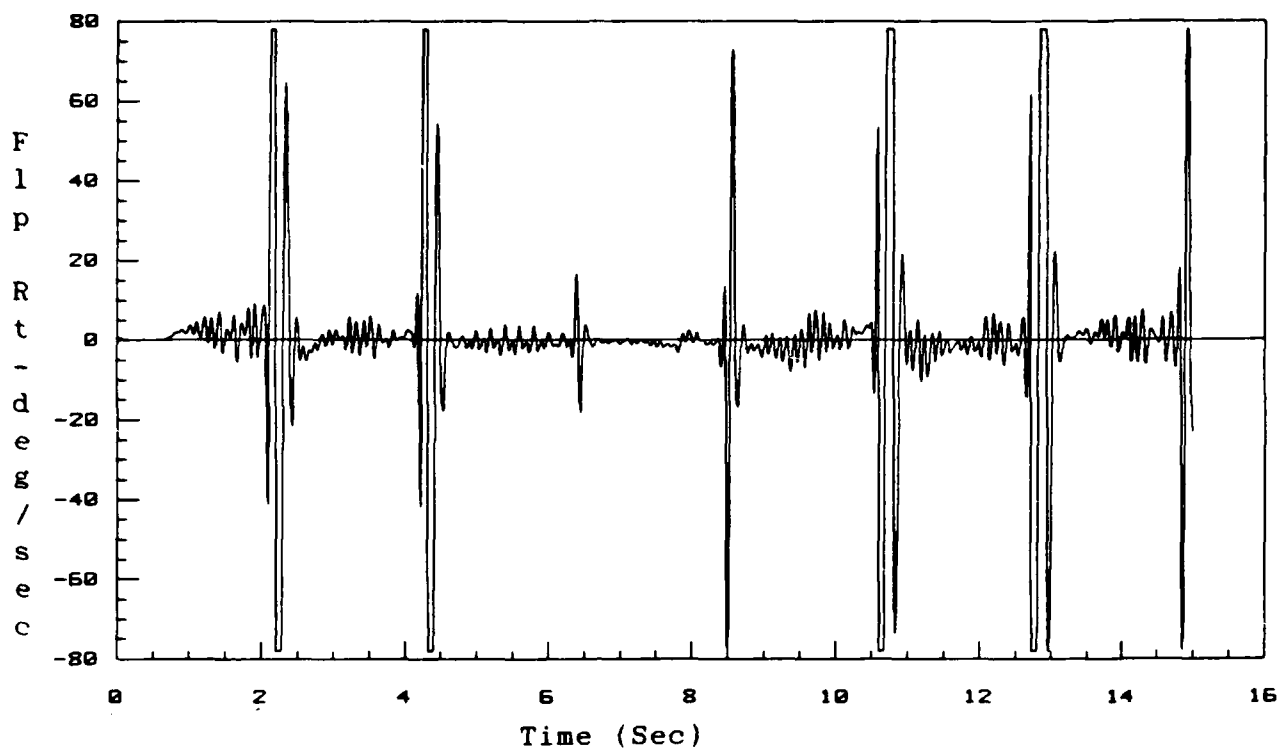


Figure 6-8. Flaperon Rate
Nominal Model: 18K 0.45M/Operating Point: 18K 0.45M

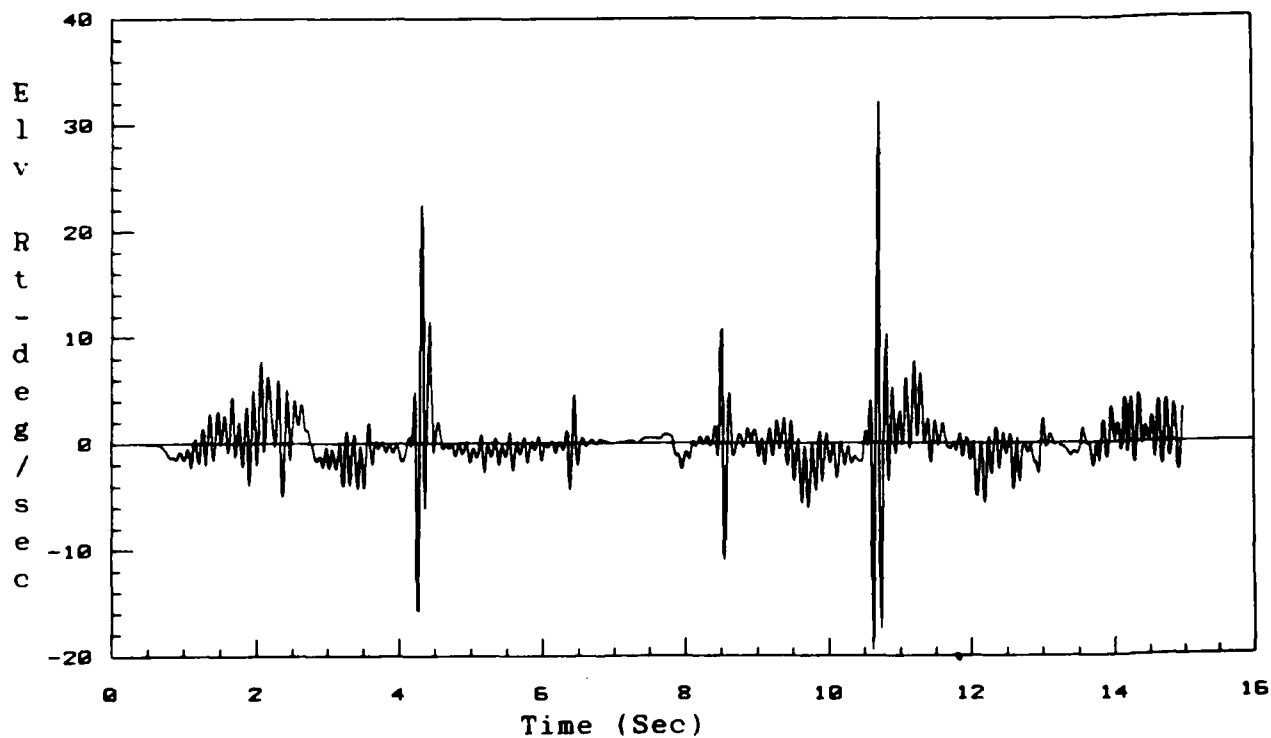


Figure 6-9. Elevator Rate
Nominal Model: 18K 0.45M/Operating Point: 18K 0.45M

showed that the performance criteria had been violated.

Figures 6-10 through 6-15 present the data when the aircraft model is set at 18,000 ft and 0.8 Mach. From the data, it can be seen that the performance criteria is violated and hence the model is outside the performance boundary for the nominal model of 18,000 ft, 0.45 Mach.

Following the simulations at 18,000 ft, the control law gains for the nominal plant were used with aircraft plant models at altitudes of 10,000, 14,000, 22,000, 26,000, 30,000, 34,000 and 38,000 ft with Mach number varying from 0.25 to 0.9 in increments of .05. The flight conditions that failed to meet the performance criteria are found in Table 6-1. Using the data presented in Table 6-1, the performance boundary for the 18,000 ft, 0.45 Mach nominal point can be obtained and is shown on Figure 6-16. From Figure 6-16, it can be seen that the performance boundary encompasses the dynamic pressure line on which the nominal plant resides. Figure 6-16 also shows that the upper and lower limits of the performance boundary are contingent on the differential of dynamic pressure between the nominal model and the actual flight condition model.

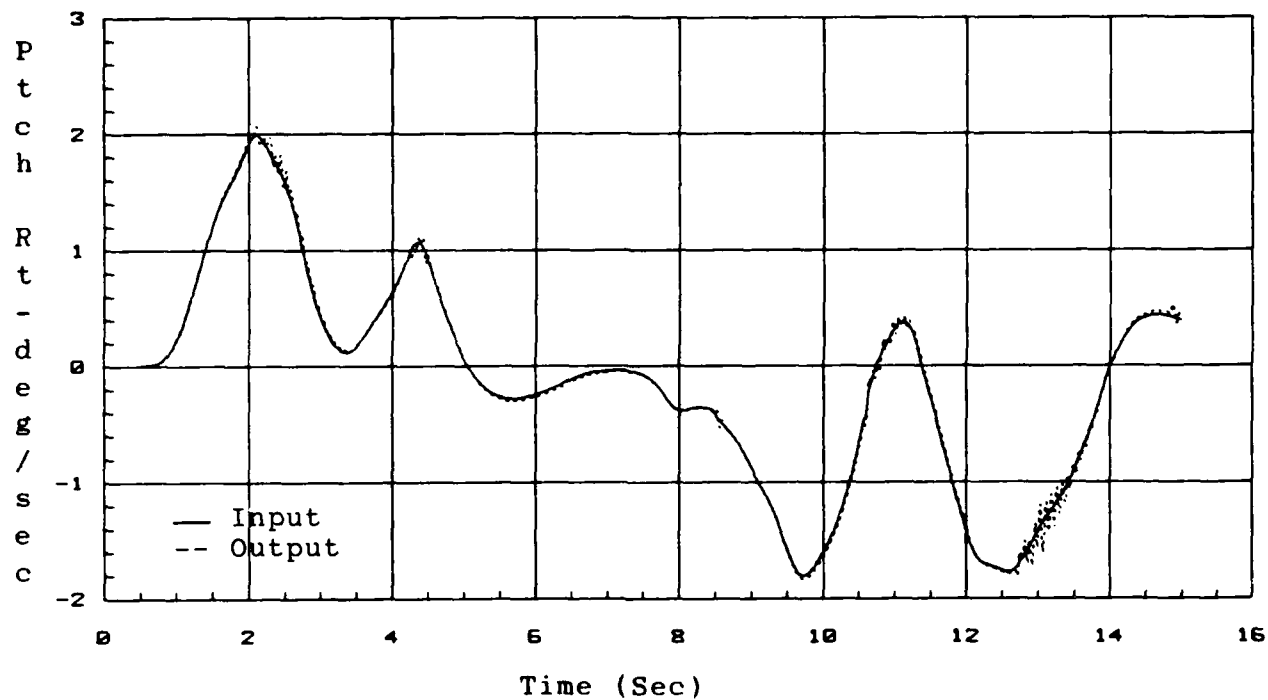


Figure 6-10. Pitch Rate Response
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

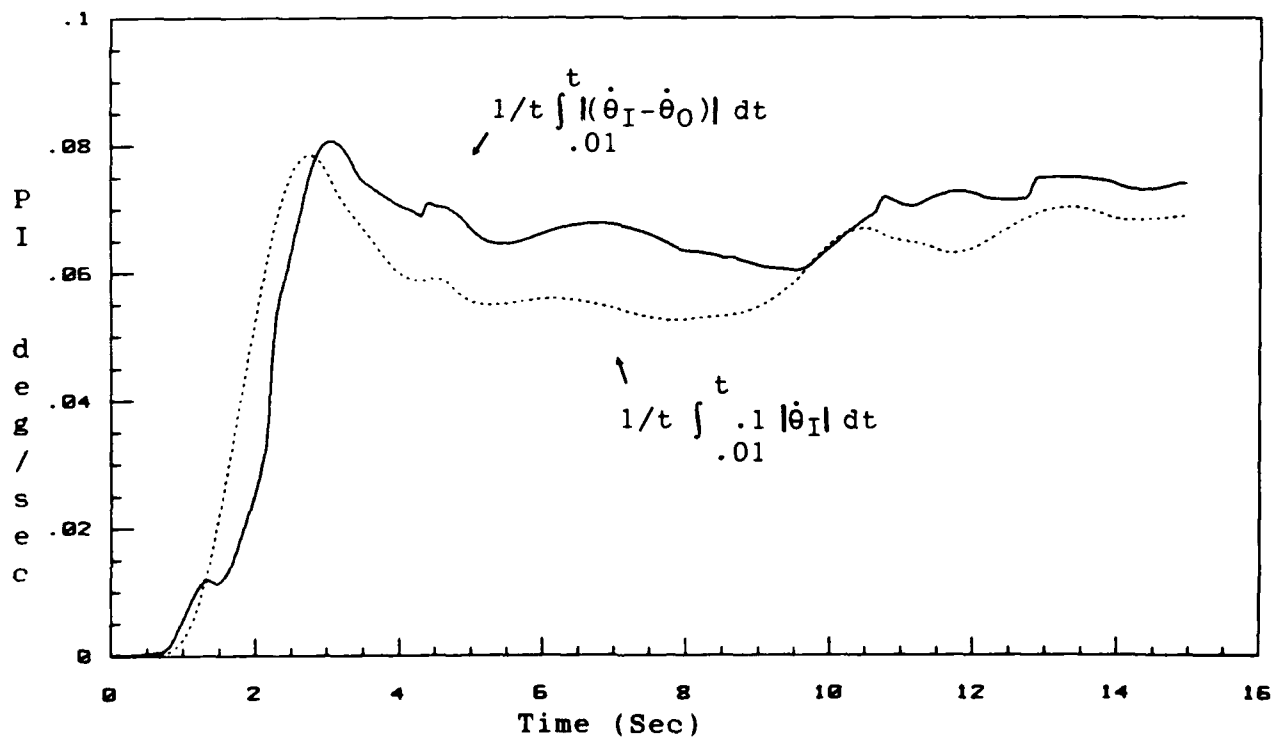


Figure 6-11. Pitch Rate Performance Criterion
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

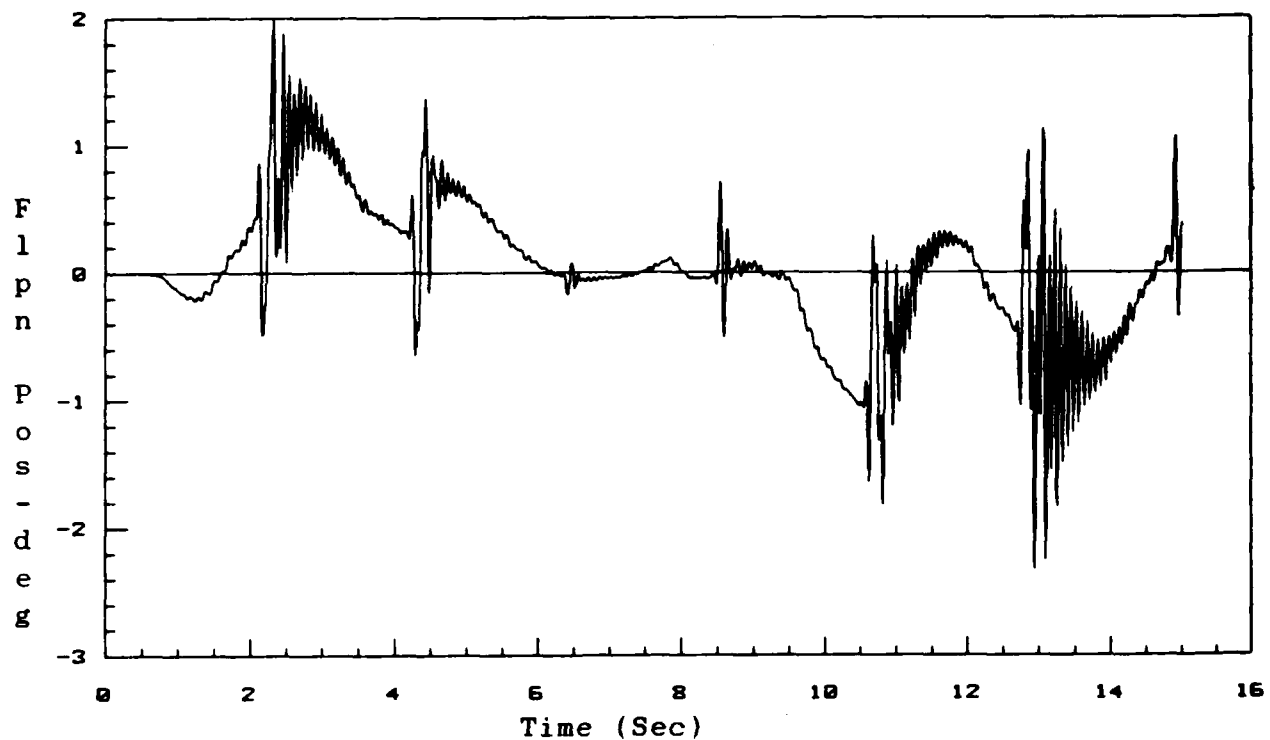


Figure 6-12. Flaperon Position
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

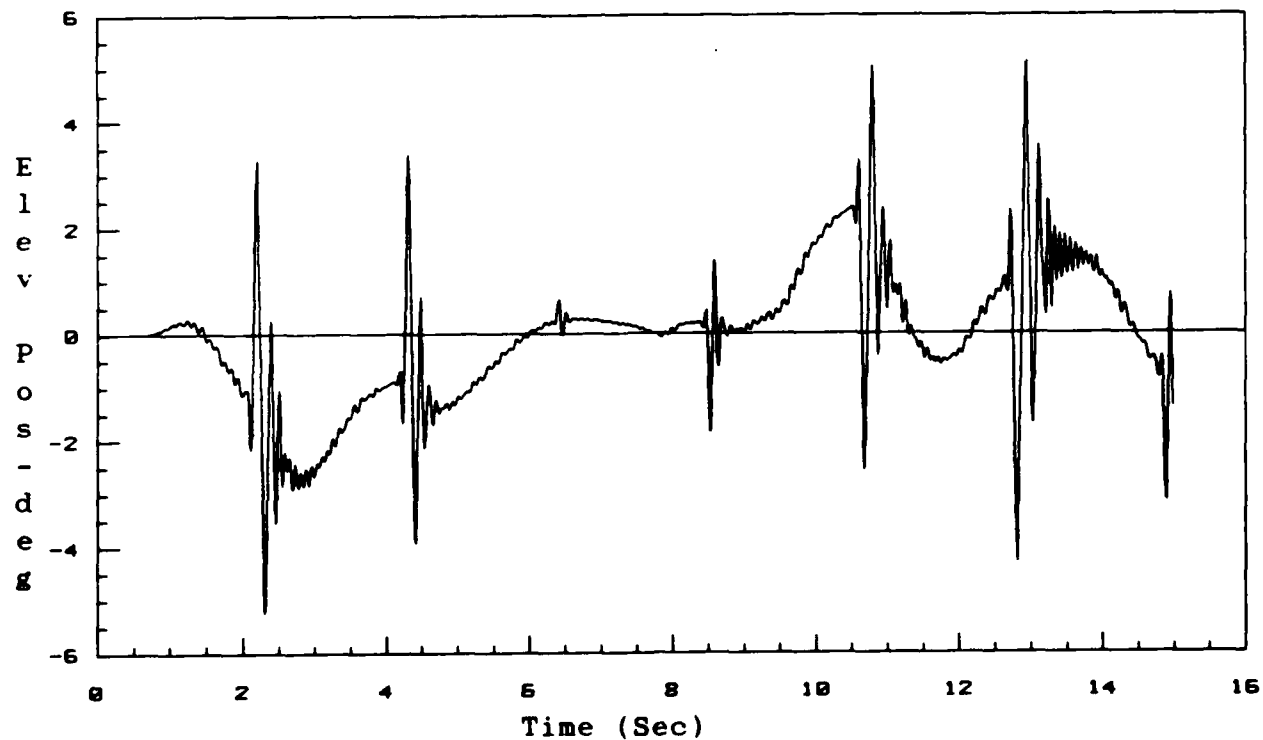


Figure 6-13. Elevator Position
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

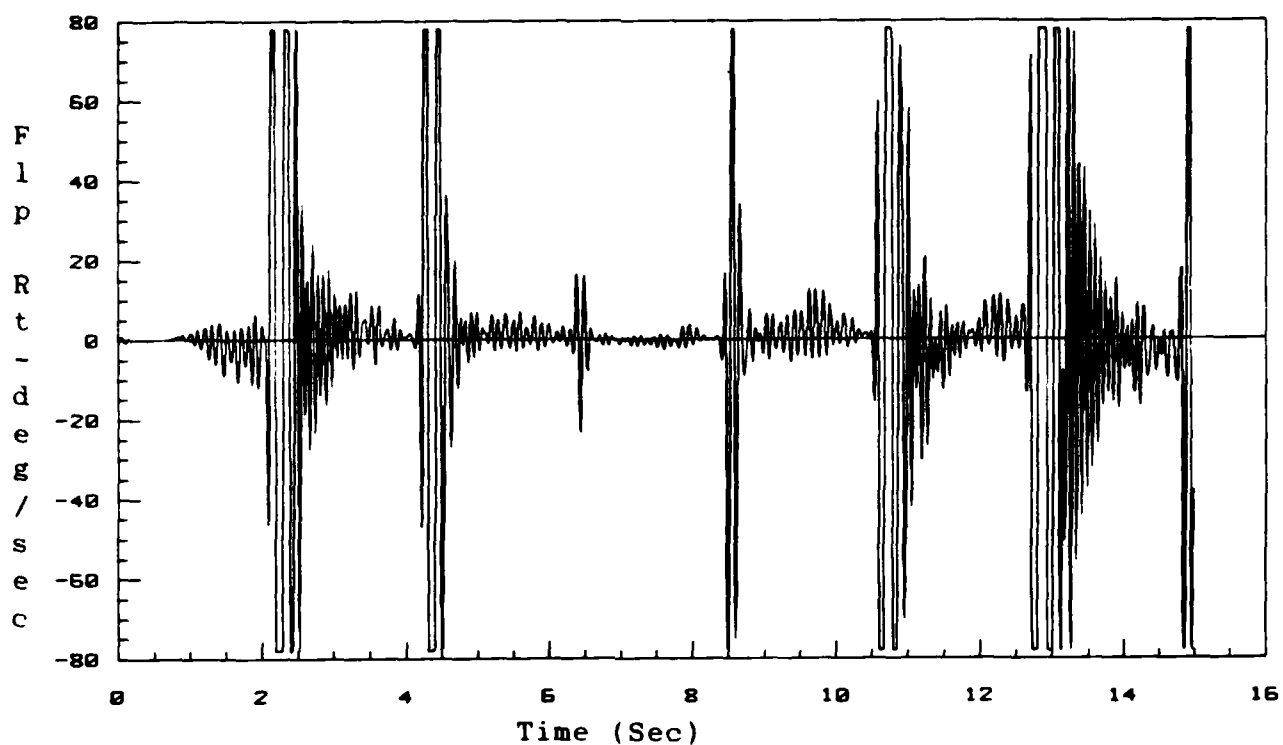


Figure 6-14. Flaperon Rate
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

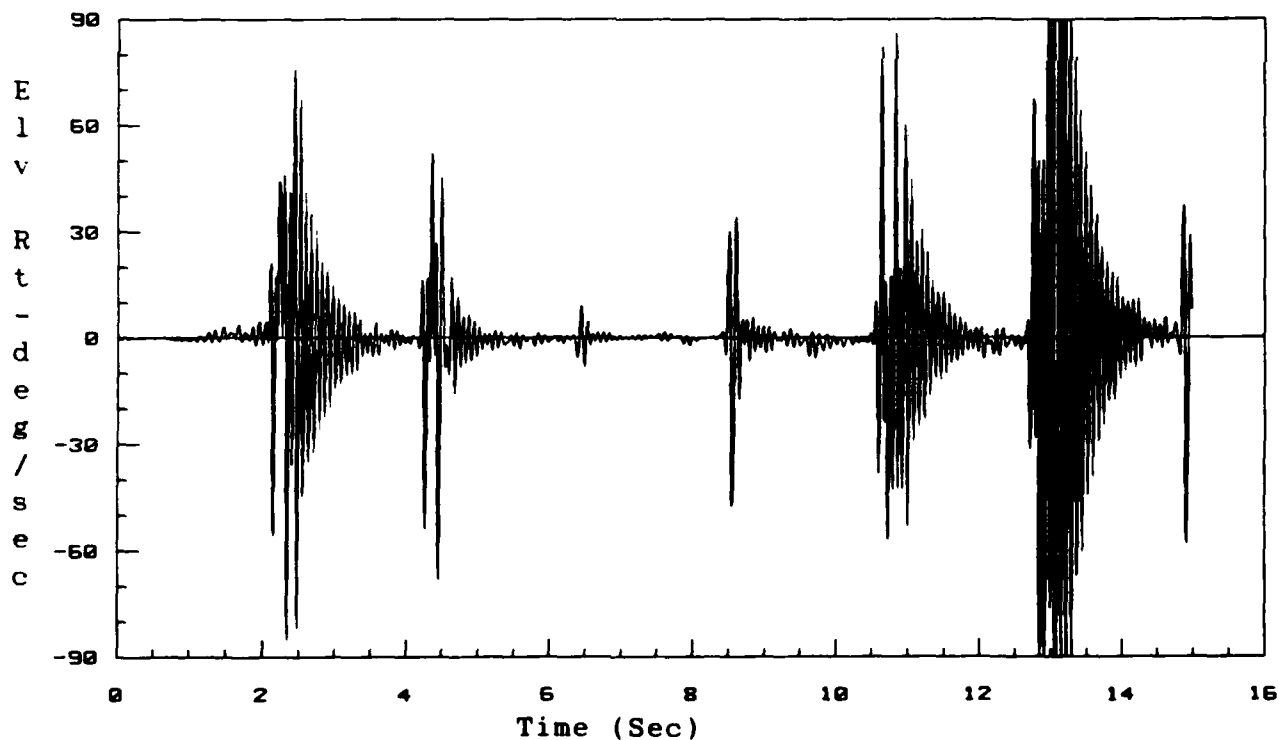


Figure 6-15. Elevator Rate
Nominal Model: 18K 0.45M/Operating Point: 18K 0.8M

Table 6-1

Flight Conditions Failing Performance Criteria
Nominal Flight Condition = 18,000 feet 0.45 Mach

Flight Conditions of Failure		
Altitude (Kft)	Mach Number	Dynamic Pressure
10	0.25	63.68
10	0.70	499.27
14	0.30	78.35
14	0.75	489.69
18	0.30	66.62
18	0.80	473.75
22	0.35	76.71
22	0.85	452.45
26	0.40	84.30
26	0.90	426.79
30	0.45	89.26
34	0.50	91.61
38	0.55	91.61

Due to the fact that the performance boundary above seemed to be dependent on dynamic pressure, the next flight condition was chosen so that the dynamic pressure was approximately the same as the 18,000 ft 0.45 Mach point. This led to choosing the next flight condition at 38,000 ft, 0.7 Mach. Flight conditions that failed to meet the performance criteria for this nominal flight condition are given in Table 6-2 and the resulting performance boundary is presented on Figure 6-17. Comparing the performance boundaries of

Table 6-2

Flight Conditions Failing Performance Criteria
Nominal Flight Condition = 38,000 feet 0.7 Mach

Flight Conditions of Failure		
Altitude (Kft)	Mach	Dynamic Pressure
10	0.25	63.68
10	0.70	499.27
14	0.30	78.35
14	0.75	489.69
18	0.35	90.68
18	0.80	473.75
22	0.35	76.71
22	0.85	452.45
26	0.40	84.30
26	0.90	426.79
30	0.45	89.26
34	0.50	91.61
38	0.55	91.61

Figures 6-16 and 6-17 reveals that they are essentially the same. This indicates that, from a performance criteria standpoint, nominal models with approximately the same dynamic pressure yield the same performance boundaries. Several other performance boundaries were obtained for nominal models with similar dynamic pressures. These flight conditions along with the associated dynamic pressures are listed in Table 6-3. The result of obtaining performance boundaries for the flight conditions in Table 6-3 showed that, for

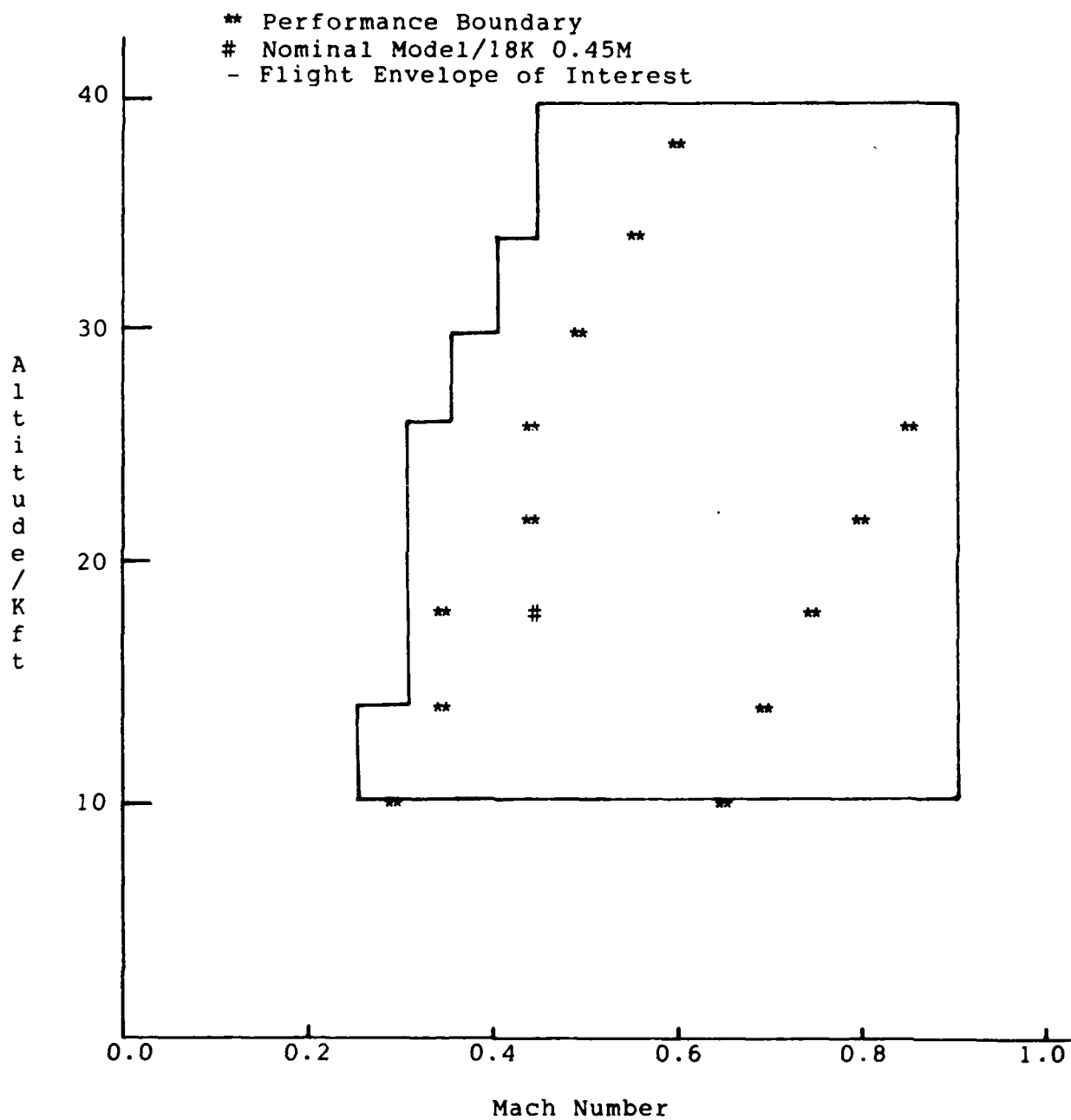


Figure 6-16. Performance Boundary for
Nominal Model at 18K 0.45M

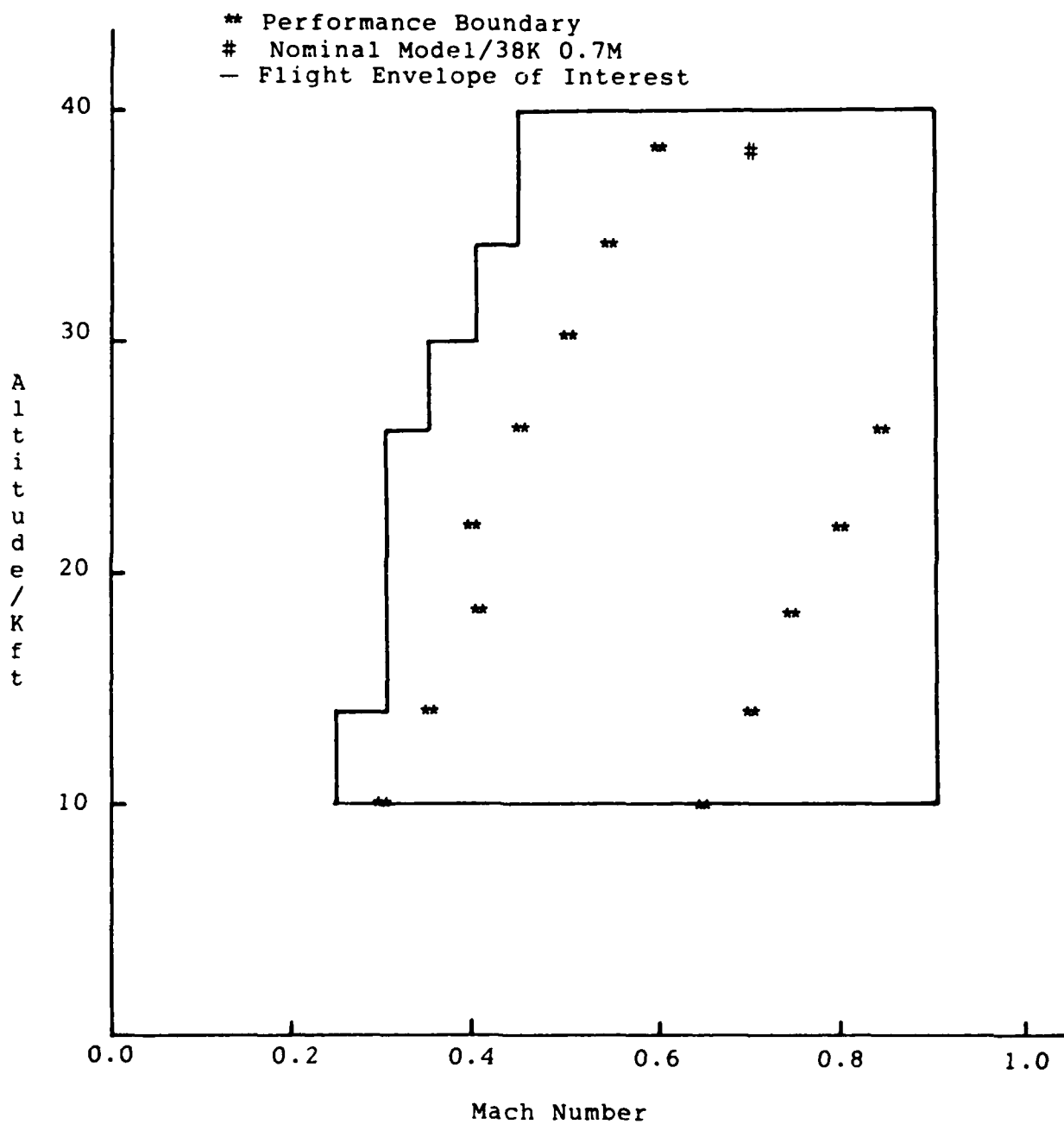


Figure 6-17. Performance Boundary for
Nominal Model at 38K 0.7M

Table 6-3.

Flight Conditions of Nominal Models With
Similar Dynamic Pressures

Case	Altitude (Kft)	Mach	Dynamic Pressure
1	22	0.65	264.58
	34	0.85	264.75
2	14	0.70	426.57
	26	0.90	426.79
3	26	0.50	131.73
	34	0.60	131.92
4	26	0.75	296.38
	34	0.90	296.82
5	30	0.50	110.19
	34	0.55	110.85
6	34	0.50	91.61
	38	0.55	91.61
7	34	0.75	206.12
	10	0.45	206.33
8	26	0.60	189.69
	22	0.55	189.43

each of the presented cases, models with similar dynamic pressures yield the same set of performance boundaries.

Another point that became evident was that, when running a fixed gain simulation, if the dynamic pressure of the aircraft model differed significantly from that of the nominal

model the performance criteria would be violated.

The fact that aircraft models with similar dynamic pressures yield equivalent performance boundaries is also indicated by examining the equations for the dimensionalized derivatives of the aircraft state space model. Recall that the numbers generated by implementing the dimensionalized derivative equations are inserted directly into the A matrix of the aircraft state space model. By examining the dimensionalized derivative equations, it can be seen that dynamic pressure is present in all but one of the equations. Not only is dynamic pressure present in the dimensionalized derivative equations, it is also the dominant variable of the parameters that are not constant. Therefore, dynamic pressure can be seen to have a large influence on the equivalency of one aircraft model with another.

Upon comparing the fixed gain simulation data generated for the 18,000 ft altitude, 0.45 Mach and 38,000 ft altitude, 0.7 Mach flight conditions, an interesting observation was made. Although the performance boundaries were equivalent for both flight conditions, the responses of the control surfaces differed for equivalent simulations. As an example, consider the flight condition of 26,000 ft altitude

and 0.45 Mach. Figures 6-18 through 6-23 show the responses for the case where the nominal model was 18,000 ft altitude, 0.45 Mach, and Figures 6-24 through 6-29 present data for the case where the nominal model was 38,000 ft altitude and 0.7 Mach. Comparison of the elevator and flaperon surface deflections for each nominal case shows that, although the general responses are similar, the nominal model of 38,000 ft altitude and 0.7 Mach has noisier surface deflection responses. The elevator deflection rates are much larger for the 38,000 ft, 0.7 Mach case and the flaperon deflection rates reach the rate limits more of the time. Because the control surfaces respond differently for the same flight conditions, the equivalence of one nominal model with another is based on more than just dynamic pressure. If dynamic pressure were the sole measure of model equivalence, not only should the performance boundaries be the same for nominal models with the same dynamic pressure, but the surface deflections and rates should be also. This would dictate that the eigenvalues of the A matrices of the nominal state space models and the control matrices be equal. The eigenvalues for the 18,000 ft, 0.45 Mach and 38,000 ft, 0.7 Mach flight conditions are found in Table 6-4. From the

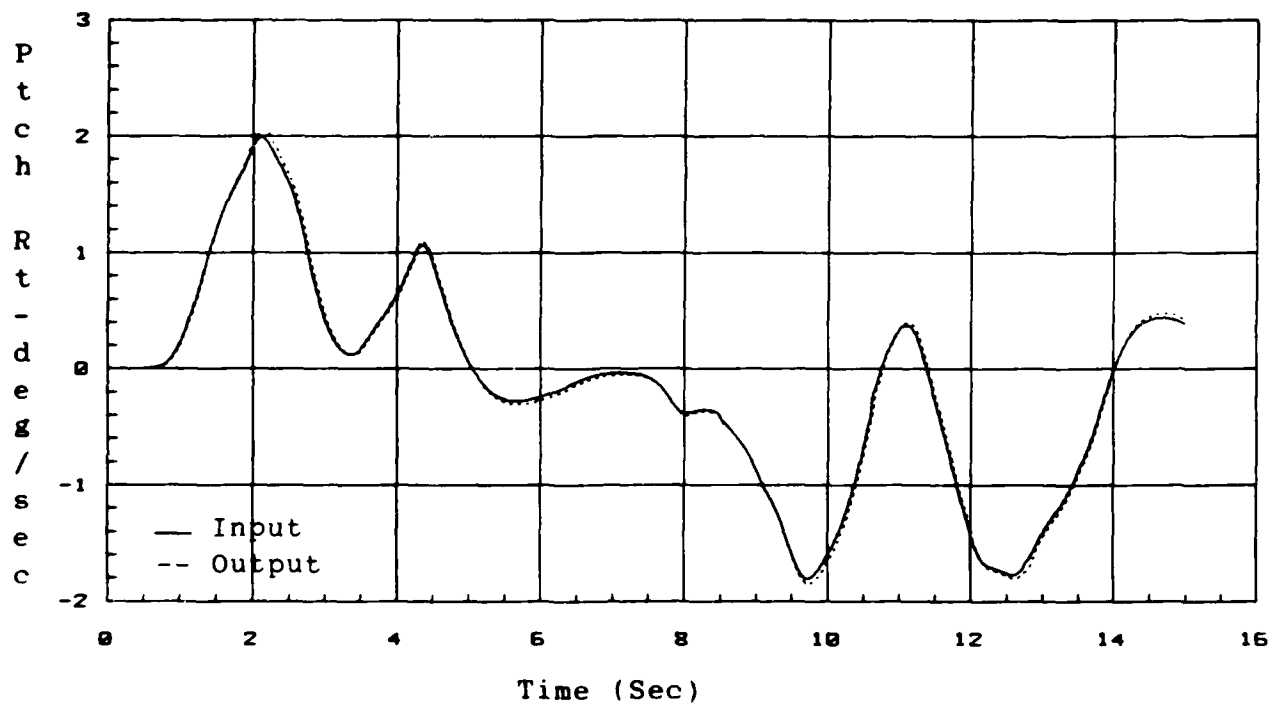


Figure 6-18. Pitch Rate Response
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

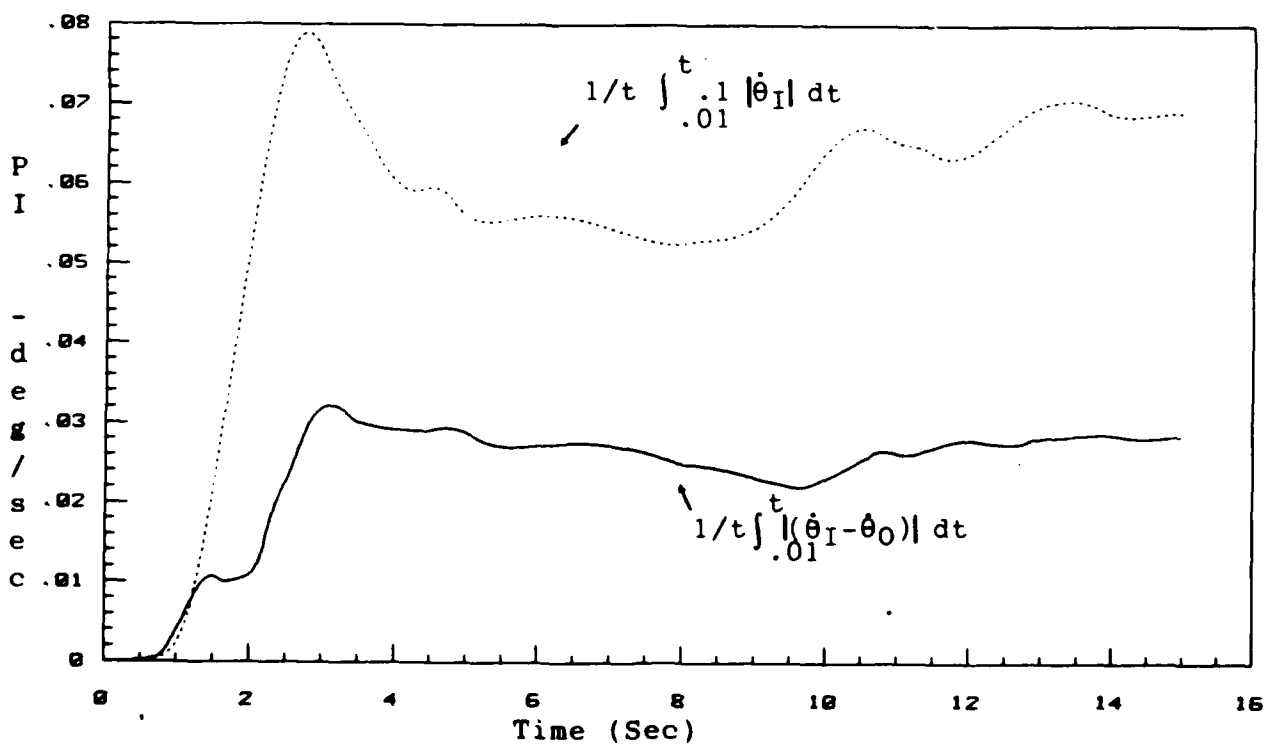


Figure 6-19. Pitch Rate Performance Criterion
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

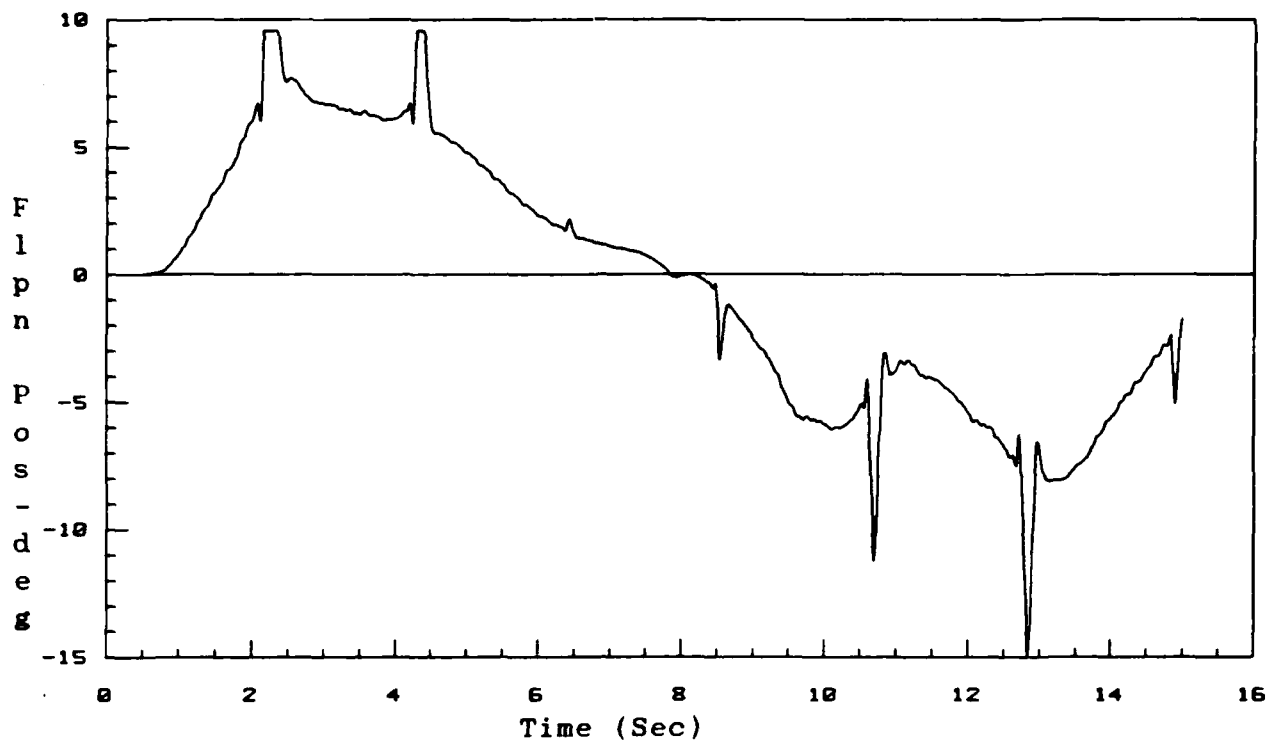


Figure 6-20. Flaperon Position
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

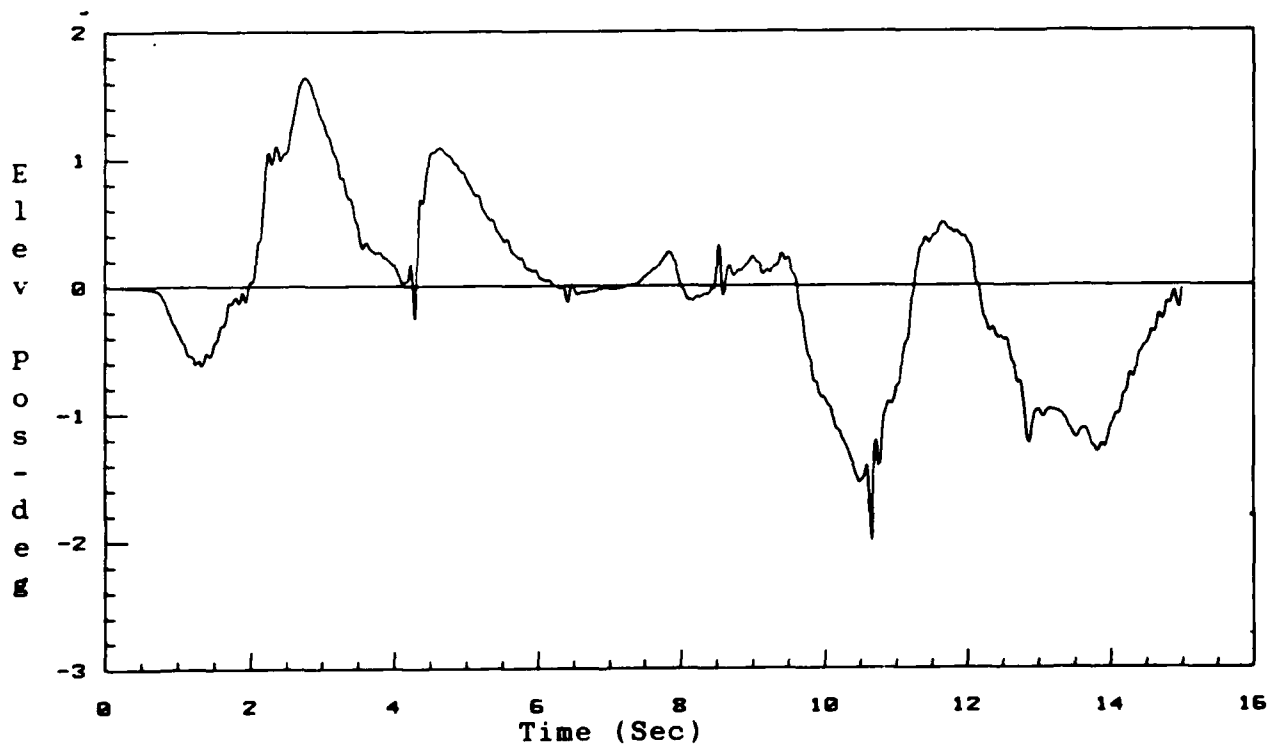


Figure 6-21. Elevator Position
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

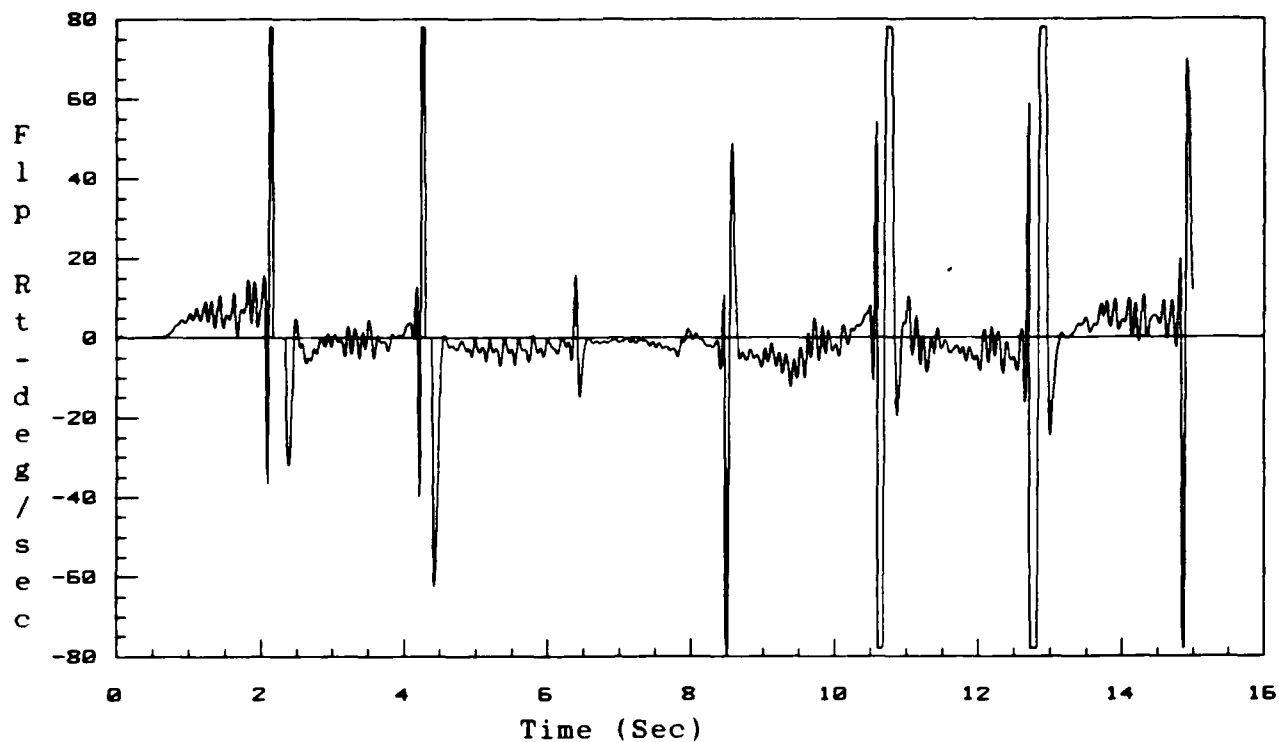


Figure 6-22. Flaperon Rate
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

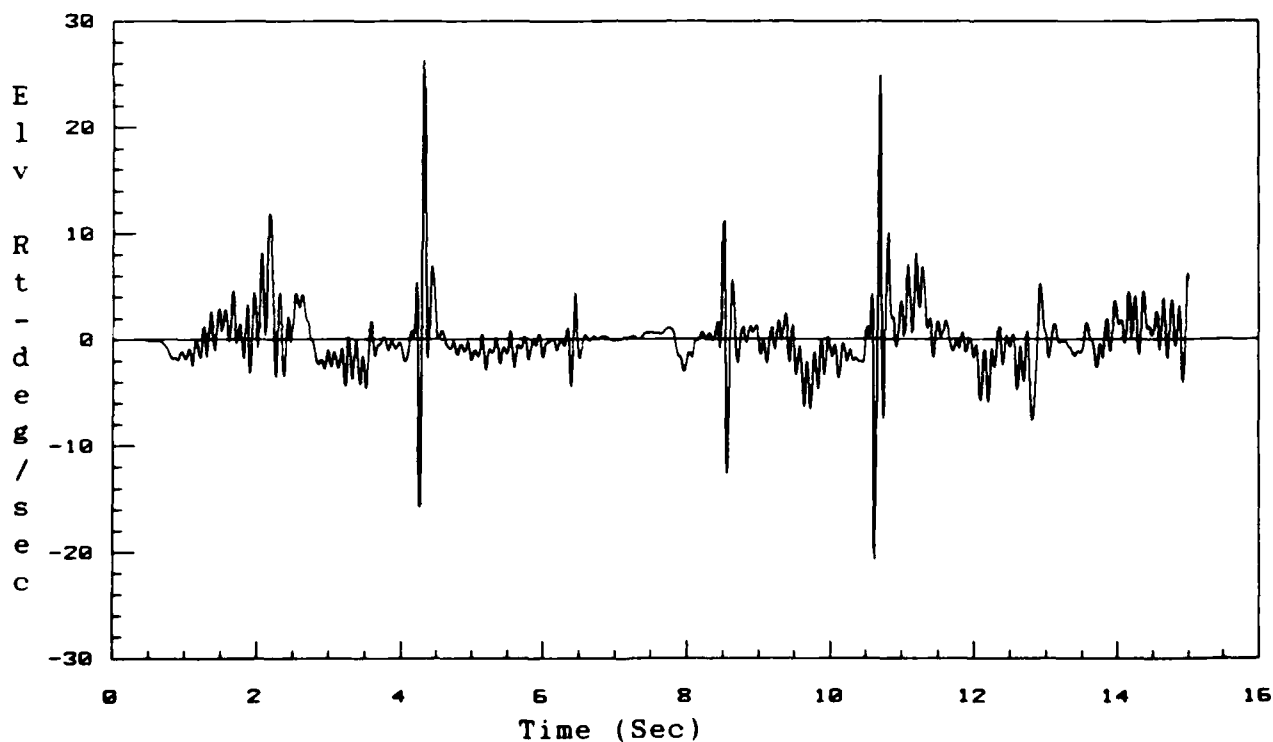


Figure 6-23. Elevator Rate
Nominal Model: 18K 0.45M/Operating Point: 26K 0.45M

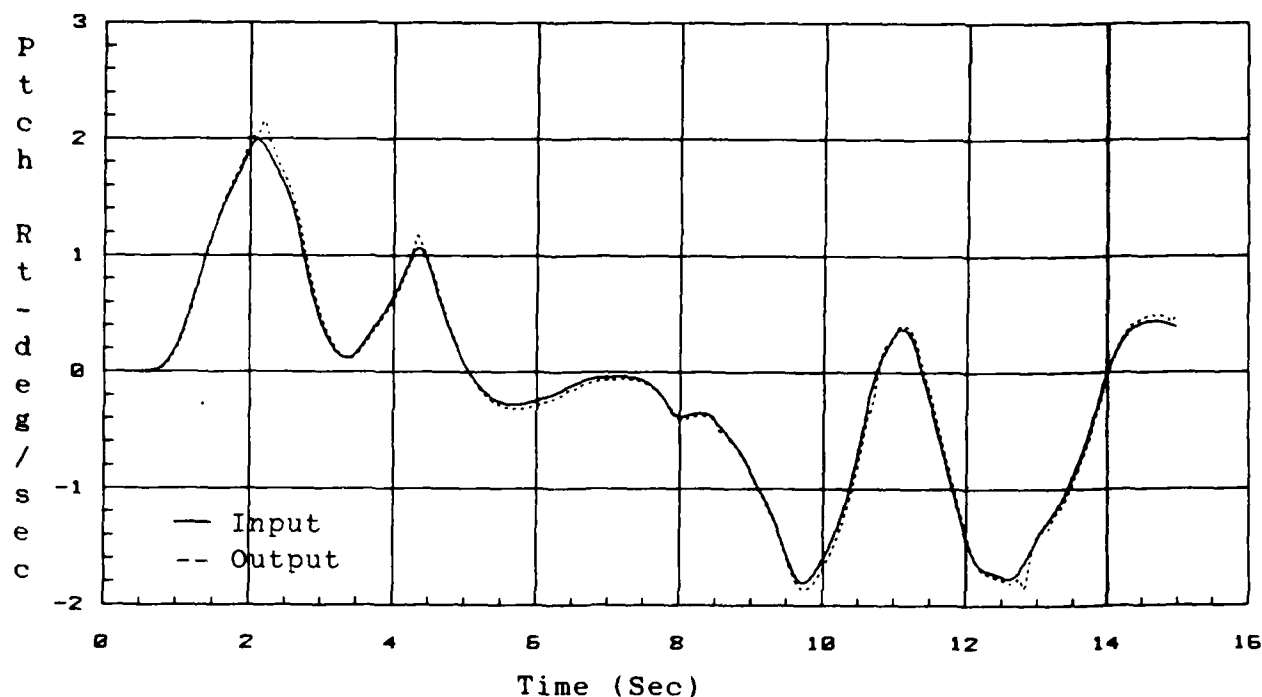


Figure 6-24. Pitch Rate Response
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

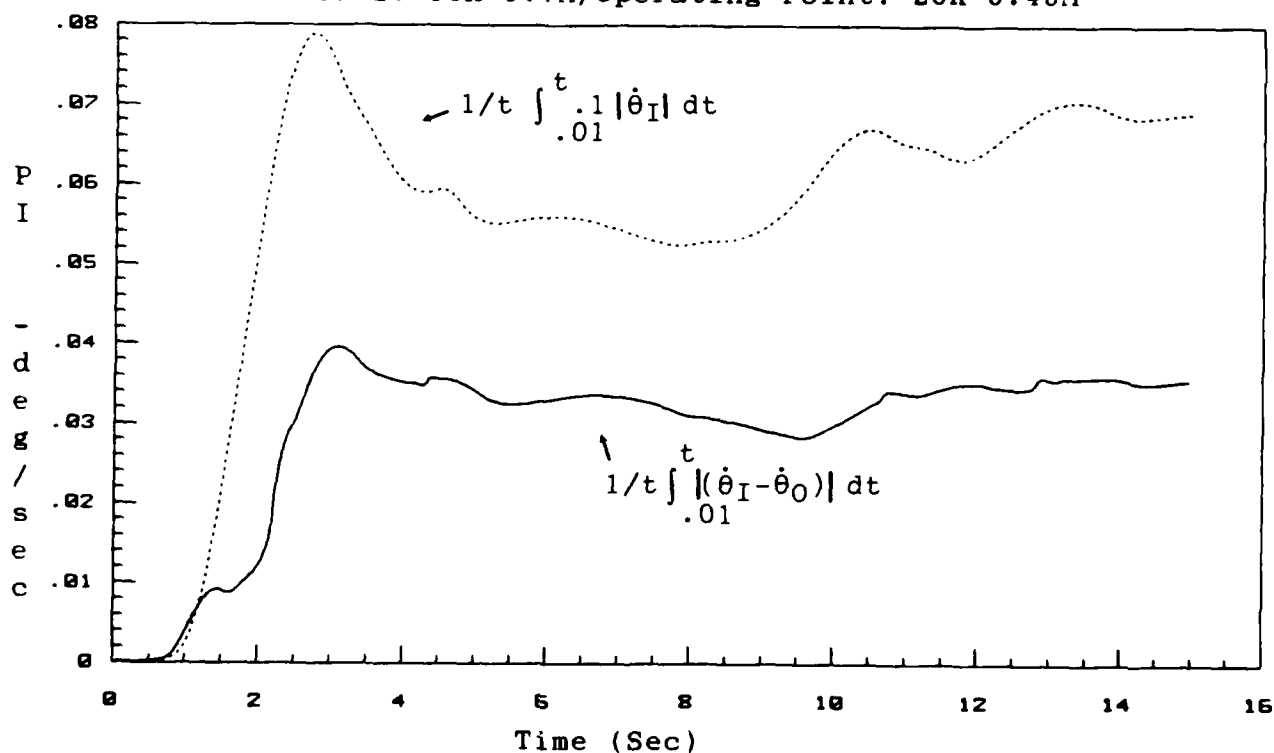


Figure 6-25. Pitch Rate Performance Criterion
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

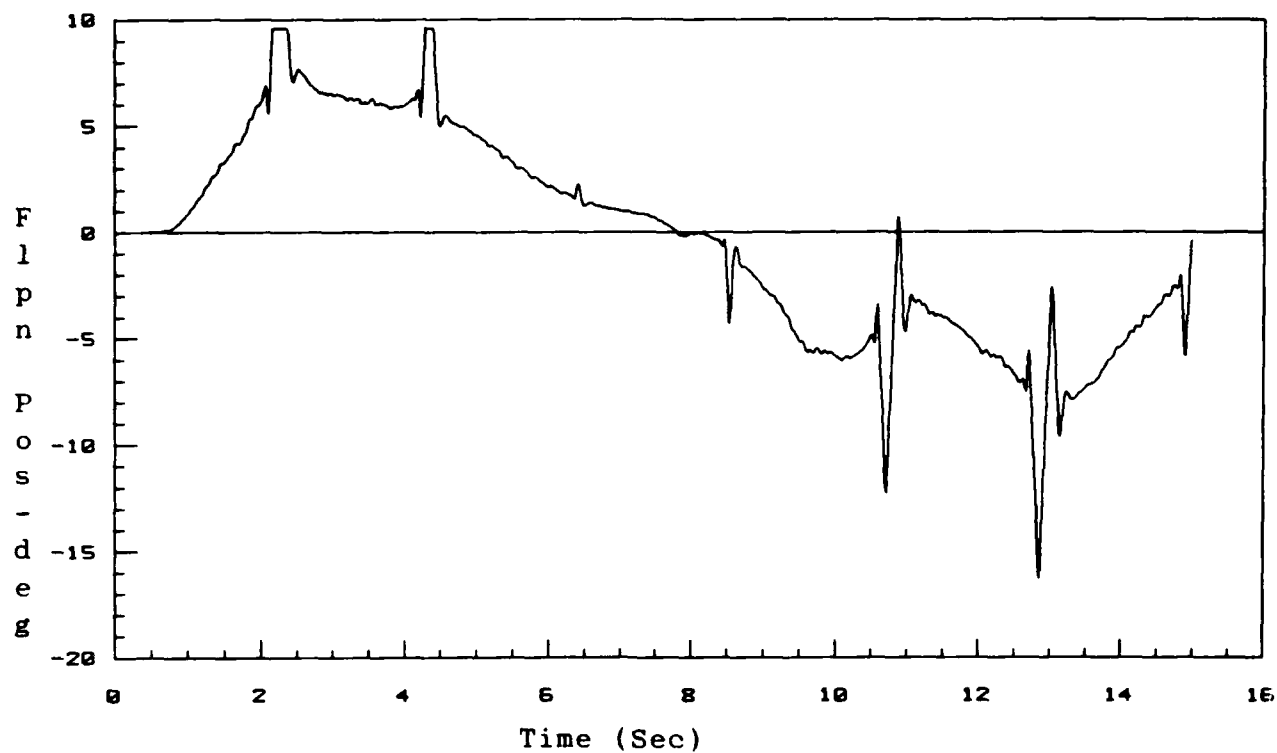


Figure 6-26. Flaperon Position
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

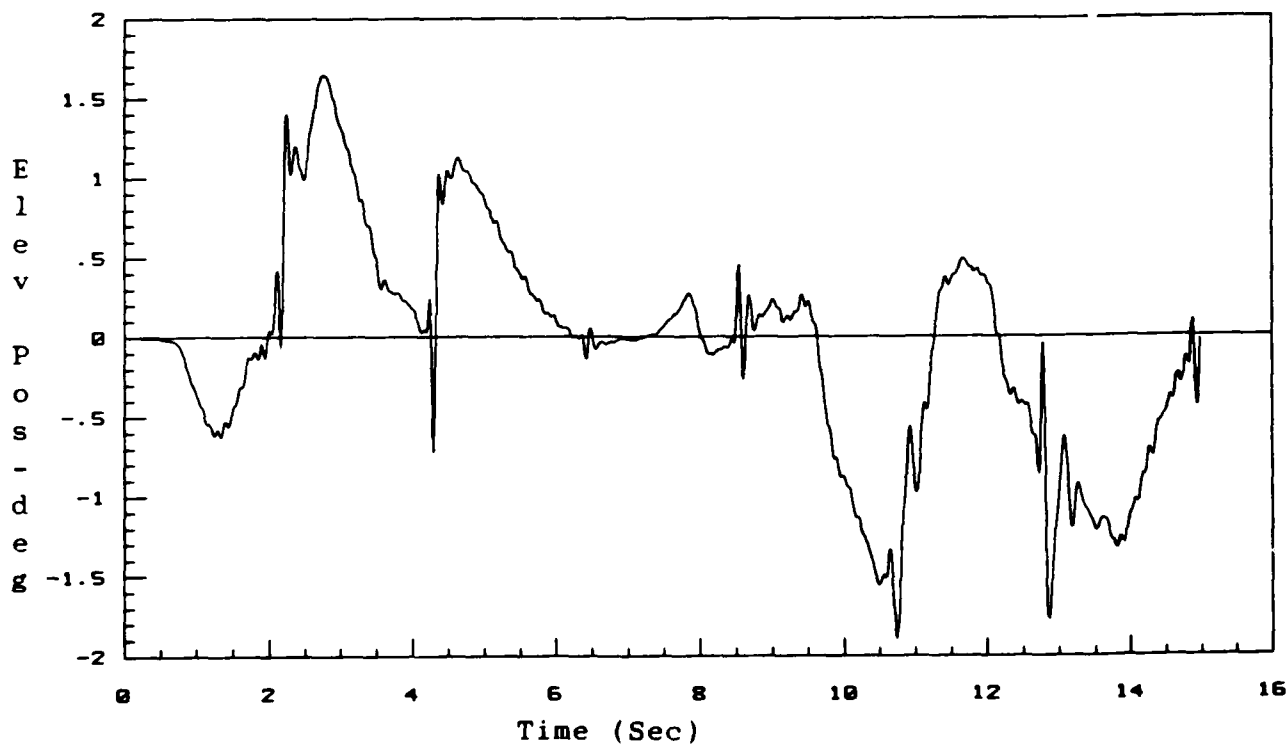


Figure 6-27. Elevator Position
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

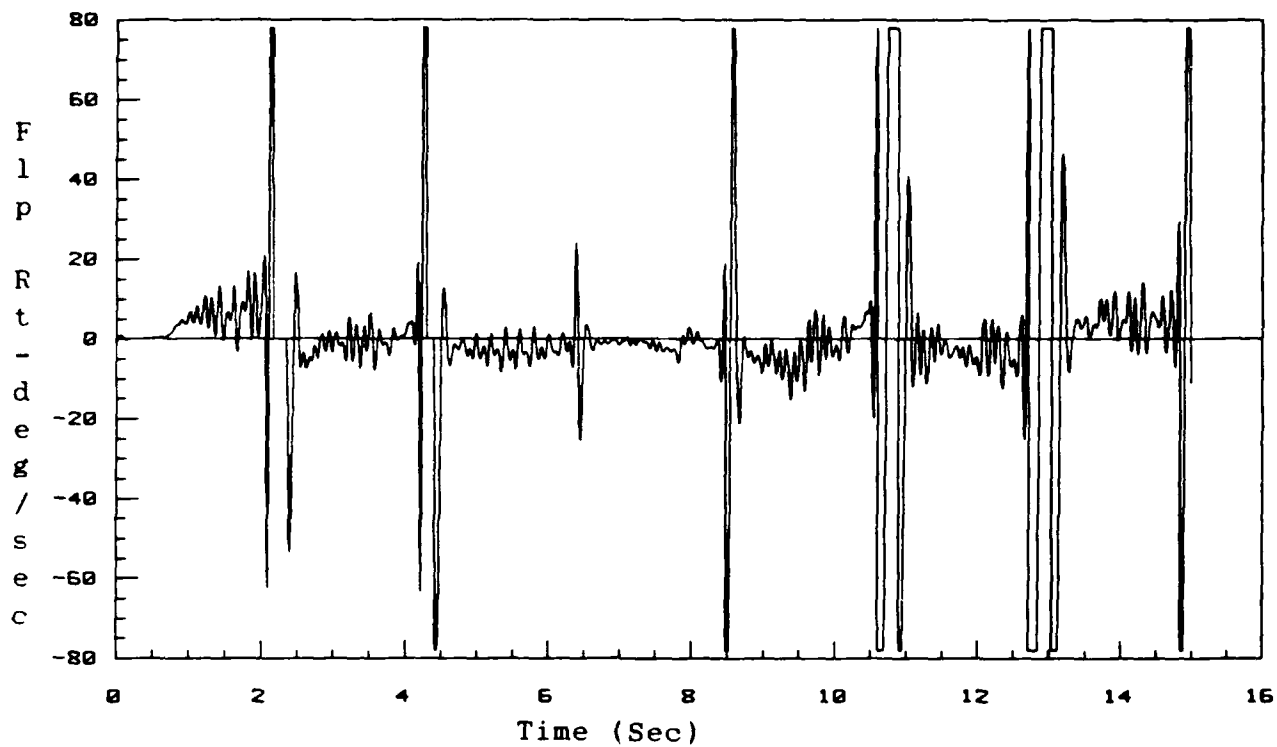


Figure 6-28. Flaperon Rate
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

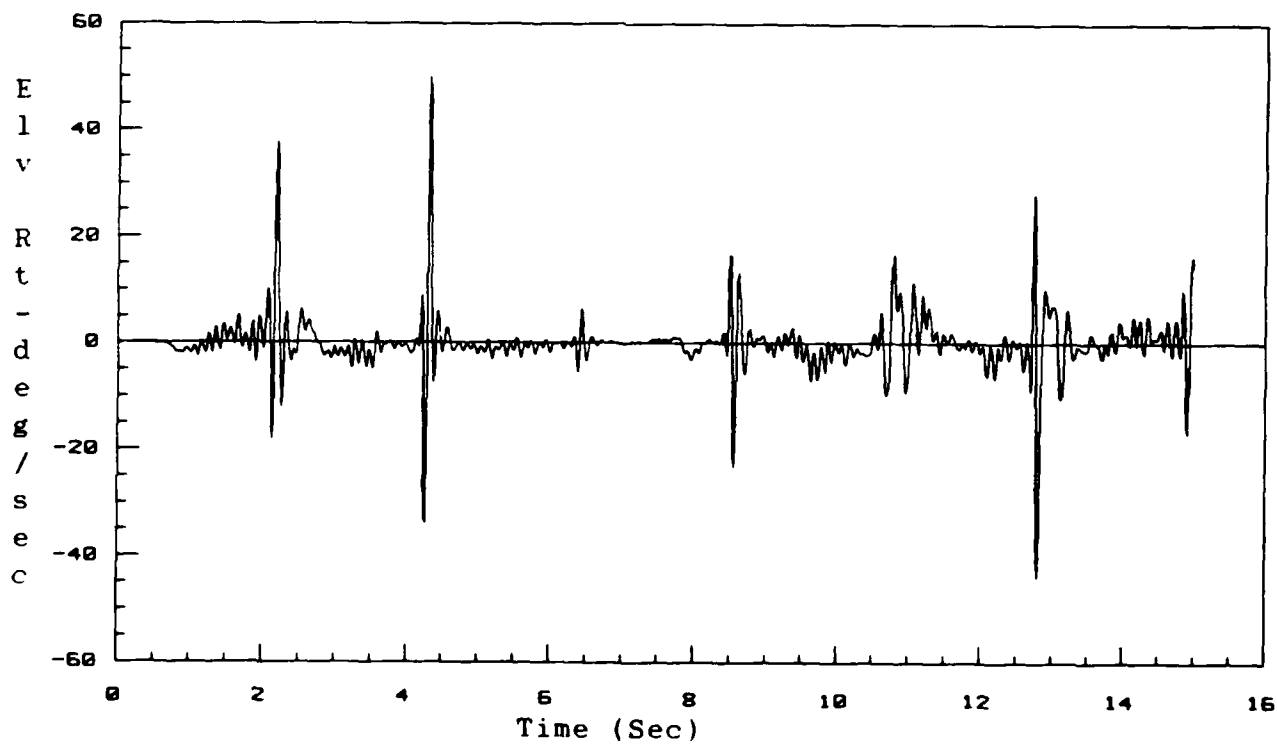


Figure 6-29. Elevator Rate
Nominal Model: 38K 0.7M/Operating Point: 26K 0.45M

Table 6-4

Eigenvalues for the A Matrix of State Space Model

Flight Condition	Eigenvalues
38,000 ft	-0.01254387 + 0.09246732 i
0.7 Mach	-0.01254387 - 0.09246732 i
Dynamic	1.11849823 + 0.00000000 i
Pressure=148.4	-1.79023020 + 0.00000000 i
18,000 ft	-0.00386373 + 0.06747448 i
0.45 Mach	-0.00386373 - 0.06747448 i
Dynamic	-2.05216607 + 0.00000000 i
Pressure=149.9	1.11328404 + 0.00000000 i

data shown on Table 6-4, it is apparent that the eigenvalues for the A matrices of flight conditions with similar values of dynamic pressure are not equivalent. This conclusion was further verified by examining the eigenvalues of the models shown on Table 6-3. A comparison of the B matrices (see Appendix A) for flight conditions with similar dynamic pressures, shows they are not equivalent either.

The reason for the differences in the control surface deflections for flight conditions with similar dynamic pressures can be found in the aerodynamic data. When examining the aerodynamic data obtained for the flight conditions of 18,000 ft, 0.45 Mach and 38,000 ft, 0.7 Mach, it was noted

that the trim values of angle of attack differed. Table 6-5 shows the angle of attack for each of the flight conditions. Figure 6-30 shows that the lift coefficient varies as the angle of attack changes. When the lift coefficient changes, the stability derivatives associated with it change, and hence so do the dimensionalized derivatives. The pitching moment also changes as the lift coefficient changes. Therefore, the stability derivatives associated with the pitching moment will differ also.

From the above discussion, it can be summarized that there are some differences in the responses of the control surfaces for models having the same dynamic pressure. Even with these differences, model equivalency is primarily associated with dynamic pressure. This was shown by generating

Table 6-5

Angle of Attack for Nominal Flight Conditions

Flight Condition	Angle of Attack (deg)
38,000 ft 0.7 Mach Dynamic Pressure=148.4	5.11515
18,000 ft 0.45 Mach Dynamic Pressure=149.9	4.96835

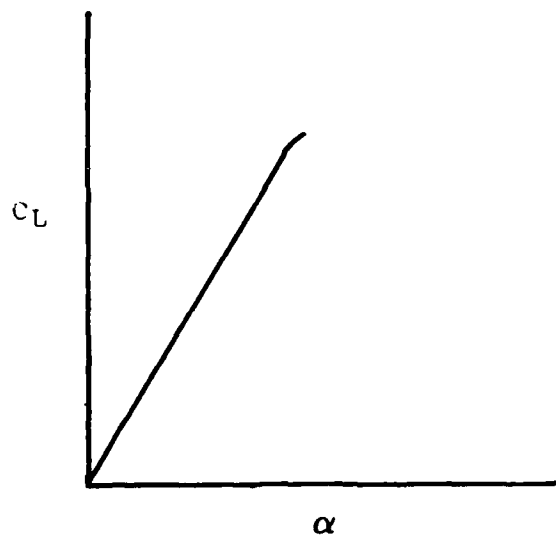


Figure 6-30. Lift Curve

performance boundaries for several pairs of models with similar values of dynamic pressure (see Table 6-3) and observing their similarity.

6.2.1.2 Control Surface Activity. The effects on the control surfaces of replacing a nominal model with plants having increased/decreased Mach number and/or altitude can be made clear by examining the changes in the aerodynamic gain. For a given simulation, the system gain is composed of the control law gains and the aerodynamic gain. The aerodynamic gain is derived from the state space model for a

specific flight condition. As an example, the aerodynamic gain for the flight condition of 30,000 ft and 1.2 Mach is shown in the discussion to follow. Recall that the open loop aircraft model is longitudinally unstable in the subsonic region. The reason the flight condition chosen was in the supersonic region is that the open loop plant is stable. Therefore, the resulting transfer functions that were used to calculate the aerodynamic gain will have all of the poles in the left half plane.

The aerodynamic gains for the 30,000 ft, 1.2 Mach flight condition were calculated for the following transfer functions: q/δ_e , q/δ_f , α/δ_e , and α/δ_f . The state aircraft state space model used was

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} M'_q & M'_\alpha \\ Z'_q & Z'_\alpha \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} M_{\delta_e} & M_{\delta_f} \\ Z_{\delta_e} & Z_{\delta_f} \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix} \quad (6-1)$$

and the output equation was

$$\begin{bmatrix} q \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} \quad (6-2)$$

The proper dimensionalized stability derivatives were obtained and inserted into the state space model above. The

desired transfer functions were then obtained and are presented below:

$$q/\delta_e = \frac{-28.3423(s + 1.07441)}{s^2 + 1.725352s + 38.81511} \quad (6-3)$$

$$q/\delta_f = \frac{-9.6179(s + .8210201)}{s^2 + 1.725352s + 38.81511} \quad (6-4)$$

$$a/\delta_e = \frac{-.1313(s + 213.8329)}{s^2 + 1.725352s + 38.81511} \quad (6-5)$$

$$a/\delta_f = \frac{-.1076(s + 88.84213)}{s^2 + 1.725352s + 38.81511} \quad (6-6)$$

The steady state aerodynamic gains were calculated for the transfer functions of Equations (6-3) through (6-6). The aerodynamic gains for the above transfer functions are presented in Table 6-6.

Next, the steady state aerodynamic gains for a flight condition at a different altitude were calculated. The flight condition was 10,000 ft and 1.2 Mach. The transfer functions for this flight condition were calculated as

Table 6-6

Aerodynamic Gains for Transfer Functions
of Equations (6-3) through (6-6)

Transfer Function	Magnitude of Gain
q/δ_e (Eq 6-3)	0.7845
q/δ_f (Eq 6-4)	0.2034
a/δ_e (Eq 6-5)	0.7237
a/δ_f (Eq 6-6)	0.2464

before and are presented in the following equations:

$$q/\delta_e = \frac{-60.5221(s + 2.150764)}{s^2 + 3.460089s + 80.19058} \quad (6-8)$$

$$q/\delta_f = \frac{-19.4011(s + 1.79332)}{s^2 + 3.460089s + 80.19058} \quad (6-9)$$

$$a/\delta_e = \frac{-.259837(s + 229.2444)}{s^2 + 3.460089s + 80.19058} \quad (6-10)$$

$$a/\delta_f = \frac{-.170677(s + 112.372)}{s^2 + 3.460089s + 80.19058} \quad (6-11)$$

As before, the steady state aerodynamic gains were calculated and are presented in Table 6-7. By comparing the aerodynamic gains of Table 6-6 and 6-7, it is seen that the gains for the pitch rate transfer functions are much larger for the altitude of 10,000 ft. The gains for the transfer functions with angle of attack as a parameter are approximately the same. Because this thesis uses pitch rate as a variable to be tracked, the fact that the aerodynamic gains for the higher altitude flight condition are larger is significant. Therefore, an increase in altitude results in an decreased aerodynamic gain and hence faster surface activity. Similar results were obtained when considering the

Table 6-7

Aerodynamic Gains for Transfer Functions
of Equations (6-8) through (6-11)

Transfer Function	Magnitude of Gain
q/δ_e (Eq 6-8)	1.6232
q/δ_f (Eq 6-9)	0.4339
α/δ_e (Eq 6-10)	0.7428
α/δ_f (Eq 6-11)	0.2392

effects on the aerodynamic gain when increasing Mach number. The data showed that, when Mach number and/or altitude increased, the aerodynamic gain decreased accordingly.

6.2.2 Two-Model Configurations

In determining the number of models required to allow the system with the multiple model algorithm to yield satisfactory tracking performance over the desired flight envelope, several two-model configurations were evaluated. The term "two-model configuration" refers to placing two aircraft models in the parallel bank to function as the secondary parameter estimators (see Figure 4-1). The two-model configuration evaluation consisted primarily of examining the amount the performance boundaries were required to overlap to yield proper tracking performance. The two models for each configuration were chosen on different dynamic pressure lines for the reasons discussed earlier. The two-model configurations are listed on Table 6-8.

The performance boundaries were determined as described previously for each of the configurations listed in Table 6-8. Figures 6-31 through 6-35 show the performance boundaries for each of the two-model configurations.

Table 6-8

Two Model Configurations

Configuration	Nominal Model Flight Conditions		
	Alt (Kft)	Mach	Dynamic Pressure
1	26	0.55	154.39
	18	0.80	473.75
2	10	0.50	254.73
	10	0.75	573.14
3	38	0.65	127.95
	10	0.65	430.50
4	10	0.50	254.73
	10	0.65	430.50
5	10	0.35	124.81
	10	0.75	573.14

From Figures 6-31 through 6-35 it is apparent that the flight envelope of interest can be encompassed by the union of the performance boundaries of two nominal models (depending on the position of the individual performance boundaries). However, in attempting to cover the desired flight envelope with two models, several observations were made as to the effects of model placement on the performance of the multiple model algorithm.

Due to the fact that the multiple model algorithm

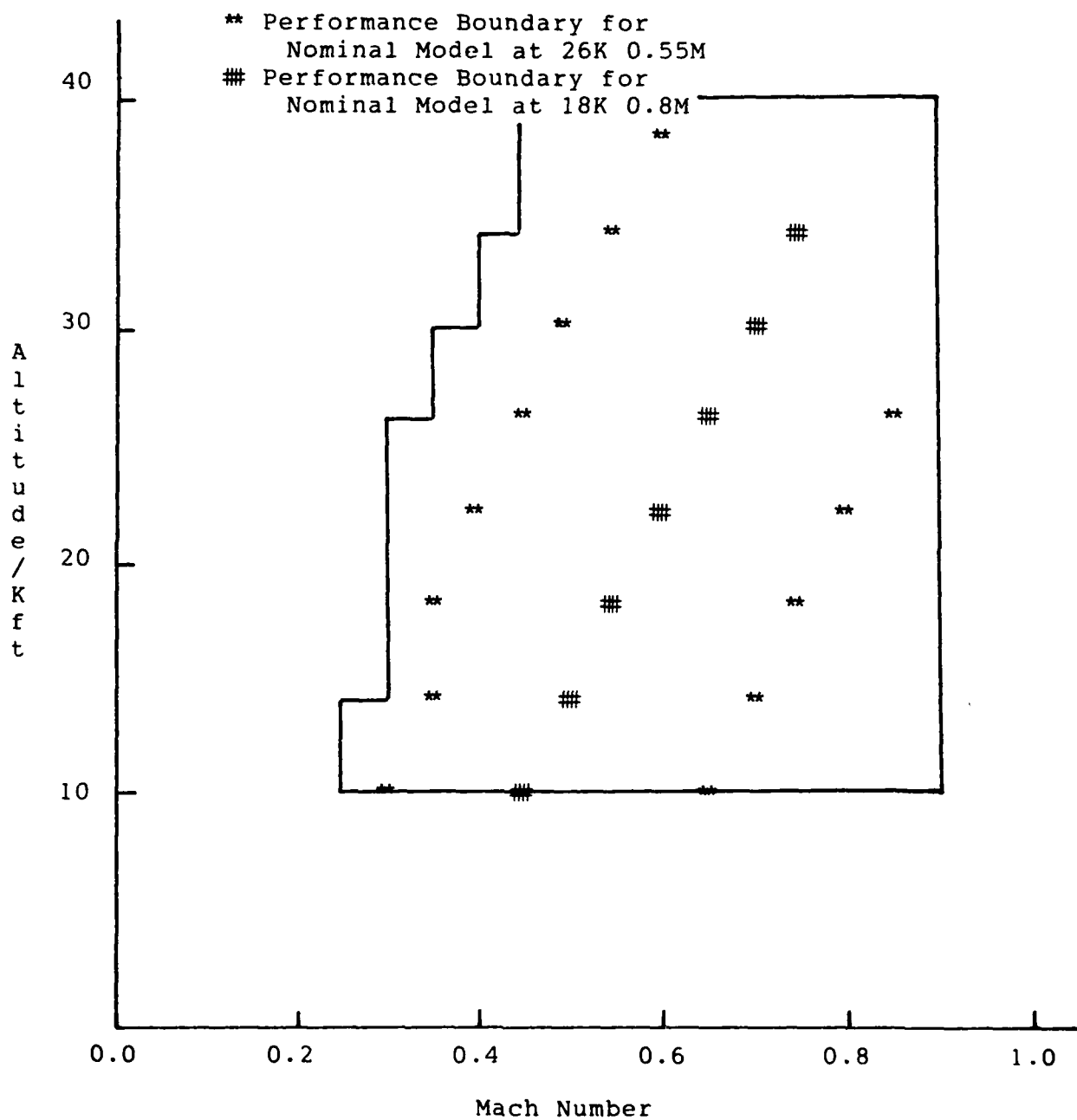


Figure 6-31. Performance Boundaries
for Configuration 1

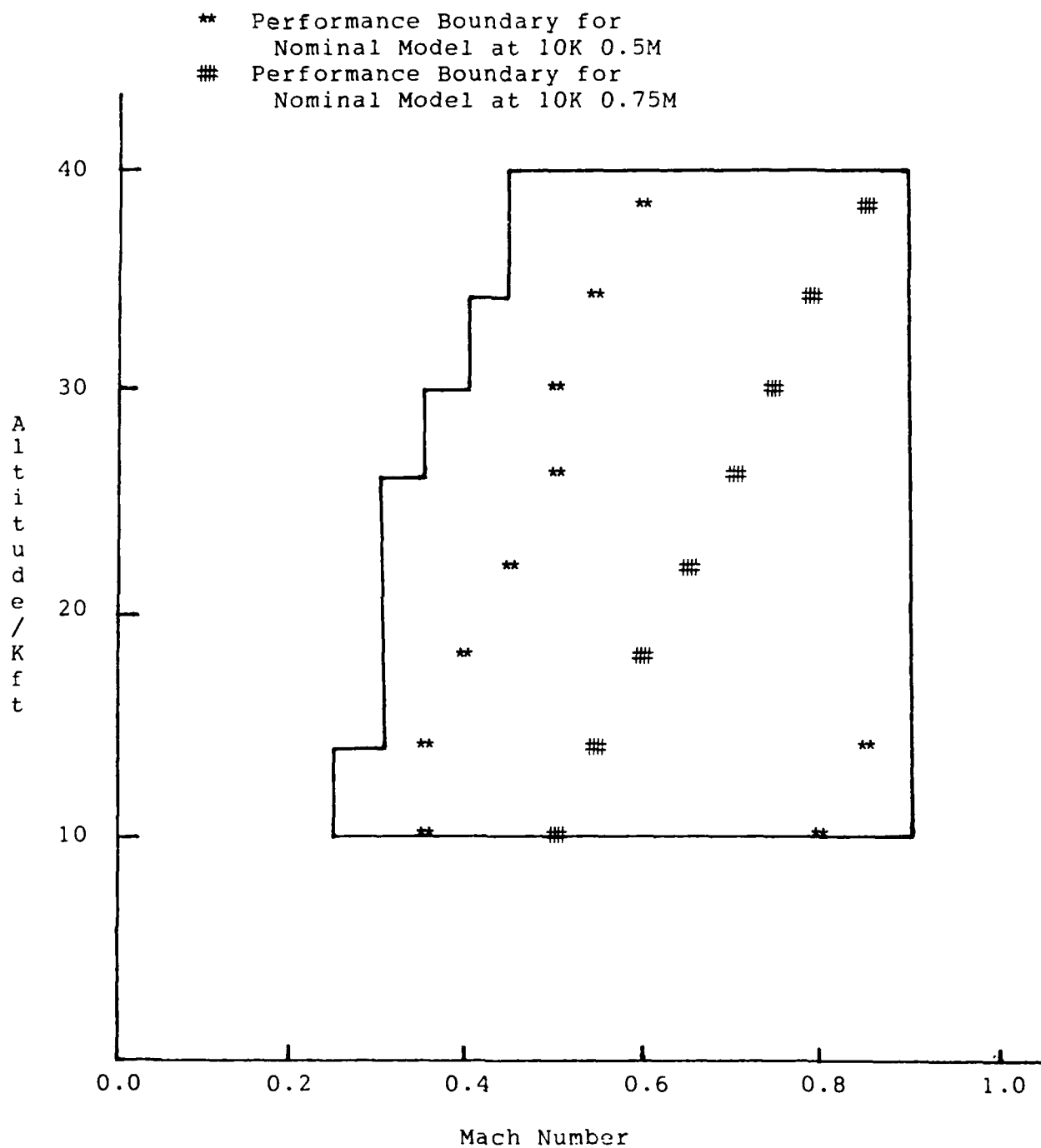


Figure 6-32. Performance Boundaries
for Configuration 2

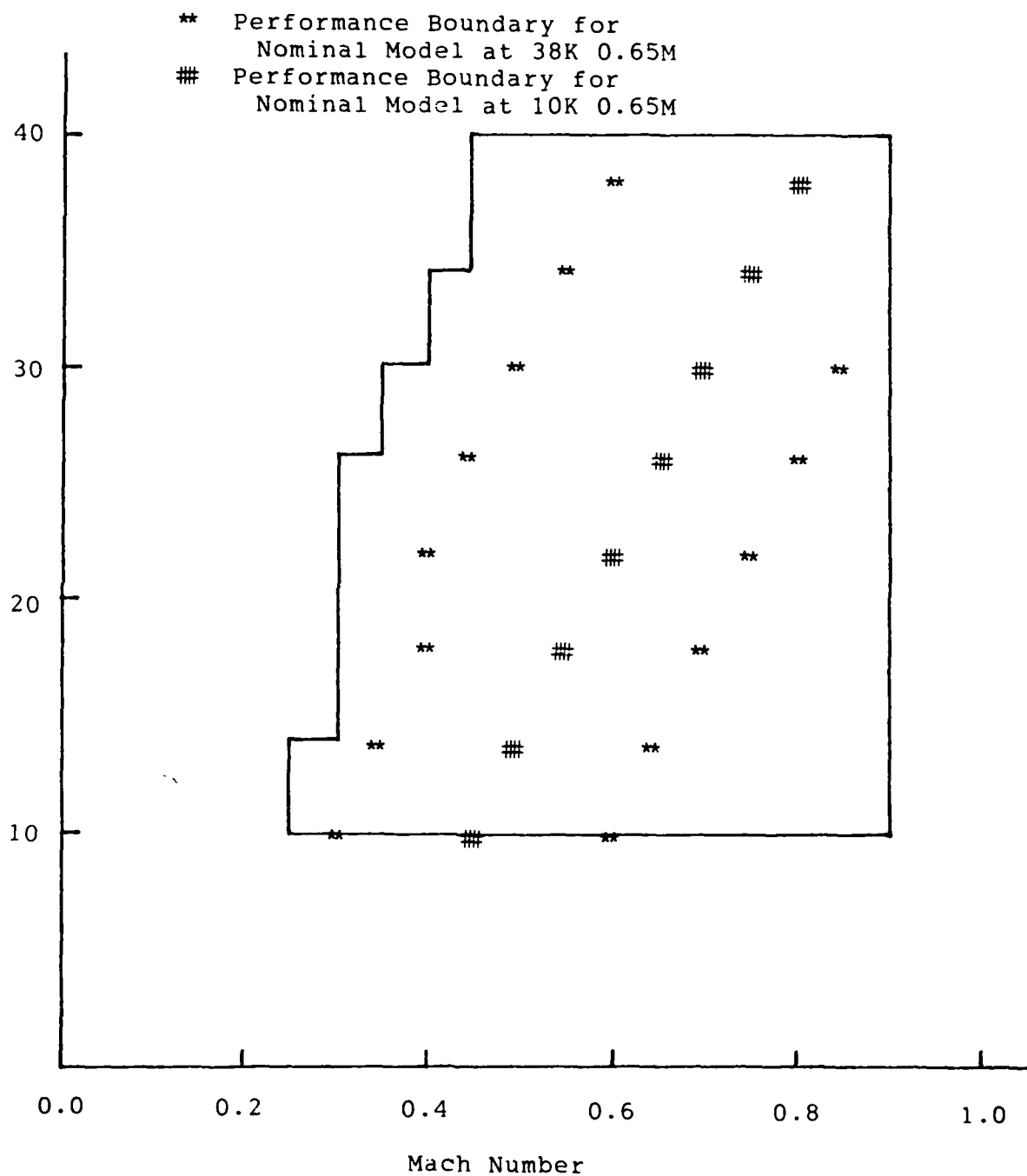


Figure 6-33. Performance Boundaries
for Configuration 3

- ** Performance Boundary for
Nominal Model at 10K 0.5M
- ## Performance Boundary for
Nominal Model at 10K 0.65M

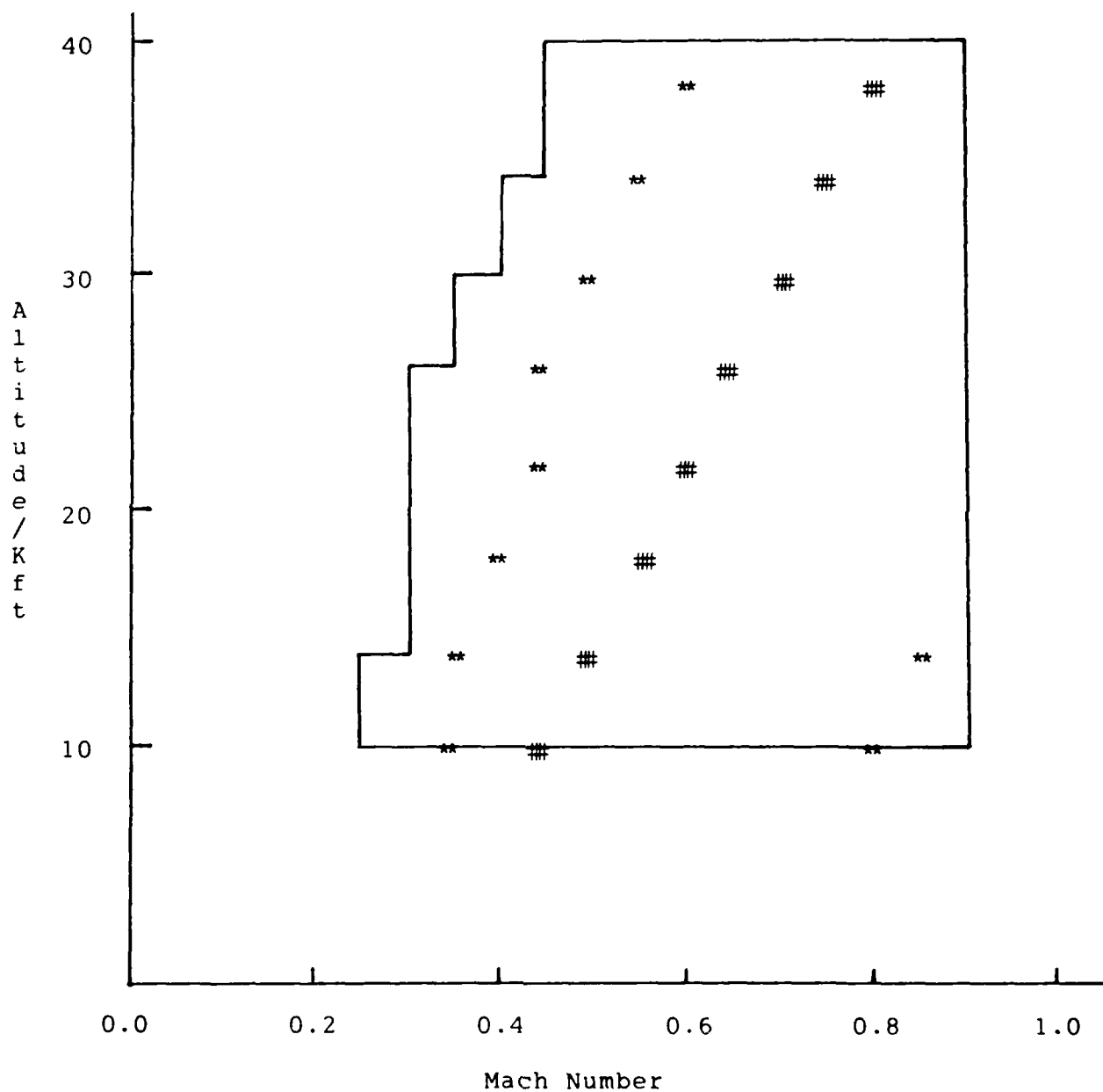


Figure 6-34. Performance Boundaries
for Configuration 4

** Performance Boundary for
 Nominal Model at 10K 0.35M
 # Performance Boundary for
 Nominal Model at 10K 0.75M

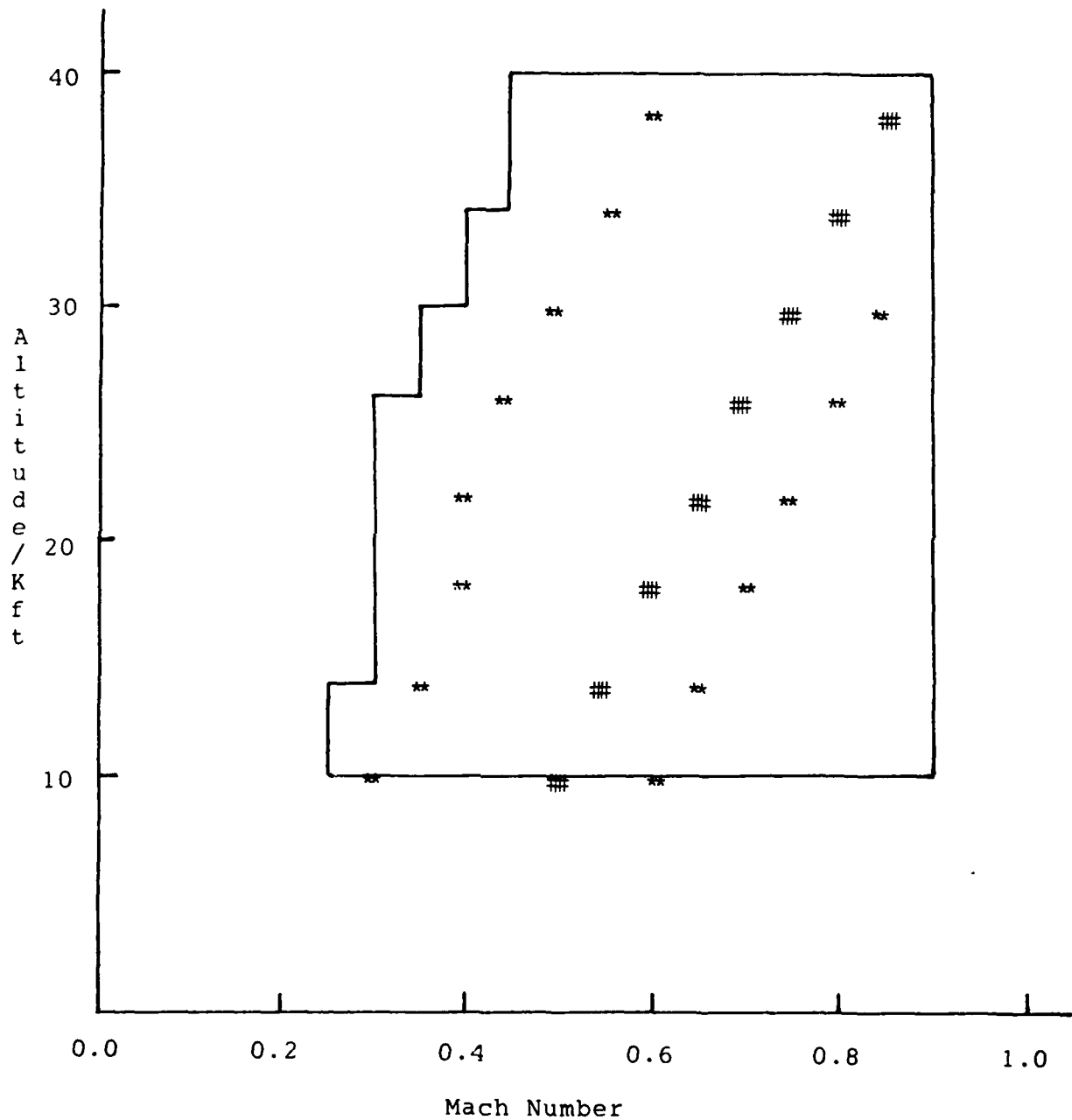


Figure 6-35. Performance Boundaries for Configuration 5

arrives at the estimated value of the B_1 portion of the parameter vector by a blending process of the hypothesis probabilities (see Chapter 4), the amount of overlap of the performance boundaries is important. Figures 6-36 through 6-44 show the significance of the overlap of the performance boundaries. It is seen from Figures 6-36 through 6-44 that although the operating point of each of the examples lies within the performance boundaries (see Figures 6-32 through 6-34) the control law gains that were implemented caused the performance criteria to be violated.

The reason that the performance criteria were violated, was due to the manner in which the multiple model algorithm was implemented. By looking at the plots of the hypothesis probabilities (see Figures 6-38,41,44), it can be seen that the multiple model algorithm calculated a probability of approximately one for one of the two models, while the other model was given a probability of approximately zero. If the probabilities were exactly one and zero for the respective cases, the time responses for the adaptive case would be the same as those generated for the fixed gain case. The fact that the responses were different for the adaptive versus the fixed gain system showed that the probabilities were not

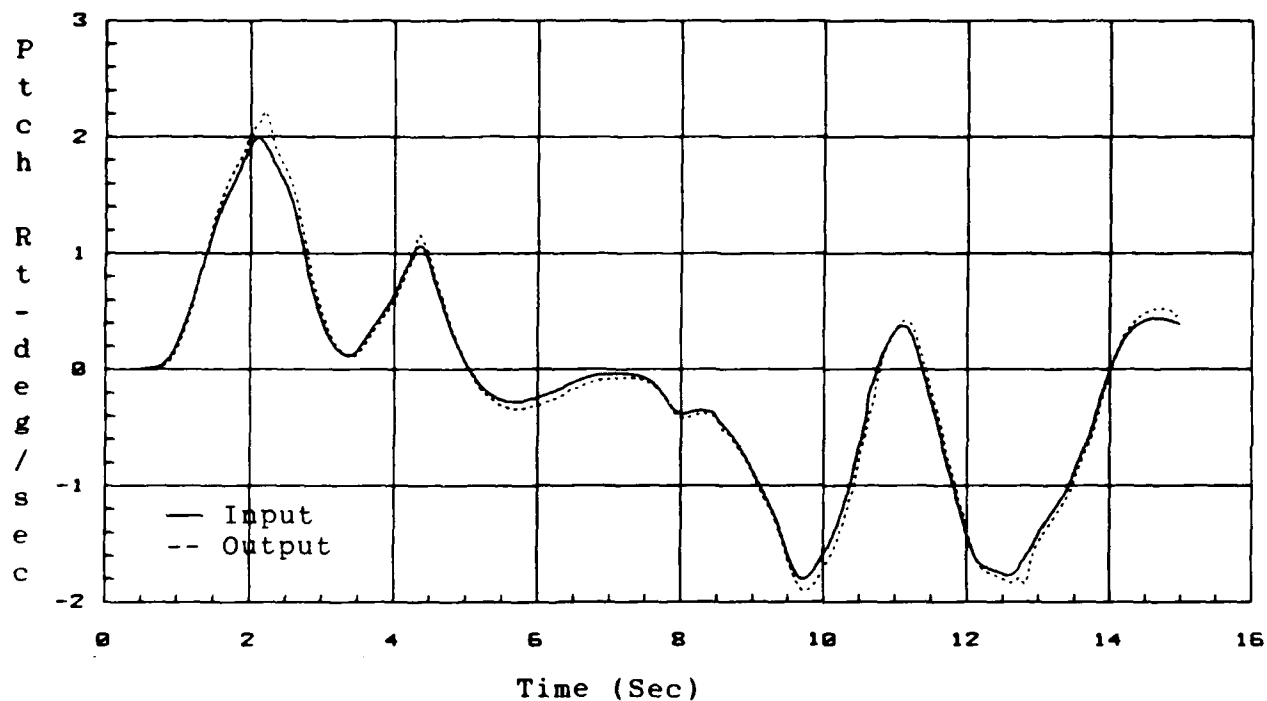


Figure 6-36. Pitch Rate Response
Configuration 2/Operating Point: 14K 0.35M

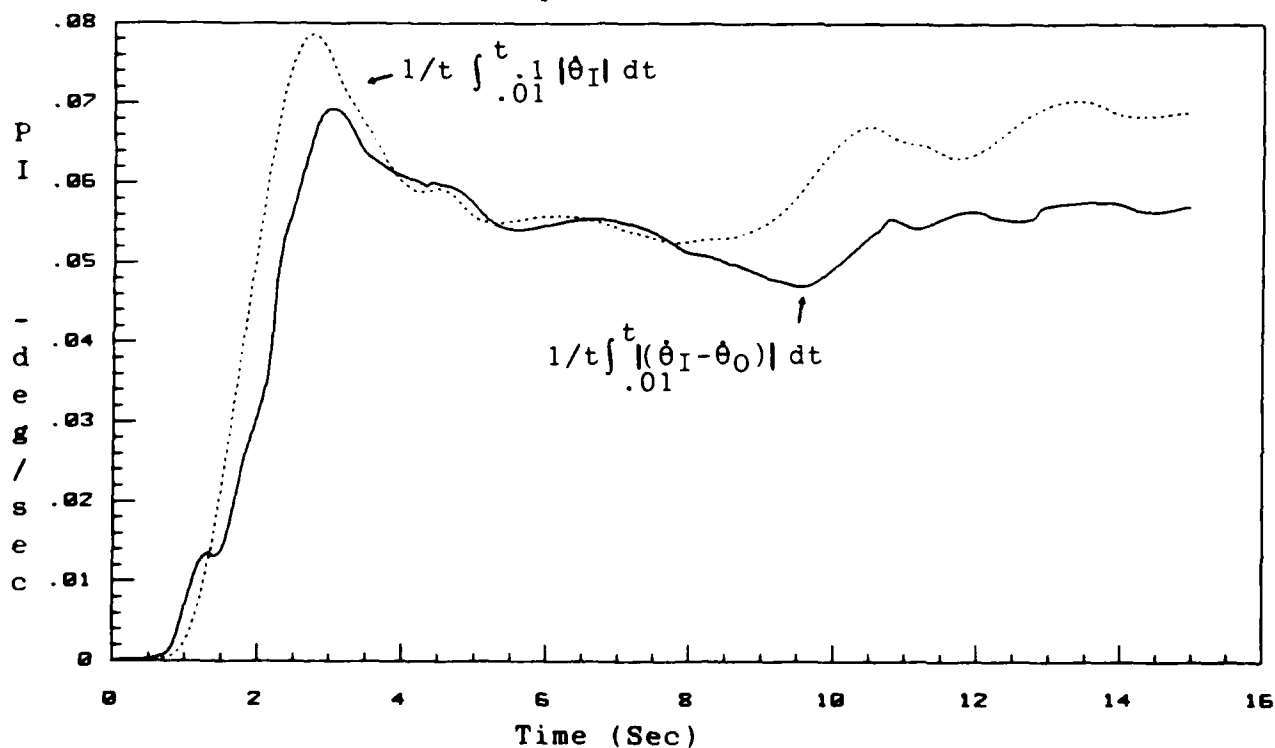


Figure 6-37. Pitch Rate Performance Criterion
Configuration 2/Operating Point: 14K 0.35M

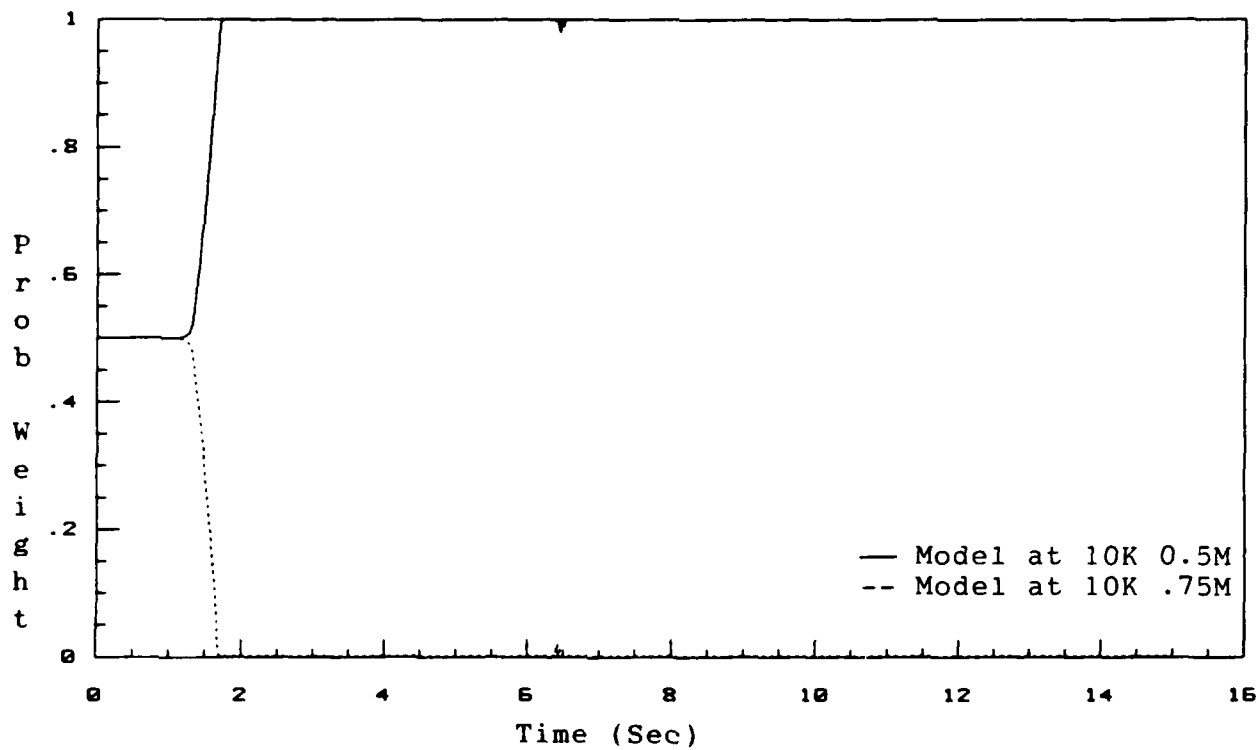


Figure 6-38. Model Probability Weightings
Configuration 2/Operating Point: 14K 0.35M

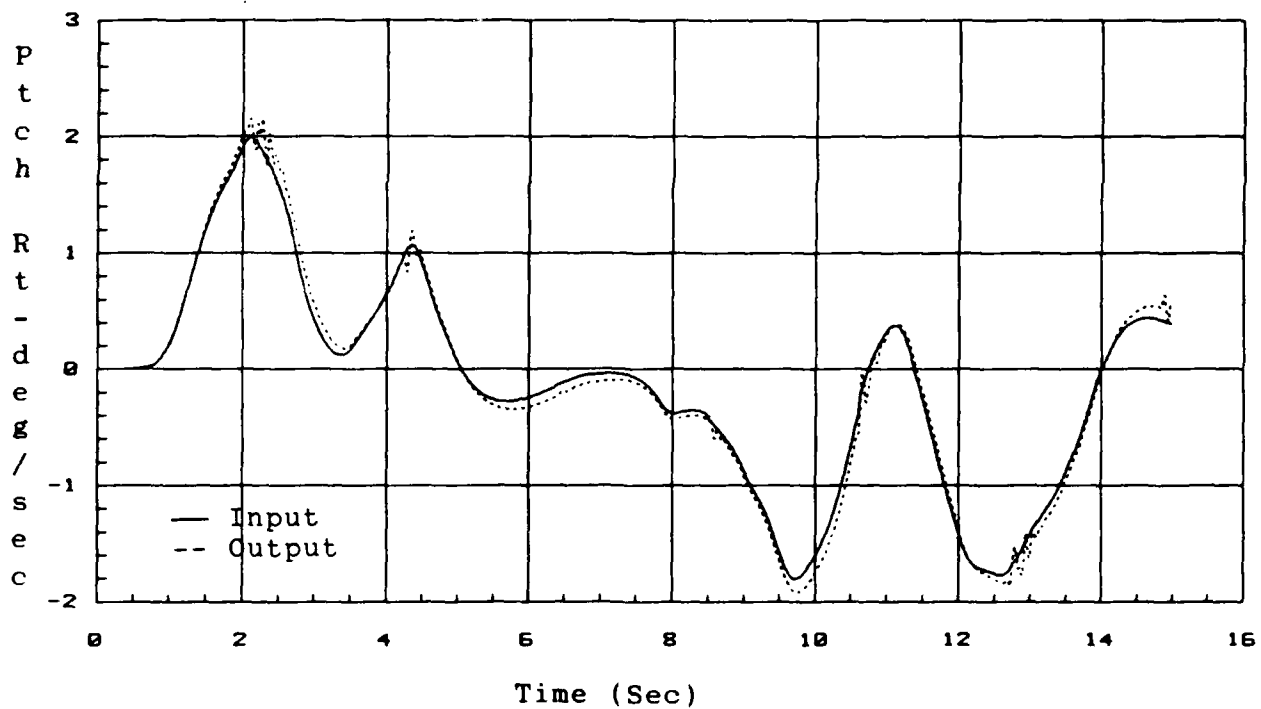


Figure 6-39. Pitch Rate Response
Configuration 4/Operating Point: 14K 0.9M

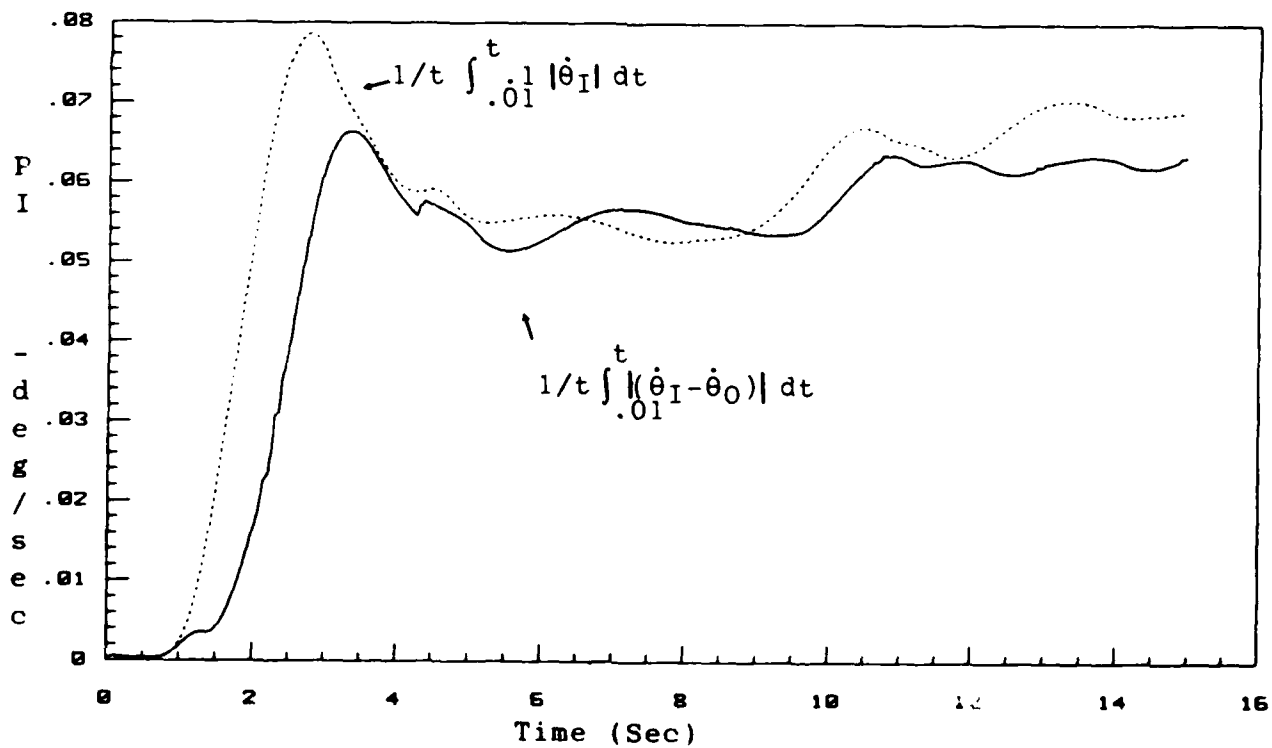


Figure 6-40. Pitch Rate Performance Criterion
Configuration 4/Operating Point: 14K 0.9M

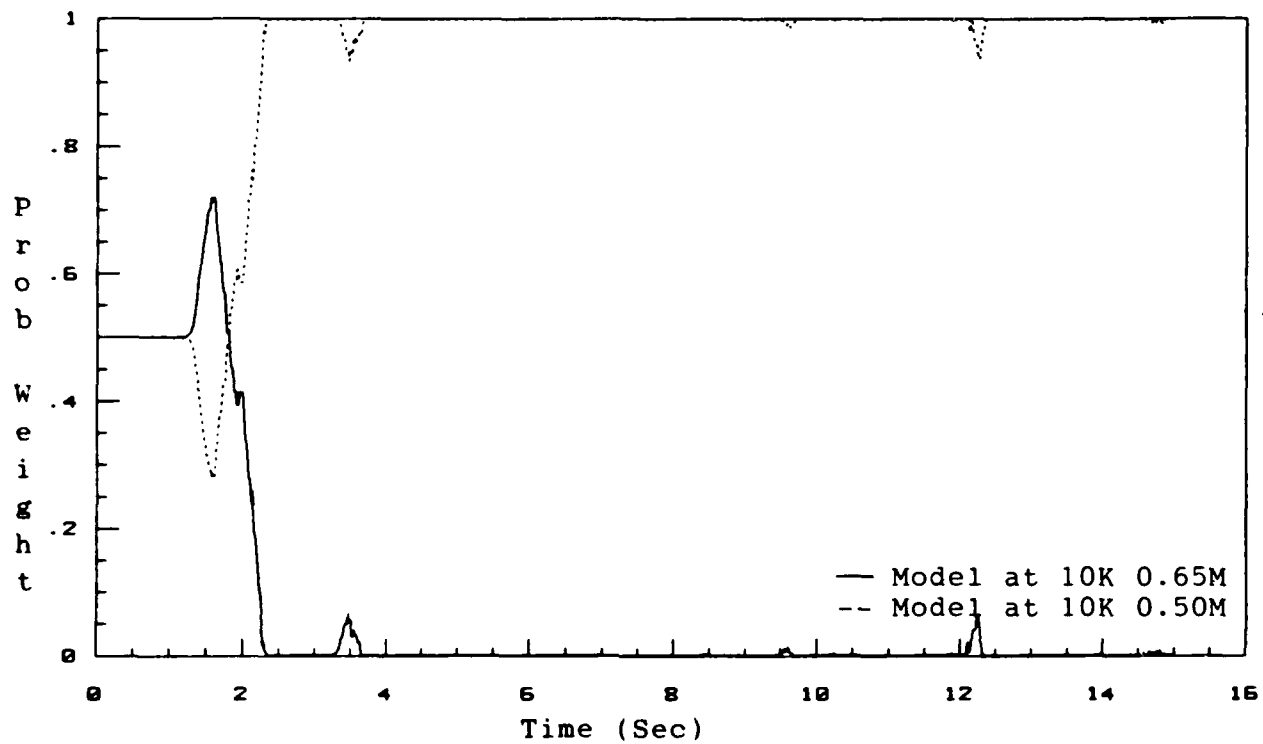


Figure 6-41. Model Probability Weightings
Configuration 4/Operating Point: 14K 0.9M

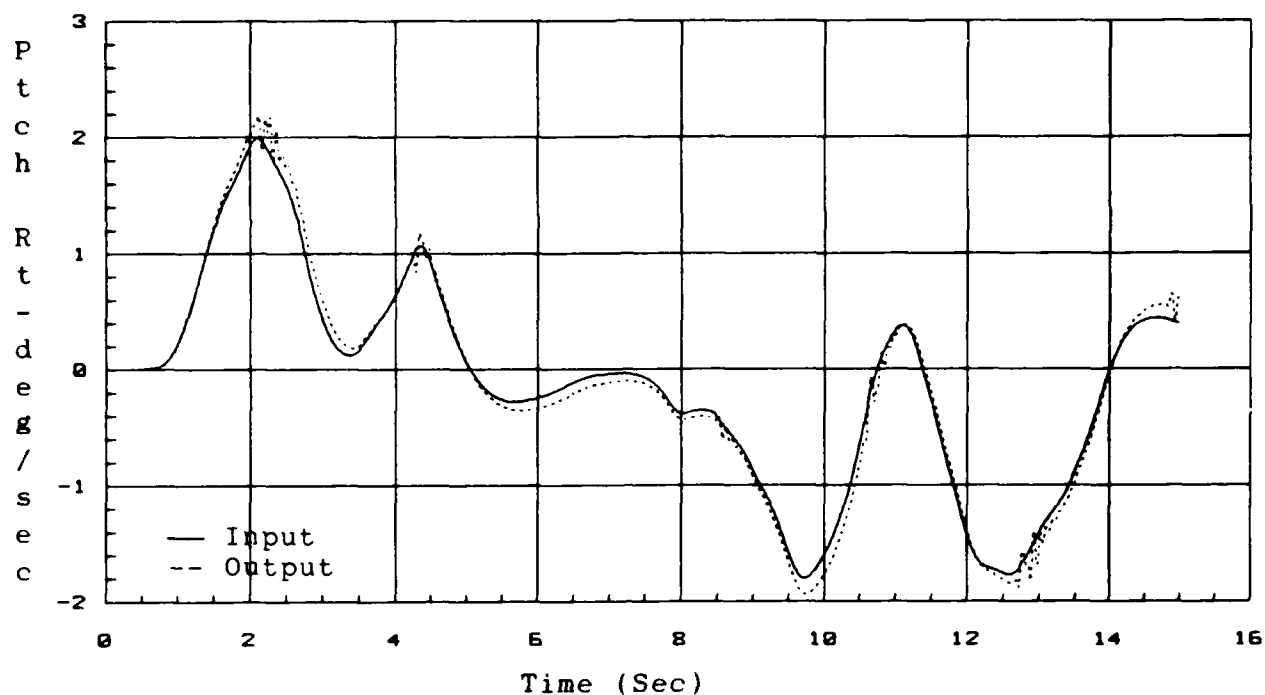


Figure 6-42. Pitch Rate Response
Configuration 3/Operating Point: 10K 0.9M

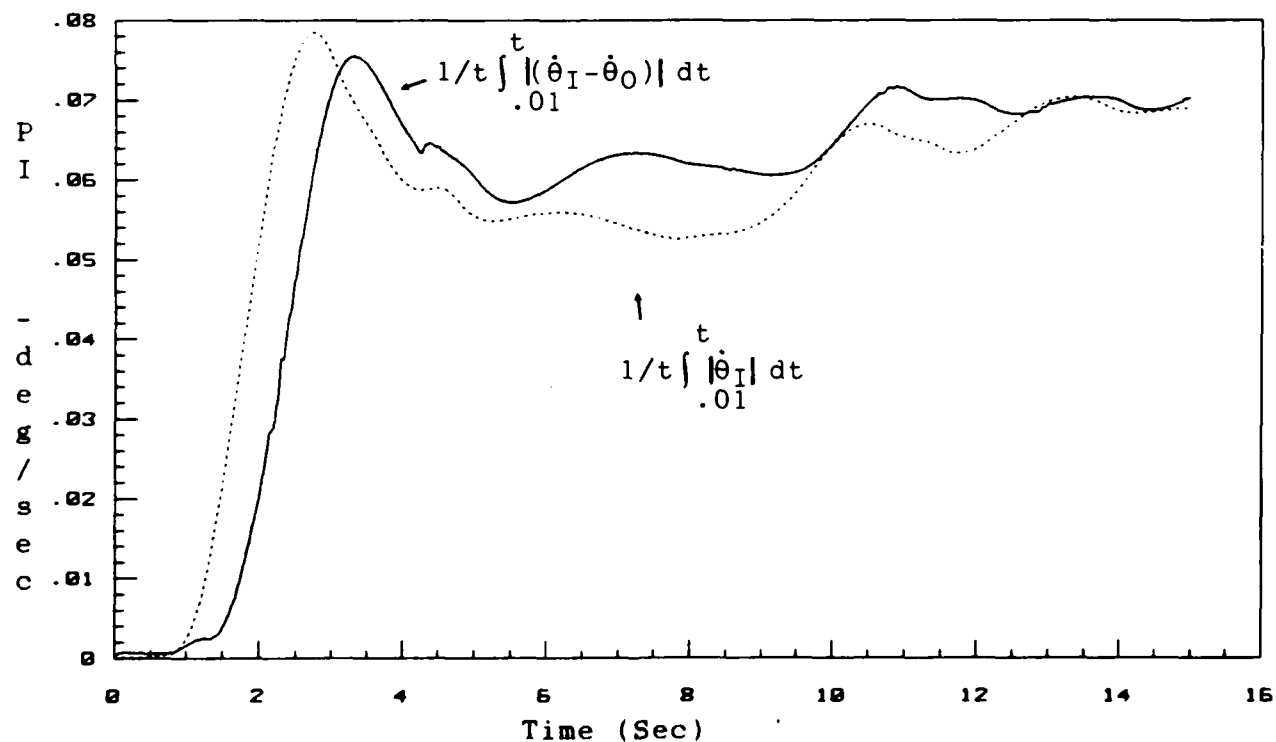


Figure 6-43. Pitch Rate Performance Criterion
Configuration 3/Operating Point: 10K 0.9M

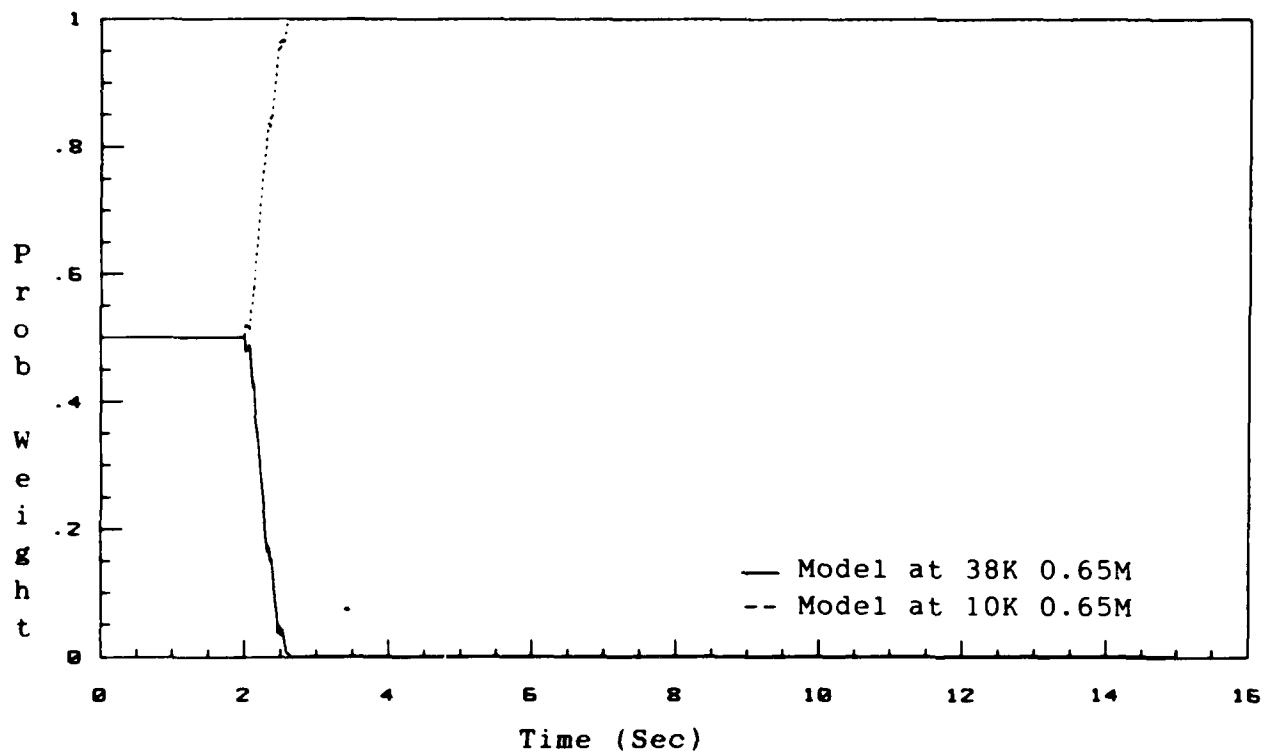


Figure 6-44. Model Probability Weightings
Configuration 3/Operating Point: 10K 0.9M

exactly equal to one and zero.

The multiple model algorithm is implemented with a lower probability bound so the probability of any given candidate model will not be set to zero. The reason for this is that the multiple model algorithm is updated recursively and, if a candidate model's probability goes to zero, the contribution of that model will be eliminated from that point forward.

Lower bounding the candidate model probabilities means that every candidate model will have an effect on the control law gains to some degree. This means that, even if the operating point of the system is at a candidate model position the control law gains will differ slightly from the gains of the specific candidate model. When the candidate models are placed on constant dynamic pressure lines that are 'too far apart' and the operating point is close to the performance boundary of one of the models (see Figure 6-32) the gains may become such that the performance criteria is violated. By referring to Figure 6-32, it can be seen that the operating point is well outside the performance boundary of model 2 and on the boundary of model 1. Although model 1 has a high probability associated with it as

the correct model, by including the small contribution of the control law gains associated with model 2 the performance criteria was violated. This is shown in the data presented on Figures 6-36 through 6-38.

By running simulations with different values of the lower bound for the weighting factors, it was seen that there is an interplay between the amount of overlap required for the performance boundaries and the lower bound size. The smaller the value of the lower bound the less overlap was required of the performance boundaries (so that satisfactory tracking performance would be obtained for all the flight conditions within the union of the performance boundaries).

By using a lower bound value for the weighting factors of 0.01 it was determined that the candidate models need to be placed so that there is a sufficient amount of overlap between the performance boundaries. As shown on Figure 6-45, when the boundaries overlap it is preferable to have the boundary of one model well inside the performance boundary of the second model to minimize the problem discussed above. Also, the position of the candidate models that will cover the outer boundaries of the desired flight envelope

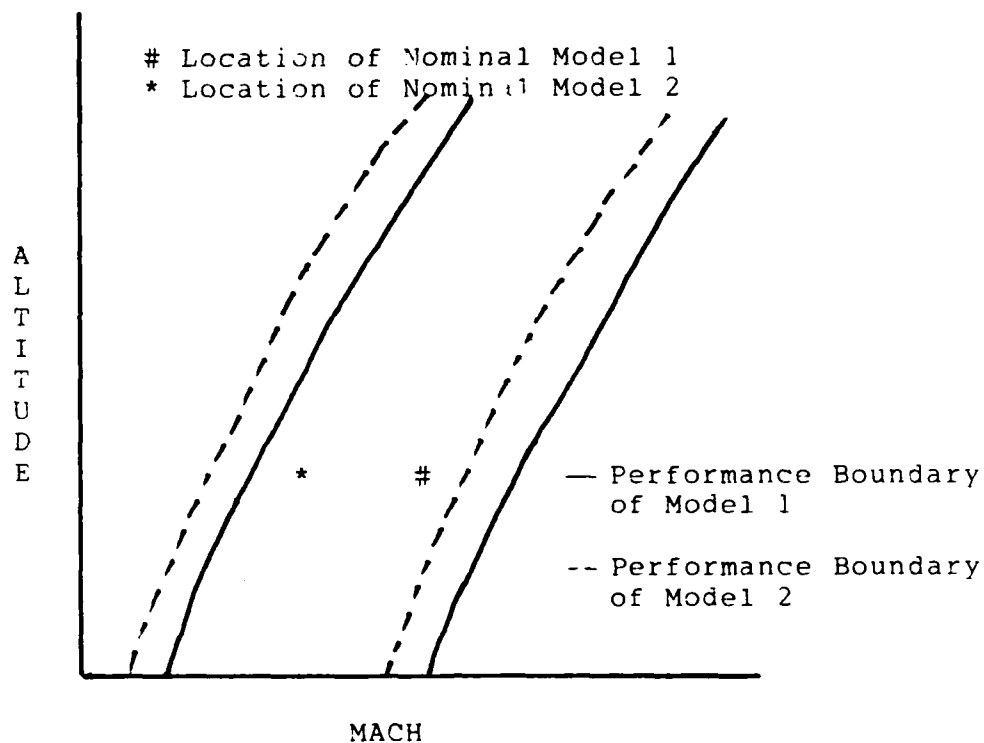


Figure 6-45. Performance Boundary Overlap

need to be placed so the outer boundaries of the flight envelope are well inside the specific candidate models' performance boundaries (see Figure 6-45). Because the model placement requirements could be better achieved with three models instead of two, several three-model configurations were evaluated in an attempt to achieve the desired tracking performance over the flight envelope of interest.

6.2.3 Three-Model Configurations

The three-model configurations that were evaluated are presented in Table 6-9. A graphical presentation of the three-model configurations are given in Figure 6-46. From the beginning of the three-model configuration evaluation, it was clear that the desired tracking performance could be obtained over the flight envelope of interest with three models.

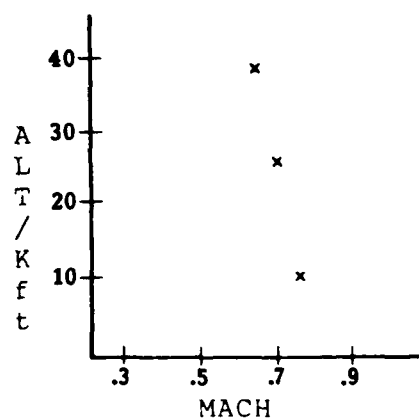
The first three-model configurations that were evaluated were configurations 6, 7, and 8 (see Table 6-9). The models for configurations 6, 7, and 8, were chosen so that the values of dynamic pressures for the candidate models would be as close as possible with the data used for this thesis. The reason for doing this was to examine the effect of choosing different nominal models (with essentially the same dynamic pressures) on the multiple model algorithm's performance.

The performance boundaries for configurations 6, 7, and 8 are presented on Figures 6-47, 6-48, and 6-49. They show that the performance boundaries for each configuration are approximately the same. This equivalency of performance boundaries is due to the fact that each configuration con-

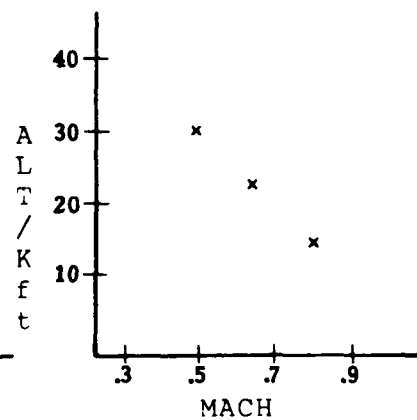
Table 6-9

Three-Model Configurations

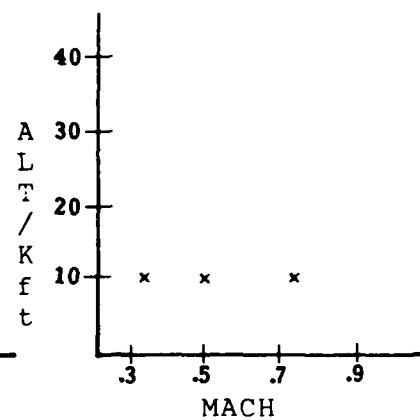
Configuration	Nominal Model Flight Conditions		
	Alt (Kft)	Mach	Dynamic Pressure
6	38	0.65	127.95
	26	0.70	258.18
	10	0.75	573.14
7	30	0.50	110.19
	22	0.65	264.58
	14	0.80	557.18
8	10	0.35	124.81
	10	0.50	254.73
	10	0.75	573.14
9	22	0.45	126.81
	22	0.65	264.58
	22	0.90	507.24
10	10	0.35	124.81
	38	0.80	193.82
	10	0.75	573.14
11	10	0.35	124.81
	22	0.65	264.58
	10	0.75	573.14
12	10	0.35	124.81
	26	0.70	258.18
	10	0.75	573.14
13	22	0.45	126.81
	22	0.65	264.58
	10	0.75	573.14
14	22	0.45	126.81
	10	0.50	254.73
	10	0.75	573.14



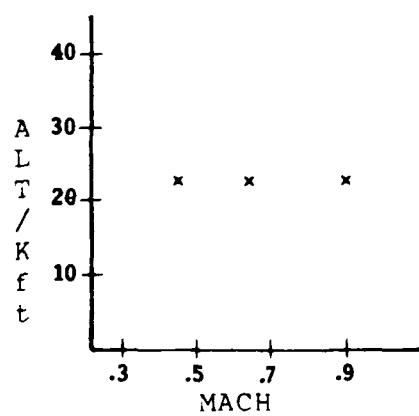
Configuration 6



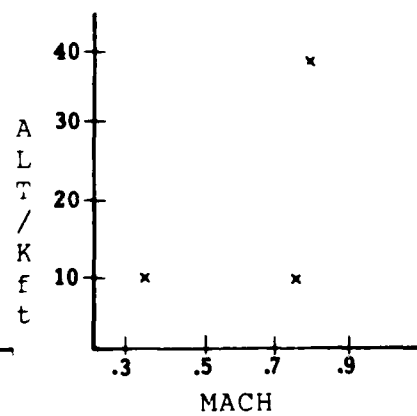
Configuration 7



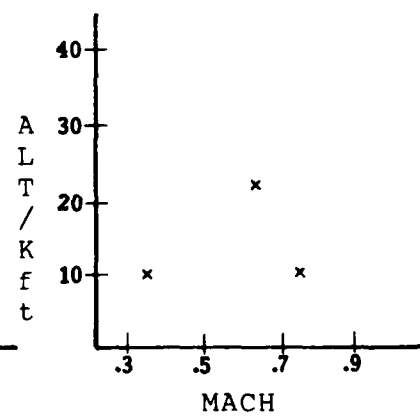
Configuration 8



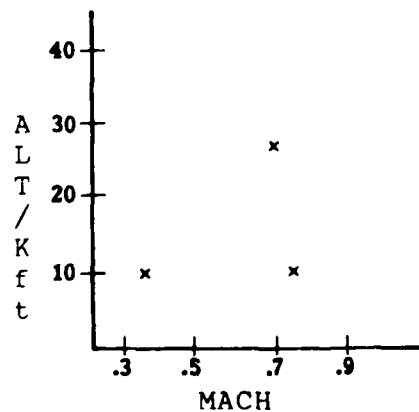
Configuration 9



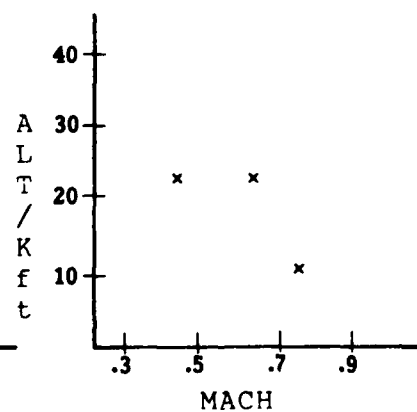
Configuration 10



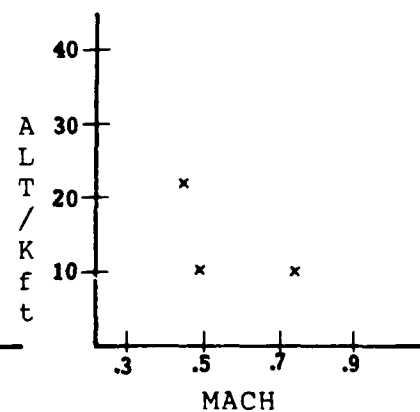
Configuration 11



Configuration 12



Configuration 13



Configuration 14

Figure 6-46. Three-Model Configurations

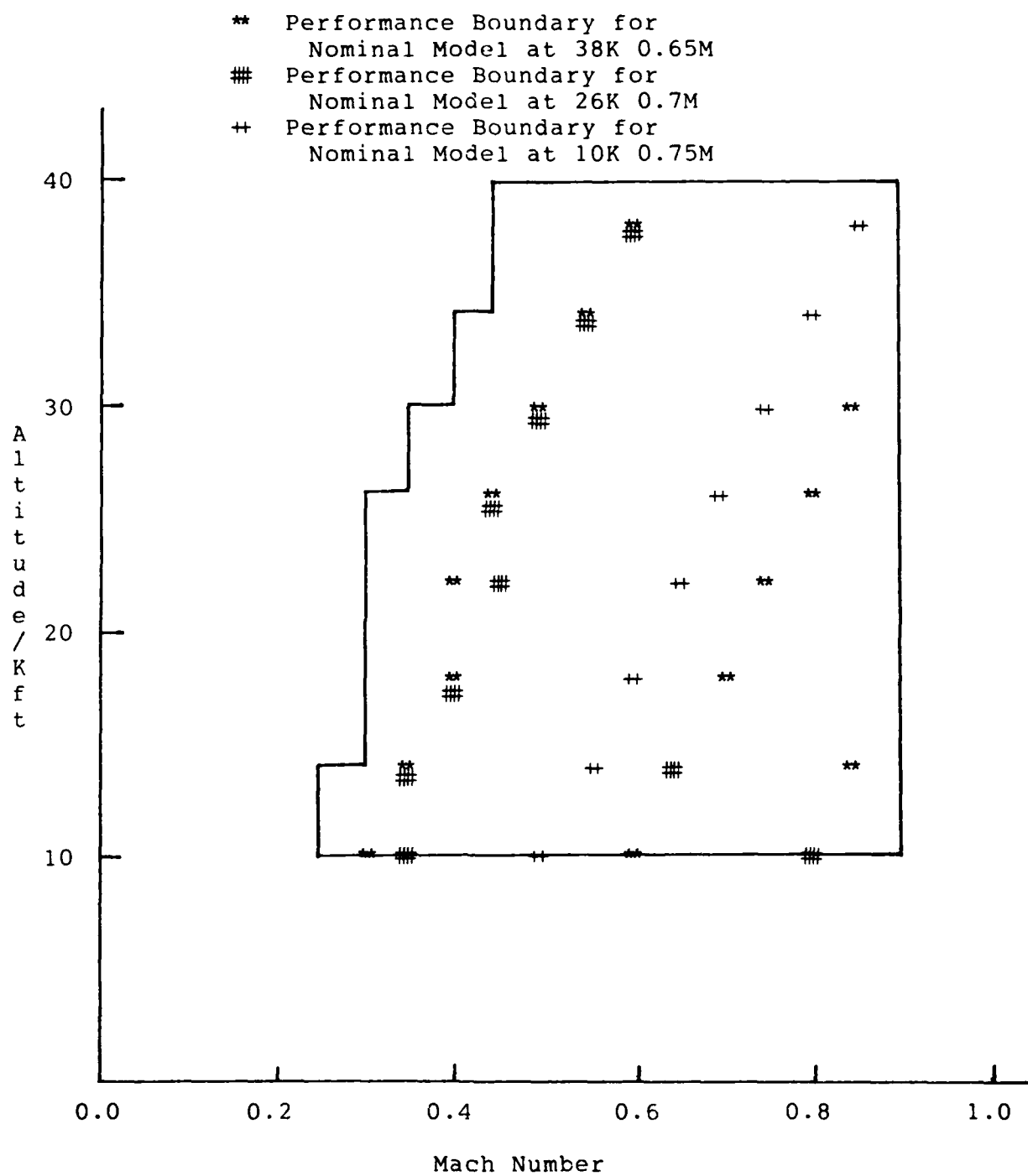


Figure 6-47. Performance Boundaries for Configuration 6

- ** Performance Boundary for
Nominal Model at 30K 0.5M
- ## Performance Boundary for
Nominal Model at 22K 0.65M
- ++ Performance Boundary for
Nominal Model at 14K 0.80M

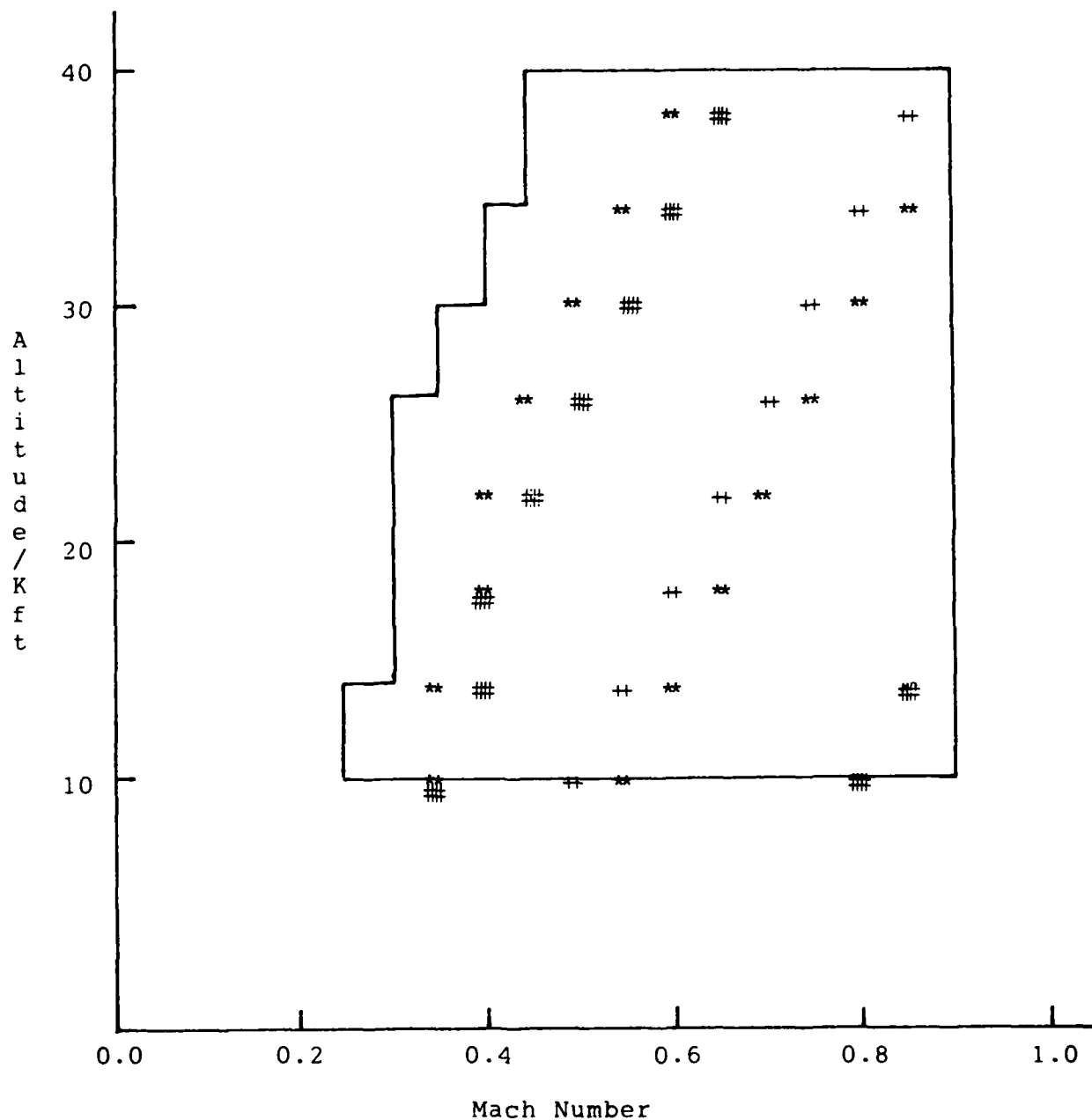


Figure 6-48. Performance Boundaries
for Configuration 7

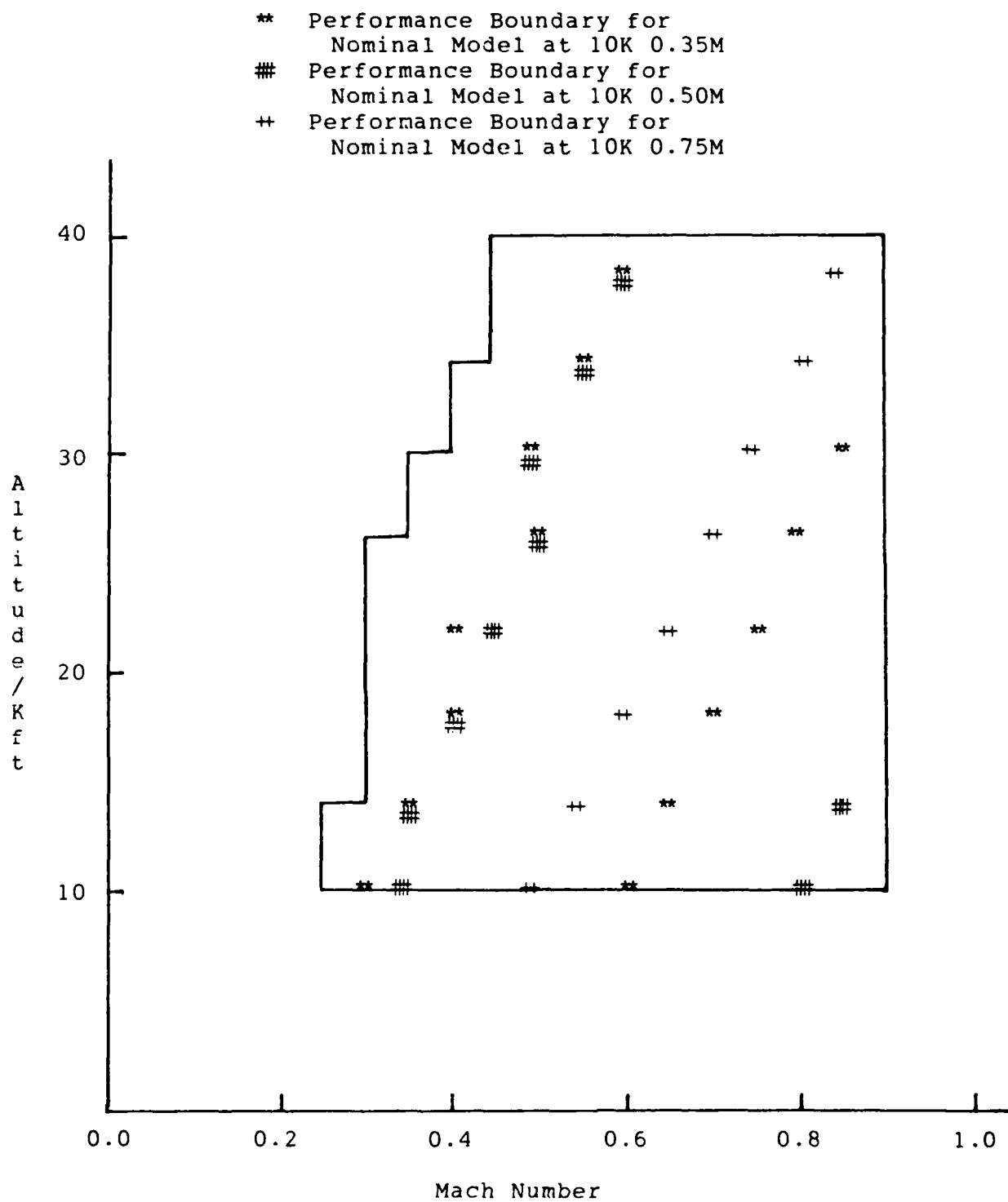


Figure 6-49. Performance Boundaries
for Configuration 8

sists of models with approximately the same dynamic pressures. The reason for the equivalency of the performance boundaries of models with the same dynamic pressure was discussed earlier in this chapter.

Following the establishment of the performance boundaries of configurations 6, 7, and 8, simulations were run using the multiple model algorithm for every flight condition in the flight envelope of interest. The results of those simulations are presented on Tables 6-10 through 6-12 and on Figures 6-50 through 6-52.

Figures 6-50 through 6-52 show graphically which model or models the multiple model algorithm selected as the model to represent the current flight condition. It should be pointed out that the performance criteria were met for every flight condition with the nominal models of configurations 6, 7, and 8. As an example of how the individual entries for each flight condition were determined for Figures 6-50 through 6-52, consider the flight condition 26,000 feet and 0.45 Mach for configuration 6. By looking at the plot of the conditional probabilities for the candidate models (see Figure 6-53), it can be seen that the probability of candidate model one (38,000 ft, 0.65 Mach) is above 0.9. There-

Table 6-10

Model Selection Data for Configuration 6

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
10K	0.25	63.7	28K	0.40	64.3	16K	0.45	194.5	0	0.50	509.5	1
10K	0.30	91.7	28K	0.35	36.3	16K	0.40	166.5	0	0.45	481.4	1
10K	0.35	124.8	28K	0.30	3.1	16K	0.30	133.7	0	0.40	448.3	1
10K	0.30	163.0	28K	0.25	35.1	16K	0.30	95.2	0	0.35	410.1	2,1
10K	0.45	206.3	28K	0.20	78.3	16K	0.25	51.8	0	0.30	366.8	2
10K	0.50	254.7	28K	0.15	126.8	16K	0.20	3.5	0	0.25	318.4	2
10K	0.55	308.2	28K	0.10	180.3	16K	0.15	50.0	0	0.20	264.9	2
10K	0.60	366.8	28K	0.05	238.9	16K	0.10	108.6	0	0.15	206.3	2,3
10K	0.65	430.5	28K	0.00	302.6	16K	0.05	172.3	0	0.10	142.6	3,2
10K	0.70	499.3	28K	0.05	371.3	16K	0.00	241.1	0	0.05	73.9	3
10K	0.75	573.1	28K	0.10	445.2	16K	0.05	315.0	0	0.00	00.0	3
10K	0.80	652.2	28K	0.15	524.2	16K	0.10	393.9	0	0.05	78.9	3
10K	0.85	736.2	28K	0.20	608.2	16K	0.15	478.0	0	0.10	163.0	3
10K	0.90	825.3	28K	0.25	697.4	16K	0.20	567.1	0	0.15	252.2	3
14K	0.25	54.4	24K	0.40	73.4	12K	0.45	203.8	4K	0.50	518.7	1
14K	0.30	78.4	24K	0.35	49.6	12K	0.40	179.8	4K	0.45	494.8	1
14K	0.35	106.6	24K	0.30	21.3	12K	0.35	151.5	4K	0.40	466.5	1
14K	0.40	139.3	24K	0.25	11.3	12K	0.30	118.9	4K	0.35	433.9	1
14K	0.45	176.2	24K	0.20	48.3	12K	0.25	81.9	4K	0.30	396.9	2,1
14K	0.50	217.6	24K	0.15	89.7	12K	0.20	40.5	4K	0.25	355.5	2
14K	0.55	263.3	24K	0.10	135.4	12K	0.15	5.2	4K	0.20	309.8	2
14K	0.60	313.4	24K	0.05	185.5	12K	0.10	55.2	4K	0.15	259.7	2
14K	0.65	367.8	24K	0.00	239.9	12K	0.05	109.6	4K	0.10	205.3	2
14K	0.70	426.6	24K	0.05	298.6	12K	0.00	168.5	4K	0.05	146.6	3,2
14K	0.75	489.7	24K	0.10	361.7	12K	0.05	231.5	4K	0.00	83.5	3,2
14K	0.80	557.2	24K	0.15	429.2	12K	0.10	298.9	4K	0.05	15.9	3
14K	0.85	628.9	24K	0.20	501.0	12K	0.15	370.8	4K	0.10	55.8	3
14K	0.90	705.1	24K	0.25	577.2	12K	0.20	446.9	4K	0.15	132.0	3
18K	0.30	66.6	20K	0.35	61.3	8K	0.40	191.6	8K	0.45	506.5	1
18K	0.35	90.7	20K	0.30	37.3	8K	0.35	167.5	8K	0.40	482.5	1

Tab. 6-10 (cont)

Model Selection Data for Configuration 1

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
18K	0.40	118.4	20K	0.25	9.5	8K	0.30	139.7	8K	0.35	454.7	1
18K	0.45	149.9	20K	0.20	21.9	8K	0.25	108.2	8K	0.30	423.2	1
18K	0.50	185.1	20K	0.15	57.1	8K	0.20	73.1	8K	0.25	388.1	2,1
18K	0.55	223.9	20K	0.10	95.9	8K	0.15	34.3	8K	0.20	349.2	2
12K	0.60	266.5	20K	0.05	138.5	8K	0.10	8.3	8K	0.15	306.6	2
18K	0.65	312.8	20K	0.00	184.8	8K	0.05	54.6	8K	0.10	260.4	2
18K	0.70	362.7	20K	0.05	234.8	8K	0.00	104.5	8K	0.05	210.4	2
18K	0.75	416.4	20K	0.10	288.4	8K	0.05	158.2	8K	0.00	156.8	2,3
18K	0.80	473.8	20K	0.15	345.4	8K	0.10	215.2	8K	0.05	99.8	3,2
18K	0.85	534.8	20K	0.20	406.9	8K	0.15	276.6	8K	0.10	38.3	3
13K	0.90	599.6	20K	0.25	471.6	8K	0.20	341.4	8K	0.15	26.5	3
22K	0.30	56.4	16K	0.35	71.6	4K	0.40	201.8	12K	0.45	516.8	1
22K	0.35	76.7	16K	0.30	51.2	4K	0.35	181.5	12K	0.40	496.4	1
22K	0.40	100.2	16K	0.25	27.8	4K	0.30	58.0	12K	0.35	472.9	1
22K	0.45	126.8	16K	0.20	1.1	4K	0.25	131.4	12K	0.30	446.3	1
22K	0.50	156.6	16K	0.15	28.6	4K	0.20	101.6	12K	0.25	416.6	1
22K	0.55	189.4	16K	0.10	61.5	4K	0.15	68.8	12K	0.20	383.7	2,1
22K	0.60	225.4	16K	0.05	97.5	4K	0.10	32.7	12K	0.15	347.7	2
22K	0.65	264.6	16K	0.00	136.6	4K	0.05	6.4	12K	0.10	308.6	2
22K	0.70	306.9	16K	0.05	178.9	4K	0.00	48.7	12K	0.05	266.3	2
22K	0.75	352.2	16K	0.10	224.3	4K	0.05	94.1	12K	0.00	220.9	2
22K	0.80	400.8	16K	0.15	272.8	4K	0.10	142.6	12K	0.05	172.4	2,3
22K	0.85	452.5	16K	0.20	324.5	4K	0.15	194.3	12K	0.10	120.7	2,3
22K	0.90	507.2	16K	0.25	379.3	4K	0.20	249.1	12K	0.15	65.9	3,2
26K	0.30	47.4	12K	0.35	80.5	0	0.40	210.8	16K	0.45	525.7	1
26K	0.35	64.5	12K	0.30	63.4	0	0.35	193.6	16K	0.40	508.6	1
26K	0.40	84.3	12K	0.25	43.7	0	0.30	173.9	16K	0.35	488.8	1
26K	0.45	106.7	12K	0.20	21.3	0	0.25	151.5	16K	0.30	466.4	1
26K	0.50	131.7	12K	0.15	3.8	0	0.20	126.5	16K	0.25	441.8	1
26K	0.55	159.4	12K	0.10	31.4	0	0.15	98.8	16K	0.20	413.8	1

Table 6-10 (cont)

Model Selection Data for Configuration 6

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
26K	0.60	189.7	12K	0.05	61.7	0	0.10	68.5	16K	0.15	383.5	2,1
26K	0.65	222.6	12K	0.09	94.7	0	0.05	35.6	16K	0.10	350.5	2
26K	0.70	258.2	12K	0.05	130.2	0	0.00	0.00	16K	0.05	315.0	2
26K	0.75	296.4	12K	0.10	168.4	0	0.05	38.2	16K	0.00	276.8	2
26K	0.80	337.2	12K	0.15	209.3	0	0.10	79.0	16K	0.05	235.9	2,3
26K	0.85	380.7	12K	0.20	252.7	0	0.15	122.5	16K	0.10	192.5	2,3
26K	0.90	426.8	12K	0.25	298.8	0	0.20	168.6	16K	0.15	146.4	2,3
30K	0.35	53.9	12K	0.30	74.0	4K	0.35	204.2	20K	0.40	519.1	1
30K	0.40	76.5	8K	0.25	51.4	4K	0.30	181.7	20K	0.35	496.6	1
30K	0.45	89.3	8K	0.20	38.7	4K	0.25	168.9	20K	0.30	483.9	1
30K	0.50	110.2	8K	0.15	17.7	4K	0.20	148.0	20K	0.25	462.9	1
30K	0.55	133.3	8K	0.10	5.4	4K	0.15	124.9	20K	0.20	439.8	1
30K	0.60	158.7	8K	0.05	30.7	4K	0.10	99.5	20K	0.15	414.5	1
30K	0.65	186.2	8K	0.00	58.3	4K	0.05	71.9	20K	0.10	386.9	1
30K	0.70	216.0	8K	0.05	88.0	4K	0.00	42.2	20K	0.05	357.2	1,2
30K	0.75	247.9	8K	0.10	119.9	4K	0.05	10.3	20K	0.00	325.2	2,1
30K	0.80	282.1	8K	0.15	154.1	4K	0.10	23.9	20K	0.05	291.1	2,1
30K	0.85	318.5	8K	0.20	190.5	4K	0.15	60.3	20K	0.10	254.7	2,3
30K	0.90	357.0	8K	0.25	229.1	4K	0.20	98.9	20K	0.15	216.1	2,3
34K	0.40	58.6	4K	0.25	69.3	8K	0.30	199.6	14K	0.35	514.5	1
34K	0.45	74.2	4K	0.20	53.8	8K	0.25	184.0	14K	0.30	498.9	1
34K	0.50	91.6	4K	0.15	36.3	8K	0.20	166.6	14K	0.25	481.5	1
34K	0.55	110.9	4K	0.10	17.1	8K	0.15	147.3	14K	0.20	462.3	1
34K	0.60	131.9	4K	0.05	4.0	8K	0.10	126.3	14K	0.15	441.2	1
34K	0.65	154.8	4K	0.00	26.9	8K	0.05	103.4	14K	0.10	418.3	1
34K	0.70	179.6	4K	0.05	51.6	8K	0.00	78.6	14K	0.05	393.6	1
34K	0.75	206.1	4K	0.10	78.2	8K	0.05	52.0	14K	0.00	367.0	1
34K	0.80	234.5	4K	0.15	106.6	8K	0.10	23.7	14K	0.05	338.6	2,1
34K	0.85	264.8	4K	0.20	136.8	8K	0.15	6.6	14K	0.10	308.4	2,1
34K	0.90	296.8	4K	0.25	168.9	8K	0.20	38.6	14K	0.15	276.3	2,3

Table 5-10 (cont)
Model Selection Data for Configuration 5

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
38K	0.45	61.3	0	0.20	66.6	12K	0.25	196.9	18K	0.30	511.8	1
38K	0.50	75.7	0	0.15	52.2	12K	0.20	182.5	18K	0.25	497.4	1
38K	0.55	91.6	0	0.10	36.3	12K	0.15	166.5	18K	0.20	481.5	1
38K	0.60	109.0	0	0.05	18.9	12K	0.10	149.2	18K	0.15	464.1	1
38K	0.65	128.0	0	0.00	0.0	12K	0.05	130.2	18K	0.10	445.2	1
38K	0.70	148.4	0	0.05	20.0	12K	0.00	109.8	18K	0.05	424.7	1
38K	0.75	170.4	0	0.10	42.4	12K	0.05	87.8	18K	0.00	402.8	1
38K	0.80	193.8	0	0.15	65.9	12K	0.10	64.4	18K	0.05	379.3	1
38K	0.85	218.8	0	0.20	90.9	12K	0.15	39.4	18K	0.10	354.3	2,1
38K	0.90	245.3	0	0.25	117.4	12K	0.20	12.9	18K	0.15	327.8	2,1

Table 6-11
Model Selection Data for Configuration 7

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
10K	0.25	63.7	4K	0.55	493.5	12K	0.40	200.9	20K	0.25	46.5	3
10K	0.30	91.7	4K	0.50	465.4	12K	0.35	172.9	20K	0.20	18.5	3
10K	0.35	124.8	4K	0.45	432.4	12K	0.30	139.8	20K	0.15	14.6	3
10K	0.40	163.0	4K	0.40	394.1	12K	0.25	101.6	20K	0.10	52.8	3,2
10K	0.45	206.3	4K	0.35	350.8	12K	0.20	58.3	20K	0.05	96.1	2
10K	0.50	254.7	4K	0.30	302.4	12K	0.15	9.9	20K	0.00	144.5	2
10K	0.55	308.2	4K	0.25	248.9	12K	0.10	43.6	20K	0.05	198.0	2
10K	0.60	366.8	4K	0.20	190.4	12K	0.05	102.2	20K	0.10	256.6	2,1
10K	0.65	430.5	4K	0.15	126.7	12K	0.00	165.9	20K	0.15	320.3	1,2
10K	0.70	499.3	4K	0.10	57.9	12K	0.05	234.7	20K	0.20	389.1	1
10K	0.75	573.1	4K	0.05	16.0	12K	0.10	308.6	20K	0.25	462.9	1
10K	0.80	652.1	4K	0.00	94.9	12K	0.15	387.5	20K	0.30	541.9	1
10K	0.85	736.2	4K	0.05	179.0	12K	0.20	471.6	20K	0.35	625.9	1
10K	0.90	825.3	4K	0.10	268.2	12K	0.25	560.7	20K	0.40	715.1	1
14K	0.25	54.5	0	0.55	502.8	8K	0.40	210.2	16K	0.25	55.8	3
14K	0.30	78.4	0	0.50	478.8	8K	0.35	186.2	16K	0.20	31.8	3
14K	0.35	106.6	0	0.45	450.5	8K	0.30	157.9	16K	0.15	3.6	3
14K	0.40	139.3	0	0.40	417.9	8K	0.25	125.3	16K	0.10	29.1	3
14K	0.45	176.3	0	0.35	380.9	8K	0.20	88.3	16K	0.05	66.1	2,3
14K	0.50	217.6	0	0.30	339.5	8K	0.15	46.9	16K	0.00	107.5	2
14K	0.55	263.3	0	0.25	293.9	8K	0.10	1.2	16K	0.05	153.2	2
14K	0.60	313.4	0	0.20	243.7	8K	0.05	48.8	16K	0.10	203.2	2
14K	0.65	367.8	0	0.15	189.4	8K	0.00	103.2	16K	0.15	257.6	2
14K	0.70	426.6	0	0.10	130.6	8K	0.05	161.9	16K	0.20	316.4	1,2
14K	0.75	489.7	0	0.05	67.5	8K	0.10	225.1	16K	0.25	379.5	1
14K	0.80	557.2	0	0.00	0.0	8K	0.15	292.6	16K	0.30	446.9	1
14K	0.85	629.0	0	0.05	71.8	8K	0.20	364.4	16K	0.35	518.8	1
14K	0.90	705.2	0	0.10	148.0	8K	0.25	440.6	16K	0.40	494.9	1
18K	0.30	66.6	4K	0.50	490.5	4K	0.35	198.0	12K	0.20	43.5	3
18K	0.35	90.7	4K	0.45	466.5	4K	0.30	173.9	12K	0.15	19.5	3

Table 6-11 (cont)
Model Selection Data for Configuration 7

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
18K	0.40	118.4	4K	0.40	438.7	4K	0.25	146.1	12K	0.10	8.3	3
18K	0.45	149.9	4K	0.35	407.3	4K	0.20	114.7	12K	0.05	39.7	3
18K	0.50	185.1	4K	0.30	372.1	4K	0.15	79.5	12K	0.00	74.9	2,3
18K	0.55	223.9	4K	0.25	332.2	4K	0.10	40.7	12K	0.05	113.7	2
18K	0.60	266.15	4K	0.20	399.7	4K	0.05	1.9	12K	0.10	156.3	2
18K	0.65	312.8	4K	0.15	244.4	4K	0.00	48.2	12K	0.15	202.6	2
18K	0.70	362.7	4K	0.10	194.5	4K	0.05	98.1	12K	0.20	252.5	2
18K	0.75	416.4	4K	0.05	140.8	4K	0.10	151.8	12K	0.25	306.2	2,1
18K	0.80	473.8	4K	0.00	83.4	4K	0.15	209.2	12K	0.30	363.6	1
18K	0.85	534.8	4K	0.05	22.3	4K	0.20	270.2	12K	0.35	424.6	1
18K	0.90	599.5	4K	0.10	42.4	4K	0.25	335.0	12K	0.40	489.4	1
22K	0.30	56.7	8K	0.50	500.8	0	0.35	208.2	8K	0.20	53.8	3
22K	0.35	76.7	8K	0.45	480.5	0	0.30	187.9	8K	0.15	33.5	3
22K	0.40	100.2	8K	0.40	457.0	0	0.25	164.4	8K	0.10	10.0	3
22K	0.45	126.8	8K	0.35	430.4	0	0.20	137.8	8K	0.05	16.6	3
22K	0.50	156.6	8K	0.30	400.6	0	0.15	108.0	8K	0.00	46.4	3
22K	0.55	189.4	8K	0.25	367.7	0	0.10	75.2	8K	0.05	79.2	2,3
22K	0.60	225.4	8K	0.20	331.7	0	0.30	39.1	8K	0.10	115.3	2
22K	0.65	264.6	8K	0.15	292.6	0	0.00	0.0	8K	0.15	154.4	2
22K	0.70	306.9	8K	0.10	250.3	0	0.05	42.3	8K	0.20	196.7	2
22K	0.75	352.3	8K	0.05	204.9	0	0.10	87.7	8K	0.25	242.1	2
22K	0.80	400.8	8K	0.00	156.4	0	0.15	136.2	8K	0.30	290.6	2,1
22K	0.85	454.5	8K	0.05	104.7	0	0.20	187.9	8K	0.35	342.3	1,2
22K	0.90	507.2	8K	0.10	49.9	0	0.25	242.7	8K	0.40	397.1	1
26K	0.30	47.4	12K	0.50	507.7	4K	0.35	217.2	4K	0.20	62.8	3
26K	0.35	64.6	12K	0.45	492.7	4K	0.30	200.3	4K	0.15	45.6	3
26K	0.40	84.3	12K	0.40	472.9	4K	0.25	180.3	4K	0.10	25.9	3
26K	0.45	106.7	12K	0.35	450.5	4K	0.20	157.9	4K	0.05	3.5	3
26K	0.50	131.7	12K	0.30	425.4	4K	0.15	132.9	4K	0.00	21.5	2
26K	0.55	159.4	12K	0.25	397.8	4K	0.10	105.2	4K	0.05	49.2	3

Table 6-11 (cont)
Model Selection Data for Configuration 7

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
26K	0.60	189.7	12K	0.20	367.5	4K	0.05	74.9	4K	0.10	79.2	3,2
26K	0.65	222.6	12K	0.15	334.5	4K	0.00	41.9	4K	0.15	112.4	2
26K	0.70	258.2	12K	0.10	298.9	4K	0.05	5.4	4K	0.20	148.0	2
26K	0.75	296.4	12K	0.05	260.8	4K	0.10	31.8	4K	0.25	186.2	2
26K	0.80	337.2	12K	0.00	219.9	4K	0.15	72.6	4K	0.30	227.0	2,1
26K	0.85	380.7	12K	0.05	176.5	4K	0.20	116.1	4K	0.35	270.5	2,1
26K	0.90	426.8	12K	0.10	130.4	4K	0.25	162.2	4K	0.40	316.6	1,2
30K	0.35	53.9	16K	0.45	503.2	8K	0.30	210.6	0	0.15	56.2	3
30K	0.40	70.5	16K	0.40	486.6	8K	0.25	194.1	0	0.10	39.7	3
30K	0.45	89.3	16K	0.35	467.9	8K	0.20	175.3	0	0.05	20.9	3
30K	0.50	110.2	16K	0.30	446.9	8K	0.15	154.4	0	0.00	0.0	3
30K	0.55	133.3	16K	0.25	423.8	8K	0.10	131.3	0	0.05	23.1	3
30K	0.60	158.7	16K	0.20	398.5	8K	0.05	105.9	0	0.10	48.5	3
30K	0.65	186.2	16K	0.15	370.9	8K	0.00	78.4	0	0.15	76.0	3
30K	0.70	215.9	16K	0.10	341.2	8K	0.05	48.6	0	0.20	105.8	3,2
30K	0.75	247.9	16K	0.05	309.2	8K	0.10	16.7	0	0.25	137.7	2,3
30K	0.80	284.1	16K	0.00	275.1	8K	0.15	17.5	0	0.30	171.9	2
30K	0.85	318.5	16K	0.05	238.7	8K	0.20	53.9	0	0.35	208.3	2
30K	0.90	357.0	16K	0.10	200.1	8K	0.25	92.5	0	0.40	246.8	2,1
34K	0.40	58.6	20K	0.40	498.5	12K	0.25	205.9	4K	0.10	51.6	3
34K	0.45	74.2	20K	0.35	483.0	12K	0.20	190.4	4K	0.05	36.0	3
34K	0.50	91.6	20K	0.30	465.6	12K	0.15	172.9	4K	0.00	18.6	3
34K	0.55	110.9	20K	0.25	446.3	12K	0.10	153.7	4K	0.05	0.7	3
34K	0.60	131.9	20K	0.20	425.2	12K	0.05	132.7	4K	0.10	21.7	3
34K	0.65	154.8	20K	0.15	402.3	12K	0.00	109.8	4K	0.15	44.6	3
34K	0.70	179.6	20K	0.10	377.6	12K	0.05	85.0	4K	0.20	69.4	3
34K	0.75	206.1	20K	0.05	351.0	12K	0.10	58.5	4K	0.25	95.9	3
34K	0.80	234.5	20K	0.00	322.6	12K	0.15	30.1	4K	0.30	124.3	2,3
34K	0.85	264.8	20K	0.05	292.4	12K	0.20	0.2	4K	0.35	154.6	2,3
34K	0.90	296.8	20K	0.10	260.3	12K	0.25	32.2	4K	0.40	186.6	2,1

Table 6-11 (cont)
Model Selection Data for Configuration 7

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
									ALT	MACH	Q	
38K	0.45	61.3	24K	0.35	495.8	16K	0.20	203.3	8K	0.05	48.9	3
38K	0.50	75.7	24K	0.30	481.5	16K	0.15	188.9	8K	0.00	34.5	3
38K	0.55	91.6	24K	0.25	465.6	16K	0.10	172.9	8K	0.05	18.6	3
38K	0.60	109.0	24K	0.20	448.1	16K	0.05	155.6	8K	0.10	1.2	3
38K	0.65	127.9	24K	0.15	429.2	16K	0.00	136.6	8K	0.15	17.8	3
38K	0.70	148.4	24K	0.10	408.8	16K	0.05	116.2	8K	0.20	38.2	3
38K	0.75	170.4	24K	0.05	386.8	16K	0.10	94.2	8K	0.25	60.2	3
38K	0.80	193.8	24K	0.00	363.3	16K	0.15	70.8	8K	0.20	83.6	3
38K	0.85	218.8	24K	0.05	338.4	16K	0.20	45.8	8K	0.35	108.6	2,3
38K	0.90	245.3	24K	0.10	311.9	16K	0.25	19.3	8K	0.40	135.1	2,3

Table 6-12

Model Selection Data for Configuration 8

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
10K	0.25	63.7	0	0.10	61.1	0	0.25	191.1	0	0.05	509.5	1
10K	0.30	91.7	0	0.05	33.1	0	0.20	163.0	0	0.45	481.4	1
10K	0.35	124.8	0	0.00	0.0	0	0.15	129.9	0	0.40	448.3	1
10K	0.40	163.0	0	0.05	38.2	0	0.10	91.7	0	0.35	410.1	1
10K	0.45	206.3	0	0.10	81.5	0	0.05	48.4	0	0.30	366.8	2
10K	0.50	254.7	0	0.15	129.9	0	0.00	0.0	0	0.25	318.4	2
10K	0.55	308.2	0	0.20	183.4	0	0.05	53.5	0	0.20	264.9	2
10K	0.60	366.8	0	0.25	242.0	0	0.10	112.1	0	0.15	206.3	2
10K	0.65	430.5	0	0.30	305.7	0	0.15	175.8	0	0.10	142.6	2,3
10K	0.70	499.3	0	0.35	374.5	0	0.20	244.5	0	0.05	73.9	3
10K	0.75	573.1	0	0.40	448.3	0	0.25	318.4	0	0.00	0.0	3
10K	0.80	652.1	0	0.45	527.3	0	0.30	397.4	0	0.05	79.0	3
10K	0.85	736.2	0	0.50	611.4	0	0.35	481.1	0	0.10	163.0	3
10K	0.90	825.3	0	0.55	701.5	0	0.40	570.6	0	0.15	252.2	3
14K	0.25	54.4	4K	0.10	70.4	4K	0.25	200.3	4K	0.50	518.7	1
14K	0.30	78.4	4K	0.05	46.5	4K	0.20	176.4	4K	0.45	494.8	1
14K	0.35	106.6	4K	0.00	18.2	4K	0.15	148.1	4K	0.40	466.5	1
14K	0.40	139.3	4K	0.05	14.5	4K	0.10	115.5	4K	0.35	433.9	1
14K	0.45	176.3	4K	0.10	51.5	4K	0.05	78.4	4K	0.30	396.9	1
14K	0.50	217.6	4K	0.15	92.8	4K	0.00	37.1	4K	0.25	355.5	2
14K	0.55	263.3	4K	0.20	136.5	4K	0.05	8.6	4K	0.20	309.8	2
14K	0.60	313.4	4K	0.25	188.6	4K	0.10	58.7	4K	0.15	259.7	2
14K	0.65	367.8	4K	0.30	242.0	4K	0.15	113.1	4K	0.10	205.3	2
14K	0.70	426.6	4K	0.35	301.8	4K	0.20	171.8	4K	0.05	146.6	2,3
14K	0.75	489.7	4K	0.40	364.9	4K	0.25	234.9	4K	0.00	83.5	3
14K	0.80	557.2	4K	0.45	432.4	4K	0.30	302.4	4K	0.05	15.9	3
14K	0.85	628.9	4K	0.50	504.2	4K	0.35	374.3	4K	0.10	55.8	3
14K	0.90	705.2	4K	0.55	580.3	4K	0.40	450.4	4K	0.15	132.0	3
18K	0.30	66.6	8K	0.05	58.2	8K	0.20	188.1	8K	0.45	506.5	1
18K	0.35	90.7	8K	0.00	34.1	8K	0.15	164.1	8K	0.40	482.5	1

Table 6-12 (cont)
Model Selection Data for Configuration 8

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
			ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
18K	0.40	118.4	8K	0.05	6.4	8K	0.10	136.3	8K	0.35	454.7	1
18K	0.45	149.9	8K	0.10	25.1	8K	0.05	104.8	8K	0.30	423.2	1
18K	0.50	185.1	8K	0.15	60.3	8K	0.00	69.7	8K	0.25	388.1	1
18K	0.55	225.9	8K	0.20	99.1	8K	0.05	30.8	8K	0.20	349.2	2
18K	0.60	266.5	8K	0.25	141.7	8K	0.10	11.8	8K	0.15	306.7	2
18K	0.65	312.8	8K	0.30	187.9	8K	0.15	58.0	8K	0.10	260.4	2
18K	0.70	362.7	8K	0.35	237.9	8K	0.20	107.9	8K	0.05	210.4	2
18K	0.75	416.4	8K	0.40	291.6	8K	0.25	161.7	8K	0.00	156.8	2,3
18K	0.80	473.8	8K	0.45	348.9	8K	0.30	219.0	8K	0.05	99.4	3,2
18K	0.85	534.8	8K	0.50	410.0	8K	0.35	280.1	8K	0.10	38.3	3,2
18K	0.90	599.6	8K	0.55	474.8	8K	0.40	344.9	8K	0.15	26.5	3
22K	0.30	56.4	12K	0.05	68.5	12K	0.20	198.4	12K	0.45	516.8	1
22K	0.35	76.7	12K	0.00	48.1	12K	0.15	178.0	12K	0.40	496.4	1
22K	0.40	100.2	12K	0.05	24.6	12K	0.10	154.5	12K	0.35	472.9	1
22K	0.45	126.8	12K	0.10	2.0	12K	0.05	127.9	12K	0.30	446.3	1
22K	0.50	156.6	12K	0.15	31.8	12K	0.00	98.2	12K	0.25	416.6	1
22K	0.55	189.4	12K	0.20	64.6	12K	0.05	65.3	12K	0.20	383.7	1
22K	0.60	225.4	12K	0.25	100.6	12K	0.10	29.3	12K	0.15	347.7	2
22K	0.65	264.6	12K	0.30	139.8	12K	0.15	9.8	12K	0.10	308.6	2
22K	0.70	306.9	12K	0.35	182.0	12K	0.20	52.1	12K	0.05	266.3	2
22K	0.75	352.3	12K	0.40	227.4	12K	0.25	97.5	12K	0.00	220.9	2
22K	0.80	400.8	12K	0.45	275.9	12K	0.30	146.1	12K	0.05	172.4	2,3
22K	0.85	452.5	12K	0.50	327.6	12K	0.35	197.7	12K	0.10	120.7	2,3
22K	0.90	507.2	12K	0.55	382.4	12K	0.40	252.5	12K	0.15	65.9	3,2
26K	0.30	47.4	16K	0.05	77.4	16K	0.20	207.3	16K	0.45	525.7	1
26K	0.35	64.5	16K	0.00	60.3	16K	0.15	190.2	16K	0.40	508.6	1
26K	0.40	84.3	16K	0.05	40.5	16K	0.10	170.4	16K	0.35	488.8	1
26K	0.45	106.7	16K	0.10	18.1	16K	0.05	148.0	16K	0.30	466.4	1
26K	0.50	131.7	16K	0.15	6.9	16K	0.00	123.0	16K	0.25	441.4	1
26K	0.55	159.4	16K	0.20	34.6	16K	0.05	95.3	16K	0.20	413.8	1

Table 6-12 (cont)

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
ALT	MACH	Q	Alt	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
26K	0.60	189.7	16K	0.25	64.8	16K	0.10	65.0	16K	0.15	383.5	1
26K	0.65	222.6	16K	0.30	97.8	16K	0.15	32.1	16K	0.10	350.5	1,2
26K	0.70	258.2	16K	0.35	133.4	16K	0.20	3.5	16K	0.05	314.9	2,1
26K	0.75	296.4	16K	0.40	171.6	16K	0.25	41.6	16K	0.00	276.8	2,1
26K	0.80	337.2	16K	0.45	212.4	16K	0.30	82.5	16K	0.05	235.9	2,1
26K	0.85	380.7	16K	0.50	255.9	16K	0.35	125.9	16K	0.10	192.5	2,3
26K	0.90	426.8	16K	0.55	301.9	16K	0.40	172.1	16K	0.15	146.4	2,3
30K	0.35	53.9	20K	0.00	70.8	20K	0.15	200.7	20K	0.40	519.2	1
30K	0.40	70.5	20K	0.05	54.3	20K	0.10	184.2	20K	0.35	502.6	1
30K	0.45	89.3	20K	0.10	35.5	20K	0.05	165.5	20K	0.30	483.9	1
30K	0.50	110.2	20K	0.15	14.6	20K	0.00	144.5	20K	0.25	462.9	1
30K	0.55	133.3	20K	0.20	8.5	20K	0.05	121.4	20K	0.20	439.8	1
30K	0.60	158.7	20K	0.25	33.9	20K	0.10	96.1	20K	0.15	414.5	1
30K	0.65	186.2	20K	0.30	61.4	20K	0.15	68.5	20K	0.10	386.9	1
30K	0.70	215.9	20K	0.35	91.2	20K	0.20	38.8	20K	0.05	357.2	1,2
30K	0.75	247.9	20K	0.40	123.1	20K	0.25	6.8	20K	0.00	325.2	1,2
30K	0.80	282.1	20K	0.45	157.3	20K	0.30	27.4	20K	0.05	291.1	2,1
30K	0.85	318.5	20K	0.50	193.7	20K	0.35	63.7	20K	0.10	254.7	2,1
30K	0.90	357.0	20K	0.55	232.2	20K	0.40	102.3	20K	0.15	216.1	2,3
34K	0.40	58.6	14K	0.05	66.2	14K	0.10	196.1	14K	0.35	514.5	1
34K	0.45	74.2	14K	0.10	50.6	14K	0.05	180.5	14K	0.30	498.9	1
34K	0.50	91.6	14K	0.15	33.2	14K	0.00	163.1	14K	0.25	41.5	1
34K	0.55	110.9	14K	0.20	14.0	14K	0.05	143.9	14K	0.20	462.3	1
34K	0.60	131.9	14K	0.25	7.1	14K	0.10	122.8	14K	0.15	441.2	1
34K	0.65	154.8	14K	0.30	30.1	14K	0.15	99.9	14K	0.10	418.3	1
34K	0.70	179.6	14K	0.35	54.7	14K	0.20	75.2	14K	0.05	393.6	1
34K	0.75	206.1	14K	0.40	81.3	14K	0.25	48.6	14K	0.00	367.0	1
34K	0.80	234.5	14K	0.45	109.3	14K	0.30	20.2	14K	0.05	338.6	2,1
34K	0.85	264.8	14K	0.50	139.9	14K	0.35	10.0	14K	0.10	308.4	2,1
34K	0.90	296.8	14K	0.55	172.0	14K	0.40	42.1	14K	0.15	276.3	2,1

Table 6-12 (cont)
Model Selection Data for Configuration 8

Operating Point			Candidate Model Deltas									Selected Model
			Model 1			Model 2			Model 3			
									ALT	MACH	Q	
ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	ALT	MACH	Q	
38K	0.45	61.3	18K	0.10	63.5	18K	0.05	18K	0.30	511.8	1	
38K	0.50	75.7	18K	0.15	49.1	18K	0.00	18K	0.25	497.4	1	
38K	0.55	91.6	18K	0.20	33.2	18K	0.05	18K	0.20	481.5	1	
38K	0.60	109.0	18K	0.25	15.8	18K	0.10	18K	0.15	464.1	1	
38K	0.65	127.9	18K	0.30	3.1	18K	0.15	18K	0.10	445.2	1	
38K	0.70	148.4	18K	0.35	23.6	18K	0.20	18K	0.05	424.7	1	
38K	0.75	170.4	18K	0.40	45.5	18K	0.25	18K	0.00	402.8	1	
38K	0.80	193.8	18K	0.45	69.0	18K	0.30	18K	0.05	379.3	1	
38K	0.85	218.8	18K	0.50	94.0	18K	0.35	18K	0.10	354.3	1,2	
38K	0.90	245.3	18K	0.55	120.5	18K	0.40	18K	0.15	327.8	1,2	

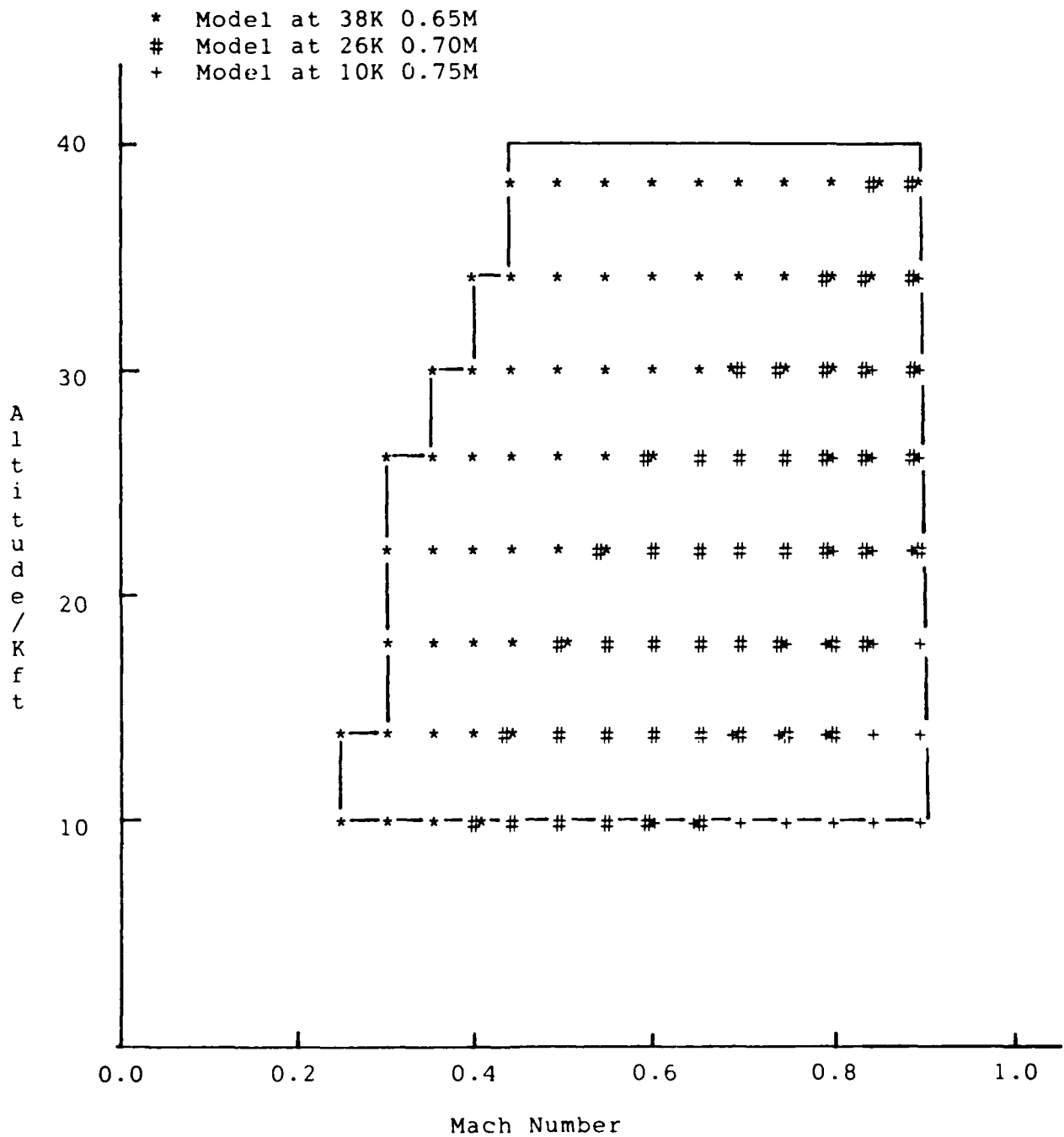


Figure 6-50. Model Selection Data
 for Configuration 6

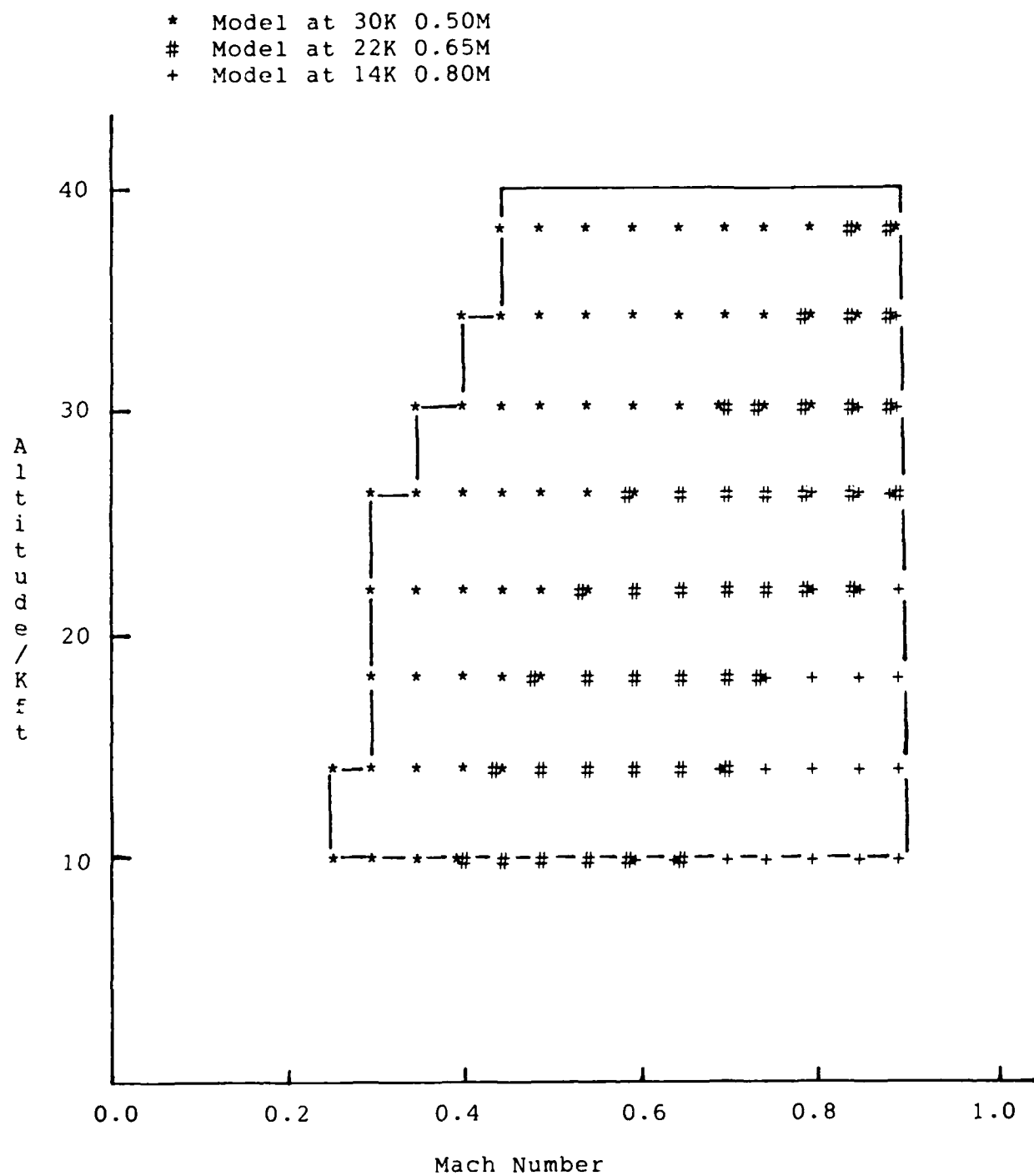


Figure 6-51. Model Selection Data
 for Configuration 7

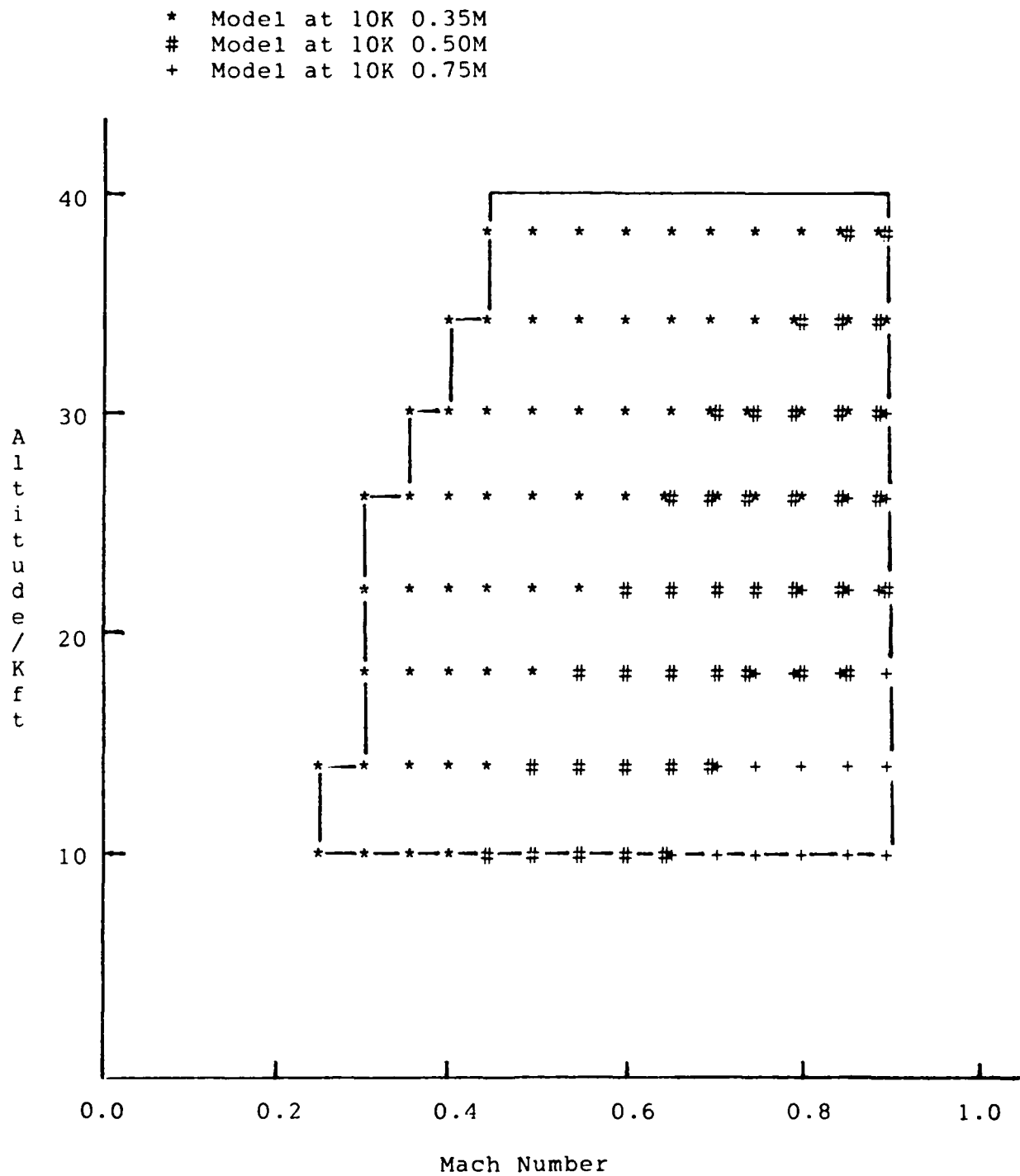


Figure 6-52. Model Selection Data
 for Configuration 8

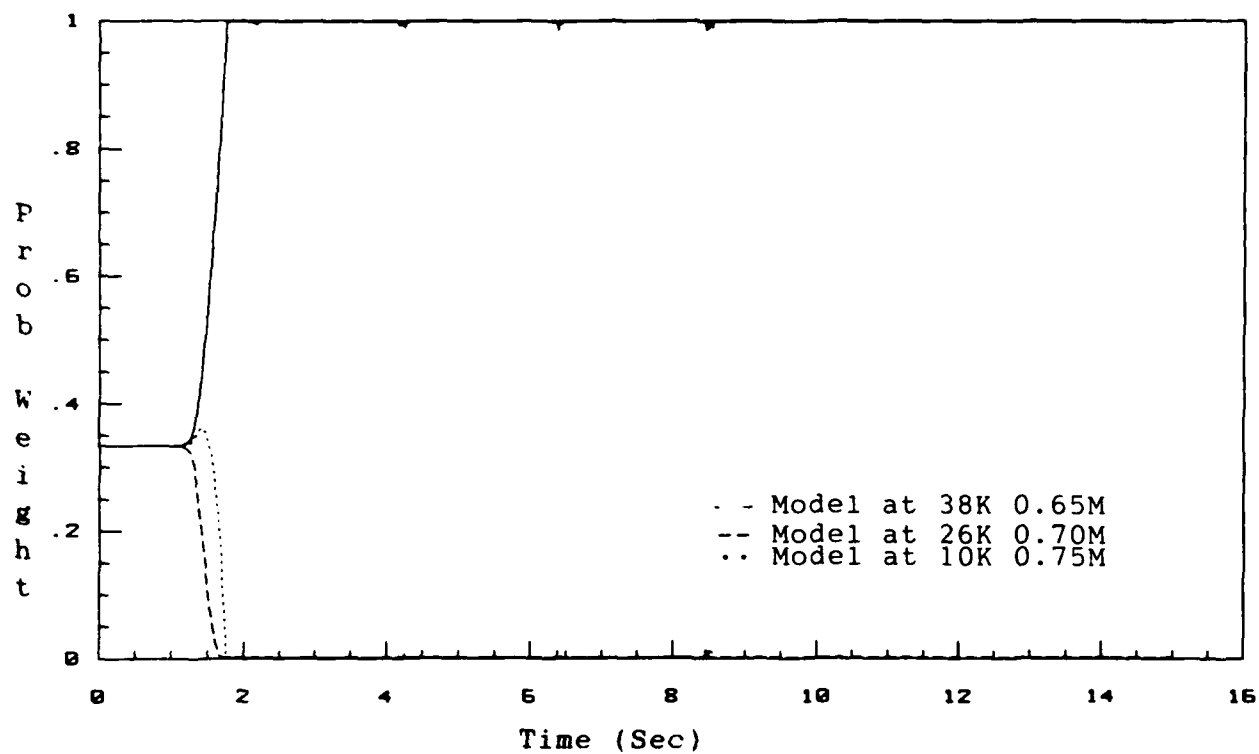


Figure 6-53. Model Probability Weightings
Configuration 6/Operating Point: 26K 0.45M

fore a * is placed at the flight condition of 26,000 feet, 0.45 Mach on Figure 6-50. If the conditional probability fell below 0.9 for a candidate model during the running of a simulation, then the two models with the highest probability were placed at the position of the flight condition of interest. As an example, the conditional probabilities for the flight condition of 26,000 feet, 0.65 Mach (configuration 8) are shown on Figure 6-54. Figure 6-54 shows that the conditional probability of model one falls below 0.9 and that the models with the most significant probabilities are models one and two, respectively. Therefore, a ** was placed on the location of the 26,000 feet, 0.65 Mach flight condition of Figure 6-52. This annotation delineated which candidate model(s) had the highest conditional probability at the given flight condition.

By comparing Figures 6-50 through 6-52, it can be seen that the multiple model algorithm performance was similar for configurations 6, 7, and 8. This similarity indicated that the conditional probability calculated for a candidate model was associated with dynamic pressure. Tables 6-10 through 6-12 show the candidate models that were selected as closest to the actual flight condition as well as the dif-

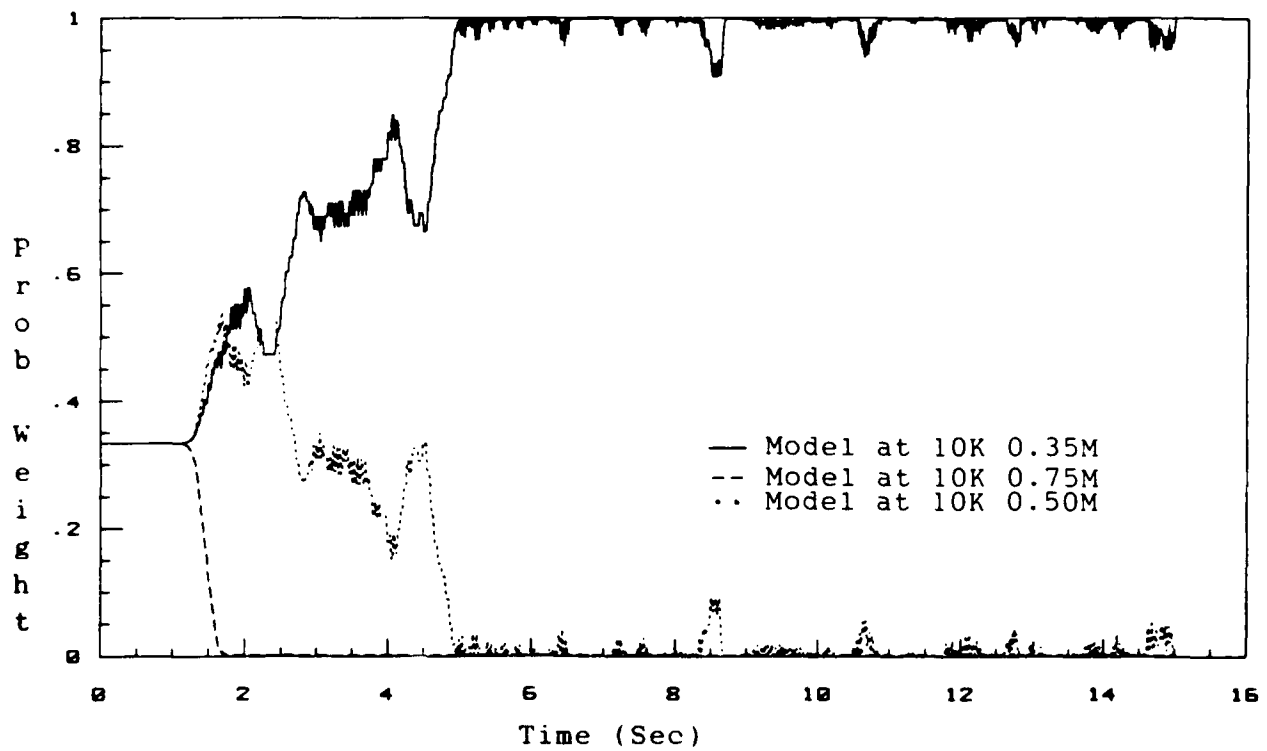


Figure 6-54. Model Probability Weightings
Configuration 8/Operating Point: 26K 0.65M

ferences between the dynamic pressure of the flight condition and the candidate models. The data showed that, in over ninety percent of the three hundred cases run, the candidate model that received the highest conditional probability was the model with a dynamic pressure closest to the dynamic pressure of the actual flight condition. Again, this pointed out the fact that model equivalency is largely dependent on dynamic pressure.

As just discussed, configurations 6, 7, and 8 all yielded satisfactory tracking performance (from a performance criteria standpoint) over the flight envelope of interest. However, when comparing the performance index responses of configurations 6, 7, and 8, it was determined that configuration 8 resulted in the least tracking error for a majority of the flight conditions. Table 6-13 presents a representative sample of the performance index results for configurations 6, 7, and 8. Configuration 6 had the largest tracking error as determined by the performance index for all of the flight conditions. As seen from Table 6-13, configuration 8 was the best of the configurations labeled as 6, 7, and 8 and configuration 6 was the least desirable.

In an effort to obtain a three-model configuration that

Table 6-13

Performance Index Comparison of Selected
Flight Conditions

Flight Condition		Configuration with Minimal Error		
Alt (Kft)	Mach	6	7	8
10	0.35			X
10	0.65			X
10	0.90		X	
14	0.35			X
14	0.65			X
14	0.90		X	
22	0.40			X
22	0.65			X
22	0.90		X	
26	0.45			X
26	0.65			X
26	0.90			X
30	0.50			X
30	0.90			X
38	0.60			X
38	0.90			X

yielded better performance than configuration 8, several other three-model configurations were examined. Specifically, these were configurations 9 through 14 as given on Table 6-9. In each case, the overall performance of configuration 8 proved to be the superior configuration.

Configuration 9 had performance index responses that were very similar to those of configuration 8. Although configuration 8 did have a minimal amount of error most of

the time, configuration 9 had less error at the low altitude/high Mach flight conditions. Configurations 11, 13, and 14 had performance index responses that were less than those for configuration 8 at 38,000 ft, 0.9 Mach. However, configuration 8 had better responses over the rest of the flight envelope. So, from a performance criteria standpoint, configuration 8 was the best configuration of those investigated with configuration 9 yielding similar performance.

In an effort to better the performance of configuration 8, a four-model configuration was evaluated with all of the models at 10,000 ft and Mach numbers of 0.35, 0.5, 0.75, and 0.9, respectively. However, the performance of this configuration was not any better than that afforded by the three models of configuration 8.

6.2.4 Control Law Gain Adjustment. In his thesis, Barfield (21) noted that although the Porter design technique yields exceptionally fast aircraft responses, many times this is achieved through large and extremely fast surface movements. After a review of several applications of the design technique, it became apparent that for a given design, the

larger the control law gains, the faster the surface movements. Reaching either the position limits or the rate limits is a condition that should be avoided. Reaching a position limit prevents the input command from being achieved. Reaching rate limits results in the system becoming nonlinear and can result in an increase in phase lag in a feedback system. This phase lag increase has been shown to cause loss of control in aircraft with a full authority fly-by-wire system (21).

Recall, from Chapter 3, that the control law gains are given by

$$\underline{K}_1 = \underline{H}^{-1}(T)\underline{\xi} \quad (6-12)$$

$$\underline{K}_2 = \rho \underline{K}_1 \quad (6-13)$$

where the diagonal weighting matrix, $\underline{\xi}$, can be altered to achieve desired tracking characteristics and the parameter, ρ , is a constant which assigns the ratio of proportional to integral control.

For all of the simulations that were accomplished up to this part of this thesis, the control law design parameters that were implemented were those used by Pineiro and Berens. The control law design parameters had the following values:

$$\underline{\xi} = \begin{bmatrix} 0.4 & 0.0 \\ 0.0 & 0.7 \end{bmatrix} \quad (6-14)$$

and

$$\rho = 0.8 \quad (6-15)$$

As is shown earlier, configuration 8 had satisfactory performance over the flight envelope of interest (from a performance index standpoint). However, for several of the flight conditions the rate limits were encountered for both the elevators and flaperons. In an effort to reduce the control surface activity for the multiple model algorithm, an investigation into the effects of altering the control law design parameters was performed.

To begin, the effects of altering the control law design parameters on the performance of a fixed gain simulation were investigated. The nominal flight condition chosen for this part of the analysis was 22,000 ft, 0.65 Mach. The responses for the fixed gain simulation using the control law design parameters values as shown above are shown Figures 6-55 through 6-60. As can be seen on Figure 6-60, the rate limits for the flaperons were encountered several times. For the reasons given above, this is a condition that should be avoided.

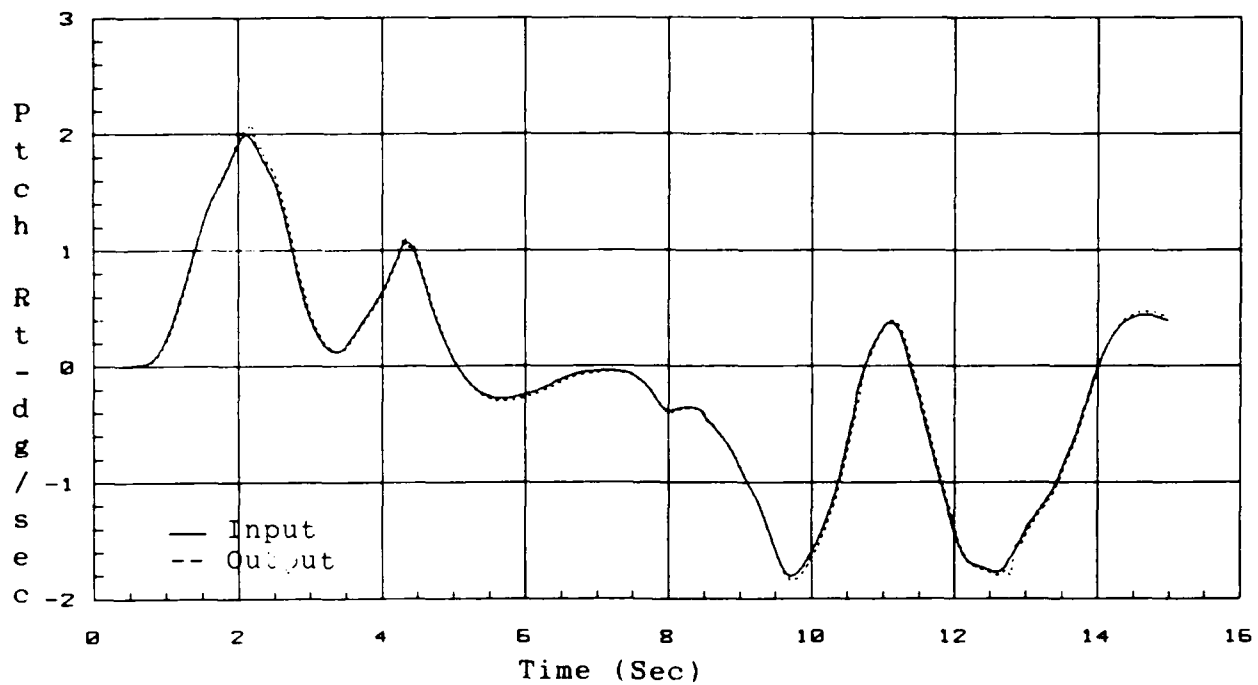


Figure 6-55. Pitch Rate Response
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

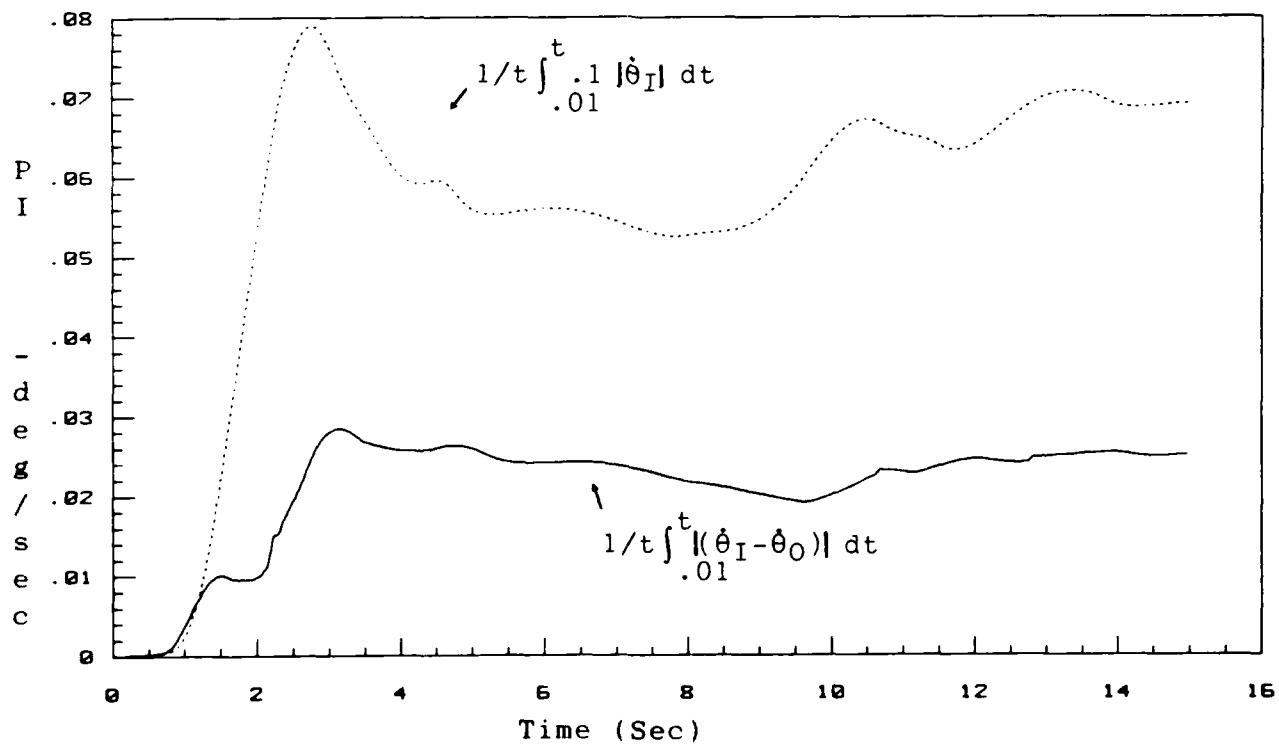


Figure 6-56. Pitch Rate Performance Criterion
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

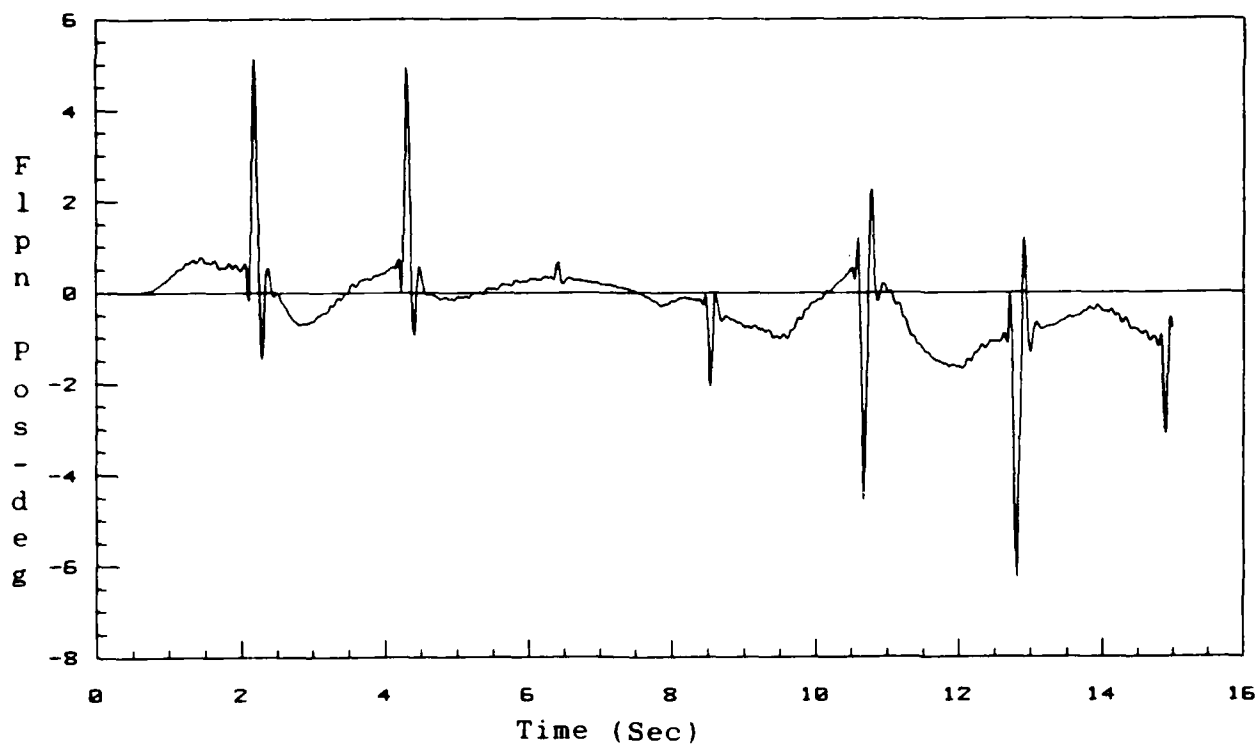


Figure 6-57. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

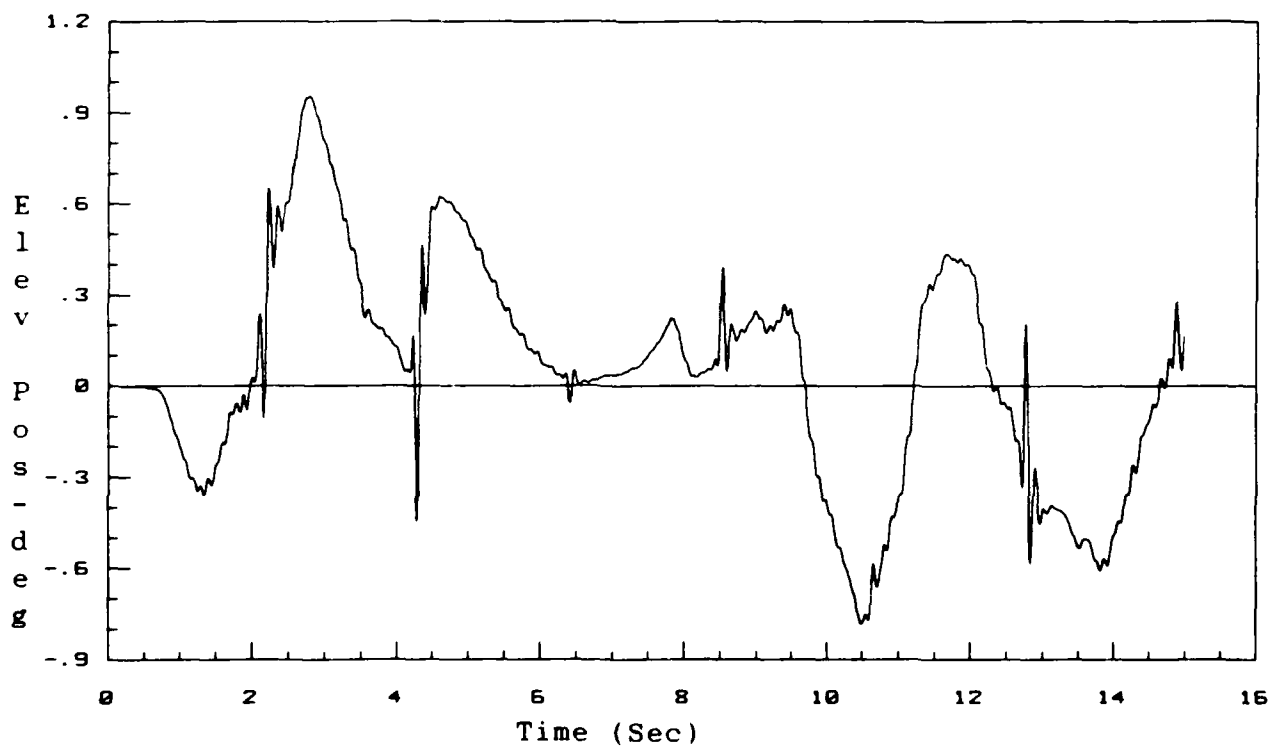


Figure 6-58. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

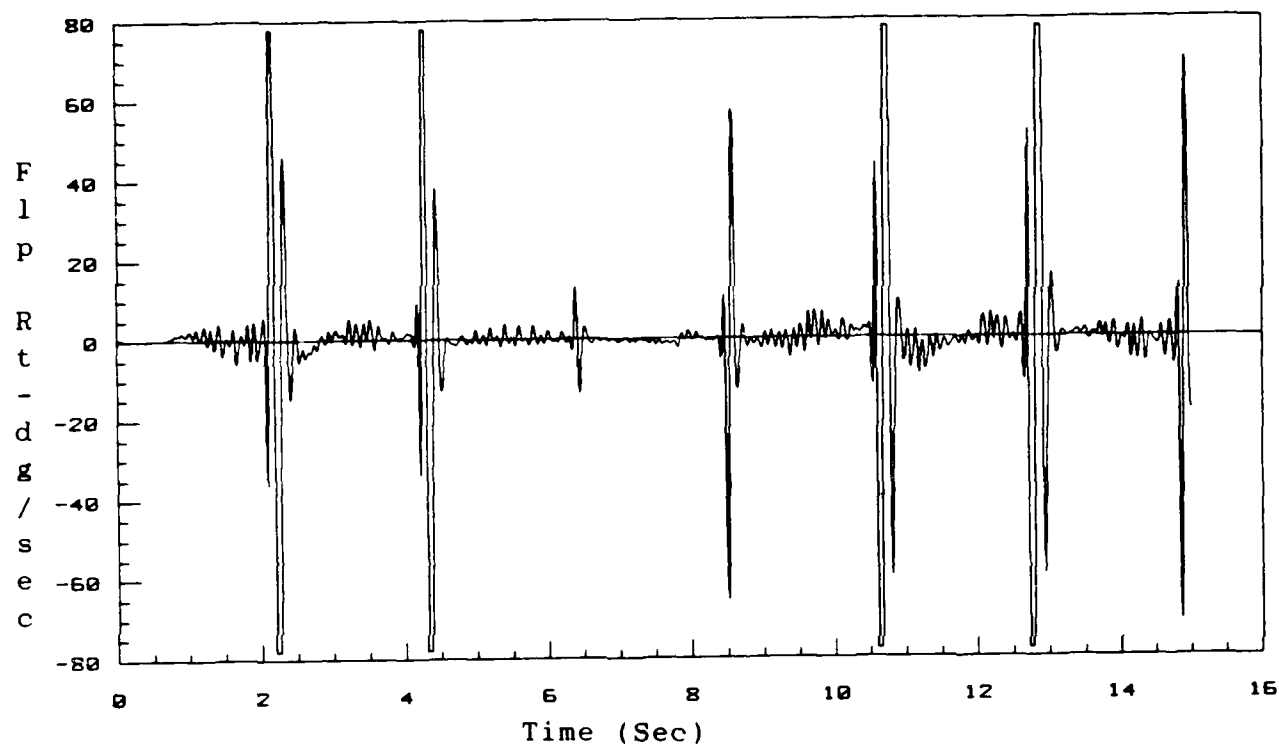


Figure 6-59. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

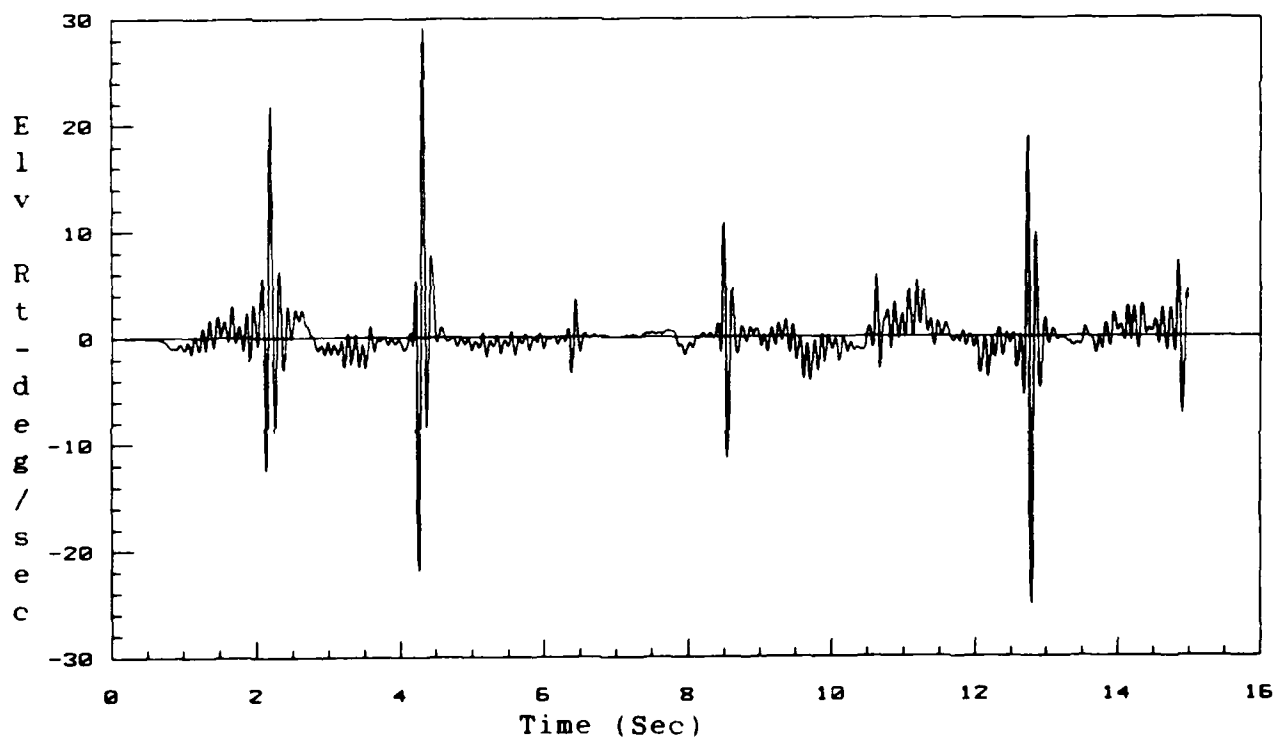


Figure 6-60. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.8

Table 6-14 lists the simulations that were run subsequent to the initial 22,000 ft, 0.65 Mach simulation and the respective values of the control law design parameters. The entries labeled Sigma 1 and Sigma 2 refer to the 1,1 and 2,2 elements of the diagonal weighting matrix, respectively while the term rho refers to the design parameter, ρ .

As can be seen from Table 6-14, simulations 1 and 2 were run with varying values of the parameter, ρ , simulations 3 through 6 were performed with different values of the 1,1 element of the weighting matrix, and simulations 7 through 9 were accomplished with different values of the 2,2 element of the weighting matrix.

Table 6-14

Design Parameter Values for Simulations
at 22,000 ft, 0.65 Mach

Simulation Number	Control Law Design Parameters		
	Sigma 1	Sigma 2	Rho
1	0.40	0.70	0.40
2	0.40	0.70	0.10
3	0.80	0.70	0.80
4	0.20	0.70	0.80
5	0.10	0.70	0.80
6	0.05	0.70	0.80
7	0.40	1.50	0.80
8	0.40	0.35	0.80
9	0.40	0.10	0.80

Decreasing the parameter, ρ , resulted in an increase in the tracking error as shown on the performance index responses. Figures 6-61 through 6-66 present the simulation results when the parameter, ρ , was set to 0.4 (simulation 1 on Table 6-14). By comparing the responses of the elevators and flaperons for the original simulation and those of simulation 1, it can be seen that there is little difference in control surface performance. Therefore, decreasing the ratio of proportional to integral control results in an increase in the tracking error as determined by the performance index while resulting in minimal changes in the control surface performances.

Figures 6-67 through 6-72 present the responses for simulation 3 on Table 6-14. As shown on Figure 6-68, increasing the value of Sigma 1 resulted in an increase in the performance error. Also, the increase in Sigma 1 resulted in the control surfaces becoming more active as evidenced by the deflection rates. The negative aspect of this is that the control surfaces spent a larger amount of time at the rate limit. Conversely, reducing Sigma 1 as in simulation 5 (see Figures 6-73 through 6-78) resulted in a performance error that was essentially the same as that of the original

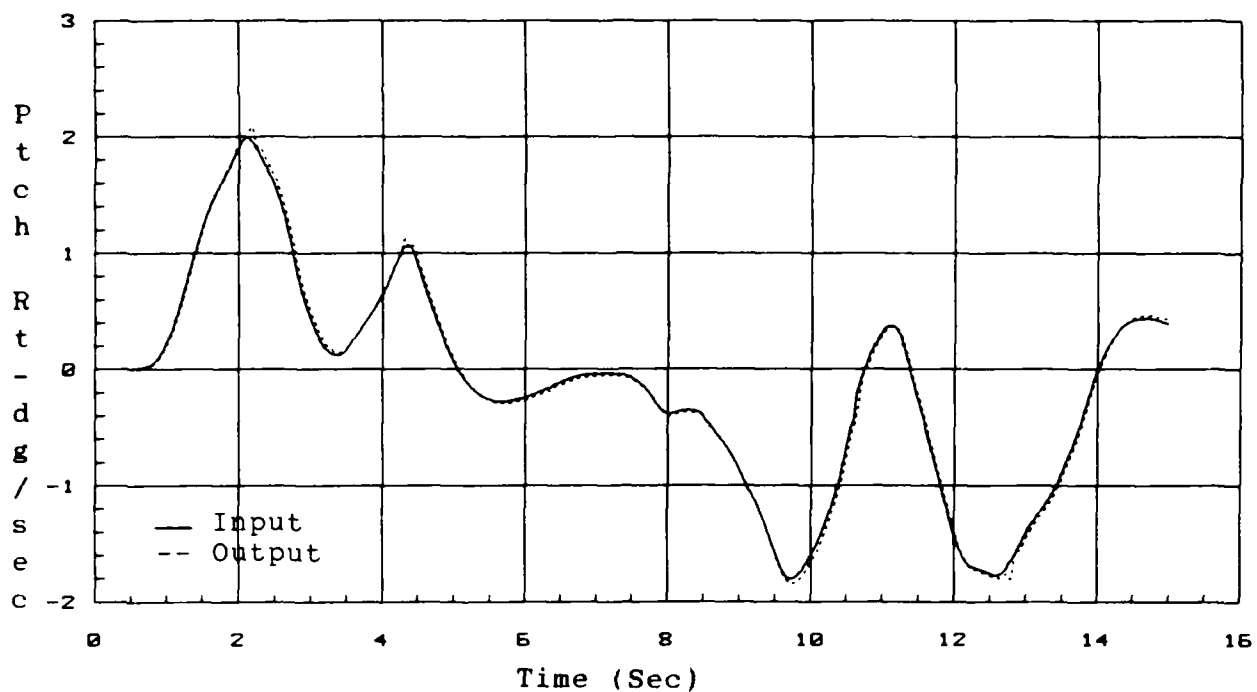


Figure 6-61. Pitch Rate Response
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

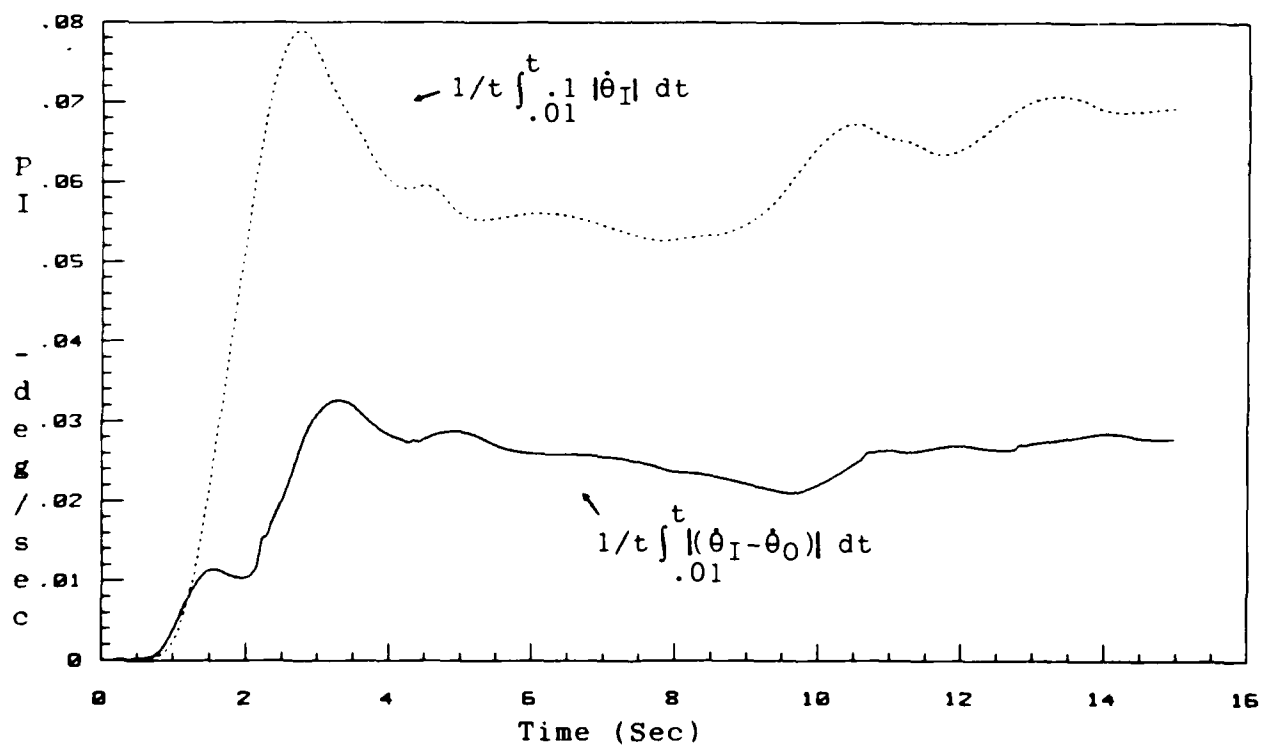


Figure 6-62. Pitch Rate Performance Criterion
 Nominal Model: 22K 0.65M/Operating Model: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

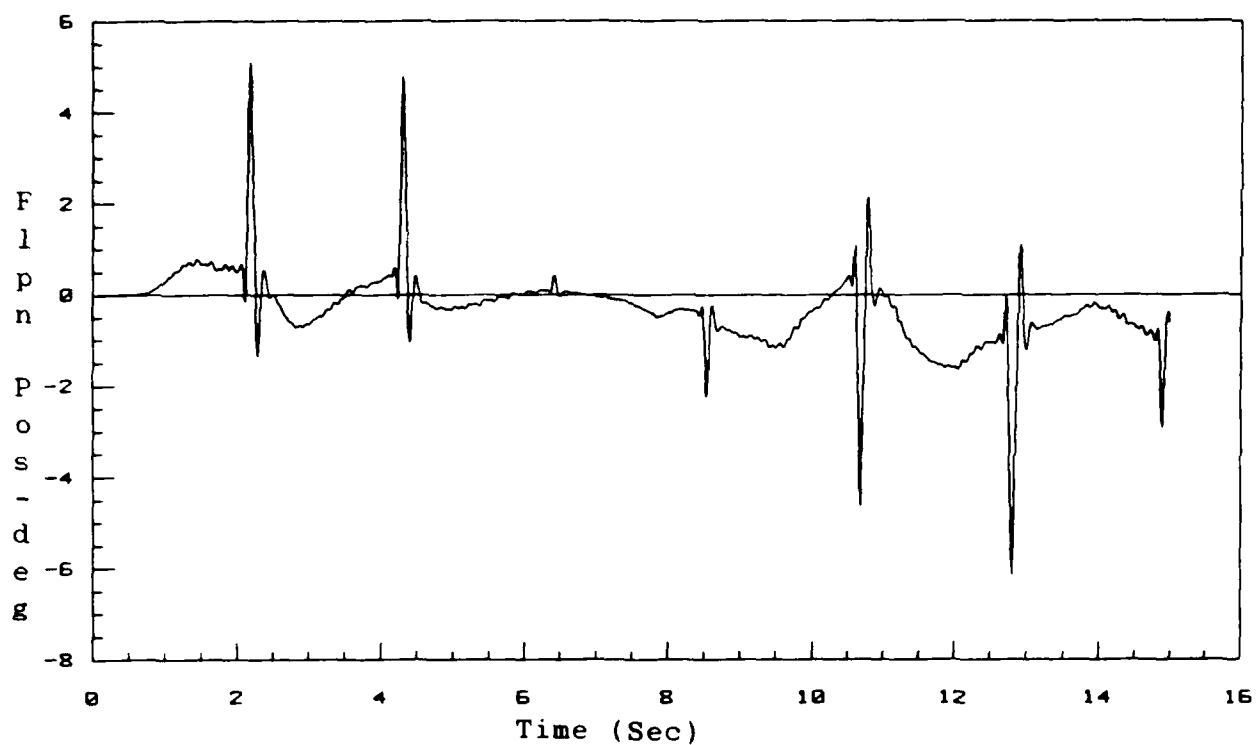


Figure 6-63. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

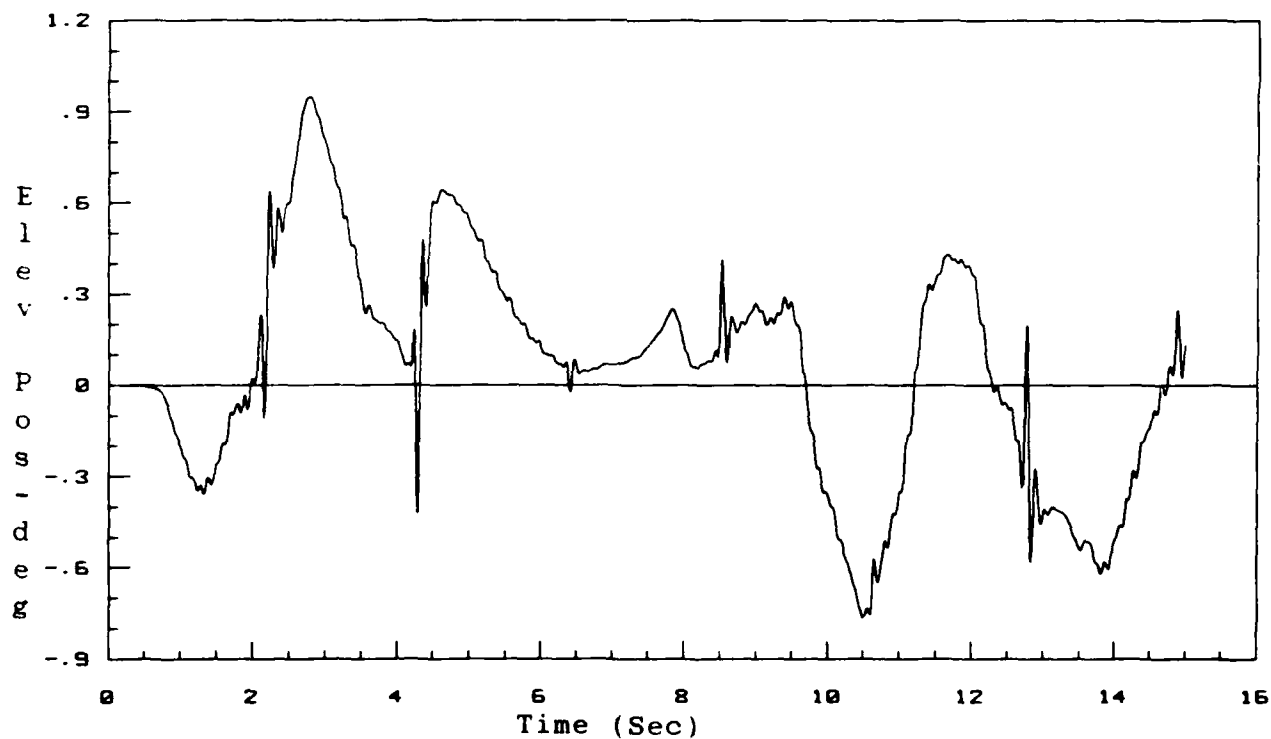


Figure 6-64. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

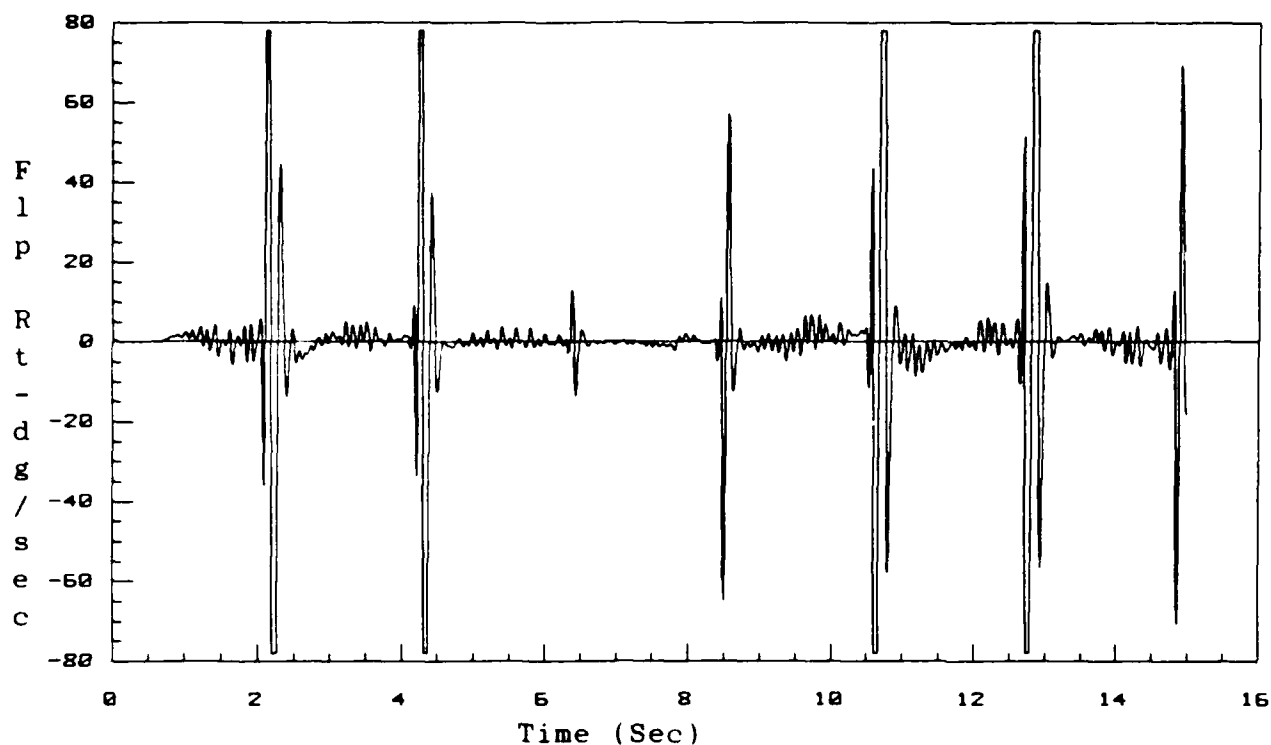


Figure 6-65. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

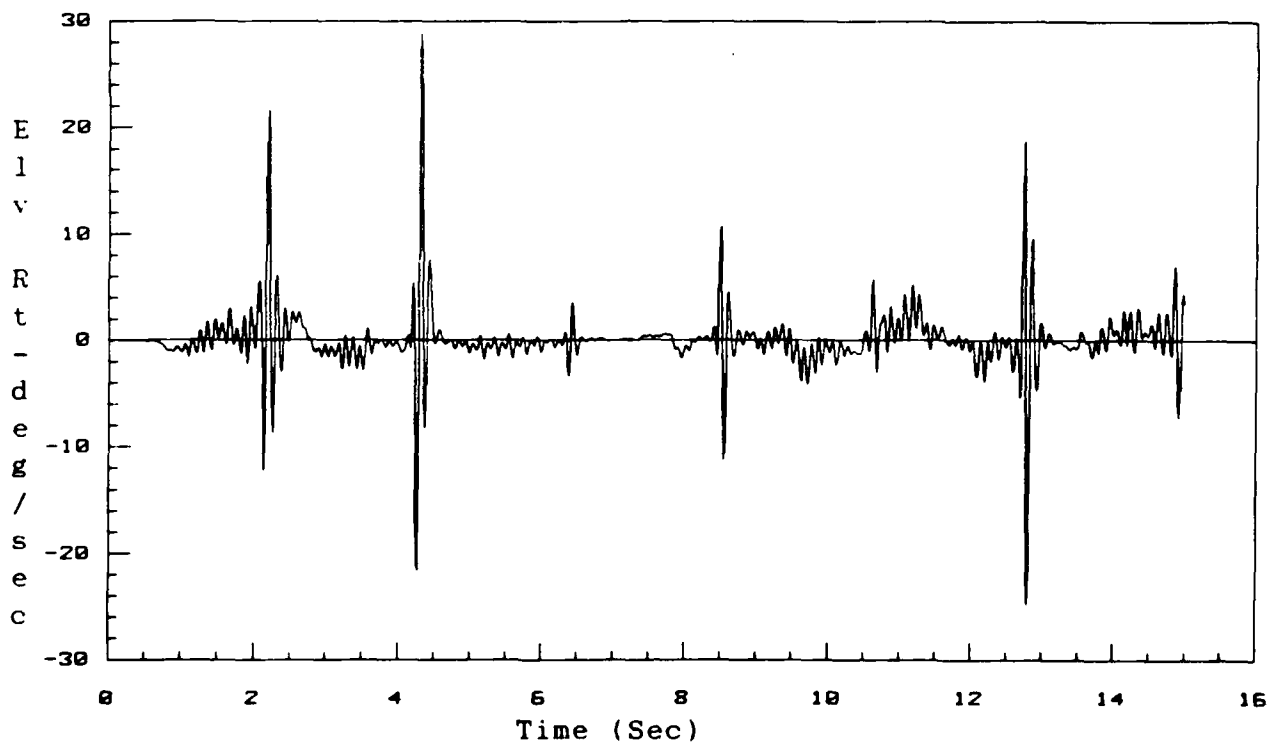


Figure 6-66. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.7 RHO=0.4

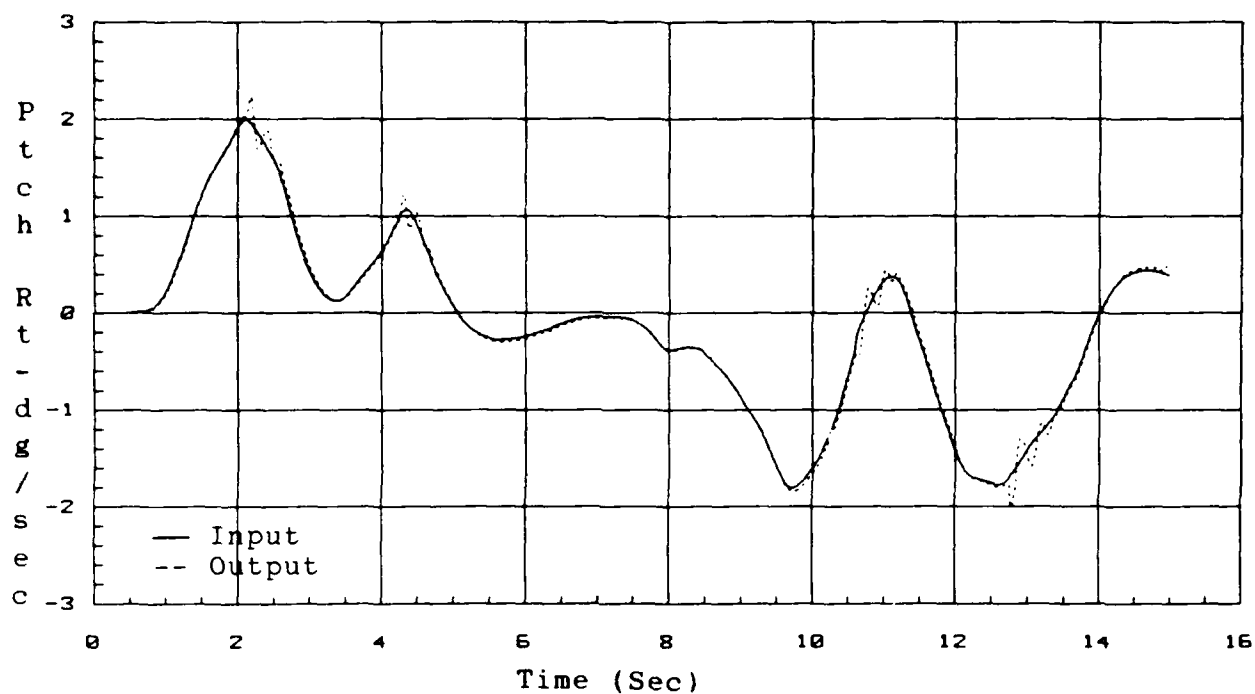


Figure 6-67. Pitch Rate Response
Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
SIG1=0.8 SIG2=0.7 RHO=0.8

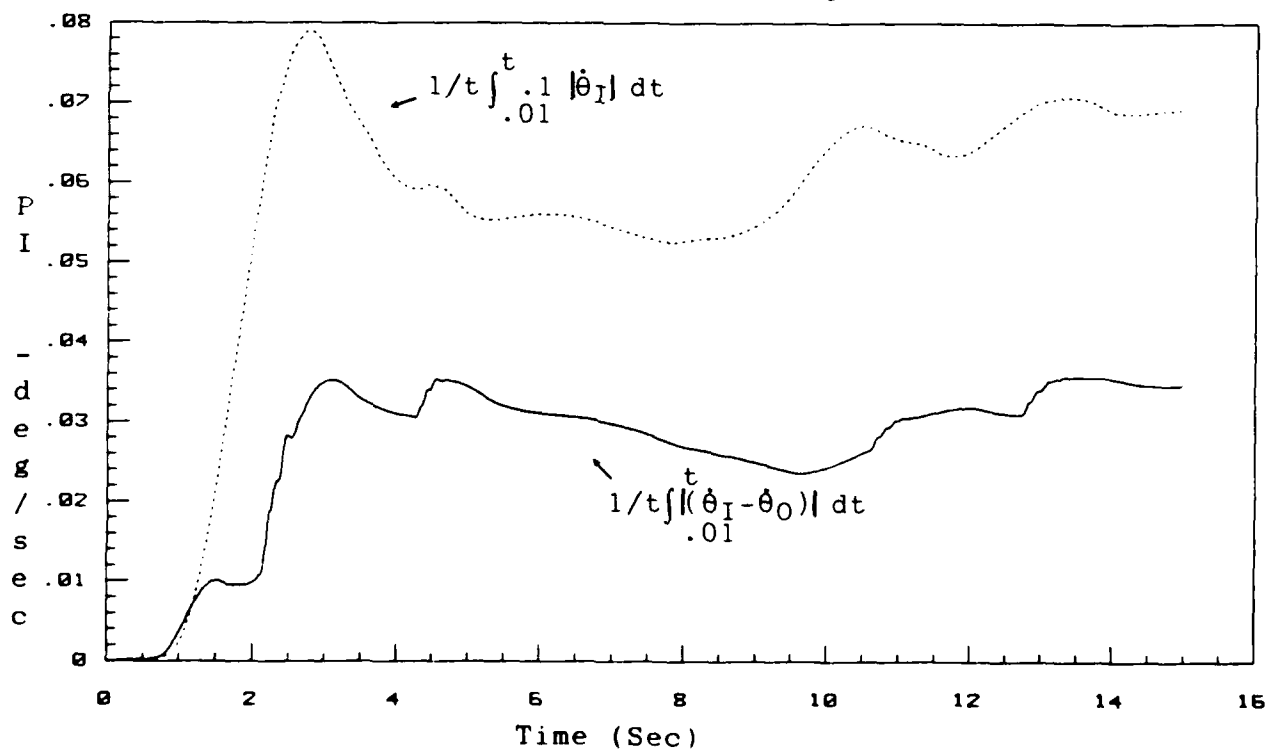


Figure 6-68. Pitch Rate Performance Criterion
Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
SIG1=0.8 SIG2=0.7 RHO=0.8

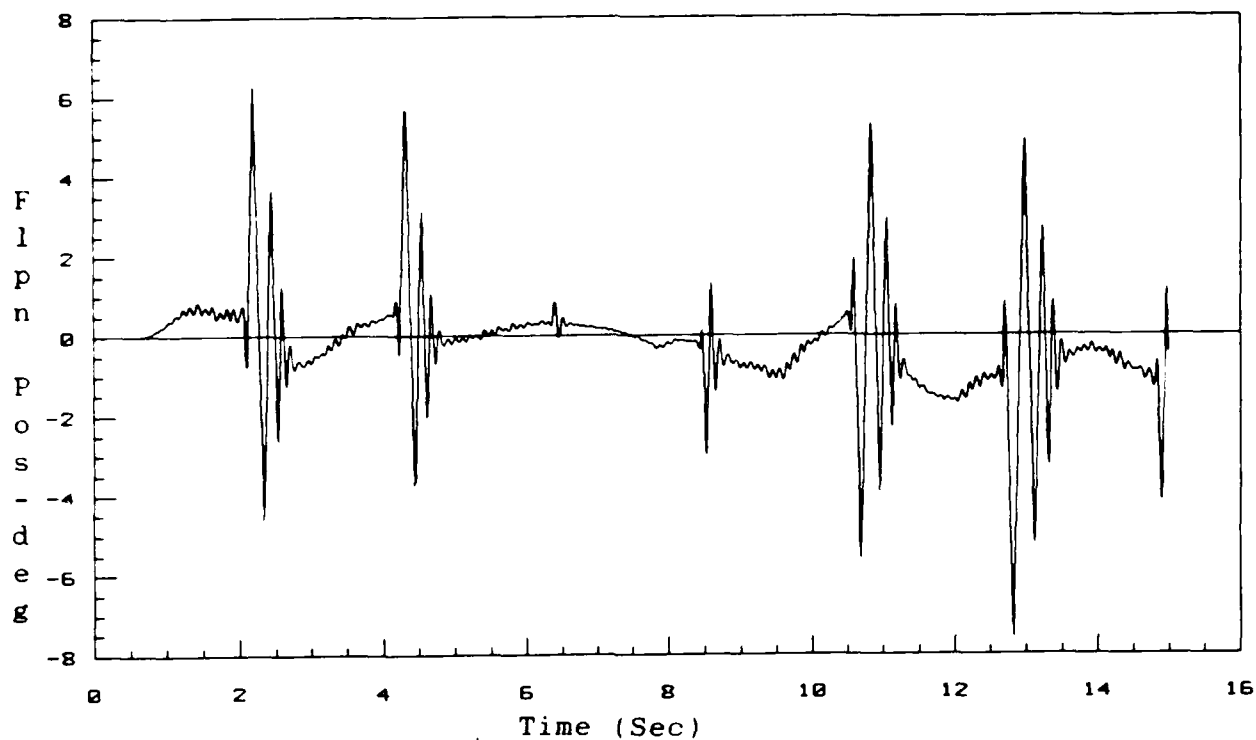


Figure 6-69. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.8 SIG2=0.7 RHO=0.8

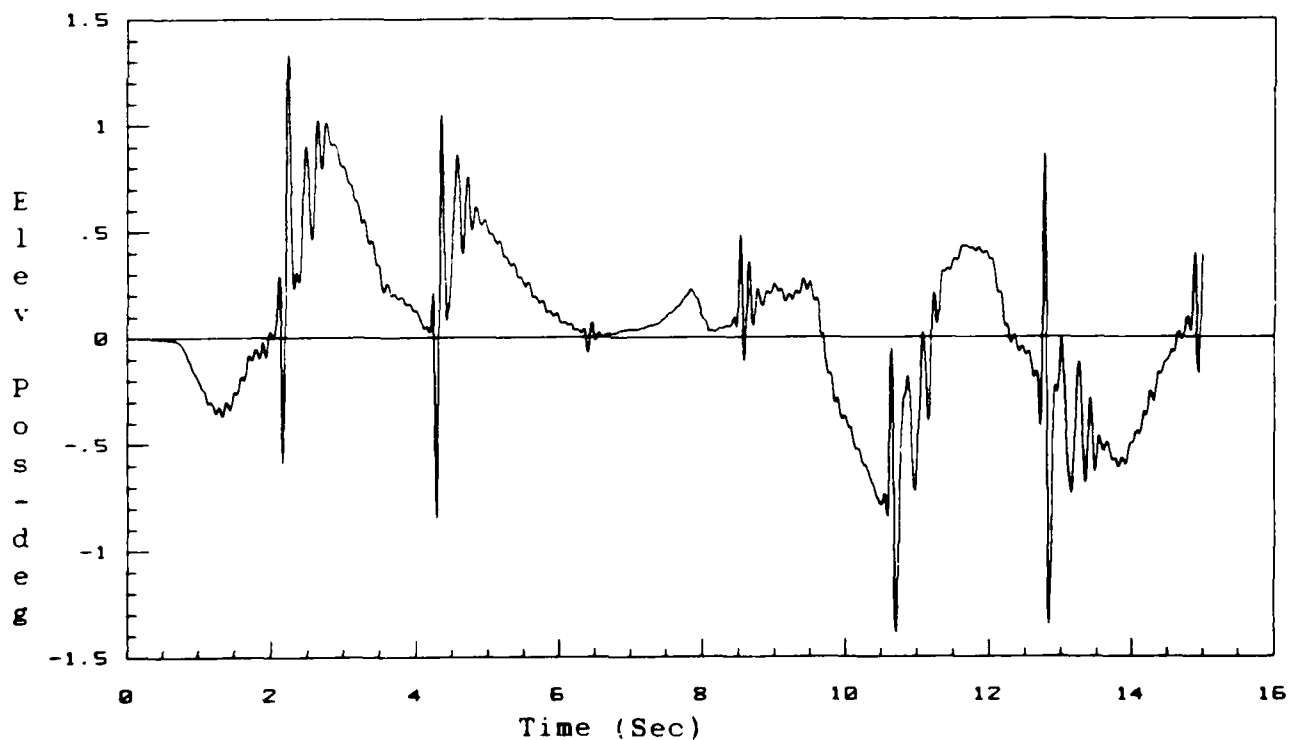


Figure 6-70. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.8 SIG2=0.7 RHO=0.8

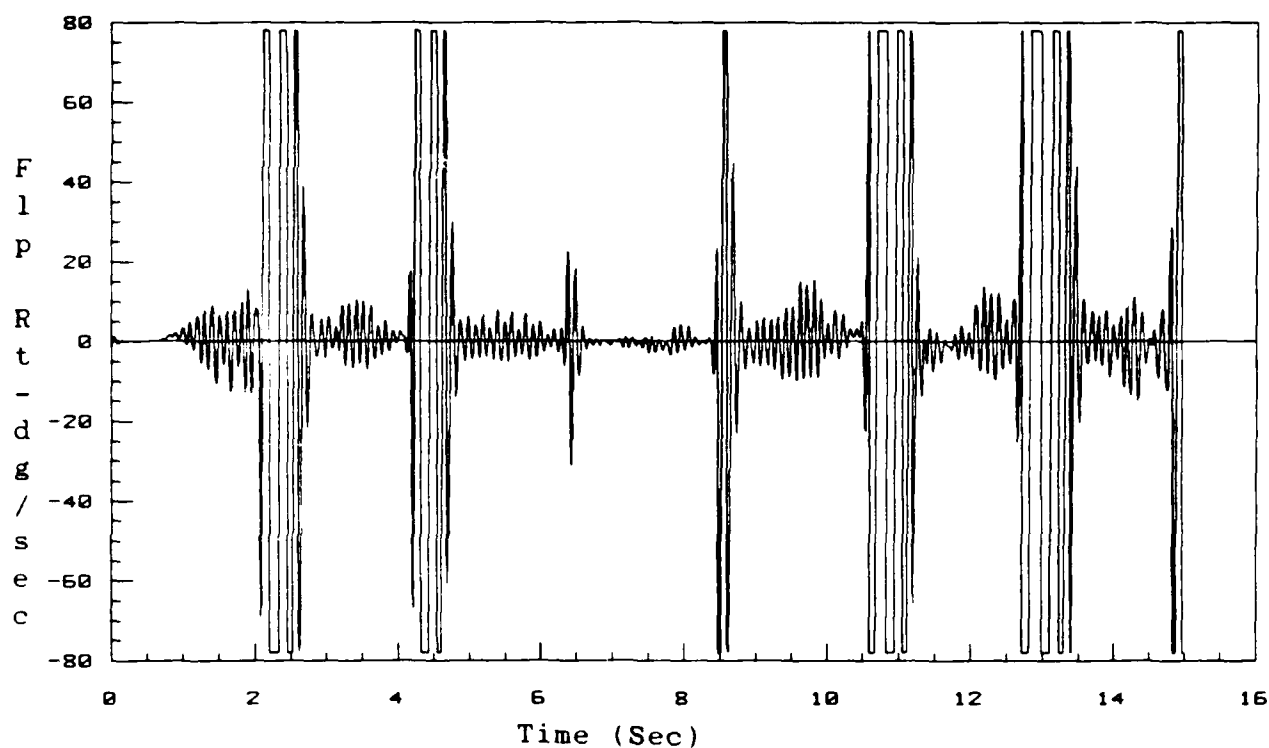


Figure 6-71. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.8 SIG2=0.7 RHO=0.8

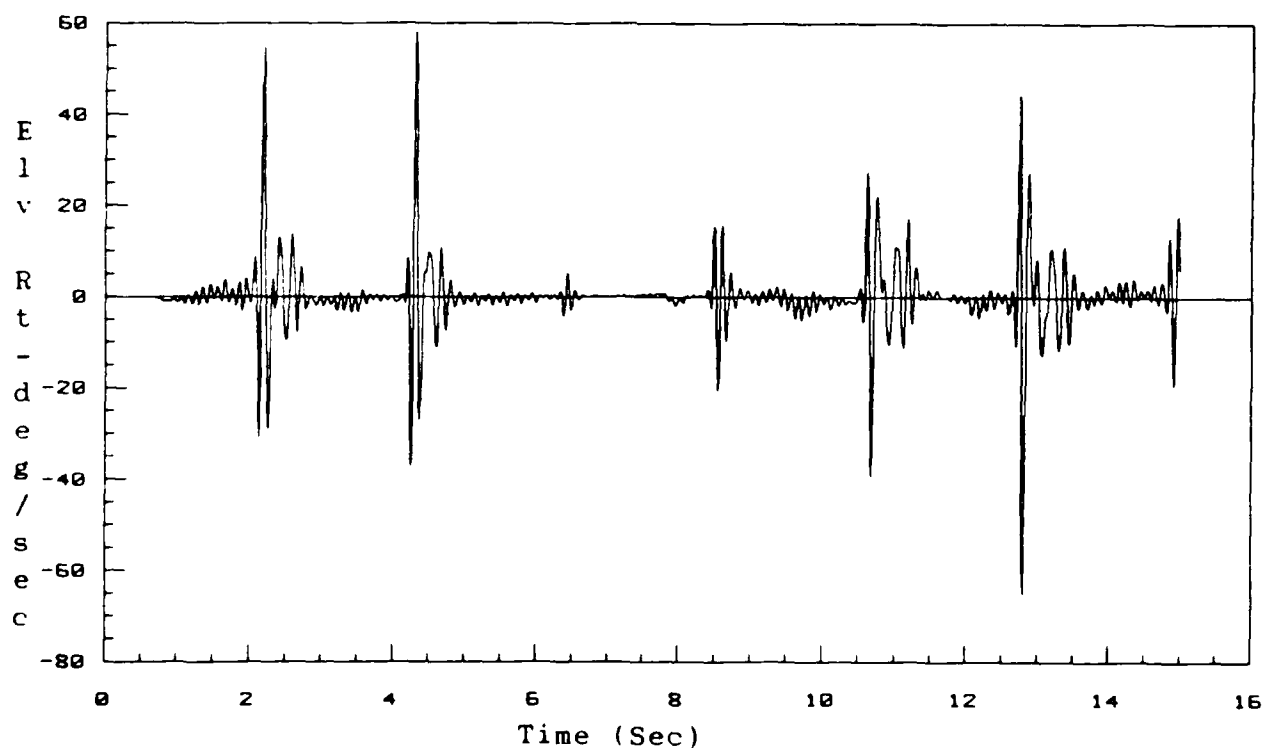


Figure 6-72. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.8 SIG2=0.7 RHO=0.8

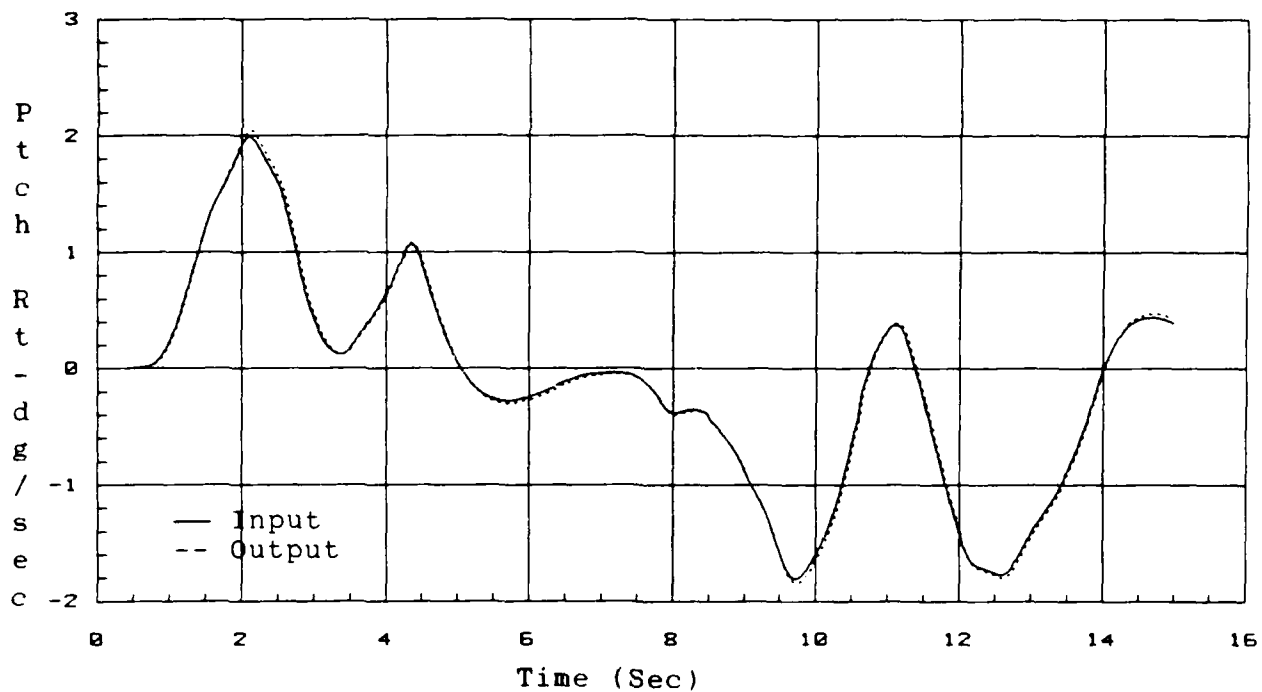


Figure 6-73. Pitch Rate Response
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

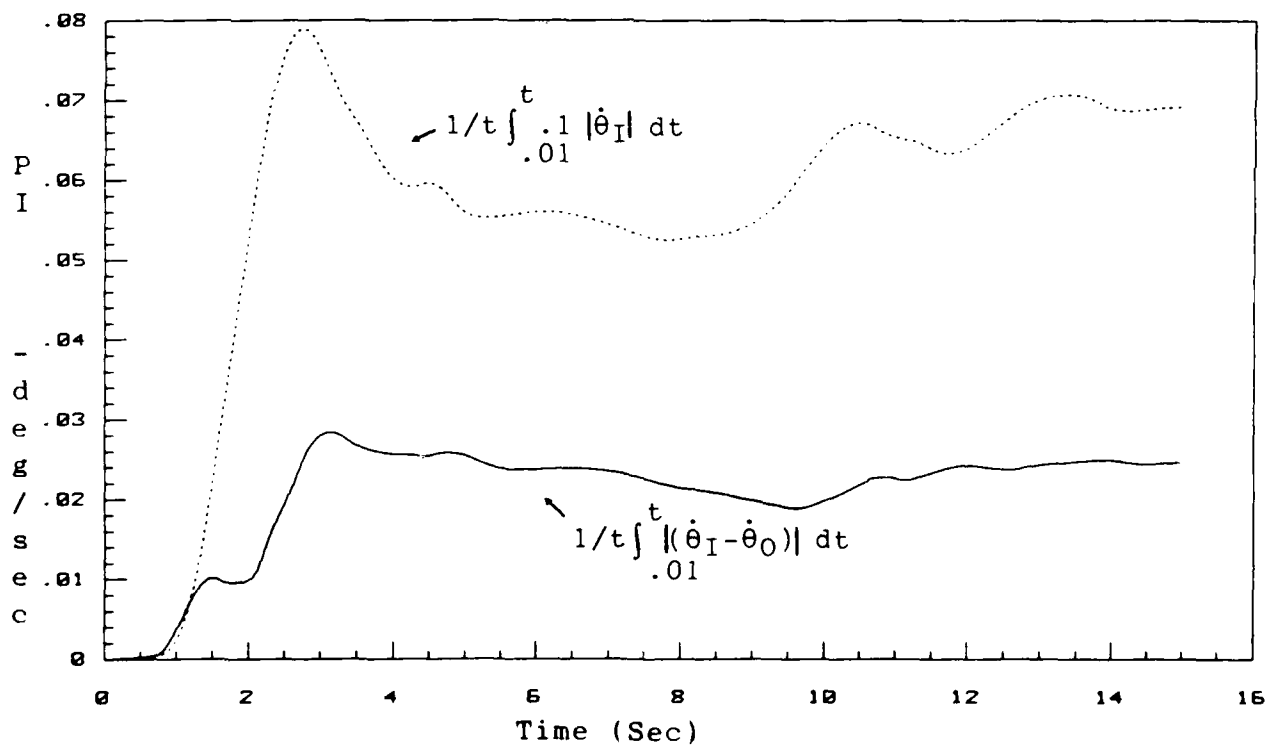


Figure 6-74. Pitch Rate Performance Criterion
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

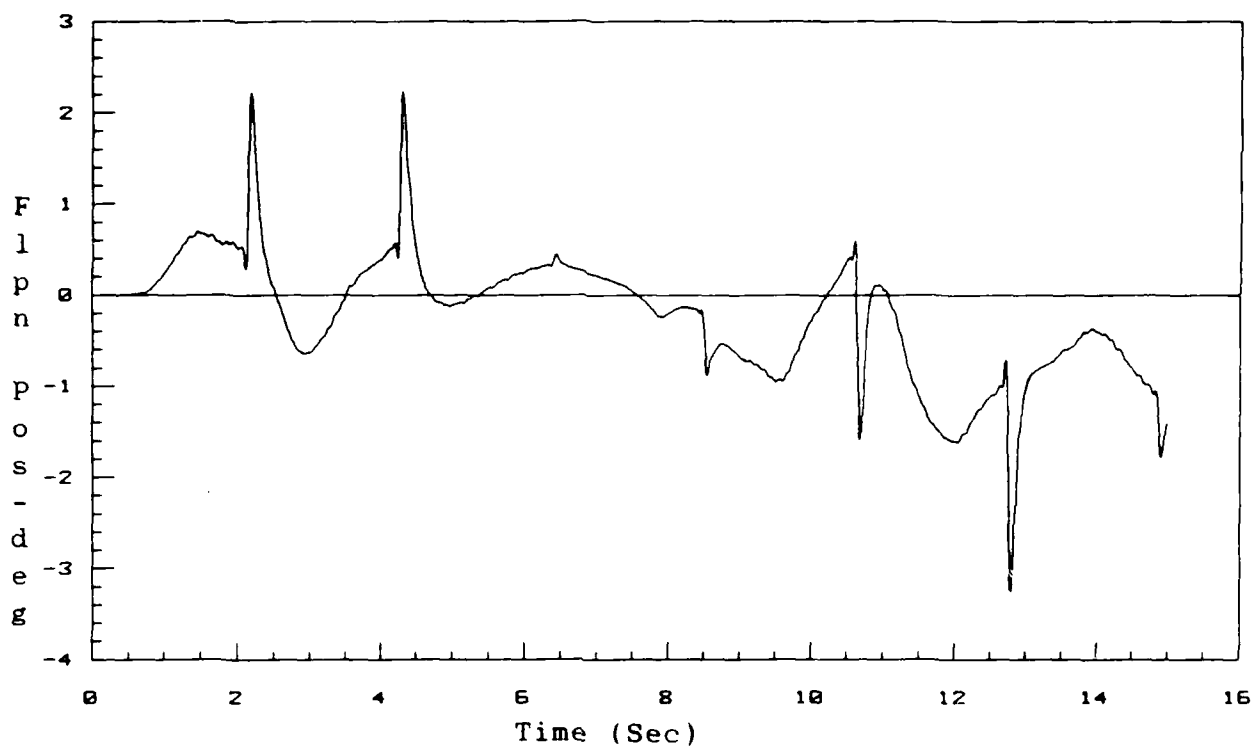


Figure 6-75. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

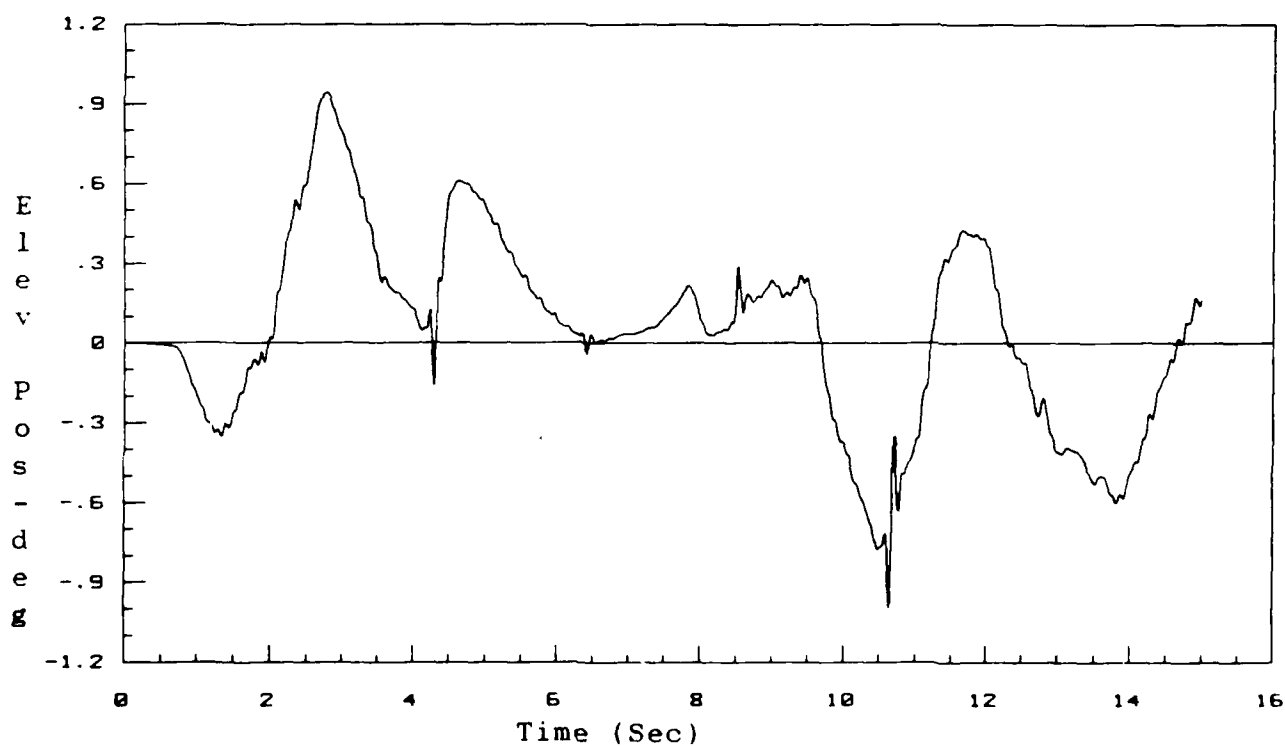


Figure 6-76. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

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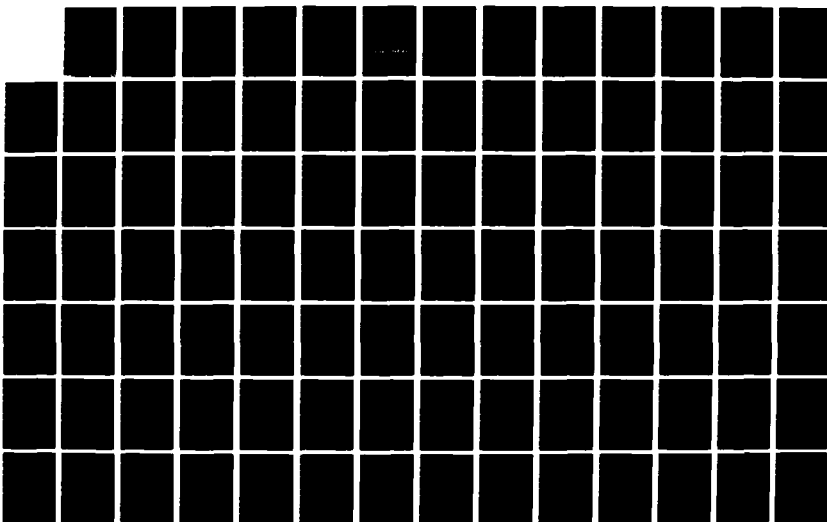
MODEL SELECTION FOR THE MULTIPLE MODEL ADAPTIVE
ALGORITHM FOR IN-FLIGHT SIMULATION(U) AIR FORCE INST OF
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AFIT/GE/ENG/87D-48

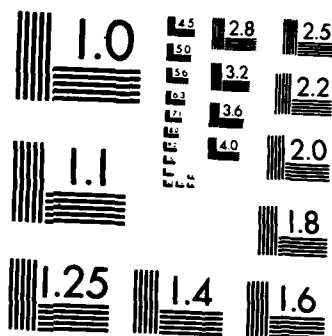
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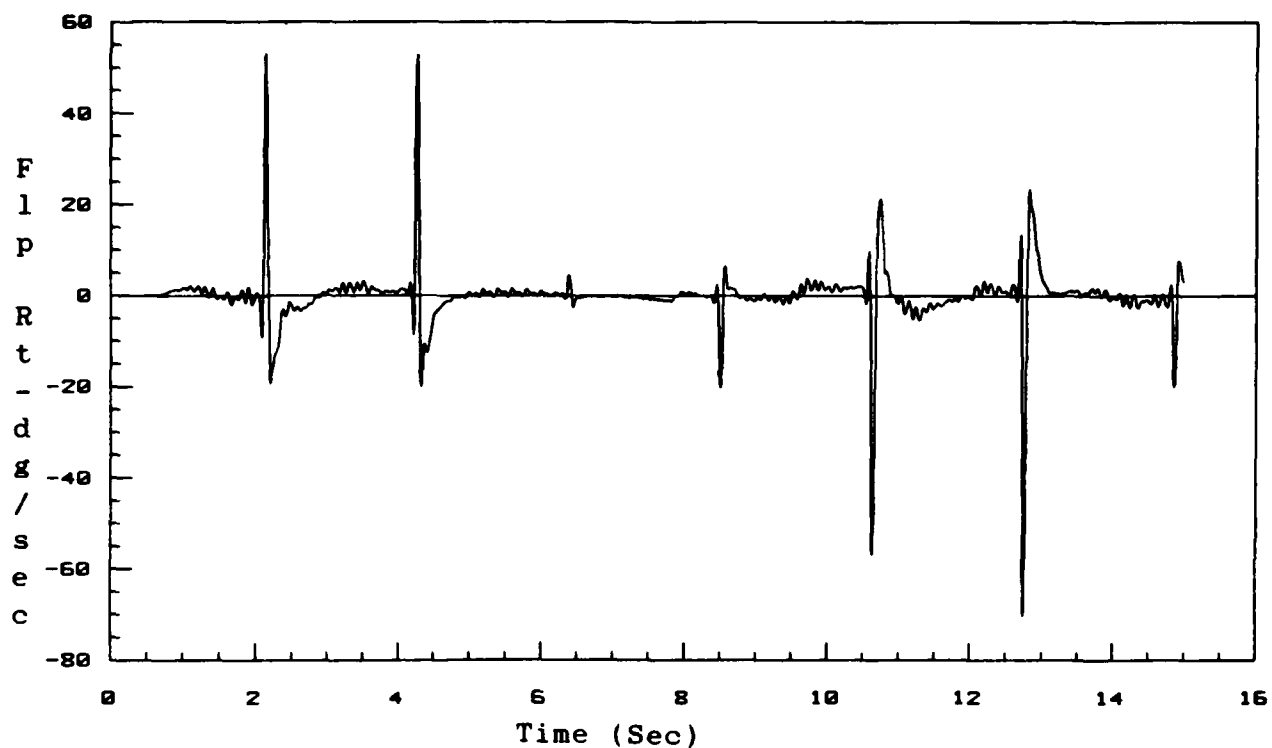


Figure 6-77. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

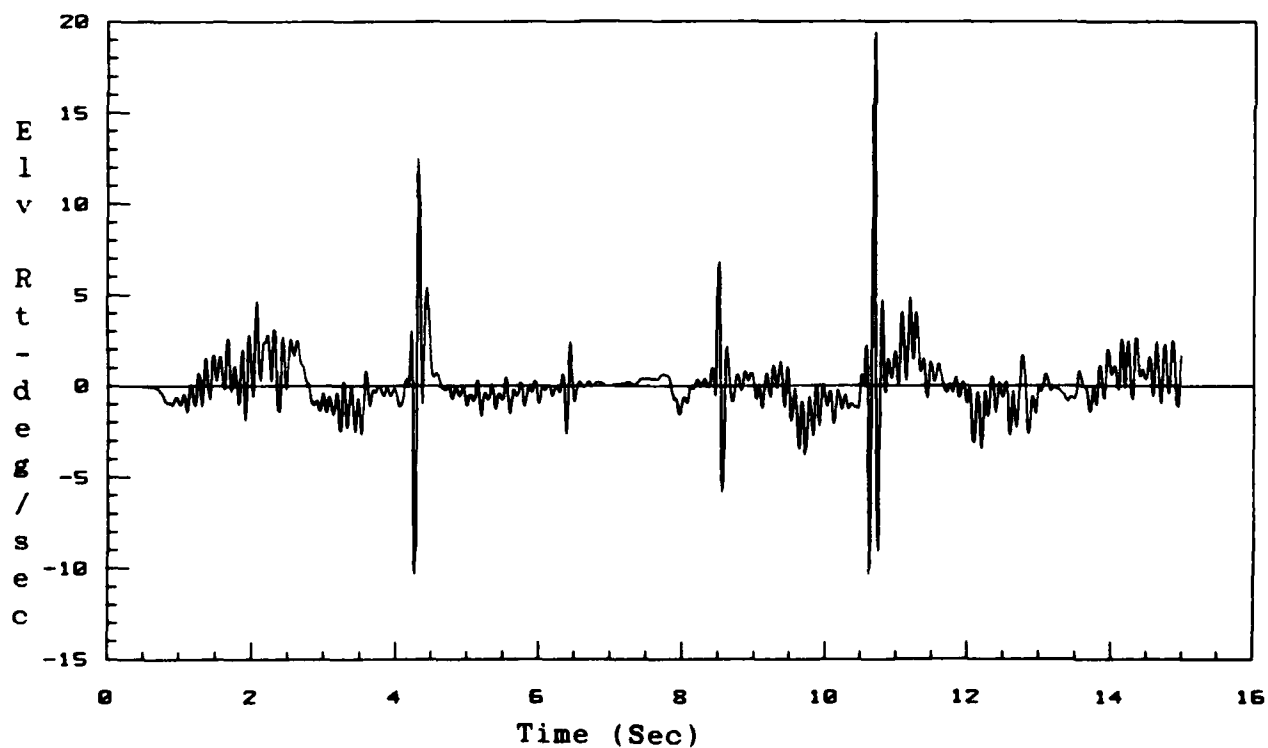


Figure 6-78. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.1 SIG2=0.7 RHO=0.8

case. However, the elevator and flaperon rates were much improved. The flaperon rates never reached rate limit for the entire simulation. The important point here is that, although the performance index was not affected significantly, decreasing Sigma 1 resulted in improved control surface activity.

Increasing Sigma 2 resulted in less error as shown on Figures 6-79 through 6-84 for simulation 7, but the control surfaces were more active, as can be seen on the elevator and flaperon positions and rates. Decreasing Sigma 2 had the effect of increasing the error as measured by the performance criteria. This is shown on Figures 6-85 through 6-90 which are simulations for simulation 9 on Table 6-14. Reducing Sigma 2 did not have much effect on the control surface rates, although Figure 6-89 shows that the amount of flaperon required to follow the input increased. The data showed that the ability of the output to tightly track the input was affected significantly by Sigma 2.

Following the determination of the effects of varying the control law design parameters, Sigma 1 was set equal to 0.1 to improve the control surface responses. With Sigma 1 equal to 0.1, simulations were run to determine the perfor-

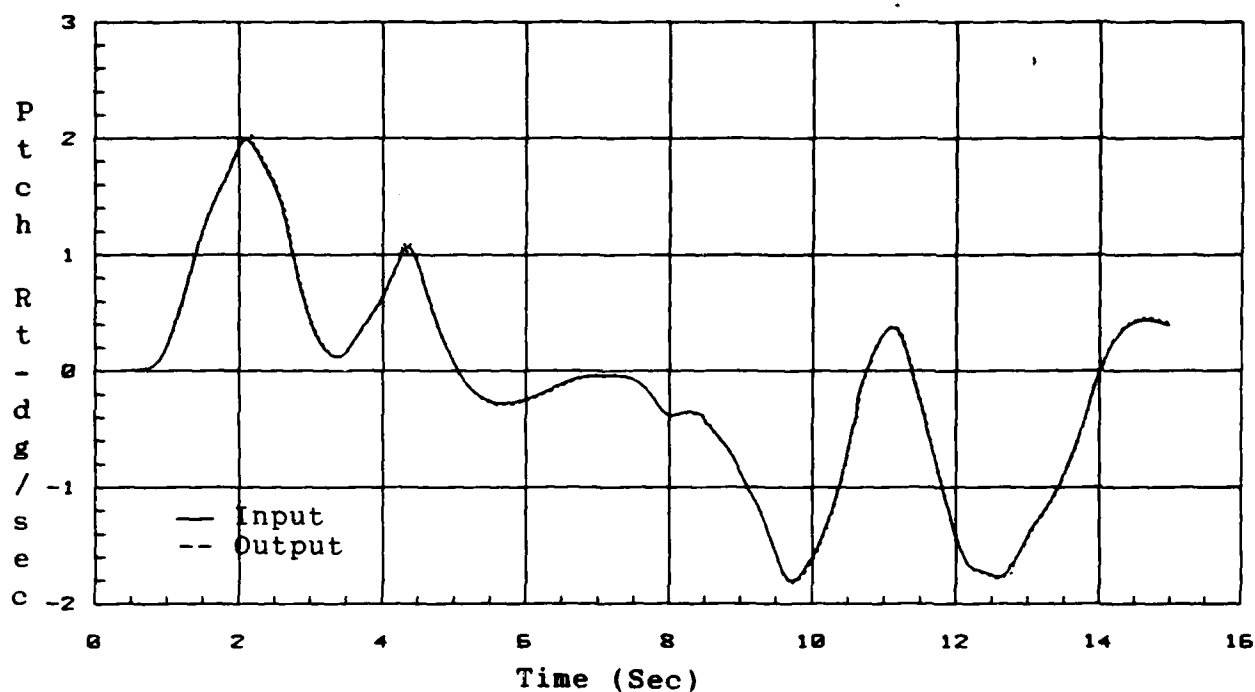


Figure 6-79. Pitch Rate Response
Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
SIG1=0.4 SIG2=1.5 RHO=0.8

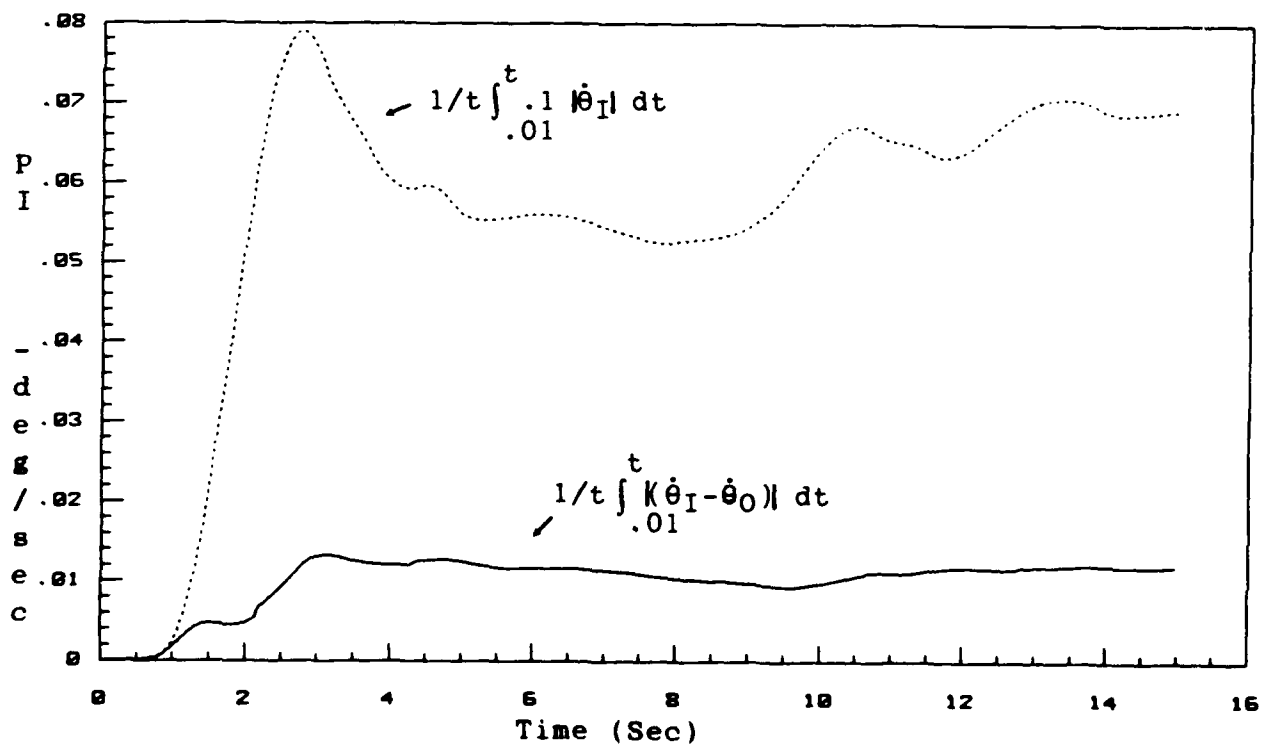


Figure 6-80. Pitch Rate Performance Criterion
Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
SIG1=0.4 SIG2=1.5 RHO=0.8

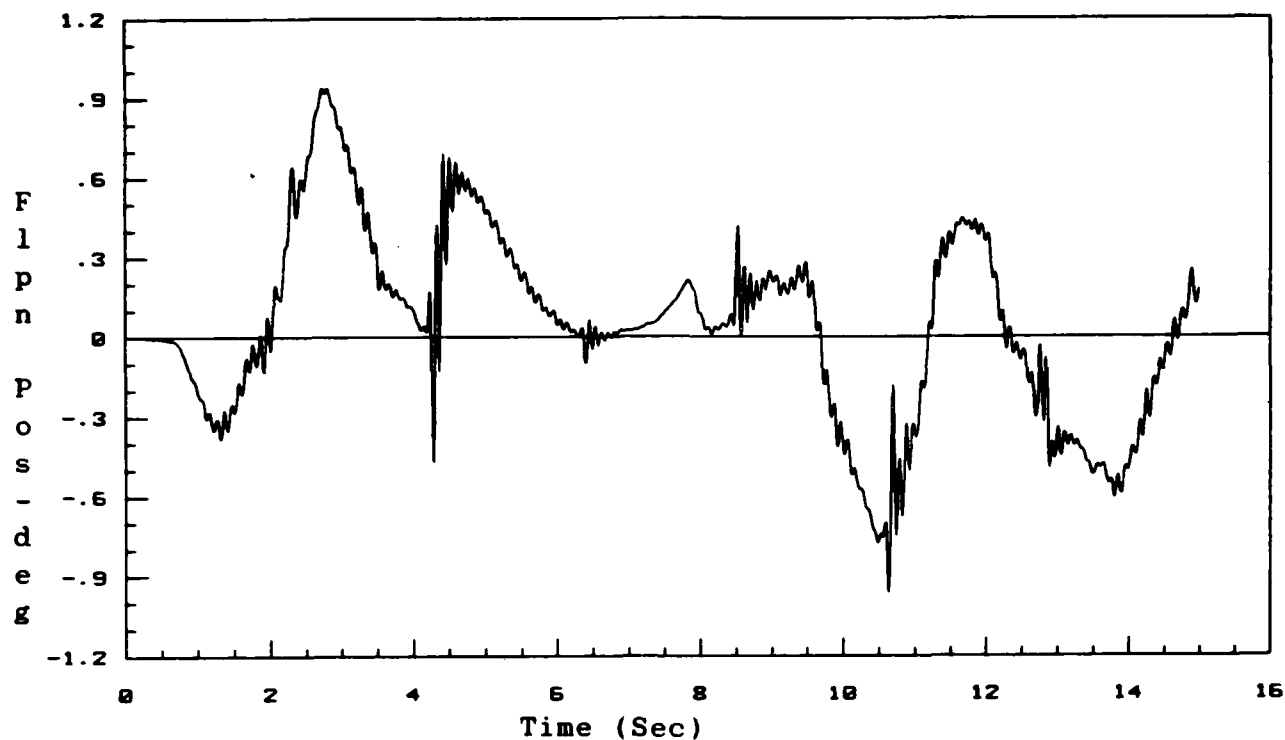


Figure 6-81. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=1.5 RHO=0.8

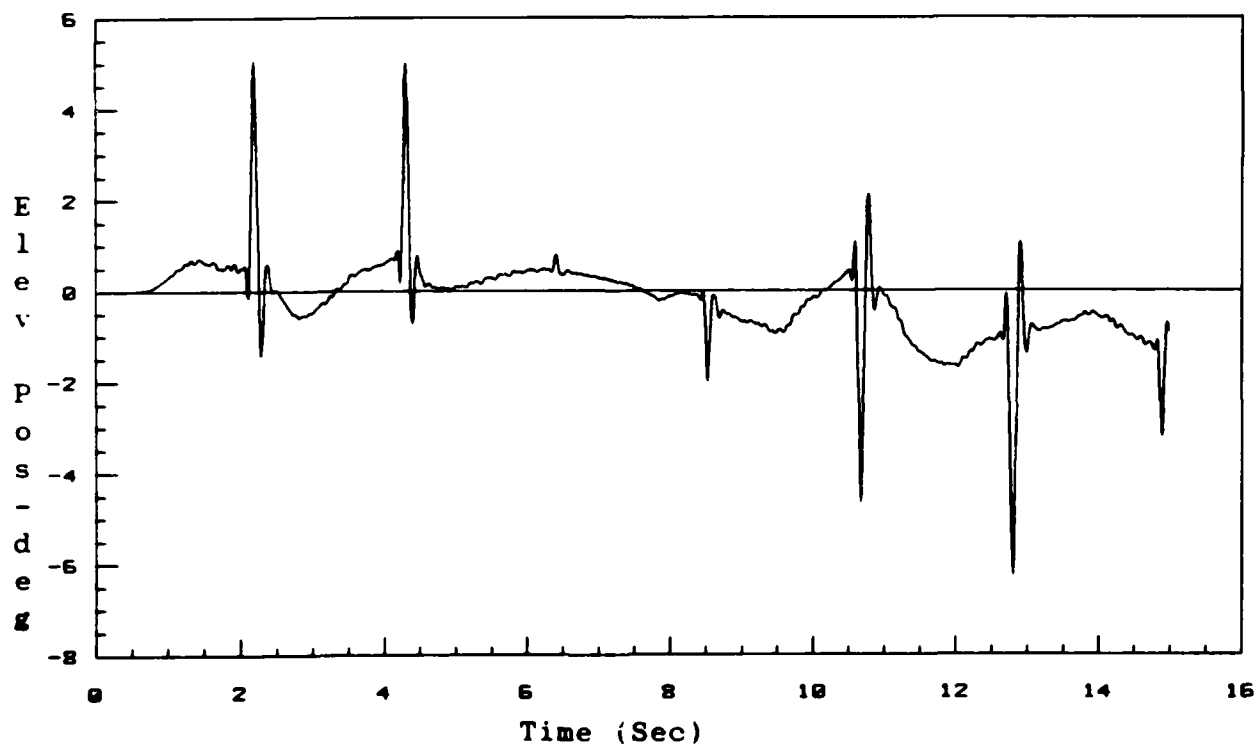


Figure 6-82. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=1.5 RHO=0.8

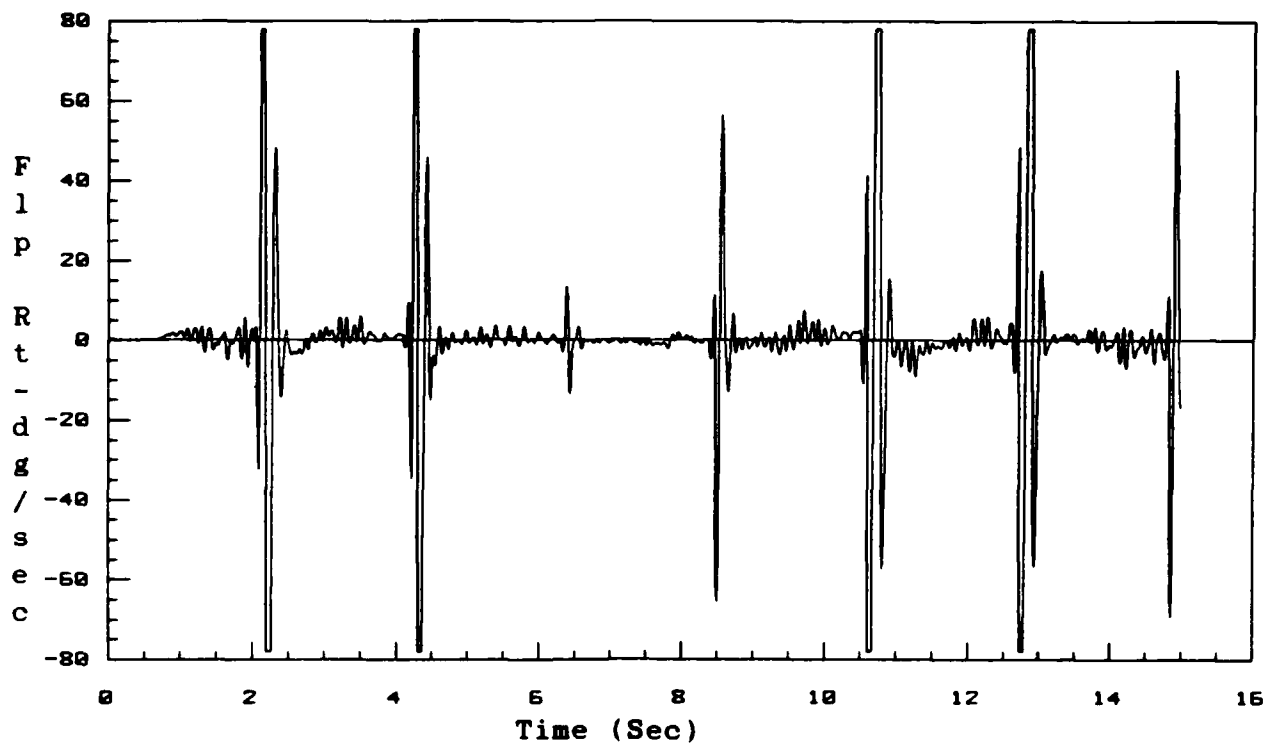


Figure 6-83. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SUG2=1.5 RHO=0.8

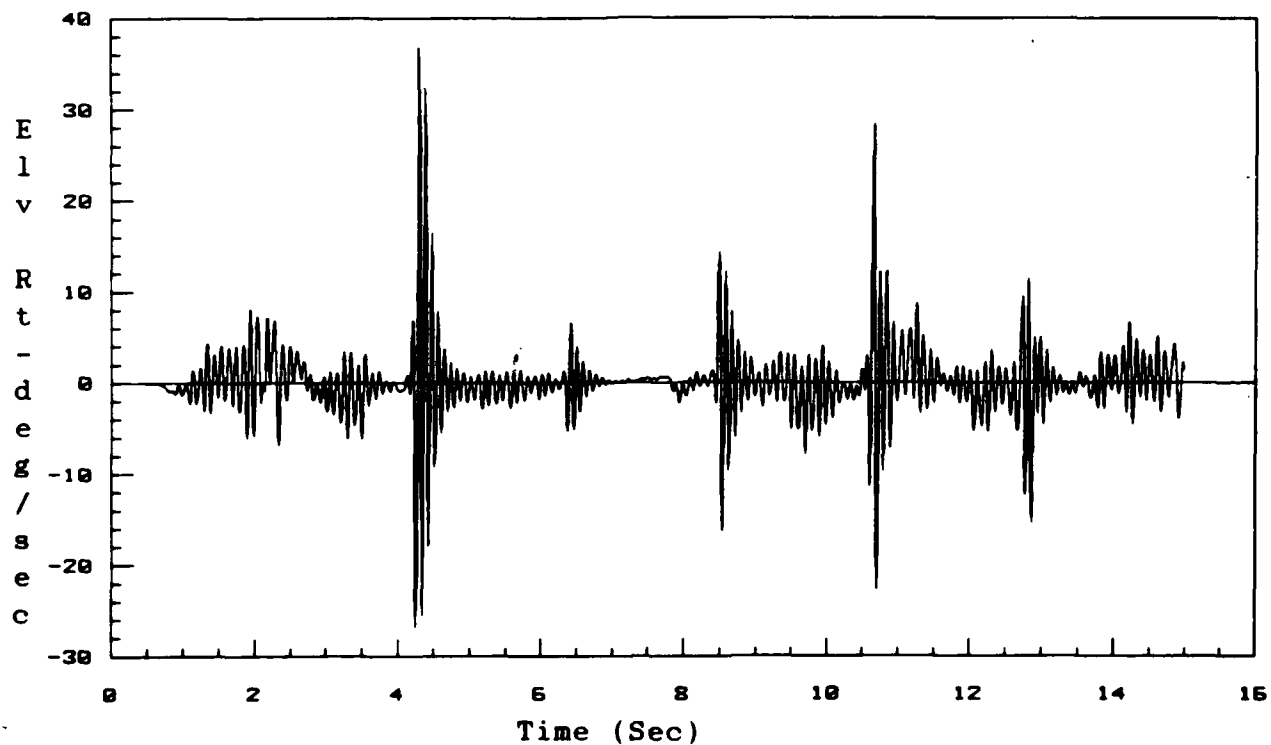


Figure 6-84. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=1.5 RHO=0.8

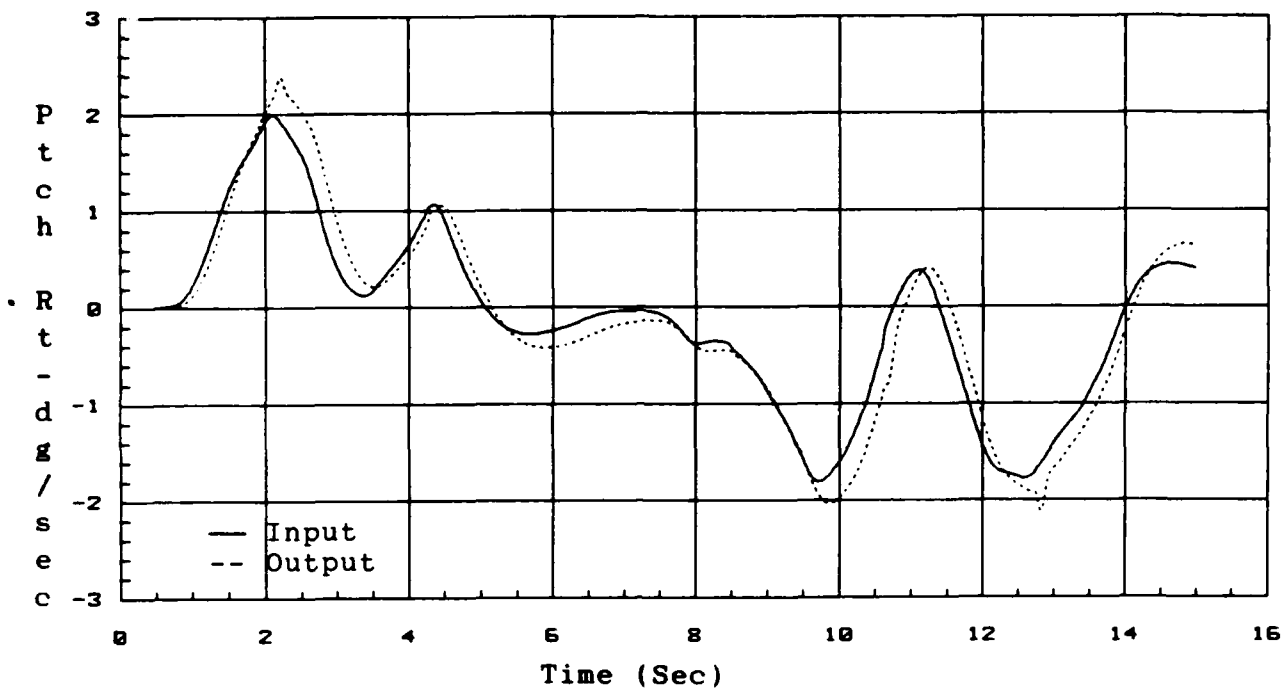


Figure 6-85. Pitch Rate Response
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

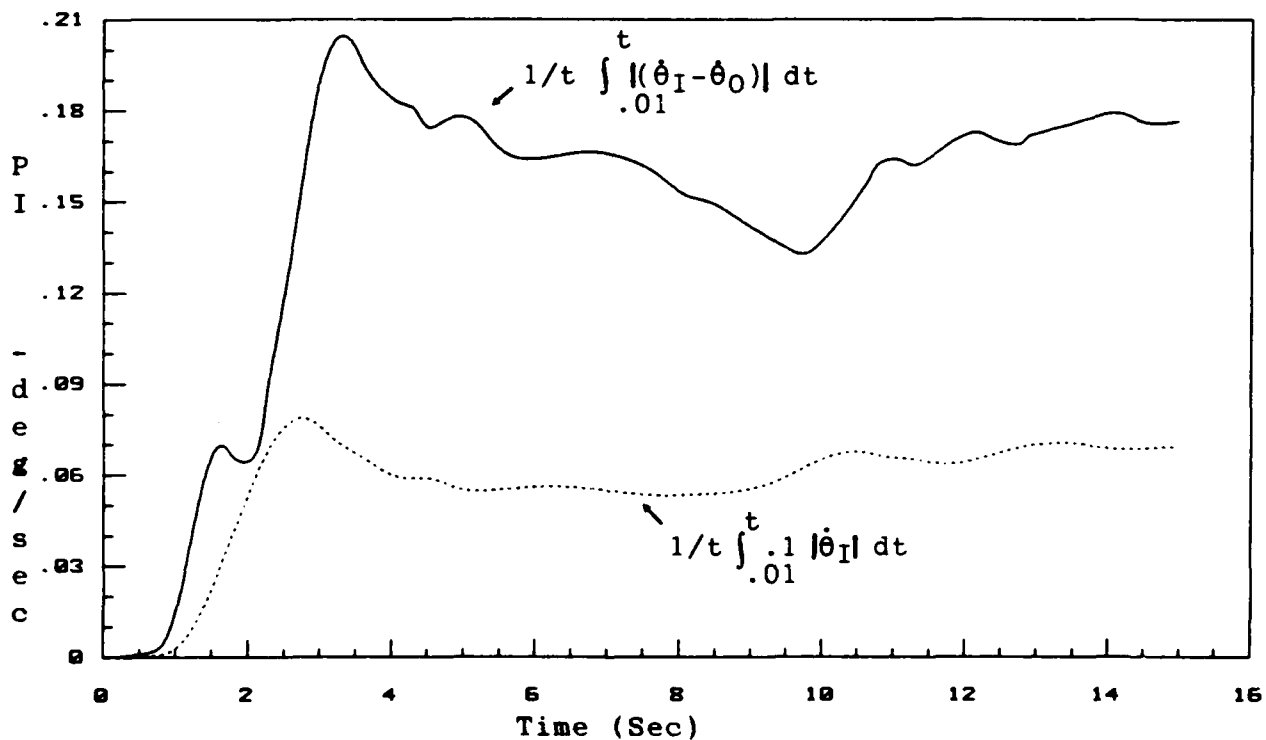


Figure 6-86. Pitch Rate Performance Criterion
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

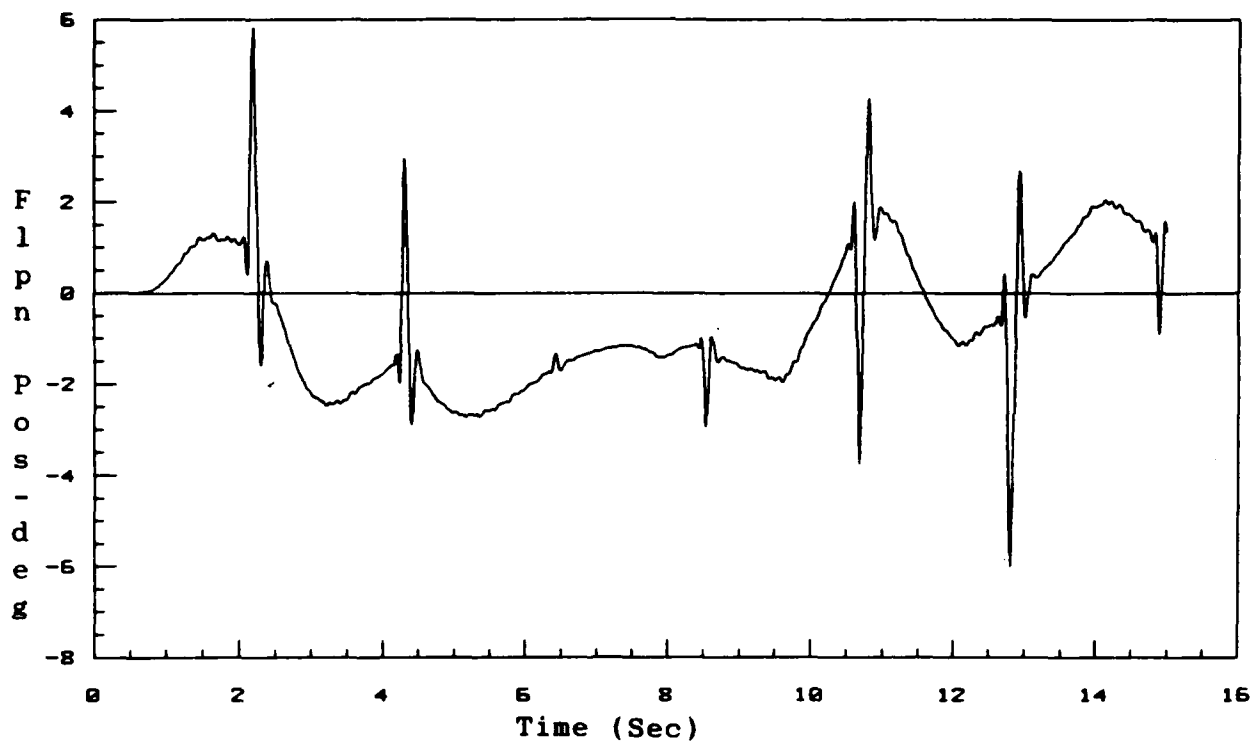


Figure 6-87. Flaperon Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

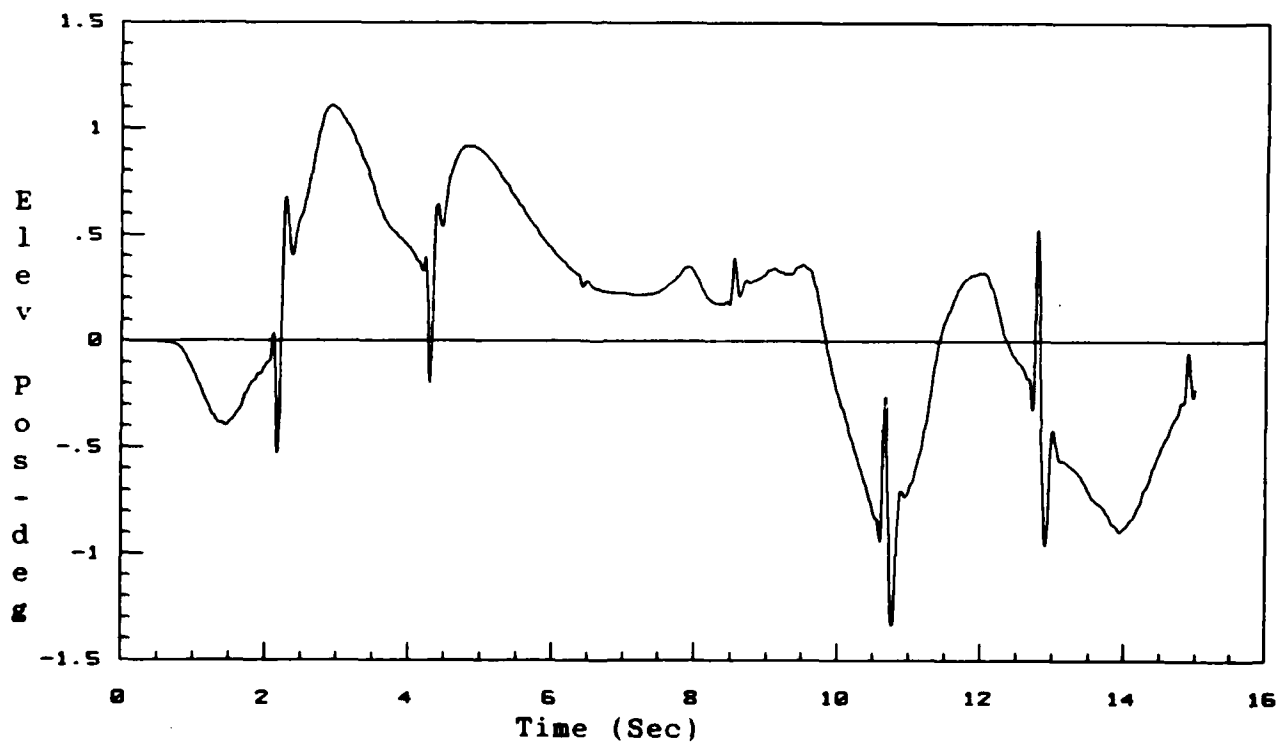


Figure 6-88. Elevator Position
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

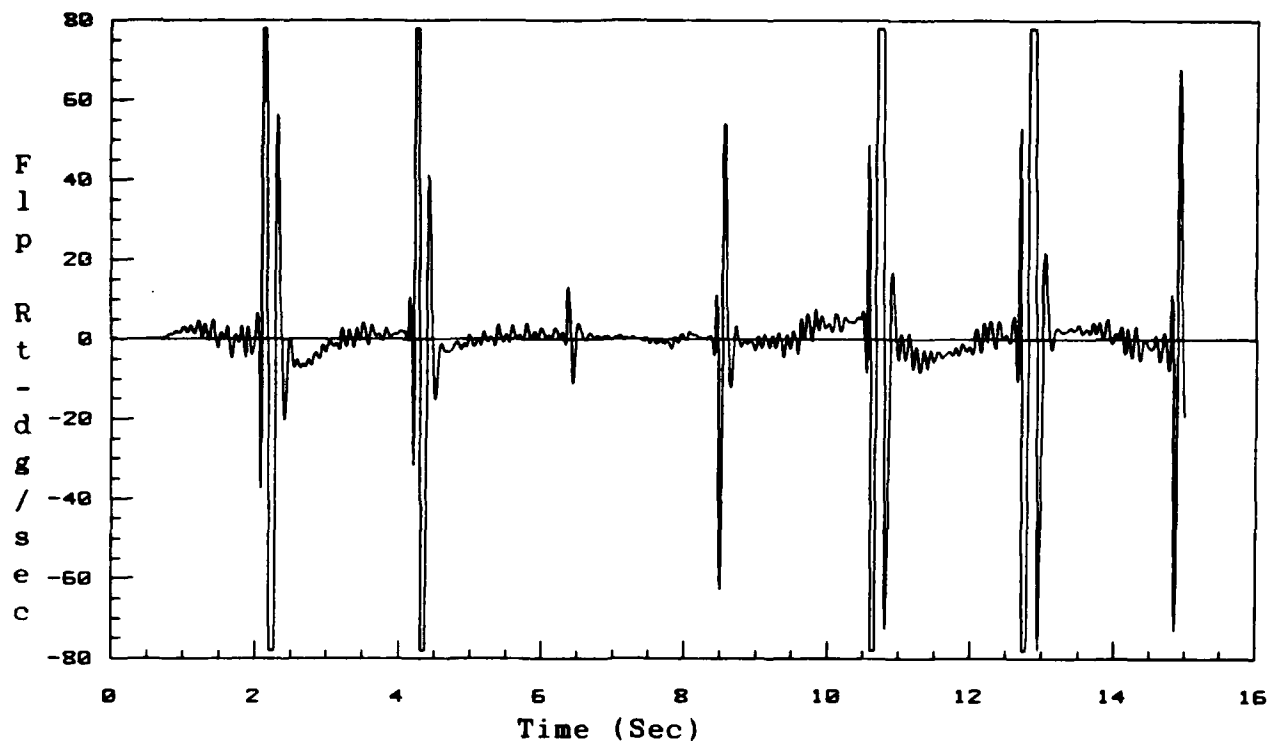


Figure 6-89. Flaperon Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

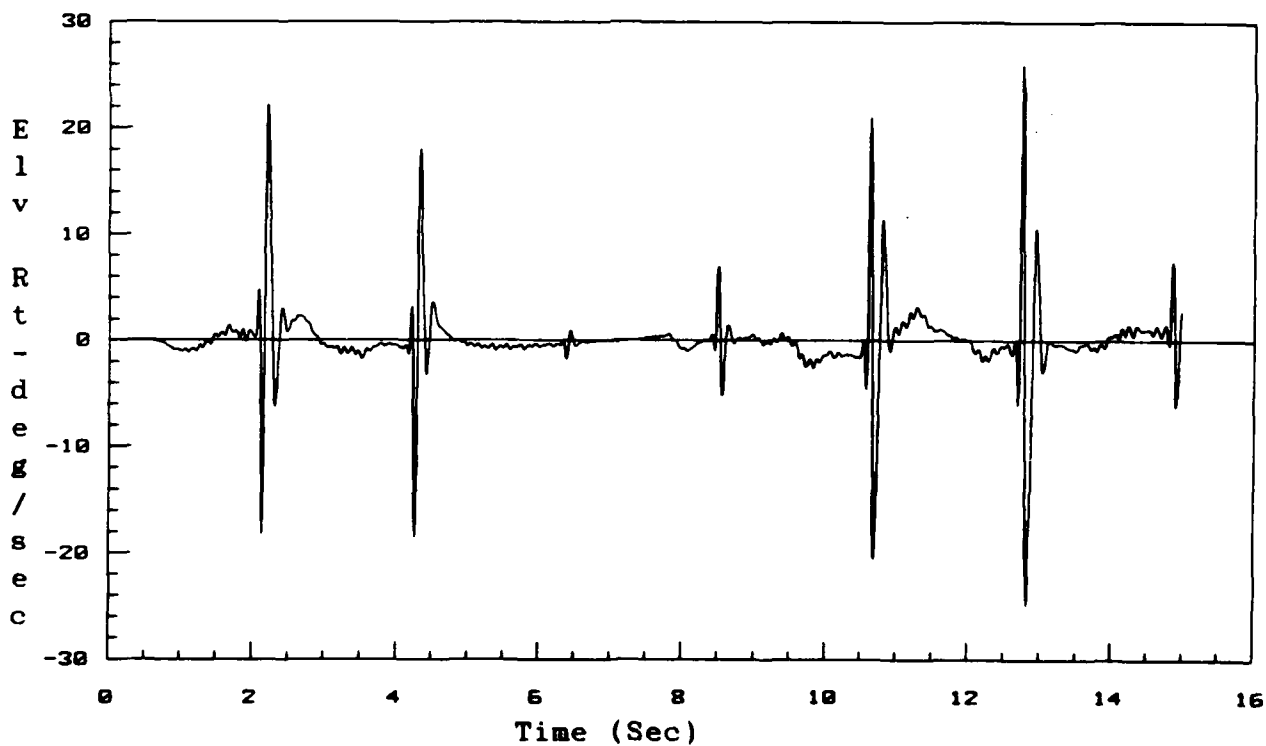


Figure 6-90. Elevator Rate
 Nominal Model: 22K 0.65M/Operating Point: 22K 0.65M
 SIG1=0.4 SIG2=0.1 RHO=0.8

mance boundary for the nominal flight condition of 22,000 ft, 0.65 Mach. The data obtained from these simulations showed that the performance boundary was not affected by the reduction of the value of Sigma 1. The performance boundary for the decreased Sigma 1 case was the same as that shown on Figure 6-48. Figure 6-48 shows the performance boundary for the 22,000 ft, 0.65 Mach flight condition with the control law design parameters as given in Equations (6-14) and (6-15). Performance boundaries for nominal flight conditions of 18,000 ft, 0.45 Mach and 38,000 ft, 0.7 Mach were also determined with Sigma 1 set equal to 0.1. As was the case earlier, the performance boundaries with Sigma 1 equal to 0.1 were the same as those determined previously.

Due to the fact that the performance boundaries remained the same when decreasing the value of Sigma 1, the three-model configuration, configuration 8, had performance boundaries that covered the flight envelope of interest as before (see Figure 6-49). By using configuration 8 as the models in the parallel bank of secondary estimators for the multiple model algorithm (with Sigma 1=0.1), satisfactory tracking performance over the flight envelope of interest (from a performance criteria standpoint) was achieved as

before. However, the control surface responses were much improved with the value of Sigma 1 set equal to 0.1. In contrast to the simulations for configuration 8 with the gains of Equations (6-14) and (6-15), the rate limits were never encountered when Sigma 1 was set equal to 0.1. Figures 6-91 through 6-102 present simulation data for configuration 8 with Sigma 1 equal to 0.1. Note the responses of the control surfaces and the fact that they do not reach the rate limits for either the elevators or flaperons.

6.2.5 Sensor Noise Effects

To determine the effects of sensor noise on the performance on the multiple model algorithm as the means of parameter identification, simulations were run where independent white, gaussian noise was injected into each of the quantities of interest. The noise levels used in this thesis are realistic noise levels for a comparable aircraft (Grumman F-14 Tomcat) and are presented in Table 6-15.

To gain some insight into the effects of noise on the multiple model algorithms performance, the two-model configuration shown in Table 6-16 was considered. For the configuration in Table 6-16, an operating point of 10,000 ft, 0.35 Mach was selected which was one of the models in

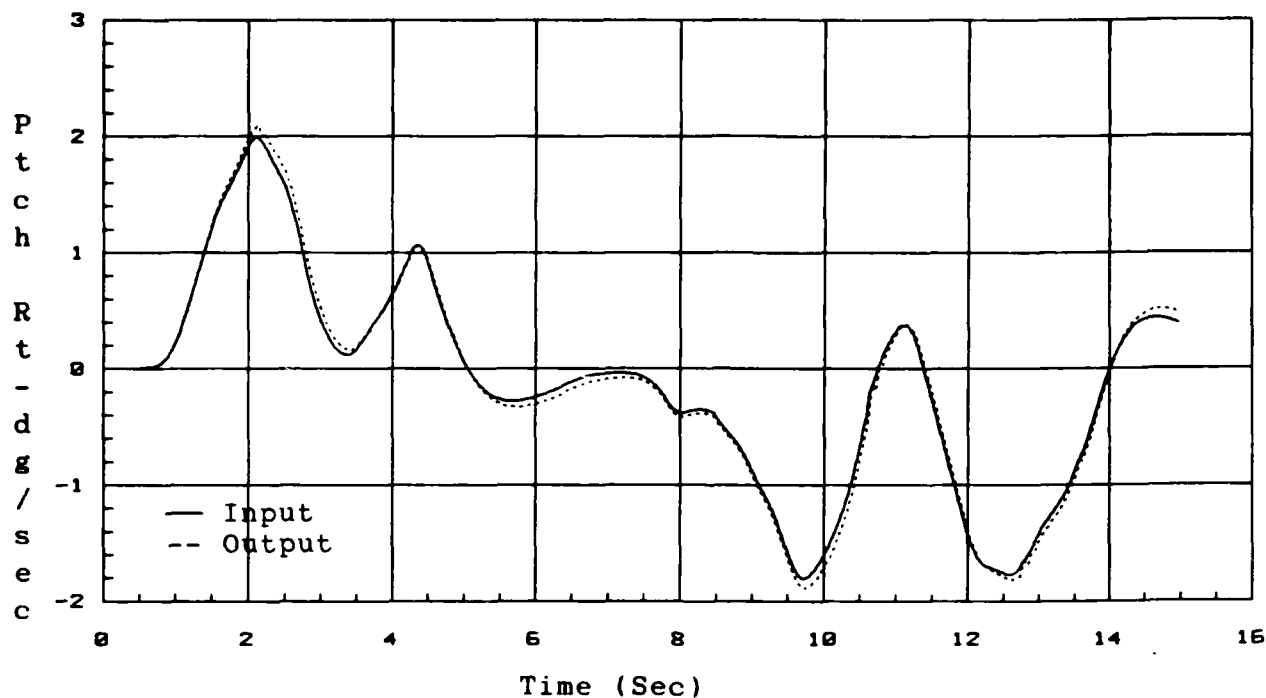


Figure 6-91. Pitch Rate Response
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

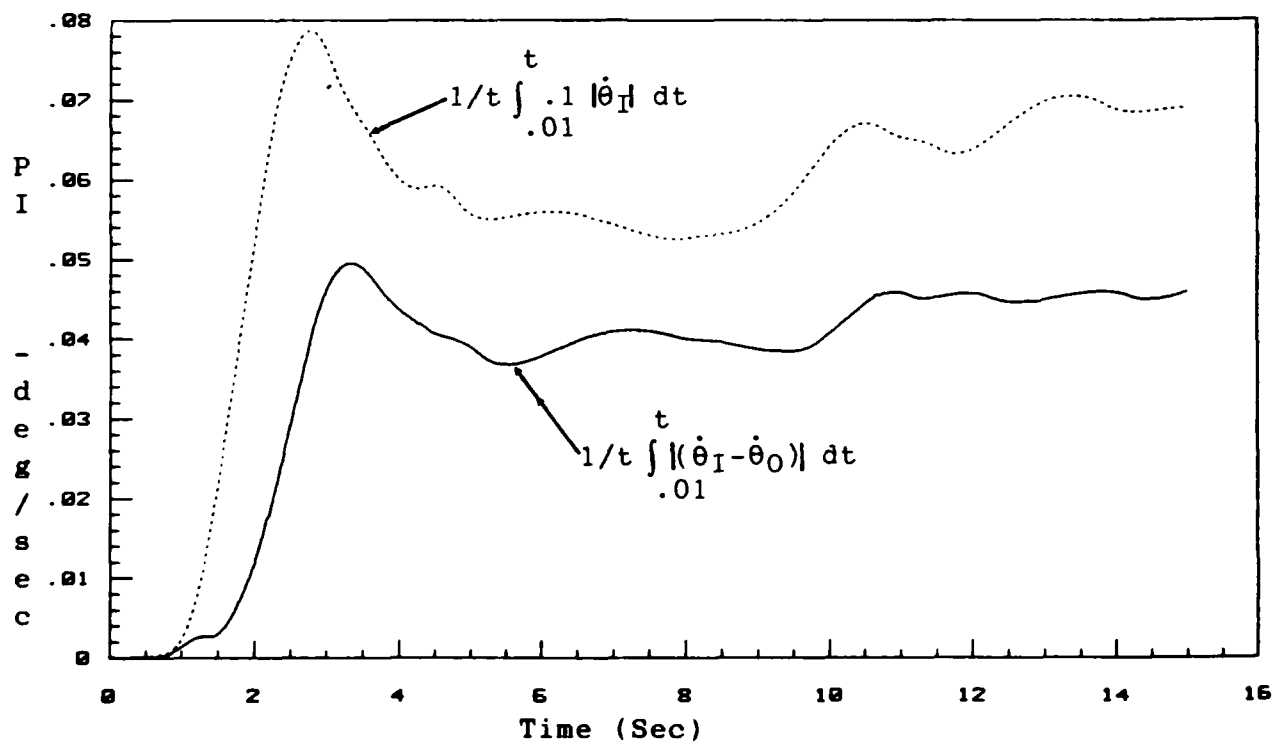


Figure 6-92. Pitch Rate Performance Criterion
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

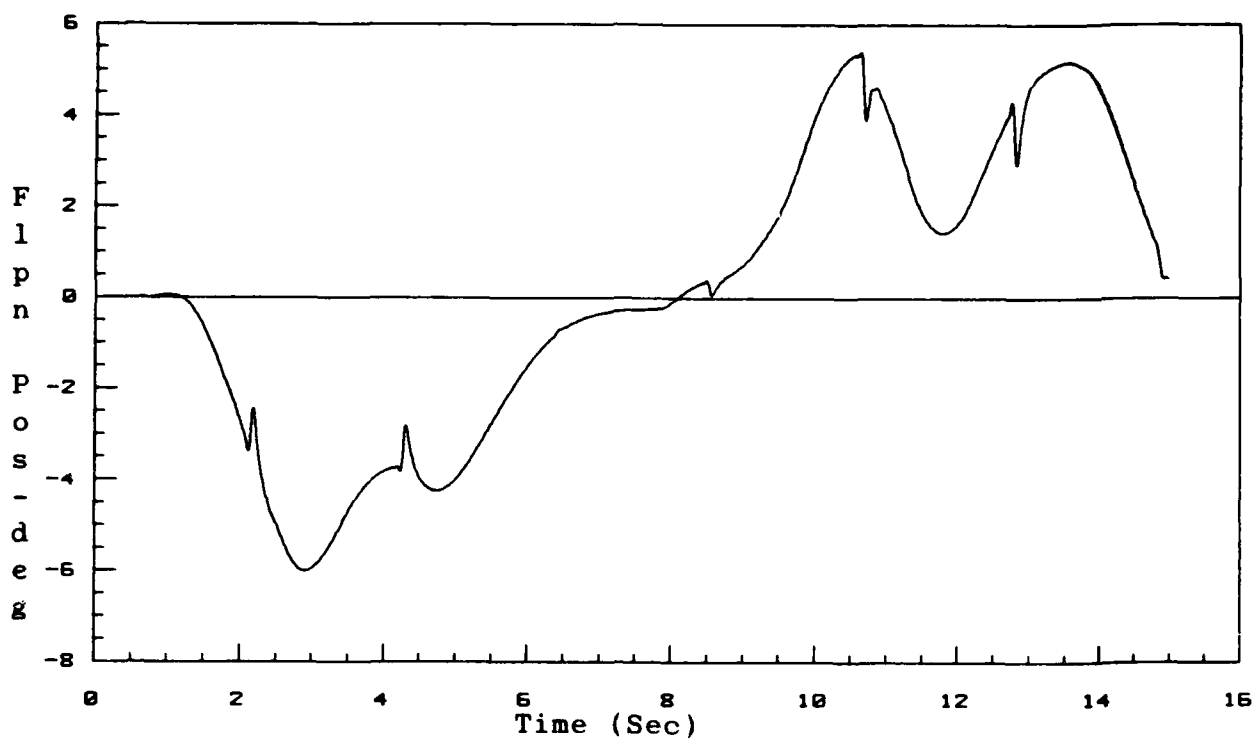


Figure 6-93. Flaperon Position
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

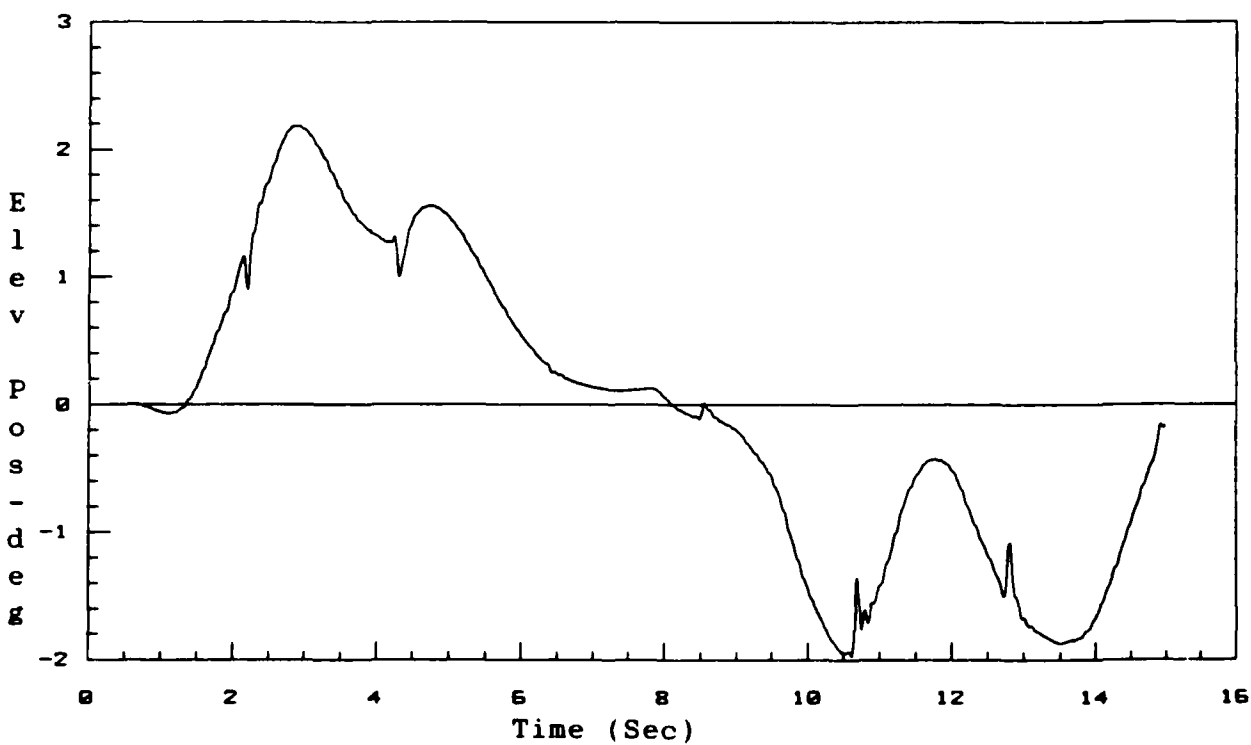


Figure 6-94. Elevator Position
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

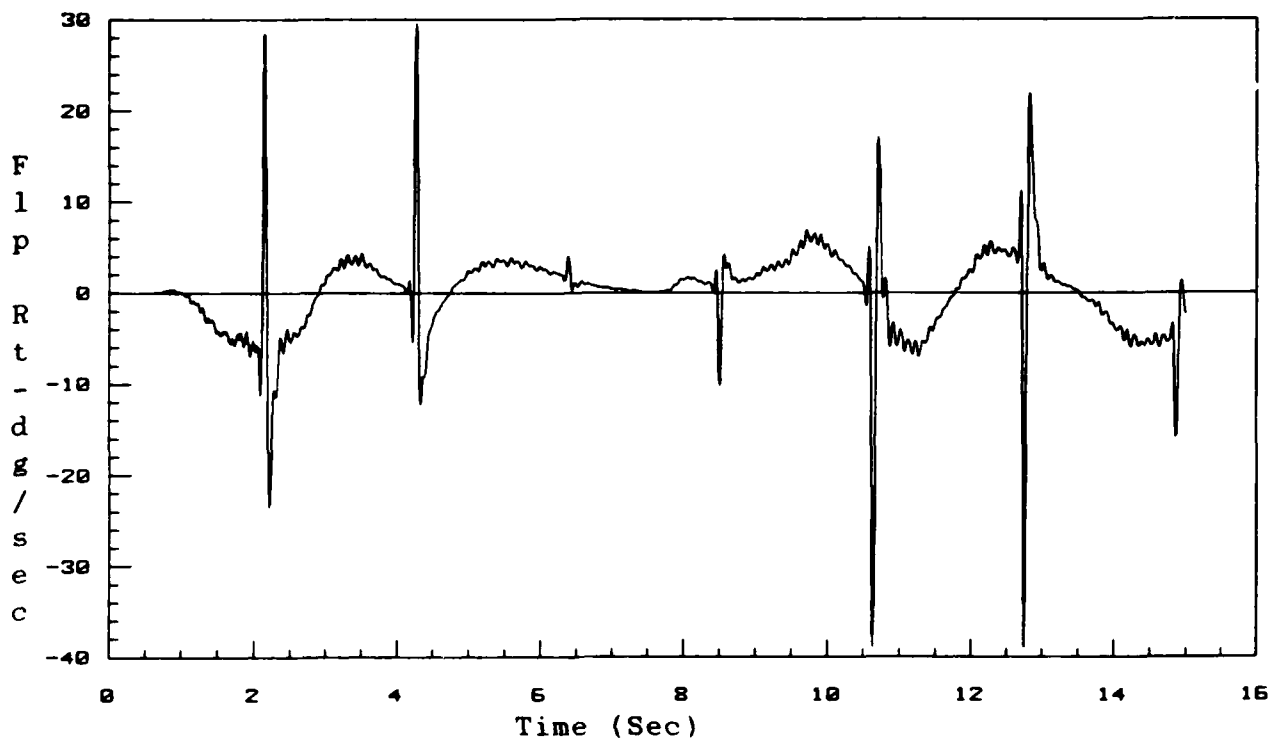


Figure 6-95. Flaperon Rate
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

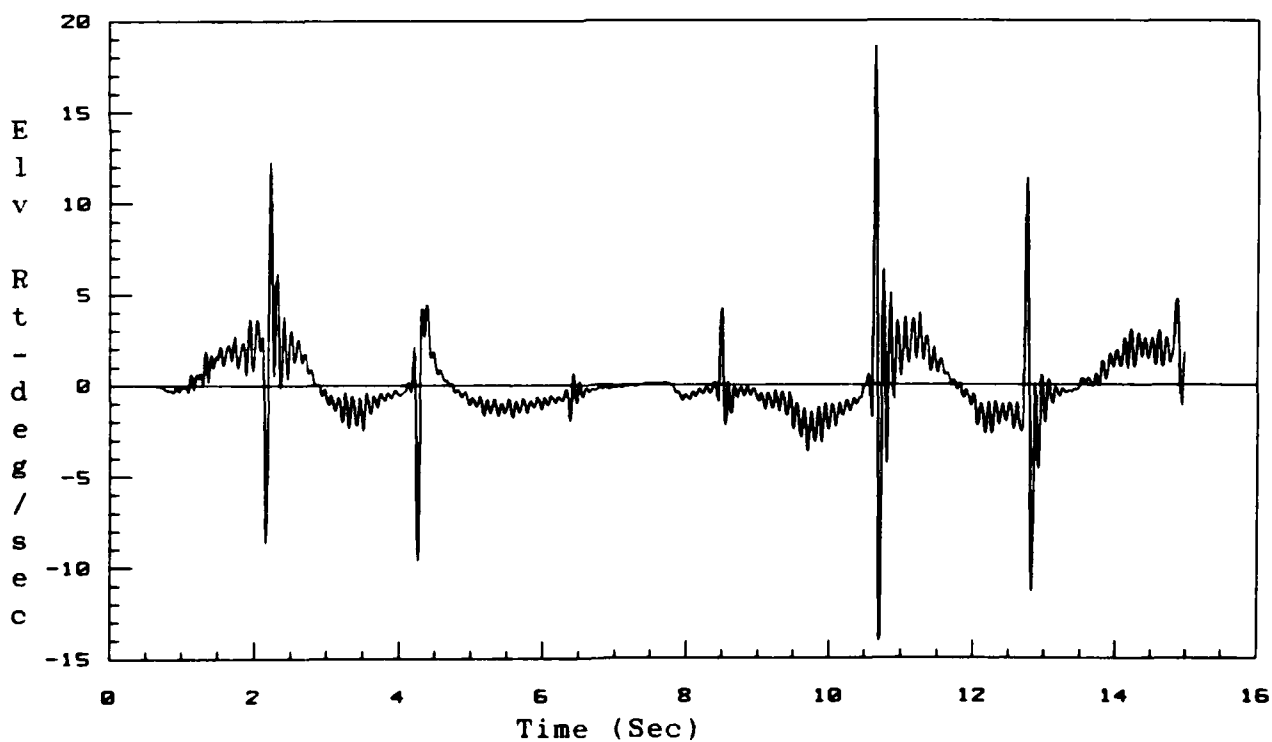


Figure 6-96. Elevator Rate
Configuration 8/Operating Point: 10K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

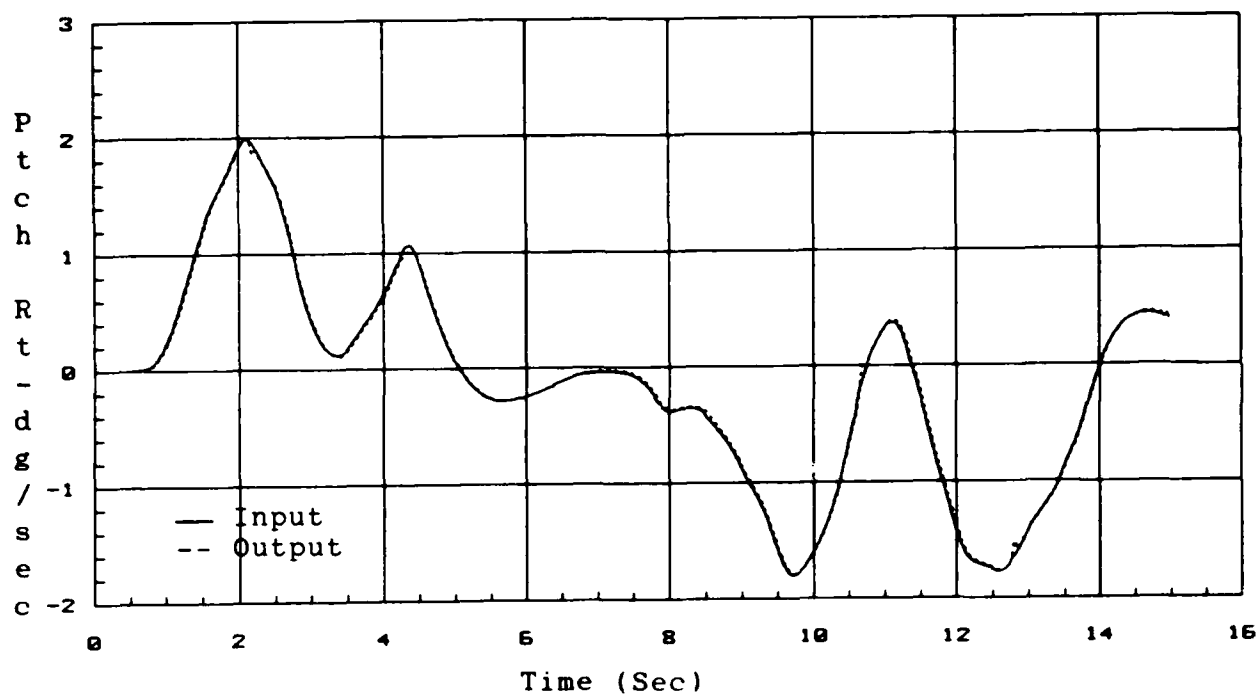


Figure 6-97. Pitch Rate Response
Configuration 8/Operating Point: 38K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.7

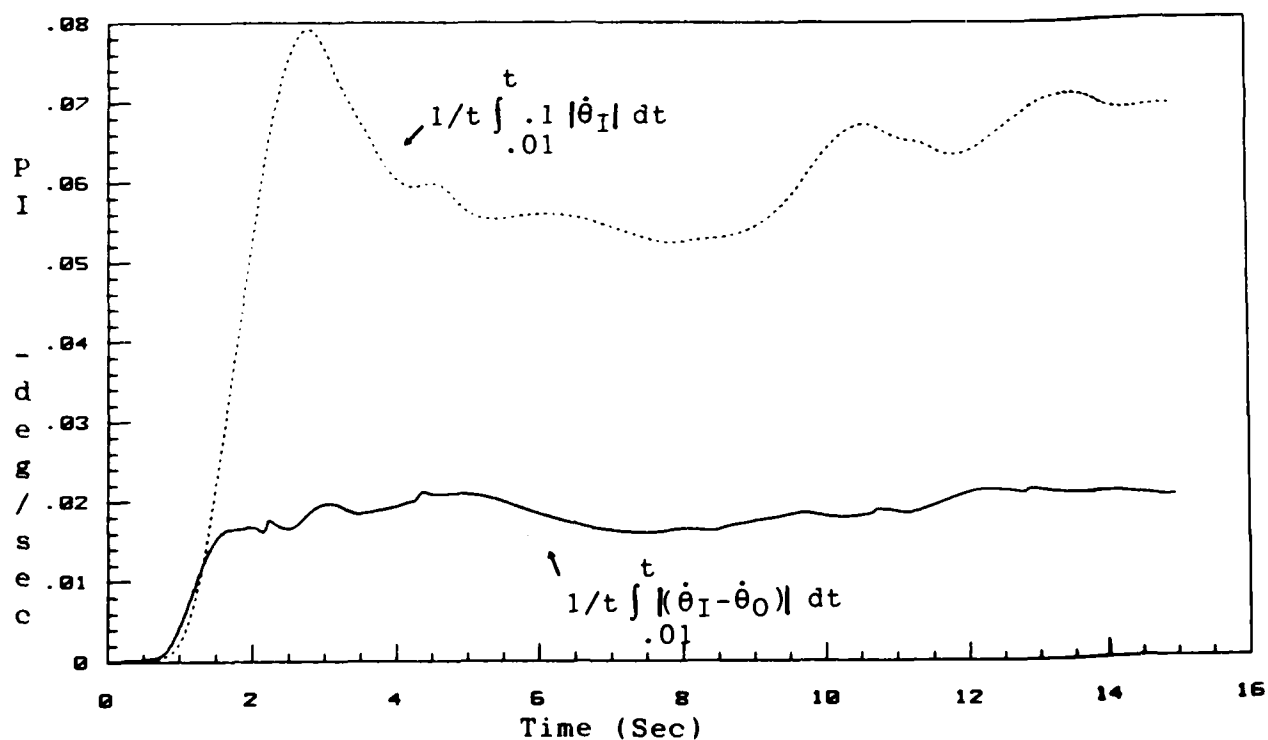


Figure 6-98. Pitch Rate Performance Criterion
Configuration 8/Operating Point: 38K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

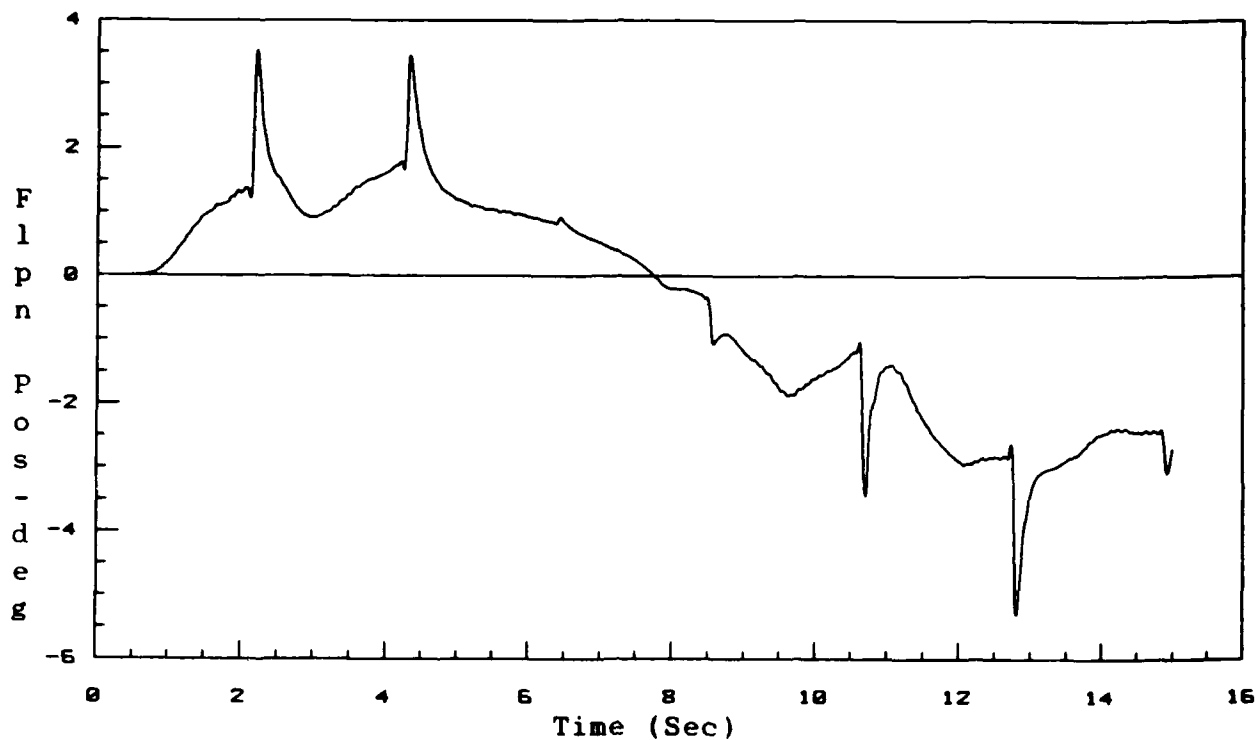


Figure 6-99. Flaperon Position
 Configuration 8/Operating Point: 38K 0.9M
 SIG1=0.1 SIG2=0.7 RHO=0.8

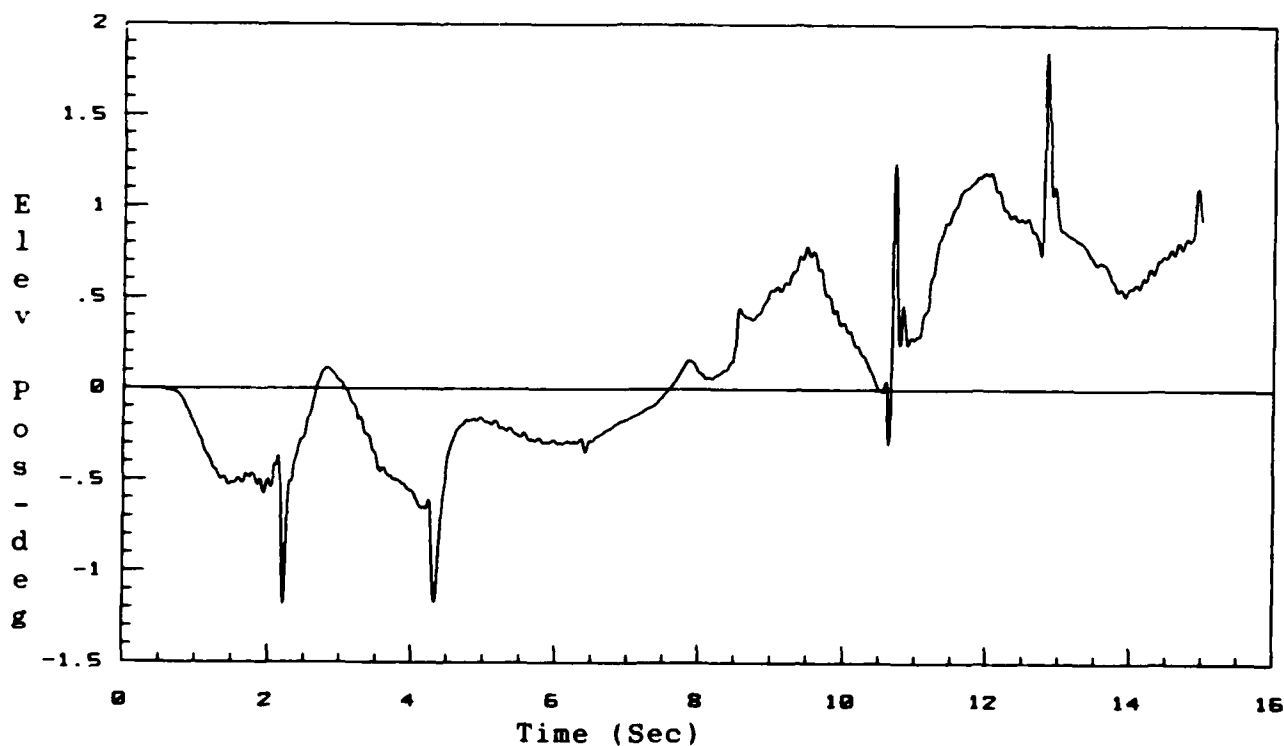


Figure 6-100. Elevator Position
 Configuration 8/Operating Point: 38K 0.9M
 SIG1=0.1 SIG2=0.7 RHO=0.8

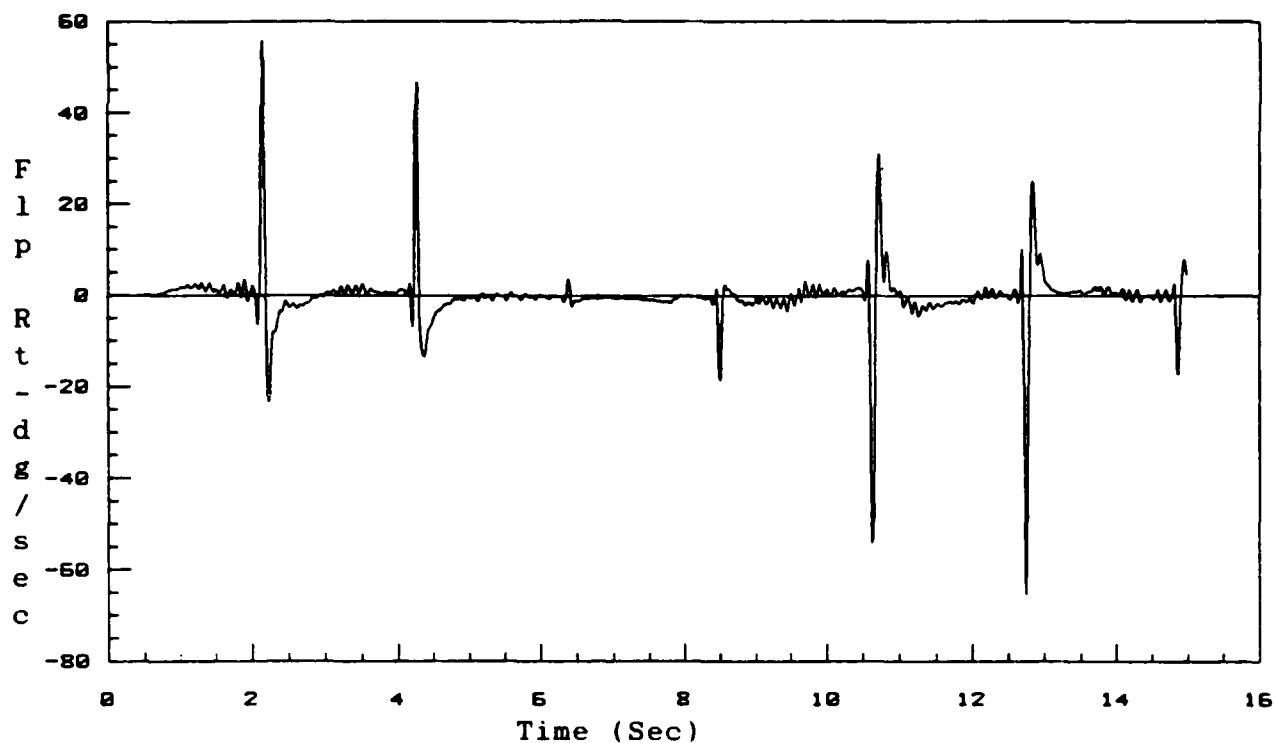


Figure 6-101. Flaperon Rate
Configuration 8/Operating Point: 38K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

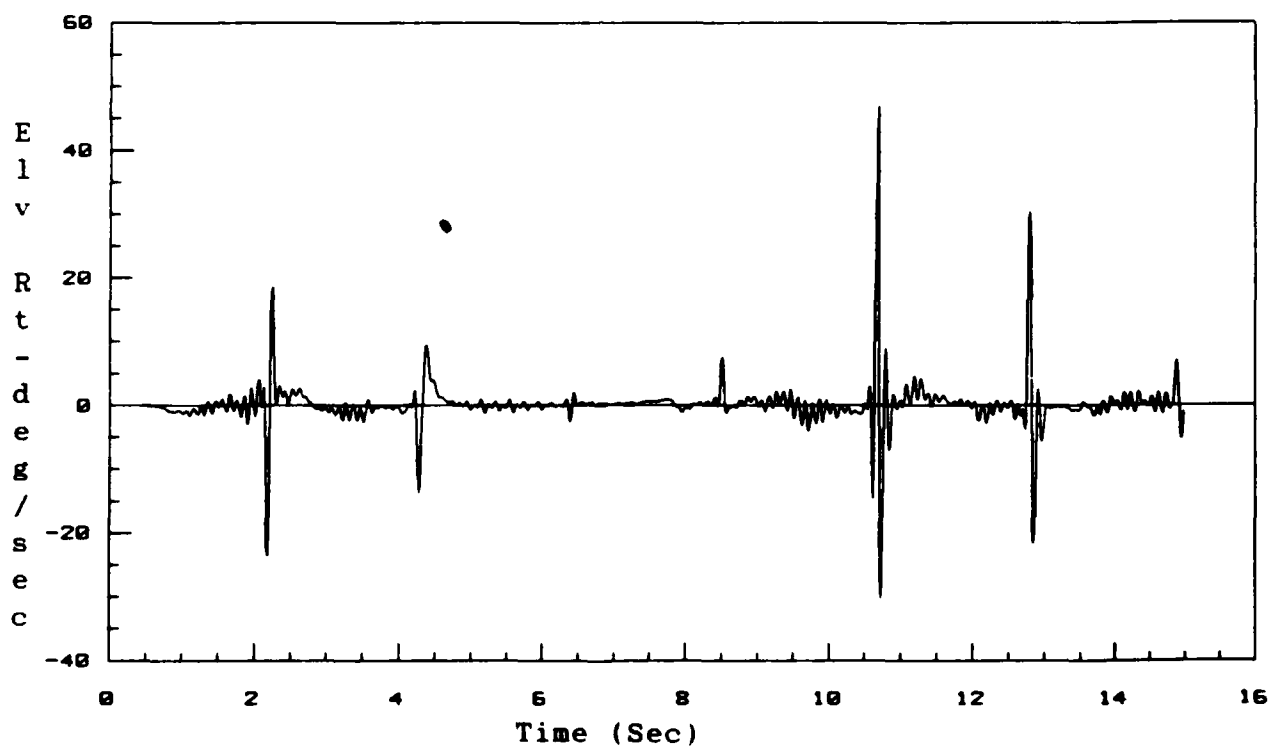


Figure 6-102. Elevator Rate
Configuration 8/Operating Point: 38K 0.9M
SIG1=0.1 SIG2=0.7 RHO=0.8

Table 6-15

Sensor Noise Data
(units are degrees)

Measured Quantity	Noise Mean	Variance
q	0.0	7.438E-8
θ	0.0	4.886E-7
α	0.0	3.404E-6

the bank of secondary estimators. As a baseline, a simulation without sensor noise was run. The weights associated with each candidate model as well as the prediction error variances are shown in Figures 6-103 through 6-107. From Figure 6-103, it can be seen that the probability of the model associated with the 10,000 ft, 0.35 Mach flight con-

Table 6-16

Two-Model Configuration for Noise Consideration

Model	Flight Condition
1	10,000 ft 0.35 Mach
2	10,000 ft 0.50 Mach

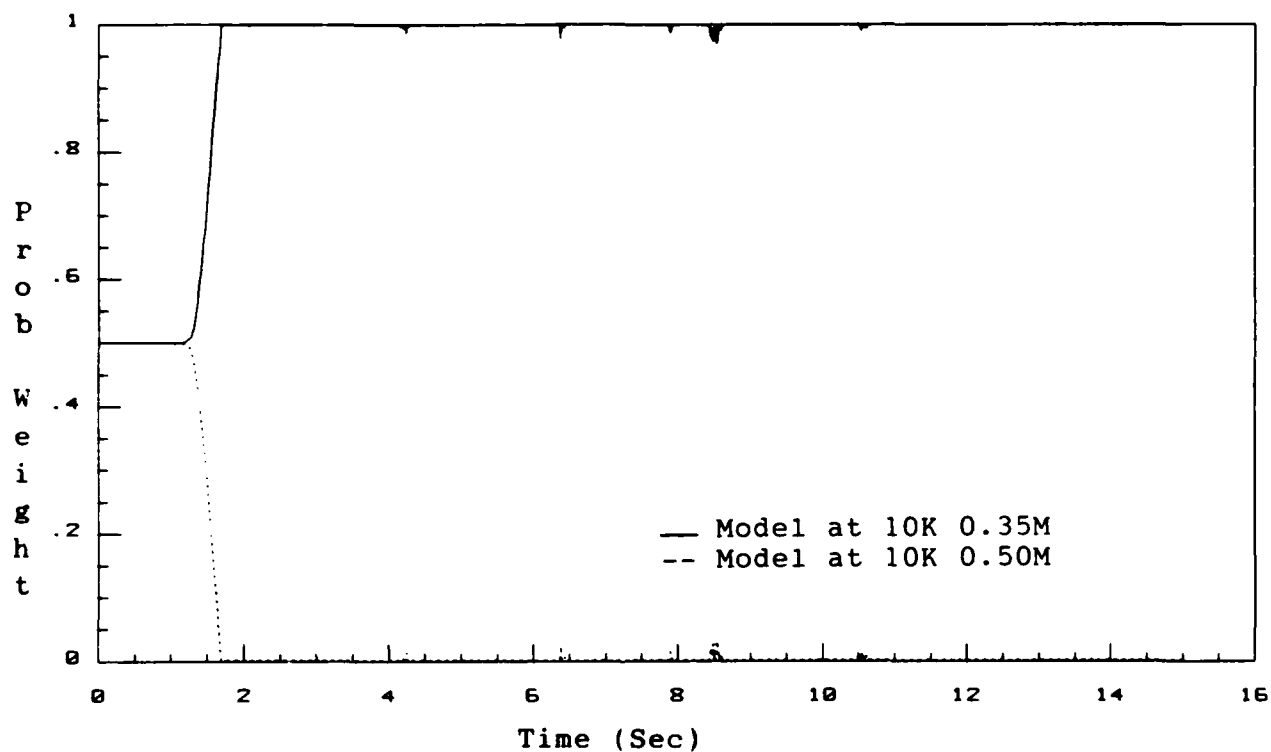


Figure 6-103. Model Probability Weightings
Operating Point: 10K .35M

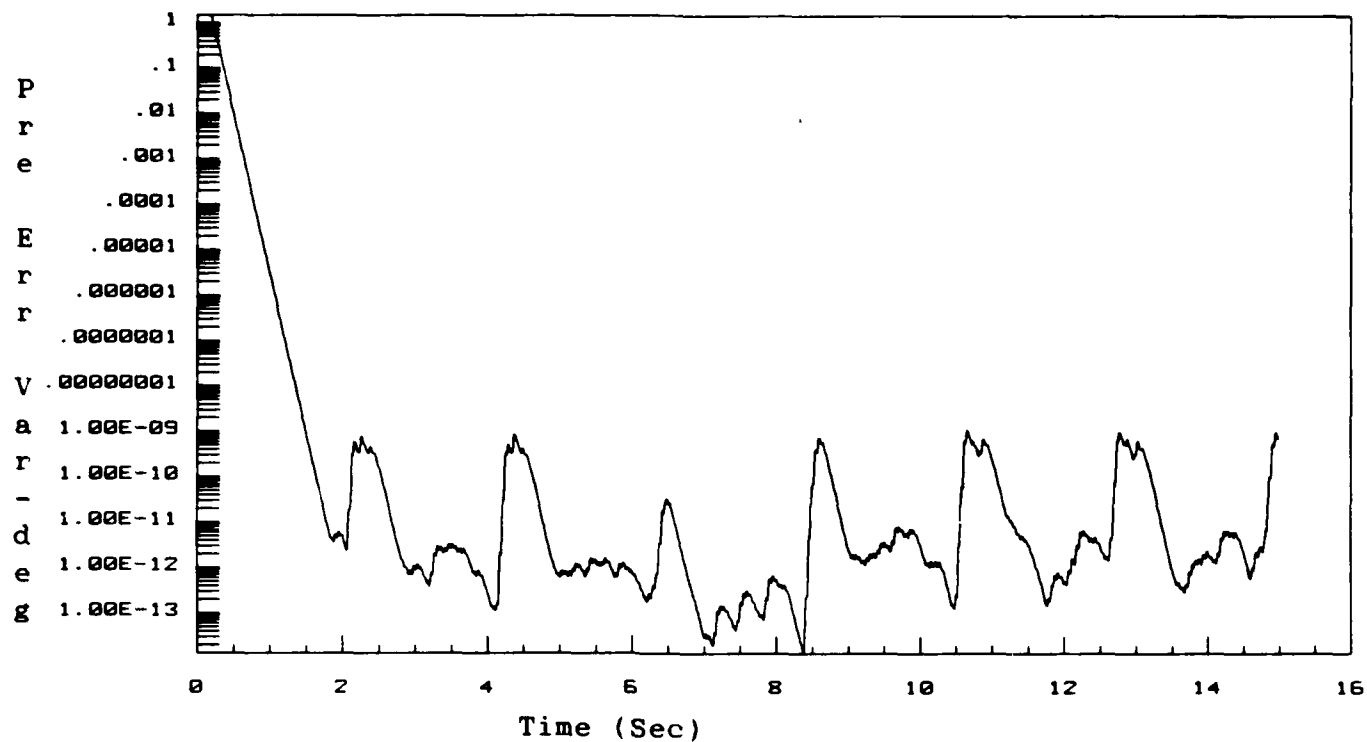


Figure 6-104. Prediction Error Variance (1,1) for Model 1
Operating Point: 10K 0.35M

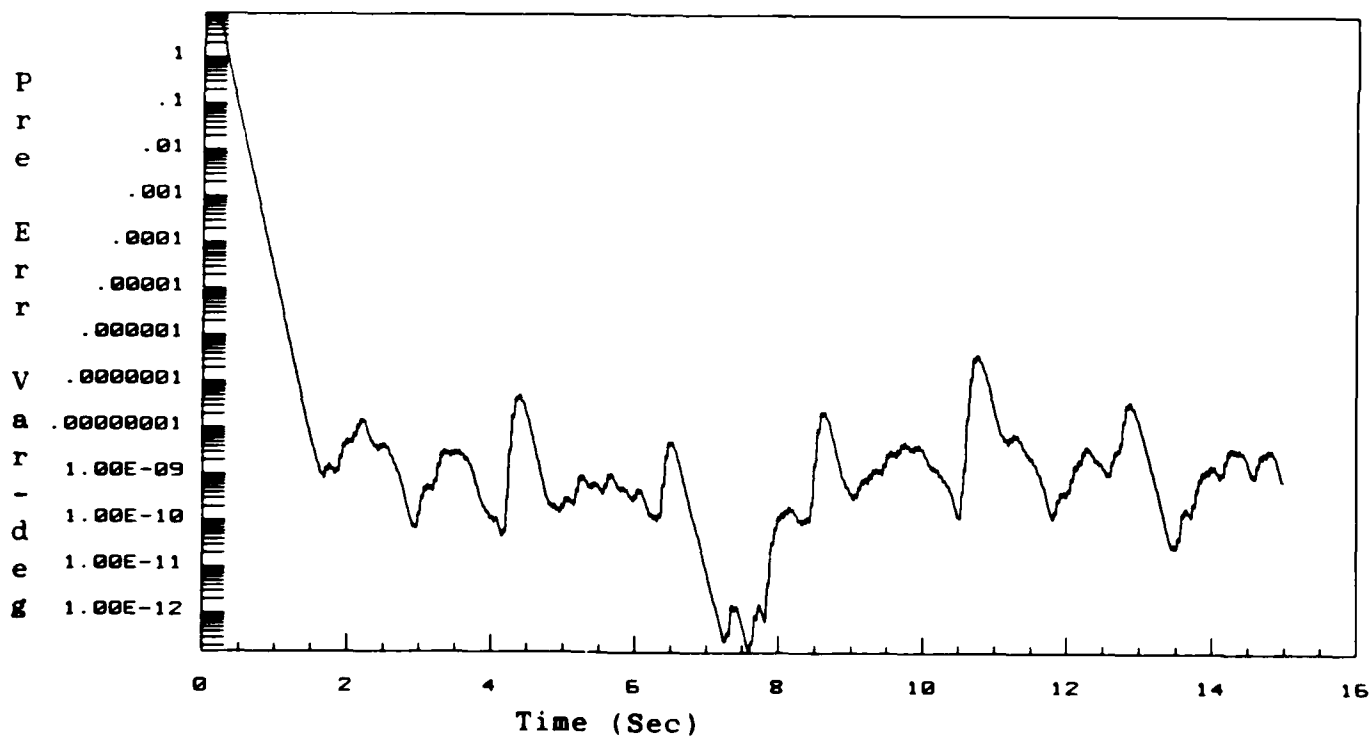


Figure 6-105. Prediction Error Variance (2,2) for Model 1
Operating Point: 10K 0.35M

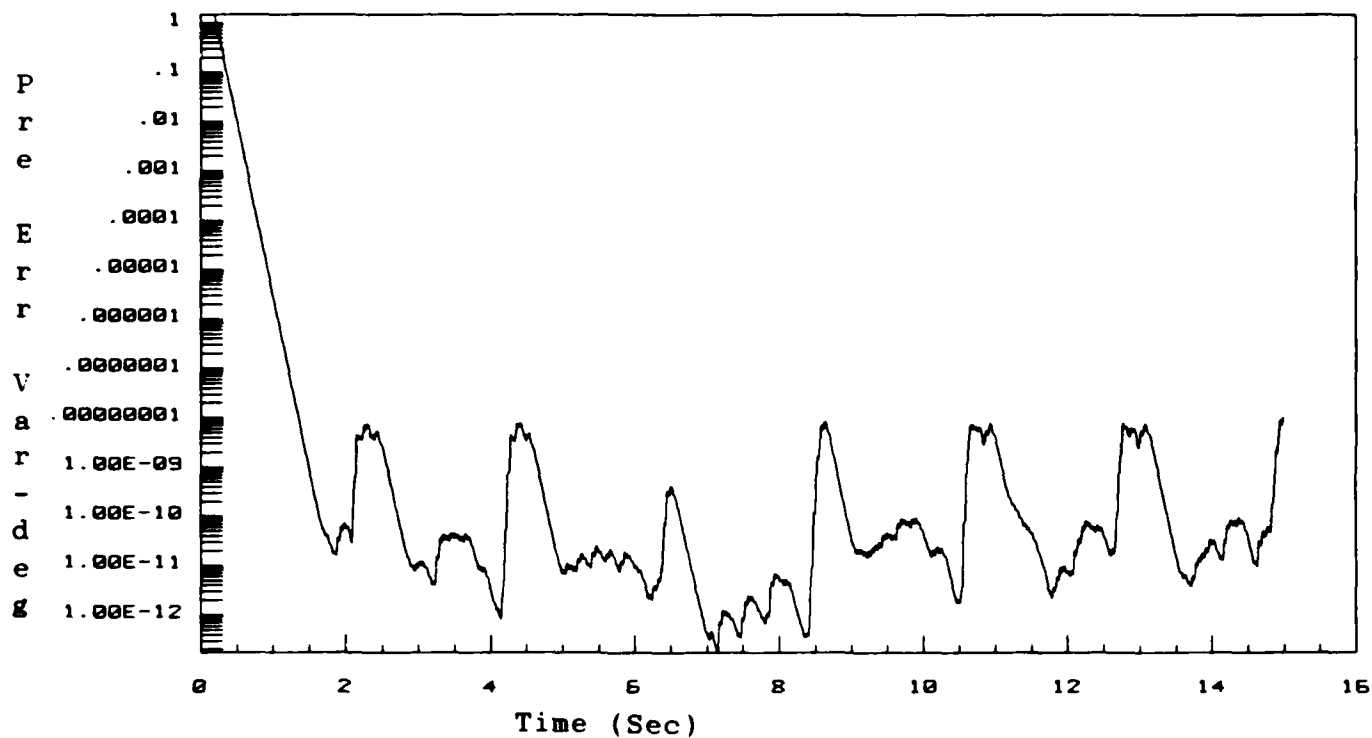


Figure 6-106. Prediction Error Variance (1,1) for Model 2
Operating Point: 10K 0.35M

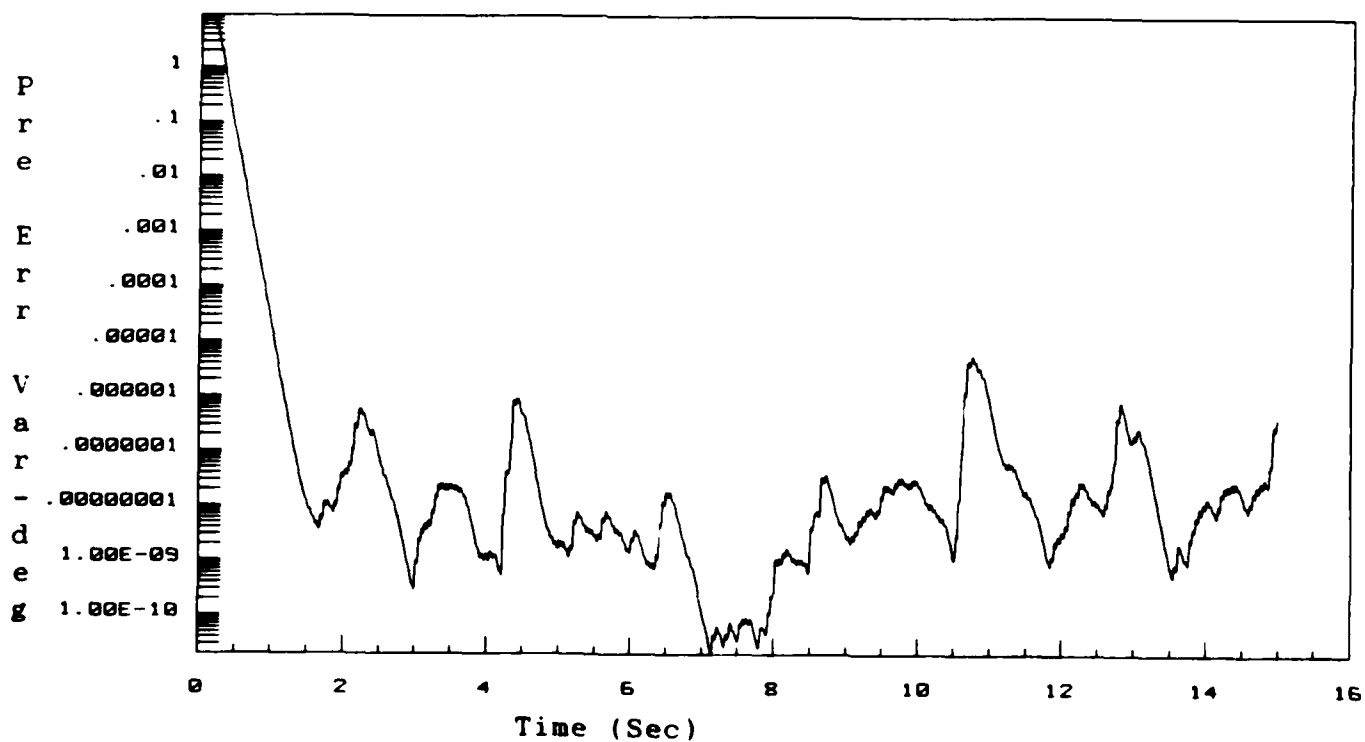


Figure 6-107. Prediction Error Variance (2,2) for Model 2
Operating Point: 10K 0.35M

figuration is very high. In comparing the prediction error variances for model 1 and model 2, Figures 6-104 through 6-107 show that the variances associated with model 1 are smaller, indicating that it is the better of the two models.

Adding the sensor noise levels as given in Table 6-15 and running a simulation for the configuration in Table 6-16 resulted in the data presented on Figures 6-108 through 6-112. The performance of the multiple model algorithm as shown on the probability curve of Figure 6-108 is very similar to that of Figure 6-103 (same conditions with no noise). The prediction error variances on Figures 6-109 through 6-112 have responses that are very similar to the no noise case, but are of increased magnitude. Figures 6-113 through 6-117 show a simulation that was performed with the sensor noise figures set at one-hundred times those shown in Table 6-15. The probability weighting curve shown on Figure 6-113 shows the effect of adding too much sensor noise on the performance of the multiple model algorithm. Instead of model 1 being selected as the correct model with a probability of almost one, a substantial contribution of model 2 is now present. The reason for this can be seen by comparing the prediction error variances as shown in Figures 6-114

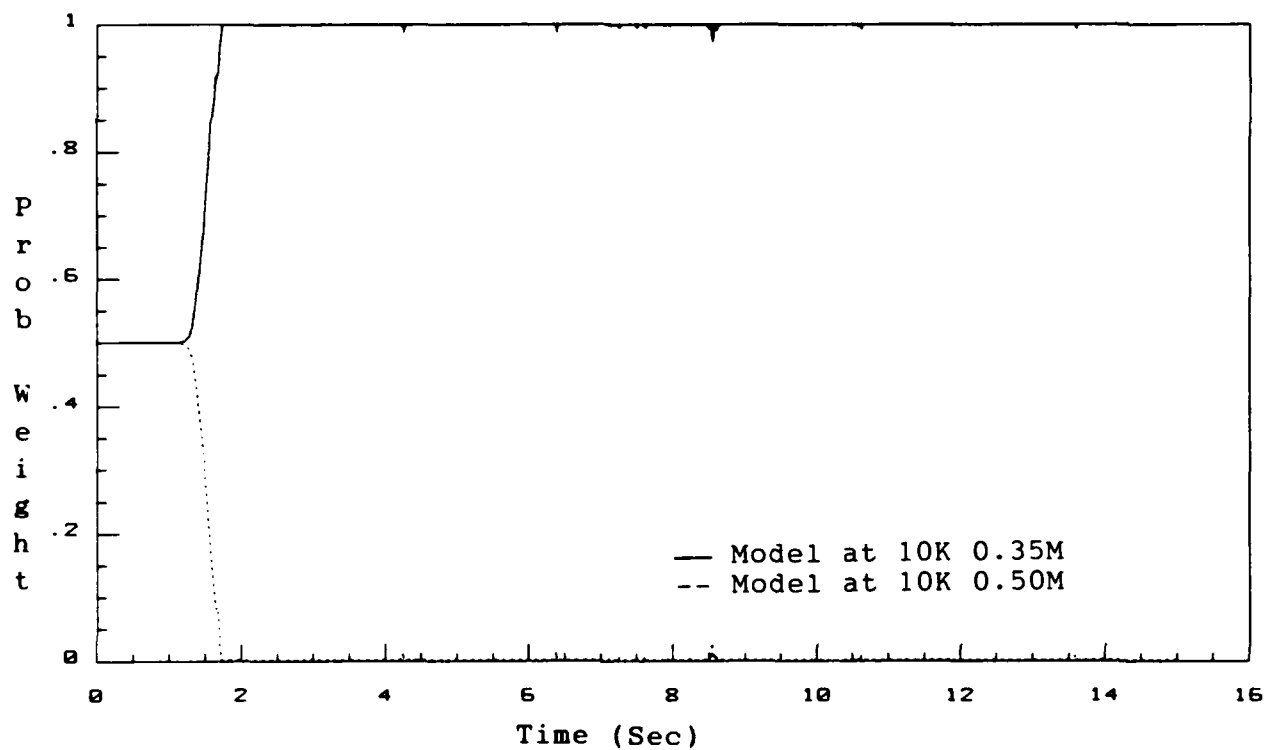


Figure 6-108. Model Probability Weightings
Sensor Noise/Operating Point: 10K 0.35M

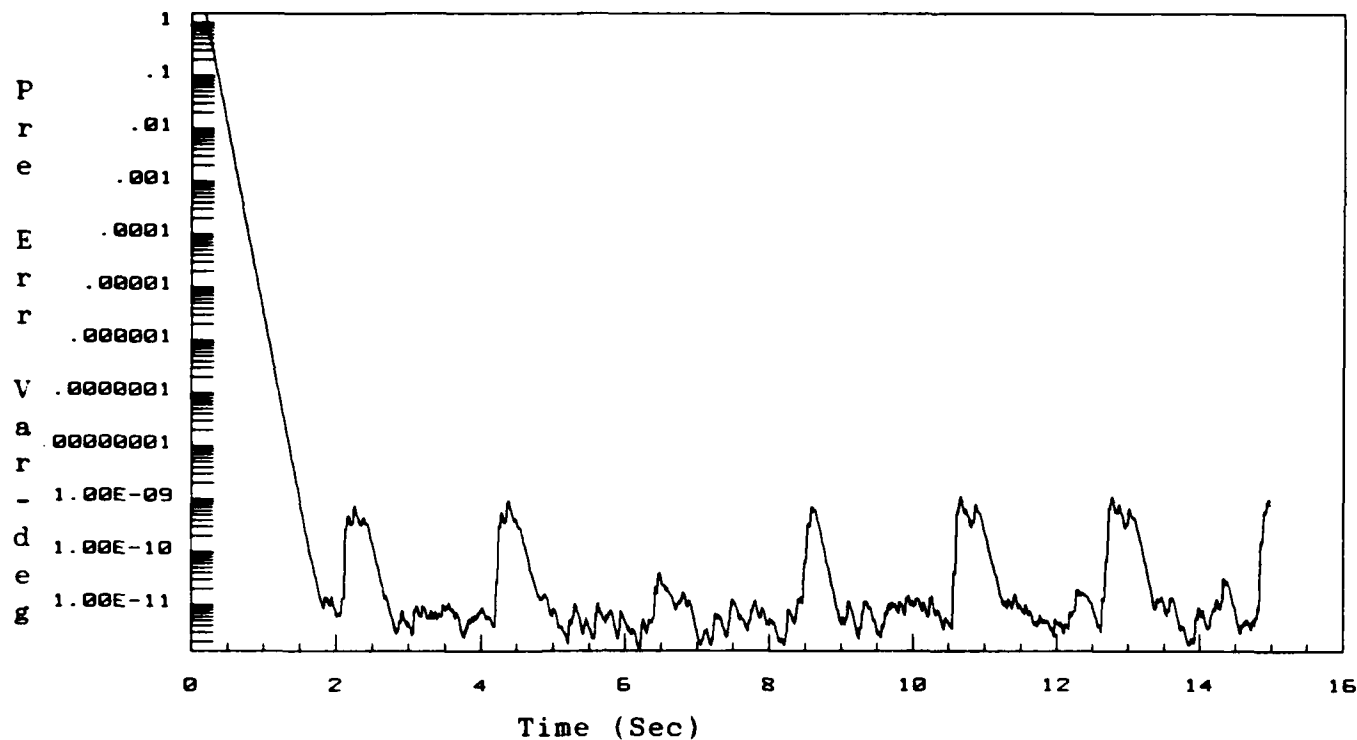


Figure 6-109. Prediction Error Variance (1,1) for Model 1
Sensor Noise/Operating Point: 10K 0.35M

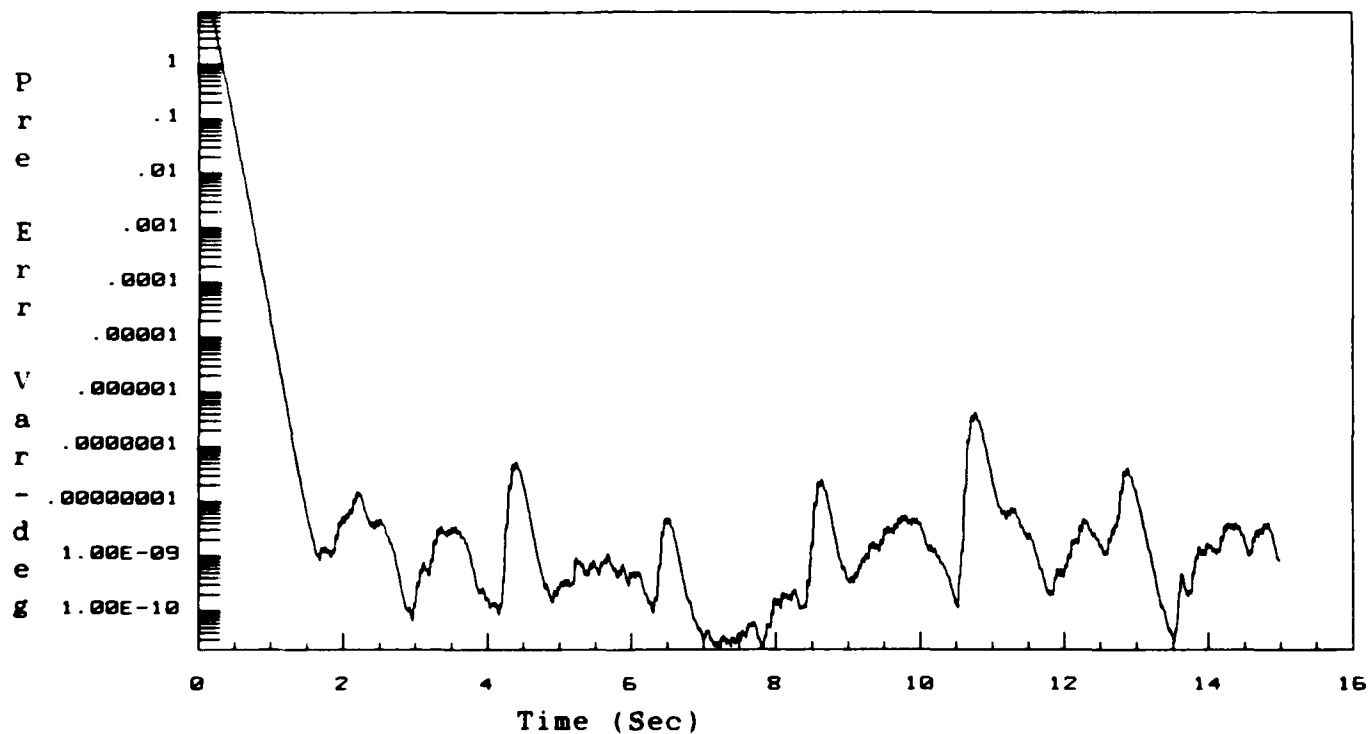


Figure 6-110. Prediction Error Variance (2,2) for Model 1
Sensor Noise/Operating Point: 10K 0.35M

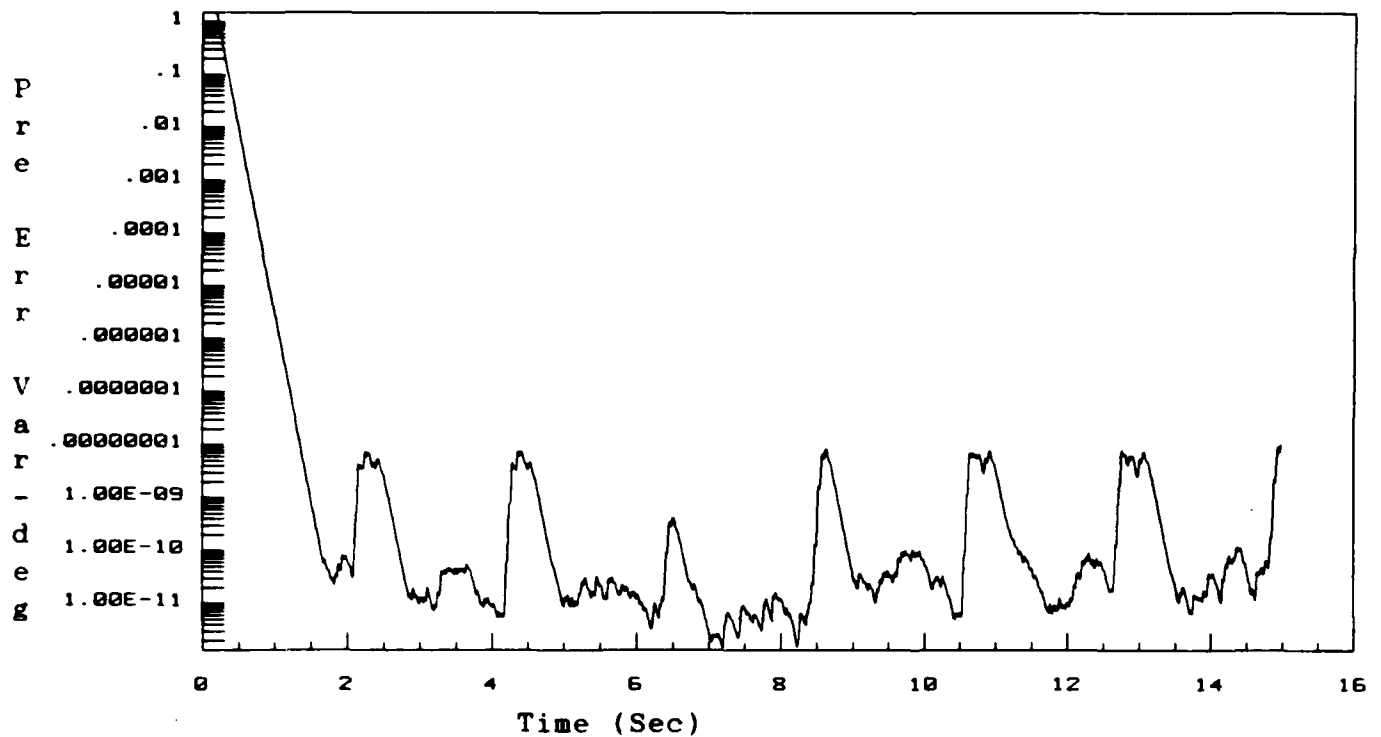


Figure 6-111. Prediction Error Variance (1,1) for Model 1
Sensor Noise/Operating Point: 10K 0.35M

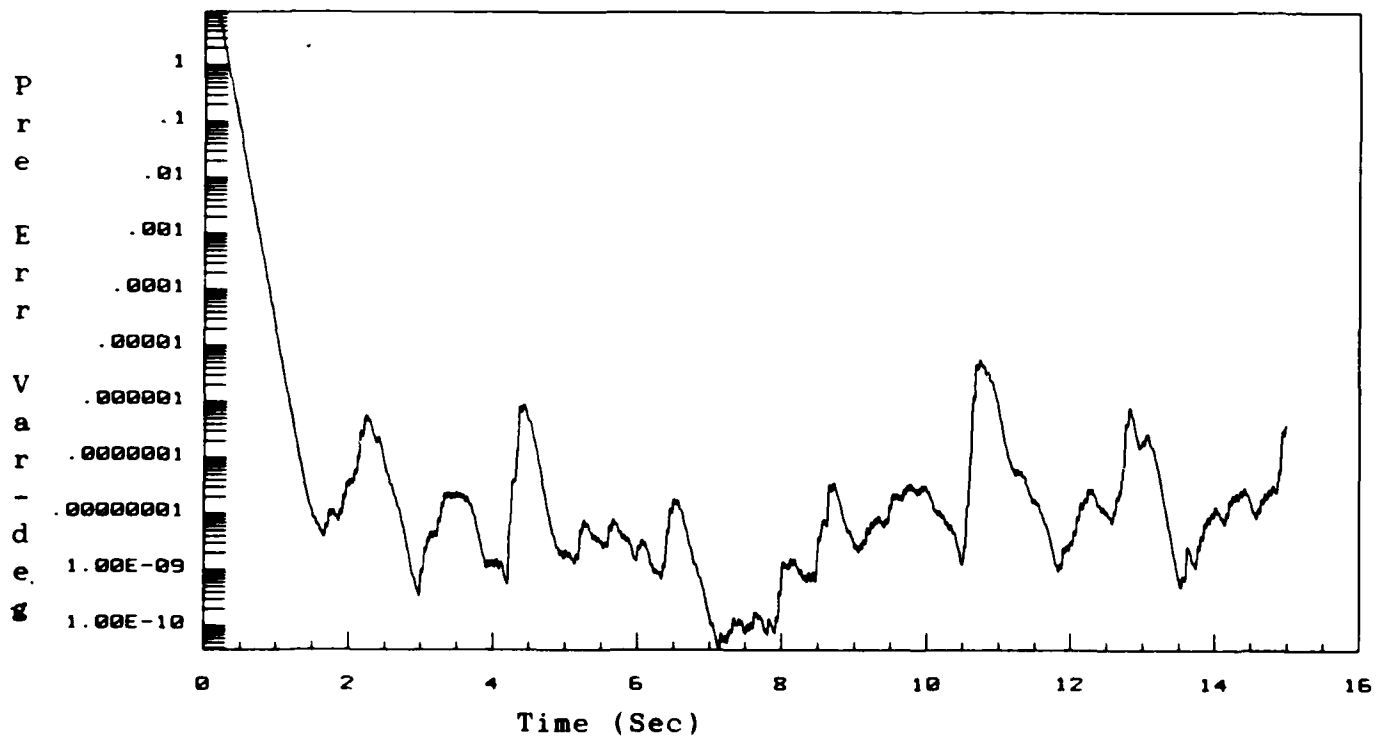


Figure 6-112. Prediction Error Variance (2,2) for Model 2
Sensor Noise/Operating Point: 10K 0.35M

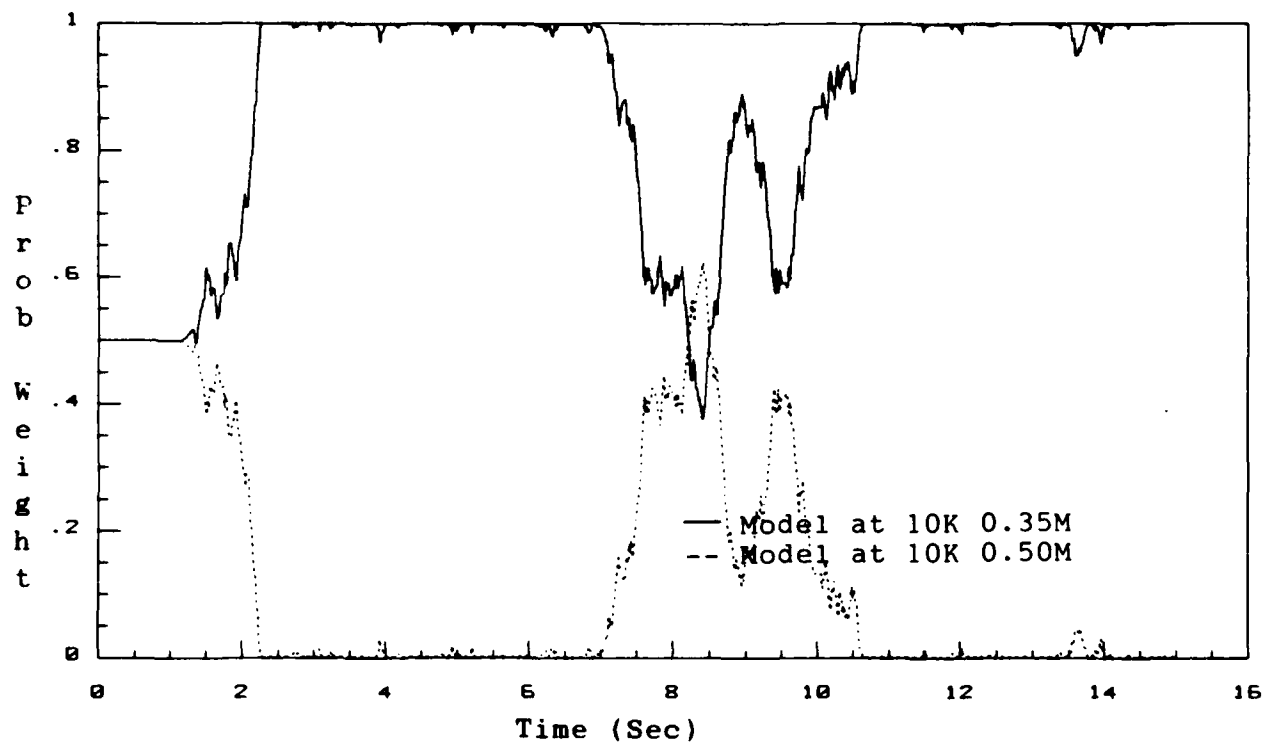


Figure 6-113. Model Probability Weightings
Sensor Noise(*100)/Operating Point: 10K 0.35M

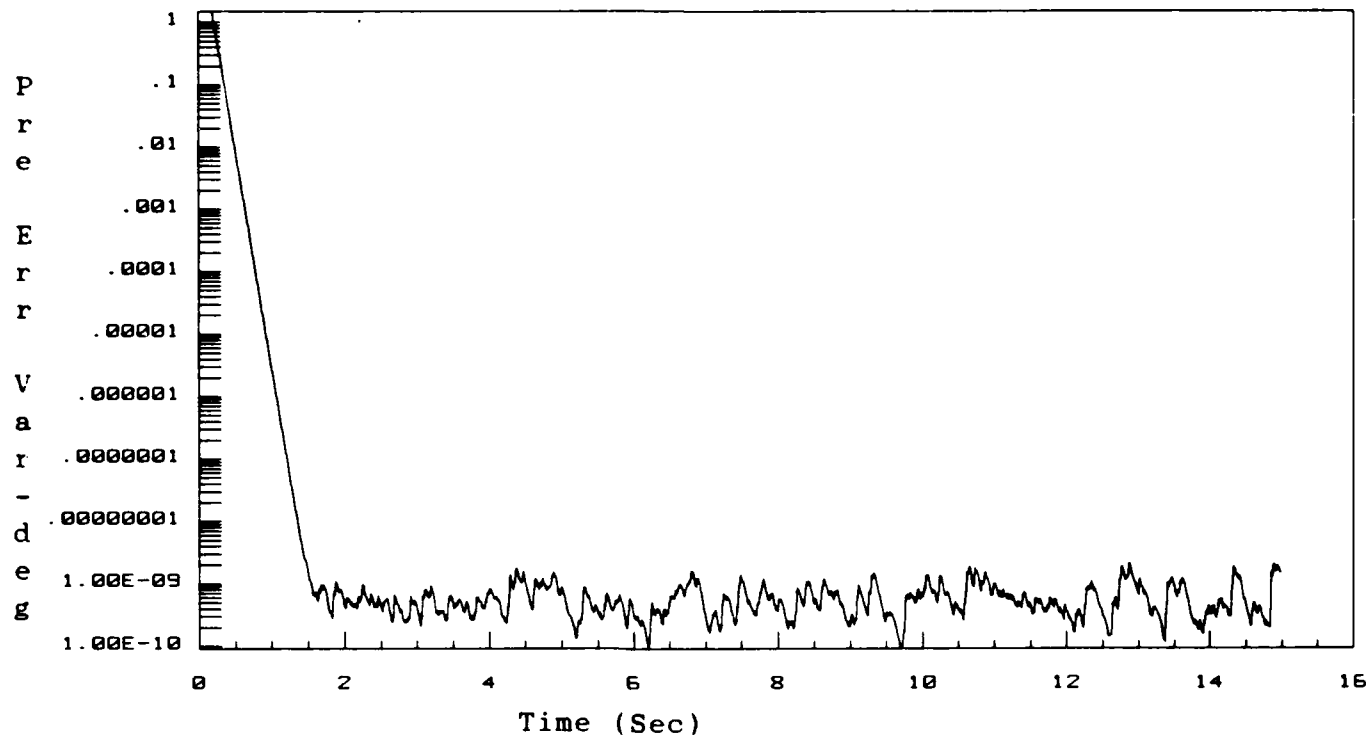


Figure 6-114. Prediction Error Variance (1,1) for Model 1
Sensor Noise(*100)/Operating Point: 10K 0.35M

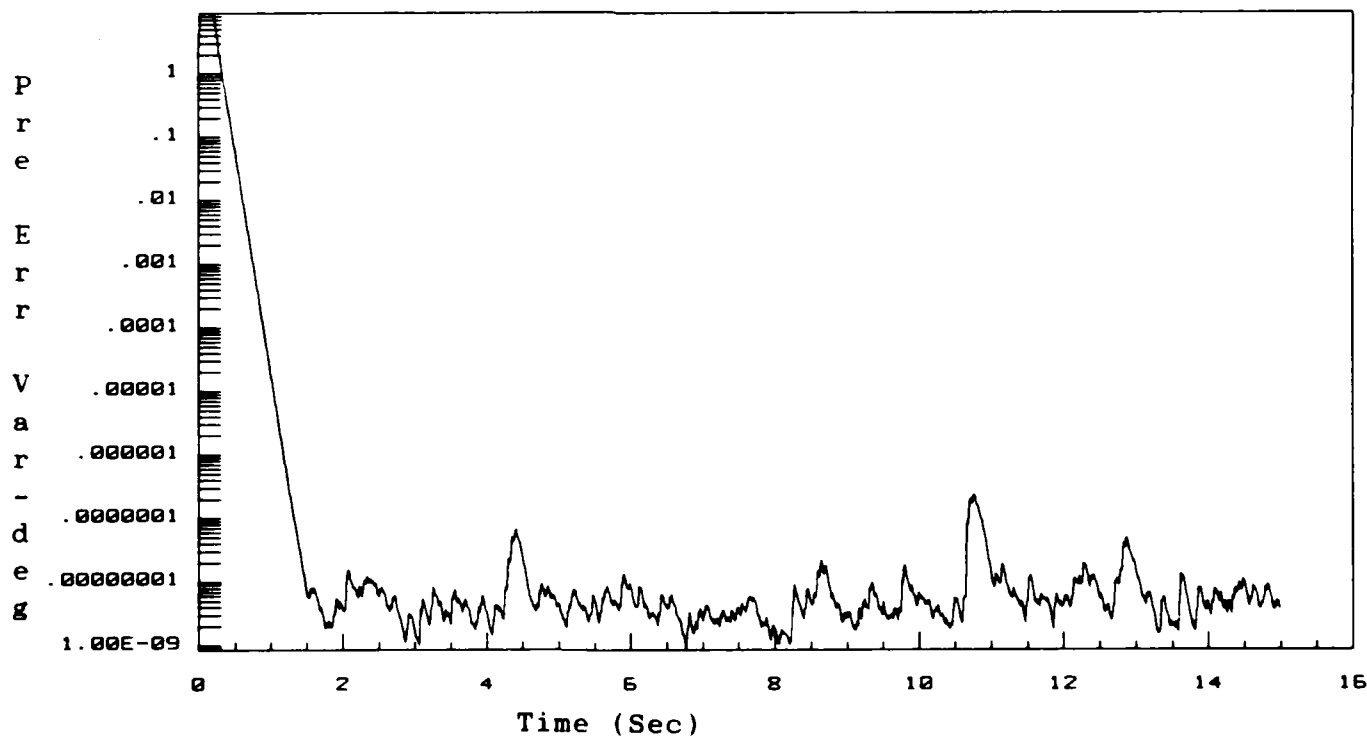


Figure 6-115. Prediction Error Variance (2,2) for Model 1
Sensor Noise(*100)/Operating Point: 10K 0.35M

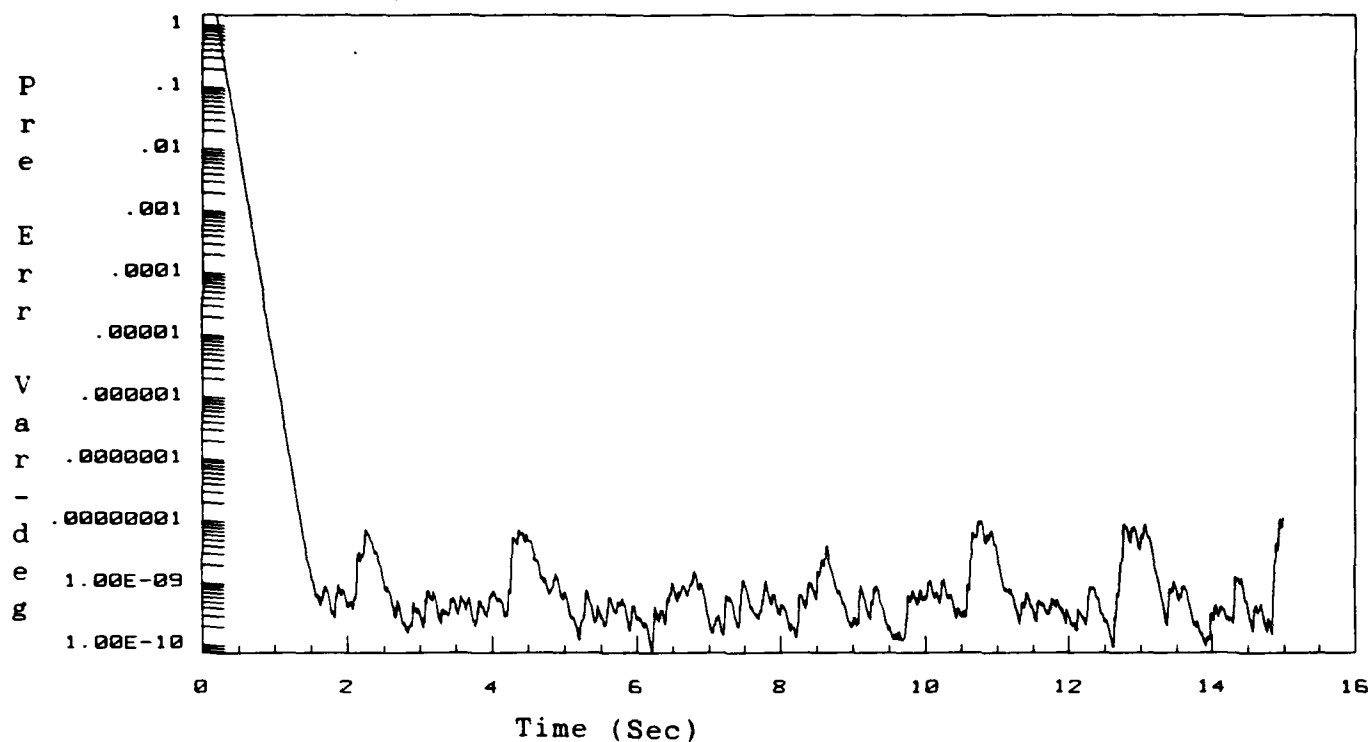


Figure 6-116. Prediction Error Variance (1,1) for Model 2
Sensor Noise(*100)/Operating Point: 10K 0.35M

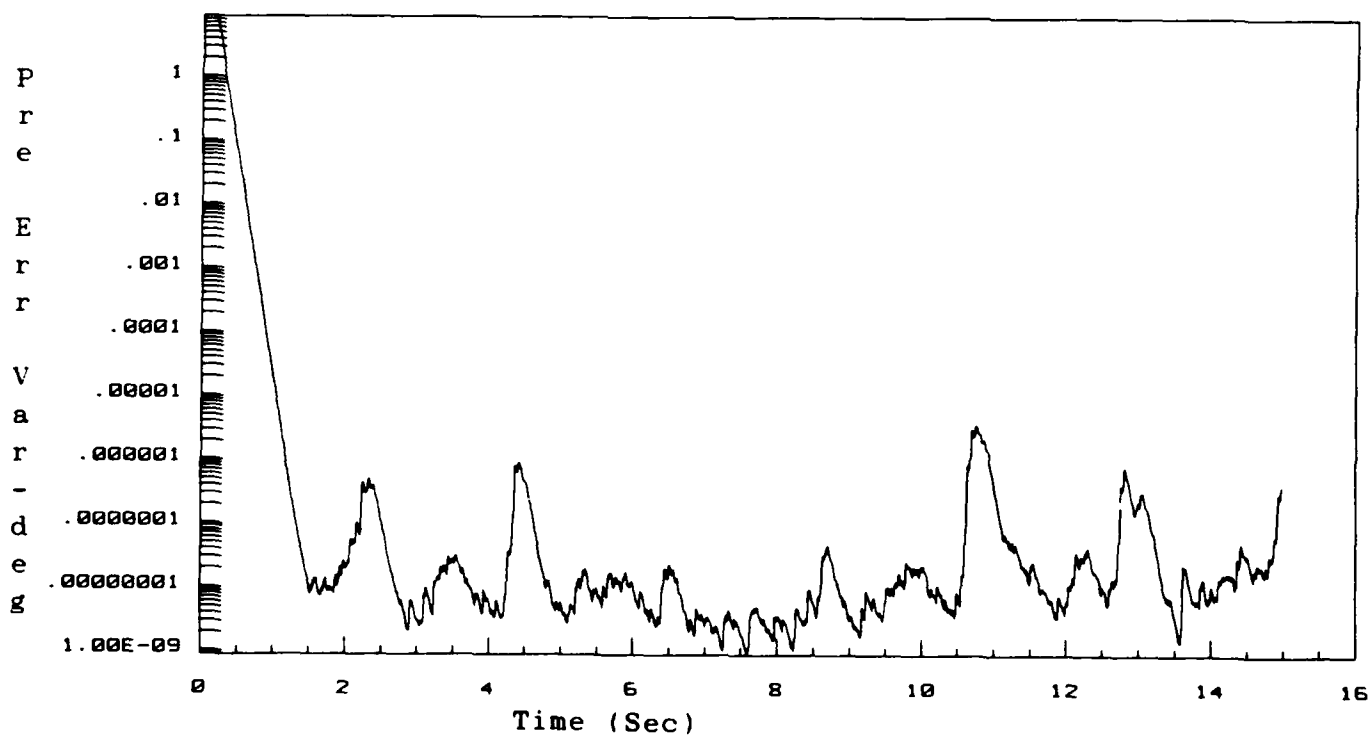


Figure 6-117. Prediction Error Variance (2,2) for Model 2
Sensor Noise(*100)/Operating Point: 10K 0.35M

through 6-117. By comparing Figures 6-114 and 6-116 (prediction error variance for pitch rate for models 1 and 2 respectively), one can see that the variances are very similar for the time of 7 through 10 seconds. Comparing Figure 6-115 to 6-117 (prediction error variance for flight path for models 1 and 2, respectively), reveals the same effect. This effect can be attributed to the sensor noise being added to the system. The noise is actually masking the residuals, thereby leading to a similarity in prediction error variances and hence to degraded multiple model algorithm performance. Comparison of the prediction error variances of the no-noise case of Figures 6-104 through 6-107 with those of Figures 6-114 through 6-117 shows clearly that the addition of sensor noise can mask the residuals, and if sufficiently high noise is added, the prediction error variances will all have the same relative responses. This similarity of response will result in degraded performance by the multiple model algorithm in assigning the weighting factors to the proper models.

The second two-model configuration evaluated is shown in Table 6-17. The two models were spaced farther apart than those of the two-model configuration presented in Table

Table 6-17

Two Model Configuration for Noise Consideration

Model	Flight Condition
1	10,000 ft 0.35 Mach
2	10,000 ft 0.75 Mach

6-16 to ascertain the effects of model spacing in relation to noise resistance. Figures 6-118 through 6-122 present simulation data for the two-model configuration with the actual flight condition set at 10,000 ft, 0.35 Mach. Figure 6-118 shows a very high probability associated with model 1, which is the model associated with the 10,000 ft, 0.35 Mach flight condition. Allowing the sensor noise to take on values one-hundred times those presented in Table 6-15 and running a simulation resulted in the data presented on Figures 6-123 through 6-127. The data shows that the probability weighting curve (Figure 6-123) indicates that model 1 is the correct model, as was the case with no noise. A comparison of the prediction error variances for models 1 and 2 (Figures 6-124 through 6-127) shows that the variances associated with model 1 are smaller than those of model 2.

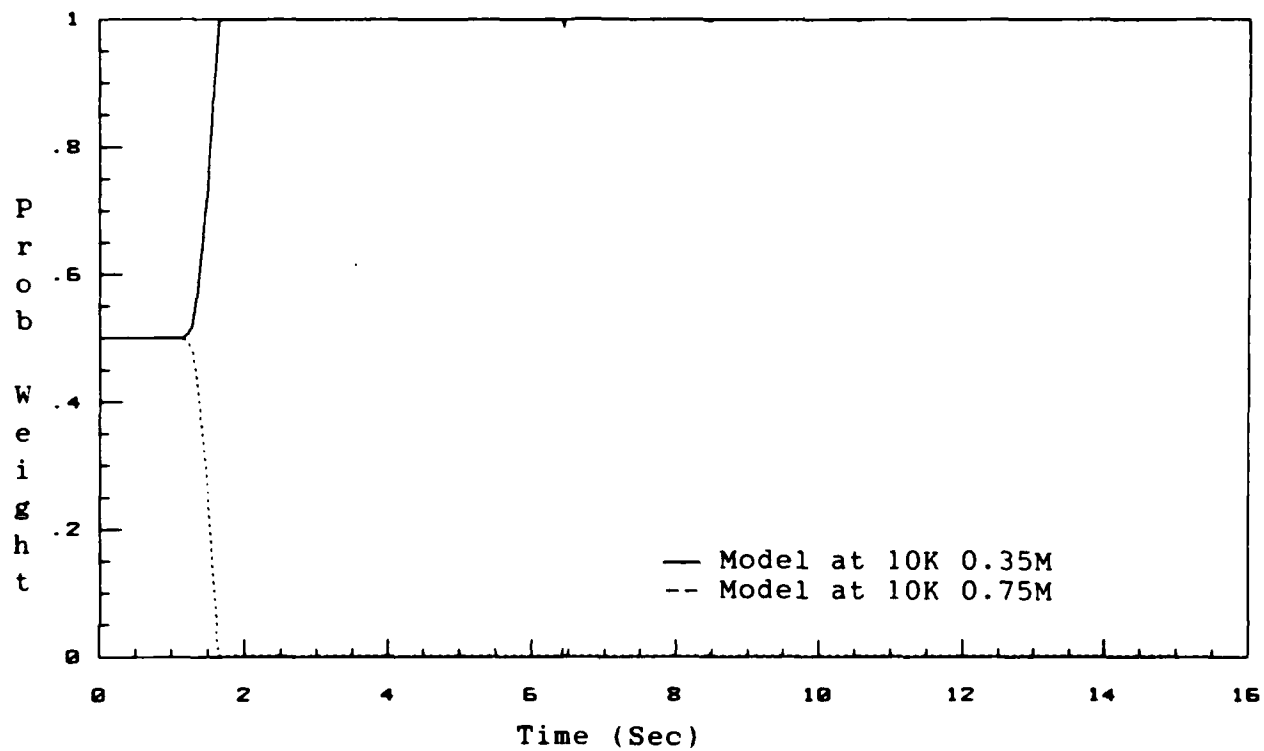


Figure 6-118. Model Probability Weightings
Operating Point: 10K 0.35M

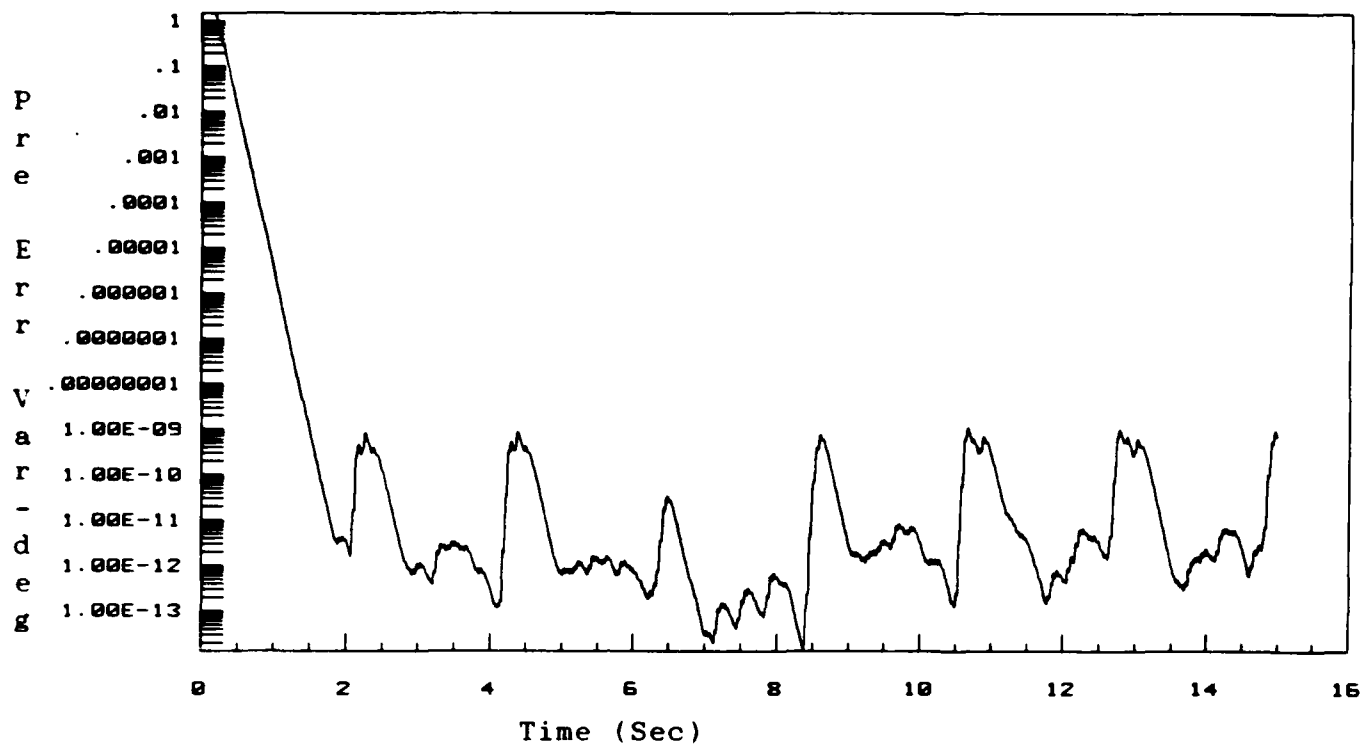


Figure 6-119. Prediction Error Variance (1,1) for Model 1
Operating Point: 10K 0.35M

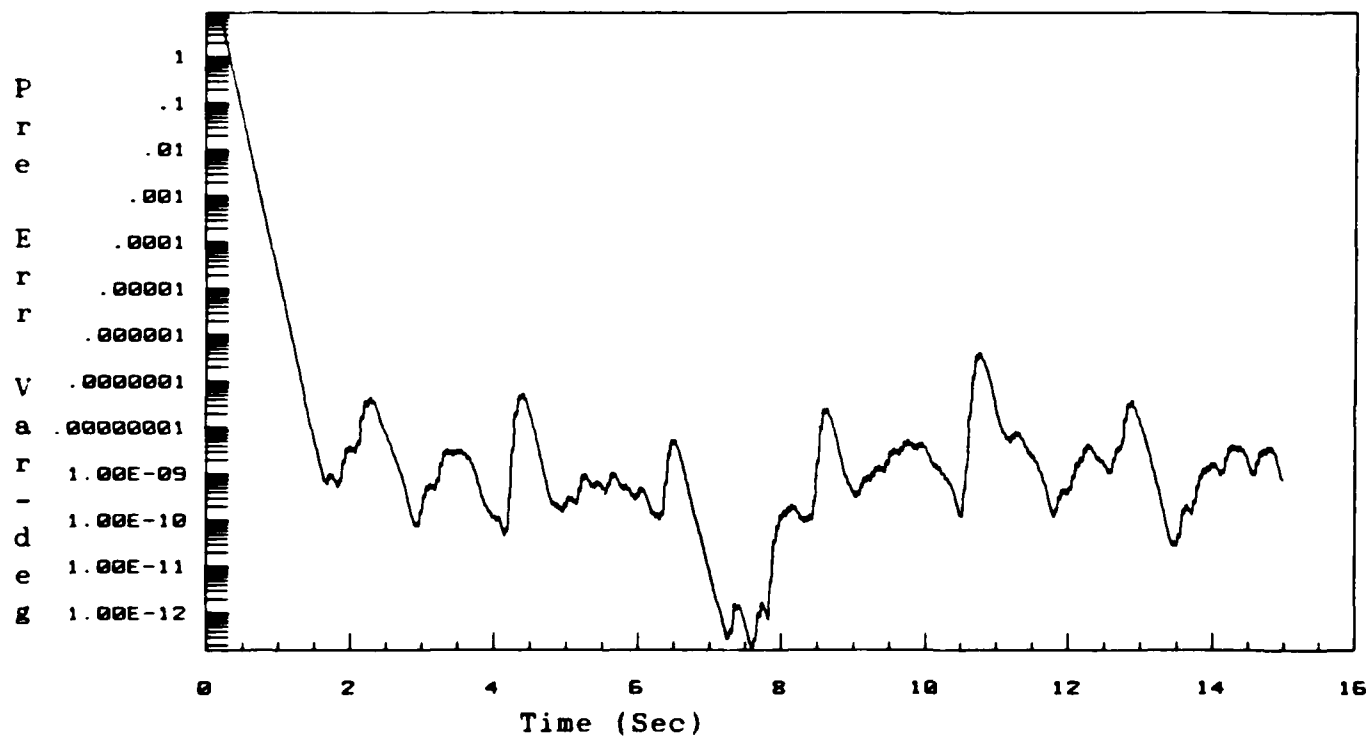


Figure 6-120. Prediction Error Variance (2,2) for Model 1
Operating Point: 10K 0.35M

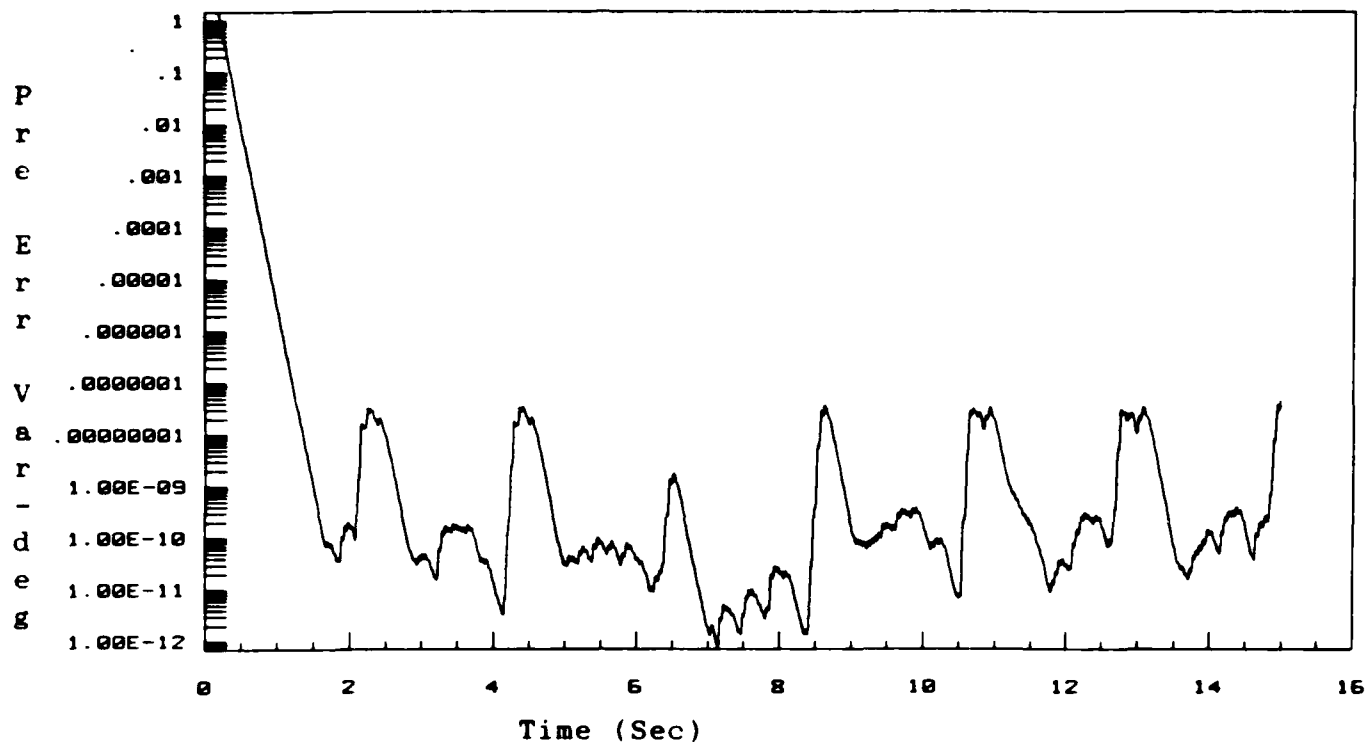


Figure 6-121. Prediction Error Variance (1,1) for Model 2
Operating Point: 10K 0.35M

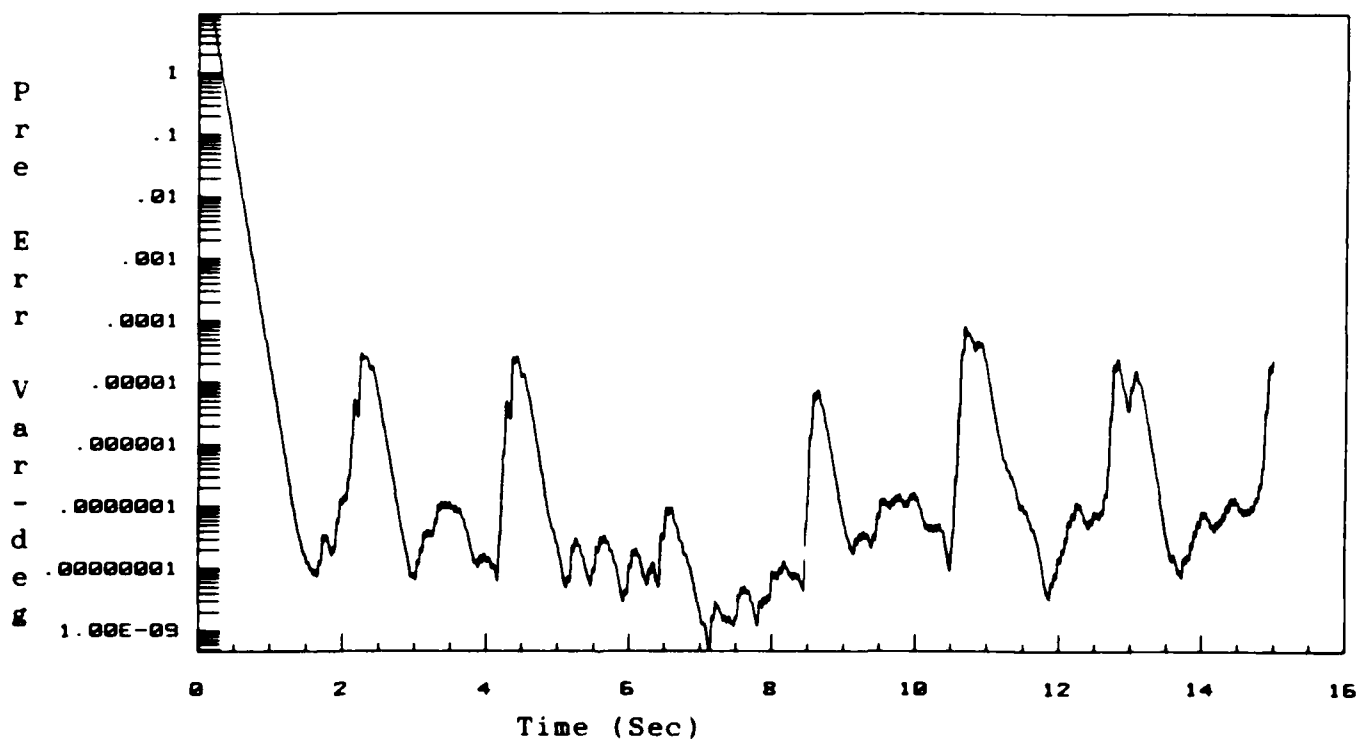


Figure 6-122. Prediction Error Variance (2,2) for Model 2
Operating Point: 10K 0.35M

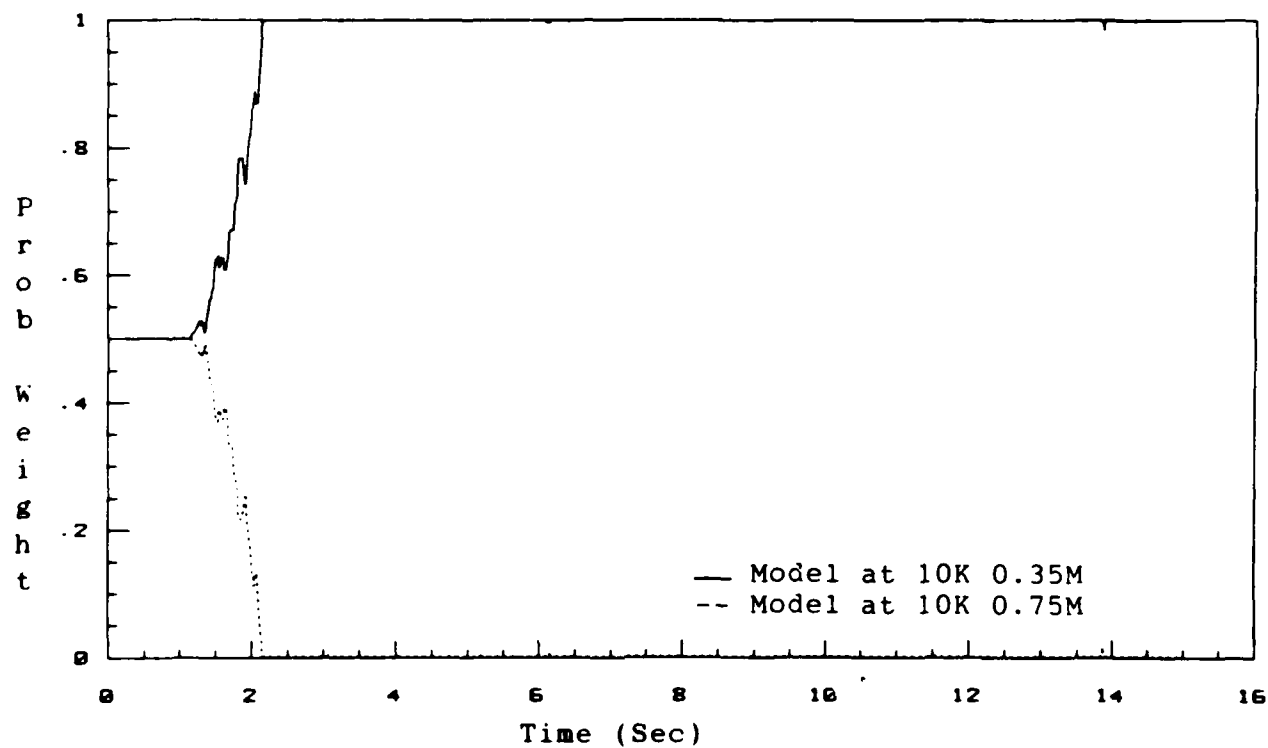


Figure 6-123. Model Probability Weightings
Sensor Noise(*100)/Operating Point: 10K 0.35M

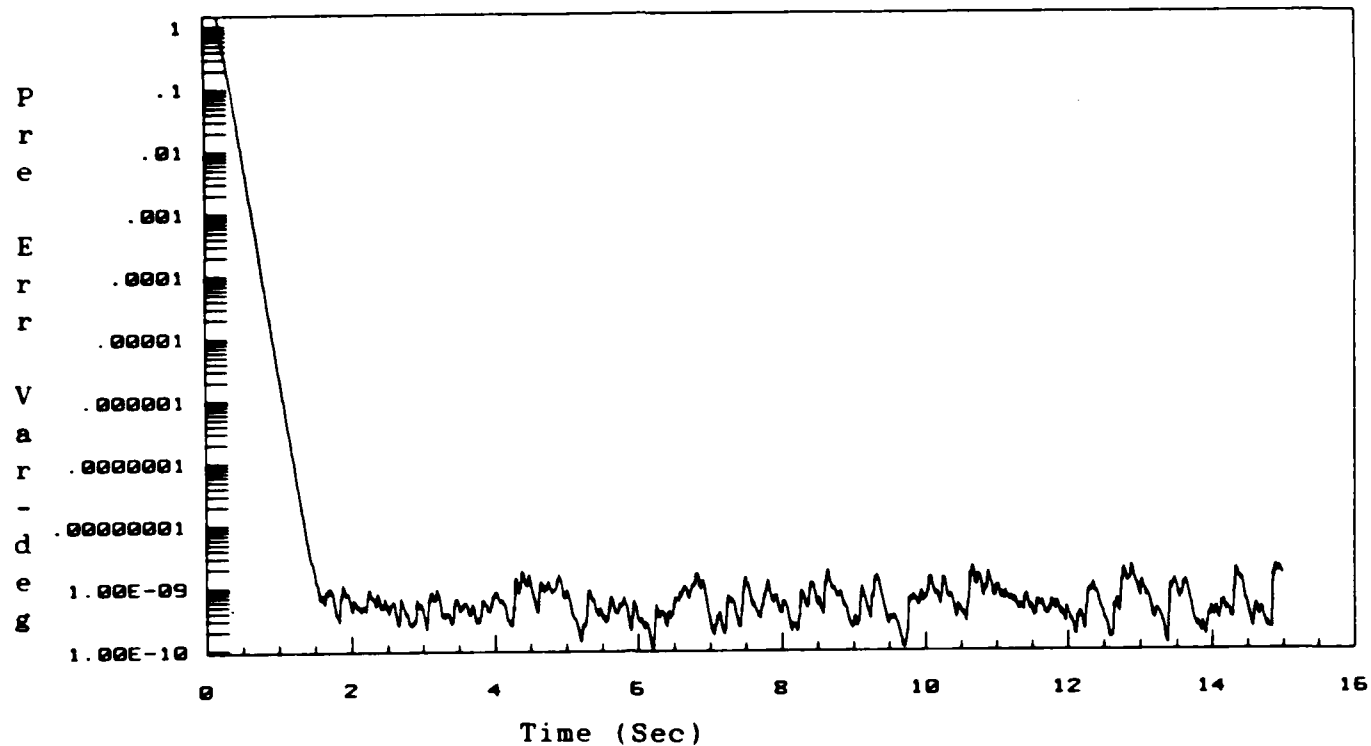


Figure 6-124. Prediction Error Variance (1,1) for Model 1
Sensor Noise(*100)/Operating Point: 10K 0.35M

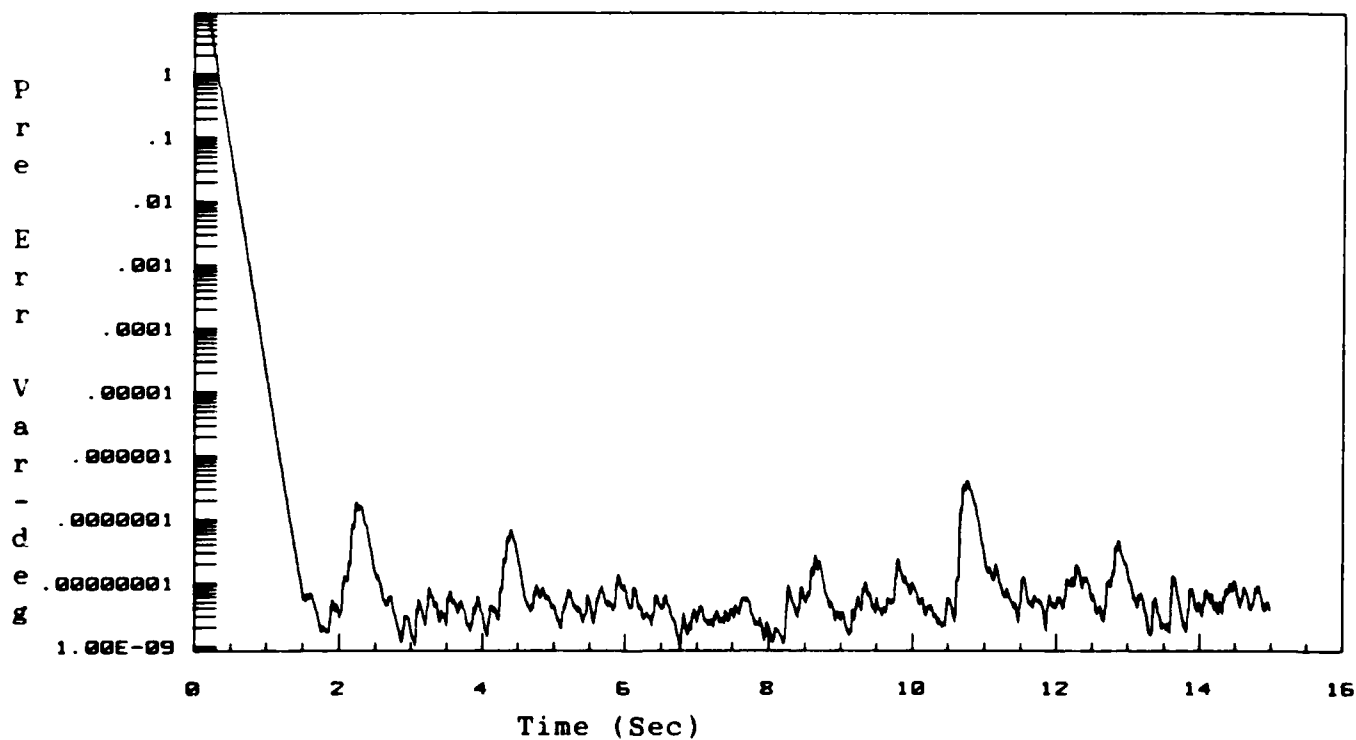


Figure 6-125. Prediction Error Variance (2,2) for Model 1
Sensor Noise(*100)/Operating Point: 10K 0.35M

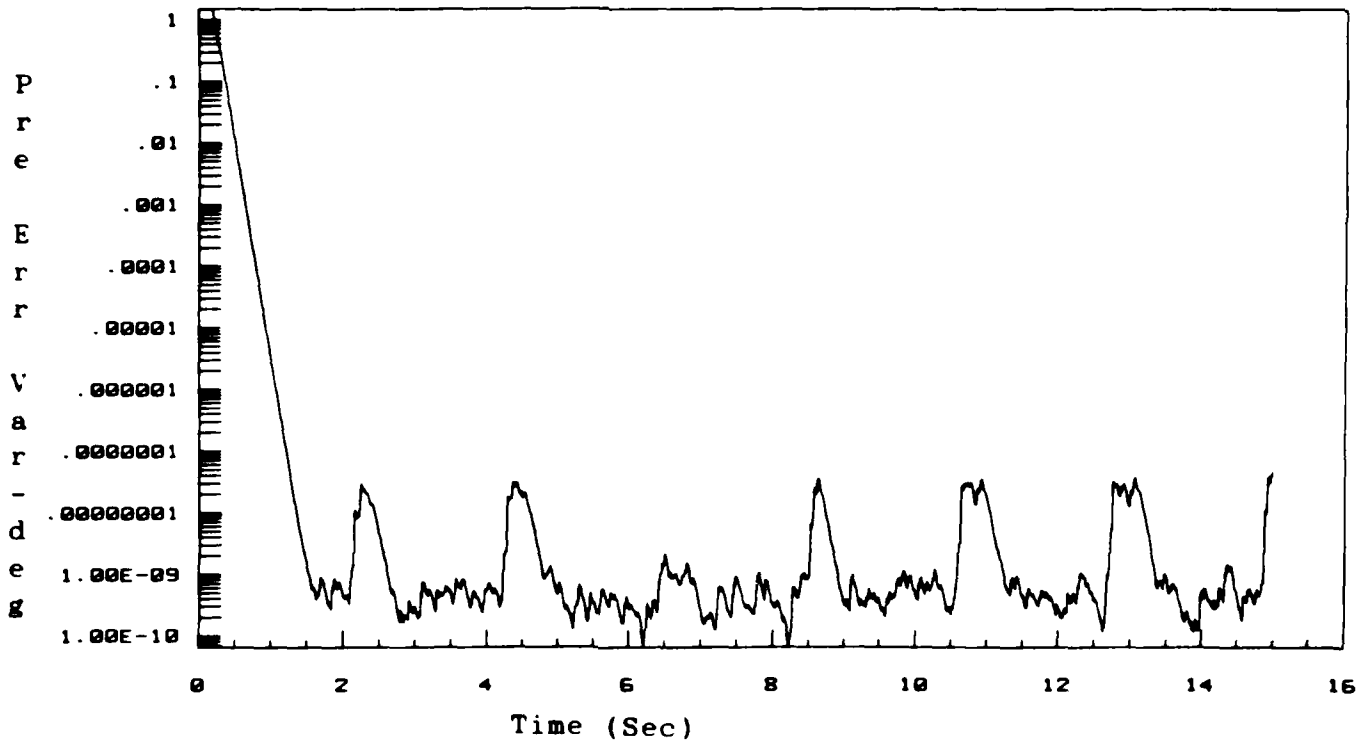


Figure 6-126. Prediction Error Variance (1,1) for Model 2
Sensor Noise(*100)/Operating Point: 10K 0.35M

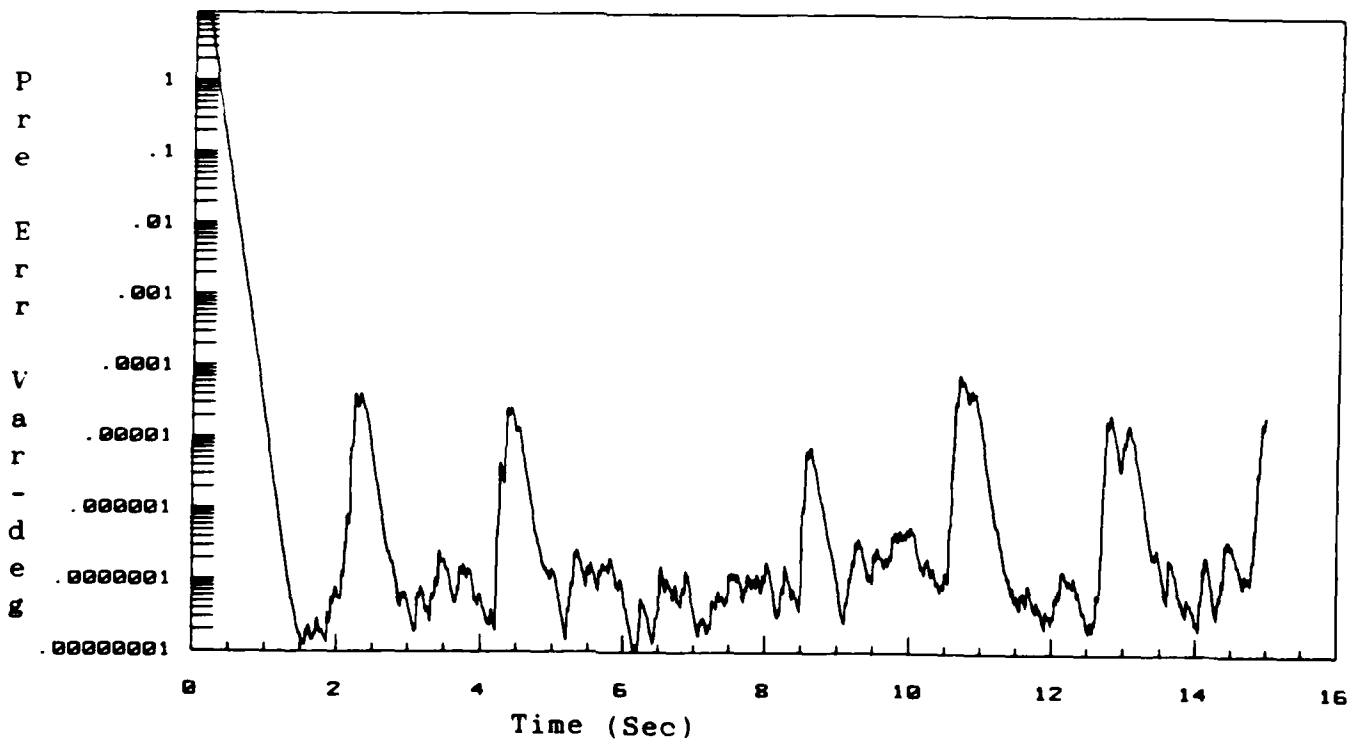


Figure 6-127. Prediction Error Variance (2,2) for Model 2
Sensor Noise(*100)/Operating Point: 10K 0.35M

Even with the sensor noise figures increased by a factor of one-hundred, the multiple model algorithm's performance has not been severely degraded, and the correct model has been selected. This is in contrast to the results obtained for the two-model configuration of Table 6-16. For the configuration of Table 6-16, an increase in the sensor noise figures of one-hundred times resulted in degraded multiple model algorithm performance. The reason for the performance difference between the two-model configurations of Tables 6-16 and 6-17 is due to the spacing of the models. The models of Table 6-13 are spaced farther apart (in dynamic pressure) than those of Table 6-16 and therefore the difference of the two models as seen in the residuals is more significant. Therefore, the error variance term introduced by a model not being the best fitting model (see Equation (5-9)) is still the dominant term and a difference in the prediction error variance responses is the result. This clearly points out the fact that, the farther apart the models of the multiple model algorithm are placed (from a dynamic pressure standpoint), the more noise resistance is possible.

Following the simulations for the two-model configurations, simulations using configuration 8 (see Table 6-9)

were run. Recall that configuration 8 was the three-model configuration that gave the best performance from a performance criteria standpoint. The purpose of these simulations was to see if the performance of three-model configuration was severely degraded in the presence of realistic sensor noise. Several flight conditions were selected as operating points, the first of which was 26,000 ft, and 0.9 Mach. The data presented on Figures 6-128 through 6-134 are for the case where no sensor noise was added. The probability weighting curve (see Figure 6-128) shows that models 2 and 3 contribute significantly to the solution of finding the correct model. Figures 6-129 through 6-134 show the prediction error variances for the respective models. Figure 6-135 shows the probability weighting curve for the sensor noise values presented in Table 6-15. From this figure, it can be seen that although models 2 and 3 are still the most heavily weighted models, model 3 has more weight than it did in the simulation without noise. Further increasing the sensor noise to one-hundred times the values in Table 6-15 results in the data on Figures 6-136 through 6-142. The probability weighting curve (see Figure 6-136) is now drastically different from the case with no noise, and model 3

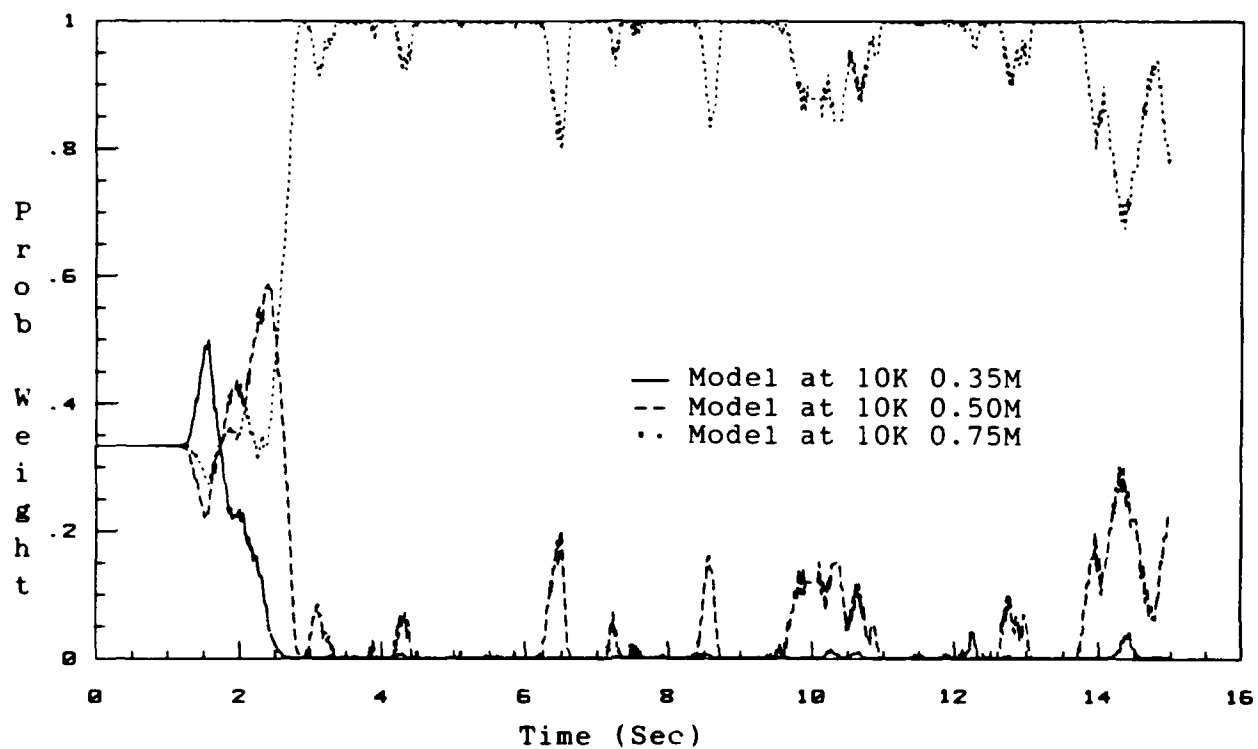


Figure 6-128. Model Probability Weightings
Configuration 8/Operating Point: 26K 0.9M

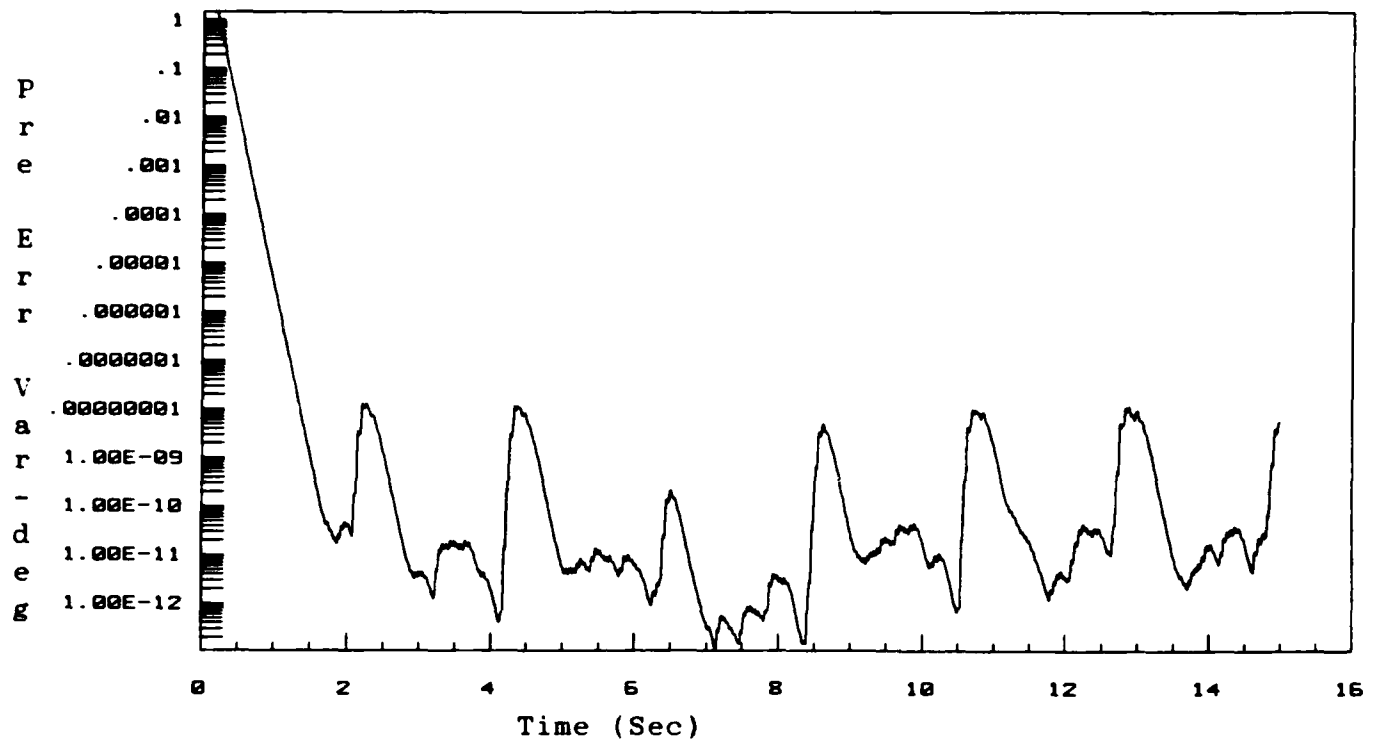


Figure 6-129. Prediction Error Variance (1,1) for Model 1
Configuration 8/Operating Point: 26K 0.9M

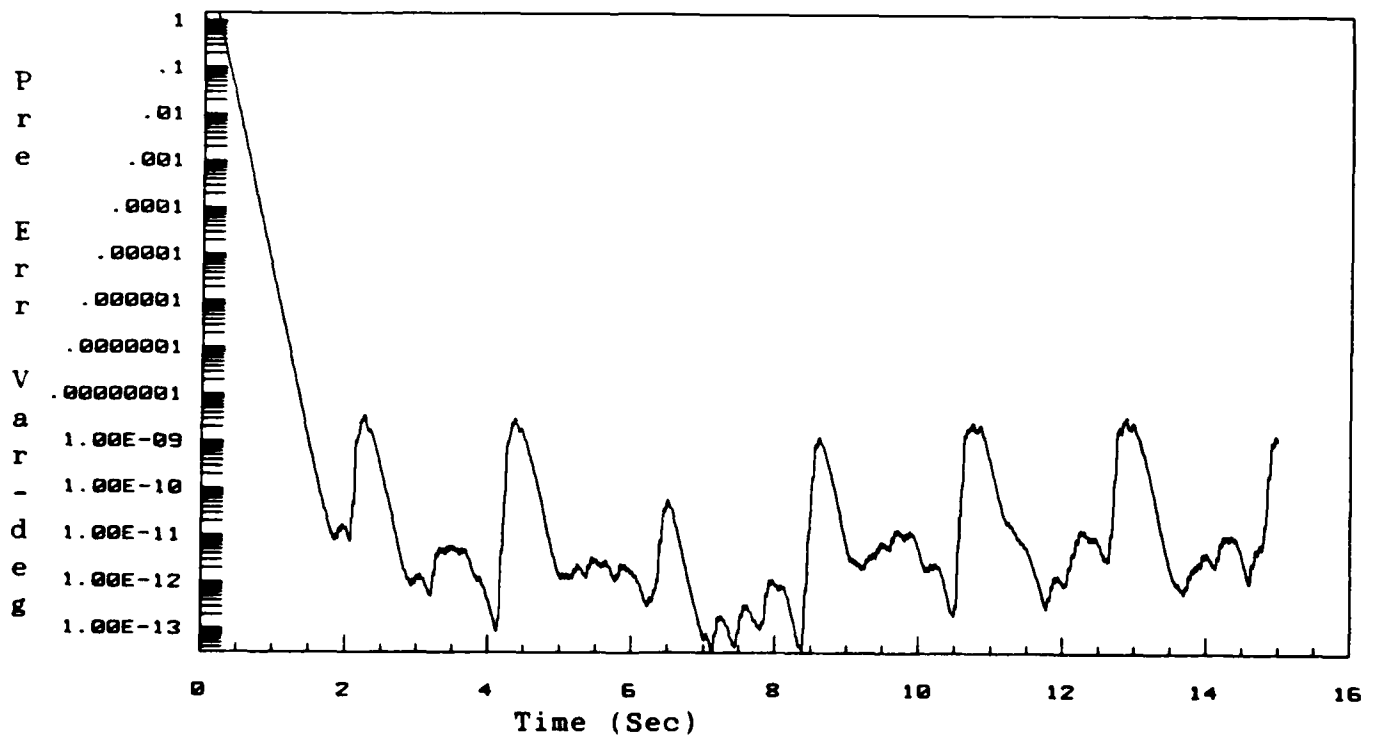


Figure 6-130. Prediction Error Variance (2,2) for Model 1
Configuration 8/Operating Point: 26K 0.9M

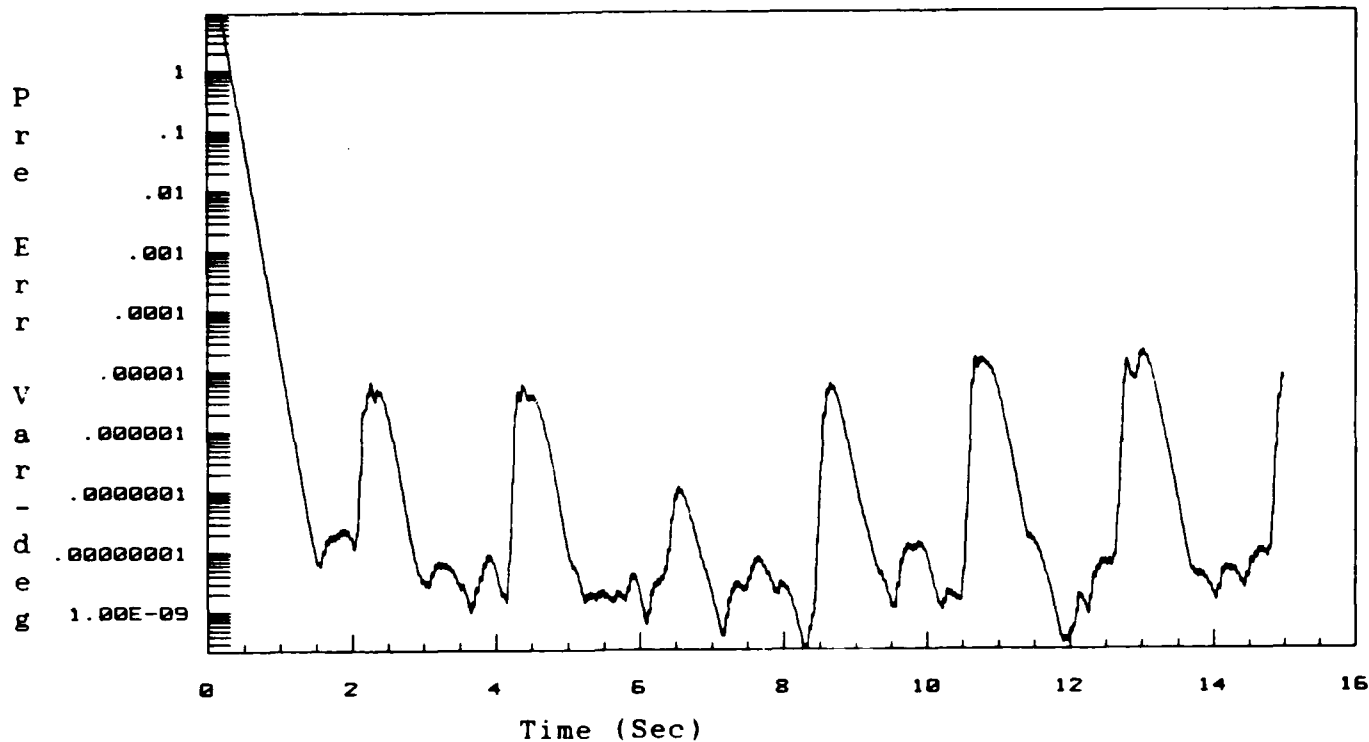


Figure 6-131. Prediction Error Variance (1,1) for Model 2
Configuration 8/Operating Point: 26K 0.9M

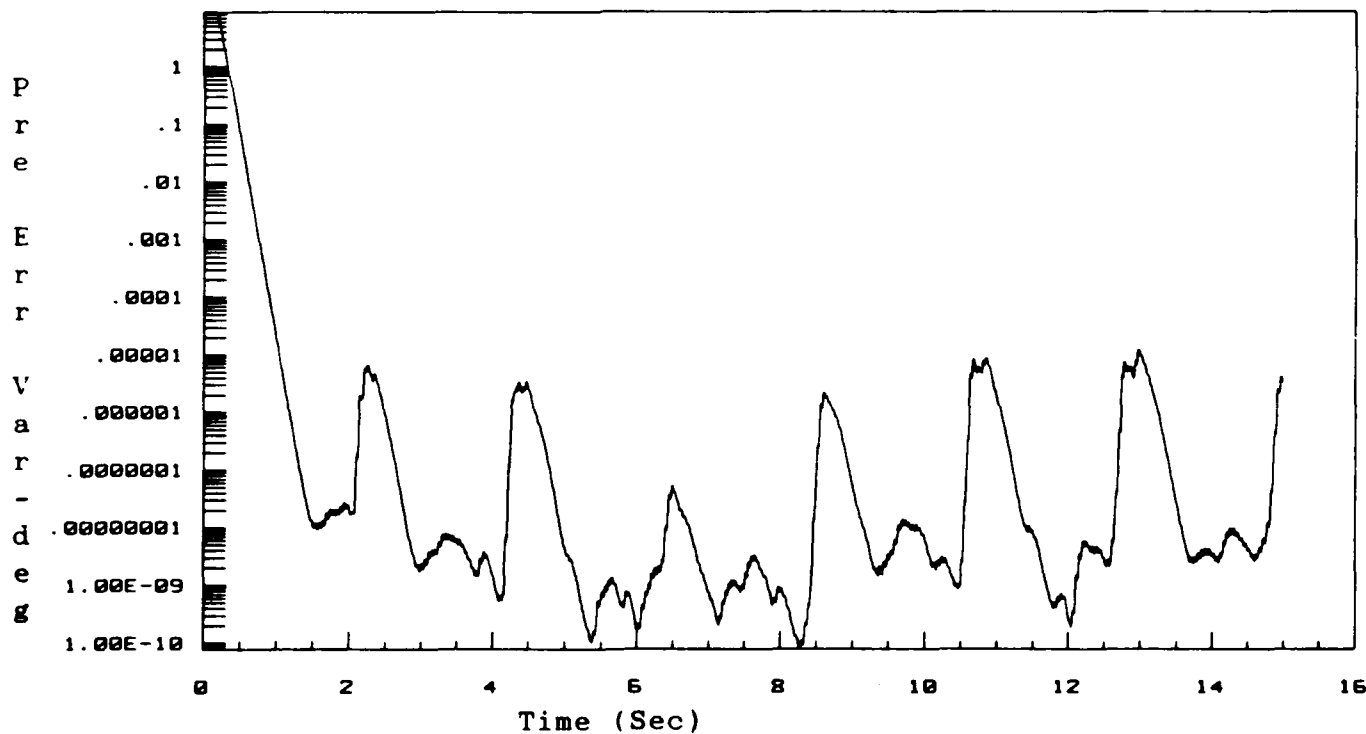


Figure 6-132. Prediction Error Variance (2,2) for Model 2
Configuration 8/Operating Point: 26K 0.9M

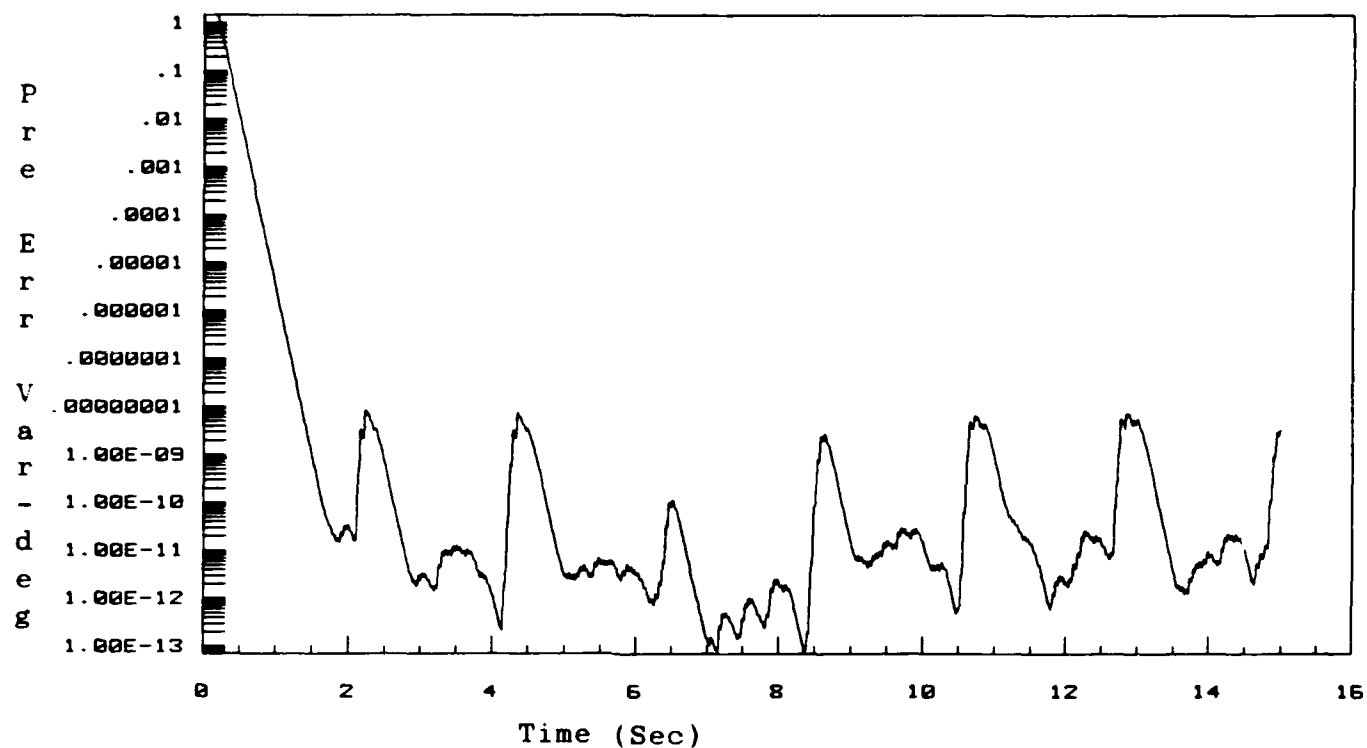


Figure 6-133. Prediction Error Variance (1,1) for Model 3
Configuration 8/Operating Point: 26K 0.9M

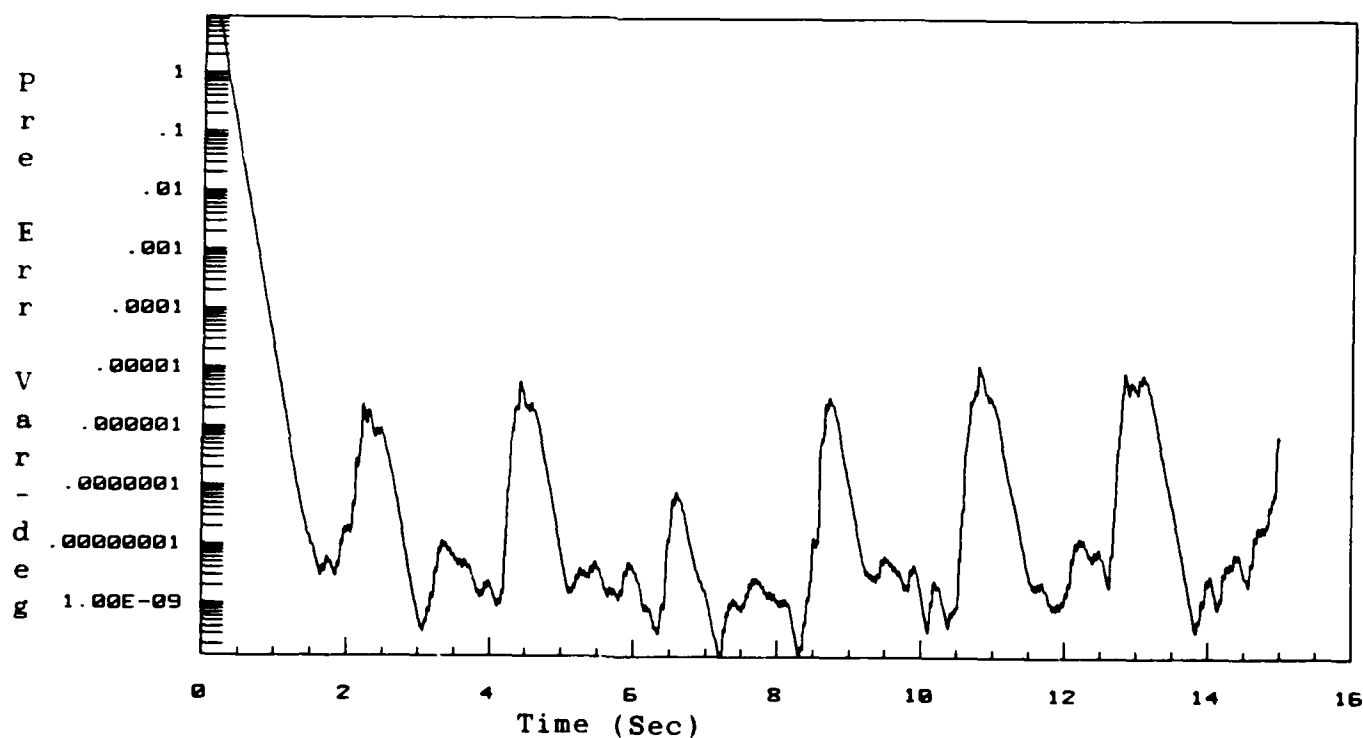


Figure 6-134. Prediction Error Variance (2,2) for Model 3
Configuration 8/Operating Point: 26K 0.9M

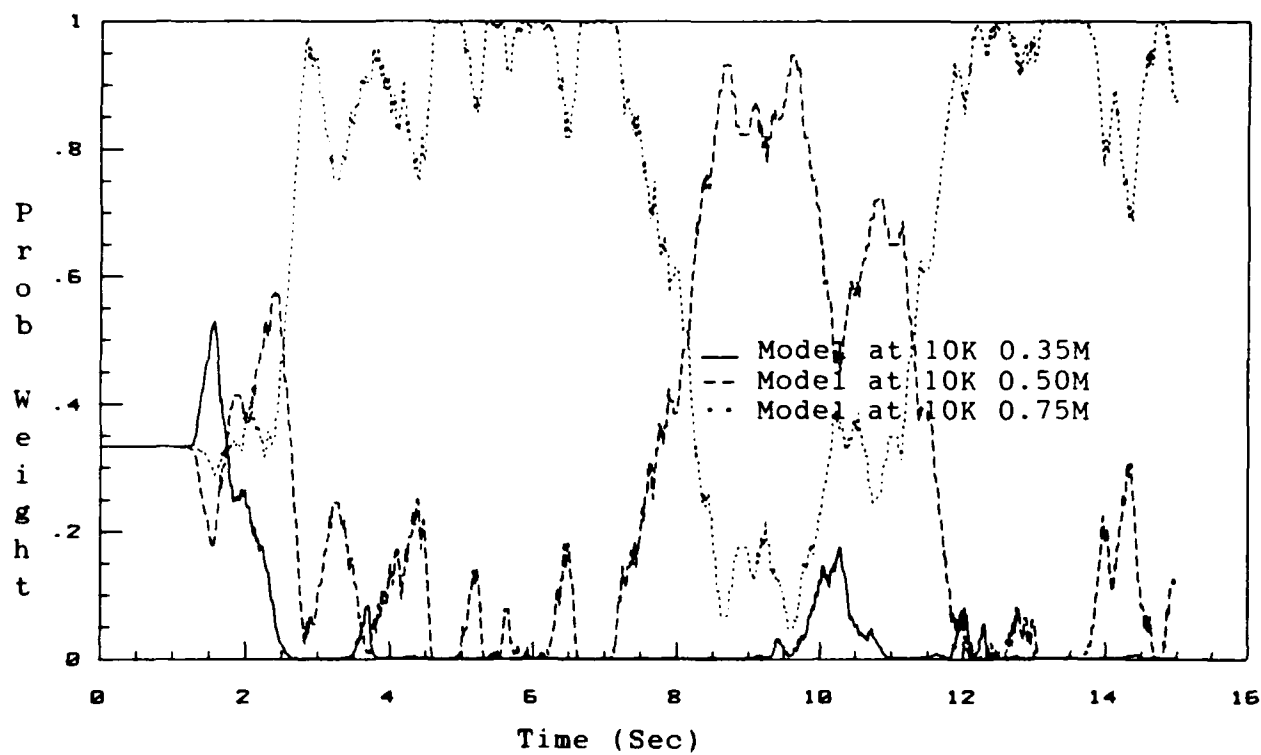


Figure 6-135. Model Probability Weightings
Configuration 8/Sensor Noise/Operating Point: 26K 0.9M

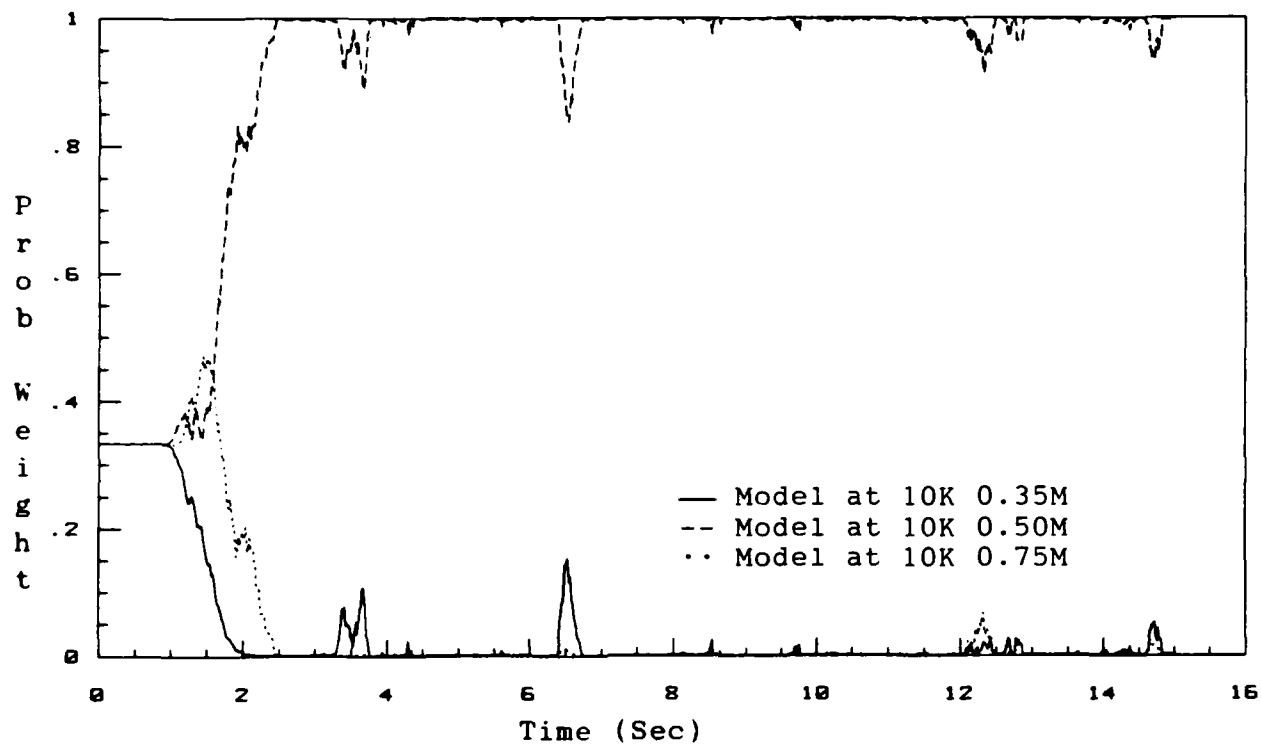


Figure 6-136. Model Probability Weightings
 Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

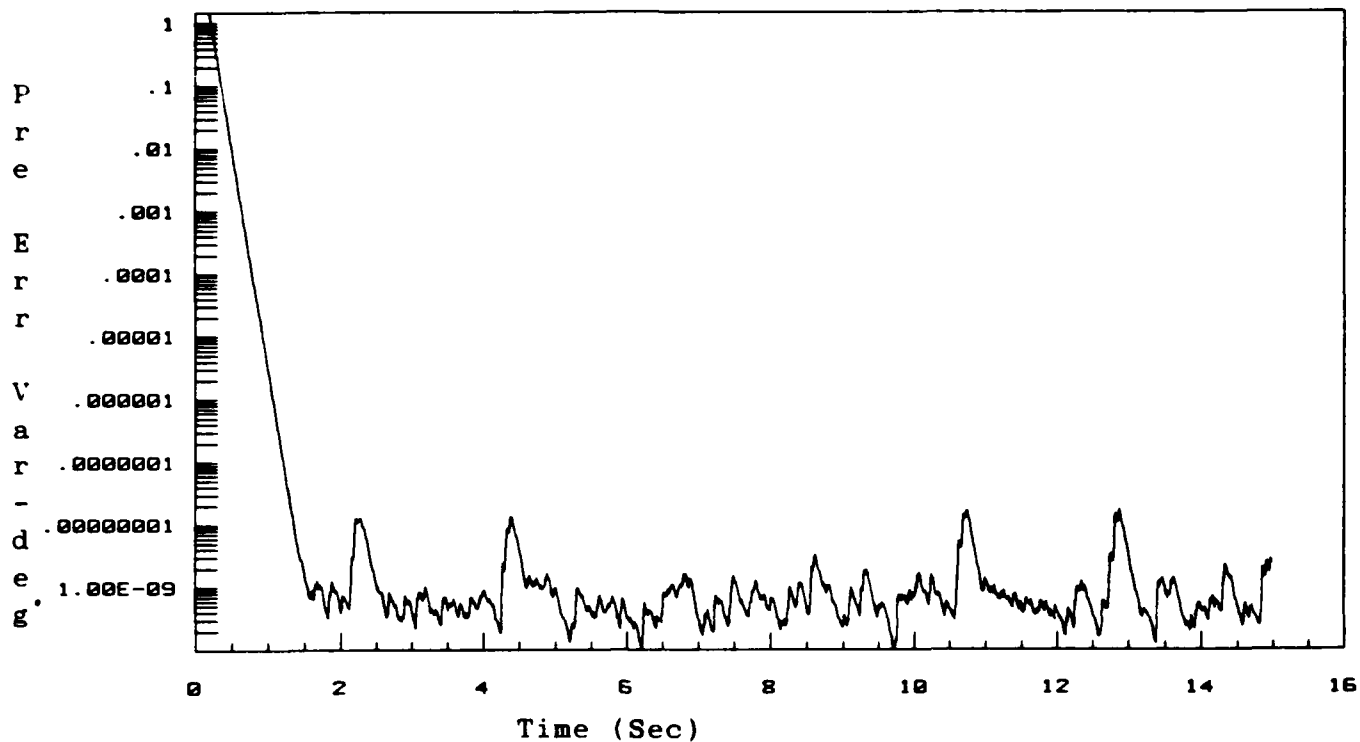


Figure 6-137. Prediction Error Variance (1,1) for Model 1
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

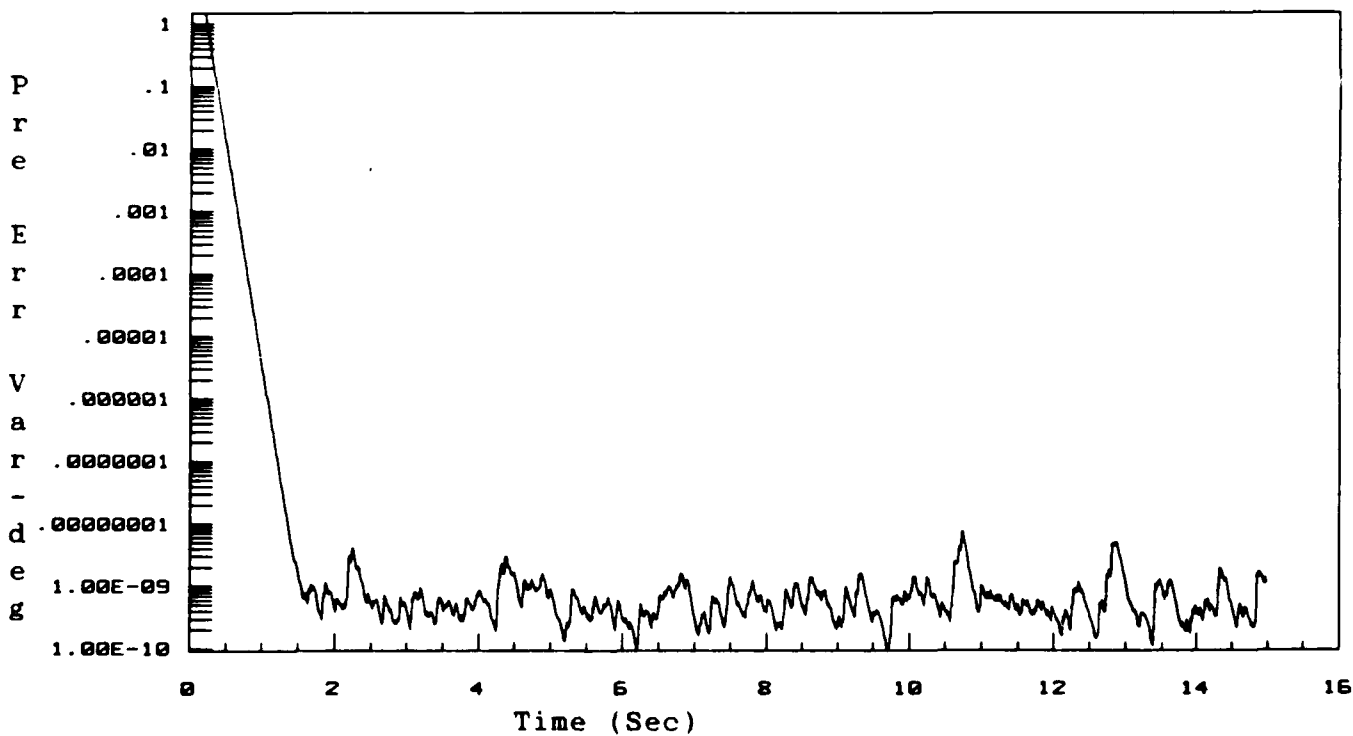


Figure 6-138. Prediction Error Variance (2,2) for Model 1
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

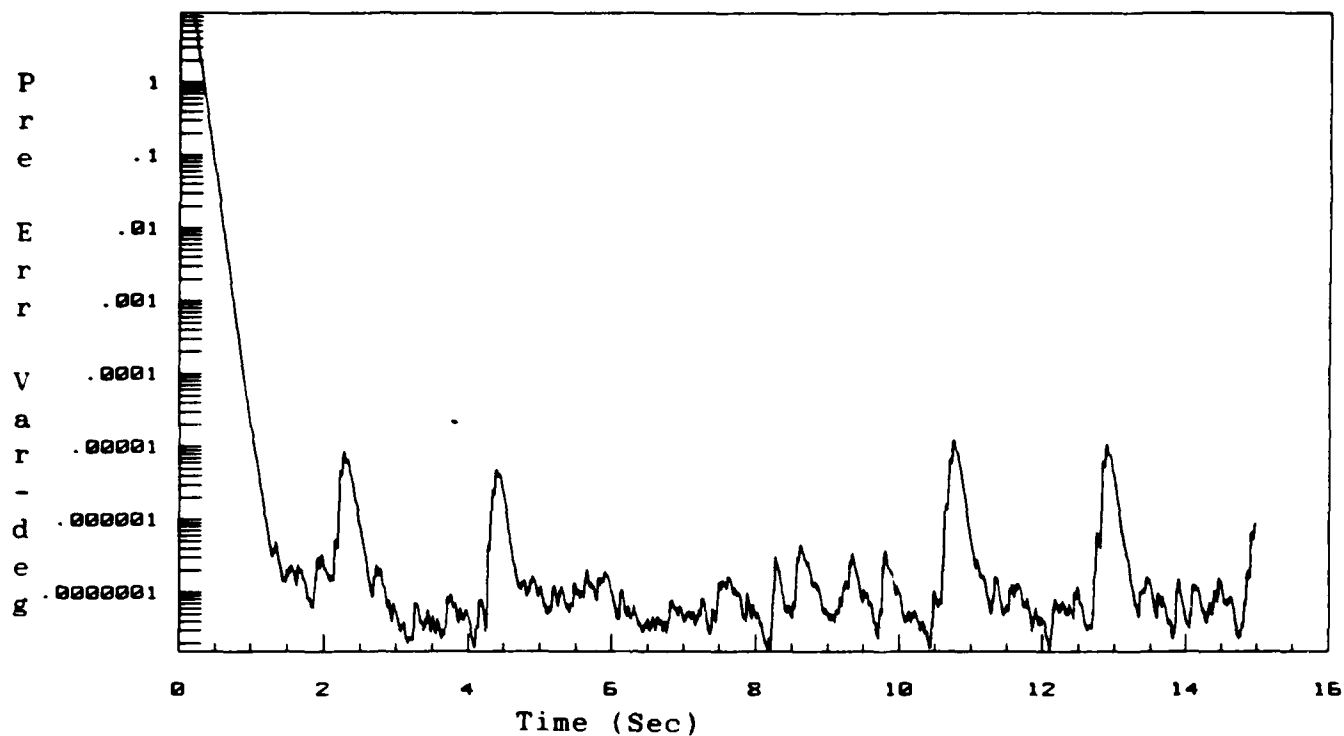


Figure 6-139. Prediction Error Variance (1,1) for Model 2
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

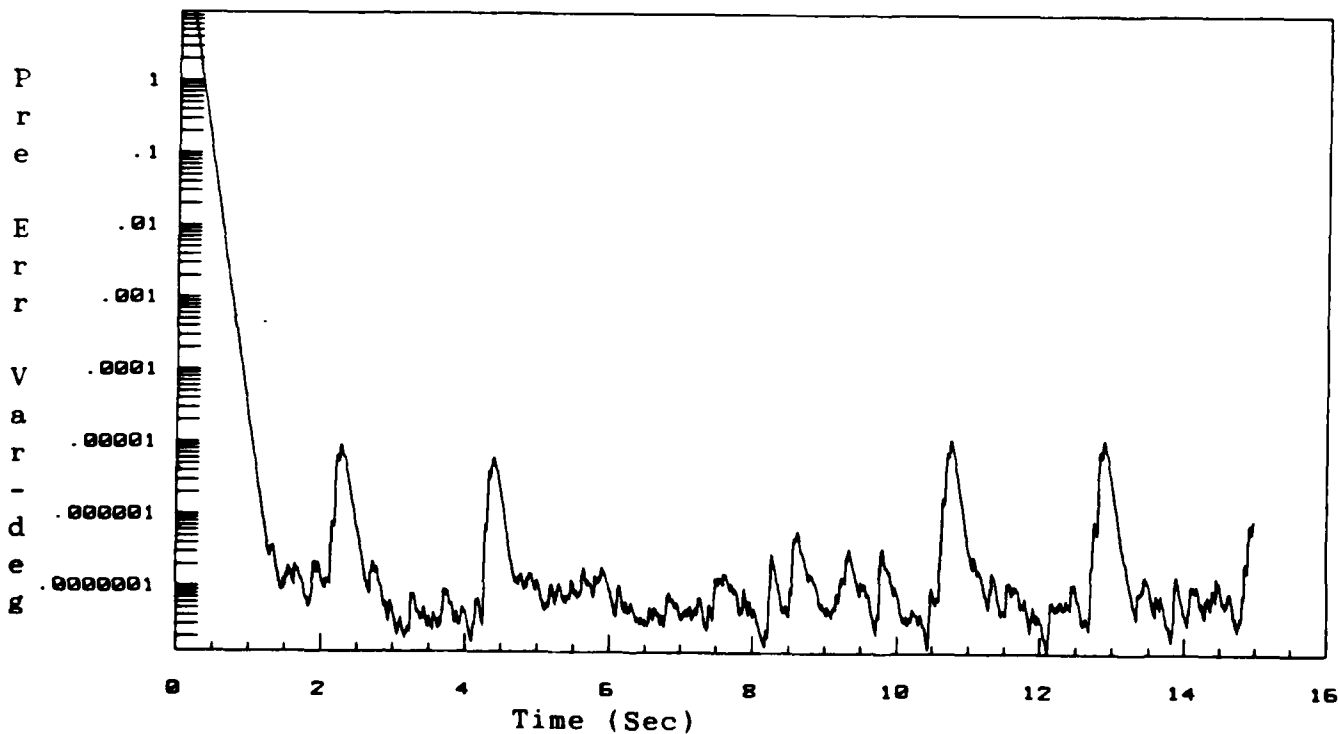


Figure 6-140. Prediction Error Variance (2,2) for Model 2
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

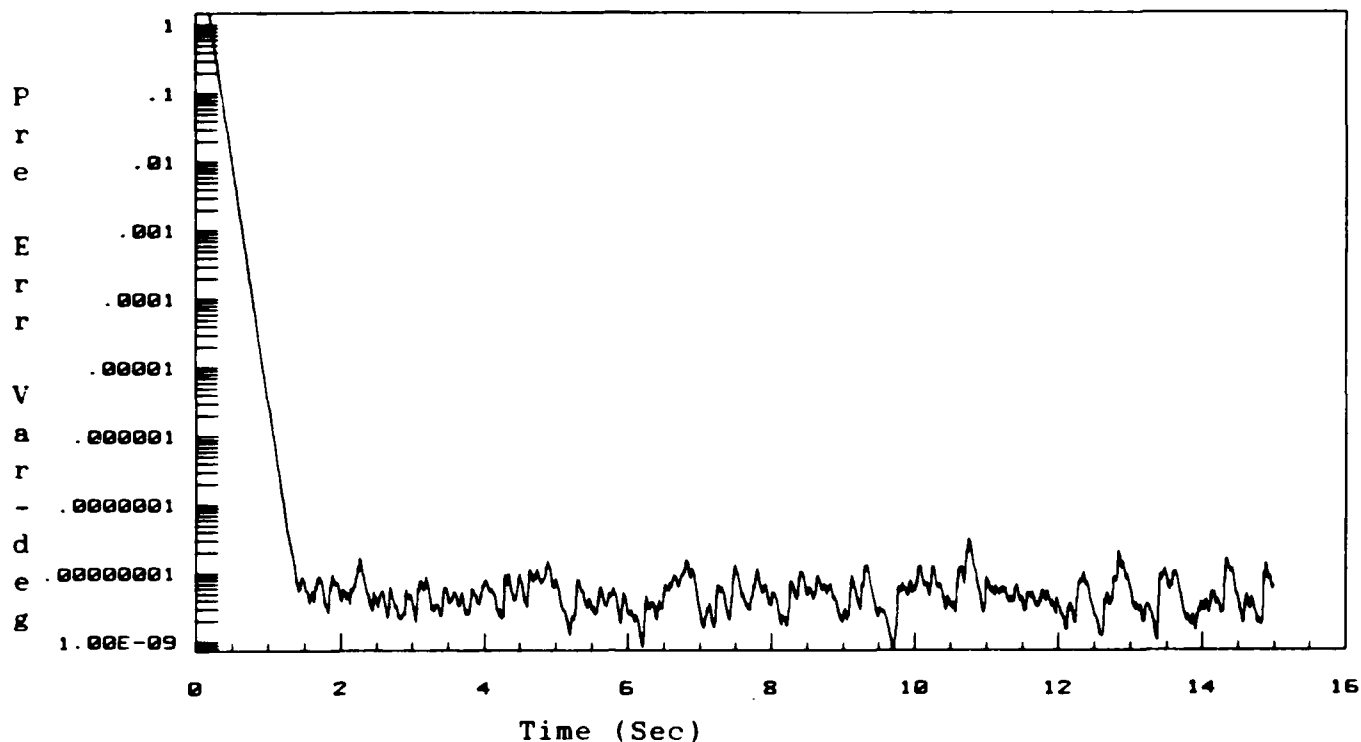


Figure 6-141. Prediction Error Variance (1,1) for Model 3
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

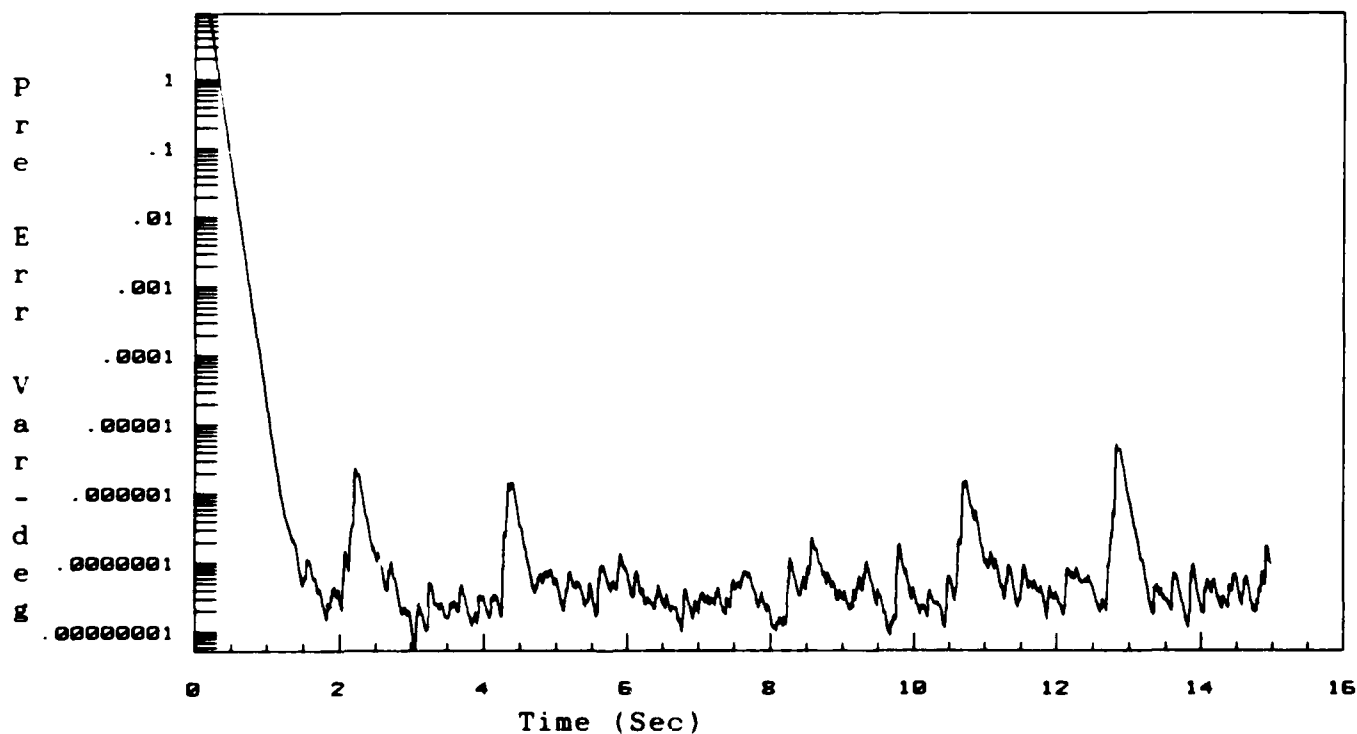


Figure 6-142. Prediction Error Variance (2,2) for Model 3
Configuration 8/Sensor Noise(*100)/Operating Point: 26K 0.9M

has been selected as the most correct model. The prediction error variances as shown in Figures 6-136 through 6-142 also have much different responses from those that were obtained from the simulations without noise. Several other simulations were run with the same general results. Figures 6-143 through 6-146 present the probability weighting curves for increasing values of sensor noise for the case where the actual flight condition was at 38,000 ft, 0.6 Mach. As can be seen from the weighting curves of Figures 6-143 through 6-146, increasing the level of sensor noise results in increasing degradation of the multiple model algorithm's performance. From the numerous simulations accomplished, it was determined that the multiple model algorithm was more sensitive to sensor noise when the probability weighting curves for no noise had two models with significant influence on determining the correct model. In contrast, the multiple model algorithm is less sensitive to sensor noise when the actual flight condition is such that the probability weighting calculation for no noise yields a single model with a probability near one. The simulation results for configuration 8 showed that the multiple model algorithm yielded satisfactory results for the sensor noise levels

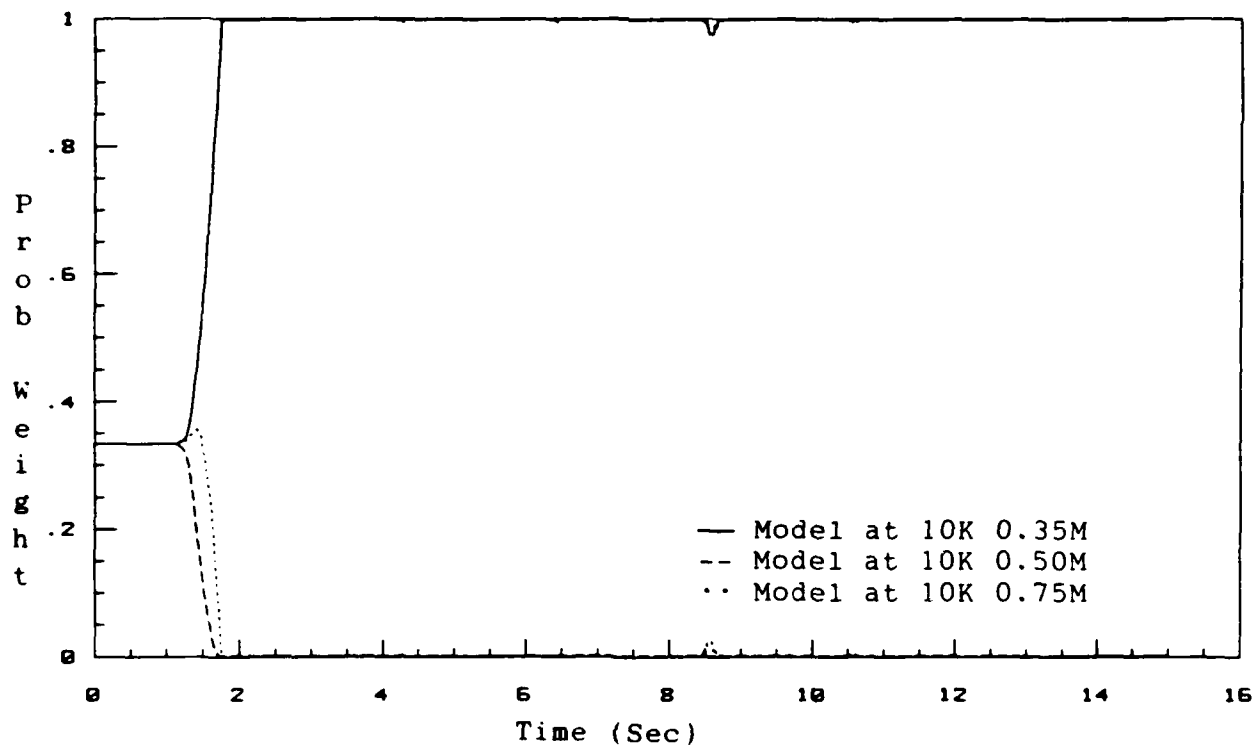


Figure 6-143. Model Probability Weightings
Configuration 8/Operating Point: 38K 0.6M

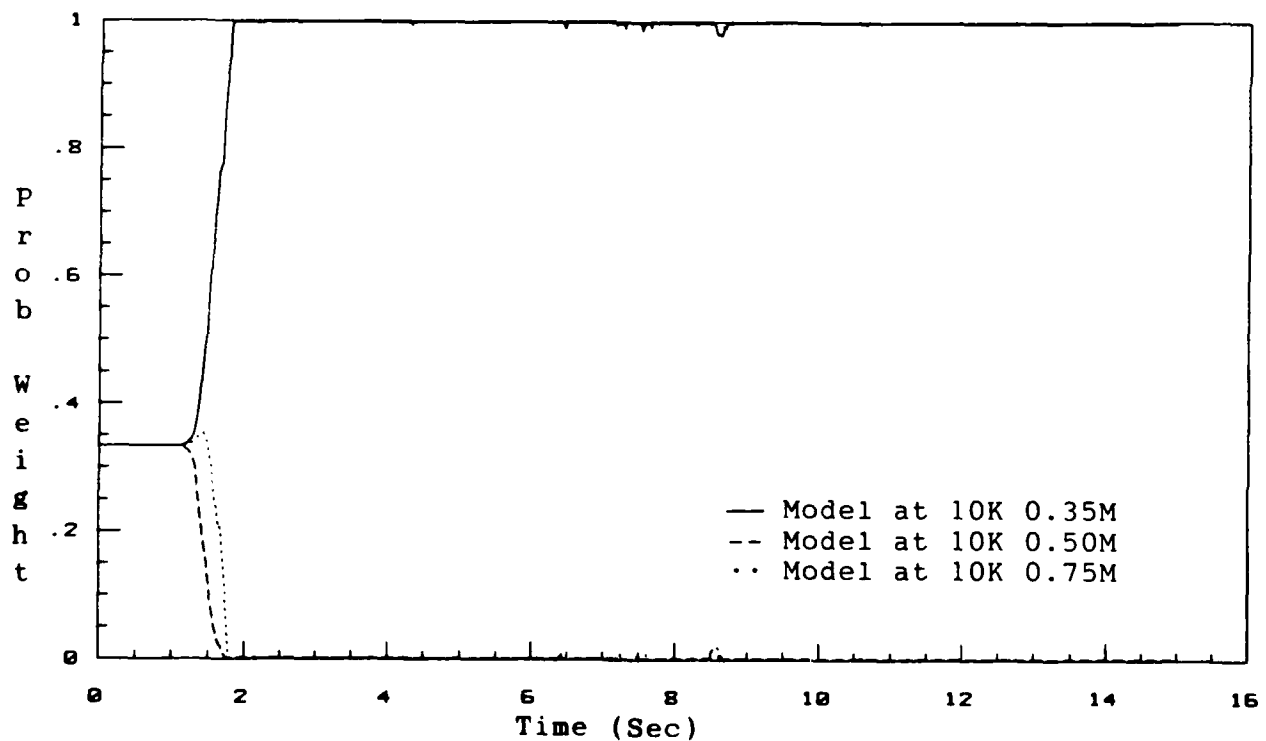


Figure 6-144. Model Probability Weightings
Configuration 8/Sensor Noise/Operating Point: 38K 0.6M

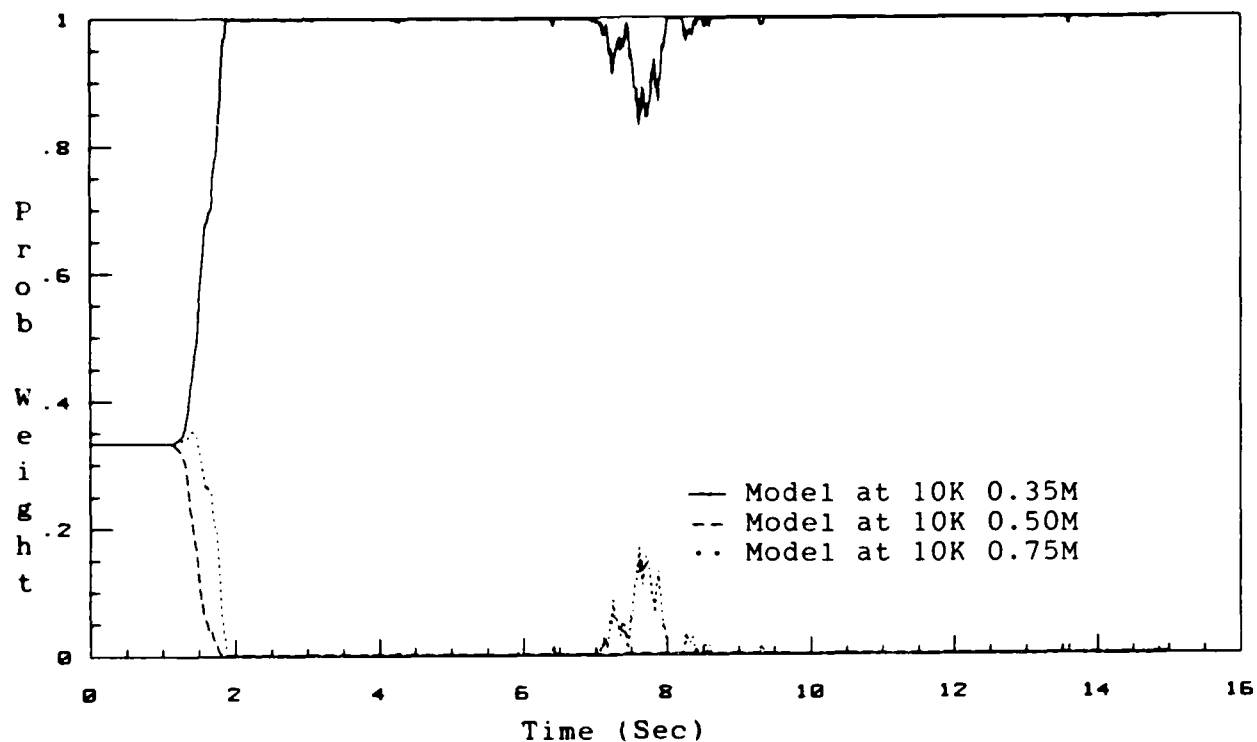


Figure 6-145. Model Probability Weightings
Configuration 8/Sensor Noise (*10)/Operating Point: 38K 0.6M

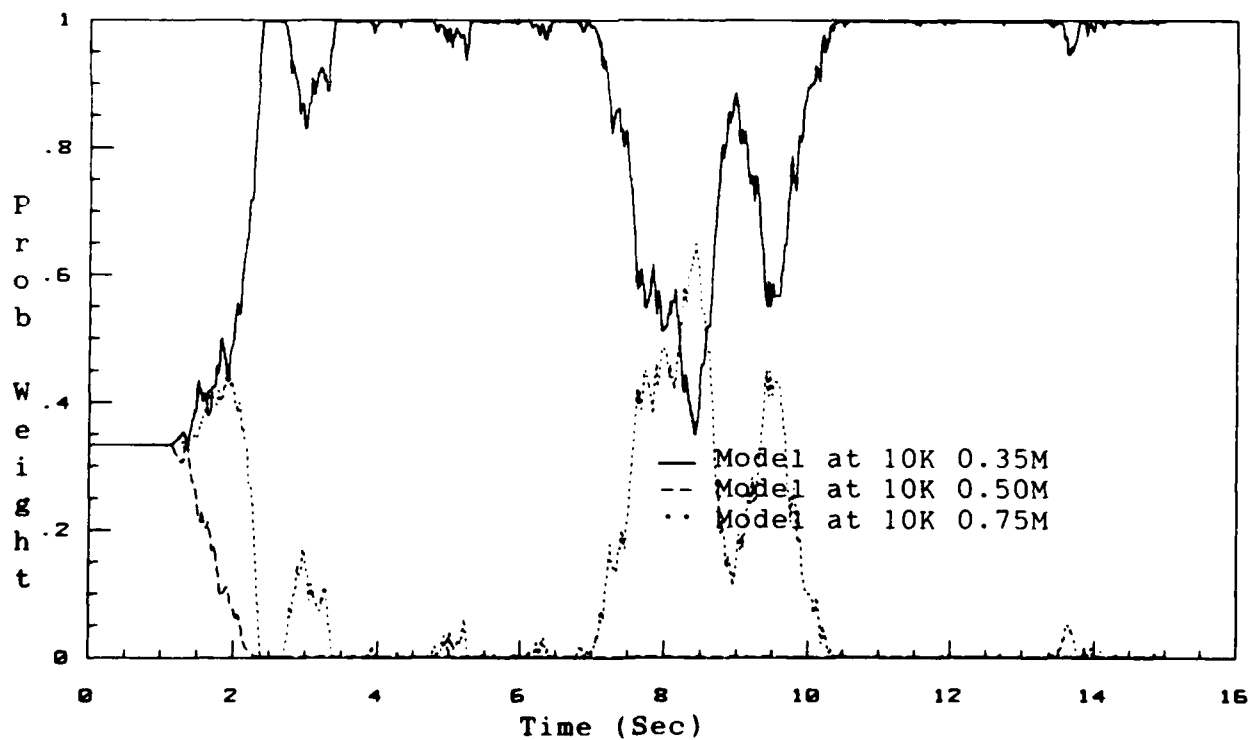


Figure 6-146. Model Probability Weightings
Configuration 8/Sensor Noise(*100)/Operating Point: 38K 0.6M

given in Table 6-15. However, increasing the noise levels beyond those in Table 6-15 resulted in serious performance degradation of the multiple model algorithm.

6.3 Summary

This chapter presents the results that were obtained during the accomplishment of this research effort. Using several nominal models as data points, performance boundaries are generated in an effort to determine regions of robustness for the given models, and observations about the primary factor in determining model equivalence are made. The effects of varying the flight condition on the control surface performance is also evaluated. Next, simulations with the multiple model algorithm are accomplished. At first, two models are placed in the secondary estimator bank, to determine the required amount of overlap of the performance boundaries of the elemental models to achieve the desired tracking performance. Following the evaluation of the two-model configurations, several three-model configurations are evaluated and are found to yield satisfactory tracking performance over the desired flight envelope. In an effort to minimize the amount of time the control

surfaces reached the rate limits, the design parameters of the controller are adjusted. Finally, the effects of sensor noise on the performance of the multiple model algorithm are evaluated. The next chapter of this thesis presents the conclusions drawn from the results detailed in this chapter. Recommendations for further study are also presented.

7. Conclusions and Recommendations

7.1 Introduction

As mentioned several times throughout this thesis, the primary objective of this research is to determine the effects of model placement on the performance of the multiple model algorithm within an adaptive control law. Several hundred simulations were run which are the basis for the results presented in Chapter 6. The following list of conclusions presents the highlights of the results in Chapter 6. The recommendations outline further areas of study in this area of research.

7.2 Conclusions

The following is a list of the highlights of the results obtained in the course of this research effort.

1. Performance criteria must be selected prior to determining the performance robustness of a nominal aircraft model. The performance criteria dictate the degree to which the plant parameters can vary before unacceptable performance is reached and therefore influence the performance boundaries for a given nominal model.

2. In determining the performance boundaries for sev-

eral different nominal models, it has been learned that the quantity model equivalence is most dependent on is dynamic pressure. By taking several different models on approximately the same dynamic pressure line, it is shown that the performance boundaries are the same.

3. Although the performance boundaries for nominal models on the same dynamic pressure line are the same, the control surface responses may be different. It is shown that the reason for this is the different trim values of angle-of-attack. It is also shown that control surface performance is influenced by the aerodynamic gains associated with the actual flight condition. By decreasing altitude and/or increasing Mach number, the aerodynamic gain increases, causing the control surfaces to respond faster. This information is useful when tuning the controller. After selection of the models for the multiple model algorithm, the controller can be tuned at the flight condition with the worst case aerodynamic gains.

4. When trying to achieve satisfactory tracking performance over the flight envelope of interest with two models in the multiple model algorithm's secondary bank, it is seen that the amount of overlap of the performance bound-

aries is very important. Due to the fact that the multiple model adaptive estimator arrives at a parameter estimate by weighting the "correctness" of the models in the bank, the final parameter estimate is a blend of those models' parameters. Because of the weighted blending of parameters of each of the models in the bank, a significant amount of overlap of the performance boundaries is required to yield satisfactory tracking performance. However, it is also shown that the lower bound on the computed probabilities affects the amount of overlap required of the performance boundaries. The amount of overlap increases as the value of the lower bound becomes larger. This performance boundary overlap requirement leads to the use of three models as secondary parameter estimators in lieu of two.

5. Due to the robustness of the controller to variations in the plant parameters, the flight envelope of interest can be modelled with three nominal models in the bank of secondary estimators in the multiple model algorithm. The robustness of the control law allows a large parameter space to be modelled by a relatively small number of discrete models while yielding excellent tracking performance.

6. In the analysis of the three-model configurations,

it is shown that, regardless of the altitudes or Mach numbers of the nominal models in the secondary estimator bank of the multiple model algorithm, the model assigned the highest weighting is the model with a value of dynamic pressure closest to that of the actual flight condition.

7. The limitation on how close the candidate models can be placed is shown to be dependent on the amount of sensor noise added to the longitudinal states. Sensor noise tends to mask the residuals generated for each of the models in the secondary estimator bank and hence degrades the multiple model algorithm's performance. It is shown that with greater separation between dynamic pressure parameterization the susceptibility of the multiple model algorithm to the sensor noise is decreased.

From the above list of conclusions, it can be seen that model placement for the multiple model algorithm is dependent on many factors. First of all, performance criteria must be selected, which in turn determine the region of performance robustness of a given nominal model. The performance criteria also have a large effect on the number of models (for the multiple model algorithm's secondary estimation bank) that are necessary to achieve the desired track-

ing performance over the specified flight envelope. Less demanding performance criteria would result in larger regions of performance robustness and hence fewer models would be required to model the desired parameter space.

The amount of overlap of the performance boundaries is shown to require a tradeoff of conflicting requirements. These conflicting requirements are the result of wanting a large amount of overlap for enhanced tracking performance, yet desiring minimal overlap for sensor noise resistance. The results show that the minimum amount of overlap that is acceptable is that which yields satisfactory tracking performance over the flight envelope of interest. The maximum amount of overlap is dictated by the expected levels of sensor noise. Therefore, once performance criteria have been established and sensor noise figures determined, an analysis as to how far apart in dynamic pressure the secondary estimator models need to be can be accomplished.

7.3 Recommendations for Further Study

This thesis presents an initial evaluation of the factors that are important in the selection of models for the multiple model algorithm. The following list presents areas

of interest that could be investigated to further this effort.

1. The flight envelope of interest should be expanded to include transonic and supersonic flight conditions. The purpose of investigating this expanded flight envelope would be to verify if model equivalence is based primarily on dynamic pressure or on some other criteria. Model selection in this portion of the flight envelope might have different characteristics from those addressed in this thesis.

2. The multiple model algorithm could be tailored so that the gains implemented in the controller would not be a weighted gain matrix of the models in the bank, but would be the set of gains associated with the model with the highest probability. This would seem to have the effect of not requiring much overlap of the performance boundaries and therefore would provide greater sensor noise resistance.

3. Although the general effects of sensor noise on model selection is addressed in this effort, further research in this area needs to be accomplished to include an extensive Monte Carlo analysis.

Appendix A

This appendix presents the A and B matrices of the state space models for each of the flight conditions that are considered in the flight envelope of interest. In addition, the aircraft parameters that are constant for each flight condition are presented. This data was obtained from the Flight Dynamics Laboratory at Wright-Patterson AFB, OH.

Constant Aircraft Parameters

S (wing reference area - ft²) = 300.00

c (wing mean aerodynamic chord - ft) = 11.32

ACGW (aircraft gross weight - lbs) = 21505.00

LOADFAC (load factor - g) = 1.0

b (wing span - ft) = 30.00

Inertias:

Ixx (slug - ft²) = 14224.04

Iyy (slug - ft²) = 61352.40

Izz (slug - ft²) = 72651.75

Flight Condition: 10,000 ft, 0.9 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1640	-0.0154	43.4875	-24.0510	B = -0.9568	19.0164
-0.0008	0.0000	-2.0397	0.9999	-0.2063	-0.3610
0.0003	-0.0005	6.9843	-0.9769	-30.8710	-9.4022

Flight Condition: 10,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1625	-0.0156	39.7461	-24.4587	B = -0.4927	15.6819
-0.0009	0.0000	-1.8507	1.0000	-0.3380	-0.1833
0.0003	-0.0010	7.1505	-0.9116	-26.8660	-7.5573

Flight Condition: 10,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1603	-0.0149	35.8901	-25.1231	B = -0.2750	12.8572
-0.0011	-0.0001	-1.6730	1.0000	-0.1833	-0.3151
0.0003	-0.0010	7.1562	-0.8477	-23.1933	-5.9530

Flight Condition: 10,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1584	-0.0138	33.9936	-25.1510	B = 0.6016	10.4851
-0.0012	-0.0001	-1.5069	1.0000	-0.1662	-0.2979
0.0003	-0.0010	6.8526	-0.7854	-19.8129	-4.6123

Flight Condition: 10,000 ft, 0.7 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1575	-0.0118	30.9741	-24.1602	B = 2.2460	3.8331
-0.0014	-0.0001	-1.3407	1.0000	-0.1547	-0.2636
0.0003	-0.0010	6.5776	-0.7244	-16.7132	-3.1283

Flight Condition: 10,000 ft, 0.65 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1550	-0.0112	30.6762	-24.0485	B = 3.5408	-5.0936
-0.0016	-0.0001	-1.1975	1.0000	-0.1432	-0.2406
0.0003	-0.0010	6.1077	-0.6648	-13.9687	-1.6329

Flight Condition: 10,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1506	-0.0108	32.2231	-24.6667	B =	4.3316	-6.2338
	-0.0019	-0.0001	-1.0657	1.0000		-0.1317	-0.2349
	0.0003	-0.0004	5.4030	-0.6067		-11.6081	-0.9454

Flight Condition: 10,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1429	-0.0099	32.7156	-26.0481	B =	4.4748	-5.0133
	-0.0024	-0.0001	-0.9855	1.0000		-0.1261	-0.2120
	0.0003	0.0002	4.3259	-0.5577		-9.7689	-0.7850

Flight Condition: 10,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1305	-0.0102	32.2804	-28.0069	B =	4.3889	-4.1826
	-0.0031	-0.0001	-0.8995	1.0000		-0.1146	-0.1891
	0.0004	0.0002	3.5523	-0.5124		-8.0844	-0.5730

Flight Condition: 10,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1092	-0.0100	31.8221	-30.7710	B =	3.7471	-3.3174
	-0.0042	-0.0001	-0.8079	1.0000		-0.1089	-0.1662
	0.0004	0.0001	3.0825	-0.4675		-6.5832	-0.3724

Flight Condition: 10,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0708	-0.0102	27.0092	-34.5258	B =	2.6872	-2.1371
	-0.0060	-0.0002	-0.6933	1.0000		-0.0974	-0.1432
	0.0005	0.0001	2.7158	-0.4228		-5.1967	-0.1833

Flight Condition: 10,000 ft, 0.35 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9931	-0.0109	19.7499	-40.0088	B =	1.9309	-3.3690
	-0.0090	-0.0002	-0.5672	1.0000		-0.0859	-0.1031
	0.0006	0.0001	2.0512	-0.3905		-3.9362	0.0344

Flight Condition: 10,000 ft, 0.3 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -31.8063	-0.0127	19.4232	-48.9139	B = 1.6157	-3.3289
-0.0150	-0.0003	-0.4526	1.0000	-0.0745	-0.0630
0.0008	0.0001	1.8793	-0.3862	-2.9794	0.3724

Flight Condition: 10,000 ft, 0.25 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -31.2389	0.0075	11.6253	-65.0974	B = -0.2005	-2.1257
-0.0286	-0.0004	-3.5626	1.0000	-0.0630	-0.0573
0.0003	0.0001	0.6131	-0.4665	-2.0684	0.2349

Flight Condition: 14,000 ft, 0.9 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1621	-0.0129	37.7866	-25.9832	B = -0.9397	16.5356
-0.0009	0.0000	-1.7819	1.0000	-0.1776	-0.3266
0.0003	-0.0005	5.6207	-0.8498	-26.6024	-8.1932

Flight Condition: 14,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1599	-0.0132	34.5150	-26.6752	B = -0.6303	13.6708
-0.0011	-0.0001	-1.6157	1.0000	-0.1662	-0.3037
0.0003	-0.0009	5.8270	-0.7927	-23.1246	-6.5776

Flight Condition: 14,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1569	-0.0126	34.6410	-27.6587	B = -0.5386	11.2529
-0.0012	-0.0001	-1.4610	1.0000	-0.1604	-0.2807
0.0003	-0.0009	5.7353	-0.7369	-19.9447	-5.1738

Flight Condition: 14,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1549	-0.0115	32.4638	-27.4111	B = 6.4171	9.2246
-0.0014	-0.0001	-1.3178	1.0000	-0.1489	-0.2636
0.0003	-0.0009	5.4947	-0.6826	-16.9997	-4.0164

Flight Condition: 14,000 ft, 0.7 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1532	-0.0099	29.7479	-26.7275	B =	1.9881	0.7391
	-0.0016	-0.0001	-1.1745	1.0000		-0.1375	-0.2291
	0.0003	0.0010	5.2196	-0.6294		-14.3182	-2.4981

Flight Condition: 14,000 ft, 0.65 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1490	-0.0102	30.6876	-27.1841	B =	3.2771	-6.7577
	-0.0018	-0.0001	-1.0509	1.0000		-0.1255	-0.2087
	0.0003	-0.0010	4.8210	-0.5777		-11.9849	-1.2483

Flight Condition: 14,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1426	-0.0098	31.8444	-28.1420	B =	3.8231	-4.9670
	-0.0022	-0.0001	-0.9408	1.0000		-0.1164	-0.2058
	0.0003	-0.0004	4.2991	-0.5277		-9.9623	-0.8613

Flight Condition: 14,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1322	-0.0087	31.8936	-29.7446	B =	3.9308	-4.2254
	-0.0028	-0.0001	-0.8662	1.0000		-0.1089	-0.1853
	0.0003	0.0002	3.5602	-0.4883		-8.3808	-0.6284

Flight Condition: 14,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1151	-0.0088	31.5910	-32.1056	B =	3.8867	-3.4343
	-0.0037	-0.0001	-0.7893	1.0000		-0.1007	-0.1649
	0.0004	0.0001	3.0968	-0.4492		-6.9347	-0.4166

Flight Condition: 14,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0859	-0.0085	28.8189	-35.3574	B =	3.2049	-2.1644
	-0.0050	-0.0002	-0.6937	1.0000		-0.0926	-0.1440
	0.0004	0.0001	2.8036	-0.4105		-5.6439	-0.1982

Flight Condition: 14,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0320	-0.0090	22.3943	-39.8968	B =	2.2655	-1.8722
	-0.0071	-0.0002	-0.5802	1.0000		-0.0834	-0.1248
	0.0005	0.0001	2.3182	-0.3763		-4.4398	-0.1450

Flight Condition: 14,000 ft, 0.35 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9175	-0.0104	17.6824	-46.9432	B =	1.9270	-4.4673
	-0.0109	-0.0002	-0.4728	1.0000		-0.0730	-0.0672
	0.0006	0.0001	1.8844	-0.3560		-3.4005	0.2782

Flight Condition: 14,000 ft, 0.3 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.6421	-0.0061	17.8659	-57.9781	B =	0.6524	-2.4686
	-0.0183	-0.0003	-0.3965	1.0000		-0.0632	-0.0569
	0.0007	0.0001	1.3232	-0.3805		-2.5582	0.3444

Flight Condition: 14,000 ft, 0.25 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-30.8164	0.0196	8.2127	-77.3577	B =	-0.9529	-1.4814
	-0.0349	-0.0004	-0.3112	1.0000		-0.0558	-0.0425
	-0.0004	0.0000	0.1568	-0.4605		-1.7615	0.1750

Flight Condition: 18,000 ft, 0.9 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1594	-0.0107	33.1373	-28.3554	B =	-0.9453	14.3090
	-0.0010	0.0000	-1.5498	1.0000		-0.1563	-0.2896
	0.0003	-0.0004	4.4287	-0.7359		-22.8082	-7.1031

Flight Condition: 18,000 ft, 0.85 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1564	-0.0110	33.7813	-29.3747	B =	-0.7707	11.8716
	-0.0012	-0.0001	-1.4085	1.0000		-0.1469	-0.2698
	0.0003	-0.0009	4.6260	-0.6862		-19.8074	-5.7001

Flight Condition: 18,000 ft, 0.8 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1523	-0.0106	33.8536	-30.7268	B =	-0.7800	9.8209
	-0.0014	-0.0001	-1.2760	1.0000		-0.1376	-0.2503
	0.0003	-0.0008	4.5196	-0.6377		-17.0651	-4.4756

Flight Condition: 18,000 ft, 0.75 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1502	-0.0095	31.0658	-30.1219	B =	0.6827	8.0728
	-0.0016	-0.0001	-1.1479	1.0000		-0.1269	-0.2323
	0.0003	-0.0009	4.3139	-0.5906		-14.5146	-3.4819

Flight Condition: 18,000 ft, 0.7 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1471	-0.0085	29.1475	-29.9090	B =	1.8901	-2.0696
	-0.0018	-0.0001	-1.0273	1.0000		-0.1168	-0.1970
	0.0003	-0.0010	4.0470	-0.5446		-12.2165	-1.9520

Flight Condition: 18,000 ft, 0.65 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1408	-0.0093	30.5283	-30.8667	B =	3.0186	-5.9469
	-0.0022	-0.0001	-0.9214	1.0000		-0.1091	-0.1829
	0.0003	-0.0010	3.7710	-0.5006		-10.2335	-1.0895

Flight Condition: 18,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1318	-0.0088	31.2822	-32.1163	B =	3.3522	-4.1756
	-0.0026	-0.0001	-0.8267	1.0000		-0.1011	-0.1787
	0.0003	-0.0004	3.4283	-0.4603		-8.5088	-0.6662

Flight Condition: 18,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1173	-0.0075	31.1300	-34.1163	B =	3.4816	-3.4524
	-0.0033	-0.0001	-0.7600	1.0000		-0.0945	-0.1602
	0.0003	0.0001	2.9861	-0.4264		-7.1575	-0.4477

Flight Condition: 18,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0936	-0.0075	29.8492	-36.9519	B =	-3.4330	-2.4262
	-0.0044	-0.0001	-0.6817	1.0000		-0.0874	-0.1415
	0.0004	0.0001	2.7699	-0.3929		-5.9225	-0.2430

Flight Condition: 18,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0531	-0.0073	24.1220	-40.7828	B =	2.6617	-1.8464
	-0.0059	-0.0002	-0.5797	1.0000		-0.0803	-0.1238
	0.0004	0.0001	2.4959	-0.3597		-4.8122	-0.1299

Flight Condition: 18,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9747	-0.0082	18.7640	-46.5509	B =	1.9813	-3.1842
	-0.0086	-0.0002	-0.4866	1.0000		-0.0720	-0.0895
	0.0005	0.0001	1.8995	-0.3399		-3.7773	0.0107

Flight Condition: 18,000 ft, 0.35 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.8054	-0.0099	19.3550	-55.4223	B =	1.5535	-3.0116
	-0.0133	-0.0002	-0.3987	1.0000		-0.0632	-0.0564
	0.0006	0.0000	1.9253d	-0.3394		-2.9367	0.3274

Flight Condition: 18,000 ft, 0.3 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.3589	0.0030	10.9466	-70.7275	B =	-0.1042	-2.2708
	-0.0230	-0.0003	-0.3144	1.0000		-0.0552	-0.0518
	0.0004	0.0001	0.7266	-0.3968		-2.1851	0.2266

Flight Condition: 22,000 ft, 0.9 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1556	-0.0089	32.7182	-31.2667	B =	-0.9673	12.3245
	-0.0012	0.0000	-1.3438	1.0000		-0.1355	-0.2575
	0.0003	-0.0004	3.4854	-0.6342		-19.4559	-6.1317

Flight Condition: 22,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1516	-0.0092	33.1820	-32.6659	B = -0.9029	10.2662
-0.0014	-0.0001	-1.2226	1.0000	-0.1272	-0.2392
0.0003	-0.0008	3.6189	-0.5912	-16.8790	-4.9160

Flight Condition: 22,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1465	-0.0088	32.6936	-34.0209	B = -0.7153	8.5536
-0.0016	-0.0001	-1.1084	1.0000	-0.1188	-0.2213
0.0003	-0.0008	3.4979	-0.5493	-14.5178	-3.8630

Flight Condition: 22,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1439	-0.0079	29.7815	-33.3828	B = 0.7297	3.9226
-0.0018	-0.0001	-0.9977	1.0000	-0.1091	-0.1963
0.0003	-0.0009	3.2901	-0.5087	-12.3246	-2.7491

Flight Condition: 22,000 ft, 0.7 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1385	-0.0079	29.2537	-33.8576	B = 1.9675	-5.0544
-0.0021	-0.0001	-0.8954	1.0000	-0.1012	-0.1655
0.0003	-0.0009	3.1065	-0.4698	-10.3758	-1.4094

Flight Condition: 22,000 ft, 0.65 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1295	-0.0085	30.1848	-35.1522	B = 2.8210	-5.1979
-0.0025	-0.0001	-0.8049	1.0000	-0.0944	-0.1580
0.0003	-0.0009	2.9456	-0.4336	-8.6946	-0.8203

Flight Condition: 22,000 ft, 0.6 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1167	-0.0077	30.6728	-36.8395	B = 2.9755	-3.3906
-0.0031	-0.0001	-0.7235	1.0000	-0.0875	-0.1537
0.0003	-0.0004	2.7733	-0.4003	-7.2322	-0.4626

Flight Condition: 22,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0964	-0.0064	29.8824	-39.3114	B =	3.0743	-2.6019
	-0.0039	-0.0001	-0.6564	1.0000		-0.0817	-0.1371
	0.0003	0.0001	2.6459	-0.3714		-6.0831	-0.2751

Flight Condition: 22,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0631	-0.0063	25.0218	-42.7143	B =	2.9967	-1.7863
	-0.0052	-0.0001	-0.5651	1.0000		-0.0756	-0.1206
	0.0004	0.0001	2.5765	-0.3428		-5.0334	-0.1229

Flight Condition: 22,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0028	-0.0067	19.2694	-47.7705	B =	2.3106	-1.6663
	-0.0072	-0.0002	-0.4761	1.0000		-0.0692	-0.1055
	0.0004	0.0001	2.0689	-0.3229		-4.0799	-0.1306

Flight Condition: 22,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.8847	-0.0080	16.6361	-55.2176	B =	1.9790	-4.2280
	-0.0105	-0.0002	-0.3987	1.0000		-0.0618	-0.0573
	0.0005	0.0000	1.8429	-0.3112		-3.2305	0.2286

Flight Condition: 22,000 ft, 0.35 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.6324	-0.0055	16.7535	-66.1551	B =	0.6164	-2.4477
	-0.0163	-0.0003	-0.3430	1.0000		-0.0544	-0.0507
	0.0005	0.0001	1.2263	-0.3337		-2.4932	0.2732

Flight Condition: 22,000 ft, 0.3 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-30.9268	0.0149	9.3812	-86.2052	B =	-0.8816	-1.3931
	-0.0287	-0.0003	-0.2812	1.0000		-0.0489	-0.0387
	-0.0001	0.0000	0.2887	-0.4023		-1.8403	0.1853

Flight Condition: 26,000 ft, 0.9 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1504	-0.0074	32.3066	-34.8809	B = -1.0182	10.5448
-0.0014	0.0000	-1.1604	1.0000	-0.1167	-0.2272
0.0003	-0.0004	2.7299	-0.5435	-16.4857	-5.2653

Flight Condition: 26,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1447	-0.0077	32.6038	-36.7208	B = -1.0436	8.8273
-0.0016	-0.0001	-1.0571	1.0000	-0.1095	-0.2104
0.0003	-0.0008	2.7933	-0.5065	-14.2904	-4.2162

Flight Condition: 26,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1395	-0.0074	31.1195	-37.4928	B = -0.3898	6.8315
-0.0018	-0.0001	-0.9584	1.0000	-0.1016	-0.1936
0.0003	-0.0008	2.6541	-0.4705	-12.2624	-3.2875

Flight Condition: 26,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1353	-0.0064	28.4955	-37.2294	B = 0.7821	-0.1848
-0.0021	-0.0001	-0.8636	1.0000	-0.0932	-0.1621
0.0003	-0.0009	2.4161	-0.4362	-10.3980	-2.0645

Flight Condition: 26,000 ft, 0.7 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1264	-0.0071	29.1362	-38.5424	B = 2.0781	-5.9435
-0.0025	-0.0001	-0.7770	1.0000	-0.0872	-0.1388
0.0003	-0.0009	2.3830	-0.4040	-8.7604	-0.9883

Flight Condition: 26,000 ft, 0.65 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1136	-0.0077	29.7819	-40.3133	B = 2.6433	-4.2897
-0.0030	-0.0001	-0.6998	1.0000	-0.0813	-0.1354
0.0003	-0.0009	2.3370	-0.3742	-7.3461	-0.6062

Flight Condition: 26,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0952	-0.0068	29.3384	-42.5014	B =	2.6237	-2.6088
	-0.0037	-0.0001	-0.6217	1.0000		-0.0753	-0.1308
	0.0003	-0.0004	2.7507	-0.3472		-6.1146	-0.2784

Flight Condition: 26,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0663	-0.0054	24.8845	-45.5421	B =	2.6719	-1.6807
	-0.0047	-0.0001	-0.5397	1.0000		-0.0704	-0.1156
	0.0003	0.0001	2.5580	-0.3227		-5.1425	-0.1073

Flight Condition: 26,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0164	-0.0057	19.6408	-50.0936	B =	2.5507	-1.6174
	-0.0063	-0.0001	-0.4608	1.0000		-0.0651	-0.1021
	0.0004	0.0001	2.1749	-0.3043		-4.2555	-0.1150

Flight Condition: 26,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9272	-0.0064	16.5774	-56.4384	B =	2.0763	-3.6976
	-0.0087	-0.0002	-0.3935	1.0000		-0.0594	-0.0654
	0.0004	0.0001	1.8059	-0.2912		-3.4448	0.0732

Flight Condition: 26,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.7515	-0.0072	19.8437	-65.6687	B =	1.1662	-2.6180
	-0.0128	-0.0002	-0.3387	1.0000		-0.0528	-0.0485
	0.0005	0.0001	1.8617	-0.2993		-2.7267	0.2641

Flight Condition: 26,000 ft, 0.35 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.3002	0.0039	8.4819	-82.7277	B =	-0.1514	-2.5498
	-0.0210	-0.0002	-0.2573	1.0000		-0.0474	-0.0417
	0.0003	0.0000	0.6050	-0.3517		-2.1251	0.2159

Flight Condition: 26,000 ft, 0.3 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -30.2511	0.0360	-2.5161	-105.4813	B = -0.7843	-2.4663
-0.0361	-0.0003	-0.1702	1.0000	-0.0449	-0.0252
-0.0005	0.0000	-0.3671	-0.3849	1.5641	0.2375

Flight Condition: 30,000 ft, 0.9 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1428	-0.0061	31.8627	-39.4235	B = -1.1166	8.9296
-0.0016	-0.0001	-0.9970	1.0000	-0.0997	-0.1976
0.0003	-0.0004	2.1557	-0.4624	-13.8388	-4.4916

Flight Condition: 30,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1351	-0.0064	31.9249	-41.5880	B = -1.1622	6.8048
-0.0019	-0.0001	-0.9095	1.0000	-0.0935	-0.1813
0.0003	-0.0007	2.1452	-0.4300	-11.9926	-3.5649

Flight Condition: 30,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1298	-0.0057	29.5791	-41.7366	B = -0.0915	5.2296
-0.0021	-0.0001	-0.8244	1.0000	-0.0862	-0.1654
0.0003	-0.0007	1.9695	-0.4002	-10.2681	-2.7791

Flight Condition: 30,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1221	-0.0056	28.2165	-42.3777	B = 1.1123	-3.6678
-0.0024	-0.0001	-0.7437	1.0000	-0.0797	-0.1288
0.0003	-0.0008	1.8191	-0.3719	-8.7032	-1.4175

Flight Condition: 30,000 ft, 0.7 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1089	-0.0065	29.1151	-44.3135	B = 2.1226	-4.9686
-0.0029	-0.0001	-0.6688	1.0000	-0.0748	-0.1178
0.0003	-0.0008	1.8816	-0.3456	-7.3431	-0.7534

Flight Condition: 30,000 ft, 0.65 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0904	-0.0071	27.4961	-46.6131	B =	2.4433	-3.3079
	-0.0036	-0.0001	-0.5917	1.0000		-0.0697	-0.1142
	0.0003	-0.0009	2.1986	-0.3217		-6.1665	-0.3962

Flight Condition: 30,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0640	-0.0060	23.9335	-49.3741	B =	2.2557	-1.5574
	-0.0045	-0.0001	-0.5044	1.0000		-0.0646	-0.1090
	0.0003	-0.0004	2.5022	-0.3004		-5.1389	-0.0785

Flight Condition: 30,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0197	-0.0049	19.1986	-53.6084	B =	2.2337	-1.5512
	-0.0058	-0.0001	-0.4346	1.0000		-0.0604	-0.0970
	0.0003	0.0001	2.2250	-0.2848		-4.3218	-0.0947

Flight Condition: 30,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9453	-0.0055	16.3750	-59.3404	B =	2.1582	-2.3112
	-0.0077	-0.0001	-0.3781	1.0000		-0.0559	-0.0773
	0.0004	0.0001	1.7976	-0.2731		-3.5762	-0.0735

Flight Condition: 30,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.8042	-0.0063	18.5182	-67.9412	B =	1.3656	-2.8570
	-0.0109	-0.0002	-0.3214	1.0000		-0.0503	-0.0460
	0.0004	0.0000	2.0308	-0.2759		-2.8834	0.2386

Flight Condition: 30,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.5127	-0.0017	12.3981	-80.8201	B =	0.1373	-2.5923
	-0.0163	-0.0002	-0.2733	1.0000		-0.0448	-0.0431
	0.0004	0.0000	0.8712	-0.3038		-2.2822	0.1731

Flight Condition: 30,000 ft, 0.35 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -30.8031	0.0187	8.5235	-102.0108	B = -0.8702	-1.7816
-0.0267	-0.0002	-0.2365	1.0000	-0.0417	-0.0303
-0.0002	0.0000	0.1110	-0.3536	1.7818	0.1746

Flight Condition: 34,000 ft, 0.9 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1324	-0.0049	31.3064	-44.7614	B = -1.1430	6.7633
-0.0019	-0.0001	-0.8530	1.0000	-0.0847	-0.1694
0.0002	-0.0003	1.6794	-0.3892	-11.5448	-3.7862

Flight Condition: 34,000 ft, 0.85 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1226	-0.0049	30.8306	-46.9536	B = -0.9896	5.6691
-0.0022	-0.0001	-0.7789	1.0000	-0.0791	-0.1555
0.0002	-0.0006	1.6010	-0.3626	-9.9930	-3.0202

Flight Condition: 34,000 ft, 0.8 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1161	-0.0043	28.1732	-46.9351	B = 0.1797	0.7674
-0.0025	-0.0001	-0.7061	1.0000	-0.0726	-0.1297
0.0002	-0.0007	1.3805	-0.3383	-8.5408	-2.0317

Flight Condition: 34,000 ft, 0.75 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.1027	-0.0052	27.7174	-48.8071	B = 1.4527	-5.4077
-0.0029	-0.0001	-0.6320	1.0000	-0.0681	-0.1018
0.0002	-0.0008	1.5937	-0.3154	-7.2383	-0.9031

Flight Condition: 34,000 ft, 0.7 Mach

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
A = -32.0835	-0.0060	25.3429	-51.3329	B = 2.1302	-3.6767
-0.0035	-0.0001	-0.5556	1.0000	-0.0638	-0.0986
0.0003	-0.0008	1.9931	-0.2944	-6.1176	-0.4988

Flight Condition: 34,000 ft, 0.65 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0568	-0.0065	22.2663	-54.2504	B =	2.2139	-2.3546
	-0.0043	-0.0001	-0.4779	1.0000		-0.0595	-0.0951
	0.0003	-0.0009	2.2699	-0.2756		-5.1454	-0.2089

Flight Condition: 34,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0148	-0.0057	17.9322	-58.3606	B =	1.8413	-1.4707
	-0.0054	-0.0001	-0.4001	1.0000		-0.0553	-0.0907
	0.0003	-0.0004	2.2327	-0.2647		-4.2934	-0.0763

Flight Condition: 34,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9478	-0.0047	15.7087	-63.7817	B =	1.8574	-1.3914
	-0.0071	-0.0001	-0.3535	1.0000		-0.0517	-0.0810
	0.0003	0.0001	1.8135	-0.2546		-3.6097	-0.1241

Flight Condition: 34,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.8282	-0.0056	16.7474	-71.7205	B =	1.3982	-3.5083
	-0.0096	-0.0001	-0.3003	1.0000		-0.0472	-0.0436
	0.0004	0.0000	2.0240	-0.2551		-2.9656	0.1677

Flight Condition: 34,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.6185	-0.0046	14.3289	-81.8577	B =	0.5153	-2.5631
	-0.0135	-0.0002	-0.2700	1.0000		-0.0424	-0.0408
	0.0004	0.0000	1.0496	-0.2694		-2.3903	0.1522

Flight Condition: 34,000 ft, 0.4 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.0439	0.0105	8.0406	-103.9423	B =	-0.7184	-2.4958
	-0.0216	-0.0002	-0.2205	1.0000		-0.0393	-0.0297
	0.0000	0.0000	0.3937	-0.3236		-1.9481	0.2287

Flight Condition: 38,000 ft, 0.9 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1172	-0.0038	30.8408	-51.7604	B =	-1.1699	5.5244
	-0.0022	-0.0001	-0.7213	1.0000		-0.0709	-0.1430
	0.0002	-0.0003	1.2864	-0.3225		-9.5743	-3.1572

Flight Condition: 38,000 ft, 0.85 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.1070	-0.0037	28.9780	-53.1253	B =	-0.4986	4.7349
	-0.0025	-0.0001	-0.6545	1.0000		-0.0657	-0.1320
	0.0002	-0.0006	1.2741	-0.3015		-8.2649	-2.5621

Flight Condition: 38,000 ft, 0.8 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0951	-0.0037	26.2610	-54.2606	B =	0.6600	-3.3294
	-0.0029	-0.0001	-0.5870	1.0000		-0.0608	-0.0950
	0.0002	-0.0007	1.4028	-0.2820		-7.0516	-1.2971

Flight Condition: 38,000 ft, 0.75 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0744	-0.0048	23.0363	-57.1571	B =	1.6924	-3.8681
	-0.0035	-0.0001	-0.5114	1.0000		-0.0574	-0.0835
	0.0002	-0.0007	1.8436	-0.2638		-5.9839	-0.5967

Flight Condition: 38,000 ft, 0.7 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0459	-0.0057	20.1578	-60.4982	B =	2.0538	-3.0343
	-0.0042	-0.0001	-0.4428	1.0000		-0.0538	-0.0816
	0.0002	-0.0008	2.0441	-0.2484		-5.0675	-0.3653

Flight Condition: 38,000 ft, 0.65 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-32.0028	-0.0065	16.3861	-64.9370	B =	1.9161	-2.1631
	-0.0053	-0.0001	-0.3751	1.0000		-0.0503	-0.0788
	0.0003	-0.0009	1.9789	-0.2394		-4.2700	-0.2208

Flight Condition: 38,000 ft, 0.6 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.9389	-0.0057	14.5232	-70.2645	B =	1.4981	-1.3132
	-0.0067	-0.0001	-0.3194	1.0000		-0.0467	-0.0747
	0.0003	-0.0005	1.8149	-0.2341		-3.5656	-0.1188

Flight Condition: 38,000 ft, 0.55 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.8311	-0.0049	15.5895	-77.8026	B =	1.2700	-3.6987
	-0.0088	-0.0001	-0.2740	1.0000		-0.0433	-0.0398
	0.0003	0.0000	2.0114	-0.2346		-2.9784	0.1275

Flight Condition: 38,000 ft, 0.5 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.6553	-0.0048	14.8821	-87.0302	B =	0.5640	-2.5569
	-0.0119	-0.0001	-0.2523	1.0000		-0.0391	-0.0376
	0.0003	0.0000	1.2438	-0.2439		-2.4339	0.1274

Flight Condition: 38,000 ft, 0.45 Mach

	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000
A =	-31.1779	0.0064	6.1007	-108.6833	B =	-0.4276	-2.9478
	-0.0182	-0.0002	-0.1880	1.0000		-0.0365	-0.0273
	0.0001	0.0000	0.4764	-0.2895		-2.0557	0.2277

Appendix B

This appendix presents the mathematical development of obtaining the Nth order autoregressive vector difference equation that results in a set of parameters representing an aircraft model at a given flight condition. This appendix was taken largely from material presented by Pineiro (1).

The difference equation model used in this thesis for the open-loop longitudinal dynamics of the host airplane is based on the discrete state and output relationships given in Equations (2-8) and (2-9) (Equations (2-8) and (2-9) repeated here for reading continuity):

$$\underline{x}((k + 1)T) = \underline{\Phi} \underline{x}(kT) + \underline{\Psi} \underline{u}(kT) \quad (A-1)$$

$$\underline{y}(kT) = \underline{C} \underline{x}(kT) \quad (A-2)$$

By taking the Z transform of Equations (A-1) and (A-2) and obtaining a transfer function model, the desired input-output relationship can be obtained.

$$\mathcal{Z}\{\underline{x}(k + 1)\} = z\underline{IX}(z) = \underline{\Phi}\underline{X}(z) + \underline{\Psi}\underline{U}(z) \quad (A-3)$$

$$\mathcal{Z}\{\underline{y}(k)\} = \underline{Y}(z) = \underline{C}\underline{X}(z) \quad (A-4)$$

Rearranging equation (A-3) yields

$$z\underline{IX}(z) - \underline{\Phi}\underline{X}(z) = \underline{\Psi}\underline{U}(z) \quad (A-5)$$

Therefore,

$$[z\underline{I} - \underline{\Phi}]\underline{X}(z) = \underline{\Psi}\underline{U}(z) \quad (A-6)$$

$$\underline{X}(z) = [z\underline{I} - \underline{\Phi}]^{-1}\underline{\Psi}\underline{U}(z) \quad (A-7)$$

By substituting Equation (A-2) into Equation (A-7) and per-

forming some matrix manipulation the following expression for $\underline{Y}(z)$ is obtained

$$\underline{Y}(z) = \underline{C}[z\underline{I} - \underline{\Phi}]^{-1} \underline{\Psi} \underline{U}(z) \quad (\text{A-8})$$

$$= \underline{G}(z) \underline{U}(z) \quad (\text{A-9})$$

The $m \times 1$ vector $\underline{Y}(z)$, the $m \times m$ matrix $\underline{G}(z)$, and the $m \times 1$ vector $\underline{U}(z)$ are given by

$$\underline{Y}(z) = \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ \vdots \\ Y_m(z) \end{bmatrix} \quad (\text{A-10})$$

$$\underline{G}(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) & \dots & G_{1m}(z) \\ G_{21}(z) & G_{22}(z) & \dots & G_{2m}(z) \\ \dots & \dots & \dots & \dots \\ G_{m1}(z) & G_{m2}(z) & \dots & G_{mm}(z) \end{bmatrix} \quad (\text{A-11})$$

$$\underline{U}(z) = \begin{bmatrix} U_1(z) \\ U_2(z) \\ \vdots \\ U_m(z) \end{bmatrix} \quad (\text{A-12})$$

The elements of $\underline{G}(z)$, denoted $G_{ij}(z)$, are the transfer functions relating the output Y_i to the control input U_j and are of the form

$$G_{ij}(z) = \frac{b_1 z^w + b_2 z^{w-1} + \dots + b_w z + b_{w+1}}{z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n} \quad (w < n) \quad (A-13)$$

By dividing the numerator and denominator of each transfer function of $\underline{G}(z)$ by z^n , the transfer function matrix is transformed into the delay operator form given by

$$G_{ij}(z) = \frac{b_1 z^{w-n} + b_2 z^{w-n-1} + \dots + b_{w+1} z^{-n}}{1 + a_1 z^{-1} + \dots + a_{n-1} z^{-n+1} + a_n z^{-n}} \quad (w < n) \quad (A-14)$$

By grouping the coefficients with the same amount of delay in every transfer function $G_{ij}(z)$ in $\underline{G}(z)$, and taking the inverse Z transform, the Nth order autoregressive difference equation of the open-loop plant is obtained

$$\begin{aligned} \underline{y}(kT) = & \underline{B}_1 \underline{u}((k-1)T) - \underline{A}_1 \underline{y}((k-1)T) + \dots \\ & + \underline{B}_n \underline{u}((k-N)T) - \underline{A}_n \underline{y}((k-N)T) + \underline{e}(kT) \end{aligned} \quad (A-15)$$

or

$$\underline{y}(kT) = \underline{\Upsilon}^T(kT) \underline{\theta} + \underline{e}(kT) \quad (A-16)$$

where $\underline{e}(kT)$ is an equation error term assumed to be a zero-mean Gaussian white noise vector added to account for modeling errors, $\underline{\Upsilon}^T(kT) \in R^{m \times 1}$ is a matrix of past values of

$\{\underline{y}(kT) \text{ and } \underline{u}(kT)\}$, and the matrices $A_i \in R^{m \times m}$ ($i=1,2,\dots,N$),
 $B_i \in R^{m \times m}$ ($i=1,2,\dots,N$) and the vector $\underline{\theta} \in R^1$ are the parameters of the Nth order difference equation.

Appendix C

This Appendix presents the position limits for the elevators and flaperons for each flight condition. As discussed in Chapter 2, the reason for the different position limits is due to the fact that the trim values for the elevator and flaperon positions differ for each flight condition.

Flight Condition		Elevator Position	Flaperon Position
Alt (Kft)	Mach	Limits	Limits
10	.90	27.3171,-22.6829	22.0000,-21.0000
10	.85	27.1407,-22.8259	22.0000,-21.0000
10	.80	27.0100,-22.9900	22.0000,-21.0000
10	.75	26.9343,-23.0657	21.5916,-21.4084
10	.70	26.9769,-23.0231	20.5543,-22.4457
10	.65	26.9687,-23.0313	20.5019,-22.4981
10	.60	26.9011,-23.0988	18.4067,-24.5933
10	.55	27.0884,-22.9116	17.2362,-25.7638
10	.50	27.2365,-22.7635	15.9609,-27.0390
10	.45	27.3375,-22.6625	14.4776,-28.5224
10	.40	27.3085,-22.6915	12.6417,-30.3583
10	.35	27.1404,-22.8597	10.1102,-32.8898
10	.30	26.1426,-23.8574	6.0729,-36.9271
10	.25	25.1583,-24.8417	5.0000,-38.0000
14	.90	27.2899,-22.7100	22.0000,-21.0000
14	.85	27.1375,-22.8625	22.0000,-21.0000
14	.80	26.9598,-23.0402	22.0000,-21.0000
14	.75	26.9439,-23.0561	21.3222,-21.6778
14	.70	26.9699,-23.0301	20.2337,-22.7664
14	.65	26.9715,-23.0285	19.0859,-23.0141
14	.60	26.8681,-23.1319	17.9156,-25.0844
14	.55	27.0560,-22.9440	16.6666,-26.3333
14	.50	27.2062,-22.7939	15.2683,-27.7317
14	.45	27.2496,-22.7504	13.6179,-29.6321
14	.40	27.1438,-22.8562	11.5134,-31.4867
14	.35	26.7824,-23.2176	8.4601,-34.5399
14	.30	25.3368,-24.6633	5.0000,-38.0000
14	.25	25.2301,-24.7699	5.0000,-38.0000
18	.90	27.2598,-22.7402	22.0000,-21.0000
18	.85	27.0957,-22.9043	22.0000,-21.0000
18	.80	26.9013,-23.0987	22.0000,-21.0000
18	.75	26.9586,-23.0414	20.9971,-22.0029
18	.70	26.9793,-23.0207	19.8341,-23.1659
18	.65	26.9669,-23.0331	18.5947,-24.4053
18	.60	26.8530,-23.1470	17.3436,-25.6564
18	.55	27.0466,-22.9534	15.9830,-27.0170
18	.50	27.1503,-22.8497	14.4367,-28.5633
18	.45	27.1075,-22.6925	12.5850,-30.4150
18	.40	26.9715,-23.0285	10.0985,-32.9015
18	.35	26.1469,-23.8531	6.4130,-36.5870

18	.30	25.0717,-24.9282	5.0000,-38.0000
22	.90	27.2262,-22.7738	22.0000,-21.0000
22	.85	27.0477,-22.9532	22.0000,-21.0000
22	.80	26.8873,-23.1127	21.8248,-21.1753
22	.75	26.9801,-23.0199	20.6028,-22.3972
22	.70	27.0164,-22.9836	19.3353,-23.6647
22	.65	27.0194,-22.9806	18.0135,-24.9865
22	.60	26.8667,-23.1333	16.6537,-26.3463
22	.55	27.0119,-22.9880	15.1578,-27.8422
22	.50	27.0199,-22.9801	13.5678,-29.5678
22	.45	26.9190,-23.0810	11.2427,-31.7573
22	.40	26.5847,-23.4153	8.2382,-33.7618
22	.35	25.3709,-24.6792	5.0000,-38.0000
22	.30	25.1478,-24.8523	5.0000,-38.0000
26	.90	27.1877,-22.8123	22.0000,-21.0000
26	.85	26.9906,-23.0094	22.0000,-21.0000
26	.80	26.9366,-23.0634	21.4198,-21.5802
26	.75	27.0291,-22.9709	20.1317,-22.8683
26	.70	27.1280,-22.8720	18.7323,-24.2677
26	.65	27.0930,-22.9170	17.3037,-25.6963
26	.60	26.8501,-23.1499	15.8139,-27.1861
26	.55	26.8977,-23.1023	14.1526,-28.8474
26	.50	26.8249,-23.1751	12.1327,-30.8673
26	.45	26.7082,-23.2918	9.5537,-33.4464
26	.40	25.7882,-24.2118	5.9549,-37.0451
26	.35	25.0313,-24.9687	5.0000,-38.0000
26	.30	25.1150,-24.8850	5.0000,-38.0000
30	.90	27.1405,-22.8595	22.0000,-21.0000
30	.85	26.9340,-23.0660	22.0000,-21.0000
30	.80	27.0269,-22.9731	20.9211,-22.0789
30	.75	27.1622,-22.8378	19.5016,-23.4984
30	.70	27.2342,-22.7658	17.9817,-25.0183
30	.65	27.1207,-22.8793	16.4251,-26.5749
30	.60	26.7404,-23.2596	14.7789,-28.2211
30	.55	26.6819,-23.3181	12.8383,-30.1617
30	.50	26.6237,-23.3767	10.4812,-32.5189
30	.45	26.0799,-23.9201	7.2889,-35.7111
30	.40	25.1127,-24.8873	5.0000,-38.0000
30	.35	24.9556,-25.0444	5.0000,-38.0000
34	.90	27.0970,-22.9030	22.0000,-21.0000
34	.85	26.9523,-23.0477	21.7643,-21.2357
34	.80	27.1475,-22.8525	20.3042,-22.6952
34	.75	27.3222,-22.6778	18.7056,-24.2944
34	.70	27.3322,-22.6678	17.0547,-25.9453

34	.65	27.0784,-22.9216	15.3424,-27.6576
34	.60	26.5003,-23.4997	13.4120,-29.5880
34	.55	26.4581,-23.5419	11.1563,-31.8437
34	.50	26.1650,-23.8349	8.2467,-34.7533
34	.45	25.2234,-24.7766	5.0000,-38.0000
34	.40	24.8747,-25.1254	5.0000,-38.0000
38	.90	27.0418,-22.9582	22.0000,-21.0000
38	.85	27.1003,-22.8997	21.1088,-21.8912
38	.80	27.3384,-22.6616	19.4861,-23.5136
38	.75	27.4664,-22.5336	17.7164,-25.2836
38	.70	27.3436,-22.6564	15.8939,-27.1061
38	.65	26.9336,-23.0665	13.8919,-29.1081
38	.60	26.2547,-23.7453	11.6644,-31.3356
38	.55	26.0784,-23.9316	8.9181,-34.0819
38	.50	25.2212,-24.7788	5.5550,-37.4450
38	.45	24.9137,-25.0863	5.0000,-38.0000

Appendix D

This Appendix presents the parameter vectors for the flight conditions specifically addressed in this thesis. Recall that the parameter vector is a set of parameters associated with a given flight condition that are used in the autoregressive difference equation form of the aircraft model.

Flight Condition: 10,000 ft, 0.35 Mach

$A_1 = -3.990568$	$\underline{B}_1 = 0.000842$	0.001033
	-0.039268	0.000310
$A_2 = 5.971522$	$\underline{B}_2 = -0.002524$	-0.003096
	0.117563	-0.000950
$A_3 = -3.971342$	$\underline{B}_3 = 0.002520$	0.003092
	-0.117320	0.000969
$A_4 = 0.990387$	$\underline{B}_4 = -0.000838$	-0.001029
	0.039025	-0.000329

Flight Condition: 10,000 ft, 0.40 Mach

$A_1 = -3.989059$	$\underline{B}_1 = 0.000957$	0.001442
	-0.051883	-0.001841
$A_2 = 5.966940$	$\underline{B}_2 = -0.002871$	-0.004321
	0.155263	0.005471
$A_3 = -3.966701$	$\underline{B}_3 = 0.002866$	0.004315
	-0.154875	-0.005419
$A_4 = 0.988821$	$\underline{B}_4 = -0.000952$	-0.001436
	0.051495	0.001789

Flight Condition: 10,000 ft, 0.50 Mach

$A_1 = -3.986201$	$\underline{B}_1 = 0.001149$	0.001891
	-0.080647	-0.005754
$A_2 = 5.958298$	$\underline{B}_2 = -0.003451$	-0.005664
	0.241174	0.017146
$A_3 = -3.957994$	$\underline{B}_3 = 0.003444$	0.005655
	-0.240404	-0.017027
$A_4 = 0.985896$	$\underline{B}_4 = -0.001143$	-0.001881
	0.079878	0.005636

Flight Condition: 10,000 ft, 0.65 Mach

$A_1 = -3.981994$	$\underline{B}_1 = 0.001427$	0.002386
	-0.139296	-0.016365
$A_2 = 5.945443$	$\underline{B}_2 = -0.004298$	-0.007146
	0.416132	0.048754
$A_3 = -3.944927$	$\underline{B}_3 = 0.004279$	0.007130
	-0.414375	-0.048414
$A_4 = 0.981466$	$\underline{B}_4 = -0.001417$	-0.002370
	0.137539	0.016024

Flight Condition: 10,000 ft, 0.75 Mach

$A_1 = -3.977787$	$\underline{B}_1 = 0.001672$	0.002931
	-0.197434	-0.046029
$A_2 = 5.932799$	$\underline{B}_2 = -0.005032$	-0.008782
	0.589214	0.137196
$A_3 = -3.932239$	$\underline{B}_3 = 0.005019$	0.008757
	-0.586125	-0.136303
$A_4 = 0.977226$	$\underline{B}_4 = -0.001658$	-0.002908
	0.194345	0.045136

Flight Condition: 10,000 ft, 0.80 Mach

$A_1 = -3.975528$	$\underline{B}_1 = 0.001793$	0.003139
	-0.231040	-0.059402
$A_2 = 5.926021$	$\underline{B}_2 = -0.005404$	-0.009403
	0.689126	0.176989
$A_3 = -3.925456$	$\underline{B}_3 = 0.005388$	0.009376
	-0.685132	-0.175772
$A_4 = 0.974963$	$\underline{B}_4 = -0.001778$	-0.003112
	0.227046	0.058185

Flight Condition: 10,000 ft, 0.90 Mach

$A_1 = -3.970230$	$\underline{B}_1 = 0.002019$	0.003569
	-0.307304	-0.093715
$A_2 = 5.911380$	$\underline{B}_2 = -0.006103$	-0.010695
	0.915522	0.278993
$A_3 = -3.910892$	$\underline{B}_3 = 0.006083$	0.010660
	-0.909132	-0.276841
$A_4 = 0.970134$	$\underline{B}_4 = -0.002000$	-0.003534
	0.300914	0.091562

Flight Condition: 14,000 ft, 0.80 Mach

$A_1 = -3.978575$	$\underline{B}_1 = 0.001567$	0.002800
	-0.198759	-0.051639
$A_2 = 5.935265$	$\underline{B}_2 = -0.004720$	-0.008388
	0.593277	0.154001
$A_3 = -3.934805$	$\underline{B}_3 = 0.004709$	0.008367
	-0.590276	-0.153085
$A_4 = 0.978115$	$\underline{B}_4 = -0.001555$	-0.002779
	0.195758	0.050722

Flight Condition: 18,000 ft, 0.45 Mach

$A_1 = -3.990800$	$\underline{B}_1 = 0.000800$	0.001234
	-0.048047	-0.001312
$A_2 = 5.972188$	$\underline{B}_2 = -0.002400$	-0.003699
	0.143841	0.003898
$A_3 = -3.971961$	$\underline{B}_3 = 0.002397$	0.003694
	-0.143541	-0.003859
$A_4 = 0.905786$	$\underline{B}_4 = -0.000797$	-0.001229
	0.047746	0.001274

Flight Condition: 18,000 ft, 0.80 Mach

$A_1 = -3.981308$	$\underline{B}_1 = 0.001363$	0.002486
	-0.170151	-0.044673
$A_2 = 5.943556$	$\underline{B}_2 = -0.004105$	-0.007449
	0.508218	0.133337
$A_3 = -3.943189$	$\underline{B}_3 = 0.004096$	0.007433
	-0.505982	-0.132653
$A_4 = 0.980940$	$\underline{B}_4 = -0.001355$	-0.002470
	0.167915	0.043990

Flight Condition: 22,000 ft, 0.45 Mach

$A_1 = -3.992165$	$\underline{B}_1 = 0.000690$	0.001052
	-0.040741	-0.001314
$A_2 = 5.976305$	$\underline{B}_2 = -0.002071$	-0.003153
	0.122015	0.003916
$A_3 = -3.976116$	$\underline{B}_3 = 0.002068$	0.003149
	-0.121805	-0.003888
$A_4 = 0.991975$	$\underline{B}_4 = -0.000688$	-0.001048
	0.040531	0.001286

Flight Condition: 22,000 ft, 0.65 Mach

$A_1 = -3.987867$	$\underline{B}_1 = 0.000938$	0.001573
	-0.086776	-0.008208
$A_2 = 5.963342$	$\underline{B}_2 = -0.002819$	-0.004714
	0.259598	0.024513
$A_3 = -3.963082$	$\underline{B}_3 = 0.002815$	0.004706
	-0.258867	-0.024401
$A_4 = 0.987607$	$\underline{B}_4 = -0.000934$	-0.001566
	0.086045	0.008096

Flight Condition: 22,000 ft, 0.90 Mach

$A_1 = -3.992314$	$\underline{B}_1 = 0.001351$	0.002570
	-0.194202	-0.061218
$A_2 = 5.976828$	$\underline{B}_2 = -0.004055$	-0.007702
	0.581808	0.183375
$A_3 = -3.976713$	$\underline{B}_3 = 0.004050$	0.007692
	-0.581010	-0.183095
$A_4 = 0.992199$	$\underline{B}_4 = -0.001345$	-0.002559
	0.193404	0.060938

Flight Condition: 26,000 ft, 0.70 Mach

$A_1 = -3.988397$	$\underline{B}_1 = 0.000867$	0.001382
	-0.087440	-0.009880
$A_2 = 5.964985$	$\underline{B}_2 = -0.002607$	-0.004142
	0.261618	0.029530
$A_3 = -3.964777$	$\underline{B}_3 = 0.002603$	0.004137
	-0.260914	-0.029421
$A_4 = 0.988189$	$\underline{B}_4 = -0.000864$	-0.001376
	0.086737	0.009770

Flight Condition: 30,000 ft, 0.50 Mach

$A_1 = -3.993622$	$\underline{B}_1 = 0.000557$	0.000771
	-0.035718	-0.000740
$A_2 = 5.980699$	$\underline{B}_2 = -0.001673$	-0.002313
	0.107009	0.002205
$A_3 = -3.980531$	$\underline{B}_3 = 0.001671$	0.002311
	-0.106863	-0.002189
$A_4 = 0.993454$	$\underline{B}_4 = -0.000556$	-0.000769
	0.035572	0.000724

Flight Condition: 34,000 ft, 0.60 Mach

$A_1 = -3.988733$	$B_1 = 0.000786$	0.001548
	-0.099757	-0.030160
$A_2 = 5.966067$	$B_2 = -0.002364$	-0.004641
	0.298482	0.090220
$A_3 = -3.965934$	$B_3 = 0.002362$	0.004636
	-0.297690	-0.089960
$A_4 = 0.988600$	$B_4 = -0.000783$	-0.001542
	0.098966	0.029899

Flight Condition: 34,000 ft, 0.85 Mach

$A_1 = -3.993531$	$B_1 = 0.000551$	0.000905
	-0.042884	-0.000771
$A_2 = 5.980380$	$B_2 = -0.001654$	-0.002714
	0.128468	0.002292
$A_3 = -3.980166$	$B_3 = 0.001652$	0.002711
	-0.128282	-0.002268
$A_4 = 0.993317$	$B_4 = -0.000549$	-0.000903
	0.042698	0.000748

Flight Condition: 38,000 ft, 0.60 Mach

$A_1 = -3.994599$	$B_1 = 0.000466$	0.000745
	-0.035619	-0.001193
$A_2 = 5.983623$	$B_2 = -0.001398$	-0.002236
	0.106733	0.003562
$A_3 = -3.983447$	$B_3 = 0.001397$	0.002234
	-0.106609	-0.003545
$A_4 = 0.994423$	$B_4 = -0.000465$	-0.000744
	0.035495	0.001176

Flight Condition: 38,000 ft, 0.65 Mach

$A_1 = -3.994002$	$\underline{B}_1 = 0.000501$	0.000786
	-0.042655	-0.002213
$A_2 = 5.981814$	$\underline{B}_2 = -0.001504$	-0.002357
	0.127793	0.006616
$A_3 = -3.981620$	$\underline{B}_3 = 0.001503$	0.002355
	-0.127620	-0.006592
$A_4 = 0.993808$	$\underline{B}_4 = -0.000500$	-0.000784
	0.042482	0.002189

Flight Condition: 38,000 ft, 0.70 Mach

$A_1 = -3.993253$	$\underline{B}_1 = 0.000536$	0.000814
	-0.050619	-0.003657
$A_2 = 5.979562$	$\underline{B}_2 = -0.001611$	-0.002441
	0.151622	0.010938
$A_3 = -3.979365$	$\underline{B}_3 = 0.001609$	0.002439
	-0.151384	-0.010905
$A_4 = 0.993056$	$\underline{B}_4 = -0.000535$	-0.000812
	0.050382	0.003624

Flight Condition: 38,000 ft, 0.80 Mach

$A_1 = -3.991436$	$\underline{B}_1 = 0.000605$	0.000946
	-0.070422	-0.012959
$A_2 = 5.974184$	$\underline{B}_2 = -0.001818$	-0.002837
	0.210843	0.038788
$A_3 = -3.974057$	$\underline{B}_3 = 0.001816$	0.002835
	-0.210419	-0.038698
$A_4 = 0.991310$	$\underline{B}_4 = -0.000603$	-0.000943
	0.069998	0.012869

Flight Condition: 38,000 ft, 0.90 Mach

$$A_1 = -3.989684$$

$$\underline{B}_1 = \begin{array}{l} 0.000705 \\ -0.095594 \end{array}$$

$$\begin{array}{l} 0.001424 \\ -0.031531 \end{array}$$

$$A_2 = 5.968947$$

$$\underline{B}_2 = \begin{array}{l} -0.002120 \\ 0.286084 \end{array}$$

$$\begin{array}{l} -0.004271 \\ 0.094348 \end{array}$$

$$A_3 = -3.968841$$

$$\underline{B}_3 = \begin{array}{l} 0.002118 \\ -0.285385 \end{array}$$

$$\begin{array}{l} 0.004267 \\ -0.094101 \end{array}$$

$$A_4 = 0.989578$$

$$\underline{B}_4 = \begin{array}{l} -0.000703 \\ 0.094895 \end{array}$$

$$\begin{array}{l} -0.001420 \\ 0.031285 \end{array}$$

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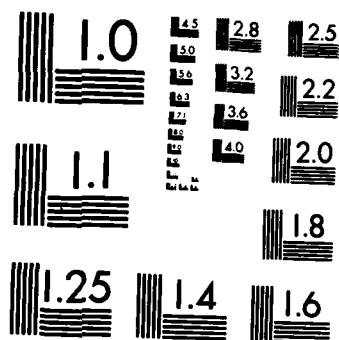
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This thesis extends the research accomplished by Capt Pineiro and Lt Berens in the area of adaptive algorithm implementation. Specifically, this thesis explores the performance characteristics of the multiple model estimation algorithm and how they influence the selection of aircraft models to allow the parameter adaptive control system to maintain tracking performance over a desired portion of the flight envelope. The aircraft dynamic equations used are those of the AFTI/F-16 and the control law design is based on the method developed by Professor Porter.

Numerous fixed gain simulations are presented in order to determine the performance robustness to plant variations of selected flight conditions. From these simulations, performance or robustness boundaries for each nominal are determined. Data is then generated to ascertain the effect on the control surfaces' responses of replacing the nominal model of a performance boundary with another model within the same boundary. That leads to the determination of the critical factors to be considered in choosing an aircraft model to represent a specific set of performance boundaries.

Following the determination as to what factors are important in establishing performance bounds for a fixed gain system, model selection for the multiple model algorithm is evaluated. In attempting to select a set of aircraft models that would ensure an acceptable level of tracking performance over the desired flight envelope several two-, three-, and four-model configurations are examined. Each configuration is evaluated to determine the amount of overlap of performance bounds required for proper tracking performance, the amount of the flight envelope that is covered, and control surface performance.

After selecting a set of aircraft models that results in the best overall system response, the effect of adjusting the control law gains on the performance of the multiple model estimation algorithm is evaluated. By assuming that all states are accessible, sensor noise is then added to each of the longitudinal states to study how noise impacts model selection. A set of models that produces acceptable tracking performance over the desired flight envelope and the most immunity to sensor noise is then selected.

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