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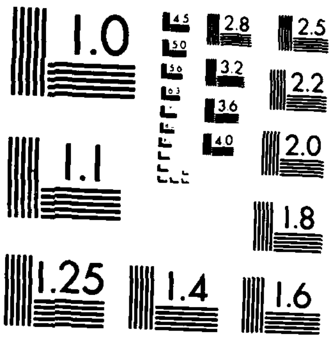

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AFIT/GE/ENG/87D-69

MULTIVARIABLE CONTROL LAW DESIGN FOR THE AFTI/F-16 WITH A FAILED CONTROL SURFACE USING A PARAMETER-ADAPTIVE CONTROLLER

THESIS

Julio E. Velez
Captain, USAF

AFIT/GE/ENG/87D-69

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MULTIVARIABLE CONTROL LAW DESIGN FOR THE AFTI/F-16 WITH A FAILED CONTROL SURFACE USING A PARAMETER-ADAPTIVE CONTROLLER

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

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Julio E. Velez, B.E.

Captain, USAF

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Julio E. Velez

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A	Continuous-time plant matrix
A^{-1}	Inverse of A
α	Angle of attack
α_T	Trim angle of attack
B	Continuous-time input matrix
b	Wing span
β	Sideslip angle
C	Output matrix
c	Mean aerodynamic cord
CG	Center of gravity
Deg	Degrees
δc	Canard deflections
δDF	Differential flaperon deflection
δDT	Differential horizpntal tail deflection
δe_l	Left horizontal tail deflection
δe_r	Right horizontal tail deflecton
δf_l	Left flaperon deflection
δf_r	Right flaperon deflection
δr	Rudder deflection
δT	Change in thrust
$\underline{e}$	Error
$\underline{\eta}$	Zero mean Gaussian white noise vector
ft	Feet
G_{ij}	Transfer function for output i, and input j
g	Gravity constant - 32.2 ft/sec
γ	Flight path angle
$K1$	Proportional control law feedback gain matrix
$K2$	Integral control law feedback gain matrix
Lat	Lateral
lbs	Pounds
Long	Longitudinal
l	Number of outputs
Π	weighting matrix (Pi)

ϕ	Roll angle
φ	Measurement matrix
$\dot{\phi}$	Roll rate
ρ	Ratio of proportional to integral feedback
q	Dynamic pressure
$\underline{r}$	Input vector
r	Yaw rate
$\dot{r}$	Yaw rate
S	Surface area
s	Laplace operator
sec	Seconds
sin	Sine
Σ	weighting matrix (sigma)
σ	Elements of Σ
T	Sampling period
Θ	Parameter vector
θ	Pitch angle
$\dot{\theta}$	Pitch rate
U	Veolocity along x-axis
$\underline{u}$	Input vector
V_T	Trimmed forward Velocity
W	Aircraft weight
$\underline{x}$	State Vector
$\underline{y}$	Output vector
$\underline{\varsigma}$	Digital integral of the error

Abstract

Multivarible control laws are designed for the Advanced Fighter Technology Integration F-16 (AFTI/F-16). Digital Proportional plus Integral (PI) output feedback controllers are designed using a method developed by Professor Brian Porter of the University of Salford, England and the computer aided design program MATRIX_X.

The AFTI/F-16 control inputs used for this design are individually controlled flaperons, individually controlled elevators, and engine thrust. The aircraft dynamics are linearized about one flight condition and are presented in both a state space form and as an autoregressive difference equation.

Both fixed gain and adaptive PI controllers are designed for a plant were the number of outputs are not equal to the number of inputs (rectangular plant). Simulations are conducted for a healthy and a failed aircraft model. The failure consists of reducing the left elevator effectiveness by 50%. When the fixed gain controller is used for the flight control system, the simulation reveals the fact that the aircraft failure causes the output responses to diverge. If provided with a "persistently exciting" input, the adaptive controller prevents the aircraft failure simulations from diverging and going unstable. However, additional testing and/or tuning of the adaptive controller is required to determine and enhance the stability of the adaptive controller.

MULTIVARIABLE CONTROL LAW DESIGN FOR THE AFTI/F-16 WITH A FAILED CONTROL SURFACE USING A PARAMETER-ADAPTIVE CONTROLLER

I. Introduction

1.1 Background

Adaptive controls have the capability of adapting to a changing environment through an internal process of measurement, evaluation, and adjustment. This gives the adaptive system the ability to determine the optimal gain settings of flight control systems at various flight conditions without requiring extensive ground or flight testing (1:199).

A parameter-adaptive control system, using recursive identification as the adaptation mechanism, was demonstrated by Capt L.A. Pineiro (2). His demonstration investigated the tracking performance effectiveness of parameter-adaptive control laws in a model-following mode. The parameter-adaptive capabilities were evaluated using linearized longitudinal equations of motion of the Advanced Fighter Technolgy Integration F-16 (AFTI/F-16) at a flight condition of 0.9 Mach at 20,000ft mean sea level (MSL)(4). Use of this adaptive system permitted tracking fidelity to be maintained despite parameter deviations. A square plant was used for this design; that is, a plant where the number of inputs equal the number of outputs. One of Capt Pineiro's recommendations was that reconfiguration strategies be investigated.

The ability of an aircraft to reconfigure its flight control laws and maintain stability after failure, or degradation, of a control surface is an important topic for the Air Force (3). Advanced aircraft, such as the AFTI/F-16, have enhanced combat effectiveness due to the use of Control Configured Vehicle (CCV) capabilities. This capability allows the AFTI/F-16 to perform unconventional maneuvers, such as translation and fuselage pointing (4). This is possible because the aircraft is equipped with additional control surfaces, such as canards, and independent operation of the left and right ailerons and elevators. This feature enables the surfaces which were previously used only to produce longitudinal movements to also produce lateral movement, and vice versa. These control surfaces can also be used to compensate for control surface failure, or damage, by applying forces and moments required to maintain stability and control (4).

1.2 Problem

The purpose of this thesis is first to design both a fixed gain and a parameter-adaptive controller for a plant where the number of inputs does not equal the number of outputs (rectangular plant). The second purpose is to compare the stabilitly and time response of these controllers with respect to reconfiguration control.

1.3 Summary of Current Knowledge

This section presents background information on Porter's modern control law design technique, adaptive systems, and reconfigurable aircraft flight controls.

1.3.1 Porter Method. Control laws can be categorized as conventional or modern. Conventional control law methodology was developed up to and including the late 1950s, and the control law methodology developed since then is referred to as modern control laws (5:12). D'Azzo and Houpis (5:525) state that conventional control laws can be effectively used for single-input single-output systems, but are difficult to use for multiple-input multiple output (MIMO) systems. Because of this difficulty, modern control laws are used for some MIMO systems (5:12). Two major categories of modern control laws are optimal control and eigenstructure assignment. Under eigenstructure assignment there are techniques of state feedback and output feedback. The remainder of this section reviews an output feedback multivariable design technique developed by Professor Brian Porter of the University of Salford, England (6).

The Porter technique uses output feedback with a high-gain error actuated controller. Output feedback is desirable over state feedback since it is not usually possible to sense, or reconstruct, the required states for a state feedback system (7). High-gain forces the closed-loop response of the system to approach an asymptotic structure with two distinct modes. These are referred to as fast and slow modes by Porter and Manganas (8). For a system in which the first Markov parameter (explained in Section 3.3) has full rank, the slow mode is characterized as uncontrollable and unobservable; therefore, the response of the closed-loop system is dominated by the fast modes. As implied by its name, the fast mode gives the system a fast tracking capability as the gain increases. The main objective of the design is to determine the controller gain values that produce satisfactory responses and simultaneously decouples the system. These gains are normally fixed and are valid only for the plant conditions originally designed, plus some small plant deviation. However, if there are large plant parameter changes, such as surface failures, then new gain values must be inserted into the controller. Jones and Porter (9) state that this reinsertion is required due to the degradation in the closed-loop response. An alternate scheme is to use an adaptive controller that automatically changes these values as the plant parameters change.

1.3.2 Adaptive Controls. Research into adaptive control was motivated in the 1950s by high performance aircraft requirements (10:471). Astrom stated that the constant gain linear systems which work satisfactorily in one operating condition may not work well in a different operating condition (10:471). In fact, according to Blakelock, the aircraft transfer function varies as the airspeed and altitude change(1:199). Due to this constant gain limitation, a system that works over a wider range of operating points was required. This requirement led to the design of adaptive flight controls. There are several types of adaptive controllers (regulators). Three of the simplest controllers are gain scheduling, model reference, and self-tuning controllers (10:471). The remainder of this section briefly discusses gain scheduling and model reference controllers. Self-tuning regulators (STR) are then discussed in greater detail.

In gain scheduling, a variable which gives an indication of the change in the plant parameters is used to modify the system gains. In flight control systems two variables are often used to schedule the gains, Mach number and dynamic pressure. A drawback to gain scheduling is that it is an open-loop solution and may require fine tuning during both ground and flight testing (10:472; 1:199). The ability to quickly change the gains of the system is an advantage of gain scheduling.

Model reference controllers use a reference model plant and compare the r sponse of the plant to the reference model. Any difference between the reference model and the plant are nulled out; therefore, the controller attempts to force the process to behave like the reference model. Isermann state that the problem with this scheme is that the reference model may not be the optimal model at the given time (11:514). Quick adaptation and the ability to use nonlinear stability theories are advantages of model reference controllers (11:514).

The self-tuning regulator. shown in Figure 1-1, consists of an inner loop between the process and the regulator and an outer loop which controls the regulator parameters. Also, the self-tuning regulator contains a recursive parameter estimator. Many parameter estimation schemes have been used, including: recursive least squares (RLS), recursive maximum likelihood, stochastic approximation, and others (11:515). An important requirement when using these schemes is for the input signal to the plant to exhibit a "persistently exciting" characteristic in order to prevent poor parameter estimations (12:598).

Pineiro (2) uses a modified RLS algorithm developed by Hagglund (13). This algorithm discounts old data based on the information obtained from the new data. This allows updates of the parameter estimates only as new information is obtained (13).

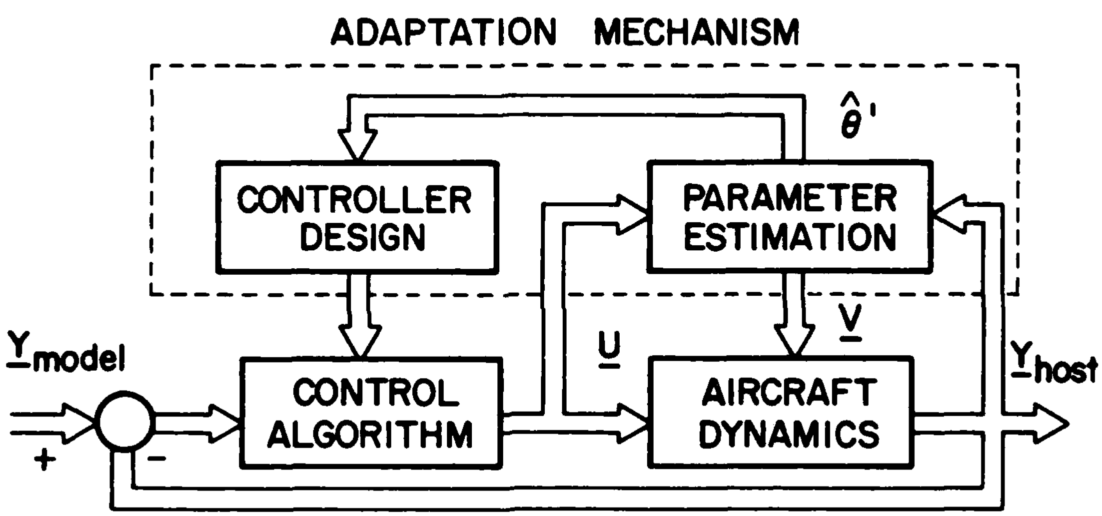


Fig. 1-1 Self-Tuning Regulator
Source 2

1.3.3 Reconfigurable Flight Controls. The design of fixed gain reconfigurable flight control laws has been demonstrated for the AFTI/F-16. Specifically, in his masters thesis (4), Eslinger designed controllers for both a model of a healthy AFTI/F-16 with all the control surfaces operational and for a model of an AFTI/F-16 with a failed right horizontal tail. Porter and Frigg also demonstrated the effective use of an adaptive controller with PI control laws in a reconfigurable scheme (14). They demonstrated the successful application of an adaptive system in a multivariable plant that had an actuator failure.

1.4 Assumptions

For this thesis design, the linearized equations of motion for the AFTI/F-16 are used. The following basic assumptions are made (1; 4; 15):

1) The aircraft has constant mass and is a rigid body.

2) The earth's surface is an inertial frame of reference.

3) The atmosphere is assumed fixed with respect to the earth.

4) Aerodynamics are constant for a given Mach number and altitude.

5) Linearization about a nominal point is acceptable.

1.5 Limitations

In order to limit the scope of the thesis to a workable level, the effects of noise, delays, or sensor dynamics were not addressed. Also, due to problems encountered during this research, it is not possible to add the actuator dynamics to this simulation. This problem is discussed in Chapter 4, Section 4.4.2.

1.6 Approach

To accomplish the purpose of this thesis, the objectives listed below were accomplished.

OBJECTIVE 1. Design a Multiple-Input Multiple-Output (MIMO) fixed gain control law for the AFTI/F-16 using MATRIX_X (24) for implementation and simulation.

The control laws for a healthy aircraft are first designed using Porter's high-gain error-actuated controller method and the computer-aided control design program called MATRIX_X. A linear model of the AFTI/F-16 is used. The aircraft control surfaces consist of two individually controlled flaperons, and two individually controlled horizontal tails. In addition, engine thrust is used as a control input. The controlled output variables are flight-path angle, sideslip angle, and yaw rate. A standard day flight condition of 0.9 Mach at 20000 feet MSL is used. The design for a healthy aircraft is then evaluated with a failed control surface model. The failed model consisted of a failed left horizontal tail. This failure is a partial failure, where the effectiveness of the control surface is reduced to fifty percent. Originally, additional surface failures and total surface failures were planned. However, since the adaptive controller (see Objective 2) did not work properly, the addition failure data was not required.

OBJECTIVE 2. Design a MIMO parameter-adaptive controller for the AFIT F-16 on MATRIX_X and compare this design to the fixed gain design.

This is accomplished by replacing the fixed parameter portion of the MIMO design (Objective 1) with a parameter-adaptive design for the AFIT F-16 on $MATRIX_X$. Data are obtained for the same flight condition and failure that are used during the fixed gain evaluation.

1.7 Summary

This chapter presents background information on adaptive systems and their use. Porter's modern control law design technique and its use in MIMO systems are also discussed. In addition, the purpose and specific objectives of this thesis are introduced.

1.8 Overview

This thesis contains five chapters and five appendices. Chapter 2 contains a description of the AFTI/F-16. Detailed multivariable control law theory and a brief description of adaptive control law theory is then presented in Chapter 3. The simulation procedures and results are contained in Chapter 4. Conclusions and recommendations are presented in Chapter 5.

II. Aircraft Description and Models

2.1 Introduction

This section presents a brief description of the AFTI/F-16 aircraft. Then the aircraft models used for this thesis are presented. These models consist of both the healthy and failed models for the AFTI/F-16. Information for this chapter is obtained mainly from References 4 and 18.

2.2 Aircraft Description

The AFTI/F-16 is shown in Figure 2-1. This aircraft is a modified F-16A and is used as a testbed for the evaluation of new technologies. This aircraft is modified with a redundant digital fly-by-wire flight control system, independent operation of the trailing edge flaps (flaperons), independent operation of the horizontal tail halves, and the addition of two vertical canards. The unagumented F-16A is statically unstable in the longitudinal axis in subsonic flight. The aircraft was designed to allow it to attain higher load factors and to reduce its drag. In addition, the Dutch roll is lightly damped in subsonic flight. Therefore, the primary function of the flight control system is to stabilize the longitudinal axis and increase the Dutch roll damping.

The modifications performed on the AFTI/F-16 allow it to perform both conventional and unconventional maneuvers. The conventional maneuvers consist of coordinated turns, rolling laterally, and longitudinal pitching. Operation of the canards, the ability to operate the flap independently, and independent operation

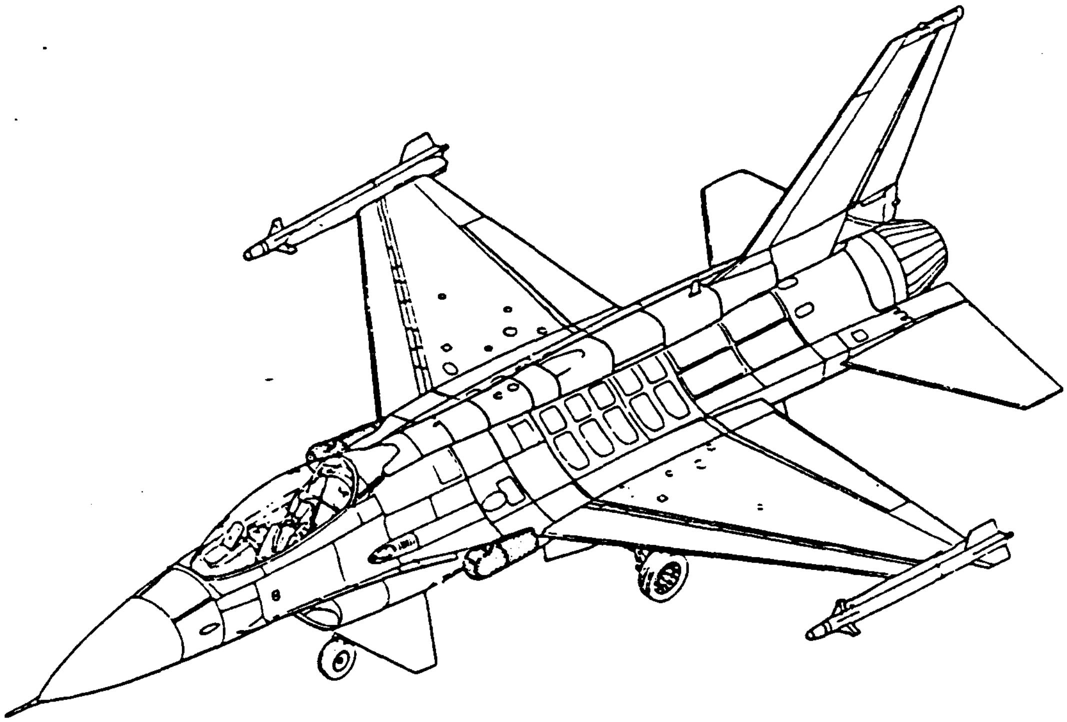


Fig. 2-1 AFTI/F-16
Source 18

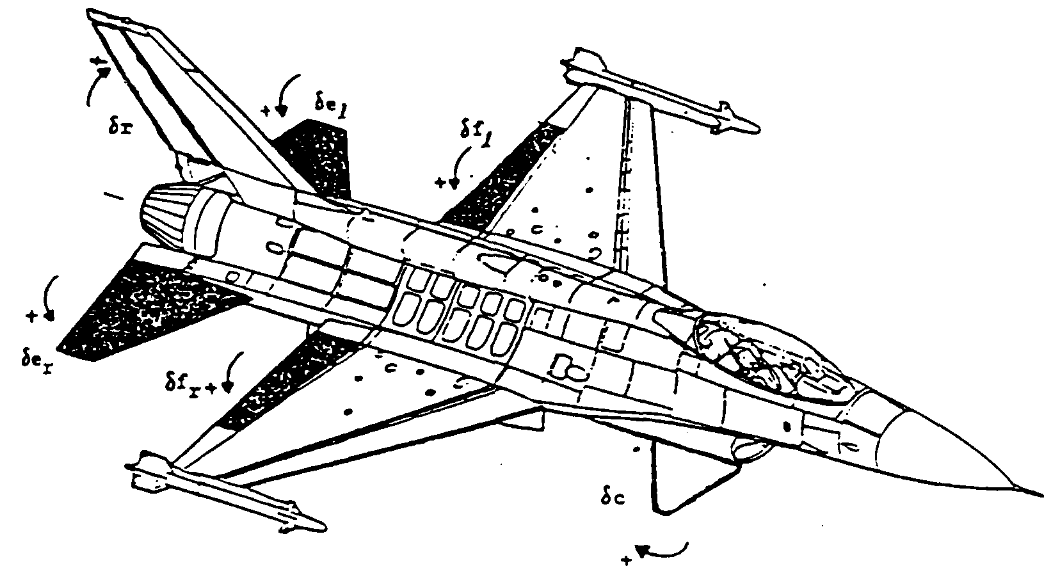
of the tail surfaces give the AFTI/F-16 the ability to decouple the aircraft force and moment equations. This decoupling allows the aircraft to perform unconventional maneuvers such as: pitch-pointing, lateral and longitudinal translation, and yaw pointing.

The AFTI/F-16 control surfaces are shown in Figure 2-2. Both the horizontal tail and flaperons serve a dual function. The horizontal tail can be either deflected symmetrically as the primary pitch control surface of the aircraft or deflected asymmetrically to augment lateral rolling. The flaperons can be either deflected symmetrically to produce lift (flaps) or deflected asymmetrically as the primary roll control surfaces. The canards have the capability to be used independently in a snowplow configuration as a speed brake or as surfaces which produce sideforces. For this thesis, only the flaperons and horizontal tails (elevators) are used as control surfaces. The control surfaces deflections are defined as follows:

δDT = Differential flaperon deflection
δDT = Differential horizontal tail deflection
δ_{fl} = Left flaperon deflection
δ_{fr} = Right flaperon deflection
δ_{el} = Left horizontal tail deflection
δ_{er} = Right horizontal tail deflection

The AFTI/F-16 flight control system is divided into four major configurations. Each of these configurations is used to support a primary mission task for the AFTI/F-16.

The first flight control configuration, called the normal mode, is used during the take-off, cruising, air-refueling and landing portion of the mission. The second



δDT is defined as: δe_r down, δe_l up

δDF is defined as: δf_r down, δf_l up

Fig. 2-2 AFTI/F-16 Control Surfaces
Source 18

configuration (air-to-air gunnery mode) is used during tasks that require precise pointing of the aircraft, such as target tracking or combat maneuvers. The third configuration (air-to-surface gunnery mode) is used during air to ground strafing attacks. The fourth configuration is the air-to-surface bombing mode; this mode allows the pilot precise control of the aircraft's velocity vector for accurate bombing. This thesis investigates the design of parameter-adaptive control laws for the air-to-air gunnery mode of the AFTI/F-16.

2.3 Aircraft Models

The state and output equations of the system (plant) to be controlled are in the form

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) \qquad (2-1)$$

$$\underline{y}(t) = C\underline{x}(t) \qquad (2-2)$$

where

A = continuous plant matrix (n x n)
B = continuous input control matrix (n x m)
C = continuous output matrix (l x n)
$\underline{x}$ = state variable vector with n states
$\underline{u}$ = input vector with m inputs
$\underline{y}$ = output vector with l outputs

The state equations are obtained from the aircraft equations of motion, which consist of equations for forces and moments acting upon the center of gravity. These equations are perturbation equations about a nominal condition. A detailed derivation of these perturbation equations is contained in Reference 15. A healthy aircraft state space model is shown in Equation (2-3). The primed terms in Equation (2-3) are the dimensionalized derivatives in the body axis. These terms are obtained from Reference 4 and are shown in Appendix A. This equation is also used for a failed surface state space model. The failed model is generated by decreasing the elements of the first column in the B matrix by 50 percent.

The output vector for this model is:

$$\underline{y}(t) = [\gamma, \beta, r]^T \tag{2-4}$$

where

$\gamma =$ Flight-path angle

$\beta =$ Sideslip angle

$r =$ Yaw rate

Flight-path angle, sideslip angle, and yaw rate are chosen as the outputs so that CB has full rank and a Proportional plus Integral (PI) controller using Porter's technique could be used. The need for these requirements is explained in greater detail in Chapter III.

III Controller Design

3.1 Introduction

This chapter presents the Proportional plus Integral (PI) controller used for this thesis. This is accomplished by first reviewing the system model presented in Chapter II. and then presenting assumptions made in the selection of the controller. The procedure used to calculate the controller gain matrices (K1 and K2). if the system matrices (A,B,C) are known. is shown. The representation of a system by use of an autoregressive model and how this model, in conjunction with the step-response matrix. is used to calculated the control law gain matrices is presented next. Finally. this chapter concludes with a brief description of the adaptive algorithm which is used for the design of an adaptive controller.

3.2 System Model

As discussed in Chapter II. the state and output equations of the system (plant) to be controlled are in the form

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) \qquad (2-1)$$

$$\underline{y}(t) = C\underline{x}(t) \qquad (2-2)$$

where

A = continuous plant matrix (n x n)

B = continuous input control matrix (n x m)

C = continuous output matrix (l x n)

$\underline{x}$ = state variable vector with n states

$\underline{u}$ = input vector with m inputs

$\underline{y}$ = output vector with l outputs

For the discrete-time case, the above model can be written in the following form (6):

$$\underline{x}[(k+1)T] = \Phi\underline{x}(kT) + \Gamma\underline{u}(kT) \qquad (3-1)$$

$$\underline{y}(kT) = C\underline{x}(kT) \qquad (3-2)$$

where

$\Phi = exp(AT)$: discrete plant matrix

$\Gamma = \int_0^T exp(A\tau)Bd\tau$: discrete input control matrix

T = sampling period

k = integer from zero to plus infinity

C = discrete output matrix

3.3 Assumptions

1. Some of the eigenvalues of A may lie in the right-half plane. This permits the use of this controller on plants that are open-loop unstable.

2. The rank of the first Markov parameter (CB) must be full to use this controller. If the rank of the CB is not full, then a Proportional plus Integral plus Derivative (PID) control law can be used as shown in Reference 19.

3. The plant model is completely observable and controllable.

4. The controllability of the plant due to integral action is maintained. This requires that the matrix R have rank of ($n + l$) (7), where:

$$R = \begin{pmatrix} B & A \\ 0 & C \end{pmatrix}$$

This controllability can also be verified by ensuring the rank of the transfer function G(s) equals l .

where

$$G(s) = C[sI - A]^{-1}B \qquad (3-3)$$

and

$$G(0) = -C(A)^{-1}B \qquad (3-4)$$

3.4 Controller Gain Calculation

3.4.1 Matrix Model. For fixed-parameter plants, tracking of an input vector ($\underline{r}$) via Porter's method (8:9:14) is possible for small values of sampling periods (T) using the digital PI controller shown in Figure 3 − 1. This controller is an error-actuated digital controller which is governed by the PI control law of the form shown in Equations (3-5). Integration is approximated by the backward difference equation (Equation (3-6)).

$$\underline{u}(kT) = (1/T)K1\underline{e}(kT) + (1/T)K2\underline{\varsigma}(kT) \qquad (3-5)$$

and

$$\underline{\varsigma}[(k+1)T] = \underline{\varsigma}(kT) + (1/T)\underline{e}(kT) \qquad (3-6)$$

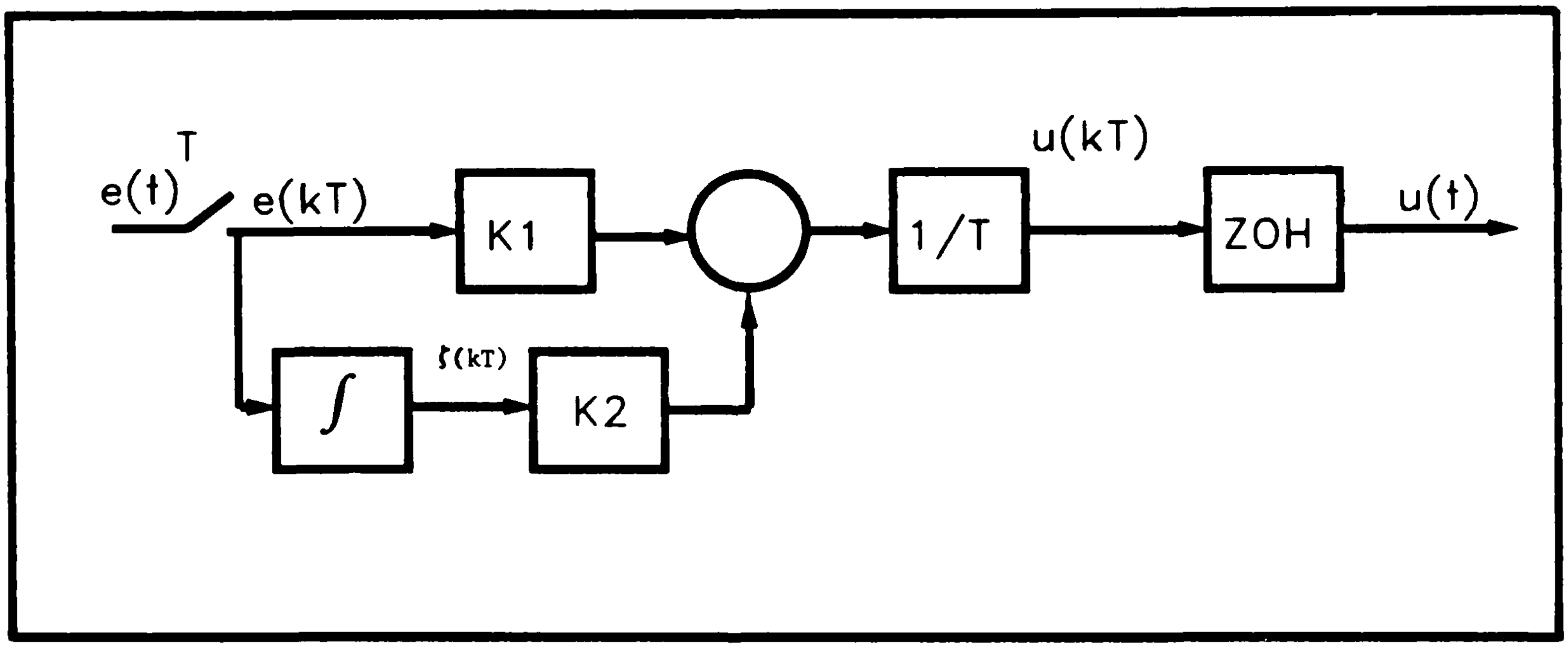


Fig. 3-1 Digital Proportional Plus Integral Controller

where

$\underline{e}(kT) = \underline{r}(kT) - \underline{y}(kT)$

$\underline{\varsigma}(kT)$ = digital integral of $\underline{e}(kT)$

$K1 = (CB)^{-1}\Sigma$

$K2 = K1\rho$

T = sampling period

Σ = diagonal weighing matrix: diag$[\sigma 1. \sigma 2, \ldots \sigma l]$

ρ = ratio of integral to proportional control

The diagonal weighting matrix Σ and the ratio of integral to proportional control ρ are two variables which the designer selects to obatain the desired response from the system.

If the system matrices are not known, a controller for the above system can also be designed using the step-response matrix (16). The step-response matrix, H(T), is the response of the system at the first sample period (t = T) due to a step being applied at time equal to zero (t = 0). The step response matrix can be obtained from Equations (2-1) and (2-2) as show below.

First, solving Equation (2-1) yields:

$$x(t) = e^{At}x(0) + \int_o^t e^{A\tau}u(t-\tau)d\tau B \qquad (3-7)$$

Substituting Equation (3-7) into Equation (2-2), with initial conditions equal to zero:

$$y(t) = C[\int_0^t e^{A\tau}d\tau B] \qquad (3-8)$$

Evaluating Equation (3-8) for a step input yields;

$$y(t) = CA^{-1}(e^{At} - I_N)B \tag{3-9}$$

Which is the definition of the step-response matrix (H(t)):

$$H(t) = CA^{-1}(e^{At} - I_N)B \tag{3-10}$$

As show in References 8, 9, 14, & 16, the controller gain matrix K1 of Equation (3-5) becomes $K1 = H(T)^{-1}\Sigma$. Where Σ is a diagonal tuning matrix selected by the designer. For small values of sampling periods (T), and a first order approximation of $e^{At} \approx I_N + AT$. Equation (3-10) can be written as:

$$H(T) \approx CA^{-1}(I_N + AT - I_N)B$$

which yields,

$$H(T) \approx TCB \tag{3-11}$$

3.4.2 Autoregressive Model. One form of expressing the input-output relationship of a system is in the form of an Nth order autoregressive difference equation (17):

$$y(kT)+A1y((k-1)T) + A2y((k-2)T)\dots ANy((k-N)T) = B1u((k-1)T)$$
$$+ B2u((k-2)T)\dots\dots BNu((k-N)T) + n(kT) \tag{3-12}$$

where

$Bi = l$ x m matrices

$Ai = l \times l$ matrices

$\underline{\epsilon}(kT)$ = zero mean Gaussian white noise vector with variance = v^2.

The order of the model (N) depends on the procedure used to generate the autoregressive model. The original algorithm used for this thesis utilized an autoregressive model derived from a transfer function representation (2). This representation can be derived by expressing Equation (3-2) as a transfer function that relates the input and output data as shown in Reference 20, page 536. This procedure shown in Reference 23 is briefly reviewed here.

The solution to Equation (3-2), with initial conditions equal to zero, is:

$$Y(z) = C[zI - \Phi]^{-1}\Gamma u(z) \qquad (3-13)$$

which can be written in the form

$$Y(z) = G(z)u(z) \qquad (3-14)$$

where

$$G(z) = C[zI - \Phi]^{-1}\Gamma \qquad (3-15)$$

$$Y(z) = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_l \end{pmatrix}$$

$$G(z) = \begin{pmatrix} G_{11}(z) & G_{12}(z) & \dots & G_{1m}(z) \\ G_{21}(z) & G_{22}(z) & \dots & G_{2m}(z) \\ \vdots & \vdots & \vdots & \\ G_{l1}(z) & \dots & \dots & G_{lm}(z) \end{pmatrix}$$

$$u(z) = \begin{pmatrix} u_1(z) \\ u_2(z) \\ \dots \\ u_m(z) \end{pmatrix}$$

where $G_{ij}(z)$ can be expressed as:

$$G_{ij}(z) = \frac{b1_{ij}z^{W} + b2_{ij}z^{W-1} + \dots + b(N-1)_{ij}z + bN}{z^{N} + a1z^{N-1} + a2z^{N-2}\dots + a(N-1)z + aN} \qquad (3-16)$$

where $N \geq W + 1$ for a proper system.

Now divide the numerator and denominator of Equation (3-16) by Z^{-N} ,which yields:

$$G_{ij}(z) = \frac{b1_{ij}z^{-1} + b2_{ij}z^{-2} + \dots + bNz^{-N}}{1 + a1z^{-1} + a2z^{-2}\dots + aNz^{-N}} \qquad (3-17)$$

Substituting Equation (3-17) into $y_i(z) = G_{ij}u_j(z)$ and rearranging yields:

$$y_i + a1z^{-1}y_i + a2z^{-2}y_i \dots + aNz^{-N}y_i = b1_{ij}z^{-1}u_j + b2_{ij}z^{-2}u_j + \dots + bN_{ij}z^{-N}u_j$$

It is important to note that the "a" coefficients of Equation (3-17) do not have subscripts. Subscripts are not required because the denominator of Equation (3 – 16) is the characteristic equation; therefore the "a" coefficients at each delay period are the same for all the individual transfer functions. That is, $a1$ of G_{11} = $a1$ of G_{23}, and $a2$ of G_{23} = $a2$ of G_{34}, and so forth.

Finally, taking the inverse Z transform (20:116), yields Equation (3-12):

$$y(kT) + A1y((k-1)T) + A2y((k-2)T) \ldots ANy((k-N)T) = B1u((k-1)T) + B2u((k-2)T) \ldots\ldots BNu((k-N)T) + n(kT) \quad (3-12)$$

The order "N" of Equation (3-12) is equal to the order of the denominator (number of states) of Equation (3-16).

Equation (3-12) can also be written as:

$$y(kT) = \varphi(kt)\Theta(kT) + \eta(kT) \quad (3-18)$$

where $\varphi(kT)$ is a matrix which contains the values of past input and output data. $\Theta(kT)$ is the parameter vector which contains the coefficients of the autoregressive equations. and $\eta(kT)$ is assumed to be a zero-mean Gaussian white-noise vector with variance σ^2 . For example. given a system which has 2 outputs. 3 inputs and a 2nd order autoregressive equation, then for the first delay (N=1) of Equation (3-18):

$\varphi(kT)$ would be written as:

$$\begin{pmatrix} u_1(k-1) & u_2(k-1) & u_3(k-1) & 0 & 0 & 0 & y_1(k-1) \\ 0 & 0 & 0 & u_1(k-1) & u_2(k-1) & u_3(k-1) & y_2(k-1) \end{pmatrix}$$

and

$$\Theta(kT) = \begin{pmatrix} bl_{11} \\ bl_{12} \\ bl_{13} \\ bl_{21} \\ bl_{22} \\ bl_{23} \\ a1 \end{pmatrix}$$

Another way of obtaining the Ai and Bi matrices of Equation (3-12) is by open-loop testing via the step-response matrix method shown in chapter 8 of Reference 19. This method requires that either the AFTI/F-16 be available for open-loop testing or the order of the autoregressive model be known so that a computer simulation of the open-loop response could be accomplished. A procedure for determining the order of the autoregressive model is shown in Reference 22. The procedure given in Reference 22 can also be used to determine the Ai and Bi matrices of equation (3-12). Therefore, the step-response method shown in Reference 19 was not used for this thesis; instead the technique explained in Reference 22 was utilized. Reference 22 procedures are outlined in Appendix B. When this method is used then the order of the autoregressive model does not necessary equal the number of states in the state-space model. The order will usually be less than the number of states in the system. In fact for the model shown in Equation (2-3), the order of the autoregressive model is four; whereas, the number of states is equal to eight. However, unlike the autoregressive equation derived from the transfer function, the Ai matrices are now fully populated. This can be seen in Appendix B.

Once the autoregressive coefficients are determined then the controller gains K1 and K2 are given in Reference 17 as follows (17):

$$K1 = H^T(T)[H(T)H^T(T)]^{-1}\Sigma \tag{3-19}$$

$$K2 = G^T(0)[G(0)G^T(0)]^{-1}\Pi \tag{3-20}$$

where

$H(T) = B1$

Σ = weighing matrix; diag$[\sigma 1, \sigma 2, ...\sigma l]$

Π = weighing matrix; diag$[\pi 1, \pi 2, \ldots \pi l]$

and

$$G(0) = (I + A1 + A2 + A3 + ...AN)^{-1}(B1 + B2 + B3....BN) \tag{3-21}$$

By applying the definition of the step-response matrix, given in Section 3.4.1, to Equation (3-12) it can be seen that B1 is the step response-matrix. This is due to the fact that all the elements of Equation (3-12) are equal to zero at time equal to "T" except B1; therefore, B1 is by definition the step-response matrix.

The autoregressive coefficients used to determine K1 and K2 can be estimated using a recursive least square estimation algorithm such as the one mentioned in the next section.

3.5 Parameter-Adaptive Algorithm

The main objective of least square estimation is to minimize (21:120)

"...the sum of squares of the differences between the actual measurement data and the proposed, or estimated, function or curve."

For this thesis the function estimated is the parameter vector $\Theta(kt)$. As explained in Section 3.42. this parameter vector is composed of the coefficients of the autoregressive model described earlier in this chapter. The ability to estimate these coefficients can then allow updating of the controller gains K1 and K2 via Equation (3-19) and Equation (3-20). This research uses a modified recursive least square estimation algorithm developed by Hagglund (13). The algorithm has been expanded from the single-input single-output case developed by Hagglund to the multiple- input multiple-output case required for this thesis. This expansion was accomplished by Pinerio (2). A detailed description of the algorithm is contained in Reference 13 and Reference 2, section 3.4. The equations implemented in the algorithm are contained in Appendix C.

3.6 Summary

The Proportional plus Integral (PI) controller used for this thesis and the procedure used for calculating the control law gain matrices. K1 and K2 are presented in this chapter. The system model. assumptions, and the procedure used to calculate K1 and K2 if the system matrices (A.B.C) are known are shown. How to calculate the controller gains via an autoregressive model in conjunction with the step-response matrix is also reviewed. Finally. this chapter concludes with a brief description of the adaptive algorithm used for the design of an adaptive controller.

IV Simulation and Results

4.1 Introduction

This chapter presents the sequence of steps taking in the simulation and testing of the fixed, and adaptive, gain digital PI controller. Also, the results of this simulation are given. The simulation sequence consists of first verifying that the model chosen does not violate the assumptions made. Then an autoregressive model is generated to permit implementation of the PI control laws. Using an autoregressive model, a fixed gain controller is designed for a healthy aircraft. Aircraft response for both a healthy and failed model are then obtained for the fixed gain controller. Finally, adaptive simulations are performed for both a healthy and a failed model.

The computer-aided design program called MATRIX_X is used for this simulation (19). MATRIX_X allows the user the option of executing a series of operations by the use of macros. These marcros are user defined text strings which can be save and executed with a single command. Throughout this simulation several macros are used; these macros are listed in Appendix D.

4.2 Assumption Verification

The macros shown in Appendix D, page D-1 through D-3 are used to verify that the assumptions made in Chapter III, section 3-3, are not violated. The nominal plant matrices are shown in Table 4-1. As can be seen in Append.x D, the nominal plant did not violate the assumptions. Once these assumptions are verified then the autoregressive model is generated.

Table 4-1 Nominal Plant Matrices

Plant Matrix (A)

0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-3.2183D+01	-5.6009D−02	3.8291D+01	-3.0138D+01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-1.1000D−03	4.5971D−05	-1.4845D+00	9.9480D−01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
3.0000D−04	-2.1010D−03	4.2717D+00	-7.7720D−01	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	3.4500D−02	-3.4536D−01	3.2600D−02	-9.9760D−01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-5.5253D+01	-2.8004D+00	1.4570D−01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	7.2370D+00	-2.3200D−02	-3.6250D−01

Input Control Matrix (B)

0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.0030D+00	1.0030D+00	1.1584D+00	1.1584D+00	1.9254D+01
-7.4600D−02	-7.4600D−02	-1.2250D−01	-1.2250D−01	0.0000D+00
-1.2029D+01	-1.2029D+01	-3.2363D+00	-3.2363D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-3.3000D−03	3.3000D−03	7.0000D−04	-7.0000D−04	0.0000D+00
6.3395D+00	-6.3395D+00	2.5519D+01	-2.5519D+01	0.0000D+00
6.4200D−01	-6.4200D−01	6.2490D−01	-6.2490D−01	0.0000D+00

4.3 Autoregressive Model Generation

The procedure to generate an autoregressive model from a state-space model is show in Appendix B. The Macros contained in pages C-5 through C-6 are used to implement this procedure. Two verifications are done to obtain confidence in the values generated from this procedure and macros. First, the value obtained for the matrix B1 is compared to the value obtained for the step response matrix given by Equation (3-11). This comparison is shown in Table 4-2. The difference between the two methods is in the fifth decimal place. Next, the value obtained for G(0) from the autoregressive model (Equation (3-21)) is compared to the value of G(0) obtained from Equation (3-4). This comparison is shown in Table 4-3. Subtracting the two calculations of G(0), reveals that the difference is in the seventh decimal place.

4.4 Fixed Gain Controller

4.4.1 Healthy Model. The nominal plant in series with the controller is simulated in $MATRIX_X$, using its system-build capability. The block diagram and interconnection of this simulation is contained in Appendix E. This simulation produces unstable responses. This instability is due to a transmission zero in the right-half plane. References 7, 25, and 26 show that the transmission zeros of a plant, where the number of inputs equal the number of outputs (square plant), can be obtained by setting the determinant of the system matrix S equal to zero.

Table 4-2 Comparison of B1 and Step-Response Matrix

B1 of Autoregressive Model

7.3442D−04	7.3442D−04	1.2143D−03	1.2143D−03	-4.4090D−08
-5.4597D−05	5.4597D−05	1.7295D−05	-1.7295D−05	0.0000D+00
6.3994D−03	-6.3994D−03	6.2089D−03	-6.2089D−03	0.0000D+00

Step-Response Matrix ($H \approx TCB$)

7.4600D−04	7.4600D−04	1.2250D−03	1.2250D−03	0.0000D+00
-3.3000D−05	3.3000D−05	7.0000D−06	-7.0000D−06	0.0000D+00
6.4200D−03	-6.4200D−03	6.2490D−03	-6.2490D−03	0.0000D+00

Difference Between B1 and Step-Response Matrix

1.1581D−05	1.1581D−05	1.0684D−05	1.0684D−05	4.4090D−08
2.1597D−05	-2.1597D−05	-1.0295D−05	1.0295D−05	0.0000D+00
2.0614D−05	-2.0614D−05	4.0103D−05	-4.0103D−05	0.0000D+00

Table 4-3 Comparison of G(0)

G(0) from Autoregressive Model

1.0834D+01	1.0834D+01	3.1894D+00	3.1894D+00	5.9969D−01
1.2604D−01	-1.2604D−01	4.9232D−01	-4.9232D−01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

G(0) using $-CA^{-1}B$

1.0834D+01	1.0834D+01	3.1894D+00	3.1894D+00	5.9969D−01
1.2604D−01	-1.2604D−01	4.9232D−01	-4.9232D−01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

where

$$S = \begin{pmatrix} sI - A & B \\ -C & 0 \end{pmatrix} \qquad (4-1)$$

Since the nominal plant for this research is a rectangular plant. Equation (4-1) cannot be used to determine the location of the transmission zeros. As suggested by Professor Porter, the controller is placed in series with the rectangular plant to form a square system (Ss). Figure 4-1 depicts this new square system. When Equation 4-1 is applied to the Ss and the determinant of Ss is set equal to zero, then eight roots are identified. Their locations are:

-1.6216D+02
-1.0096D+01
9.6680D+00
-1.0522D+00
-3.5175D−01
-5.5501D−03
-5.1412D−02
-5.5550D−02

As explained in Reference 6 and 7, the number of roots (ω) of Equation (4-1), for a square plant, are calculated as:

$$\omega = N - l \qquad (4-2)$$

where

N = number of states

l = number of outputs

Since Ss contains 11 states and 3 outputs, Equation 4-2 appears to be valid

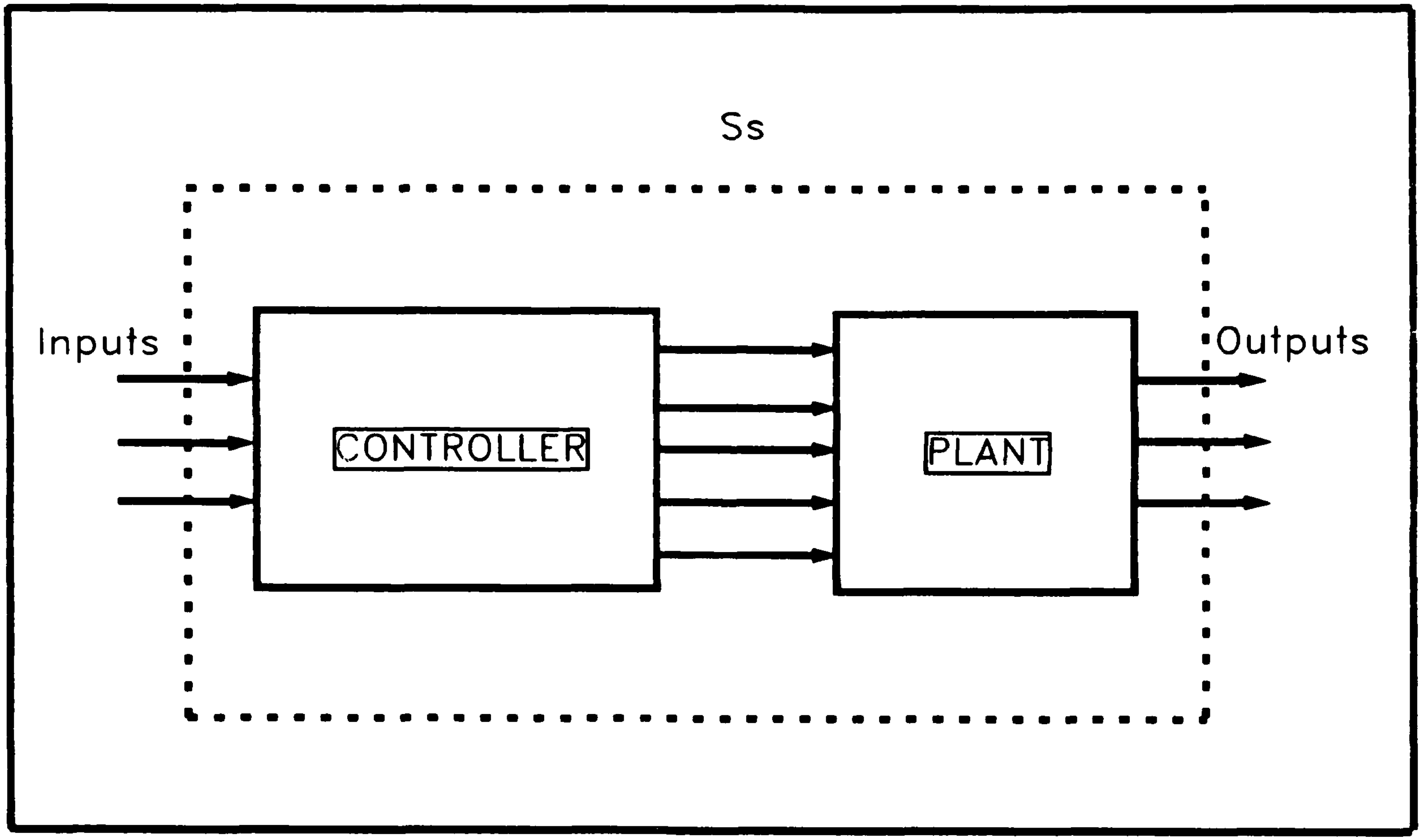


FIG. 4-1 Square System (Ss)

for this system. The above transmission zeros are calculated with the weighing matrices Pi(Π) and Sigma (Σ) equal to diag[0.9, 0.9, 0.9] and diag[0.003, 0.003, 0.095], respectively.

As shown in References 6 and 7, the slow mode roots (Z1) of a square plant for this type of PI controller are calculated as:

$$Z1 = (|\lambda K1 + K2| = 0) \qquad (4-3)$$

However when the determinant of Ss is set to zero, then both the slow mode roots and the transmission zeros appear as roots. Therefore, to determine if the root at 9.668 is a transmission zero or a slow mode root the sign of K1 is changed from plus (+) to minus (−). Since $K2 = G^T(0)[G(0)G^T(0)]^{-1}\Pi$ (Equation (3-20)), the sign change of K1 is accomplished by multplying Π by -1. The new location of the roots are now:

-1.6216D+02
9.6435D+00
-1.0117D+01
-1.0522D+00
3.5175D−01
5.5501D−03
5.1070D−02
-5.5942D−02

As can be seen, the root at 9.668 moves to 9.6435: however this root does not change sign and is still in the right-half plane. The root at 0.3517, 0.0055, and 0.05071 changed sign: these are the slow mode roots. Hence, the root at 9.6 is a transmission zero.

The value of Π is kept constant at diag[0.9:0.9:0.9] while Σ is varied to observe the impact it has on the location of the right-half transmission zero. Sigma is varied from diag[0.9.0.9.0.9] to diag[0.0.0]. The resulting transmission zero in the right-half plane goes from 9.65 to 13.45. As mentioned in chapter II. the AFTI/F-16 is longitudinally unstable (pole in the right-half plane). With a pole and a transmission zero both in the right-half plane, it is not possible to get a stable response using this type of PI controller.

Since the above system could not be stabilized. a different output vector is chosen. The second output vector consists of forward velocity, sideslip angle, and yaw rate. The output matrix is now:

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The same procedure of adding the controller in series with the plant is accomplished for this second plant. With Σ equal to diag[0.003. 0.003. 0.095] and Π equal to diag[0.9. 0.9. 0.9] the transmission zeros for this plant are located at:

-1.6216D+02	+0.0000D+00i
-7.8847D+00	+0.0000D+00i
-2.4463D−01	+1.0992D+00i
-2.4463D−01	−1.0992D+00i
-1.0522D+00	+0.0000D+00i
-3.5175D−01	+0.0000D+00i
-5.5501D−03	+0.0000D+00i
2.9025D−02	+0.0000D+00i

When the same procedure of setting all the elements of Π to -0.9 is accomplished, it is observed that the zero at 0.0029 is a slow mode root. This root is placed in the right-half plane by making the first element of Π a negative number. For Π = diag[-0.9, 0.9. 0.9] the transmission zeros are now located at:

-1.6216D+02 +0.0000D+00i
-7.8454D+00 +0.0000D+00i
-2.3610D−01 +1.1139D+00i
-2.3610D−01 −1.1139D+00i
-1.0522D+00 +0.0000D+00i
-3.5175D−01 +0.0000D+00i
-5.5501D−03 +0.0000D+00i
-2.8527D−02 +0.0000D+00i

This plant is now stable. Table 4-4 contains the values which are used to obtain a stable response for this plant.

The above system is now simulated in the system-build portion of MATRIX_X. The maneuver choosen for evaluating the system is a coordinated turn. This maneuver is accomplished by commanding beta to zero and commanding a yaw rate. The command shown in Figure 4-2 is applied to the yaw rate input. This command produces a bank-angle (ϕ) via the the relationship between yaw rate and ϕ given in Blakelock (:147) as:

$$r = \frac{g}{V} \sin\phi \tag{4 - 3}$$

where

V = forward velocity of the aircraft

g = gravitational constant (32.2 ft/sec)

Responses due to this input are shown in Figure 4-3 through Figure 4-6. Once a stable controller is designed, then the actuator dynamics are added to the simulation prior to attempting to fine tune the controller. All attempts to tune the controller with actuator dynamics added have been unsuccessful. At this time, no

Table 4-4 Closed-Loop Response - Without Actuators

$\Sigma = \text{diag}[0.03, 0.003, 0.095]$.

$\Pi = \text{diag}[-0.9, 0.9, 0.9]$.

Closed-Loop Poles

-2.8849D−03	+0.0000D+00i
-5.5229D−03	+0.0000D+00i
-2.4983D−01	+0.0000D+00i
-9.9236D−01	+0.0000D+00i
-1.8210D−01	+1.2683D+00i
-1.8210D−01	-1.2683D+00i
-2.4316D+00	+3.4542D+00i
-2.4316D+00	-3.4542D+00i
-5.6535D−01	+5.9474D+00i
-5.6535D−01	-5.9474D+00i
-1.0850D+01	+0.0000D+00i

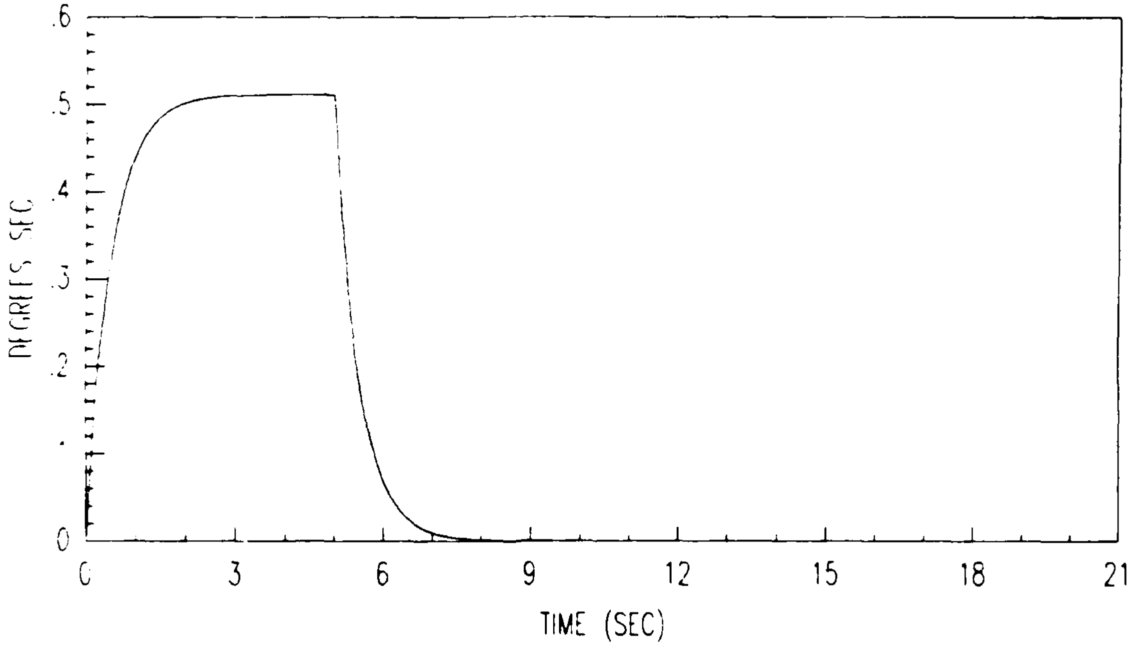


FIG. 4-2 YAW RATE COMMAND

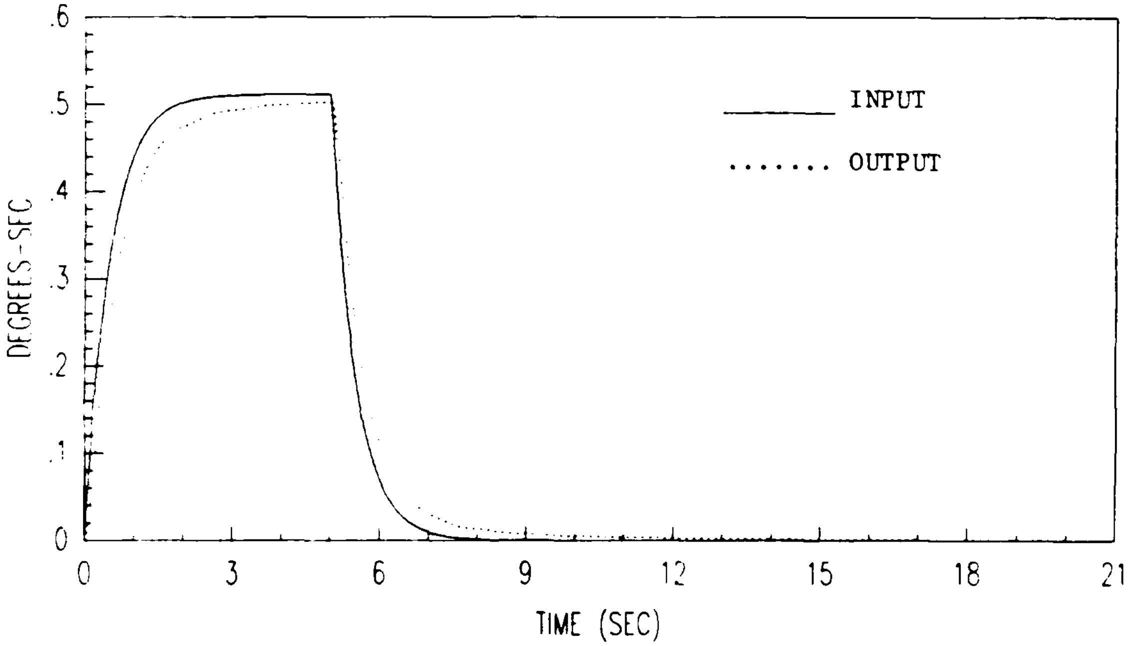


FIG. 4-3 FIXED GAIN YAW RATE RESPONSE

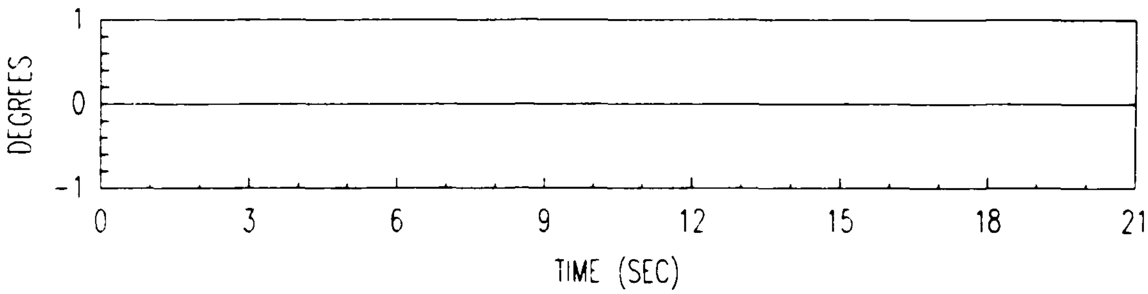


FIG. 4-4 FIXED GAIN PITCH ANGLE RESPONSE

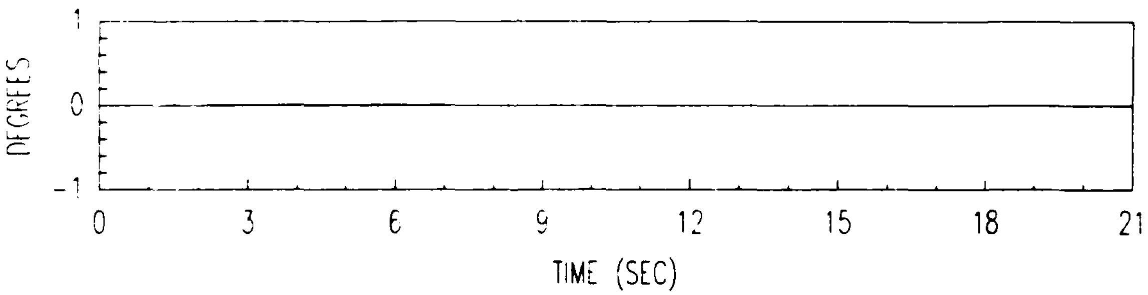


FIG. 4-5 FIXED GAIN SIDESLIP ANGLE RESPONSE

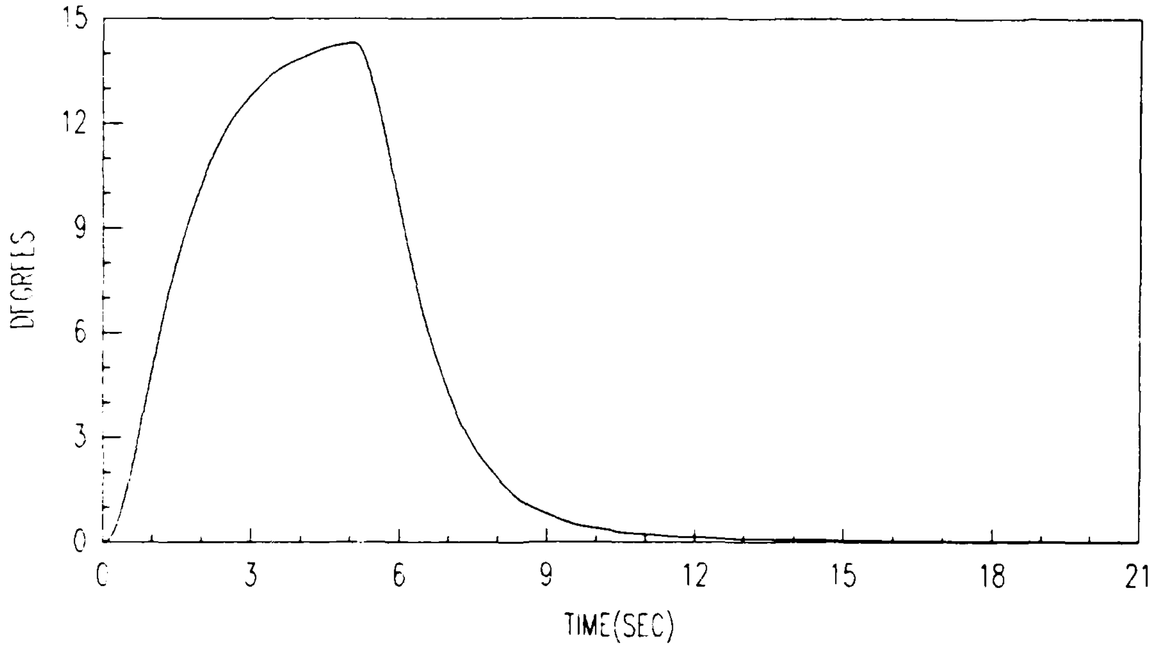


FIG. 4-6 FIXED GAIN BANK ANGLE RESPONSE

further attempts are made to tune the fixed gain controller with actuator dynamics and the research into the fixed gain failure model and the adaptive controller is continued. Therefore, fixed gain failure and adaptive controller simulations do not include actuator dynamics. However later in the research, actuators are implemented as shown in Appendix E, pages E-35 through E-38. The response of the fixed gain controller with the actuators is now stable. The responses of the aircraft, with actuators, due to the input shown in Fiqure 4-2 are contained in Figure 4-7 through 4-10.

4.4.2 Fixed Gain Failure Model. After the controller gains for the healthy model (without actuator dynamics) are obtained, then a simulation is run with the same controller gains but with the aircraft failure model in the simulation. The aircraft failure model consist of reducing the left elevator effectiveness by 50%. The same maneuver commanded for the healthy model is used for this simulation. Aircraft response data are obtained and the results are presented in Figure 4-11 through Figure 4-14. As can be seen from these plots the aircraft goes unstable due to this failure. The next step taken is to design an adaptive controller and obtain data to allow comparison with the fixed gain controller.

4.5 Adaptive Controller

Adaptive simulations are conducted after obtaining data for the fixed gain controller. First, simulations with the healthy model are accomplished. These simulations are conducted by first initializing the parameter vector with the correct values for the given plant. The first two seconds of the simulation are conducted in the fixed gain mode, then the adaptive controller is turned on.

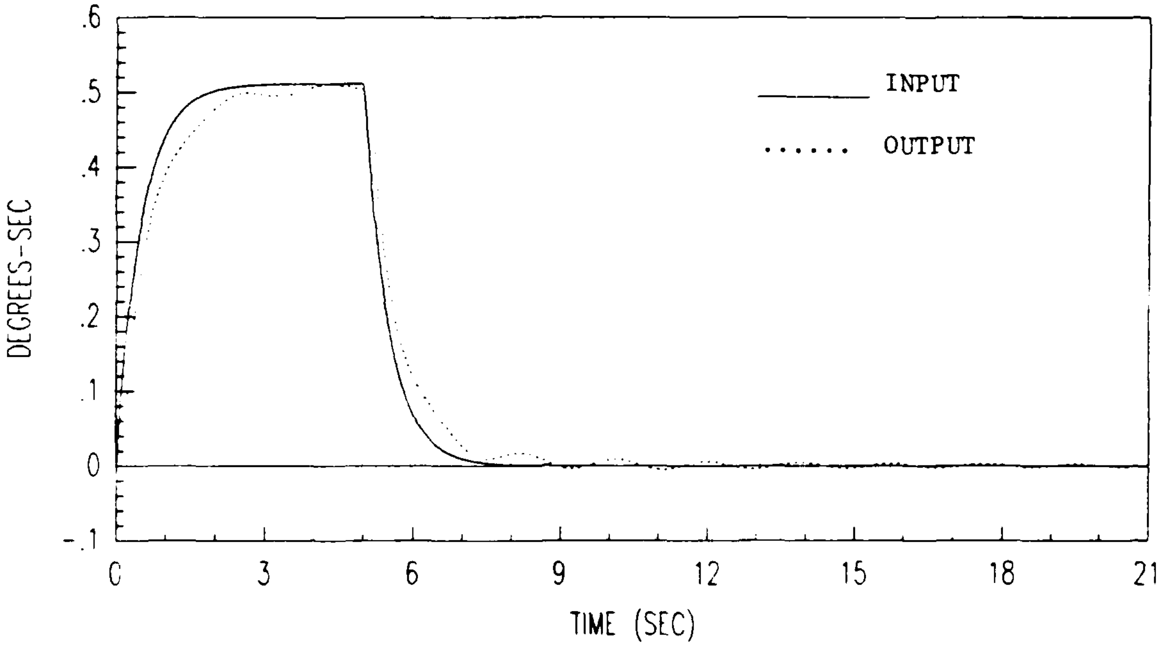


FIG. 4-7 FIXED GAIN YAW RATE RESPONSE - WITH ACTUATORS

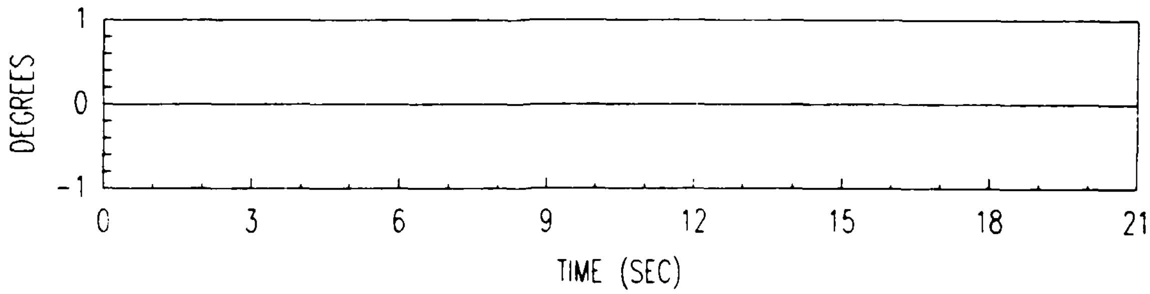


FIG. 4-8 FIXED GAIN PITCH ANGLE RESPONSE - WITH ACTUATORS

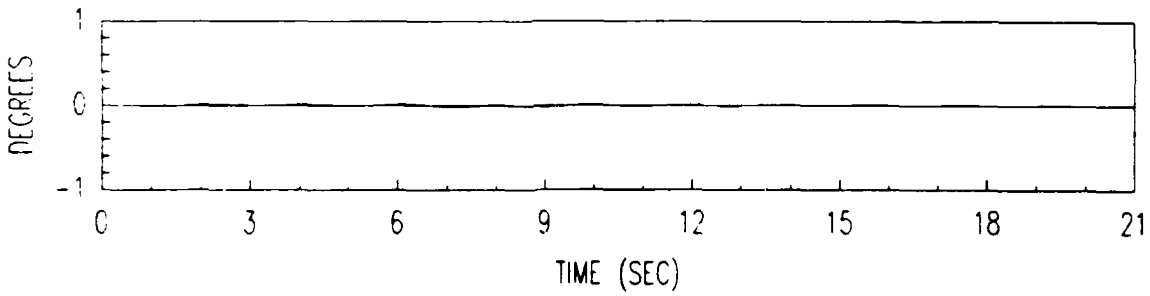


FIG. 4-9 FIXED GAIN SIDESLIP ANGLE RESPONSE - WITH ACTUATORS

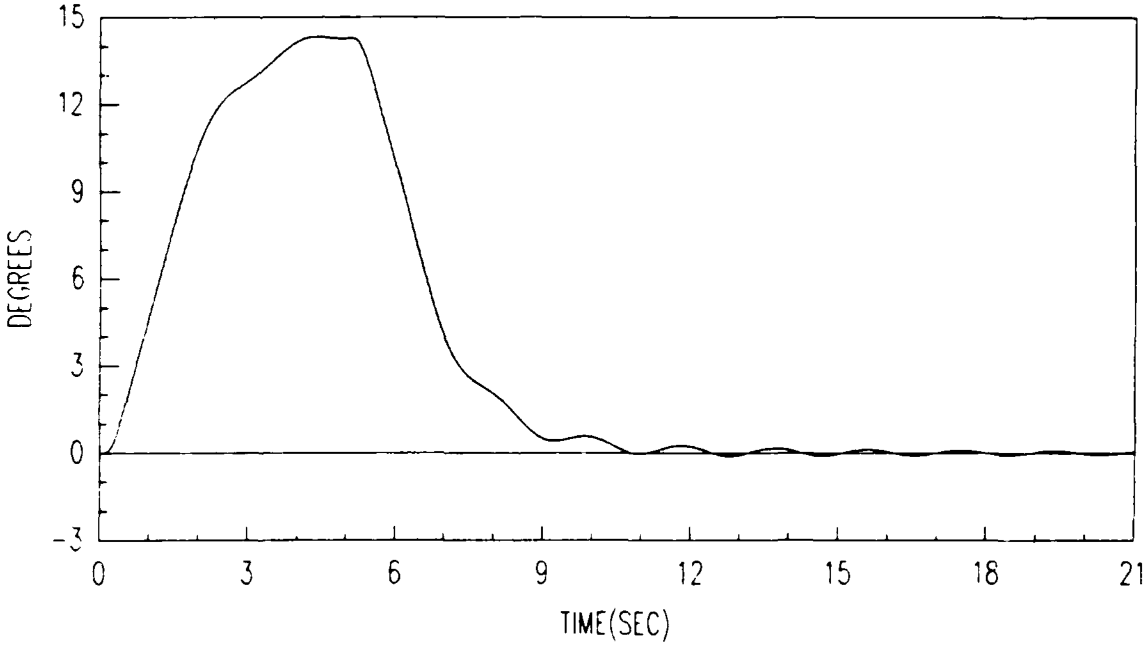


FIG. 4-10 FIXED GAIN BANK ANGLE RESPONSE - WITH ACTUATORS

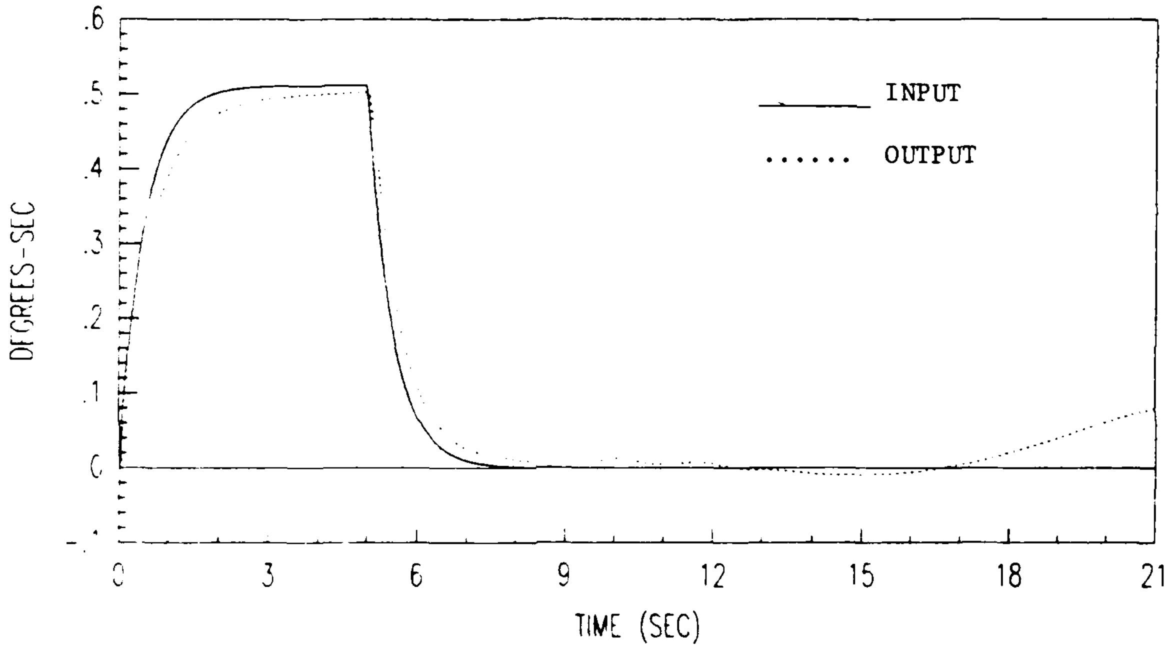


FIG. 4-11 FIXED GAIN YAW RATE RESPONSE - WITH FAILURE

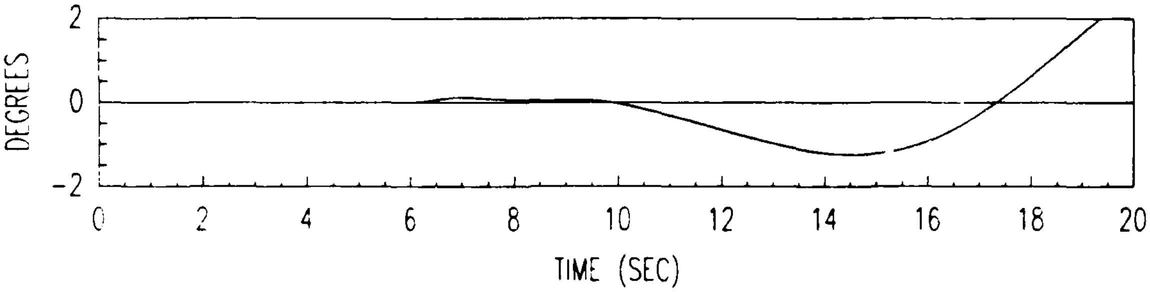


FIG. 4-12 FIXED GAIN PITCH ANGLE RESPONSE - WITH FAILURE

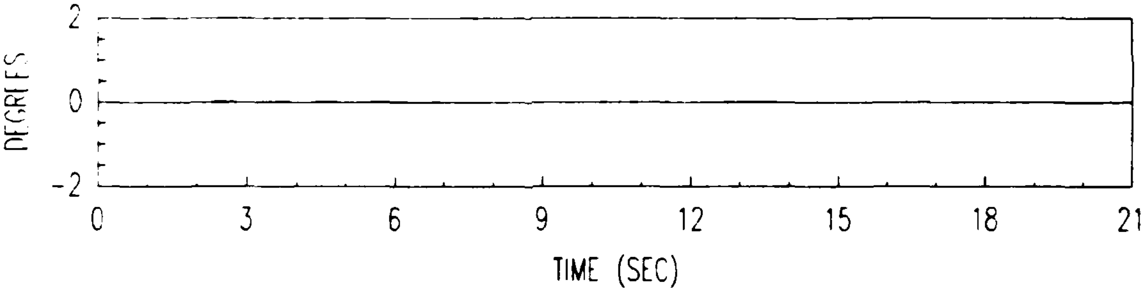


FIG. 4-13 FIXED GAIN SIDESLIP ANGLE RESPONSE - WITH FAILURE

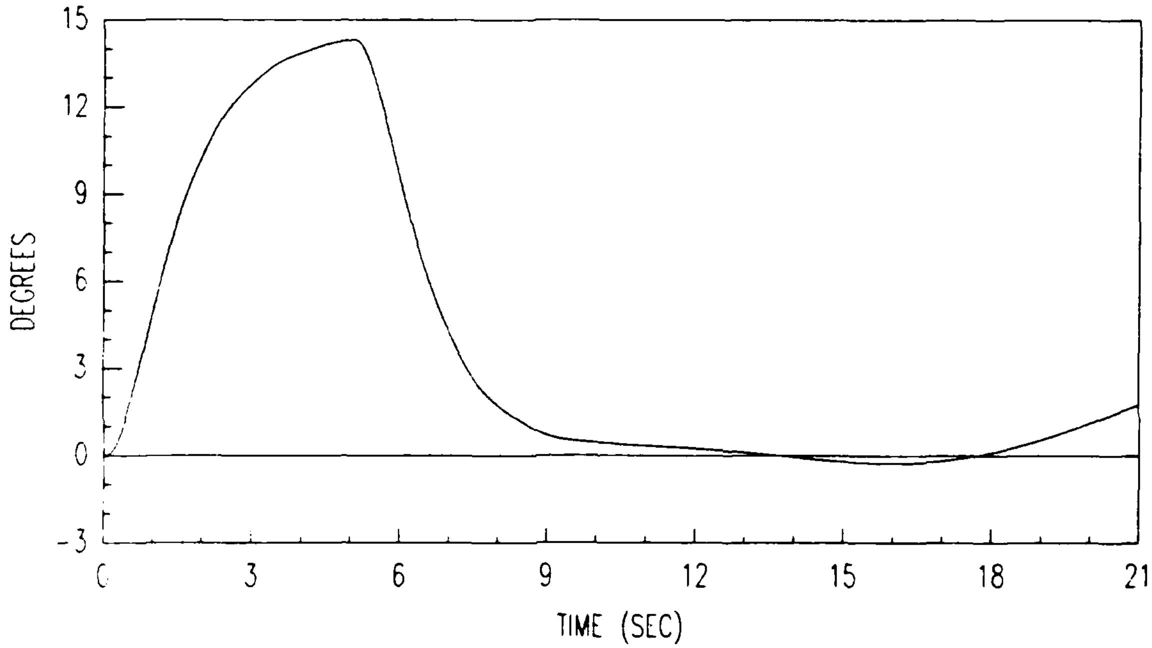


FIG. 4-14 FIXED GAIN BANK ANGLE RESPONSE - WITH FAILURE

The purpose of the adaptive controller is to identify (estimate) the first 15 elements (B1 matrix) of the parameter vector. The remaining 76 elements of the parameter vector are held constant during the simulation. The responses of the aircraft are displayed in Figure 4-15 through 4-18. Inspection of these plots reveals that the controller appears to be stable. However, inspection of the B1 matrix elements, shown in Fiqure 4-19 and Figure 4-24, reveal that the elements of B1 are very erratic after 15 seconds. In order to determine the effect this erratic estimate would have on the aircrat response if a maneuver is commanded during this time frame, the three pulse yaw rate command show in Figure 4-25 is applied as an input. Figures 4-26 through 4-29 contain the aircraft responses due to this three pulse command. As can be seen, the aircraft goes unstable. The estimates of B1 are show in Figure 4-30 through Figure 4-35. All the elements of B1 have a large burst at approximately 18 seconds. These bursts can be caused by lack of excitation in the input signal. As stated in Chapter 1, a lack of "persistently exciting" inputs can cause poor parameter estimates (12:598). Therefore to determine if the adaptive controller requires additional input excitation, the noise input shown in Figure 4-37 is applied as the yaw rate command. Aircraft responses due to this command are contained in Figure 4-36 throught Figure 4-40. These plots show that the aircraft is stable and the yaw rate channel is attempting to track the input command. The estimates of B1 shown in Figure 4-41 through 4-46 do not have the large burst observed when the three pulse input is applied. The next step taken is to fail the left elevator by 50 % and observe if the adaptive

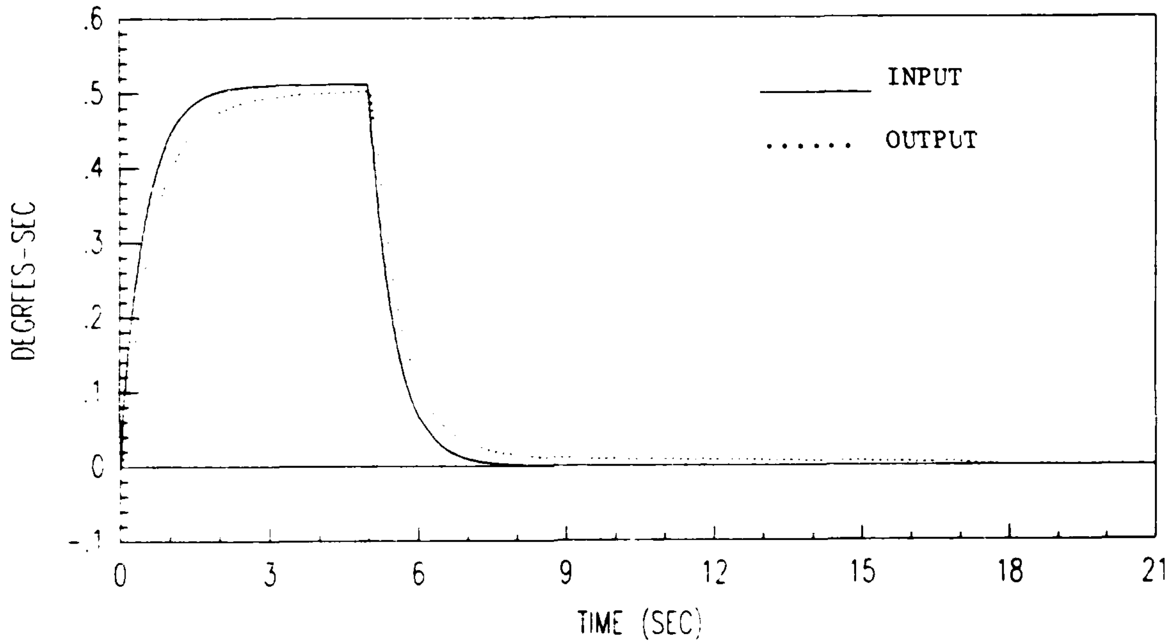


FIG. 4-15 ADAPTIVE YAW RATE RESPONSE

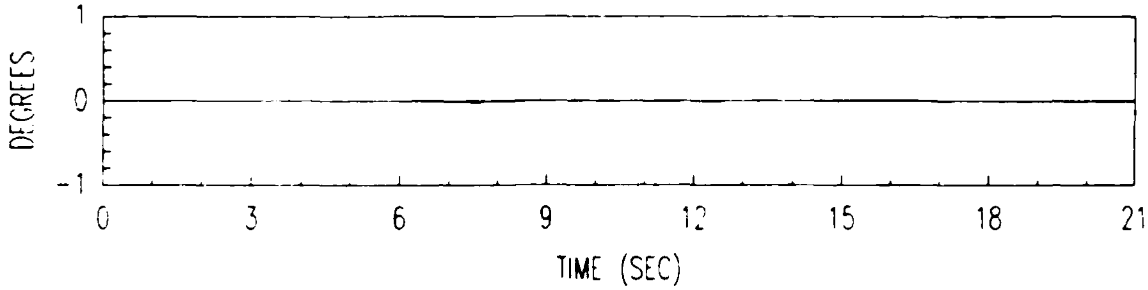


FIG. 4-16 ADAPTIVE PITCH ANGLE RESPONSE

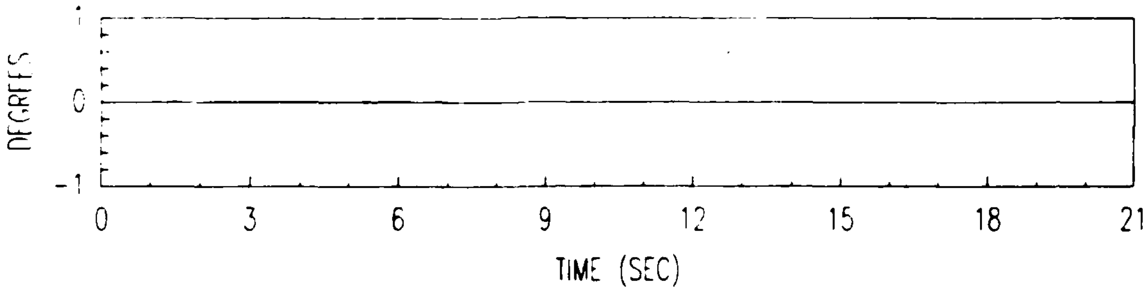


FIG. 4-17 ADAPTIVE SIDESLIP ANGLE RESPONSE

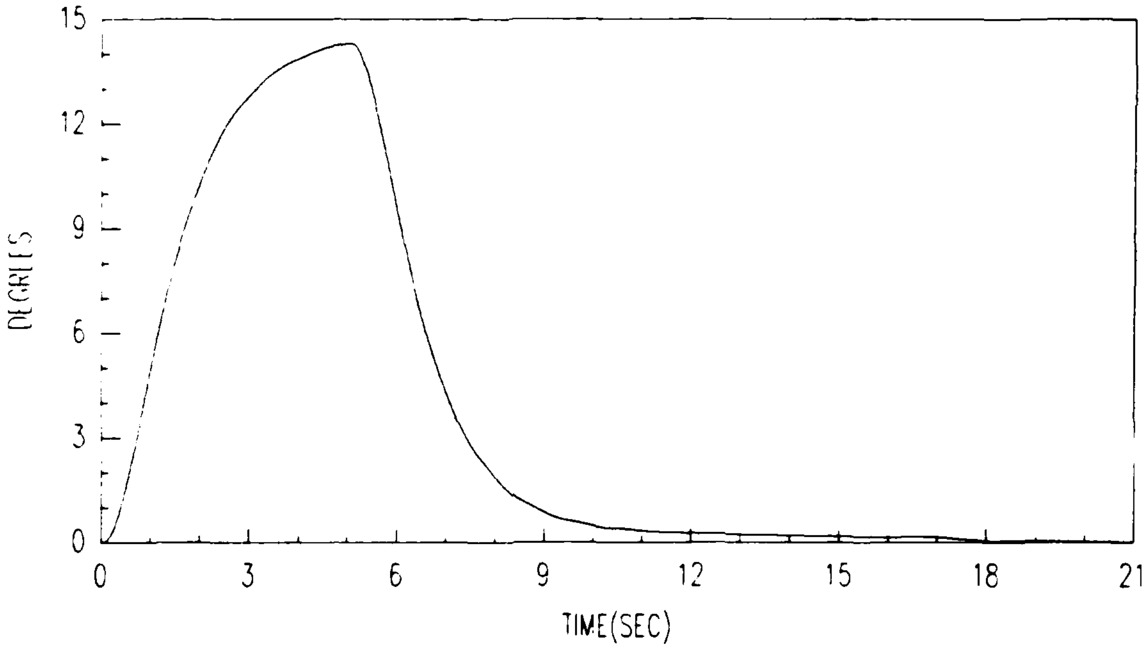


FIG. 4-18 ADAPTIVE BANK ANGLE RESPONSE

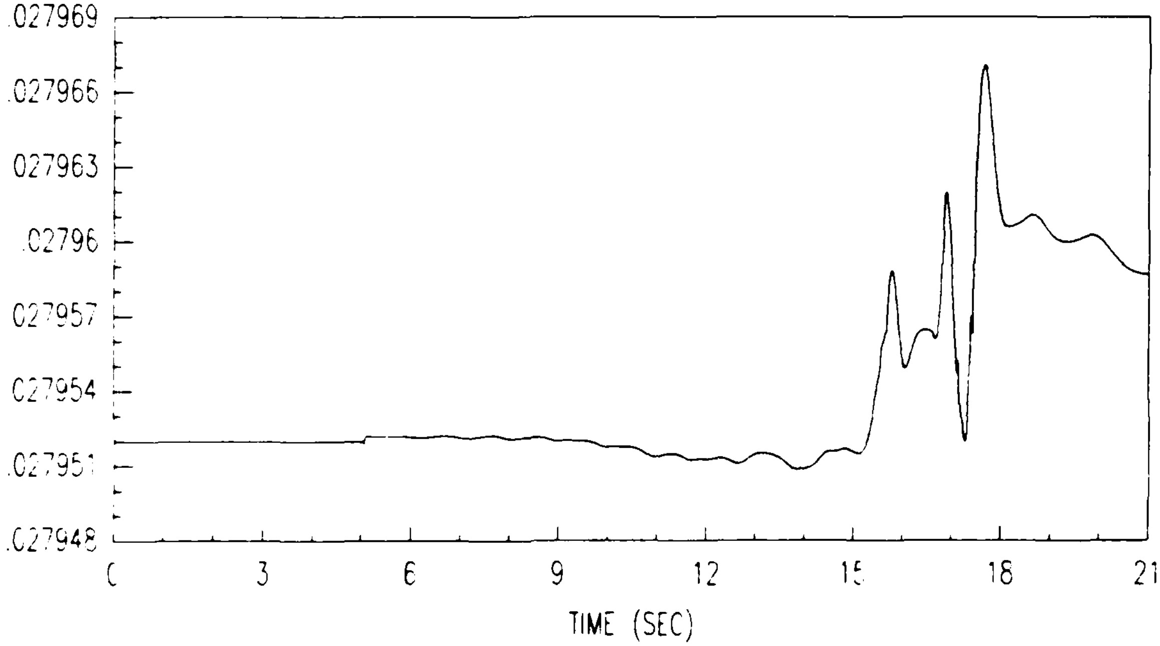


FIG. 4-19 B1(1,1) ESTIMATE - ONE PULSE COMMAND

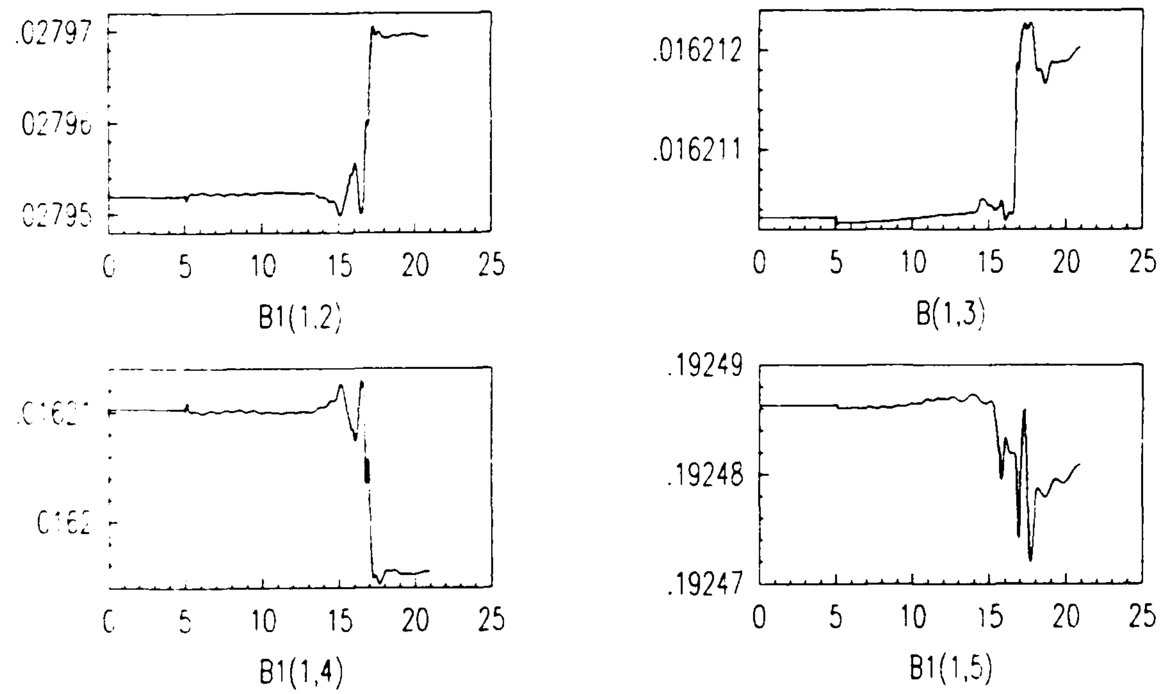


FIG. 4-20 B1(1,2) TO B1(1,5) ESTIMATES - ONE PULSE COMMAND

X AXIS = SECONDS

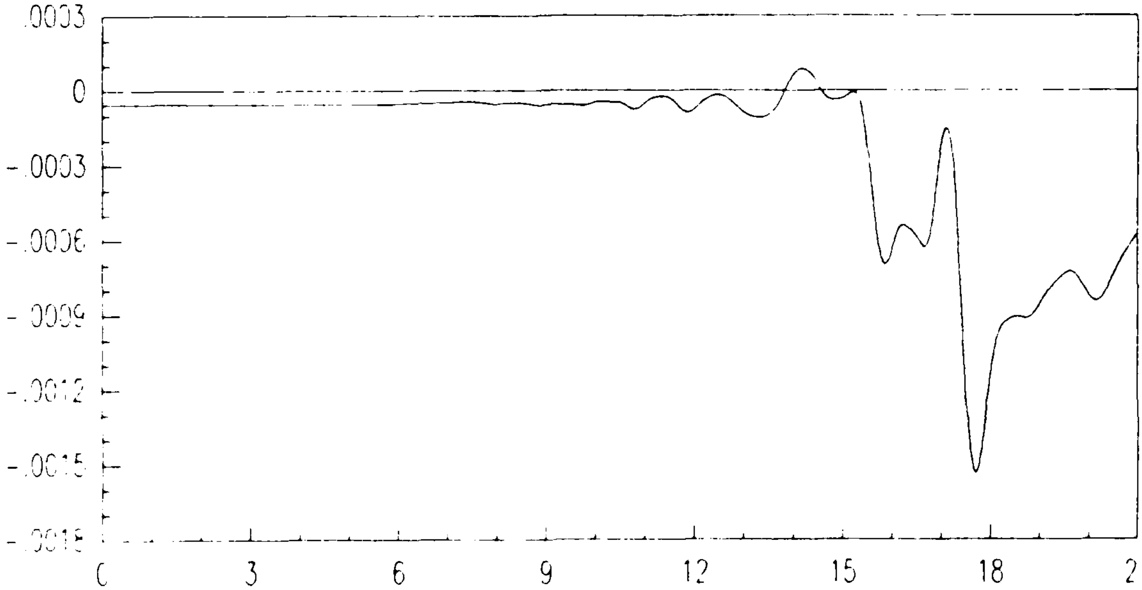


FIG. 4-21 B1(2,1) ESTIMATE - ONE PULSE COMMAND

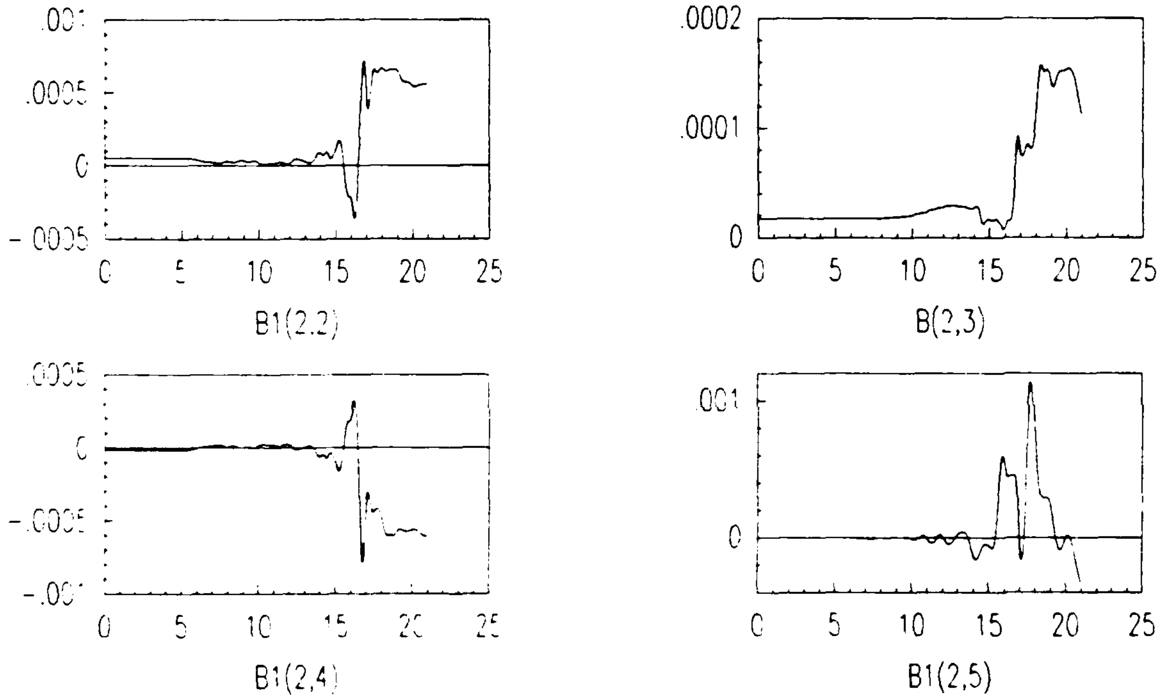


FIG. 4-22 B1(2,2) TO B1(2,5) ESTIMATES - ONE PULSE COMMAND

X AXIS = SECONDS

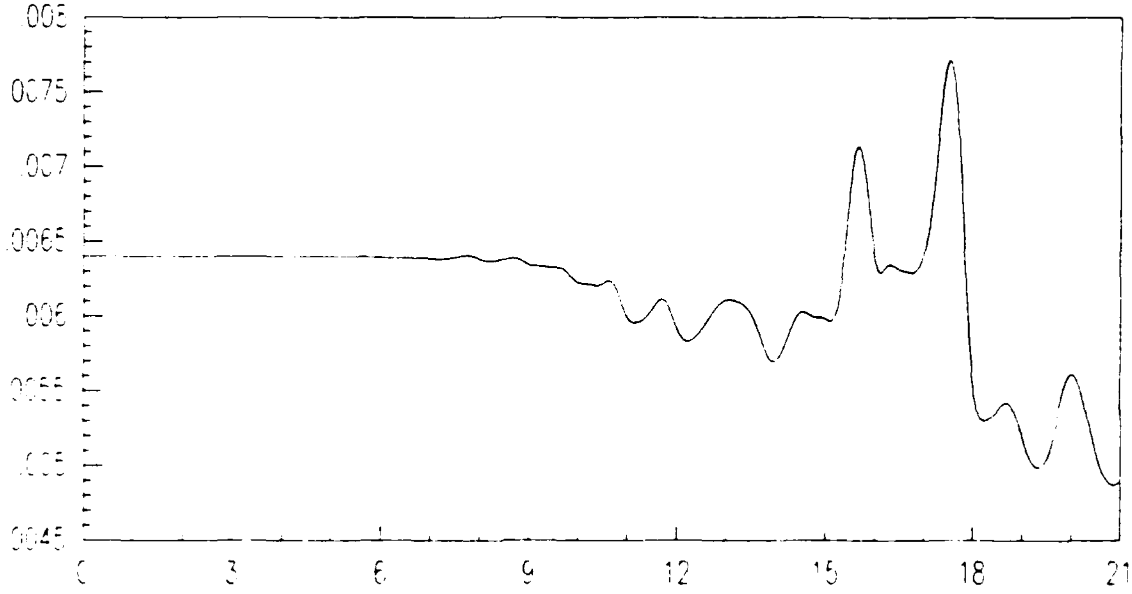


FIG. 4-23 B1(3,1) ESTIMATE - ONE PULSE COMMAND

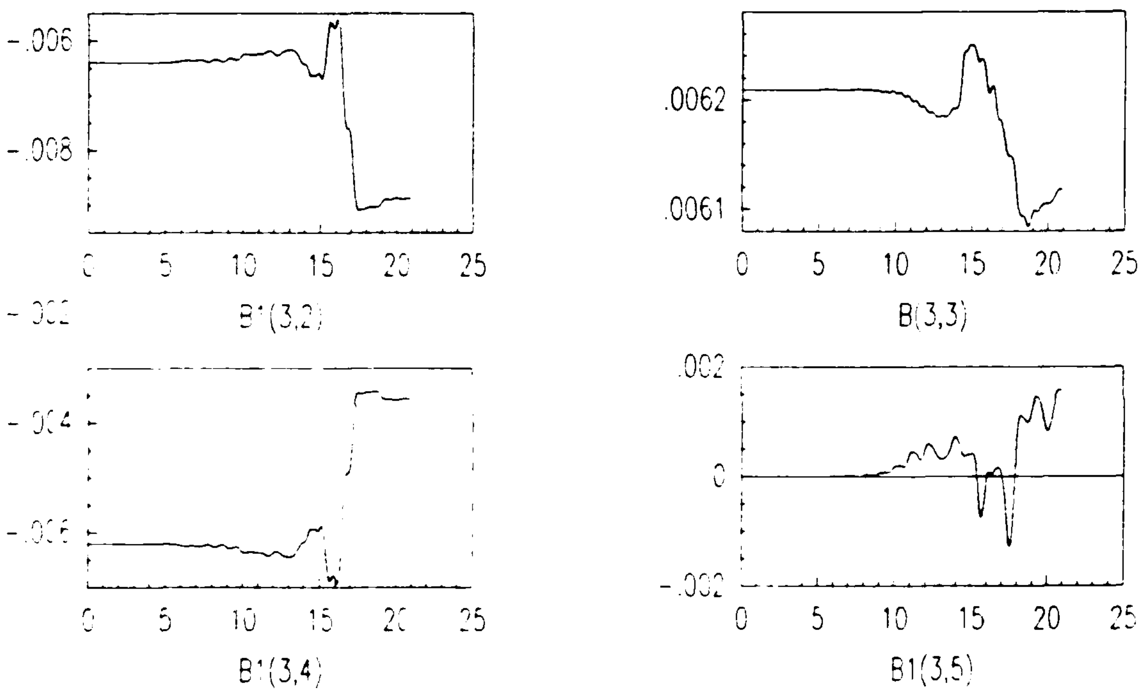


FIG. 4-24 B1(3,2) TO B1(3,5) ESTIMATES - ONE PULSE COMMAND

X AXIS = SECONDS

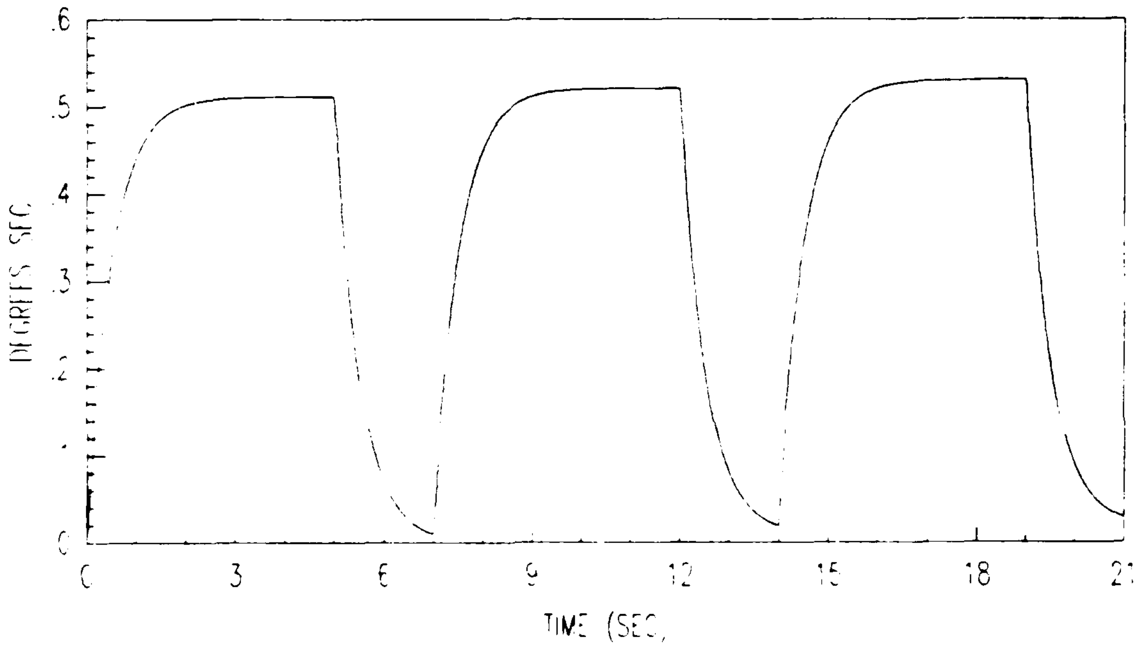


FIG. 4-25 YAW RATE COMMAND - THREE PULSES

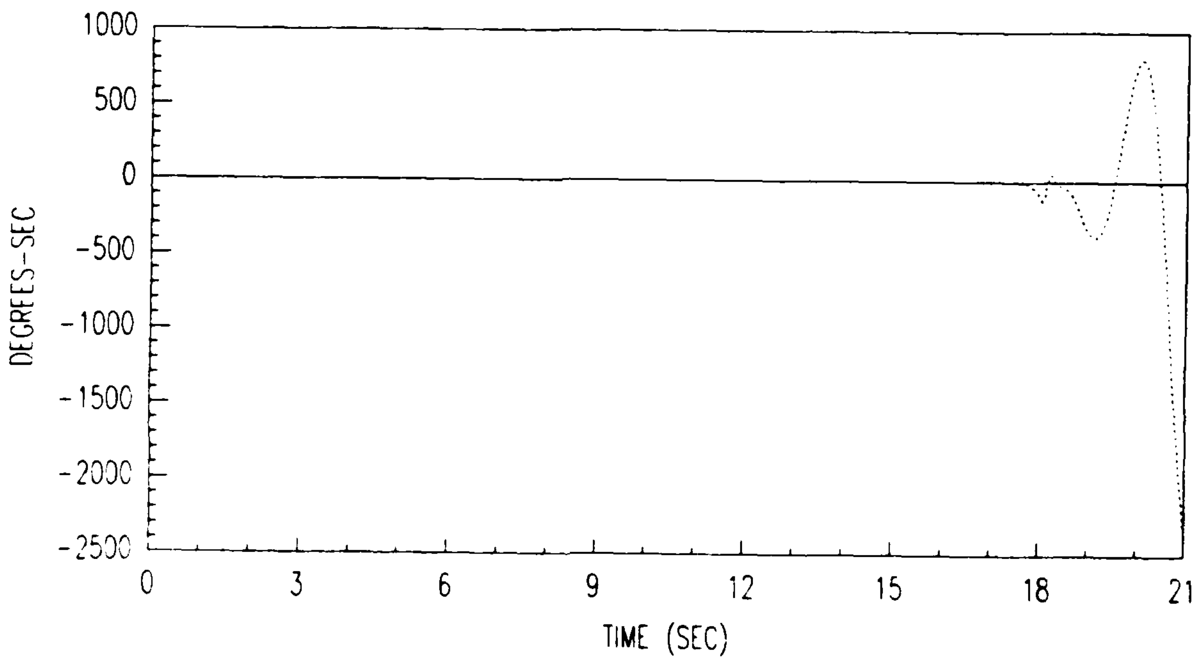


FIG. 4-26 ADAPTIVE YAW RATE RESPONSE - THREE PULSES

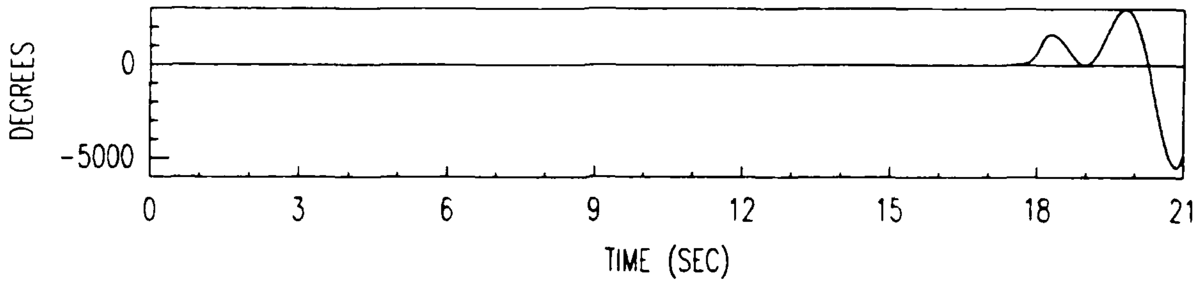


FIG. 4-27 ADAPTIVE PITCH ANGLE RESPONSE - THREE PULSES

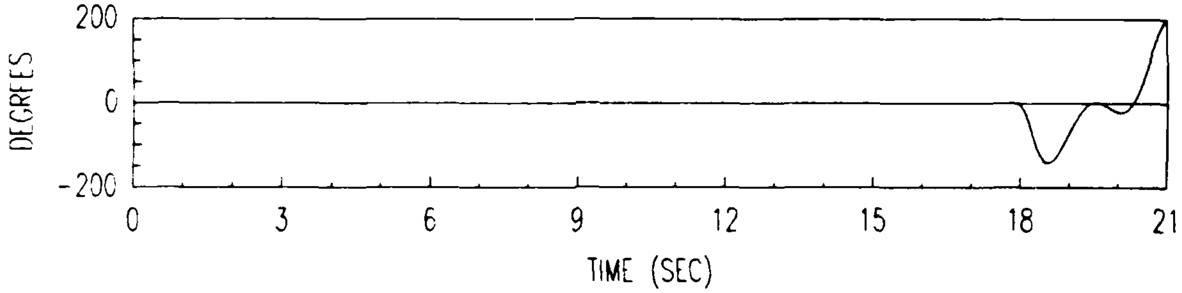


FIG. 4-28 ADAPTIVE SIDESLIP ANGLE RESPONSE - THREE PULSES

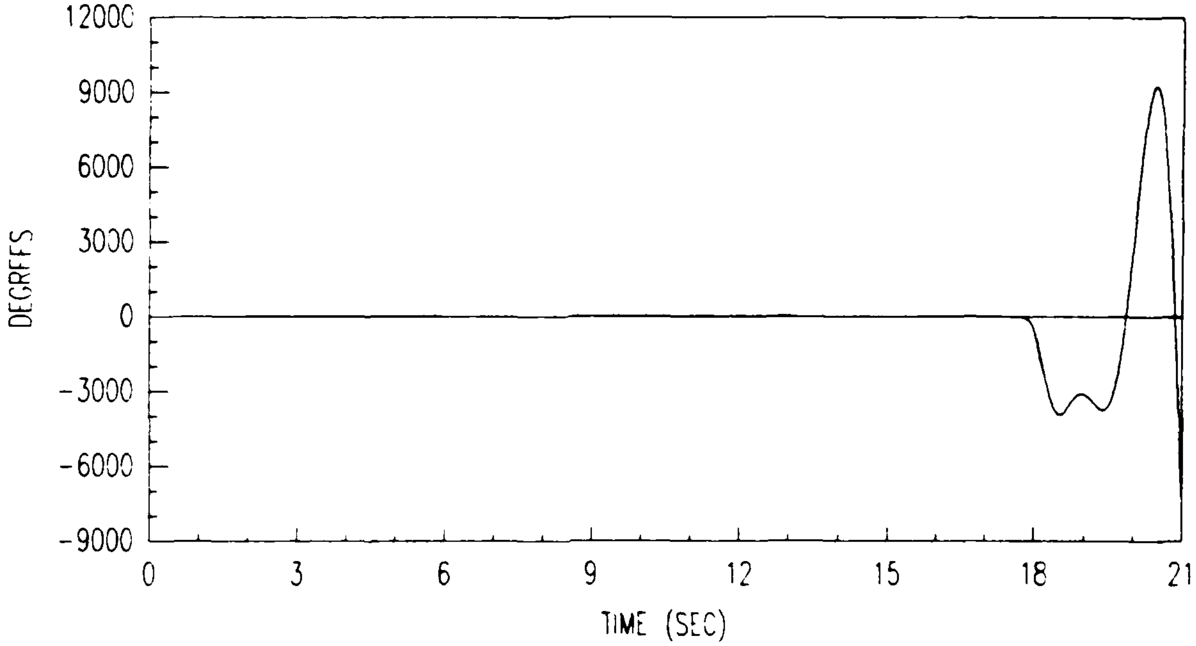


FIG. 4-29 ADAPTIVE BANK ANGLE RESPONSE - THREE PULSES

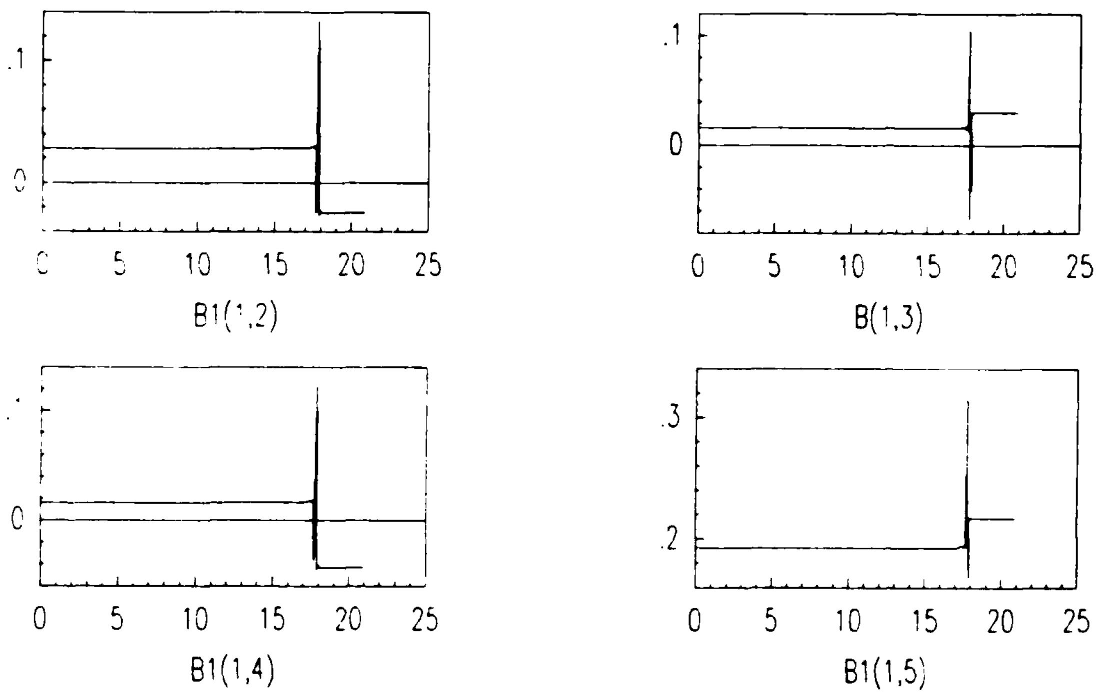


FIG. 4-31 B1(1,2) TO B1(1,5) ESTIMATES - THREE PULSE COMMAND

X AXIS = SECONDS

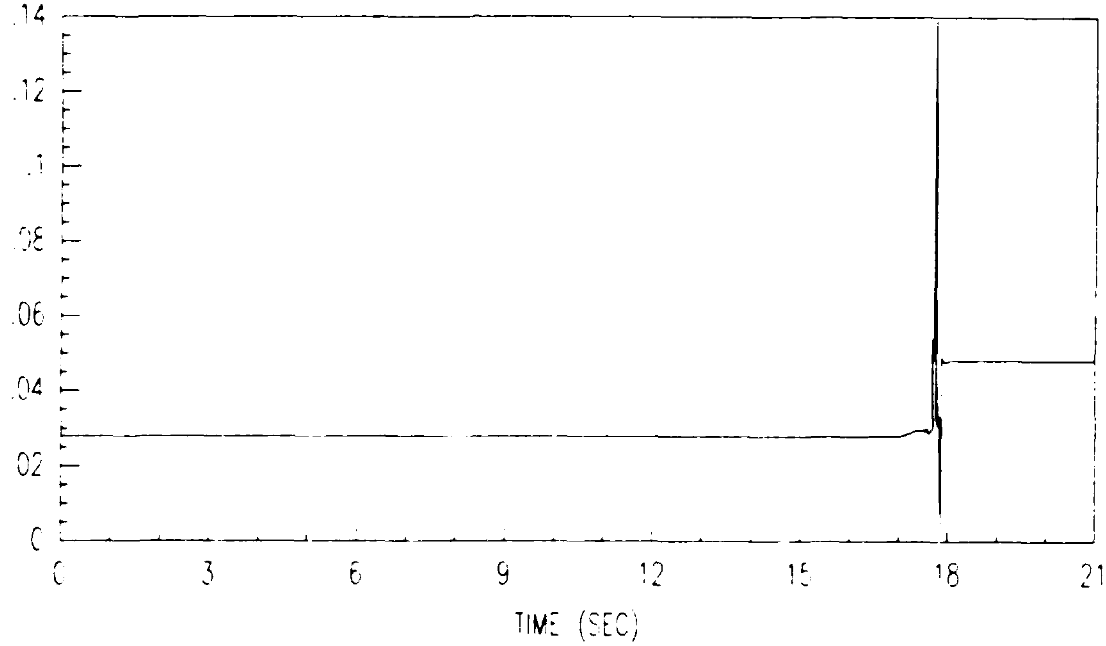


FIG. 4-30 B1(1,1) ESTIMATE - THREE PULSE COMMAND

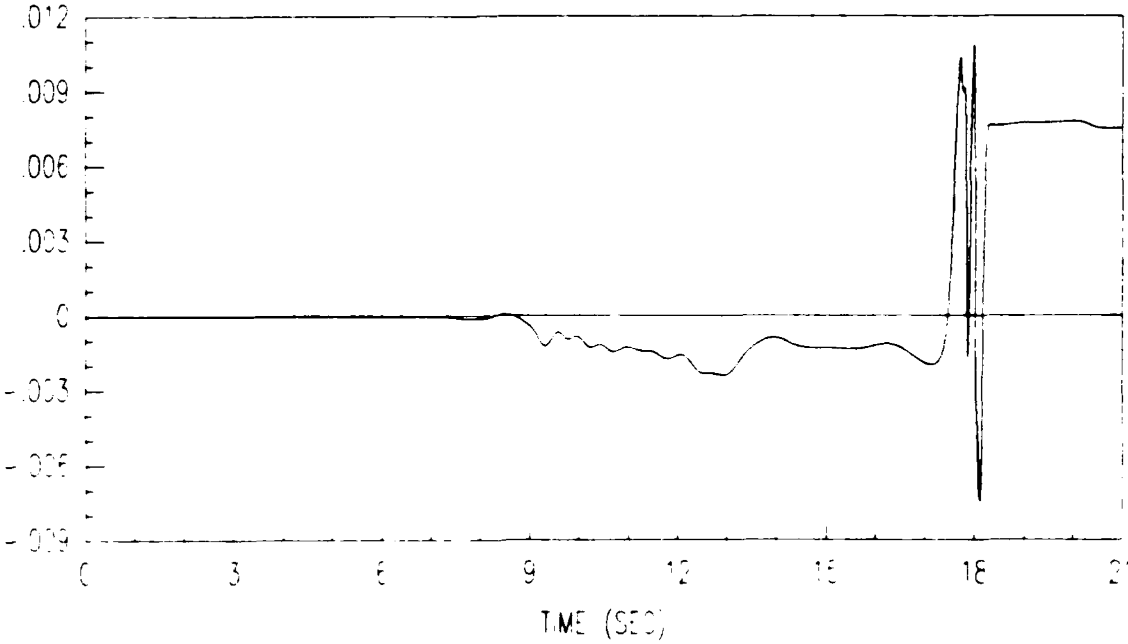


FIG. 4-32 B1(2,1) ESTIMATE - THREE PULSE COMMAND

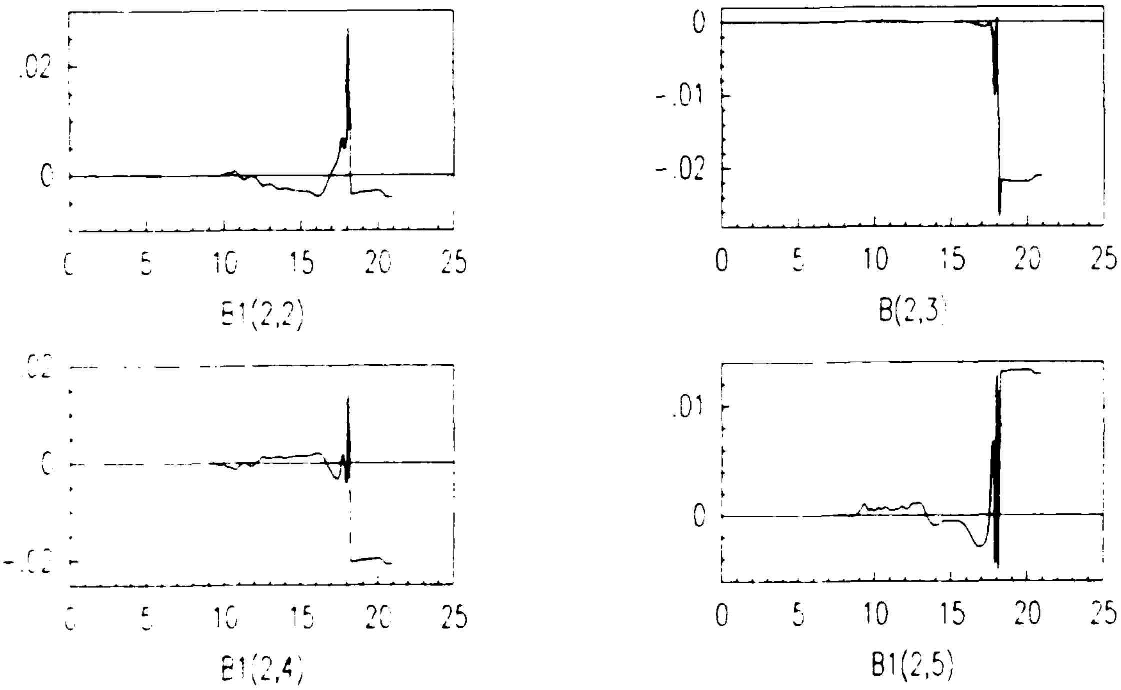


FIG 4-33 B1(2,2) TO B1(2,5) ESTIMATES - THREE PULSE COMMAND

X AXIS = SECONDS

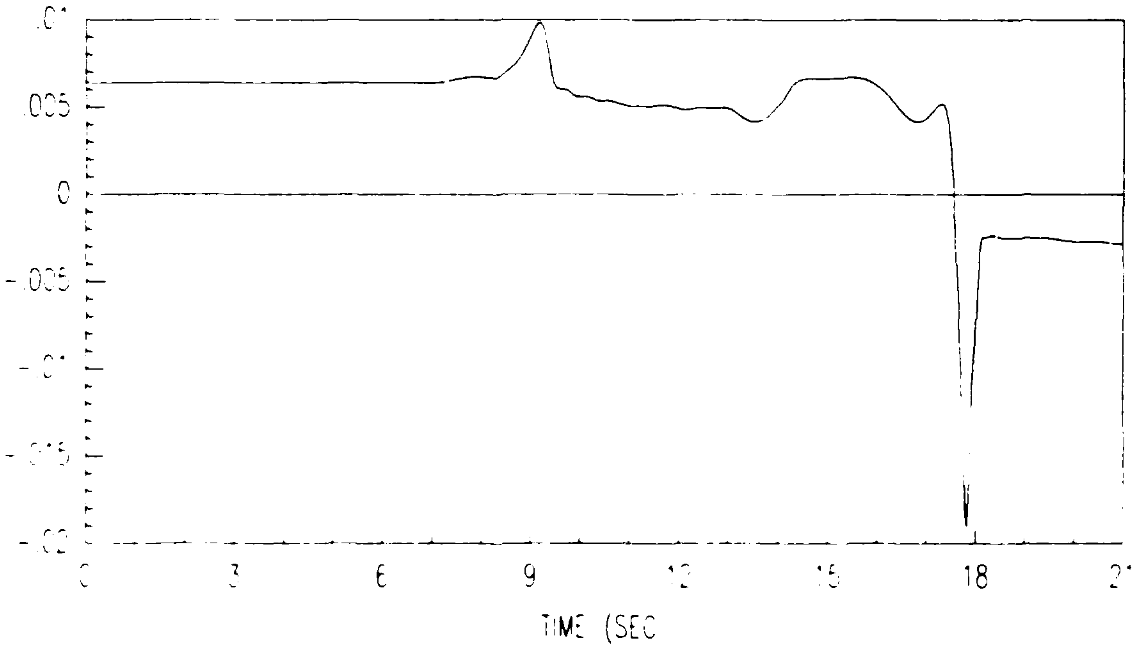


FIG. 4-34 B1(3,1) ESTIMATE - THREE PULSE COMMAND

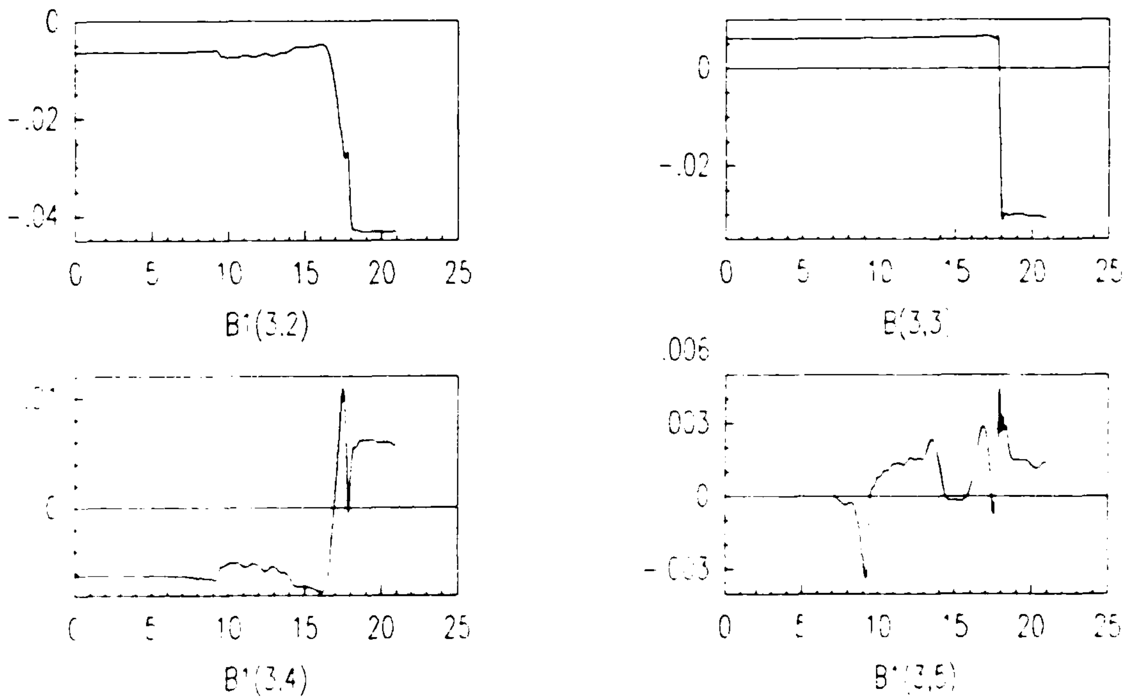


FIG 4-35 B1(3,2) TO B1(3,5) ESTIMATES - THREE PULSE COMMAND

X AXIS = SECONDS

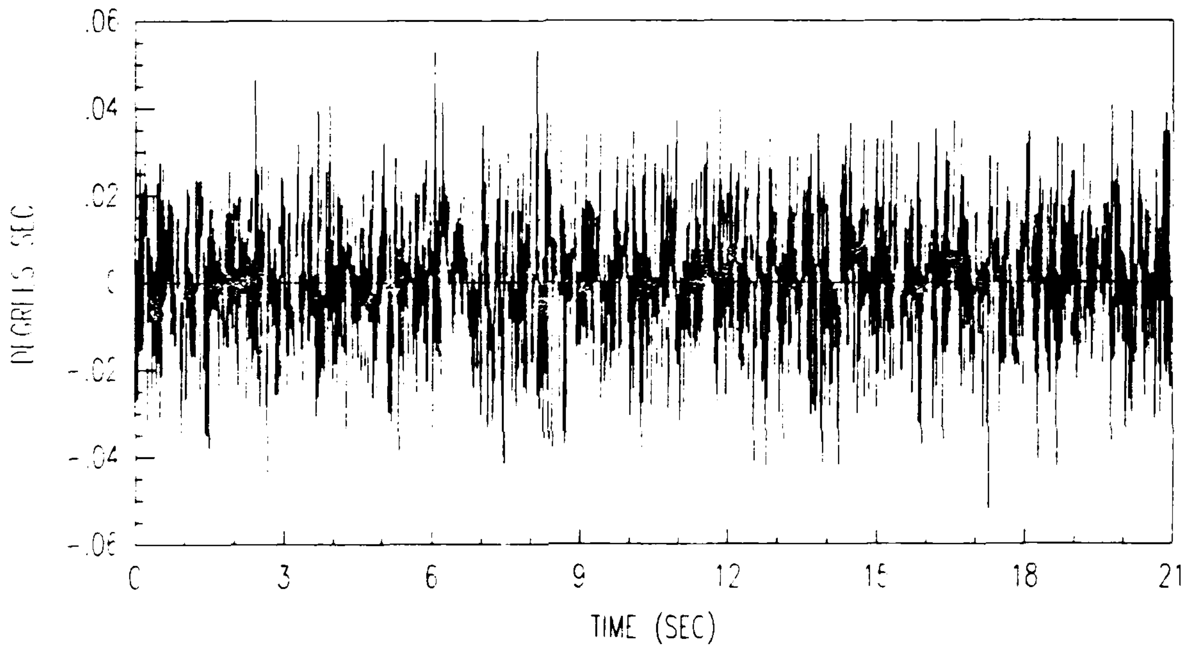


FIG. 4-36 YAW RATE NOISE COMMAND

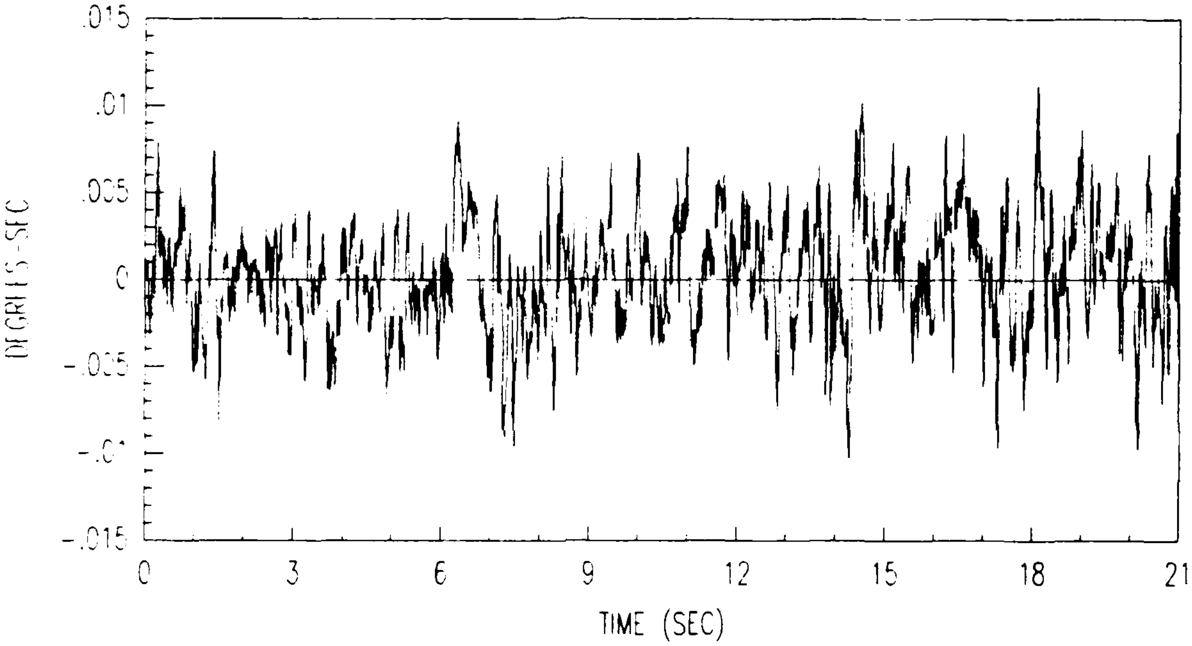


FIG. 4-37 ADAPTIVE YAW RATE RESPONSE - NOISE COMMAND

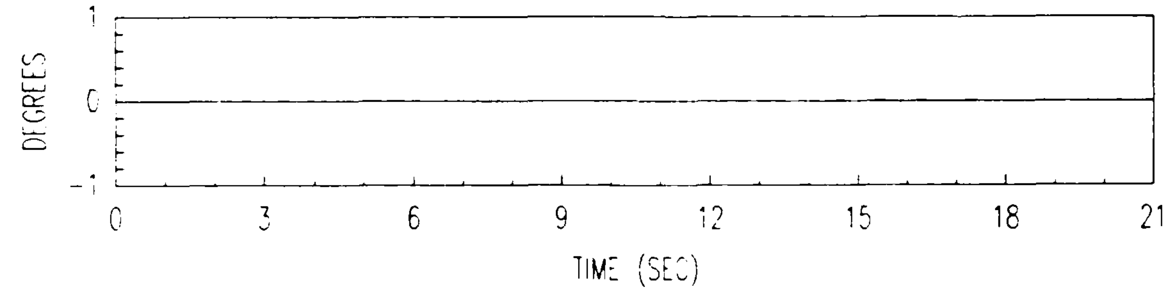


FIG. 4-38 ADAPTIVE PITCH ANGLE RESPONSE - NOISE CMD

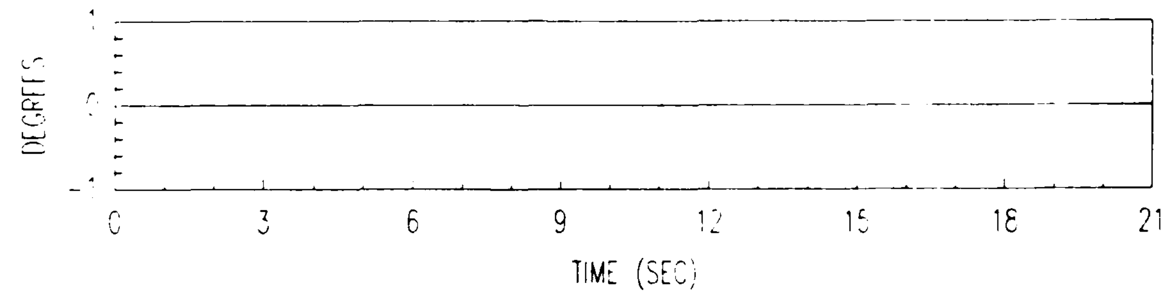


FIG. 4-39 ADAPTIVE SIDESLIP ANGLE RESPONSE - NOISE CMD

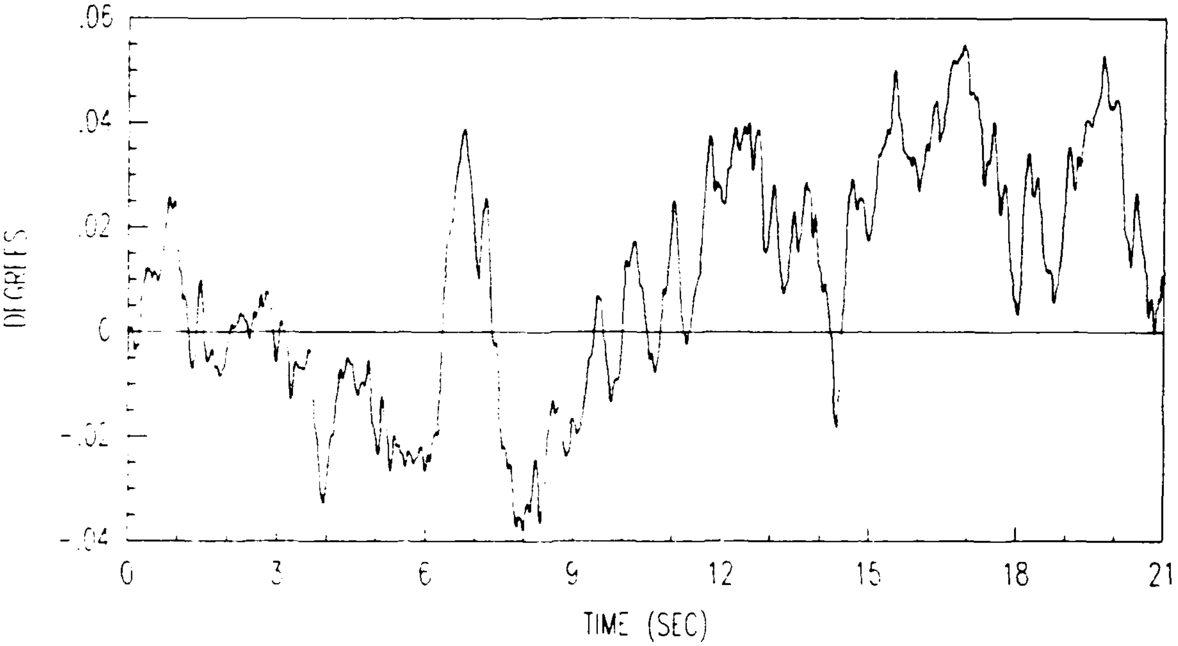


FIG. 4-40 ADAPTIVE BANK ANGLE RESPONSE - NOISE CMD

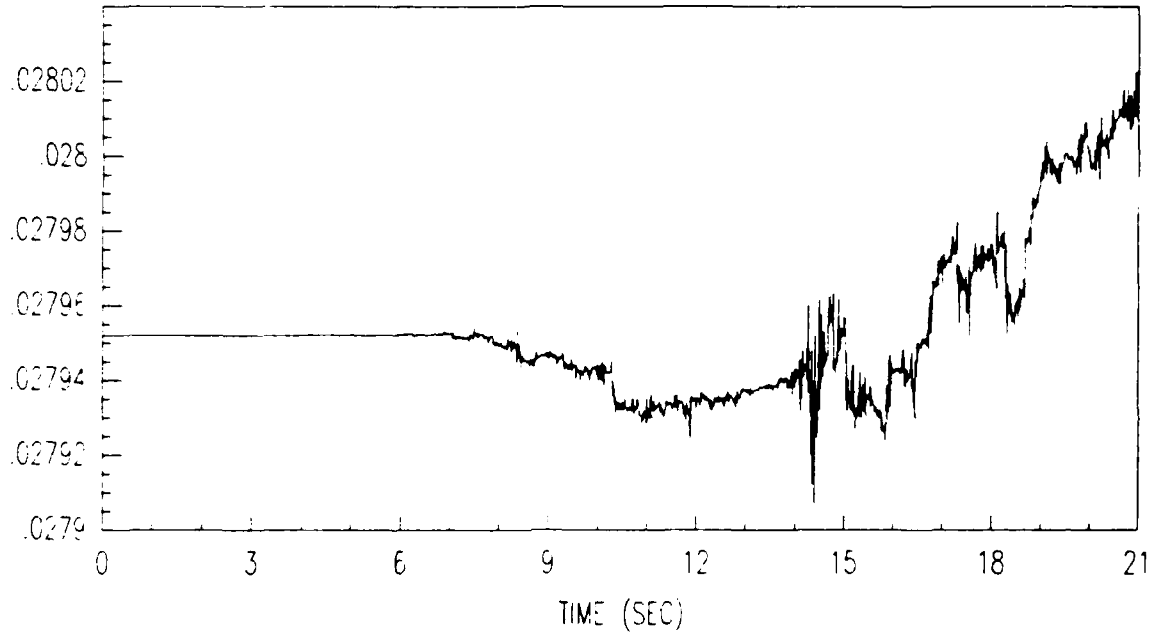


FIG. 4-41 B1(1,1) ESTIMATE - NOISE COMMAND

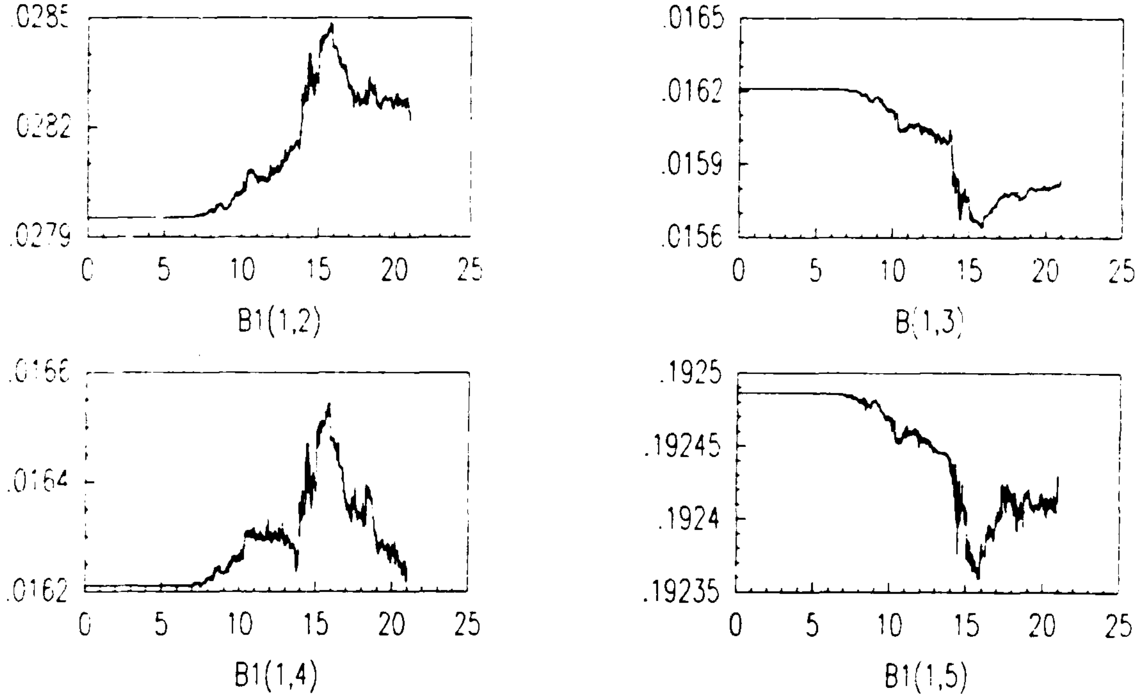


FIG. 4-42 B1(1,2) TO B1(1,5) ESTIMATES - NOISE COMMAND

X AXIS = SECONDS

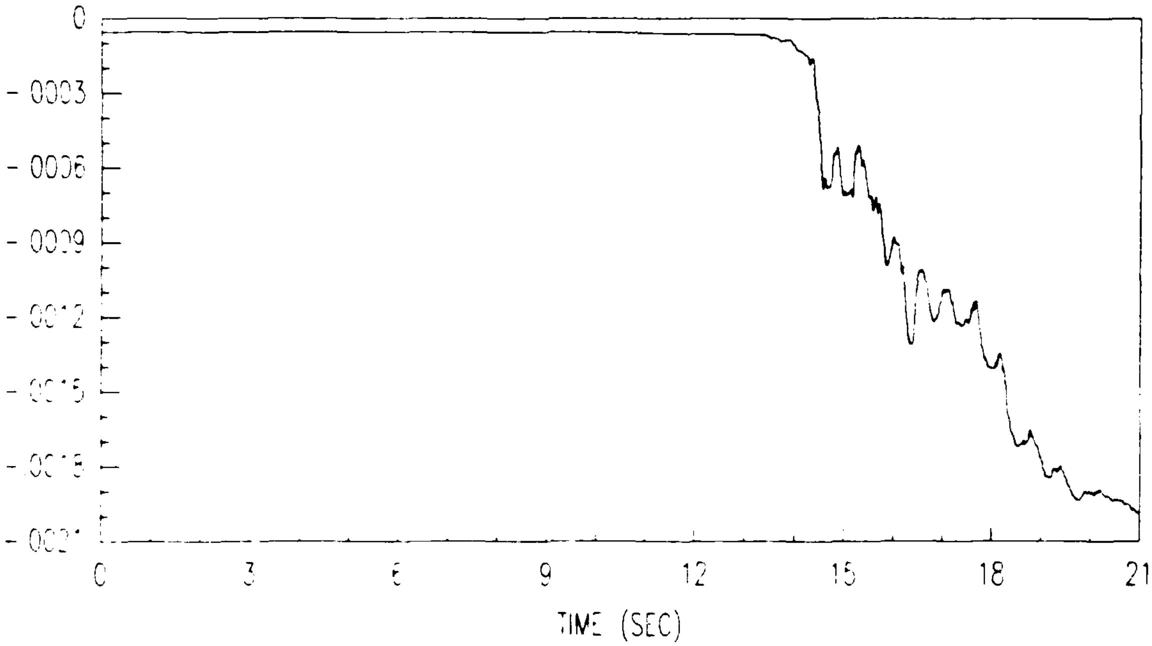


FIG 4-43 B'(2,1) ESTIMATE - NOISE COMMAND

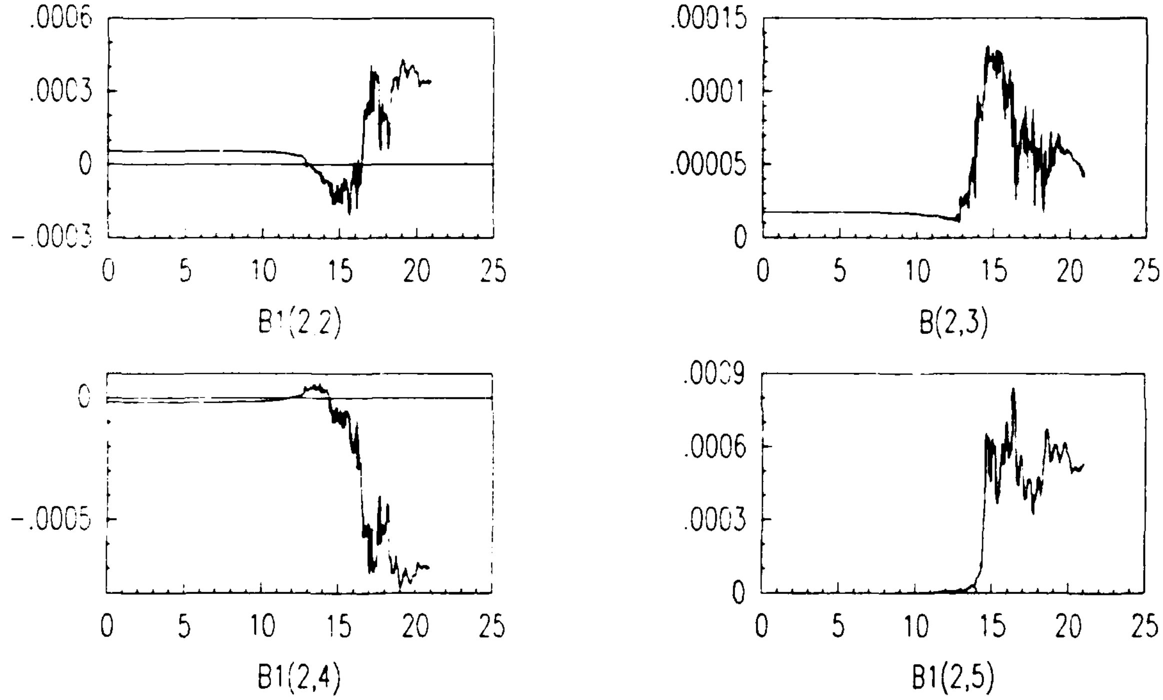


FIG. 4-44 B1(2,2) TO B1(2,5) ESTIMATES - NOISE COMMAND

X AXIS = SECONDS

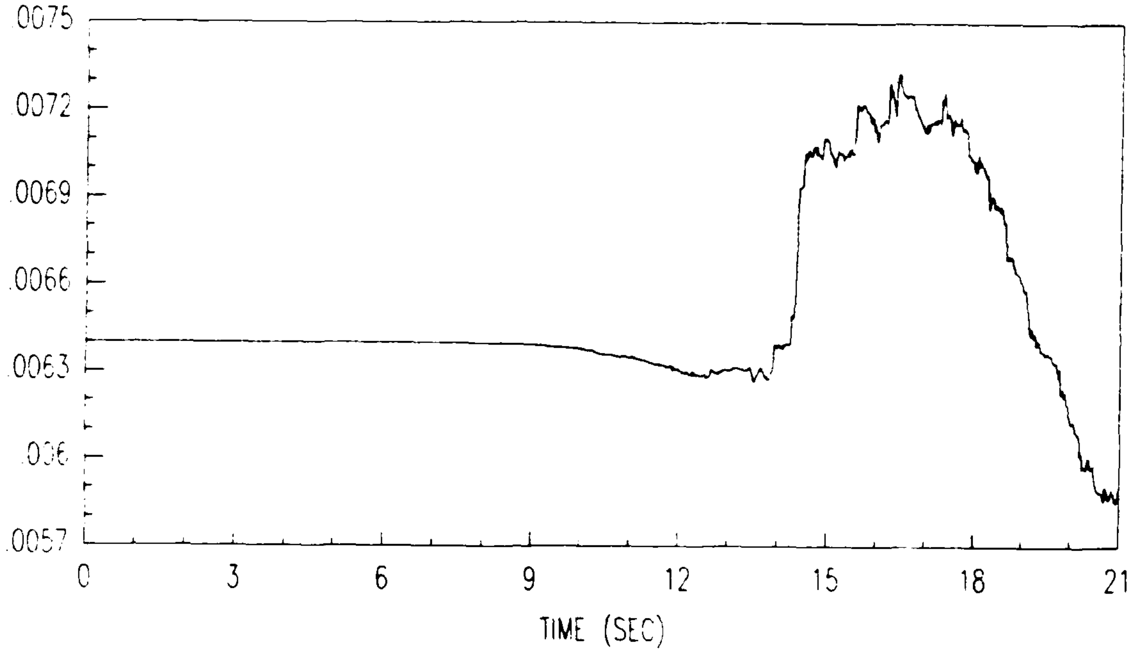


FIG. 4-45 B1(3,1) ESTIMATE - NOISE COMMAND

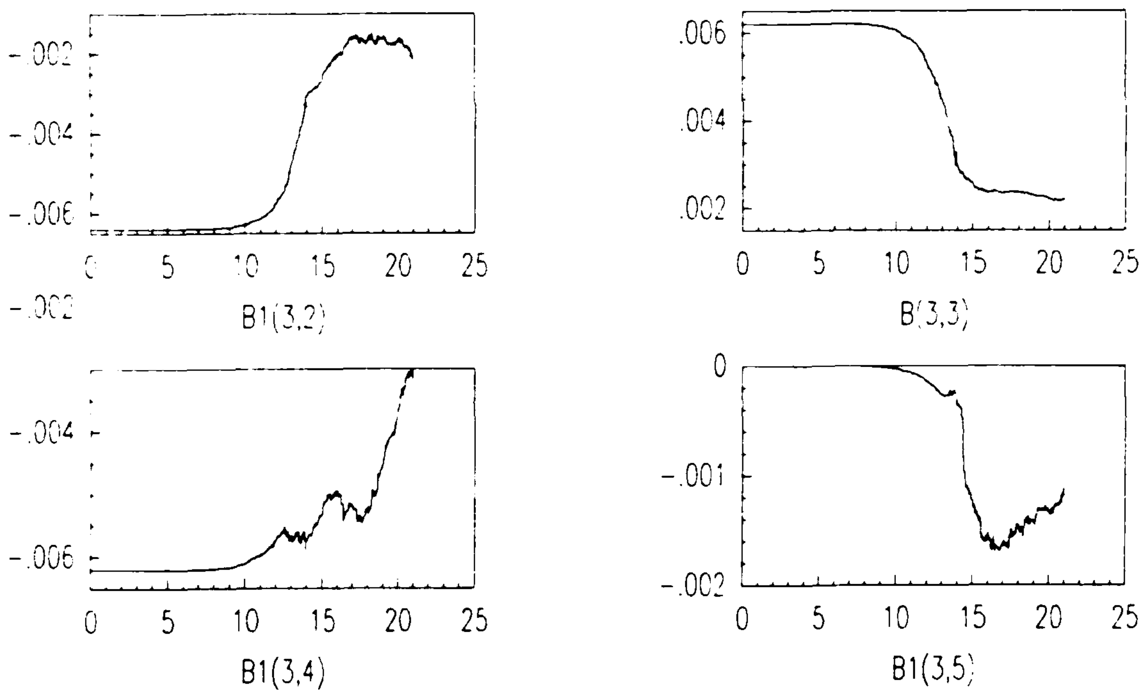


FIG. 4-46 B1(3,2) TO B1(3,5) ESTIMATES - NOISE COMMAND

X AXIS = SECONDS

controller identifies the new plant B1 matrix. The B1(1.1). B1(2.1). and B1(3.1) elements of the failed model are one half the value of the healthy model. The remaining elements of B1 are the same for both plants.

The failure is introduced six seconds into the simulation. The response of the aircraft due to this failure (with the noise input) for the fixed gain controller is required so as to allow comparison of the fixed and adaptive controller performance.

The results of this simulation for the fixed gain are shown in Figure 4-47 through Fiqure 4-50. The results of this simulation for the adaptive gain controller are shown in Figure 4-51 through Figure 4-55. Comparing the yaw rate and bank angle of both controllers reveals that the fixed gain controller is going unstable toward the end of the simulation: whereas. the adaptive controller responses are becoming more stable. The reason the adaptive controller is more stable than the fixed gain controller is because the estimates of B1(1.1) and B1(3.1) are becoming smaller. as required. However. the estimated negative value of B1(2.1) is increasing when it should have been decreasing. The value of B1(2.1) is very small when compared to the value of B1(1.1) & B(3.1) and the response of the system may be dominated by the effects of B1(1.1) and B1(3.1).

To determine if the adaptive controller is attempting to converge on the correct plant parameters requires addtional testing. Also. tuning and/or modification to the adaptive controller is required to decrease the convergence time. All of these options required more time then is presently allocated to this research: therefore.

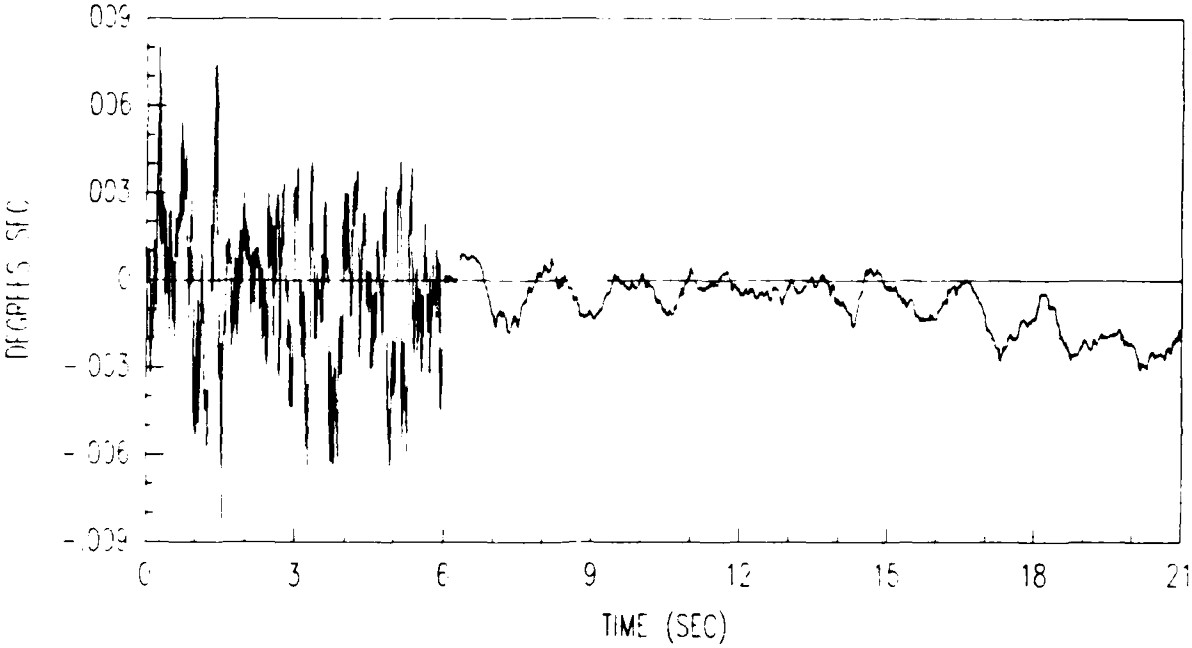


FIG. 4-47 FIXED GAIN YAW RATE RESPONSE - NOISE + FAILURE

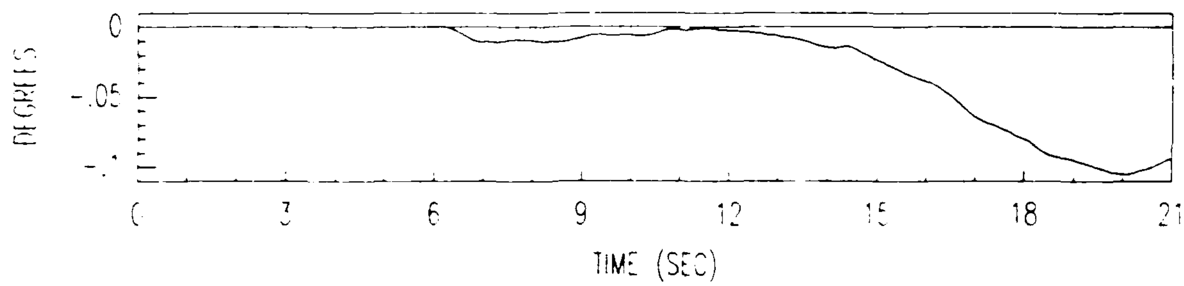


FIG. 4-48 FIXED GAIN PITCH ANGLE RESPONSE - NOISE + FAILURE

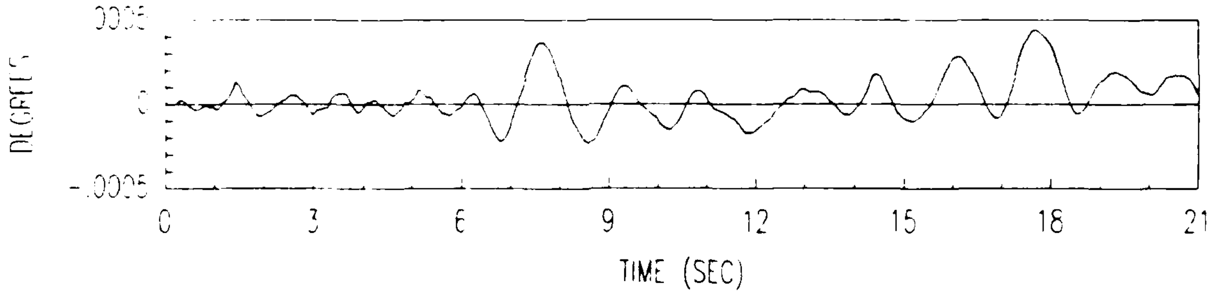


FIG. 4-49 FIXED GAIN SIDESLIP ANGLE - NOISE + FAILURE

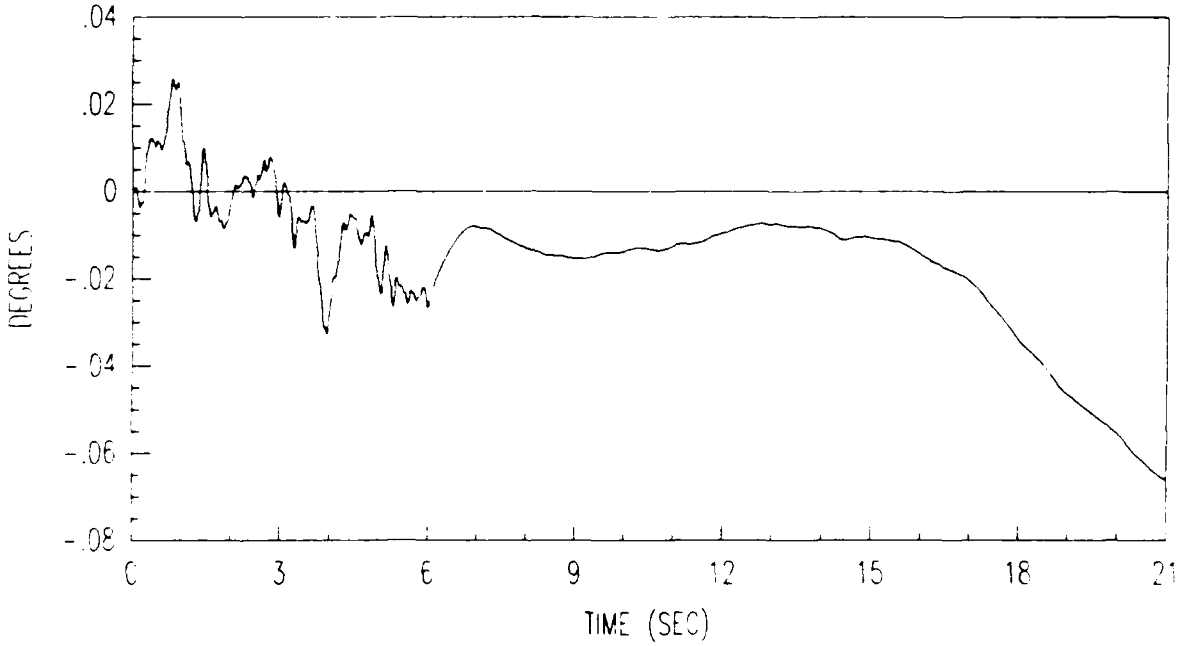
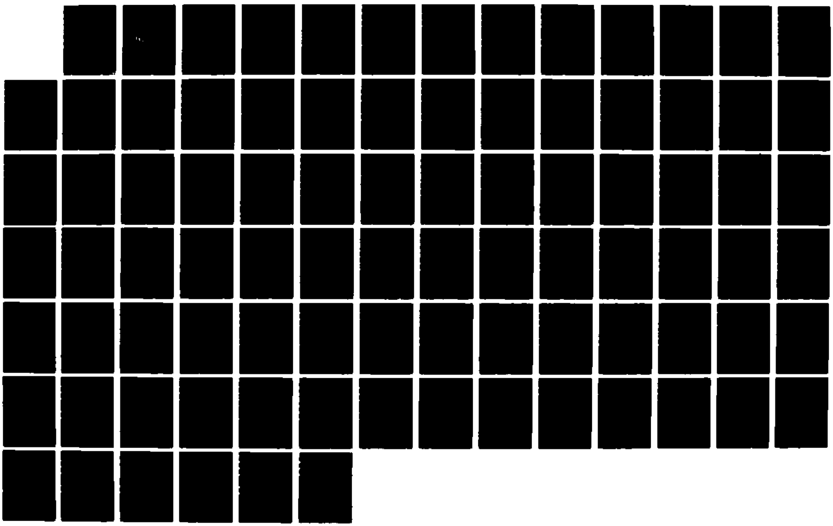
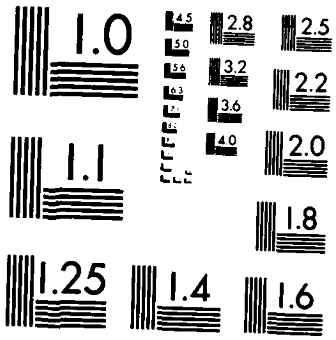


FIG. 4-50 FIXED GAIN BANK ANGLE RESPONSE - NOISE + FAILURE

AD-A189 848 MULTIVARIABLE CONTROL LAW DESIGN FOR THE AFTI/F-16 WITH 2/2
A FAILEDCCONTROL (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI J E VELEZ
UNCLASSIFIED DEC 87 AFIT/GE/ENG/87D-69 F/G 1/4 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

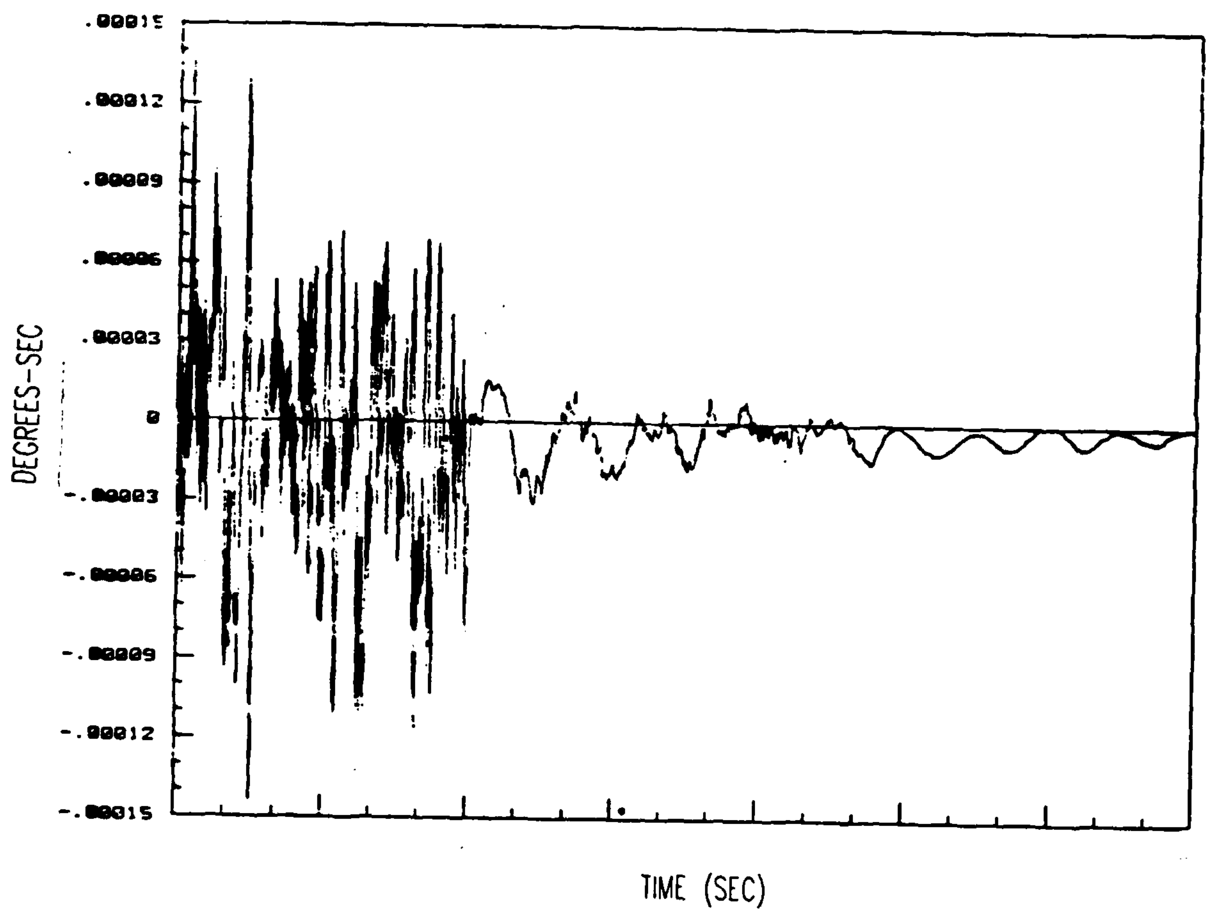
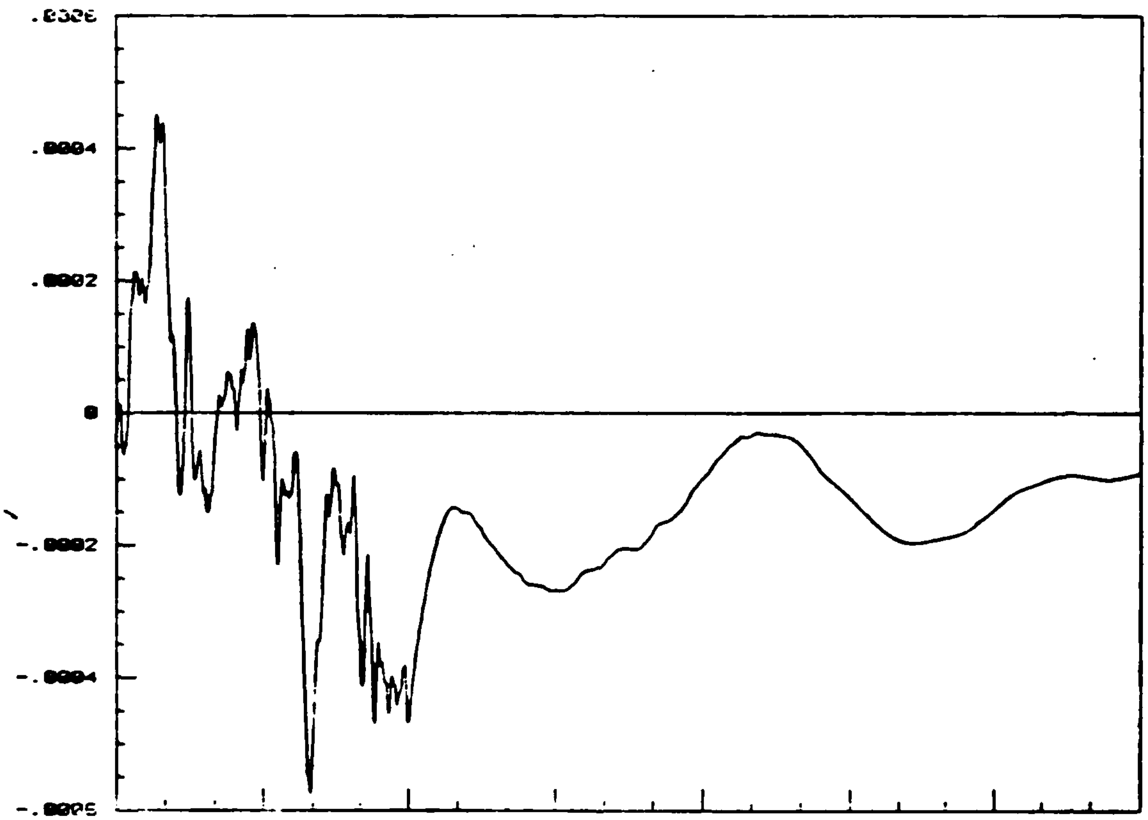


FIG. 4-51 ADAPTIVE GAIN YAW RATE - NOISE + FAILURE



TIME (SEC)

FIG. 4-52 ADAPTIVE GAIN BANK ANGLE - NOISE + FAILURE

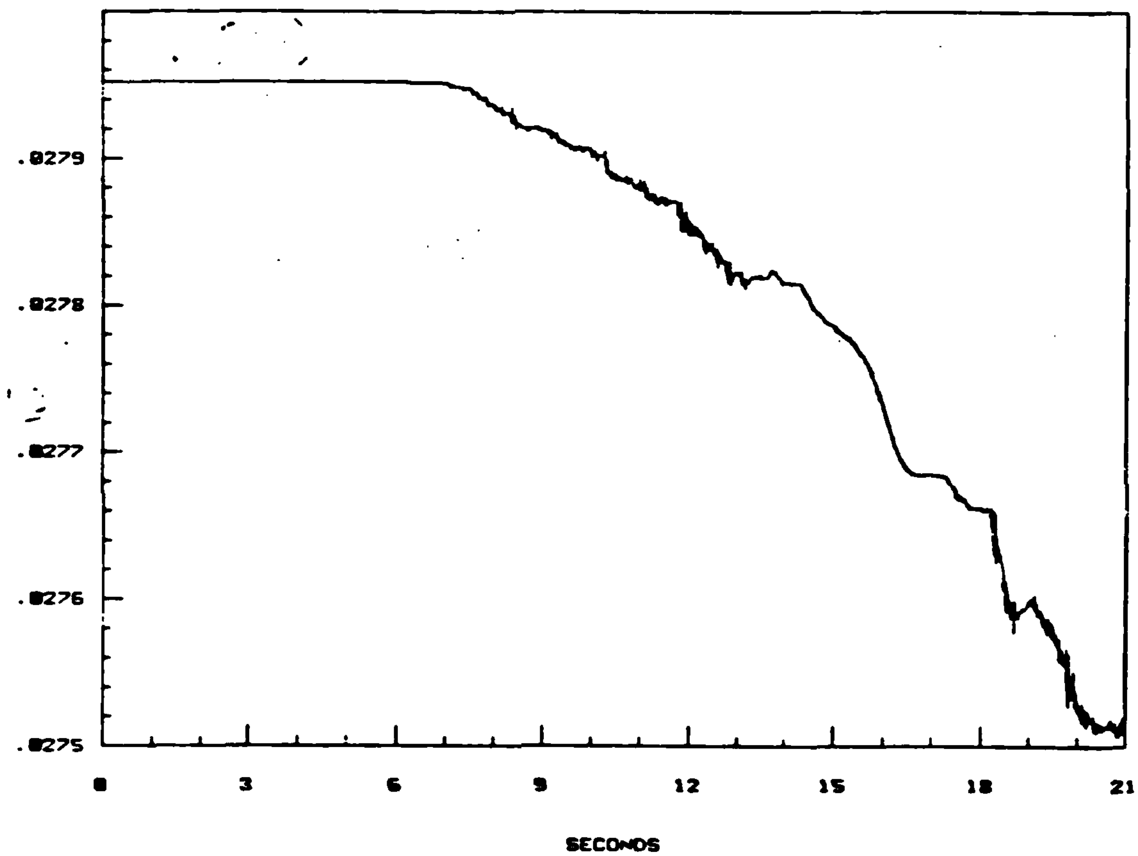


FIG. 4-53 B1(1,1) ESTIMATE - NOISE + FAILURE

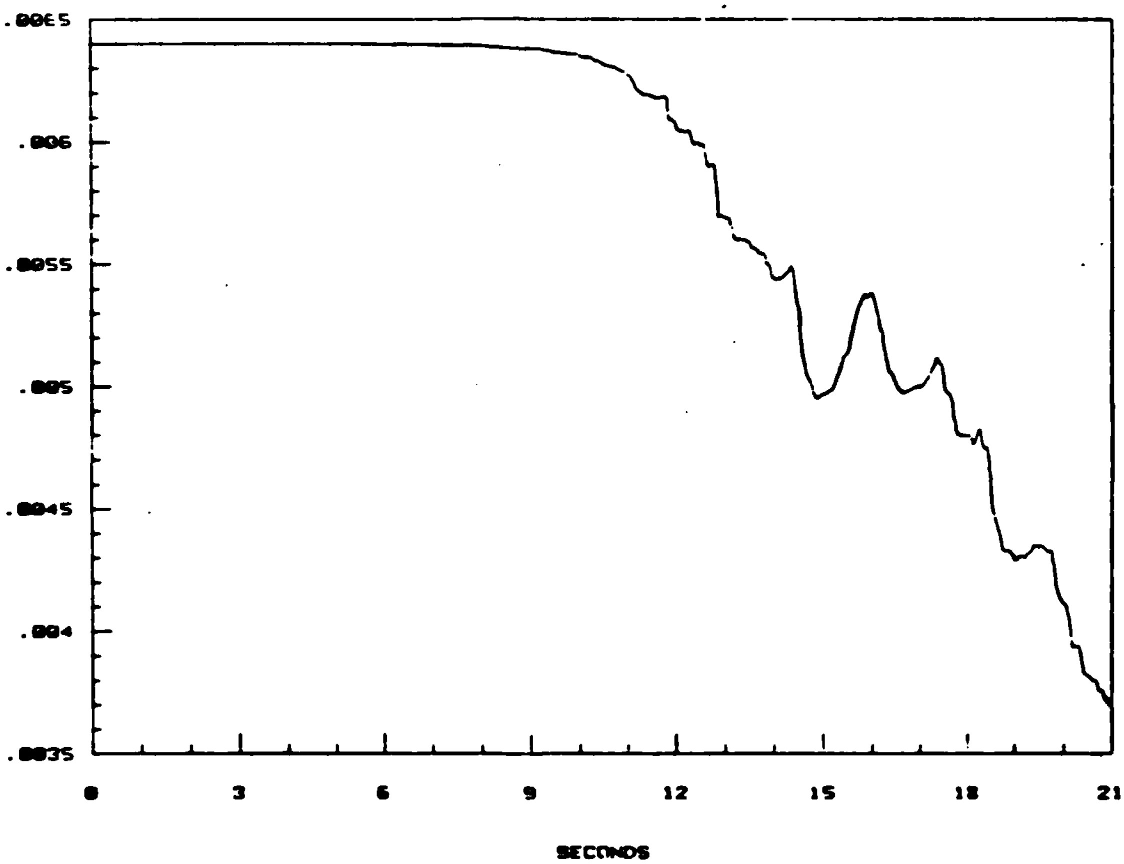


FIG. 4-54 B1(2,1) ESTIMATE - NOISE + FAILURE

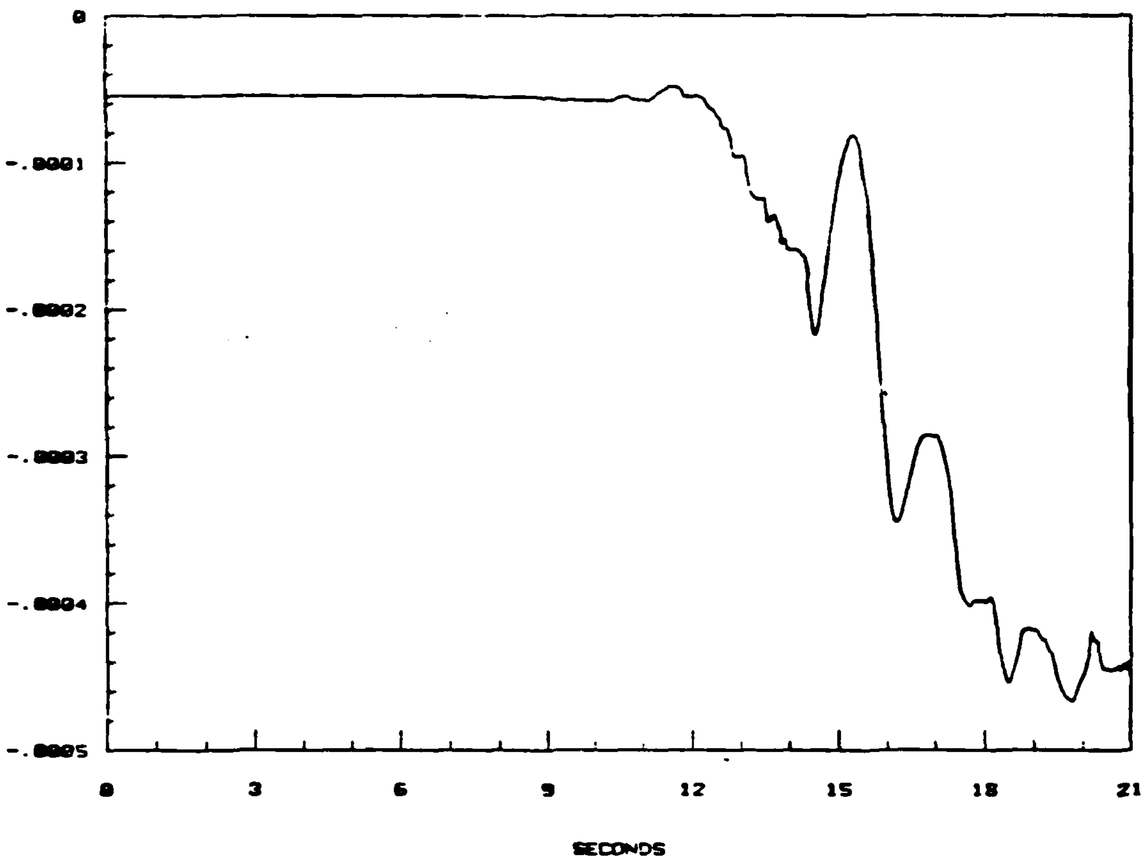


FIG. 4-55 B1(3,1) ESTIMATE - NOISE + FAILURE

further testing was abandoned and conclusions and recommendation are made from the results obtained up to this point.

Chapter V Conclusions and Recommendations

5.1 Conclusions

5.1.1 Fixed Gain Controller. In order to design a Multiple-Input Multiple-Output (MIMO) fixed gain controller for a rectangular plant, using Porter's technique, requires knownledge of the location of the transmission zeros. Specifically, all the transmission zeros must lie in the left-hand plane for this unstable plant. Location of the transmission zeros can not be determined using the procedure previously used for a square plant. However, if the controller is placed in series with the plant and a new "square system " is formed, then the procedure used for the square plant yields the location of the transmission zeros of the "square system".

5.1.2 Adaptive Controller. The MIMO adaptive gain controller required that noise be inserted as the input signal to aid the controller in identifying the plant parameters. When a failure was introduced, the adaptive controller was more stable than the fixed controller. However, it was not determined if the additional stability of the adaptive controller was caused by an attempt of the controller to identify the plant parameters, or because the controller was diverging in the direction of the new plant parameters. If the additonal stability was due to the diversion, then this controller would also become unstable.

5.2 Recommendations

Recommend that further research be conducted to determine a precise technique for calculating the transmission zero locations for a rectangular plant.

Recommend that testing and tuning of the rectangular plant with actuator position and rate limits be accomplished.

Recommend that additional tuning and or modification be performed on the adaptive algorithm to decrease the convergence time required in identifying the plant parameters.

Upon successful completion of the above recommendations, recommend that the effect of different failure models, noise, delays, and sensor dynamics, be evaluated on both a fixed gain and adaptive controller.

Appendix A: Aircraft Data

Introduction

This appendix contains the aircraft data that is used for the models of this thesis. Table A-1 contains common aircraft data. while Table A-2 contains the primed stability derivatives for the flight conditions specified in this thesis. This data was obtained and reprinted from Reference 4.

Table A-1

Aircraft Data

S	wing reference area = 300.0 ft^2
C	wing mean aerodynamic cord = 11.32 ft
B	wing span = 30.0 ft
W	weight = 21.018.0 lbs
M	mass = 652.73 slugs
I_x	x-axis moment of inertia = 10.033.4 ft^2
I_y	y-axis moment of inertia = 53.876.3 ft^2
I_z	z-axis moment of inertia = 61.278.4 ft^2
I_{xz}	product of inertia = 282.132 ft^2
$\bar{Q}$	dynamic pressure = 552.11295 lbs/ft^2
V_T	trim velocity = 933.23 ft/sec
α_T	trim angle of attack = 0.03246 radians

Table A-2

Primed Dimensional Derivatives in the Body Axis

$X'_\theta = -32.1830$	$Z'_\theta = -0.001120$	$M'_\theta = 0.000309$
$X'_u = -0.056009$	$Z'_u = 0.000045971$	$M'_u = -0.002101$
$X'_\alpha = 38.2906$	$Z'_\alpha = -1.48446$	$M'_\alpha = 4.27171$
$X'_q = -30.1376$	$Z'_q = 0.994789$	$M'_q = -0.777221$
$X'_{\delta e_t} = 2.005930$	$Z'_{\delta e_t} = -0.149227$	$M'_{\delta e_t} = -24.0581$
$X'_{\delta f_t} = 2.31681$	$Z'_{\delta f_t} = -0.244924$	$M'_{\delta f_t} = -6.47269$
$X'_{\delta_t} = 19.254$	......	$M'_{\delta_c} = -0.9864$
$Y'_\phi = 0.034486$	......	......
$Y'_\beta = -0.343554$	$L'_\beta = -55.2526$	$N'_\beta = 7.2370$
$Y'_p = 0.032636$	$L'_p = -2.80004$	$N'_p = -0.023184$
$Y'_r = 0.997556$	$L'_r = 0.145674$	$N'_r = -0.362530$
$Y'_{\delta DF} = -0.001371$	$L'_{\delta DF} = -51.0502$	$N'_{\delta DF} = -1.25006$
$Y'_{\delta DT} = 0.026609$	$L'_{\delta DT} = -50.7290$	$N'_{\delta DT} = -5.13710$
$Y'_{\delta r} = 0.037032$	$L'_{\delta r} = 10.3955$	$N'_{\delta r} = -5.80890$
$Y'_{\delta c} = 0.026734$	$L'_{\delta c} = 5.53185$	$N'_{\delta c} = 5.89254$

Appendix B: Calculation of Autoregressive Model From State-Space Model

B.1 Introduction

This appendix consist of three parts. The first part presents the procedures used in deriving the autoregressive model from a state-space model. The second part presents the MATRIX$_X$ macros used to implement this procedure and calculate the autoregressive model coefficients. The last part depicts the use of the macros to generate the three autoregressive models required for this thesis.

B.2 Autoregressive Model

The procedure to generate an autoregressive model from a state-space model is extracted from reference 22 and is condensed in this appendix . Detailed definitions, propositions, proofs and theorems are contained in reference 22.

Given a state-space representation of the form shown in chapter III, equations 2-1 and 2-2, it is possible to generate an autoregrresive model of the form:

$$A_o y(k) + A_1 y(k-1) + \ldots + A_q y(k-q) = B_1 u(k-1) + B_2 u(k-2) + \ldots + B_r u(k-r) \qquad (B-1)$$

To determine the A_i and B_i coefficients the following procedure must be accomplished.

1. Ensure that the inverse of A exists and that C is of full rank. Then form a constructibility matrix Co as:

$$
Co = \begin{pmatrix} CA^{-1} \\ CA^{-2} \\ \cdots \\ CA^{-N} \end{pmatrix} \qquad (B-2)
$$

2. Check to make sure that Co has a rank equal to the number of states in the state-space representation.

3. Form a temporary transformation matrix (TT) from the linearly independent row vectors of Co. This is accomplished by taking the first row of Co $(C_1 A^{-1})$ and making it the first row of TT. Next, examine the second row of Co $(C_2 A^{-1})$. If it is linearly independent of the row vector in TT, then make it $(C_2 A^{-1})$ the second row of TT. Continue the process with the next row of Co. If it is linearly independent of the row vectors in TT then incorporate it into TT. Once the last row of Co $(C_l A^{-N})$ has been examine and TT formed, then the next step is to rearrange the rows of TT and form a new transformation matrix (T).

4. The transformation matrix T is formed in the following manner:

$$
T = \begin{pmatrix} C_1' A^{1} \\ \vdots \\ C_1' A^{-V_1} \\ \cdots \\ \vdots \\ \cdots \\ C_l' A^{-1} \\ \vdots \\ C_l' A^{-V_l} \end{pmatrix} \qquad (B-3)
$$

where the ith constructibility index, V_i, is the smallest positive integer (greater than or equal to one) so that $C_i A^{-vi-1}$ is a linear combination of the row before it. For example, if the following four rows were examined:

$$C_1 A^{-1}, C_1 A^{-2}, C_1 A^{-3}, \text{ and } C_1 A^{-4}$$

and $C_1 A^{-4}$ is a linear combinations of the previous rows, then $V1 = 3$.

5. Ensure that $V_1 \geq V_2 \geq V_3 > ...V_l$. If this is not true, then permutate the output matrix (C) to satisfy this requirement. Also verify that $V_1 + V_2 + ... + V - l = N$.

6. Now define the following matrices:

$$\bar{C} = CT^{-1} \qquad (B-4)$$

$$\bar{B} = TB \qquad (B-5)$$

7. Form the Ai coefficients by:

$$A_i = \bar{C} Sq_i(k-i) \qquad (B-6)$$

where

$$Sp_i(k) = \begin{pmatrix} (k-1) & 0 & 0 \\ (k-2) & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ (k-V_1) & \vdots & \vdots \\ 0 & (k-1) & \vdots \\ \vdots & (k-2) & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & (k-V_2) & \vdots \\ \vdots & 0 & (k-1) \\ \vdots & \vdots & (k-2) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & (k-V_l) \end{pmatrix} \qquad (B-7)$$

For example, to generate A_1, the Sp_1 matrix needs to be formed. The Sp_1 matrix is formed by setting the "k-1" elements of Sp to 1 and all the other elements would be set to zero.

8. Form the B_i coefficients by:

$$B_i = \tilde{C} Sq_{i-1}(k-i)\bar{B} \qquad (B-8)$$

where

$$Sq \quad = \quad S^{i-1}D^i, v = \max V_i$$

and

S is block-diagonal Toeplitz given by:

$$S_i = \begin{pmatrix} 0 & \dots & 0 \\ 1 & \dots & \dots \\ 0 & 1 & 0 \end{pmatrix}$$

$S_i = 1 \dots (V_i x V_i)$

The MATRIX$-X$ macro used to implement this procedure are shown and explained in the next section.

B.3 Autoregressive Macros

The three macros used to calculated the coefficients of the autoregressive model are presented in this section. The macros are first shown in the matrix format and then they are shown in a expanded format with comments added. Use of these macros to generate the autoregressive model for the plants contained in this thesis are shown in the next section.

B.3.1 ARMAMODEL. This macro generates the construtability matrix (Co) introduced in the previous section.

```
CLEAR CO,CO=<C*INV(AD)>;FOR I=2:N,CO=<CO;C*INV(AD)**I>;END,>TMAT<;
CLEAR CO,  :Clearing the previous construtability matrix.
CO=<C*INV(AD)>;  :Forming the first row of the construtability
                  matrix.
FOR I=2:N,  :Loop for forming the construtability matrix.
            CO=<CO;C*INV(AD)**I>;
END,
>TMAT<;  : Executing the macros TMAT.
```

2. TMAT. Forms the temporary transformation matrix (T) from the independent rows of Co.

```
V1=0;CR=0;CLEAR TT,CLEAR R, TT=<CO(1,:)>;R=TT;FOR I = 2:2*N,
R=<TT;CO(I,:)>; RN=RANK(R); IF RN=I-CR THEN TT=R;ELSEIF RN<I-CR THEN
R=TT;VI=<V1;I>;V1=VI;CR=CR+1;END,END,T=R,CO,;
```

```
V1=0;  :Initializing temporary vector.
CR=0;  :Initializing counter.
CLEAR TT,  :Clearing temporary vector.
CLEAR R,  :Clearing temporary vector.
```

```
TT=<CO(1,:)>;  :Forming first row of TT.
R=TT;
FOR I = 2:2*N,  :Loop for generating R, which becomes the
                 temporary transfomation matrix.
    R=<TT;CO(I,:)>;
    RN=RANK(R);
    IF RN=I-CR THEN TT=R;
    ELSEIF RN<I-CR THEN R=TT;
    VI=<V1;I>;
    V1=VI;
    CR=CR+1;
    END,
    END,
T=R,
CO,;
```

3. BARS. Generates A, B, and C from a transformation matrix (TF). The transformation matrix must contain the independent rows of Co in the order shown in Equation (B-3).

```
CBAR=C*INV(TF);ADBAR=TF*AD*INV(TF);BDBAR=TF*BD;H=CBAR*BDBAR,;
CBAR=C*INV(TF);
ADBAR=TF*AD*INV(TF);
BDBAR=TF*BD;
H=CBAR*BDBAR,;  :Step-response matrix, also B1.
```

B.4 Autoregressive Calculations

This section uses the macros explained in the previous section to generate the autoregressive coefficients for the three models used in this thesis.

B.4.1 Plant 1.

Calculation of the autoregressive model for the first nominal plant. The output vector for the first plant consist of flight-path angle, beta, and yaw rate.

A =

COLUMNS 1 THRU 6

0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00
-3.2183D+01	-5.6009D-02	3.8291D+01	-3.0138D+01	0.0000D+00	0.0000D+00
-1.1000D-03	4.5971D-05	-1.4845D+00	9.9480D-01	0.0000D+00	0.0000D+00
3.0000D-04	-2.1010D-03	4.2717D+00	-7.7720D-01	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	3.4500D-02	-3.4536D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-5.5253D+01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	7.2370D+00

COLUMNS 7 THRU 8

0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
1.0000D+00	0.0000D+00
3.2600D-02	-9.9760D-01
-2.8004D+00	1.4570D-01
-2.3200D-02	-3.6250D-01

B =

0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.0030D+00	1.0030D+00	1.1584D+00	1.1584D+00	1.9254D+01
-7.4600D-02	-7.4600D-02	-1.2250D-01	-1.2250D-01	0.0000D+00
-1.2029D+01	-1.2029D+01	-3.2363D+00	-3.2363D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-3.3000D-03	3.3000D-03	7.0000D-04	-7.0000D-04	0.0000D+00
6.3395D+00	-6.3395D+00	2.5519D+01	-2.5519D+01	0.0000D+00
6.4200D-01	-6.4200D-01	6.2490D-01	-6.2490D-01	0.0000D+00

C =

1.	0.	-1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.

D =

0.	0.	0.	0.	0.
0.	0.	0.	0.	0.
0.	0.	0.	0.	0.

AD =

COLUMNS 1 THRU 6

1.0000D+00	-1.0473D-07	2.1198D-04	9.9620D-03	0.0000D+00	0.0000D+00
-3.2174D-01	9.9944D-01	3.7359D-01	-2.9986D-01	0.0000D+00	0.0000D+00
-1.0967D-05	3.5252D-07	9.8547D-01	9.8367D-03	0.0000D+00	0.0000D+00
6.1259D-06	-2.0914D-05	4.2236D-02	9.9247D-01	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	2.7335D-03
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	3.4435D-04	9.9610D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-9.4305D-05	-5.4378D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.2461D-05	7.2167D-02

COLUMNS 7 THRU 8

0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
9.8610D-03	1.6314D-05
3.2372D-04	-9.9390D-03
9.7230D-01	4.1578D-03
-2.1663D-04	9.9602D-01

BD =

-5.9997D-04	-5.9997D-04	-1.6149D-04	-1.6149D-04	-6.7267D-09
2.7952D-02	2.7952D-02	1.6210D-02	1.6210D-02	1.9249D-01
-1.3344D-03	-1.3344D-03	-1.3758D-03	-1.3758D-03	3.7363D-08
-1.1985D-01	-1.1985D-01	-3.2266D-02	-3.2266D-02	-2.0165D-06
3.1409D-04	-3.1409D-04	1.2641D-03	-1.2641D-03	0.0000D+00
-5.4597D-05	5.4597D-05	1.7295D-05	-1.7295D-05	0.0000D+00
6.2533D-02	-6.2533D-02	2.5165D-01	-2.5165D-01	0.0000D+00
6.3994D-03	-6.3994D-03	6.2089D-03	-6.2089D-03	0.0000D+00

]ARMAMODEL[- Invoking the macro ARMAMODEL

T =

COLUMNS 1 THRU 6

9.9999D-01	4.6324D-07	-1.0150D+00	2.2320D-05	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-3.4554D-04	1.0030D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.2505D-05	-7.2551D-02
9.9998D-01	9.3572D-07	-1.0301D+00	1.9555D-04	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-6.9197D-04	1.0051D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	5.0096D-05	-1.4542D-01
9.9997D-01	1.4208D-06	-1.0456D+00	5.2321D-04	0.0000D+00	0.0000D+00
9.9996D-01	1.9218D-06	-1.0612D+00	1.0089D-03	0.0000D+00	0.0000D+00

```
 COLUMNS     7 THRU    8
 0.0000D+00   0.0000D+00
-3.2821D-04   1.0010D-02
 2.4756D-04   1.0033D+00
 0.0000D+00   0.0000D+00
-6.6072D-04   2.0083D-02
 5.2662D-04   1.0058D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
```

Checking to make sure the constructibility indices are in the correct order.

```
 V1          =

    0.
    8.
    9.
   11.
   12.
   13.
   14.
   15.
   16.
```

V1=4, V2=2 and V3=2; This is in the proper order.

Forming the transformation matrix TF (T Final).

```
<> TF=[T(1,:);T(4,:);T(7,:);T(8,:);T(2,:);T(5,:);T(3,:);T(6,:)]

 TF          =

 COLUMNS     1 THRU    6
 9.9999D-01  4.6324D-07 -1.0150D+00  2.2320D-05  0.0000D+00  0.0000D+00
 9.9998D-01  9.3572D-07 -1.0301D+00  1.9555D-04  0.0000D+00  0.0000D+00
 9.9997D-01  1.4208D-06 -1.0456D+00  5.2321D-04  0.0000D+00  0.0000D+00
 9.9996D-01  1.9218D-06 -1.0612D+00  1.0089D-03  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -3.4554D-04  1.0030D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -6.9197D-04  1.0051D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.2505D-05 -7.2551D-02
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  5.0096D-05 -1.4542D-01

 COLUMNS     7 THRU    8
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
-3.2821D-04   1.0010D-02
-6.6072D-04   2.0083D-02
 2.4756D-04   1.0033D+00
 5.2662D-04   1.0058D+00
```

```
]BARS[ - Invoking the macro BARS

 BDBAR       =

 7.5172D-04   7.5172D-04   1.2342D-03   1.2342D-03   4.4474D-08
 7.5124D-04   7.5124D-04   1.2495D-03   1.2495D-03   1.3450D-07
 7.3257D-04   7.3257D-04   1.2601D-03   1.2601D-03   2.2663D-07
 6.9529D-04   6.9529D-04   1.2660D-03   1.2660D-03   3.2150D-07
-1.1336D-05   1.1336D-05  -3.5314D-06   3.5314D-06   0.0000D+00
 3.2105D-05  -3.2105D-05  -2.5069D-05   2.5069D-05   0.0000D+00
 6.4398D-03  -6.4398D-03   6.2903D-03  -6.2903D-03   0.0000D+00
 6.4775D-03  -6.4775D-03   6.3751D-03  -6.3751D-03   0.0000D+00

 ADBAR       =

     COLUMNS    1 THRU    6
 3.9774D+00 -5.9319D+00  3.9316D+00 -9.7709D-01  0.0000D+00  0.0000D+00
 1.0000D+00  1.9779D-13  9.6020D-14 -1.2722D-13  0.0000D+00  0.0000D+00
-9.8255D-14  1.0000D+00 -3.9568D-13  8.3267D-15  0.0000D+00  0.0000D+00
 3.7947D-13 -7.9003D-13  1.0000D+00 -4.9594D-13  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.9951D+00 -9.9707D-01
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.0000D+00  2.6522D-17
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  7.3194D-02 -7.1142D-02
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -8.6736D-18  6.9389D-18

  COLUMNS    7 THRU    8
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 1.4340D-02  -1.4252D-02
-3.0791D-17   4.7705D-18
 1.9693D+00  -9.6938D-01
 1.0000D+00   4.1633D-17

CBAR        =

     COLUMNS    1 THRU    6
 3.9774D+00 -5.9319D+00  3.9316D+00 -9.7709D-01  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.9951D+00 -9.9707D-01
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  7.3194D-02 -7.1142D-02

  COLUMNS    7 THRU    8
 0.0000D+00   0.0000D+00
 1.4340D-02  -1.4252D-02
 1.9693D+00  -9.6938D-01
```

Now the "B" coefficients are formed.

B1ARMA=CBAR*BDBAR

B1ARMA =

7.3442D-04	7.3442D-04	1.2143D-03	1.2143D-03	-4.4090D-08
-5.4597D-05	5.4597D-05	1.7295D-05	-1.7295D-05	0.0000D+00
6.3994D-03	-6.3994D-03	6.2089D-03	-6.2089D-03	0.0000D+00

B2ARMA=CBAR*SQ*BDBAR

B2ARMA =

-2.2213D-03	-2.2213D-03	-3.6398D-03	-3.6398D-03	4.3557D-08
-8.0478D-05	8.0478D-05	-8.6128D-05	8.6128D-05	0.0000D+00
-6.2418D-03	6.2418D-03	-6.0974D-03	6.0974D-03	0.0000D+00

B3ARMA=CBAR*SQ**2*BDBAR

B3ARMA =

2.2214D-03	2.2214D-03	3.6314D-03	3.6314D-03	4.3432D-08
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00

B4ARMA=CBAR*SQ**3*BDBAR

B4ARMA =

-7.3450D-04	-7.3450D-04	-1.2059D-03	-1.2059D-03	-4.3455D-08
0.0000D-00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00

Now the "A" coefficients are formed.

SP1 =

1.	0.	0.
0.	0.	0.
0.	0.	0.
0.	0.	0.
0.	1.	0.
0.	0.	0.
0.	0.	1.
0.	0.	0.

```
A1ARMA=-CBAR*SP1

 A1ARMA       =

-3.9774D+00   0.0000D+00   0.0000D+00
 0.0000D+00  -1.9951D+00  -1.4340D-02
 0.0000D+00  -7.3194D-02  -1.9693D+00

 SP2          =

     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     1.     0.
     0.     0.     0.
     0.     0.     1.
A2ARMA=-CBAR*SP2

 A2ARMA       =

 5.9319D+00   0.0000D+00   0.0000D+00
 0.0000D+00   9.9707D-01   1.4252D-02
 0.0000D+00   7.1142D-02   9.6938D-01

 SP3          =

     0.     0.     0.
     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
A3ARMA=-CBAR*SP3

 A3ARMA       =

-3.9316D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00

 SP4          =

     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
```

A4ARMA=-CBAR*SP4

A4ARMA =

9.7709D-01	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00

Calculating the steady-state transfer function (G0).

The denominator of G0 is labled G0DEN.

G0DEN=[EYE(3)+A1ARMA+A2ARMA+A3ARMA+A4ARMA]

G0DEN =

-9.2837D-10	0.0000D+00	0.0000D+00
0.0000D+00	1.9329D-03	-8.8330D-05
0.0000D+00	-2.0513D-03	9.7068D-05

The numerator of Go is labled G0N.

G0N=[B1ARMA+B2ARMA+B3ARMA+B4ARMA]

G0N =

-1.0057D-08	-1.0057D-08	-2.9606D-09	-2.9606D-09	-5.5669D-10
-1.3508D-04	1.3508D-04	-6.8833D-05	6.8833D-05	0.0000D+00
1.5762D-04	-1.5762D-04	1.1149D-04	-1.1149D-04	0.0000D+00

G0=INV(G0DEN)*G0N

G0 =

1.0833D+01	1.0833D+01	3.1891D+00	3.1891D+00	5.9964D-01
1.2604D-01	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

Checking G0 agianst -C*INV(A)*B

-C*INV(A)*B =

1.0834D+01	1.0834D+01	3.1894D+00	3.1894D+00	5.9969D-01
1.2604D-01	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

B.4.2 Plant 2. Calculation of the autoregressive model for the second plant. The output vector for this plant consist of forward velocity, beta, and yaw rate.

```
 A            =

    COLUMNS    1 THRU    6
 0.0000D+00  0.0000D+00  0.0000D+00  1.0000D+00  0.0000D+00  0.0000D+00
-3.2183D+01 -5.6009D-02  3.8291D+01 -3.0138D+01  0.0000D+00  0.0000D+00
-1.1000D-03  4.5971D-05 -1.4845D+00  9.9480D-01  0.0000D+00  0.0000D+00
 3.0000D-04 -2.1010D-03  4.2717D+00 -7.7720D-01  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  3.4500D-02 -3.4536D-01
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -5.5253D+01
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  7.2370D+00

 COLUMNS    7 THRU    8
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 1.0000D+00   0.0000D+00
 3.2600D-02  -9.9760D-01
-2.8004D+00   1.4570D-01
-2.3200D-02  -3.6250D-01

 B            =

 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
 1.0030D+00   1.0030D+00   1.1584D+00   1.1584D+00   1.9254D-01
-7.4600D-02  -7.4600D-02  -1.2250D-01  -1.2250D-01   0.0000D+00
-1.2029D+01  -1.2029D+01  -3.2363D+00  -3.2363D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
-3.3000D-03   3.3000D-03   7.0000D-04  -7.0000D-04   0.0000D+00
 6.3395D+00  -6.3395D+00   2.5519D+01  -2.5519D+01   0.0000D+00
 6.4200D-01  -6.4200D-01   6.2490D-01  -6.2490D-01   0.0000D+00

 C            =

0.    1.    0.    0.    0.    0.    0.    0.
0.    0.    0.    0.    0.    1.    0.    0.
0.    0.    0.    0.    0.    0.    0.    1.
```

```
RANK(R); IF RN=I-CR THEN TT=R;ELSEIF RN<I-CR THEN R=TT;VI=<V1;I>;V1=VI;

CR=CR+1;END,END,T=R,CO,;

 <> ]ARMAMODEL[

  T           =

     COLUMNS    1 THRU    6
 3.2192D-01  1.0006D+00 -3.9236D-01  3.0296D-01  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -3.4554D-04  1.0030D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.2505D-05  7.2551D-02
 6.4401D-01  1.0011D+00 -8.0392D-01  6.0924D-01  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -6.9197D-04  1.0051D+00
 0.0000D-00  0.0000D+00  0.0000D+00  0.0000D+00  5.0096D-05 -1.4542D-01
 9.6629D-01  1.0017D+00 -1.2351D+00  9.1906D-01  0.0000D+00  0.0000D+00
 1.2887D+00  1.0023D+00 -1.6864D+00  1.2326D+00  0.0000D+00  0.0000D+00

  COLUMNS    7 THRU    8
 0.0000D+00   0.0000D+00
-3.2821D-04   1.0010D-02
 2.4756D-04   1.0033D+00
 0.0000D+00   0.0000D+00
-6.6072D-04   2.0083D-02
 5.2662D-04   1.0058D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
```

Checking to make sure that the constructibility indices are in the proper order.

```
  V1          =

     0.
     8.
     9.
    11.
    12.
    13.
    14.
    15.
    16.

V1=4, V2=2, V3=2; THIS IS IN THE PROPER ORDER
```

FORMING THE TRANSFORMATION MATRIX TF (T FINAL)

```
TF=[T(1,:);T(4,:);T(7,:);T(8,:);T(2,:);T(5,:);T(3,:);T(6,:)]

  TF          =

    COLUMNS    1 THRU    6
 3.2192D-01  1.0006D+00 -3.9236D-01  3.0296D-01  0.0000D+00  0.0000D+00
 6.4401D-01  1.0011D+00 -8.0392D-01  6.0924D-01  0.0000D+00  0.0000D+00
 9.6629D-01  1.0017D+00 -1.2351D+00  9.1906D-01  0.0000D+00  0.0000D+00
 1.2887D+00  1.0023D+00 -1.6864D+00  1.2326D+00  0.0000D+00  0.0000D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -3.4554D-04  1.0030D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00 -6.9197D-04  1.0051D+00
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.2505D-05 -7.2551D-02
 0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  5.0096D-05 -1.4542D-01

  COLUMNS    7 THRU    8
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00
-3.2821D-04   1.0010D-02
-6.6072D-04   2.0083D-02
 2.4756D-04   1.0033D+00
 5.2662D-04   1.0058D+00

BARS =

 CBAR=C*INV(TF);ADBAR=TF*AD*INV(TF);BDBAR=TF*BD;H=CBAR*BDBAR,; <>

[BARS]

  H           =

 2.7952D-02   2.7952D-02   1.6210D-02   1.6210D-02   1.9249D-01
-5.4597D-05   5.4597D-05   1.7295D-05  -1.7295D-05   0.0000D+00
 6.3994D-03  -6.3994D-03   6.2089D-03  -6.2089D-03   0.0000D+00

  BDBAR       =

-8.0117D-03  -8.0117D-03   6.9319D-03   6.9319D-03   1.9259D-01
-4.4347D-02  -4.4347D-02  -2.4270D-03  -2.4270D-03   1.9270D-01
-8.1080D-02  -8.1080D-02  -1.1873D-02  -1.1873D-02   1.9281D-01
-1.1824D-01  -1.1824D-01  -2.1413D-02  -2.1413D-02   1.9293D-01
-1.1336D-05   1.1336D-05  -3.5314D-06   3.5314D-06   0.0000D+00
 3.2105D-05  -3.2105D-05  -2.5069D-05   2.5069D-05   0.0000D+00
 6.4398D-03  -6.4398D-03   6.2903D-03  -6.2903D-03   0.0000D+00
 6.4775D-03  -6.4775D-03   6.3751D-03  -6.3751D-03   0.0000D+00
```

ADBAR =

COLUMNS 1 THRU 6

3.9774D+00	-5.9319D+00	3.9316D+00	-9.7709D-01	0.0000D+00	0.0000D+00
1.0000D+00	-2.2737D-13	2.2737D-13	-5.6843D-14	0.0000D+00	0.0000D+00
4.5475D-13	1.0000D+00	1.1369D-12	-3.4106D-13	0.0000D+00	0.0000D+00
6.8212D-13	-1.3642D-12	1.0000D+00	-3.4106D-13	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.9951D+00	-9.9707D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	2.6522D-17
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	7.3194D-02	-7.1142D-02
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-8.6736D-18	6.9389D-18

COLUMNS 7 THRU 8

0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
1.4340D-02	-1.4252D-02
-3.0791D-17	4.7705D-18
1.9693D+00	-9.6938D-01
1.0000D+00	4.1633D-17

Now forming the "B" coefficients.

B1ARMA1= CBAR*BDBAR

B1ARMA1 =

2.7952D-02	2.7952D-02	1.6210D-02	1.6210D-02	1.9249D-01
-5.4597D-05	5.4597D-05	1.7295D-05	-1.7295D-05	0.0000D+00
6.3994D-03	-6.3994D-03	6.2089D-03	-6.2089D-03	0.0000D+00

SQ =

0.	0.	0.	0.	0.	0.	0.	0.
1.	0.	0.	0.	0.	0.	0.	0.
0.	1.	0.	0.	0.	0.	0.	0.
0.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	1.	0.

```
B2ARMA1=CBAR*SQ*BDBAR

  B2ARMA1      =

-4.7607D-02  -4.7607D-02  -3.9060D-02  -3.9060D-02  -5.7321D-01
-8.0478D-05   8.0478D-05  -8.6128D-05   8.6128D-05   0.0000D+00
-6.2418D-03   6.2418D-03  -6.0974D-03   6.0974D-03   0.0000D+00
B3ARMA1=CBAR*SQ**2*BDBAR

  B3ARMA1      =

 1.1833D-02   1.1833D-02   2.9625D-02   2.9625D-02   5.6891D-01
 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
 B4ARMA1=CBAR*SQ**3*BDBAR

  B4ARMA1      =

 7.8281D-03   7.8281D-03  -6.7731D-03  -6.7731D-03  -1.8818D-01
 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
```

Now forming the "A" coefficients.

```
  SP1          =

     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     1.     0.
     0.     0.     0.
     0.     0.     1.
     0.     0.     0.

A1ARMA1=-CBAR*SP1

  A1ARMA1      =

-3.9774D+00   0.0000D+00   0.0000D+00
 0.0000D+00  -1.9951D+00  -1.4340D-02
 0.0000D+00  -7.3194D-02  -1.9693D+00

  SP2          =

     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     1.     0.
     0.     0.     0.
     0.     0.     1.
```

```
A2ARMA1=-CBAR*SP2

 A2ARMA1      =

 5.9319D+00   0.0000D+00   0.0000D+00
 0.0000D+00   9.9707D-01   1.4252D-02
 0.0000D+00   7.1142D-02   9.6938D-01

 SP3          =

     0.     0.     0.
     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.

A3ARMA1=-CBAR*SP3

 A3ARMA1      =

-3.9316D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00

 SP4          =

     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     1.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.
     0.     0.     0.

A4ARMA1=-CBAR*SP4

 A4ARMA1      =

 9.7709D-01   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00
 0.0000D+00   0.0000D+00   0.0000D+00
```

Now calculating the steady-state transfer funciton (Go)
The denominator of Go is labled Goden.

GODEN=[EYE(3)+A1ARMA1+A2ARMA1+A3ARMA1+A4ARMA1]

GODEN =

-9.2829D-10	0.0000D+00	0.0000D+00
0.0000D+00	1.9329D-03	-8.8330D-05
0.0000D+00	-2.0513D-03	9.7068D-05

The numerator of Go is labled GON

GON = [B1ARMA1+B2ARMA1+B3ARMA1+B4ARMA1]

GON =

5.7875D-06	5.7875D-06	1.6963D-06	1.6963D-06	8.0958D-10
-1.3508D-04	1.3508D-04	-6.8833D-05	6.8833D-05	0.0000D+00
1.5762D-04	-1.5762D-04	1.1149D-04	-1.1149D-04	0.0000D+00

GO=INV(GODEN)*GON

GO =

-6.2346D+03	-6.2346D+03	-1.8273D+03	-1.8273D+03	-8.7212D-01
1.2604D-01	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

Checking Go against -C*INV(A)*B

-C*INV(A)*B =

-6.2346D+03	-6.2346D+03	-1.8273D+03	-1.8273D+03	-8.7212D-01
1.2604D-01	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
4.2874D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

B.4.3 Plant 3. Calculation of the autoregressive model for the failed plant. The output vector for this plant consist of forward velocity, beta, and yaw rate. The left elevator effectiveness is reduced by 50%. This is accomplished by multipling the corresponding B column of the healthy model by 0.50.

A =

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
-32.1830	-0.0560	38.2906	-30.1376	0.0000	0.0000	0.0000	0.0000
-0.0011	0.0000	-1.4845	0.9948	0.0000	0.0000	0.0000	0.0000
0.0003	-0.0021	4.2717	-0.7772	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0345	-0.3454	0.0326	-0.9976
0.0000	0.0000	0.0000	0.0000	0.0000	-55.2526	-2.8004	0.1457
0.0000	0.0000	0.0000	0.0000	0.0000	7.2370	-0.0232	-0.3625

B =

0.0000	0.0000	0.0000	0.0000	0.0000
0.5015	1.0030	1.1584	1.1584	19.2540
-0.0373	-0.0746	-0.1225	-0.1225	0.0000
-6.0145	-12.0291	-3.2363	-3.2363	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
-0.0016	0.0033	0.0007	-0.0007	0.0000
3.1698	-6.3395	25.5186	-25.5186	0.0000
0.3210	-0.6420	0.6249	-0.6249	0.0000

C =

0.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.

[BARS]

BDBAR =

-4.0058D-03	-8.0117D-03	6.9319D-03	6.9319D-03	1.9259D-01
-2.2174D-02	-4.4347D-02	-2.4270D-03	-2.4270D-03	1.9270D-01
-4.0540D-02	-8.1080D-02	-1.1873D-02	-1.1873D-02	1.9281D-01
-5.9119D-02	-1.1824D-01	-2.1413D-02	-2.1413D-02	1.9293D-01
-5.6678D-06	1.1336D-05	-3.5314D-06	3.5314D-06	0.0000D+00
1.6053D-05	-3.2105D-05	-2.5069D-05	2.5069D-05	0.0000D+00
3.2199D-03	-6.4398D-03	6.2903D-03	-6.2903D-03	0.0000D+00
3.2388D-03	-6.4775D-03	6.3751D-03	-6.3751D-03	0.0000D+00

B1ARMA2=CBAR*BDBAR

B1ARMA2 =

1.3976D-02	2.7952D-02	1.6210D-02	1.6210D-02	1.9249D-01
-2.7299D-05	5.4597D-05	1.7295D-05	-1.7295D-05	0.0000D+00
3.1997D-03	-6.3994D-03	6.2089D-03	-6.2089D-03	0.0000D+00

B2ARMA2=CBAR*SQ*BDBAR

B2ARMA2 =

-2.3803D-02	-4.7607D-02	-3.9060D-02	-3.9060D-02	-5.7321D-01
-4.0239D-05	8.0478D-05	-8.6128D-05	8.6128D-05	0.0000D+00
-3.1209D-03	6.2418D-03	-6.0974D-03	6.0974D-03	0.0000D+00

B3ARMA2=CBAR*SQ**2*BDBAR

B3ARMA2 =

5.9163D-03	1.1833D-02	2.9625D-02	2.9625D-02	5.6891D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00

B4ARMA2=CBAR*SQ**3*BDBAR

B4ARMA2 =

3.9141D-03	7.8281D-03	-6.7731D-03	-6.7731D-03	-1.8818D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00

The numerator of the steady-state transfer function for the second plant.

GN=[B1ARMA2+B2ARMA2+B3ARMA2+B4ARMA2]

GN =

2.8937D-06	5.7875D-06	1.6963D-06	1.6963D-06	8.0958D-10
-6.7538D-05	1.3508D-04	-6.8833D-05	6.8833D-05	0.0000D+00
7.8808D-05	-1.5762D-04	1.1149D-04	-1.1149D-04	0.0000D+00

The steady-state transfer function for the second plant is labled G02.

G02=INV(GODEN)*GN

G02 =

-3.1173D+03	-6.2346D+03	-1.8273D+03	-1.8273D+03	-8.7212D-01
6.3021D-02	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
2.1437D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

The steady state transfer function could have been calculating by taking the corresponding column of the healthy steady-state transfer function (G0 below) and multipling it by 0.50. That is, for this second plant the first column of G0 would be multiplied by 0.50 and that would generate the steady-state transfer function G02 shown above.

G0 =

-6.2346D+3	-6.2346D+03	-1.8273D+03	-1.8273D+03	-8.7212D-01
1.2604D-01	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
4.2874D-00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

Checking G02 against -C*INV(A)*B

-C*INV(A)*B =

-3.1173D+03	-6.2346D+03	-1.8273D+03	-1.8273D+03	-8.7212D-01
6.3021D-02	-1.2604D-01	4.9232D-01	-4.9232D-01	0.0000D+00
2.1437D+00	-4.2874D+00	1.1553D+01	-1.1553D+01	0.0000D+00

Appendix C: Estimation Alogrithm Equations

C.1 Introduction

This appendix contains the equations used in Reference 2. It is explained in Reference 2 that these equations are solved m (number of outputs) times per time step.

C.2 Equation Listing

At time kT ($k \geq 0$, where 0 is initiation time) calculate for I=1,..., m

$$\hat{\Theta}'_i = \hat{\Theta}'_{i-1}(kT) + \frac{1}{V_i(kT)} P_i(kT) v_i(kT) \bar{\varepsilon}_i(kT) \qquad (C-1)$$

where

$\bar{\varepsilon}_i(kT)$ is the estimated prediction error

$V_i(kT)$ is the estimated prediction error variance

$P_i(kT)$ is the estimated parameter covariance matrix

$v_i(kT)$ is the i^{th} column of measurements in $\Upsilon(kT)$

with initial conditions

$\hat{\Theta}'_0(0)$ initial presumed values of the step-response matrix elements

$V_i(0)$ initial presumed value for prediction error variance for I=1.... m

$P_0(0)$ estimated covariance of the parameter estimates at initiation time

$v_i(0)$ vector of past measurements prior to initiation of identification for i=1.... m

with design parameters

"a" desired variance of the parameter estimates

$\gamma 1, \gamma 2, \gamma$ r0 design parameters for a fault detection scheme

$\gamma 3, \tau, r1$ design parameters of the prediction error variance estimator

and recursive relationships (in proper order of ocurrence

$$\hat{\Theta}_0(kT) = \hat{\Theta}_m[(k-1)T] \qquad (C-2)$$

$$\bar{\epsilon}_i(kT) = y_i(kT) - v_i^T(kT)\hat{\Theta}_{i-1}(kT) + \Omega_i(kT) \qquad (C-3)$$

$$P_0(kT) = P_m(k-1)T \qquad (C-4)$$

$$\eta_i(kT) = v_i^T(kT)P_{i-1}(kT)v_i(kT) \qquad (C-5)$$

$$\mu_i(kT) = v_i^T(kT)P_{i-1}^2(kT)v_i(kT) \qquad (C-6)$$

$$\lambda_i(kT) = v_i^T(kT)P_{i-1}^3(kT)v_i(kT) \qquad (C-7)$$

$$\delta_{i_d}(kT) = \frac{1}{\mu_i(kT)}\left[\frac{\lambda_i(kT)}{\mu_i(kT)} - a\right] \qquad (C-8)$$

$$r_0(kT) = r_m[(k-1)T] \qquad (C-9)$$

$$V_i(kT) = \begin{cases} \gamma 3 V_i[(k-1)T] + (1-\gamma 3)\epsilon_i^2(kT-\tau) & \text{if } r_{i-1}(kT) < r1 \\ V_i[(k-1)T] & \text{if } r_{i-1}(kT) \geq r1 \end{cases} \qquad (C-10)$$

$$\alpha_{i_d}(kT) = V_i^{-1}(kT) + \frac{\delta_{i_d}(kT)}{\delta_{i_d}(kT)\eta_i(kT) - 1} \qquad (C-11)$$

$$\alpha_i(kT) = \begin{cases} 0 & \text{if } \alpha_{i_d}(kT) \leq 0 \\ \alpha_{i_d}(kT) & \text{if} 0 < \alpha_{i_d}(kT) \leq \frac{1}{\eta_i(kT)} \\ \frac{1}{\eta_i(kT)} & \text{if } \frac{1}{\eta_i(kT)} < \alpha_{i_d}(kT) \leq V_i^{-1}(kT) + \frac{1}{\eta_i(kT)} \\ 0 & \text{if } \alpha_{i_d} > V_i^{-1}(kT) + \frac{1}{\eta_i(kT)} \end{cases} \qquad (C-12)$$

$$v_i0(kT) = 1 - \frac{\eta_i(kT)}{V_i(kT) + [1 - \alpha_i(kT) V_i(kT)]\eta_i(kT)} \qquad (C-13)$$

$$\beta_i(kT) = \begin{cases} 0 & \text{if } r_{i-1}(kT) < r0 \\ \frac{V_i(kT) v_i0(kT)(r_{i-1}(kT) - R_0)}{v_i^T(kT) v_i(kT)(1 - R_0)} & \text{if } r_{i-1}(kT) \geq r0 \end{cases} \qquad (C-14)$$

$$P - i(kT) = P_{i-1}(kT) - \frac{P_{i-1}(kT) v_i(kT) v_i^T(kT) P_{i-1}(kT)}{[V_i^{-1}(kT) - \alpha_i(kT)]^{-1} + \eta_i(KT)} + \beta_i)kT)I \qquad (C-15)$$

Appendix D: Macro Lisitings

D.1 Introduction

This appendix contains two sections. The first section shows the procedure used to verify the assumptions made in Chapter III. The second section shows and explains the macros used in foming the open-loop and closed-loop transfer function when the controller, actuators, and plant are in series.

D.2 Assumptions Verification

In this section the assumptions made in Chapter III are verified. This is done by first presenting the plant matrices and then MATRIX$_x$ is used to determine the eigenvalues, check the rank of C*B, check the observability and controllability of the plant, and finally to verify that the tranmission zeros do not lie in the right-half plane.

PLANT MATRICES (0.9M, 20,000 ft)

The continuous plant matrix (A):

COLUMNS 1 THRU 6

0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00
-3.2183D+01	-5.6009D-02	3.8291D+01	-3.0138D+01	0.0000D+00	0.0000D+00
-1.1000D-03	4.5971D-05	-1.4845D+00	9.9480D-01	0.0000D+00	0.0000D+00
3.0000D-04	-2.1010D-03	4.2717D+00	-7.7720D-01	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	3.4500D-02	-3.4536D-01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	-5.5253D+01
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	7.2370D+00

COLUMNS 7 THRU 8

0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00
1.0000D+00	0.0000D+00
3.2600D-02	-9.9760D-01
-2.8004D+00	1.4570D-01
-2.3200D-02	-3.6250D-01

The continuous input matrix (B):

0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
1.0030D+00	1.0030D+00	1.1584D+00	1.1584D+00	1.9254D+01
-7.4600D-02	-7.4600D-02	-1.2250D-01	-1.2250D-01	0.0000D+00
-1.2029D+01	-1.2029D+01	-3.2363D+00	-3.2363D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00	0.0000D+00
-3.3000D-03	3.3000D-03	7.0000D-04	-7.0000D-04	0.0000D+00
6.3395D+00	-6.3395D+00	2.5519D+01	-2.5519D+01	0.0000D+00
6.4200D-01	-6.4200D-01	6.2490D-01	-6.2490D-01	0.0000D+00

The continuous output matrix (C):

1.	0.	-1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.

EIGENVALUES

Checking the eigenvalues of the A matrix.

EIG(A) =

-0.0473 + 0.1626i
-0.0473 - 0.1626i
1.0119 + 0.0000i
-3.2350 + 0.0000i
-0.3917 + 2.9615i
-0.3917 - 2.9615i
-2.6977 + 0.0000i
-0.0272 + 0.0000i

As mentioned in Chapter III, there is one pole in the right-half plane.

RANK OF C*B

Verifying that C*B has full rank.

C*B =

0.0746	0.0746	0.1225	0.1225	0.0000
-0.0033	0.0033	0.0007	-0.0007	0.0000
0.6420	-0.6420	0.6249	-0.6249	0.0000

RANK(C*B) = 3.

OBSERVABILITY and CONTROLLABILITY

Verifying that the plant is observable and controllable. A system matrix needs to be form. The system matrix S is:

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

where

$$D = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

S=[A,B;C,D]

S =

COLUMNS 1 THRU 8

0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
-32.1830	-0.0560	38.2906	-30.1376	0.0000	0.0000	0.0000	0.0000
-0.0011	0.0000	-1.4845	0.9948	0.0000	0.0000	0.0000	0.0000
0.0003	-0.0021	4.2717	-0.7772	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0345	-0.3454	0.0326	-0.9976
0.0000	0.0000	0.0000	0.0000	0.0000	-55.2526	-2.8004	0.1457
0.0000	0.0000	0.0000	0.0000	0.0000	7.2370	-0.0232	-0.3625
1.0000	0.0000	-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

COLUMNS 9 THRU 13

0.0000	0.0000	0.0000	0.0000	0.0000
1.0030	1.0030	1.1584	1.1584	19.2540
-0.0746	-0.0746	-0.1225	-0.1225	0.0000
-12.0291	-12.0291	-3.2363	-3.2363	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
-0.0033	0.0033	0.0007	-0.0007	0.0000
6.3395	-6.3395	25.5186	-25.5186	0.0000
0.6420	-0.6420	0.6249	-0.6249	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000

First, verifying that the number of observable states equal 8.

```
[SOB,NSOB]=OBSERVABLE(S,8)

 NSOB  = 8.
```

Now, verifying that the number of states controllable equal 8.

```
[SCN,NSCN]=CNTRLABLE(S,8)

 NSCN  = 8.
```

Verifying that controllability of the plant due to integral action is maintained. First R is formed as specified in Chapter III, then the rank of R is checked.

```
<> R=[B,A;D,C];  RANK(R)
```

The rank of R is 11, as required. This controllability is also checked by verifying that the rank of GO is equal to the number of outputs .

```
<>RANK(-C*INV(A)*B)
```

The rank of GO is 3 as required.

TRANSMISSION ZEROS

Verifying that there are no transmission zeros in the right-half plane.

```
<> ZEROS(S,8)

THIS SYSTEM HAS NO TRANSMISSION ZEROS
```

CONCLUSIONS

None of the assumptions listed in Chapter III are violated.

D.3 Transfer Function Calculation

This section presents and explains the macros use to determine the open-loop and close-loop transfer function generate by the series combination of the controller and plant. The macro used to generate root locus plots is also presented and explained.

There were three macros used, the first called PCONT was used to determine the continuous transfer functions. The second macro, called PCLOSE, was used to determine the discrete transferfunctions. Finally the last one (PACT) was used to determine the transfer functions when the actuators were added to the system. The first macros (PCONT) is then shown as it appears when used in MATRIX . This macro is then separated into individual commands and comments are made to explain the purpose of these commands. The two remaining macros are then displayed and only the difference between them and the first macro is esplained.

PCONT

```
 CLEAR K1,CLEAR K2,S=<A,B;C,D>;FOR I=1:5,FOR J=1:3,K1(I,J)=HI(I,J)
*SIGMA(J,1);EN D,END,K2=GO'*INV(GO*GO')*RHO;K1;K2;NUMCNTRL=<K1,K2>;
DENCNTRL=<1,0>;<SCTRL,NSCTRL>=SFORM(NUMCNTRL,DENCNTRL,3);<SOPL,NSOPL>=
SERIES(SCTRL,NSCTRL,S,NS);<NUMOPL,DENOPL>=TFORM(SOPL,NSOPL);<AOPL,BOPL>=
SPLIT(SOPL,NSOPL);<SCL,NSCL>=FEEDBACK(SOPL,NSOPL);<NUMCL,DENCL>=
TFORM(SCL,NSCL);POL=PVA(AOPL);<ACL,BCL>=SPLIT(SCL,NSCL);PCL=PVA(ACL);
```

```
CLEAR K1, - Initialize K1

CLEAR K2, - Initialize K2

S=<A,B;C,D>; - Form the system matrix

FOR I=1:5, - Loop used for the rows of K1

FOR J=1:3, - Loop used for the columns of K2

K1(I,J)=HI(I,J)*SIGMA(J,1); - Forming K1, using the inverse of
                              the step-response matrix.
```

```
END, - End of first loop

END, - End of second loop

K2=G0'*INV(G0*G0')*RHO; - Forming K2, using the steady-state
                          transfer funtion and Rho

K1; - Display K1 if desired

K2; - Display K2 if desired

NUMCNTRL=<K1,K2>; - Form numerator of controller transfer function

DENCNTRL=<1,0>; - Form denominator of controller transfer function

<SCTRL,NSCTRL>=SFORM(NUMCNTRL,DENCNTRL,3); - Form the system matrix
                                             of the controller

<SOPL,NSOPL>=SERIES(SCTRL,NSCTRL,S,NS); - Form the open-loop system
                                          matrix

<NUMOPL,DENOPL>=TFORM(SOPL,NSOPL); - Form the numerator and
                                     denominator of the open-loop
                                     transfer function

<AOPL,BOPL>=SPLIT(SOPL,NSOPL); - Obtain the A and B matrices of the
                                 open-loop system

<SCL,NSCL>=FEEDBACK(SOPL,NSOPL); - Form a unity feedback closed-loop
                                   system

<NUMCL,DENCL>=TFORM(SCL,NSCL); - Obtain the numerator and denominator
                                 of the close-loop transfer function


POL=PVA(AOPL); - Eigenvalues of the open-loop system

<ACL,BCL>=SPLIT(SCL,NSCL); - Obtain the A and B matrices of the
                             closed-loop system

PCL=PVA(ACL); - Eignevalues of the closed-loop system
```

PCLOSE

```
 CLEAR K1,CLEAR K2,S=<A,B;C,D>;SD=DISC(S,NS,TSAMP);<AD,BD>=SPLIT(SD,NS);
FOR I=1:5,FOR J=1:3,K1(I,J)=HI(I,J)*SIGMA(J,1);END,END,K2=G0'*INV(G0*G0')
*RHO;K1;K2;NUMCNTRL=<K1,K2>;DENCNTRL=<1,0>;<SCNTRL,NSCTRL>=SFORM(NUMCNTRL,
DENCNTRL,3);<SDCTRL,NSDCTRL>=DISC(SCNTRL,NSCTRL,TSAMP);<SOPL,NSOPL>=
SERIES(SDCTRL,NSDCTRL,SD,NS);<NUMOPL,DENOPL>=TFORM(SOPL,NSOPL);<AOPL,BOPL>=
SPLIT(SOPL,NSOPL);<SCL,NSCL>=FEEDBACK(SOPL,NSOPL);<NUMCL,DENCL>=
TFORM(SCL,NSCL);POL=PVA(AOPL);<ACL,BCL>=SPLIT(SCL,NSCL);PCL=PVA(ACL);
```

The PCLOSE macro discretizes the plant and controller to obtain the pole location in the Z plane.

PACT

```
CLEAR K1,CLEAR K2,S=<A,B;C,D>;FOR I=1:5,FOR J=1:3,K1(I,J)=HI(I,J)*SIGMA(J,1)
;END,END,K2=G0'*INV(G0*G0')*RHO;K1;K2;NUMCNTRL=<K1,K2>;DENCNTRL=<1,0>;
<SCTRL,NSCTRL>=SFORM(NUMCNTRL,DENCNTRL,3);<SCA,NSCA>=SERIES(SCTRL,NSCTRL,
S5,9);<SOPL,NSOPL>=SERIES(SCA,NSCA,S,NS);<NUMOPL,DENOPL>=TFORM(SOPL,NSOPL);
<AOPL,BOPL>=SPLIT(SOPL,NSOPL);<SCL,NSCL>=FEEDBACK(SOPL,NSOPL);<
NUMCL,DENCL>=TFORM(SCL,NSCL);POL=PVA(AOPL);<ACL,BCL>=SPLIT(SCL,NSCL);
PCL=PVA(ACL);
```

The above PACT macro adds the actuator system matrix (SCA) in series with the controller and plant.

Appendix E: MATRIX$_X$ Simulation

Introduction

This appendix contains the MATRIX$_X$ system block diagram representation and interconnection of the simulations conducted for this thesis. The block diagram is shown first, follwed by the interconnection data for the approriate block diagram. The simulation blocks are presented in the following order:

SYSTEM BLOCK

A/C BLOCK

SERVOS BLOCK

ADAPT2 BLOCK

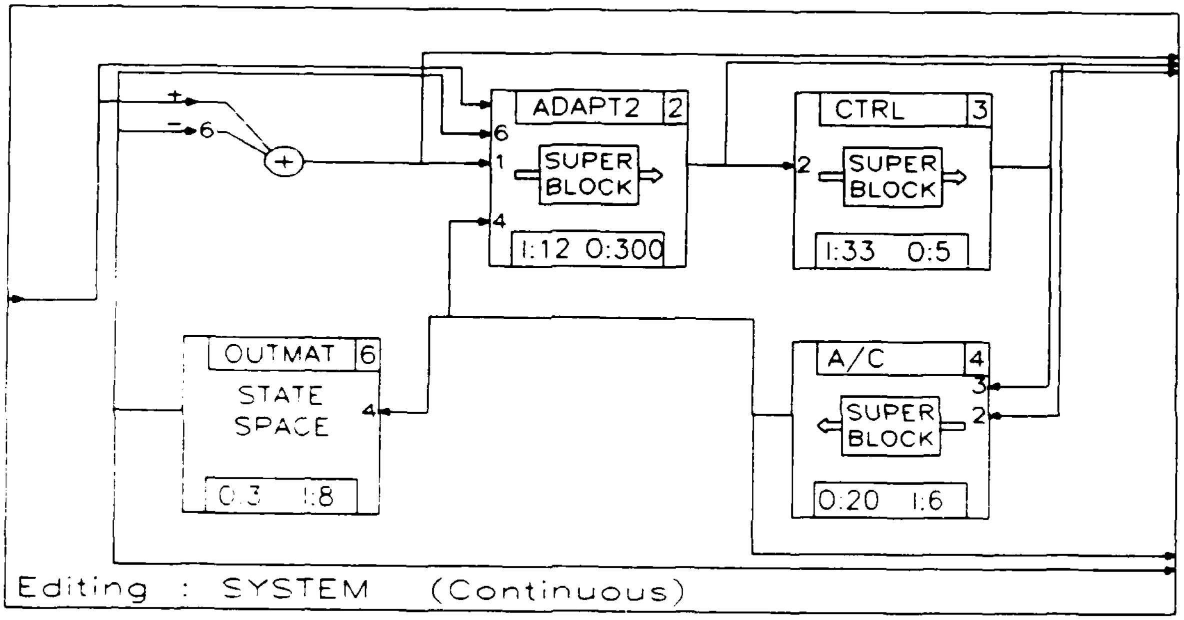
CTRL BLOCK

B MAT BLOCK

A MAT BLOCK

LMTINT BLOCK

SERVO2 BLOCK



ADAPT2
2
6
1
4
SUPER
BLOCK
I:12 O:300
CTRL
3
2
SUPER
BLOCK
I:33 O:5
OUTMAT
6
STATE
SPACE
4
O:3 I:8
A/C
4
3
2
SUPER
BLOCK
O:20 I:6
+
−
6
+
Editing : SYSTEM (Continuous)

INTERCONNECTION FOR SUPER-BLOCK "SYSTEM"

```
<BUILD> Summing Junction : IN SUM
 SIGNS       =
    1.   -1.

     Inputs : 6        Outputs : 3        States : 0

<BUILD> Super-Block : ADAPT2

     Inputs : 12       Outputs : 300      States : 0

<BUILD> Super-Block : CTRL

     Inputs : 33       Outputs : 5        States : 0

<BUILD> Super-Block : A/C

     Inputs : 6        Outputs : 20       States : 0

<BUILD> Linear Dynamic System : OUTMAT

 S           =

    0.    1.    0.    0.    0.    0.    0.    0.
    0.    0.    0.    0.    0.    1.    0.    0.
    0.    0.    0.    0.    0.    0.    0.    1.

     Inputs : 8        Outputs : 3        States : 0

Block location : 1
                   INPUTS              ||              OUTPUTS
           ----------------------------||----------------------------
           From  Name  Location  | To  || From| Name  Location    To
           ----------------------------||----------------------------
           (  1) External Input  |  1  ||  1  | ADAPT2     2     (  1)
                                 |     ||     | External Output (329)
           (  2) External Input  |  2  ||  2  | ADAPT2     2     (  2)
                                 |     ||     | External Output (330)
           (  3) External Input  |  3  ||  3  | ADAPT2     2     (  3)
                                 |     ||     | External Output (331)
           (  1) OUTMAT      6   |  4  ||
           (  2) OUTMAT      6   |  5  ||
           (  3) OUTMAT      6   |  6  ||
```

Block location : 2

INPUTS

From	Name	Location	To
(1)	IN SUM	1	1
(2)	IN SUM	1	2
(3)	IN SUM	1	3
(1)	OUTMAT	6	4
(2)	OUTMAT	6	5
(3)	OUTMAT	6	6
(1)	A/C	4	7
(2)	A/C	4	8
(3)	A/C	4	9
(4)	A/C	4	10
(5)	A/C	4	11
(4)	External Input		12

OUTPUTS

From	Name	Location	To
1	A/C	4	(6)
	External Output		(1)
2	CTRL	3	(1)
	External Output		(2)
3	CTRL	3	(2)
	External Output		(3)
4	CTRL	3	(3)
	External Output		(4)
5	CTRL	3	(4)
	External Output		(5)
6	CTRL	3	(5)
	External Output		(6)
7	CTRL	3	(6)
	External Output		(7)
8	CTRL	3	(7)
	External Output		(8)
9	CTRL	3	(8)
	External Output		(9)
10	CTRL	3	(9)
	External Output		(10)
11	CTRL	3	(10)
	External Output		(11)
12	CTRL	3	(11)
	External Output		(12)
13	CTRL	3	(12)
	External Output		(13)
14	CTRL	3	(13)
	External Output		(14)
15	CTRL	3	(14)
	External Output		(15)
16	CTRL	3	(15)
	External Output		(16)
17	CTRL	3	(16)
	External Output		(17)
18	CTRL	3	(17)
	External Output		(18)
19	CTRL	3	(18)
	External Output		(19)
20	CTRL	3	(19)
	External Output		(20)
21	CTRL	3	(20)
	External Output		(21)
22	CTRL	3	(21)
	External Output		(22)
23	CTRL	3	(22)

From	Name	Location	To
	External	Output	(23)
24	CTRL	3	(23)
	External	Output	(24)
25	CTRL	3	(24)
	External	Output	(25)
26	CTRL	3	(25)
	External	Output	(26)
27	CTRL	3	(26)
	External	Output	(27)
28	CTRL	3	(27)
	External	Output	(28)
29	CTRL	3	(28)
	External	Output	(29)
30	CTRL	3	(29)
	External	Output	(30)
31	CTRL	3	(30)
	External	Output	(31)
32	CTRL	3	(31)
	External	Output	(32)
33	CTRL	3	(32)
	External	Output	(33)
34	CTRL	3	(33)
	External	Output	(34)
35	External	Output	(35)
36	External	Output	(36)
37	External	Output	(37)
.		.	.
.		.	.
.		.	.
300	External	Output	(300)

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(2)	ADAPT2	2	1	1	A/C	4	(1)
					External	Output	(324)
(3)	ADAPT2	2	2	2	A/C	4	(2)
					External	Output	(325)
(4)	ADAPT2	2	3	3	A/C	4	(3)
					External	Output	(326)
(5)	ADAPT2	2	4	4	A/C	4	(4)
					External	Output	(327)
(6)	ADAPT2	2	5	5	A/C	4	(5)
					External	Output	(328)
(7)	ADAPT2	2	6				
(8)	ADAPT2	2	7				
(9)	ADAPT2	2	8				

From	Name	Location	To
(10)	ADAPT2	2	9
(11)	ADAPT2	2	10
(12)	ADAPT2	2	11
(13)	ADAPT2	2	12
(14)	ADAPT2	2	13
(15)	ADAPT2	2	14
(16)	ADAPT2	2	15
(17)	ADAPT2	2	16
(18)	ADAPT2	2	17
(19)	ADAPT2	2	18
(20)	ADAPT2	2	19
(21)	ADAPT2	2	20
(22)	ADAPT2	2	21
(23)	ADAPT2	2	22
(24)	ADAPT2	2	23
(25)	ADAPT2	2	24
(26)	ADAPT2	2	25
(27)	ADAPT2	2	26
(28)	ADAPT2	2	27
(29)	ADAPT2	2	28
(30)	ADAPT2	2	29
(31)	ADAPT2	2	30
(32)	ADAPT2	2	31
(33)	ADAPT2	2	32
(34)	ADAPT2	2	33

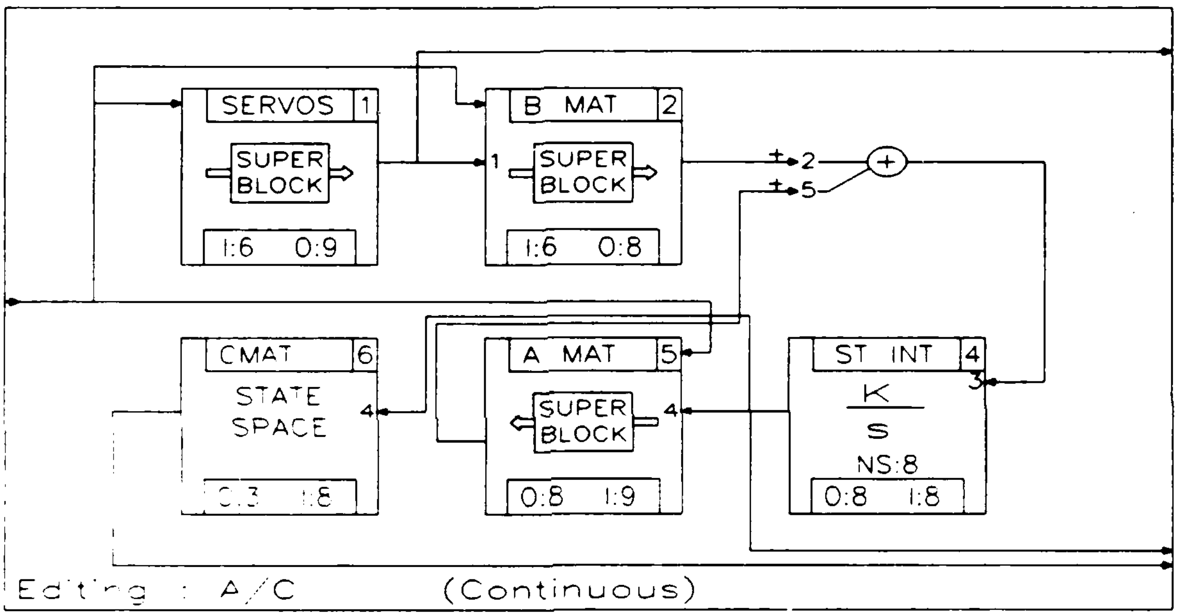
Block location : 4

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	CTRL	3	1	1	ADAPT2	2	(7)
					External	Output	(301)
(2)	CTRL	3	2	2	ADAPT2	2	(8)
					External	Output	(302)
(3)	CTRL	3	3	3	ADAPT2	2	(9)
					External	Output	(303)
(4)	CTRL	3	4	4	ADAPT2	2	(10)
					External	Output	(304)
(5)	CTRL	3	5	5	ADAPT2	2	(11)
					External	Output	(305)
(1)	ADAPT2	2	6	6	External	Output	(306)
				7	External	Output	(307)
				8	External	Output	(308)
				9	External	Output	(309)
				10	OUTMAT	6	(1)
					External	Output	(310)
				11	OUTMAT	6	(2)
					External	Output	(311)

12	OUTMAT	6	(3)	
	External	Output	(312)	
13	OUTMAT	6	(4)	
	External	Output	(313)	
14	OUTMAT	6	(5)	
	External	Output	(314)	
15	OUTMAT	6	(6)	
	External	Output	(315)	
16	OUTMAT	6	(7)	
	External	Output	(316)	
17	OUTMAT	6	(8)	
	External	Output	(317)	
18	External	Output	(318)	
19	External	Output	(319)	
20	External	Output	(320)	

Block location : 6

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(10)	A/C	4	1	1	IN SUM	1	(4)
					ADAPT2	2	(4)
					External	Output	(321)
(11)	A/C	4	2	2	IN SUM	1	(5)
					ADAPT2	2	(5)
					External	Output	(322)
(12)	A/C	4	3	3	IN SUM	1	(6)
					ADAPT2	2	(6)
					External	Output	(323)
(13)	A/C	4	4				
(14)	A/C	4	5				
(15)	A/C	4	6				
(16)	A/C	4	7				
(17)	A/C	4	8				


SERVOS 1
SUPER BLOCK
1:6 0:9
B MAT 2
SUPER BLOCK
1:6 0:8
CMAT 6
STATE SPACE
A MAT 5
SUPER BLOCK
0:8 1:9
ST INT 4
K
s
NS:8
0:8 1:8
(Continuous)

INTERCONNECTION FOR SUPER-BLOCK "A/C"

```
<BUILD> Super-Block : SERVOS

     Inputs : 6        Outputs : 9        States : 0

<BUILD> Super-Block : B MAT

     Inputs : 6        Outputs : 8        States : 0

<BUILD> Summing Junction : SUM

 SIGNS       =

    1.     1.

     Inputs : 16       Outputs : 8        States : 0

<BUILD> Integrator of Order 1 : ST INT

 GAIN        =

    1.     1.     1.     1.     1.     1.     1.     1.

 XO          =

    0.     0.     0.     0.     0.     0.     0.     0.

     Inputs : 8        Outputs : 8        States : 8

<BUILD> Super-Block : A MAT

     Inputs : 9        Outputs : 8        States : 0

<BUILD> Linear Dynamic System : CMAT

 S           =

    0.     1.     0.     0.     0.     0.     0.     0.
    0.     0.     0.     0.     0.     1.     0.     0.
    0.     0.     0.     0.     0.     0.     0.     1.

     Inputs : 8        Outputs : 3        States : 0
```

Block location : 1

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External Input		1	1	B MAT	2	(1)
					External Output		(1)
(2)	External Input		2	2	B MAT	2	(2)
					External Output		(2)
(3)	External Input		3	3	B MAT	2	(3)
					External Output		(3)
(4)	External Input		4	4	B MAT	2	(4)
					External Output		(4)
(5)	External Input		5	5	B MAT	2	(5)
					External Output		(5)
(6)	External Input		6	6	External Output		(6)
				7	External Output		(7)
				8	External Output		(8)
				9	External Output		(9)

Block location : 2

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	SERVOS	1	1	1	SUM	3	(1)
(2)	SERVOS	1	2	2	SUM	3	(2)
(3)	SERVOS	1	3	3	SUM	3	(3)
(4)	SERVOS	1	4	4	SUM	3	(4)
(5)	SERVOS	1	5	5	SUM	3	(5)
(6)	External Input		6	6	SUM	3	(6)
				7	SUM	3	(7)
				8	SUM	3	(8)

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	B MAT	2	1	1	ST INT	4	(1)
(2)	B MAT	2	2	2	ST INT	4	(2)
(3)	B MAT	2	3	3	ST INT	4	(3)
(4)	B MAT	2	4	4	ST INT	4	(4)
(5)	B MAT	2	5	5	ST INT	4	(5)
(6)	B MAT	2	6	6	ST INT	4	(6)
(7)	B MAT	2	7	7	ST INT	4	(7)
(8)	B MAT	2	8	8	ST INT	4	(8)

(1)	A MAT	5	9				
(2)	A MAT	5	10				
(3)	A MAT	5	11				
(4)	A MAT	5	12				
(5)	A MAT	5	13				
(6)	A MAT	5	14				
(7)	A MAT	5	15				
(8)	A MAT	5	16				

Block location : 4

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	SUM	3	1	1	A MAT	5	(1)
					CMAT	6	(1)
					External Output		(10)
(2)	SUM	3	2	2	A MAT	5	(2)
					CMAT	6	(2)
					External Output		(11)
(3)	SUM	3	3	3	A MAT	5	(3)
					CMAT	6	(3)
					External Output		(12)
(4)	SUM	3	4	4	A MAT	5	(4)
					CMAT	6	(4)
					External Output		(13)
(5)	SUM	3	5	5	A MAT	5	(5)
					CMAT	6	(5)
					External Output		(14)
(6)	SUM	3	6	6	A MAT	5	(6)
					CMAT	6	(6)
					External Output		(15)
(7)	SUM	3	7	7	A MAT	5	(7)
					CMAT	6	(7)
					External Output		(16)
(8)	SUM	3	8	8	A MAT	5	(8)
					CMAT	6	(8)
					External Output		(17)

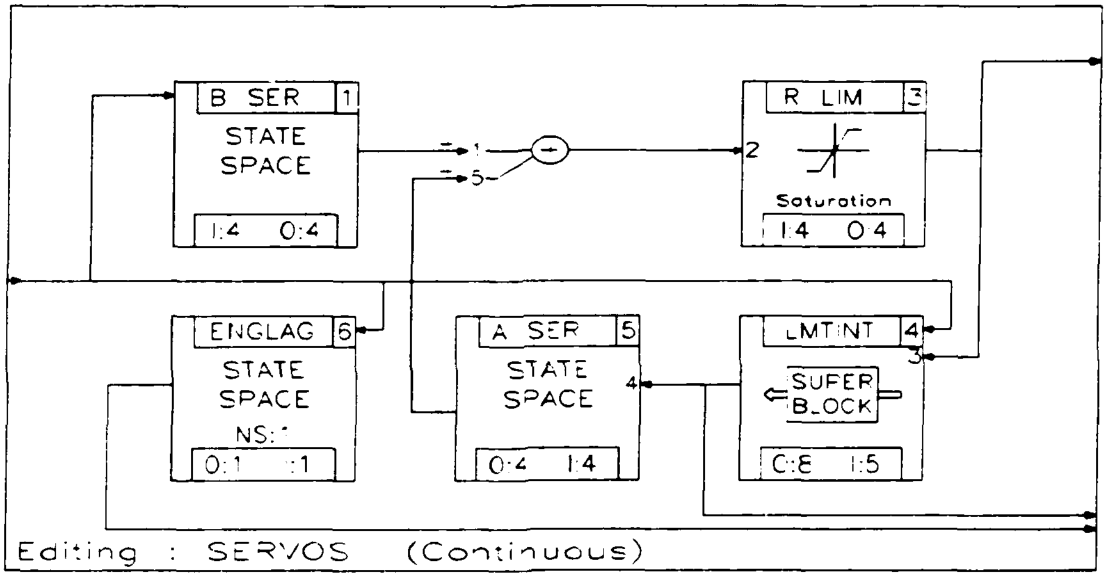
Block location : 5

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	ST INT	4	1	1	SUM	3	(9)
(2)	ST INT	4	2	2	SUM	3	(10)
(3)	ST INT	4	3	3	SUM	3	(11)
(4)	ST INT	4	4	4	SUM	3	(12)

From	Name	Location	To	From	Name	Location	To
(5)	ST INT	4	5	5	SUM	3	(13)
(6)	ST INT	4	6	6	SUM	3	(14)
(7)	ST INT	4	7	7	SUM	3	(15)
(8)	ST INT	4	8	8	SUM	3	(16)
(6)	External Input		9				

Block location : 6

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	ST INT	4	1	1	External Output		(18)
(2)	ST INT	4	2	2	External Output		(19)
(3)	ST INT	4	3	3	External Output		(20)
(4)	ST INT	4	4				
(5)	ST INT	4	5				
(6)	ST INT	4	6				
(7)	ST INT	4	7				
(8)	ST INT	4	8				



B SER
1
STATE SPACE
I:4 O:4
R LIM
3
Saturation
I:4 O:4
ENGLAG
6
STATE SPACE
NS:1
O:1 I:1
A SER
5
STATE SPACE
O:4 I:4
LMTINT
4
SUPER BLOCK
O:8 I:5
Editing : SERVOS (Continuous)

INTERCONNECTION FOR SUPER-BLOCK "SERVOS"

<BUILD> Linear Dynamic System : B SER

S =

1000.	0.	0.	0.
0.	1000.	0.	0.
0.	0.	1000.	0.
0.	0.	0.	1000.

Inputs : 4 Outputs : 4 States : 0

<BUILD> Summing Junction : SUM

SIGNS =

1. 1.

Inputs : 8 Outputs : 4 States : 0

BUILD> Absolute Saturation Limit : R LIM

SATURATION =

90000. 90000. 90000. 90000.

Inputs : 4 Outputs : 4 States : 0

<BUILD> Super-Block : LMTINT

Inputs : 5 Outputs : 8 States : 0

<BUILD> Linear Dynamic System : A SER

```
S          =

     -1000.       0.        0.        0.
        0.    -1000.        0.        0.
        0.        0.    -1000.        0.
        0.        0.        0.    -1000.

     Inputs : 4      Outputs : 4      States : 0
```

<BUILD> Linear Dynamic System : ENGLAG

```
S          =

   -1.    1.
    1.    0.

XO         =

    0.

     Inputs : 1      Outputs : 1      States : 1
```

Block location : 1

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External Input		1	1	SUM	2	(1)
(2)	External Input		2	2	SUM	2	(2)
(3)	External Input		3	3	SUM	2	(3)
(4)	External Input		4	4	SUM	2	(4)

Block location : 2

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	B SER	1	1	1	R LIM	3	(1)
(2)	B SER	1	2	2	R LIM	3	(2)
(3)	B SER	1	3	3	R LIM	3	(3)
(4)	B SER	1	4	4	R LIM	3	(4)
(1)	A SER	5	5				
(2)	A SER	5	6				
(3)	A SER	5	7				
(4)	A SER	5	8				

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	SUM	2	1	1	LMTINT	4	(1)
					External Output		(10)
(2)	SUM	2	2	2	LMTINT	4	(2)
					External Output		(11)
(3)	SUM	2	3	3	LMTINT	4	(3)
					External Output		(12)
(4)	SUM	2	4	4	LMTINT	4	(4)
					External Output		(13)

Block location : 4

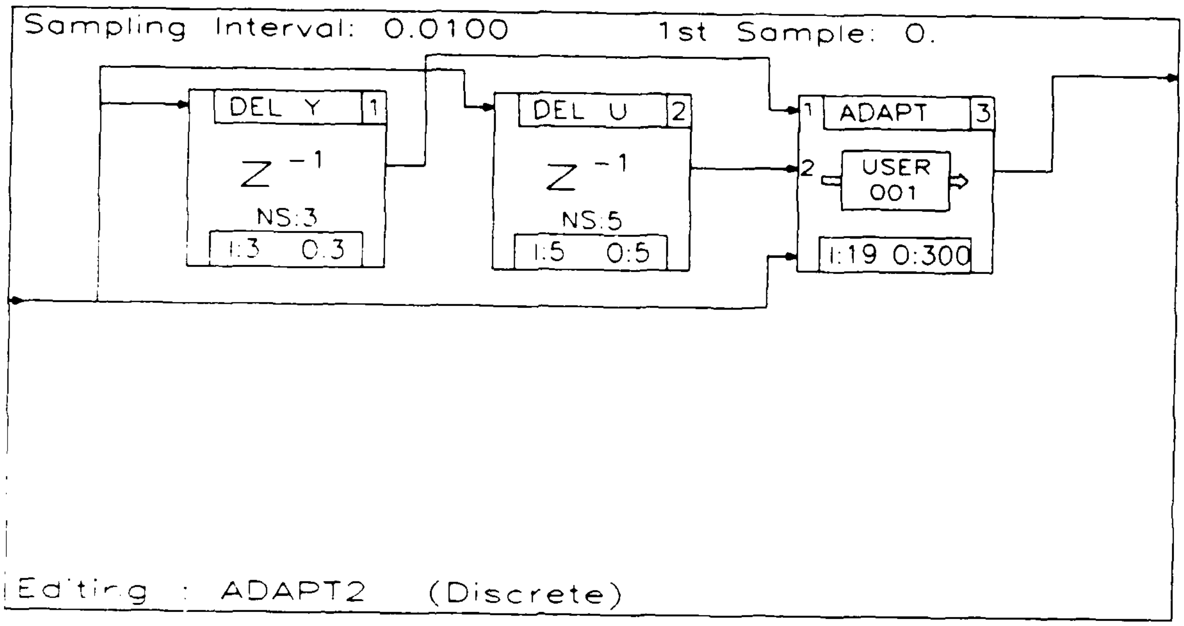
INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	R LIM	3	1	1	A SER	5	(1)
					External Output		(1)
(2)	R LIM	3	2	2	A SER	5	(2)
					External Output		(2)
(3)	R LIM	3	3	3	A SER	5	(3)
					External Output		(3)
(4)	R LIM	3	4	4	A SER	5	(4)
					External Output		(4)
(6)	External Input		5	5	External Output		(6)
				6	External Output		(7)
				7	External Output		(8)
				8	External Output		(9)

Block location : 5

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	LMTINT	4	1	1	SUM	2	(5)
(2)	LMTINT	4	2	2	SUM	2	(6)
(3)	LMTINT	4	3	3	SUM	2	(7)
(4)	LMTINT	4	4	4	SUM	2	(8)

Block location : 6

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(5)	External Input		1	1	External Output		(5)



Sampling Interval: 0.0100
1st Sample: 0.
DEL Y
1
Z⁻¹
NS:3
I:3 O:3
DEL U
2
Z⁻¹
NS:5
I:5 O:5
ADAPT
3
1
2
USER
001
I:19 O:300
Editing : ADAPT2 (Discrete)

```
INTERCONNECTION FOR SUPER-BLOCK "ADAPT2"

<BUILD> Time Delay -> z**(-K) : DEL Y

 K*TSAMP     =

   1.0000D-02

      Inputs : 3        Outputs : 3        States : 3

<BUILD> Time Delay -> z**(-K) : DEL U

 K*TSAMP     =

   1.0000D-02

      Inputs : 5        Outputs : 5        States : 5

<BUILD> User Code Function Block #1 : ADAPT
     This block has direct feed-through terms (dY/dU <> 0)

 RPAR        =

    COLUMNS    1 THRU    6
8.5000D-01  9.5000D-01  5.0000D-04  1.0000D+02  2.9000D+00  2.0000D+00

    COLUMNS    7 THRU   12
2.0000D+00  1.0000D-02  1.0000D+00  1.0000D-02  5.0000D-01  1.0000D-08

    COLUMNS   13 THRU   18
1.0000D+02  1.0000D-02  9.9000D+01  1.0000D+06  5.0000D-03  1.0000D-12

    COLUMNS   19 THRU   24
2.5000D-01  2.0000D-01  1.0000D-01  1.0000D-01 -9.0000D-01  9.0000D-01

    COLUMNS   25 THRU   30
9.0000D-01  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00

    COLUMNS   31 THRU   36
3.0000D-02  3.0000D-03  9.5000D-02  2.0000D-01  0.0000D+00  0.0000D+00

    COLUMNS   37 THRU   42
0.0000D+00  2.0000D-01  0.0000D+00  0.0000D+00  0.0000D+00  2.0000D-01

    COLUMNS   43 THRU   48
0.0000D+00  6.0000D+00  1.0000D+02  1.0000D+00  2.0000D+00  1.0000D+00
```

```
      COLUMNS  49 THRU  54
  2.0000D+00  3.0000D+00  4.0000D+00  5.0000D+00  2.7952D-02  2.7952D-02

      COLUMNS  55 THRU  60
  1.6210D-02  1.6210D-02  1.9249D-01 -5.4597D-05  5.4597D-05  1.7295D-05

      COLUMNS  61 THRU  66
 -1.7295D-05  0.0000D+00  6.3994D-03 -6.3994D-03  6.2089D-03 -6.2089D-03

      COLUMNS  67 THRU  72
  0.0000D+00 -3.9774D+00  0.0000D+00  0.0000D+00  0.0000D+00 -1.9951D+00

      COLUMNS  73 THRU  78
 -1.4340D-02  0.0000D+00 -7.3194D-02 -1.9693D+00 -4.7607D-02 -4.7607D-02

      COLUMNS  79 THRU  84
 -3.9060D-02 -3.9060D-02 -5.7321D-01 -8.0478D-05  8.0478D-05 -8.6128D-05

      COLUMNS  85 THRU  90
  8.6128D-05  0.0000D+00 -6.2418D-03  6.2418D-03 -6.0974D-03  6.0974D-03

      COLUMNS  91 THRU  96
  0.0000D+00  5.9319D+00  0.0000D+00  0.0000D+00  0.0000D+00  9.9707D-01

      COLUMNS  97 THRU 102
  1.4252D-02  0.0000D+00  7.1142D-02  9.6938D-01  1.1833D-02  1.1833D-02

      COLUMNS 103 THRU 108
  2.9625D-02  2.9625D-02  5.6891D-01  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 109 THRU 114
  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 115 THRU 120
  0.0000D+00 -3.9316D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 121 THRU 126
  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  7.8281D-03  7.8281D-03

      COLUMNS 127 THRU 132
 -6.7731D-03 -6.7731D-03 -1.8818D-01  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 133 THRU 138
  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 139 THRU 144
  0.0000D+00  9.7709D-01  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00

      COLUMNS 145 THRU 150
  0.0000D+00  0.0000D+00  0.0000D+00  0.0000D+00  1.3978D-02  2.7957D-02
```

COLUMNS 151 THRU 156

1.6214D-02 1.6214D-02 1.9253D-01 -2.7299D-05 5.4597D-05 1.7295D-05

COLUMNS 157 THRU 162

-1.7295D-05 0.0000D+00 3.1997D-03 -6.3994D-03 6.2089D-03 -6.2089D-03

COLUMNS 163 THRU 168

0.0000D+00 -3.9774D+00 0.0000D+00 0.0000D+00 0.0000D+00 -1.9951D+00

COLUMNS 169 THRU 174

-1.4340D-02 0.0000D+00 -7.3194D-02 -1.9693D+00 -2.3807D-02 -4.7613D-02

COLUMNS 175 THRU 180

-3.9068D-02 -3.9068D-02 -5.7334D-01 -4.0239D-05 8.0478D-05 -8.6128D-05

COLUMNS 181 THRU 186

8.6128D-05 0.0000D+00 -3.1209D-03 6.2418D-03 -6.0974D-03 6.0974D-03

COLUMNS 187 THRU 192

0.0000D+00 5.9319D+00 0.0000D+00 0.0000D+00 0.0000D+00 9.9707D-01

COLUMNS 193 THRU 198

1.4252D-02 0.0000D+00 7.1142D-02 9.6938D-01 5.9157D-03 1.1831D-02

COLUMNS 199 THRU 204

2.9630D-02 2.9630D-02 5.6903D-01 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 205 THRU 210

0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 211 THRU 216

0.0000D+00 -3.9316D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 217 THRU 222

0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 3.9156D-03 7.8311D-03

COLUMNS 223 THRU 228

-6.7742D-03 -6.7742D-03 -1.8822D-01 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 229 THRU 234

0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 235 THRU 240

0.0000D+00 9.7709D-01 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 241 THRU 246

0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 2.7957D-02 2.7957D-02

COLUMNS 247 THRU 252
1.6214D-02 1.6214D-02 1.9253D-01 -5.4597D-05 5.4597D-05 1.7295D-05

COLUMNS 253 THRU 258
-1.7295D-05 0.0000D+00 6.3994D-03 -6.3994D-03 6.2089D-03 -6.2089D-03

COLUMNS 259 THRU 264
0.0000D+00 -3.9778D+00 0.0000D+00 0.0000D+00 0.0000D+00 -1.9951D+00

COLUMNS 265 THRU 270
-1.4340D-02 0.0000D+00 -7.3194D-02 -1.9693D+00 -4.7613D-02 -4.7613D-02

COLUMNS 271 THRU 276
-3.9068D-02 -3.9068D-02 -5.7334D-01 -8.0478D-05 8.0478D-05 -8.6128D-05

COLUMNS 277 THRU 282
8.6128D-05 0.0000D+00 -6.2418D-03 6.2418D-03 -6.0974D-03 6.0974D-03

COLUMNS 283 THRU 288
0.0000D+00 5.9332D+00 0.0000D+00 0.0000D+00 0.0000D+00 9.9707D-01

COLUMNS 289 THRU 294
1.4252D-02 0.0000D+00 7.1142D-02 9.6938D-01 1.1831D-02 1.1831D-02

COLUMNS 295 THRU 300
2.9630D-02 2.9630D-02 5.6903D-01 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 301 THRU 306
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 307 THRU 312
0.0000D+00 -3.9329D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 313 THRU 318
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 7.8311D-03 7.8311D-03

COLUMNS 319 THRU 324
-6.7742D-03 -6.7742D-03 -1.8822D-01 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 325 THRU 330
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 331 THRU 336
0.0000D+00 9.7752D-01 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 337 THRU 340
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

IPAR =

COLUMNS 1 THRU 12

1.	4.	0.	8.	3.	1.	0.	5.	4.	0.	20.	3.

COLUMNS 13 THRU 17

200.	0.	0.	1.	0.

Inputs : 19 Outputs : 300 States : 0

Block location : 1

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(6)	External Input		1	1	ADAPT	3	(12)
(5)	External Input		2	2	ADAPT	3	(13)
(4)	External Input		3	3	ADAPT	3	(14)

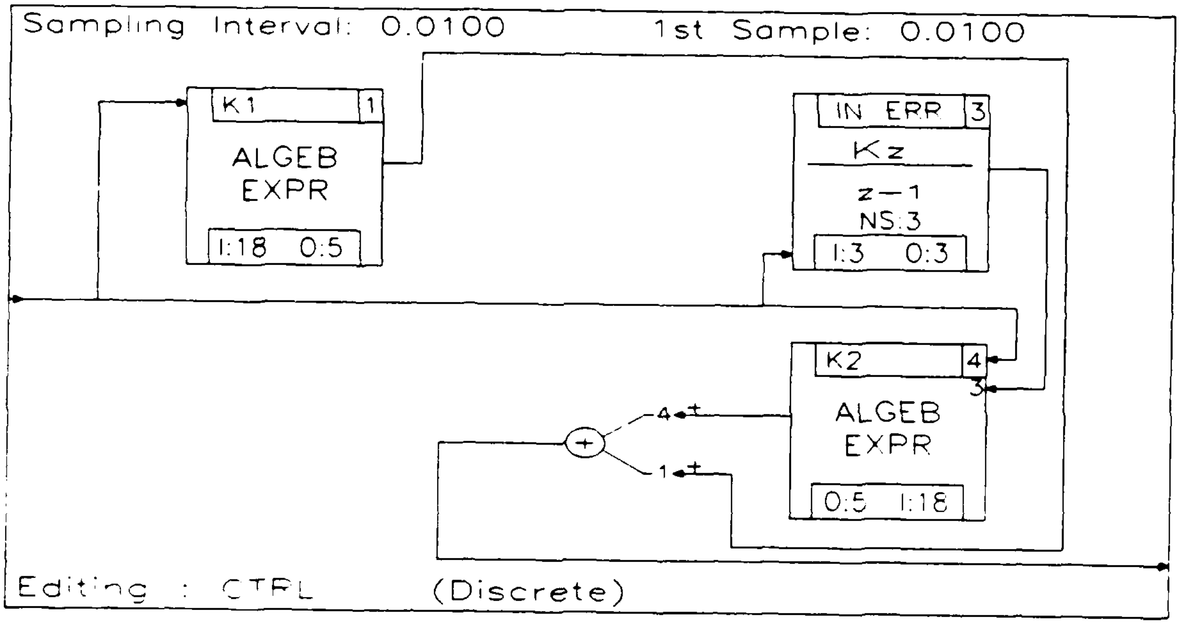
Block location : 2

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(11)	External Input		1	1	ADAPT	3	(7)
(10)	External Input		2	2	ADAPT	3	(8)
(9)	External Input		3	3	ADAPT	3	(9)
(8)	External Input		4	4	ADAPT	3	(10)
(7)	External Input		5	5	ADAPT	3	(11)

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External Input		1	1	External Output		(1)
(2)	External Input		2	2	External Output		(2)
(3)	External Input		3	3	External Output		(3)
(4)	External Input		4	4	External Output		(4)
(5)	External Input		5	5	External Output		(5)
(6)	External Input		6	6	External Output		(6)
(1)	DEL U	2	7	7	External Output		(7)
(2)	DEL U	2	8	8	External Output		(8)
(3)	DEL U	2	9	9	External Output		(9)
(4)	DEL U	2	10	10	External Output		(10)
(5)	DEL U	2	11	11	External Output		(11)

(1) DEL Y 1	12	12	External Output	(12)
(2) DEL Y 1	13	13	External Output	(13)
(3) DEL Y 1	14	14	External Output	(14)
(7) External Input	15	15	External Output	(15)
(8) External Input	16	16	External Output	(16)
(9) External Input	17	17	External Output	(17)
(10) External Input	18	18	External Output	(18)
(11) External Input	19	19	External Output	(19)
		20	External Output	(20)
		21	External Output	(21)
		.	.	.
		.	.	.
		.	.	.
		300	External Output	(300)


Sampling Interval: 0.0100
1st Sample: 0.0100
K1
1
ALGEB
EXPR
I:18 0:5
IN ERR
3
Kz
z—1
NS:3
I:3 0:3
K2
4
3
ALGEB
EXPR
0:5 I:18
4
1
Editing : CTRL
(Discrete)

INTERCONNECTION FOR SUPER-BLOCK "CTRL"

```
<BUILD> General Algebraic Equation : K1

    Y1 = U4*U1 + U5*U2 + U6*U3;
    Y2 = U7*U1 + U8*U2 + U9*U3;
    Y3 = U10*U1 + U11*U2 + U12*U3;
    Y4 = U13*U1 + U14*U2 + U15*U3;
    Y5 = U16*U1 + U17*U2 + U18*U3;

  Constants : 0    Parameters : 0    Stacksize : 3

     Inputs : 18      Outputs : 5       States : 0

<BUILD> Integrator of Order 1 : IN ERR

 GAIN        =

   1.0000D-02   1.0000D-02   1.0000D-02

 X0          =

    0.    0.    0.

     Inputs : 3       Outputs : 3       States : 3

<BUILD> General Algebraic Equation : K2

    Y1 = U4*U1 + U5*U2 + U6*U3;
    Y2 = U7*U1 + U8*U2 + U9*U3;
    Y3 = U10*U1 + U11*U2 + U12*U3;
    Y4 = U13*U1 + U14*U2 + U15*U3;
    Y5 = U16*U1 + U17*U2 + U18*U3;

  Constants : 0    Parameters : 0    Stacksize : 3

     Inputs : 18      Outputs : 5       States : 0

<BUILD> Summing Junction :

 SIGNS       =

    1.    1.

     Inputs : 10      Outputs : 5       States : 0
```

Block location : 1

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External	Input	1	1		5	(1)
(2)	External	Input	2	2		5	(2)
(3)	External	Input	3	3		5	(3)
(4)	External	Input	4	4		5	(4)
(5)	External	Input	5	5		5	(5)
(6)	External	Input	6				
(7)	External	Input	7				
(8)	External	Input	8				
(9)	External	Input	9				
(10)	External	Input	10				
(11)	External	Input	11				
(12)	External	Input	12				
(13)	External	Input	13				
(14)	External	Input	14				
(15)	External	Input	15				
(16)	External	Input	16				
(17)	External	Input	17				
(18)	External	Input	18				

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External	Input	1	1	K2	4	(1)
(2)	External	Input	2	2	K2	4	(2)
(3)	External	Input	3	3	K2	4	(3)

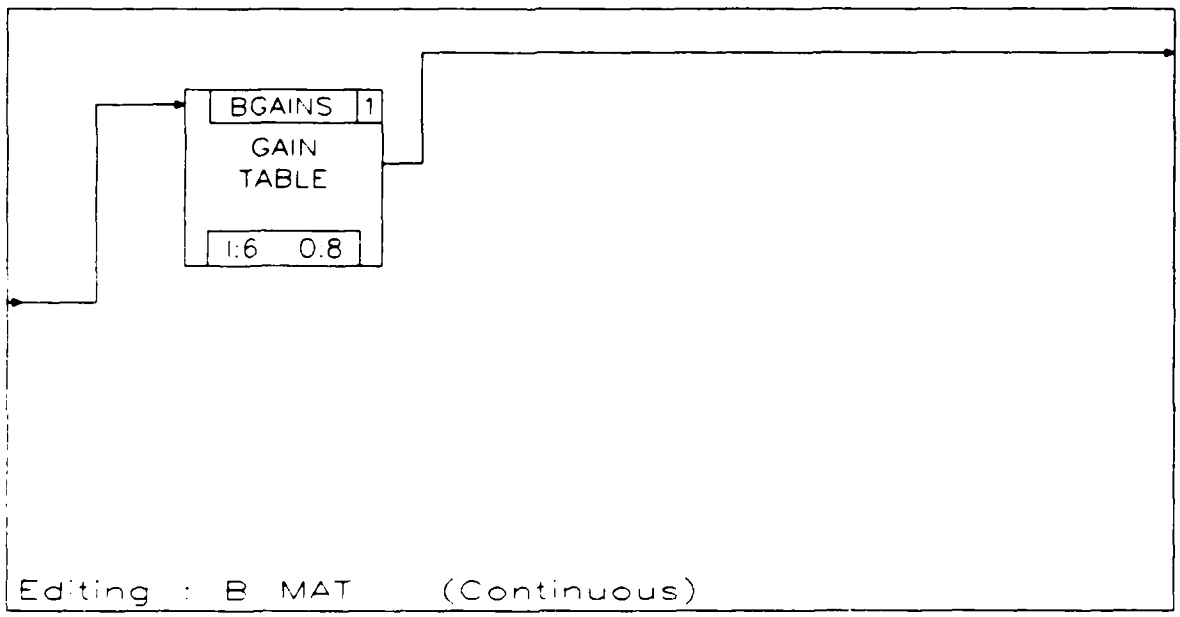
Block location : 4

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	IN ERR	3	1	1		5	(6)
(2)	IN ERR	3	2	2		5	(7)
(3)	IN ERR	3	3	3		5	(8)
(19)	External	Input	4	4		5	(9)
(20)	External	Input	5	5		5	(10)
(21)	External	Input	6				
(22)	External	Input	7				

(23)	External Input	8
(24)	External Input	9
(25)	External Input	10
(26)	External Input	11
(27)	External Input	12
(28)	External Input	13
(29)	External Input	14
(30)	External Input	15
(31)	External Input	16
(32)	External Input	17
(33)	External Input	18

Block location : 5

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	K1	1	1	1	External Output		(1)
(2)	K1	1	2	2	External Output		(2)
(3)	K1	1	3	3	External Output		(3)
(4)	K1	1	4	4	External Output		(4)
(5)	K1	1	5	5	External Output		(5)
(1)	K2	4	6				
(2)	K2	4	7				
(3)	K2	4	8				
(4)	K2	4	9				
(5)	K2	4	10				


BGAINS
1
GAIN
TABLE
1:6 0.8
Editing : B MAT (Continuous)

INTERCONNECTION FOR SUPER-BLOCK "B MAT"

<BUILD> Gain Table Scheduler : BGAINS

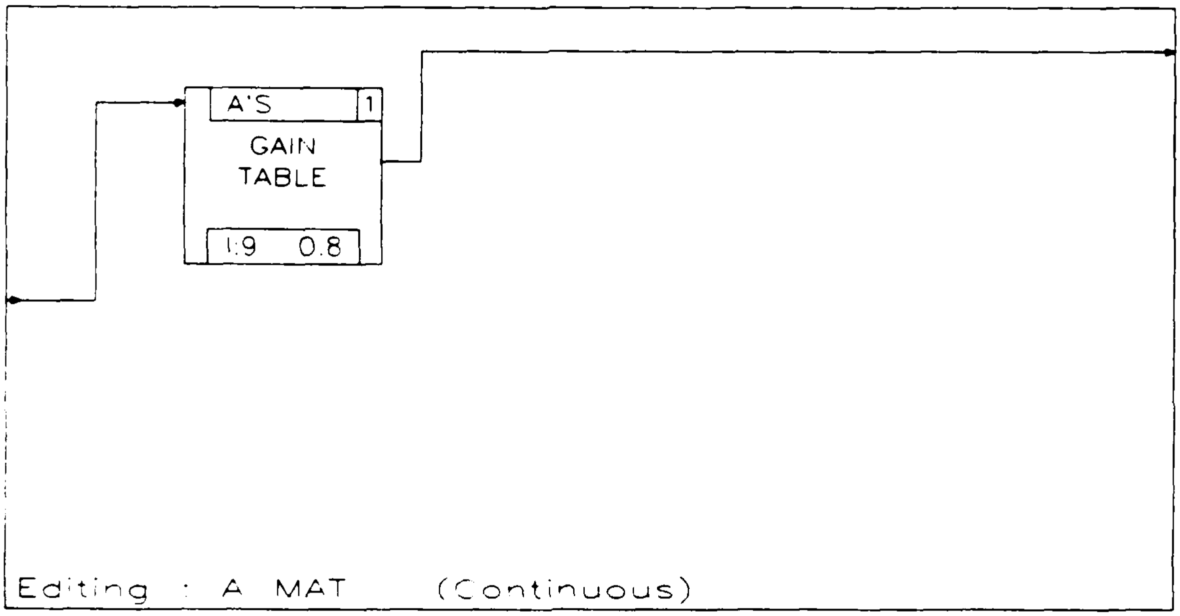
Switch Tolerance = 1.0000D-02

At least 26 more lines. Enter blank to continue output: 1

Inputs : 6 Outputs : 8 States : 0

Block location : 1

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(6)	External	Input	1	1	External	Output	(1)
(1)	External	Input	2	2	External	Output	(2)
(2)	External	Input	3	3	External	Output	(3)
(3)	External	Input	4	4	External	Output	(4)
(4)	External	Input	5	5	External	Output	(5)
(5)	External	Input	6	6	External	Output	(6)
				7	External	Output	(7)
				8	External	Output	(8)


A'S
1
GAIN
TABLE
1.9
0.8
Editing : A MAT (Continuous)

INTECONNECTION FOR SUPER-BLOCK "A MAT"

<BUILD> Gain Table Scheduler : A'S

Switch Tolerance = 1.0000D-02

GAIN TABLE =

COLUMNS 1 THRU 6

0.0000D+00	0.0000D+00	0.0000D+00	1.0000D+00	0.0000D+00	0.0000D+00
-3.2183D+01	-5.6009D-02	3.8291D+01	-3.0138D+01	0.0000D+00	0.0000D+00
-1.1000D-03	4.5971D-05	-1.4845D+00	9.9480D-01	0.0000D+00	0.0000D+00
3.0000D-04	-2.1010D-03	4.2717D+00	-7.7720D-01	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D-00	0.0000D+00	0.0000D+00
0.0000D+00	0.0000D+00	0.0000D+00	0.0000D-00	3.4500D-02	-3.4536D-01
0.0000D-00	0.0000D+00	0.0000D+00	0.0000D-00	0.0000D+00	-5.5253D+01
0.0000D-00	0.0000D+00	0.0000D+00	0.0000D-00	0.0000D+00	7.2370D+00

COLUMNS 7 THRU 8

0.0000D+00	0.0000D-00
0.0000D-00	0.0000D-00
0.0000D-00	0.0000D-00
0.0000D+00	0.0000D+00
1.0000D+00	0.0000D-00
3.2600D-02	-9.9760D-01
-2.8004D-00	1.4570D-01
-2.3200D-02	-3.6250D-01

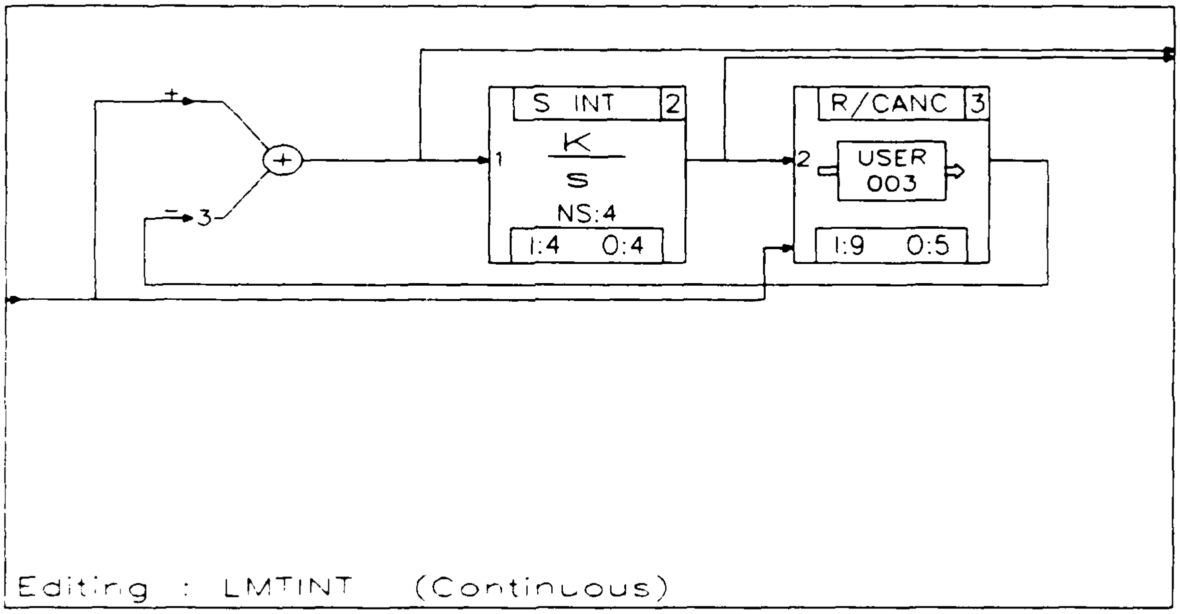
BREAKPOINT =

5.0000D-01

Inputs : 9 Outputs : 8 States : 0

Block location : 1

INPUTS From	Name	Location	To	OUTPUTS From	Name	Location	To
(9)	External	Input	1	1	External	Output	(1)
(1)	External	Input	2	2	External	Output	(2)
(2)	External	Input	3	3	External	Output	(3)
(3)	External	Input	4	4	External	Output	(4)
(4)	External	Input	5	5	External	Output	(5)
(5)	External	Input	6	6	External	Output	(6)
(6)	External	Input	7	7	External	Output	(7)
(7)	External	Input	8	8	External	Output	(8)
(8)	External	Input	9				


S INT
2
K
s
NS:4
1
I:4 0:4
R/CANC
3
2
USER
003
I:9 0:5
+
-
3
Editing : LMTINT (Continuous)

INTERCONNECTION FOR SUPER-BLOCK "LMTINT"

```
<BUILD> Summing Junction :

 SIGNS       =

    1.    -1.

     Inputs : 8       Outputs : 4       States : 0

<BUILD> Integrator of Order 1 : S INT

 GAIN        =

    1.     1.     1.     1.

 XO          =

    0.     0.     0.     0.

     Inputs : 4       Outputs : 4       States : 4

<BUILD> User Code Function Block #3 : R/CANC
     This block has direct feed-through terms (dY/dU <> 0)

 RPAR        =

    COLUMNS     1 THRU    6
  -3.9724D-03    4.7543D+03   -3.9724D-03    4.7578D+03   -3.1416D-03    3.8397D+03

    COLUMNS     7 THRU    8
  -3.1416D-03    3.8397D+03

 IPAR        =

    4.     1.

     Inputs : 9       Outputs : 5       States : 0
```

Block location : 1

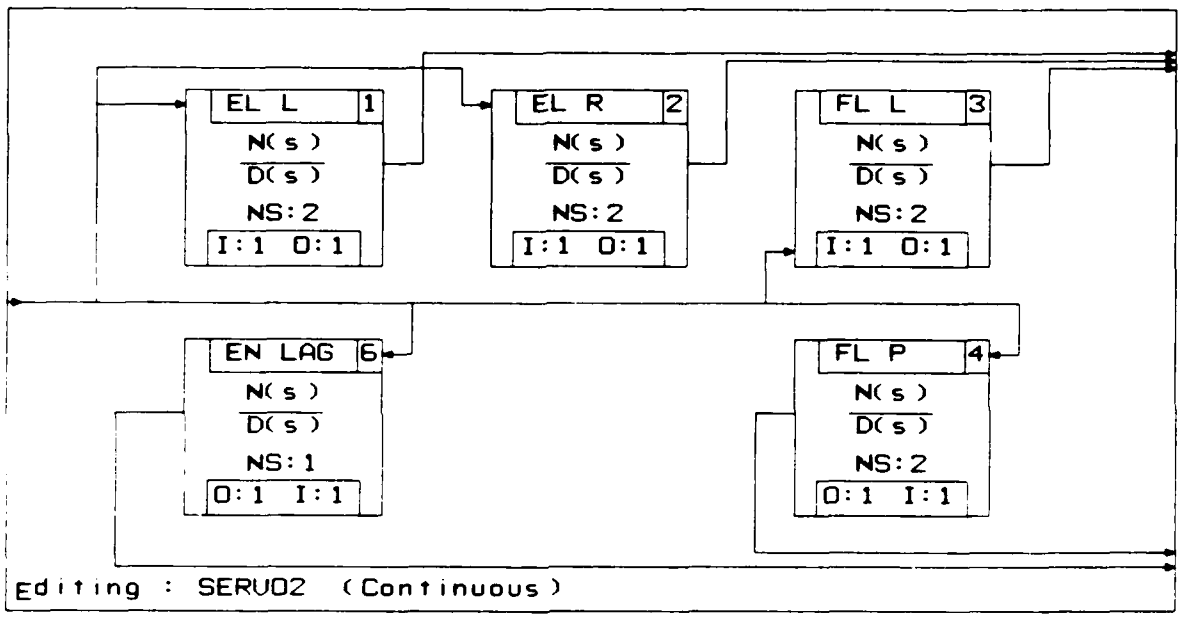
INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)	External	Input	1	1	S INT	2	(1)
					External	Output	(5)
(2)	External	Input	2	2	S INT	2	(2)
					External	Output	(6)
(3)	External	Input	3	3	S INT	2	(3)
					External	Output	(7)
(4)	External	Input	4	4	S INT	2	(4)
					External	Output	(8)
(1)	R/CANC	3	5				
(2)	R/CANC	3	6				
(3)	R/CANC	3	7				
(4)	R/CANC	3	8				

Block location : 2

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(1)		1	1	1	R/CANC	3	(2)
					External	Output	(1)
(2)		1	2	2	R/CANC	3	(3)
					External	Output	(2)
(3)		1	3	3	R/CANC	3	(4)
					External	Output	(3)
(4)		1	4	4	R/CANC	3	(5)
					External	Output	(4)

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(5)	External	Input	1	1		1	(5)
(1)	S INT	2	2	2		1	(6)
(2)	S INT	2	3	3		1	(7)
(3)	S INT	2	4	4		1	(8)
(4)	S INT	2	5	5			
(1)	External	Input	6				
(2)	External	Input	7				
(3)	External	Input	8				
(4)	External	Input	9				


EL L 1
N(s)
D(s)
NS:2
I:1 O:1
EL R 2
N(s)
D(s)
NS:2
I:1 O:1
FL L 3
N(s)
D(s)
NS:2
I:1 O:1
EN LAG 6
N(s)
D(s)
NS:1
O:1 I:1
FL P 4
N(s)
D(s)
NS:2
O:1 I:1
Editing : SERVO2 (Continuous)

INTERCONNECTION FOR SUPER-BLOCK "SERVO2"

<BUILD> Linear Dynamic System : EL L

S =

-92.	-1440.	1.
1.	0.	0.
0.	1440.	0.

XO =

0. 0.

Inputs : 1 Outputs : 1 States : 2

<BUILD> Linear Dynamic System : EL R

S =

-92.	-1440.	1.
1.	0.	0.
0.	1440.	0.

XO =

0. 0.

Inputs : 1 Outputs : 1 States : 2

<BUILD> Linear Dynamic System : FL L

S =

-92.	-1440.	1.
1.	0.	0.
0.	1440.	0.

XO =

0. 0.

Inputs : 1 Outputs : 1 States : 2

<BUILD> Linear Dynamic System : FL P

S =

-92.	-1440.	1.
1.	0.	0.
0.	1440.	0.

X0 =

0. 0.

Inputs : 1 Outputs : 1 States : 2

<BUILD> Linear Dynamic System : EN LAG

S =

-1.	1.
1.	0.

X0 =

0.

Inputs : 1 Outputs : 1 States : 1

Block location : 1

INPUTS					OUTPUTS			
From	Name	Location	To	From	Name	Location	To	
(1)	External	Input	1	1	External	Output	(1)	

Block location : 2

INPUTS					OUTPUTS			
From	Name	Location	To	From	Name	Location	To	
(2)	External	Input	1	1	External	Output	(2)	

Block location : 3

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(3)	External	Input	1	1	External	Output	(3)

Block location : 4

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(4)	External	Input	1	1	External	Output	(4)

Block location : 6

INPUTS				OUTPUTS			
From	Name	Location	To	From	Name	Location	To
(5)	External	Input	1	1	External	Output	(5)

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Vita

Captain Julio E. Velez was born on 14 August 1950 in Camuy, Puerto Rico. He enlisted in the USAF in August 1968 and was honorably discharged upon returning from Vietnam in June 1972. After receiving a Bachelor of Engineering in Electronics with honors from Pratt Institute in 1976 he worked in the private sector until accepting a commission in the USAF in 1978. After attending the officer training school at Lackland AFB, he was assigned to the 475th Test Squadron, Tyndall AFB, Fl. where he worked as a project officer. His next assignment was to the USAF Test Pilot School (TPS), where he received training as a flight test engineer. After graduating from TPS Capt Velez was assigned to the 1st Test Squadron, Clark AB, Republic of the Philippines from 1983 till 1986. He entered the School of Engineering, Air Force Institute of Technology in June 1986.

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16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Adaptive Controller, Flight Controls, Multivariable Parameter-Adaptive Algorithm, AFTI/F-16, ARMA model
01	03		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Title: MULTIVARIABLE CONTROL LAW FOR THE AFTI/F-16 WITH A FAILED CONTROL SURFACE USING A PARAMETER-ADAPTIVE CONTROLLER

Thesis Chairman: Dr. John J. D'Azzo

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Air Force Institute of Technology (AFC)
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Multivariable control laws are designed for the Advanced Fighter Technology Integration F-16 (AFTI/F-16). Both fixed gain and adaptive Proportional plus Integral (PI) controllers are designed for a plant were the number of outputs are not equal to the number of inputs (rectangular plant). A parameter estimation algorithm is used for the adaptive controller. Simulations are conducted for a healthy and a failed aircraft model. The failure consists of reducing the left elevator by 50%. When the fixed gain controller is used for the flight control system, the simulation reveals the fact that the aircraft failure causes the output responses to diverge. If provided with a "persistently exciting" input the adaptive controller prevents the aircraft failure simulation from diverging and going unstable. However, additional testing and/or tuning of the adaptive controller is required to determine and enhance the stability of the adaptive controller.

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