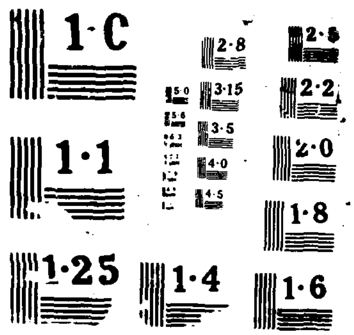
AD-A189 908 A COMPUTER STUDY OF AIR DEFENSE GUN EFFECTIVENESS(U) 1/1
NAVAL POSTGRADUATE SCHOOL MONTEREY CA H D JUNG DEC 87

UNCLASSIFIED F/G 15/3.2 NL


1·1
1·25
1·4
1·6
1·8
2·0
2·2
2·8
3·15
3·5
4·0
4·5

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A COMPUTER STUDY OF AIR DEFENSE GUN EFFECTIVENESS

by

Hyun Dae, Jung

December 1987

Thesis Advisor Robert E. Ball

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for Public Release; Distribution is Unlimited
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL (If applicable) 67	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000	7b ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000
8a NAME OF FUNDING / SPONSORING ORGANIZATION 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO / PROJECT NO / TASK NO / WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)
A COMPUTER STUDY OF AIR DEFENSE GUN EFFECTIVENESS

12 PERSONAL AUTHOR(S)
JUNG, HYUN DAE

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Master's Thesis	FROM ______ TO ______	1987 December	59

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Computer simulation of air defense for surface to aircraft.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This study describes the continuing development of a computer model for the simulation of burst kill probabilities for air defense gun systems firing projectiles at maneuvering aircraft. The computer simulation developed by Keeling is modified by adding a high explosive proximity and contact fuzed round to the simulation. The objective of this thesis is to develop and analyze the shipboard anti-air defense problem in order to choose the best air defense gun system. The air defense gun system studied consists of a 40 mm gun, firing fragmenting proximity and contact fuzed projectiles, an early warning radar system, and a fire control system. The aircraft vulnerability and the gun data used in this thesis are entirely synthetic to avoid security classification problems.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL Robert E. Ball	22b TELEPHONE (Include Area Code) (408) 646-2885 22c OFFICE SYMBOL 67Bp

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

A Computer Study of Air Defense Gun Effectiveness

by

Hyun Dae, Jung
Lieutenant Commander, Republic of Korean Navy
B.S., Korean Naval Academy, 1976

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING SCIENCE

from the

NAVAL POSTGRADUATE SCHOOL
December 1987

Author: ______________________
Hyun Dae, Jung

Approved by: ______________________
Robert E. Ball, Thesis Advisor

____
Max F. Platzer, Chairman,
Department of Aeronautics

____
Gordon E. Schacher,
Dean of Science and Engineering

ABSTRACT

This study describes the continuing development of a computer model for the simulation of burst kill probabilities for air defense gun systems firing projectiles at maneuvering aircraft. The computer simulation developed by Keeling is modified by adding a high explosive proximity and contact fuzed round to the simulation. The objective of this thesis is to develop and analyze the shipboard anti-air defense problem in order to choose the best air defense gun system. The air defense gun system studied consists of a 40 mm gun, firing fragmenting proximity and contact fuzed projectiles, an early warning radar system, and a fire control system. The aircraft vulnerability and the gun data used in this thesis are entirely synthetic to avoid security classification problems.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENT

I would like to express my sincere appreciation to my thesis advisor, Dr. Robert E. Ball, Aeronautics Engineering Department of the U. S. Naval Postgraduate School.

I thank my wife and two son for their support and patience away from our home during two and half years in the United States America.

Finally, I would also like acknowledge the Republic of Korean Navy for the opportunity to attend the Naval Postgraduate School.

I. INTRODUCTION

A. BACKGROUND

In modern times it often occurs that during battle, friendly ships are threatened by enemy aircraft. This thesis focuses on a ship's ability to defend itself against aircraft with a 40 mm gun system. The computer program presented in this paper is a modification of a simulation written in the A Programming Language(APL) by Capt. C.Keeling. [Ref. 1] Keeling's program simulates engagements between air defense gun systems firing non-explosive projectiles at maneuvering aircraft. In his thesis, Keeling suggested further research on proximity fuzed and contact fuzed high explosive(HE) rounds. This paper expands Keeling's simulation to include the HE round with both a contact fuze and a proximity fuze

The simulation shows how changing gun parameters can affect a gun system's probability of kill against an aircraft. These parameters include: target range, projectile size, ballistic dispersion, muzzle velocity, firing rate, radar error, linear fire control, type of fuze (a new feature) and aircraft probability of kill given a hit or warhead lethal radius (a new feature). The use of this program increases the information available on the effectiveness of gun systems with and without HE warheads. The ultimate measure of success in the design and performance of an air defense gun system is the ability of the system to shoot down low-flying hostile aircraft within the combat environment.

B. OBJECTIVE

The objective of this thesis is to continue the development of a computer model that will be helpful in making decisions regarding the design and operation of an air defense gun system. In this thesis, the program is used to gain insight into how each of the design parameters of the gun system affect the probability of kill and which of the parameters are most important in the gun system design for air defense. In particular, the influence of the HE round on gun effectiveness will be studied.

C. APPROACHING THE PROBLEM

The gun system is located on a ship at the origin of an X, Y, and Z coordinate system. The playing area for the model is the quarter of a hemi-sphere entirely within the first quadrant of the X, Y and Z plane as shown in Figure 1.1. The size of the playing area may be adjusted by modifying the program's source code.

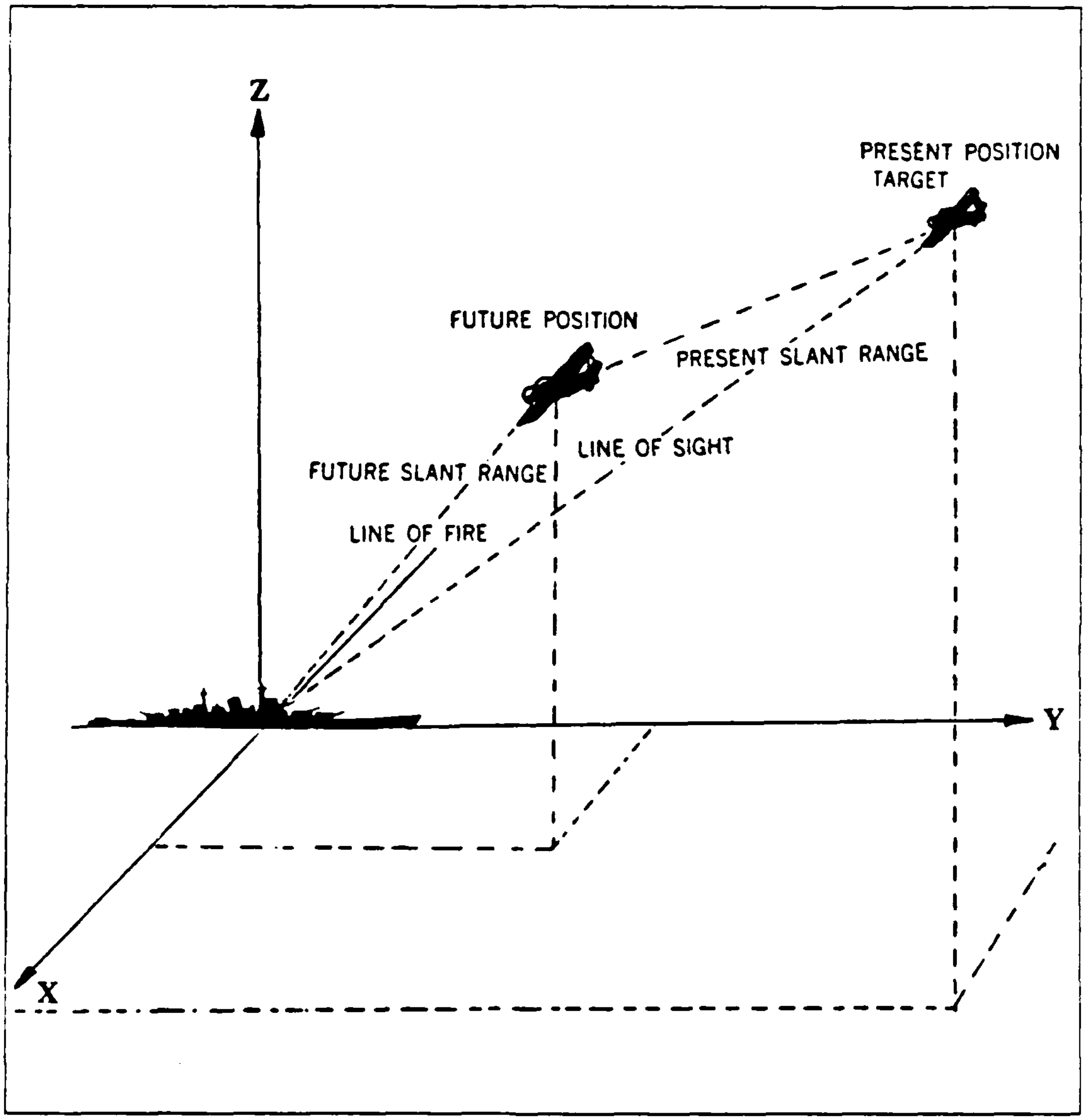


Figure 1.1 Playing Area of The X, Y & Z Plane.

The model assumes one-on-one engagements, no suppressive fire from the aircraft, and no terrain effects. The target is not necessarily attacking the gun system and may or may not maneuver while within the gun's effective range.

D. ORGANIZATION OF THE MODEL

The air defense system consists of an early warning system, a tracking radar, a weapon control system and a gun with one or more barrels. The model studied here is a twin 40 mm gun using high explosive rounds located onboard the ship at the origin of the playing area. As an aircraft enters into the radar detection envelope of the early warning radar, the aircraft is detected and identified as hostile by the ship. The tracking radar is given the target's position. This radar tracks the aircraft and continuously computes the target range, elevation, azimuth, and velocity. The gun fire control computer solves for the lead angle in azimuth and elevation based on the calculated target positional data assuming no target maneuvering. The gun is positioned at the lead angle and fires a burst of proximity or contact fuzed high explosive rounds. The projectiles fly out toward the moving target. The target continues on its randomly selected flight path. The miss distance and assumed ballistic dispersion are used in the model to determine the probability of aircraft kill for each round P_{KSS}. For the proximity fuzed warheads, the projectiles do not have to hit the target to cause damage. The probability of aircraft kill given a detonation, $P_{K\ D}$, is determined using the Carlton diffused Gaussian kill function . For the contact fuzed warheads, the projectiles must hit the target in order to cause damage. The probability the aircraft is killed given the hit on the target, $P_{K\ H}$ is represented by the two-dimensional Carlton hit function.

1. Scenario

The general approach in this model is the development of the kill probability for one gun system firing one burst at one randomly maneuvering target. The probability of kill for each round is used to compute a burst kill probability. The gun system computes target flight path characteristics, predicts a time-dependent intercept point in space, positions the gun and fires the projectiles. For contact fuzed HE rounds, the target vulnerability is represented by a vulnerable area (A_v) in a plane in space that is perpendicular to the slant range (R) between the gun and the center of the target. For the proximity fuzed warhead, the target vulnerability is represented by a lethal radius.

2. **Model Input**

Inputs are required for the gun, the fuze, the target, and ranges of engagement. Gun inputs are muzzle velocity, projectile coefficient of drag, burst size, rate of fire, and angular ballistic dispersion. The fuze input is the probability of fuzing, which is independent of range. Aircraft inputs are velocity, altitude, acceleration performance capabilities, and vulnerability data based on the type and size of projectile and fuze being used. A flight path generates, for several different types of aircraft, maneuvers using a Monte Carlo simulation.

II. PROXIMITY FUZED WARHEAD

A. BASIC THEORY

A proximity fuze detects the target and causes detonation of the high explosive warhead in the vicinity of the target. A warhead usually contains a powerful but relatively insensitive high-explosive that can only be initiated by the heat and energy from the primary explosive in the fuze [Ref. 3]. Proximity fuzes accomplish their purpose through "influence sensing" with no contact between the projectile and the target . These fuzes are actuated by some characteristic feature of the target, such as a reflected radio signal, an induced magnetic field, an interrupted light wave, a pressure measurement, or an acoustical impulse. "Proximity sensing" results in detonation of the bursting charge in the vicinity of the target or targets. A direct hit is not necessary to disable an aircraft and achieve the desired effect. An aircraft may be damaged in any part of the large volume occupied by fragments from the exploding round. Since several air targets may exist simultaneously, the sensing equipment must be able to isolate a selected target. Proximity fuzing had its origins in England early in World War II. Proximity fuzing is represented as VT fuzing (a code name used during Word War II to imply variable time fuzing) [Ref. 2 p. 76].

B. TARGET SUSCEPTIBILITY

The modeling and quantification of the individual events and elements in an air defense encounter is referred to as a susceptibility assessment. Susceptibility is the inability of an aircraft to avoid being damaged in the pursuit of its mission, to its probability of being hit [Ref. 2] The level of susceptibility of an aircraft in encounter with a threat is dependent upon three major factors:

- **The scenario** includes the physical environment in which the encounter takes **place, the** air defense deployment and activity, and the aircraft flight path and tactics, including any supporting forces.
- The air defense characteristics, operations, and lethality.
- The aircraft observables or detectable signatures, any countermeasures used, the aircraft performance capabilities, and any self-protection armament are the important factors associated with the aircraft itself.

One of the most important susceptibility measures is the closest point of approach or miss distance of a projectile to a maneuvering target.

In general, the smaller the miss distance, the more likely the aircraft will be hit. Whether or not the aircraft is hit depends upon the presented area of the aircraft. This study assumed the presented area A_p of the aircraft is based on the six cardinal sides. The most the projectile can see in the intercept plane is part of three sides. The threat aircraft used in the study has a presented area from the area of the top A_{top} (50 m^2, equal to the bottom), the area of the side A_{side} (30 m^2, the same on each side), and the area of the front A_{front} (5 m^2, equal to the rear).

C. MISS DISTANCE

As shown in Figure 2.1, the CPA is the mean value for an assumed Gaussian distribution of the projectile miss distance and the ballistic dispersion is the round to round variance about that point.

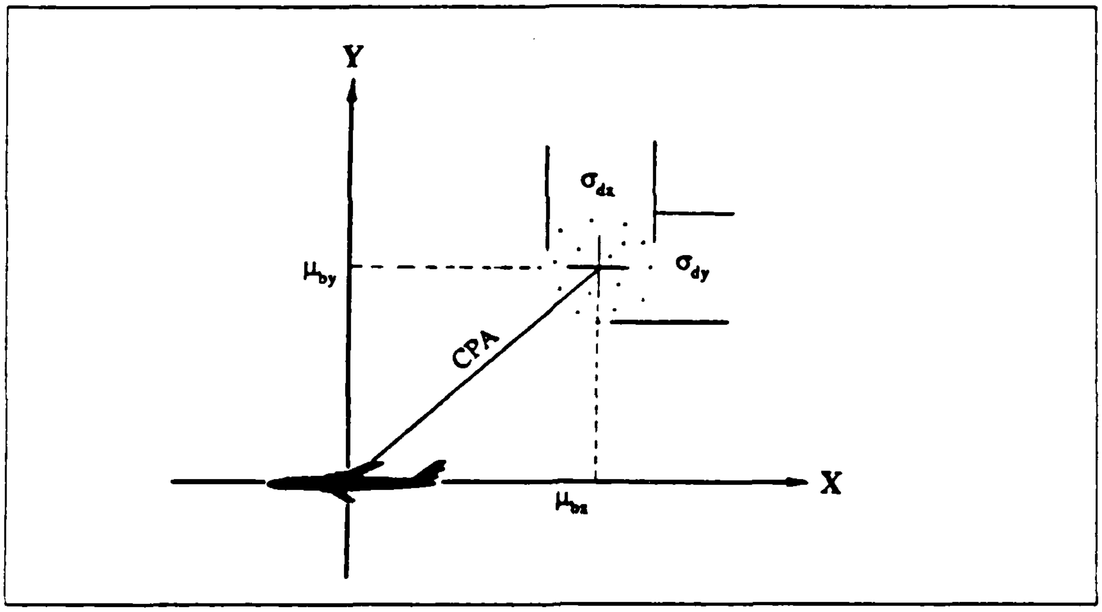


Figure 2.1 Ballistic Dispersion, Bias, and CPA.

The miss distance form for the round is given by

$$\mu_{bx}^2 + \mu_{by}^2 = CPA^2 \qquad \text{(eqn 2.1)}$$

where

- (μ_{bx}, μ_{by}) is the mean or bias in the x and y direction.

D. THE FUZE

The function of the fuze is to initiate the detonation of a warhead at a time and place such that maximum damage will be inflicted upon the target. Since the warhead damage volume is preset, the achievement of maximum damage depends on the time of fuze initiation [Ref. 3]. The fuze package consists of a safety-and-arming device to keep the weapon safe until it is deployed and clear of friendly forces, a detonator to initiate the HE charge detonation, a device that senses the presence of a target (known as the target detection device (TDD)), and a logic circuit that initiates detonation at the proper time.

The functioning of a fuze can be represented by the probability P_f which may or may not be dependent upon the miss distance. For example, a proximity fuze with an 80% probability of detonating the warhead within a miss distance range r_c of 100 ft and a 0% probability beyond r_c can be modeled in the form

$$P_f(x,y) = 0.8 \quad \text{when } (x^2 + y^2)^{1/2} \leq 100\text{ft}$$
$$P_f(x,y) = 0 \quad \text{when } (x^2 + y^2)^{1/2} > 100\text{ft}$$

E. TARGET VULNERABILITY

Vulnerability refers to the inability of the aircraft to withstand one or more hits by the damage mechanisms, to its vincibility, to its liability to serious damage or destruction when hit by gun fire [Ref. 2]. The critical components on an aircraft are those components which, if either damaged or destroyed, would lead to an aircraft kill. A general procedure has been developed for determining the critical components, their possible damage or failure modes, and the effects of the component damage or failure upon the continued operation of the aircraft. The procedure consists of

- A selection of the aircraft kill levels or categories to be considered.
- An assembly of the technical and functional description of the aircraft.
- The determination of the aircraft components of the aircraft and their damage caused failure modes for the selected kill levels.

1. State Transition Matrix Method

The study by Keeling used the state transition matrix method to describe the damage state of the aircraft after one or more hits. This method allows the consideration of redundancy in the critical components and accounts for the fact that a single shot cannot kill more than one redundant critical component. This method states that a sequence of independent events can be modeled as a Markov process. The state space of the Markov process includes kill of the aircraft, no kill of the aircraft, a state space for a kill of each of the critical redundant components, and combinations of kills of redundant critical components that eventually lead to a kill of the aircraft., The state of the target at the begining of the engagement is represented as the initial state vector S^0. One transition represents the state of the target after one random hit on the aircraft, and S^{j+1} represents the state of the target after the j^{th} hit. The matrix that accounts for the transition is $|TM|$. Thus

$$S^{j+1} = |TM| \, S^j \qquad \text{(eqn 2.2)}$$

The state transition matrix method is not appropriate for the HE round.

2. Vulnerability Model for Externally Detonating Warheads

For the proximity fuzed HE rounds, the aircraft's vulnerability can be represented by a $P_{K\ D}$ function of x and y proposed by Carlton

$$P_{K\ D} = \exp(-x^2\ r_0^2) * \exp(-y^2\ r_0^2) \qquad \text{(eqn 2.3)}$$

The scaling parameter r_0 can be related to the warhead lethal radius r_l, such that when $r = r_l$, $P_{K\ D} = 0.5$. Collocating the one-dimensional Carlton kill function through 0.5 when $r = r_l$ gives $r_0 = 1.20\ r_l$ as shown in Figure 2.2.

F. P_{KSS}

This simulation uses the circular Carlton kill function method as presented by Ball [Ref. 2 p. 320]. The equation for P_{KSS} is

$$P_{KSS} = r_0^2 P_f (\alpha^{1\ 2} \beta^{1\ 2}) * \exp(-(\mu_{bx}^2\ \alpha) - (\mu_{by}^2\ \beta)) \qquad \text{(eqn 2.4)}$$

where

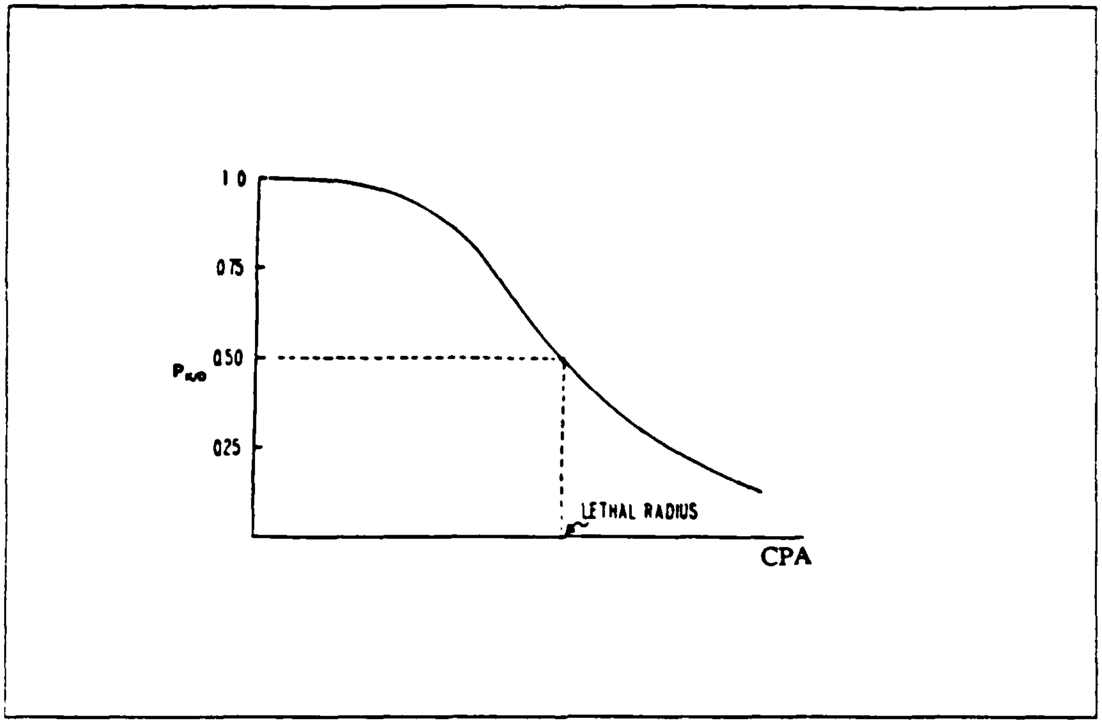


Figure 2.2 $P_{K,D}$ Function of X and Y.

- α is $(2\sigma_{dx}^2 + r_0^2)$
- β is $(2\sigma_{dy}^2 + r_0^2)$
- $(\sigma_{dx}, \sigma_{dy})$ is the ballistic dispersion of the round in the x and y direction.
- r_0^2 is a scaling parameter of the warhead lethal radius r_l such that $r_0 = 1.2r_l$.
- P_f is the probability of fuzing (assumed constant).

The calculation of the probability of kill for a variety of ranges is made simpler by using angular errors for the dispersion errors. From the Figure 2.3 it can be seen that α_{x1} and α_{x2} are the angles representing the angular standard deviation of the dispersion errors about the closest point of approach. Figure 2.3 shows that

$$\sigma_{dx} = \alpha_{dx} R \qquad \text{(eqn 2.5)}$$

where

- $\alpha_{dx1} = \alpha_{dx2} = \alpha_{dx}$
- α_{dx} is the angular ballistic dispersion in the x direction.

By the same theory in the y direction can be shown to be

$$\sigma_{dy} = \alpha_{dy} R$$

where α_{dy} is the angular ballistic dispersion in the y direction.

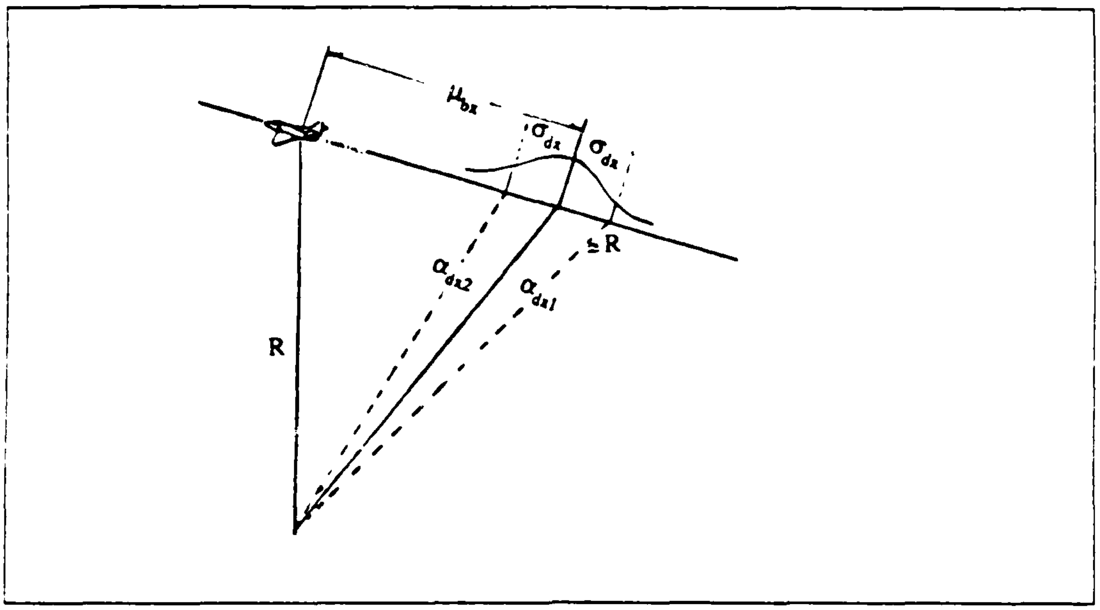


Figure 2.3 Effect of Angular Dispersion with Range.

Furthermore assumption is made that the angular ballistic dispersion as shown in Figure 2.2 is the same in both direction, hence

$$\alpha_{dx} = \alpha_{dy} = \alpha$$

The equation for the Carlton kill function model, Equation 2.3, simplifies to

$$P_{KSS} = (r_0^2 P_f / \gamma) * \exp(-(\mu_{bx}^2 + \mu_{by}^2) / (\gamma)) \qquad \text{(eqn 2.6)}$$

where

- γ is $(2\alpha^2 R^2 + r_0^2)$

The final form for the probability of kill is obtained by using Equation 2.1, giving

$$P_{KSS} = (r_0^2 P_f / \gamma) * \exp(-CPA^2 / \gamma) \qquad \text{(eqn 2.7)}$$

For this study, r_1 is taken as five meters. Thus, r_0 is six meters.

G. PROBABILITY OF KILL GIVEN A BURST

The probability of an aircraft kill for a burst of rounds is directly proportional to the probability there is a clear line of sight from the detecting element to the aircraft. Given that the aircraft has been detected, the probability that one or more propagators will be fired at the aircraft is detonated by P_F. Thus, the probability that the aircraft is killed by one shot in an encounter, $P_{K/E}$, is given [Ref. 2. p. 320] by

$$P_{K/E} = P_D * P_F * P_{KSS} \qquad \text{(eqn 2.8)}$$

- P_D is the probability that the aircraft has been detected from the start of a search up to the present time t.
- P_F is the probability that a propagator will be fired at the aircraft.
- This study assumes that $P_D * P_F = 1$.

The probability that the aircraft is killed by the sequence of N gun shots is unity minus the product of the individual probabilities of survival for each shot. Thus, for the burst of N rounds P_{KSS} in Equation 2.8 is replaced with

$$P_{K/Burst} = 1 - \prod_{i=1}^{N} \left[1 - P_{KSSi} \right] \qquad \text{(eqn 2.9)}$$

where the subscript i denotes the ith shot.

III. CONTACT FUZED WARHEAD

Contact fuzes detonate the HE warhead upon contact with the target. Detonation can be made to occur on the surface or inside the target, depending upon the design of the fuze.

A. TARGET SUSCEPTIBILITY

The presented area A_p of the aircraft is defined to be the area of the aircraft seen by the projectile in a plane normal to the path of the projectile at intercept. The procedure for determining the presented area seen by the round is determined based upon the difference between the projectile's velocity and the relative velocity of the projectile with respect to the aircraft. This procedure is described in detail in [Ref. 1].

B. TARGET VULNERABILITY

For warheads that do not use a proximity fuze, such as small arms and HE warheads with contact fuzes, a hit on the aircraft must occur to cause damage. Thus, the kill function that defines the probability the target is killed due to a propagator becomes $P_{K\ H}$ for random hits, and integration is carried out over the extent of the aircraft. In this approach, the probability the aircraft is hit by the propagator, P_H, is computed and multiplied by the probability the aircraft is killed given the random hit on the aircraft, $P_{K\ H}$. The computation of P_H is developed by Keeling and accomplished using the Carlton hit function. The aircraft vulnerability is represented by its vulnerable area A_V, centered at the aim point, and any hit on the vulnerable area causes a kill. The computation for the vulnerable area is the product of presented area A_P and the probability the aircraft is killed given the random hit on the aircraft $P_{K\ H}$ [Ref. 2]

$$A_V = P_{K\ H} * A_P \qquad \text{(eqn 3.1)}$$

This model assumes the $P_{K\ H}$ value for the round is 0.25, and A_V is given as

$$A_V = 0.25 * A_P \qquad \text{(eqn 3.2)}$$

C. P_{KSS}

This simulation uses the Carton kill function for the P_{KSS} as presented in [Ref. 2].

$$P_{KSS} = \frac{A_v}{(2\pi\sigma_x^2 + x_0^2)^{1/2}(2\pi\sigma_y^2 + y_0^2)^{1/2}} \exp\left[\frac{-\pi\mu_x^2}{(2\pi\sigma_x^2 + x_0^2)} + \frac{-\pi\mu_y^2}{(2\pi\sigma_y^2 + y_0^2)}\right] \quad \text{(eqn 3.3)}$$

The product of x_0 and y_0 can be taken as (1)equal to the presented area, A_P, or (2) equal to the vulnerable area, A_V (see Ref.2, PP 316-315). This study assumes $x_0 = y_0$ and uses A_P for the product. The means and deviations in Equation 3.3 are determined in the same manners as was done for the proximity fuzed round in Chapter II. Thus, the final form for the probability of kill given a single shot becomes

$$P_{KSS} = (.25)A_P\,(2\pi\alpha^2R^2 + A_P^2)^* \exp\{-\pi CPA^2\,(2\pi\alpha^2R^2 + A_P^2)\} \quad \text{(eqn 3.4)}$$

D. *PROBABILITY OF KILL GIVEN A BURST*

When multiple shots are considered, the single hit $P_{K\,H}$ is used for each shot. To determine the probability of an aircraft kill for a burst of rounds, the same procedure uses for the proximity fuzed warhead is used. Thus,

$$P_{K/Burst} = 1 - \prod_{i=1}^{N}\left[1 - P_{KSSi}\right] \quad \text{(eqn 3.5)}$$

IV. COMPUTER PROGRAM

Keeling's program is modified by adding subroutines PROXFUZE, CONTFUZE, BURSTKILL, and by changing the appropriate data to a 40 mm gun. This computer program consists of a main program and utilizes fifteen subroutines. The program was written in the A Programing Language (APL). The computer program listing is shown in APPENDIX A, and a flowchart of the program is depicted in Figure 4.1.

A. MAIN PROGRAM

The main program uses a DRIVER program for initiating simulation and probes the user for inputs. The program DRIVER calls the GUN subroutine for simulating gun fire solution. After the simulation has been initiated, subroutine GUN simulates the air defense engagement by utilizing the appropriate subroutines necessary to simulate the engagement of an aircraft by the on-board ship air defense gun. Subroutine GUN collects statistics on the probability of kill and where the actual slant range of intercept occurred. These data are then sent back to the main program DRIVER.

B. SUBROUTINES

1. Initiation

Subroutine GUN simulates the air defense engagement by calling the appropriate subroutines necessary to simulate the engagement of the aircraft by the on-board ship air defense gun.

2. Generation

The generation of a situation for air defense, aircraft's position, the radar located position with radar errors, and flight path maneuver of aircraft is subroutines RANGECHEK, RADAR, and AIRCRAFT.

3. Calculation

Subroutines INTERCEPT, MISSDIST, and MISSVECTOR are used to determine the predicted intercept point for the median round in the burst and the closest point of approach for each round in the burst.

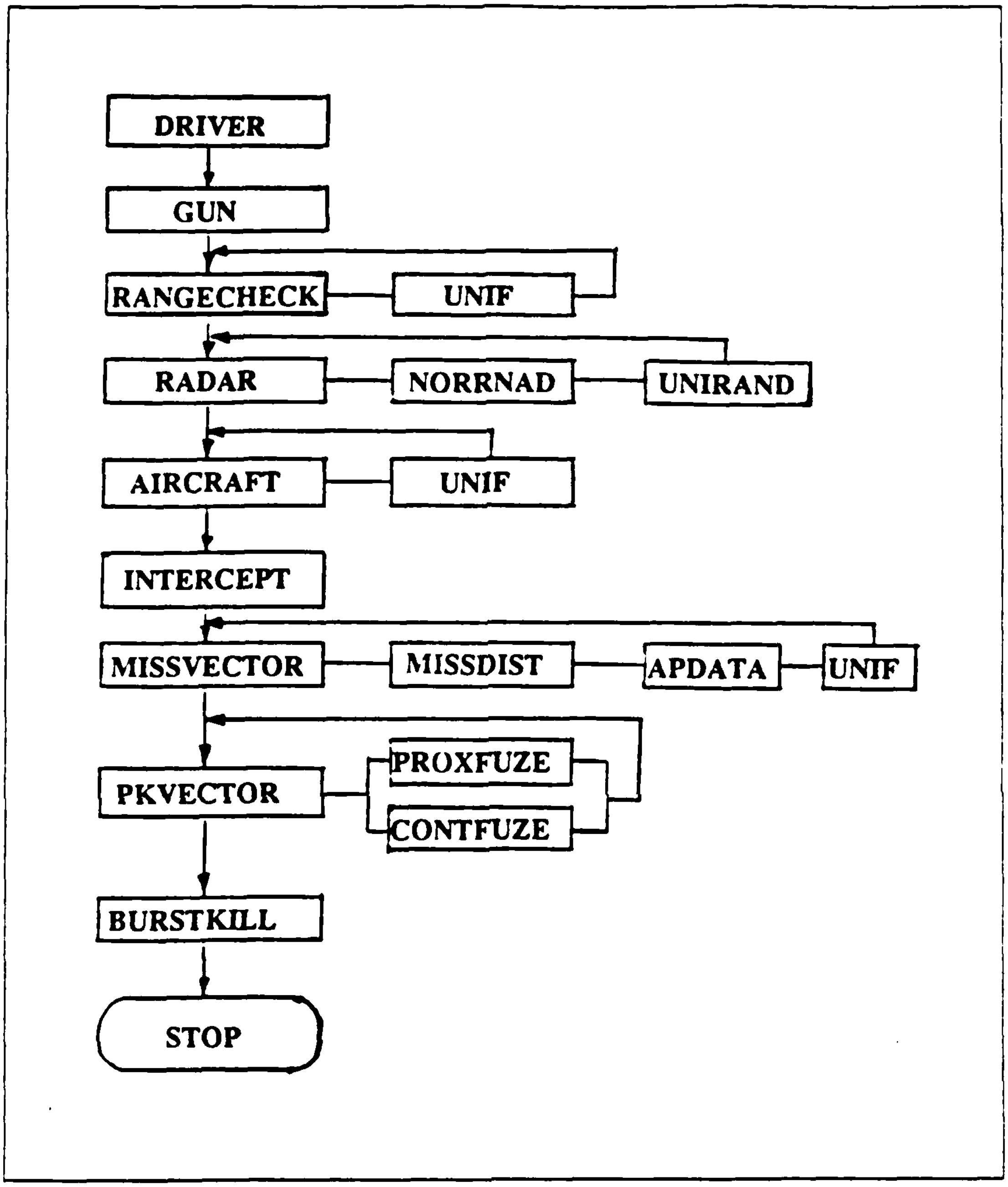


Figure 4.1 Flowchart of Computer Program..

4. Susceptibility

Subroutines APDATA, PKVECTOR, CONTFUZE, and PROXFUZE are used in determining the susceptibility of the aircraft. APDATA determines the presented area of the aircraft that is used in the Carlton kill function model in subroutine CONTFUZE. PKVECTOR stores the data for each round in the burst.

5. **Vulnerability**

Subroutines PKVECTOR, CONTFUZE, PROXFUZE, and BURSTKILL are used in determining the vulnerability of the aircraft. Subroutine PKVECTOR stores the data for each round in the probability of kill. Subroutine CONTFUZE and PROXFUZE are used in determining the probability of kill for a single shot whether or not the projectile hit the target.

6. **Random Number**

Subroutines UNIF, UNIRAND, and NORRAND are used to generate the random number required in the simulation. Subroutine UNIF generates uniform random numbers in the interval(a, b). UNIRAND generates pseudo-uniform random numbers. NORRAND is the generation of normal random numbers.

C. SUBROUTINE PROXFUZE

Subroutine PROXFUZE is used in determining the probability of kill for a single shot of the HE proximity fuzed round. This vulnerability data is used by subroutine BURSTKILL to determine the probability of kill given burst.

D. SUBROUTINE CONTFUZE

Subroutine CONTFUZE is used in determining the probability of kill for a single shot of the HE contact fuzed round. This vulnerability data is used in subroutine BURSTKILL to determine the probability of kill given burst.

E. SUBROUTINE BURSTKILL

Subroutine BURSTKILL determined the probability of kill for a given burst. This probability of kill data uses the vulnerability of the aircraft.

V. ANALYSIS AND CONCLUSIONS

This program model was initiated with the realization that the probability of kill is one measure of effectiveness that has a broad base application in the field of air defense gun system evaluation. This program for determining gun system probability of kill has simulated several parameters. These parameters describe the gun, target environments, and engagement procedures whereby one gun system encounters one maneuvering target. The data used is given in APPENDIX D. This study uses a box plot to illustrate the effects of several important parameters. This graphical imformation gives a quick sense for the distribution of P_K for the individual test values of each of the parameters. The vertical box contains the middle fifty percent of the distribution, and the individual dots show all the outliers. The mean value of the probability of kill at each of the test value settings are connected with a line. The result of each simulation is listed with the Empirical Statistics table in APPENDIX E.

A. TYPE OF FUZE

The probability of kill of the aircraft in a one-on-one encounter with both the proximity fuzed warhead and the contact fuzed warhead, $P_{K\ E}$, is shown in Figure 5.1. The effects that a proximity and a contact fuzed warhead have on the lethality of the 40 mm gun using a HE rounds can be seen in this Figure. The proximity fuzed warhead shows a higher probability of kill given a burst than the contact fuzed warhead against a maneuvering target. Consequently, the remaining studies concentrate on the proximity fuzed warhead.

B. BALLISTIC DISPERSION

Figure 5.2 shows an example of how the 40 mm gun probability of kill changes with angular dispersion. The degree of dispersion can immediately be seen, as well as how many of the data points were considered as outliers. Since the dispersion settings for optimality is arbitrary in this range, the assumption is made that the dispersion angle for the gun is 1.0 milliradians.

C. RANGE EVALUATION

The evaluation of the P_K for different range values was conducted for the maneuvering target. The maneuvering target was engaged by one burst for each range

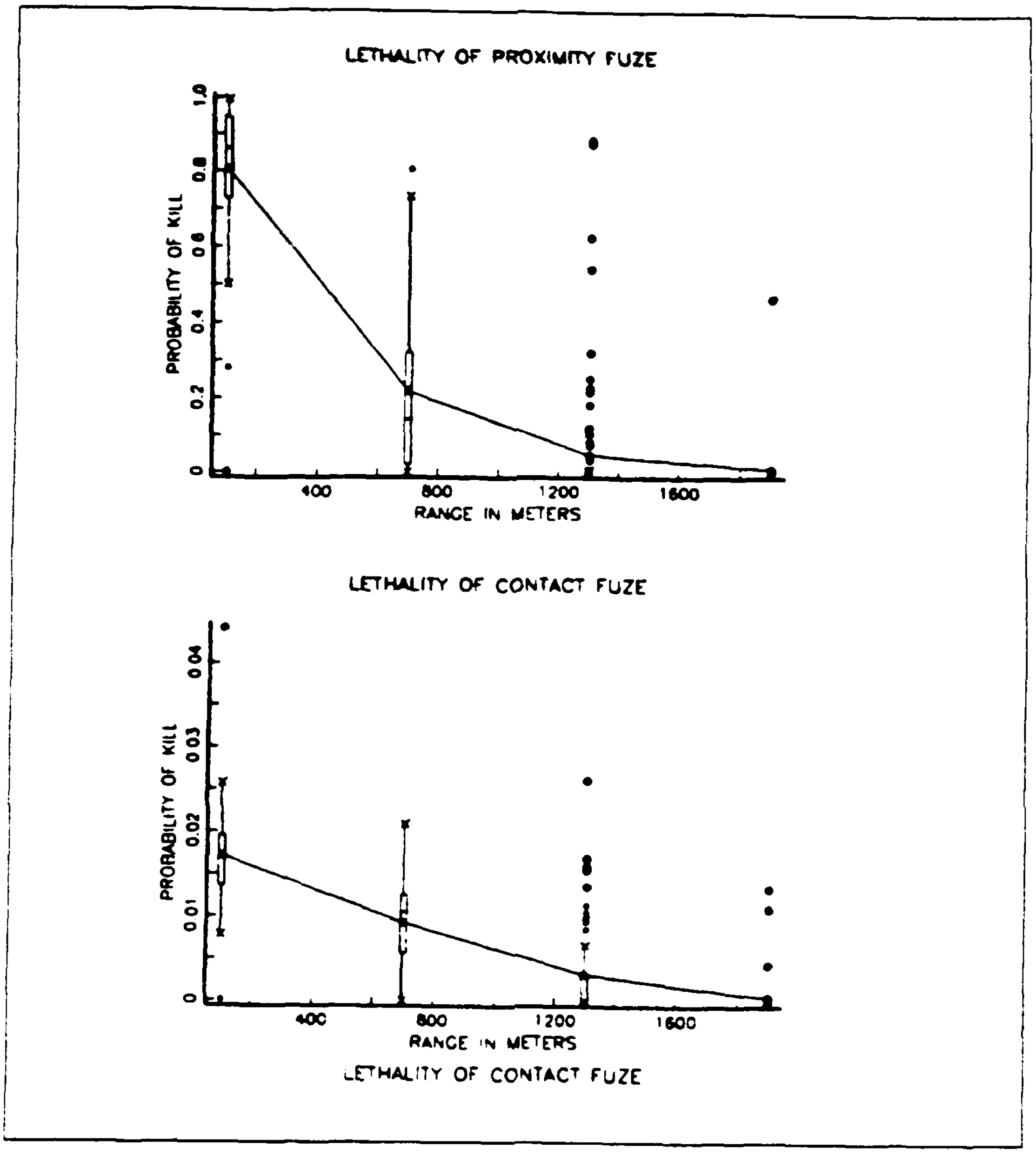


Figure 5.1 Graphical Comparison of Fuze on 40 MM gun Lethality.

value. The gun produced the highest probability of kill at the 700 meters, and the effective range was 1300meters against the maneuvering target. The air defense for a real situation requires about 2000meters for successful engagement.

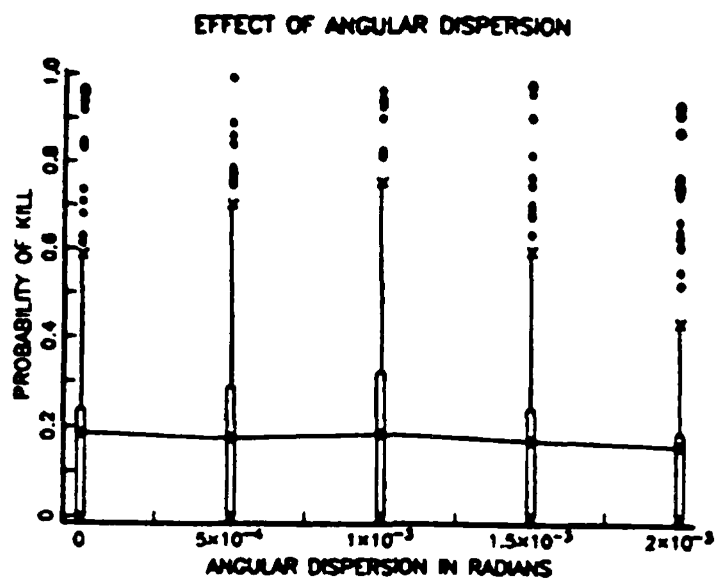


Figure 5.2 Effects of Ballistic Dispersion on 40 MM gun lethality.

D. EFFECTS OF TARGET MANEUVERS

Figure 5.4 shows the effects that a maneuvering and non -maneuvering target have on the lethality of the 40 mm gun. Against the non-maneuvering aircraft, the probability of kill given a burst is about twice that for the maneuvering target. That means that the lethality of the air defense gun requires a non linear fire control predictor based upon the tactical air combat maneuver(TACM).

E. EFFECTS OF RADAR ERROR

Figure 5.5 shows the effect of radar error on the 40 mm gun. In today's combat environment, the radar accuracy is an important factor. That means a large improvement in lethality can be obtained by reducing the radar error. The 100 percent error represents the current state of the radar system, so greater or less than 100 percent reflects great differences in lethality due to technological improvements to today's radar system.

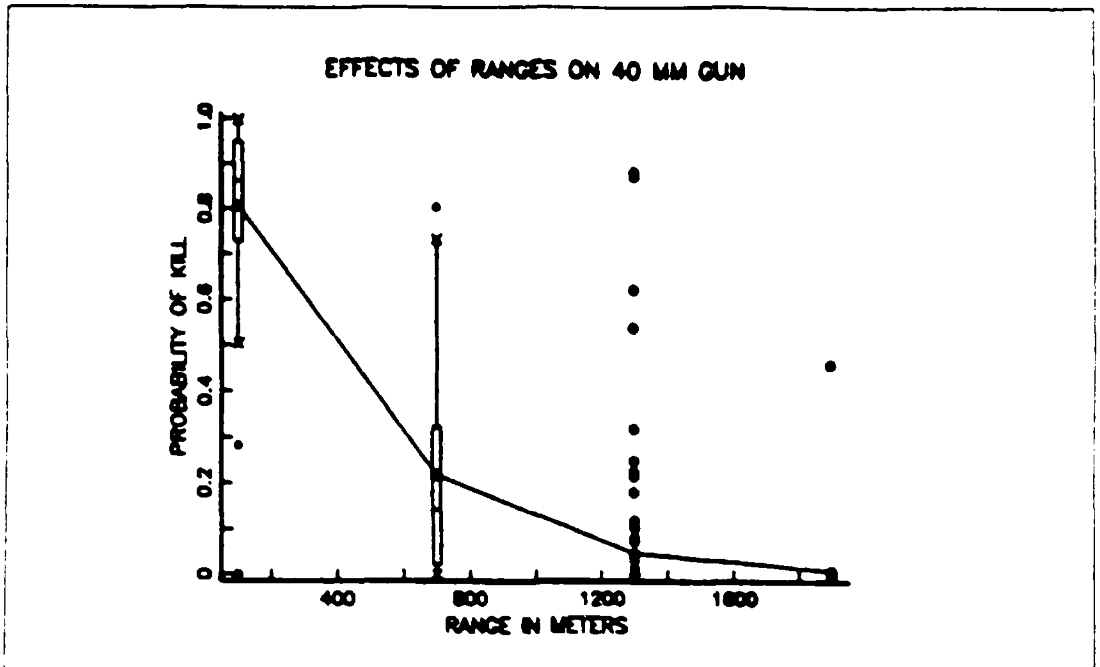


Figure 5.3 Effects of Range on 40 MM Gun Lethality.

F. EFFECTS OF MUZZLE VELOCITY AND RATE OF FIRE

Increased rate of fire and muzzle velocity can significantly improve the probability of kill for the rounds in the burst, as shown in Figures 5.6 and 5.7. Increasing the rate of fire decreases the time between projectiles, which means there are more projectiles in the area of where the aircraft is at intercept. Thus, the probability of kill is increased for each round in the burst. Increasing muzzle velocity decreases the time required to get to the target, and thereby reduces the effects of target maneuver.

G. CONCLUSIONS

This study has been conducted from a theoretical and intuitive point of view with conclusions withheld in many cases pending validation. The execution of this program demonstrated what appear to be some clear improvements is gun lethality. The sample results in each of the graphs show the difference in the probability of kill values. They represent a convenient and practical base from which to study or perform parametric analyses on existing systems or on systems still on the drawing board. The 40 mm gun with the proximity fuzed warhead considered here for the given scenario and target

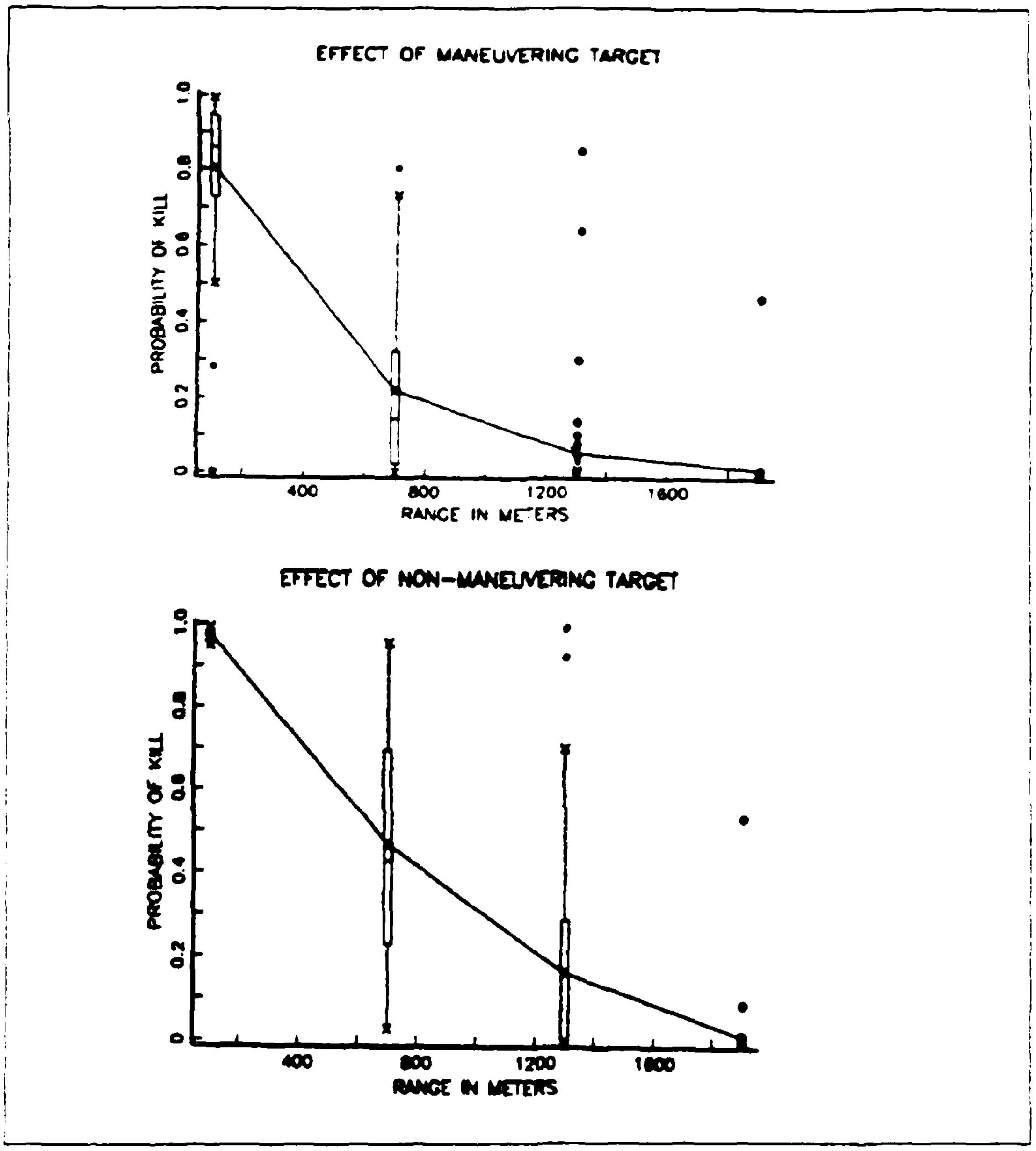


Figure 5.4 Effects of Maneuver and Non-maneuver on 40 MM Gun lethality.

vulnerability data has an effective range of about 1300 meters against a maneuvering target. This gun can be improved by improving the radar, the fire control computer, projectile muzzle velocity, gun system rate of fire. These improvements should be attempted in the order given.

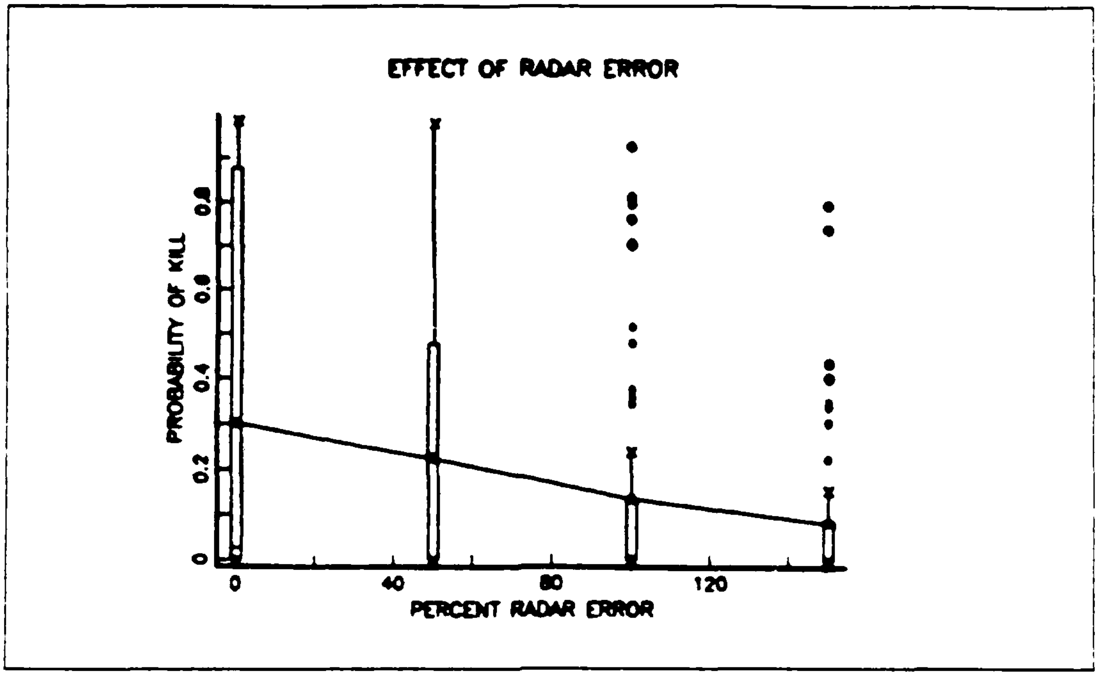


Figure 5.5 Effects of Radar Error on 40 MM Gun Lethality.

Another air defense gun system study should be conducted to determine the synergistic effects of changing more than one parameter simultaneously. The model should be further developed to account for non linear fire control. This can be easily done by changing the radar and intercept section of this program.

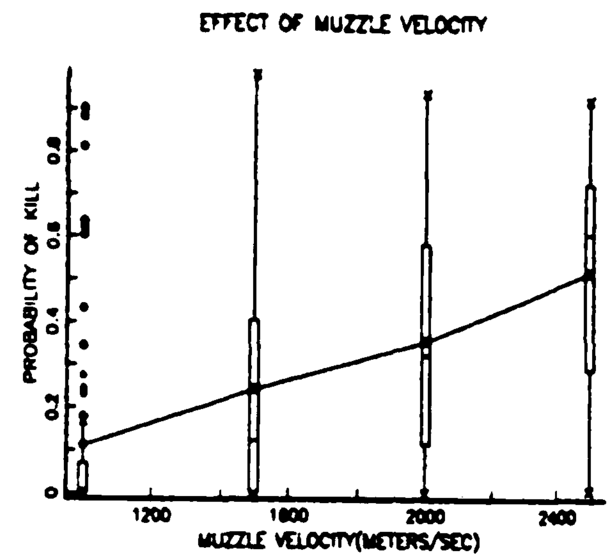


Figure 5.6 Effects of Muzzle Velocity on 40 MM Gun Lethality.

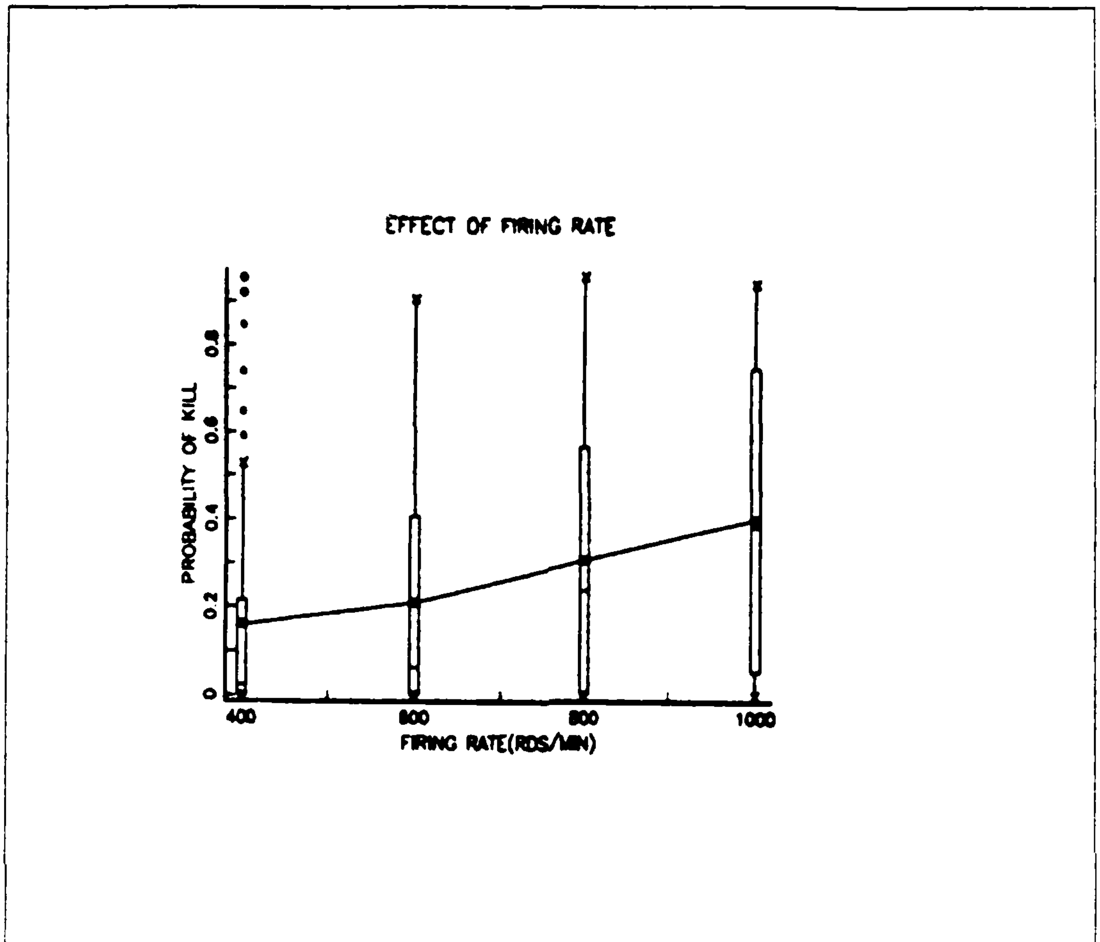


Figure 5.7 Effects of Firing Rate on 40 MM Gun Lethality.

APPENDIX A
APL COMPUTER PROGRAM

1. KEELING'S PROGRAM

```
    ∇ DRIVER;COUNT;I
⍝*************************************************
⍝********    DATA  INPUT    ********************
⍝*************************************************
⍝ 'HOW MANY DATA POINTS DO YOU WANT PER RANGE?'
 DATA←50
 DATA←DATA-1
⍝ 'WHAT IS THE GUN FIRING RATE IN ROUNDS PER MINUTE?'
 FIRERATE←1000
⍝ 'WHAT IS THE MUZZLE VELOCITY?'
 V0←1150
⍝ 'WHAT IS THE BURST SIZE?'
 BURST←5
⍝ 'WHAT IS THE ANGULAR DISPERSION IN RADIANS?'
 DISP←0.0015
⍝ 'WHAT IS THE DRAG FACTOR  B=CD×V0*0.5?'
 B←0.15
⍝ 'WHAT ARE THE RANGES YOU WANT TO LOOK AT?'
 RANGES← 1900 1300 700 100
⍝ 'WHAT ARE THE AREAS OF THE AIRCRAFT?'
 AFRT←5
 ASIDE←30
 ATOP←50
⍝*************************************************
⍝*** PROGRAM EXECUTION AND DATA COLLECTION  *******
⍝***  P IS PROBABILITY OF KILL     ******
⍝***  R SLANT IS RANGE  OF TARGET AT INTERCEPT  ***
⍝*************************************************
 R←⍳0
 P←⍳0
 I←1
 COUNT←⍴RANGES
 LOOP:RANGE←RANGES[I]
 GUN
 P←P,PKDATA
 R←R,RANGEI
 I←I+1
 →(I≤COUNT)/LOOP
⍝**************************************************
⍝** DATA CONTROL VECTOR USED FOR GRAPHS IN GRAFSTAT **
⍝**************************************************
⍝'PK='
⍝7 4 ⍕ 4 10 ⍴P
⍝'RANGE='
⍝5 0 ⍕ 4 10 ⍴R
 C←((DATA+1),(⍴RANGES))⍴RANGES
 C←,(⍉C)
    ∇

    ∇ GUN;I;VAC;BODY;TOP;SIDE;VRX;VRY;VRZ
⍝*************************************************
⍝******** SIMULATES AIR DEFENSE GUN BY CALLING *****
⍝********    APPROPRIATE SUBROUTINES      ********
⍝*************************************************
 RANGEI←⍳0
 I←0
 PKDATA←⍳0
 LOOP3:RANGECHEK
```

```
 RADAR
 AIRCRAFT
 INTERCEPT
 MISSVECTOR
 PHVECTOR
 PROBKILL
 RANGEI←RANGEI,RTGT
 PKDATA←PKDATA,PK
 I←I+1
 →(I≤DATA)/LOOP3
 ∇

 ∇ RANGECHEK;R;O;D;MAXALT;MINALT
 ⍝*************************************************
 ⍝**** DETERMINES A/C POSITION AT TIME OF FIRING **
 ⍝*************************************************
 ⍝ 'WHAT IS THE MAXIMUM ALTITUDE IN METERS?'
 MAXALT←1100
 ⍝ 'WHAT IS THE MINIMUM ALTITUDE IN METERS?'
 MINALT←90
 R←RANGE
 O← 0 1.5707963 UNIF 1
 →(R>MAXALT)/LOOP1
 ZF←(MINALT,R) UNIF 1
 →LOOP2
 LOOP1:ZF←(MINALT,MAXALT) UNIF 1
 LOOP2:D←((R*2)-(ZF*2))*0.5
 XF←D×(2○O)
 YF←D×(1○O)
 ∇

 ∇ RADAR;S2;THETAA;R;DELRANGE;DTHETA;THETAE
 ⍝***********************************************
 ⍝**** DETERMINES RADAR LOCATED TARGET BASED ***
 ⍝***** ON ERRORS INDUCED BY RADAR *********
 ⍝***********************************************
 S2←0.0012+(2÷RANGE)
 DELRANGE←1 NORRAND 0 6.2
 DELRANGE←DELRANGE×1
 DTHETA←2 NORRAND(0,S2)
 DTHETA←DTHETA×1
 R←RANGE±DELRANGE
 THETAA←¯1○(XF÷(((XF*2)+(YF*2))*0.5))
 THETAA←THETAA
 THETAE←¯1○(ZF÷RANGE)
 THETAE←THETAE
 YR←R×(2○(DTHETA[2]+THETAE))×(2○(DTHETA[1]+THETAA))
 XR←R×(2○(DTHETA[2]+THETAE))×(1○(DTHETA[1]+THETAA))
 ZR←R×(1○(DTHETA[2]+THETAE))
 ∇

 ∇ AIRCRAFT;VMIN;AXMAX;AXMIN;AZMIN;AZMAX;D;O;VZ1
 ;VZ2;MINALT;MAXALT;AXY;AZ1;AZ2;O1
 ⍝************************************************
 ⍝***** A/C PERFORMANCE DATA INPUTS **********
 ⍝************************************************
 ⍝ 'WHAT IS THE MIN ALT THE A/C IS ALLOWED TO FLY?'
 MINALT←90
 ⍝ 'WHAT IS THE MAX ALT THE GUN WILL ENGAGE?'
 MAXALT←1100
 ⍝ 'WHAT IS THE MIN VELOCITY OF A/C IN METERS/SEC?'
 VMIN←150
 ⍝ 'WHAT IS THE MAX VELOCITY OF A/C IN METERS/SEC?'
 VMAX←220
 ⍝ 'MAX A/C ACCELERATION IN Z DIRECTION IN M/SEC2?'
 AZMAX←49
 ⍝ 'MIN A/C ACCELERATION IN Z DIRECTION IN M/SEC2?'
 AZMIN←39.2
```

```
[16] ⍝ 'MAX ACCELERATION IN X OR Y DIRECTION IN M/SEC2?'
[17]  AXY←0.4×AZMAX
[18] ⍝**************************************************
[19] ⍝**** DETERMINES A/C FLIGHT PATH MANEUVER  *****
[20] ⍝*****        AT TIME OF FIRING  ***********
[21] ⍝**************************************************
[22]  VA←(VMIN,VMAX) UNIF 1
[23]  O←0 6.2831853 UNIF 2
[24]  VZ2←(((VA*2)÷2)*0.5)
[25]  VZ1←((MINALT-ZF)÷(MAXALT-MINALT))×VZ2
[26]  VZ←(VZ1,VZ2) UNIF 1
[27]  D←((VA*2)-(VZ*2))*0.5
[28]  VX←(D×(2○O[1]))
[29]  VY←(D×(1○O[1]))
[30]  AZ1←(MINALT-ZF)÷((MAXALT-MINALT)÷(AZMIN×0.6))
[31]  AZ2←0.6×AZMAX
[32]  AZ←(AZ1,AZ2) UNIF 1
[33]  O1←0 6.2831853 UNIF 1
[34]  AX←AXY×(2○O1)
[35]  AY←AXY×(1○O1)
[36]  VRZ←1 NORRAND(VZ,(0.1,VZ))
[37]  VRX←1 NORRAND(VX,(0.1,VX))
[38]  VRY←1 NORRAND(VY,(0.1,VY))
    ∇

    ∇ INTERCEPT;F1TN;BURSTIME;XF1;YF1;ZF1;RPROJ;N;FTN
[1]  ⍝**************************************************
[2]  ⍝** DETERMINES PREDICTED INTERCEPT POINT BASED  **
[3]  ⍝***  ON LINEAR FIRE CONTROL COMPUTATIONS  ******
[4]  ⍝***   AND THE NEWTON-RAPHSON TECHNIQUE *******
[5]  ⍝**************************************************
[6]   DELT←1÷(FIRERATE÷60)
[7]   BURSTIME←BURST÷(FIRERATE÷60)
[8]   N←⌊(BURST÷2)
[9]   TO←N×DELT
[10]  TAU←0
[11] LOOP1:XPI←XR+(VRX×TO)+(VRX×TAU)
[12]  YPI←YR+(VRY×TO)+(VRY×TAU)
[13]  ZPI←ZR+(VRZ×TO)+(VRZ×TAU)
[14]  RTGT←((XPI*2)+(YPI*2)+(ZPI*2))*0.5
[15]  RPROJ←(V0×TAU)÷((B×TAU)+1)
[16]  FTN←RTGT-RPROJ
[17]  F1TN←((XPI×VRX)+(YPI×VRY)+(ZPI×VRZ))÷RTGT
[18]  F1TN←F1TN-(V0÷(1+(B×TAU)*2))
[19]  →(RTGT<3000)/LOOP3
[20]  AIRCRAFT
[21] LOOP3:→((|FTN)<0.0001)/LOOP2
[22]  TAU←TAU-(FTN÷F1TN)
[23]  →LOOP1
[24] LOOP2:→0
    ∇

    ∇ MISSVECTOR
[1]  ⍝****************************************************
[2]  ⍝** CALCULATES DISTANCE FROM A/C TO MEAN THEORETICAL *
[3]  ⍝**  INTERCEPT POINT FOR EACH PROJECTILE     *******
[4]  ⍝****************************************************
[5]   APN←⍳0
[6]   VIMPN←⍳0
[7]   CPAN←⍳0
[8]   N←1
[9]   TAU0←¯0.1
[10] LOOP:MISSDIST
[11] ⍝****************************************************
[12] ⍝**  COLLECTS STATISTICS ON BIS, PRESENTED AREA,  ***
[13] ⍝**       AND BURST VELOCITY.  ****
[14] ⍝**    USED TO DETERMIN PHIT AND PKILL    *******
[15] ⍝****************************************************
[16]  CPAN←CPAN,CPA
```

```
[17] APDATA
[18] APN←APN,AP
[19] VIMPN←VIMPN,VIMP
[20] N←N+1
[21] →(N≤BURST)/LOOP
[22] →0
     ∇

     ∇ MISSDIST;A;F1;F2;F3;XA;YA;ZA;F1TN;F2TN;BT2;BT
       ;XAP;YAP;ZAP;T0
[1]  ⍝*********************************************
[2]  ⍝**** DETERMINES TIME AT WHICH CLOSEST POINT **
[3]  ⍝*****     OF APPROACH OCCURS ******
[4]  ⍝*********************************************
[5]  VPX←(XPI÷RTGT)×V0
[6]  VPY←(YPI÷RTGT)×V0
[7]  VPZ←(ZPI÷RTGT)×V0
[8]  LOOP1:BT←(B×TAU)+1
[9]  BT2←BT*2
[10] XP←(VPX×TAU)÷BT
[11] YP←(VPY×TAU)÷BT
[12] ZP←(VPZ×TAU)÷BT
[13] T0←(N-1)×DELT
[14] XA←XF+(VX×T0)+((AX÷2)×(T0*2))+((VX+(AX×T0))×TAU)
       +(AX  (TAU*2)  2)
[15] YA←YF+(VY×T0)+((AY÷2)×(T0*2))+((VY+(AY×T0))×TAU)
       +(AY  (TAU*2)  2)
[16] ZA←ZF+(VZ×T0)+((AZ÷2)×(T0*2))+((VZ+(AZ×T0))×TAU)
       +(AZ  (TAU*2)  2)
[17] XAP←XA-XP
[18] YAP←YA-YP
[19] ZAP←ZA-ZP
[20] F11←XAP×(VX+(AX×T0)+(AX×TAU)-(VPX÷BT2))
[21] F12←YAP×(VY+(AY×T0)+(AY×TAU)-(VPY÷BT2))
[22] F13←ZAP×(VZ+(AZ×T0)+(AZ×TAU)-(VPZ÷BT2))
[23] F14←((XAP*2)+(YAP*2)+(ZAP*2))*0.5
[24] F1TN←(F11+F12+F13)
[25] A←BT*3
[26] F21←((F11÷XAP)*2)+((AX+(2×VPX×B÷A))×XAP)
[27] F22←((F12÷YAP)*2)+((AY+(2×VPY×B÷A))×YAP)
[28] F23←((F13÷ZAP)*2)+((AZ+(2×VPZ÷B÷A))×ZAP)
[29] F2TN←F21+F22+F23
[30] →((TAU-TAU0)<0.0001)/LOOP2
[31] TAU0←TAU
[32] TAU←TAU-(F1TN÷F2TN)
[33] ⍝I←I+1
[34] ⍝→((LOOP2×⍳I)≥30)/LOOP2
[35] →LOOP1
[36] LOOP2:CPA←F14
[37] TAU0←0
[38] VP←V0÷(((B×TAU)+1)*2)
[39] RPROJ←(V0×TAU)÷((B×TAU)+1)
     ∇

     ∇ APDATA;VXIMP;VYIMP;VZIMP;AP1;AP2
[1]  ⍝**********************************************
[2]  ⍝** COMPUTES PRESENTED AREA OF THE AIRCRAFT **
[3]  ⍝**********************************************
[4]  VXIMP←VX-(VP×XP÷RPROJ)
[5]  VYIMP←VY-(VP×YP÷RPROJ)
[6]  VZIMP←VZ-(VP×ZP÷RPROJ)
[7]  VIMP←((VXIMP*2)+(VYIMP*2)+(VZIMP*2))*0.5
[8]  →(VIMP≤VP)/LOOP
[9]  AP1←ASIDE-((VIMP-VP)×(ASIDE-AFRT)÷(|(VA)))
[10] AP2←ATOP-((VIMP-VP)×(ATOP-AFRT)÷(|(VA)))
[11] →LOOP2
[12] LOOP:AP1←((VIMP-VP)×(ASIDE-AFRT)÷((|(VA))))+ASIDE
[13] AP2←((VIMP-VP)×(ATOP-AFRT)÷((|(VA))))+ATOP
[14] LOOP2:AP←(AP1,AP2) UNIF 1
```

```
    ∇

    ∇ PHVECTOR
    ⍝*************************************************
    ⍝** STORES PROBABILITY OF KILL FOR EACH ROUND ****
    ⍝*************************************************
     PHIT←⍳0
     N←⍴CPAN
     I←1
    LOOP:CPA←CPAN[I]
     PROBHIT
     PHIT←PHIT,PH
     I←I+1
     →(I≤N)/LOOP
    ∇

    ∇ PROBHIT;I;N
    ⍝**********************************************
    ⍝*** CALCULATES PROB OF HIT FOR A SINGLE ROUND **
    ⍝**********************************************
     R←RANGE
     A←((6.2831×(R*2)×(DISP*2))+AP)
     B←(¯3.1421×(CPA*2))
     C←(2×(R*2)×(DISP*2))+AP
     D←B÷C
     PH←(AP÷A)×(*D)
    ∇

    ∇ PROBKILL;I
    ⍝************************************************
    ⍝** DETERMINES PROBABILITY FOR A GIVEN BURST ****
    ⍝************************************************
     Y←0 0 0 1
     I←1
    LOOP1:VIMP←VIMPN[I]
    ⍝************************************************
    ⍝** MINIMUM IMPACT VELOCITY THAT CAUSES DAMAGE ****
    ⍝************************************************
     →(VIMP≤500)/STOP
     AVDATA
     PH←PHIT[I]
     →(PH<1E¯8)/LOOP2
     →((+/X)=0)/LOOP2
     MARKOV
    LOOP2:I←I+1
     →(I≤BURST)/LOOP1
    STOP:PK←(,Y)[1]
    ∇

    ∇ AVDATA;VMIN;MAXV;AVMAX;AVMIN
    ⍝*************************************************
    ⍝** BUILDS TRANSITION MATRIX FOR VELOCITY AT BURST ***
    ⍝*************************************************
    ⍝ 'AV DATA FOR MIN VELOCITY EXPRESSED AS A VECTOR?'
    ⍝AVMIN←1 0.1103 0.1103 0.0823 0 0.8897 0 0.028 0 0
       0.8897 0.028 0 0 0 0.8617
    ⍝AVMIN←1 0.1408 0.1408 0.0988 0 0.8592 0 0.0421 0 0
       0.8592 0.0421 0 0 0 0.817
     AVMIN← 1 0.2018 0.2018 0.1317 0 0.7982 0 0.0701 0 0
       0.7982 0.0701 0 0 0 0.7281
    ⍝AVMIN← 1 0.2628 0.2628 0.1646 0 0.7372 0 0.0982 0 0
       0.7372 0.0982 0 0 0 0.639
    ⍝ 'AV DATA FOR MAX VELOCITY EXPRESSED AS A VECTOR?'
    ⍝AVMAX←1 0.2073 0.2073 0.1372 0 0.7297 0 0.0701 0 0
       0.7927 0.0701 0 0 0 0.7226
    ⍝AVMAX←1 0.2341 0.2341 0.15 0 0.7659 0 0.0841 0 0
       0.7659 0.0841 0 0 0 0.6818
     AVMAX← 1 0.2018 0.2018 0.1728 0 0.7982 0 0.1122 0
```

```
      0.7982 0.1122 0 0 0 0.6028
[13] ⍝AVMAX← 1 0.3165 0.3165 0.1902 0 0.6835 0 0.1262 0 0
      0.6835 0.1262 0 0 0 0.5573
[14] ⍝ 'WHAT IS THE MINIMUM VELOCITY?'
[15] VMIN←50
[16] →(VBUST≤100)/STOP
[17] ⍝ 'WHAT IS THE MAXIMUM VELOCITY?'
[18] MAXV←V0+VMAX
[19] X←(((VBUST-VMIN)×(AVMAX-AVMIN))÷(MAXV-VMIN))+AVMIN
[20] →GO
[21] STOP:X←0
[22] GO:→0
      ∇

      ∇ MARKOV;DIMENSION;AVMATRIX;IDMATRIX;TRANSMATRIX
      ;YNEW;COUNT;PM;L;N;X1
[1]  ⍝**********************************************
[2]  ⍝**** BUILDS LETHALITY MATRIX AND TRANSITIONS ******
[3]  ⍝****  IT TO GIVE PROBABILITY OF KILL **************
[4]  ⍝**********************************************
[5]  ⍝'INPUT VULNERABLE AREA DATA AS AN ARRAY'
[6]  X1←X
[7]  L←⍴X1
[8]  DIMENSION←L*0.5
[9]  AVMATRIX←(DIMENSION,DIMENSION)⍴X1
[10] ⍝'INPUT THE INITIAL STATE VECTOR'
[11] ⍝Y←0 0 0 1
[12] ⍝'WHAT IS THE PROBABILITY OF HIT'
[13] ⍝PH←⎕
[14] AVMATRIX←AVMATRIX×PH
[15] PM←1-PH
[16] IDMATRIX←((⍳DIMENSION)∘.×(⍳DIMENSION))
[17] IDMATRIX←((DIMENSION,DIMENSION)⍴((⍳DIMENSION)*2))
      =IDMATRIX)
[18] TRANSMATRIX←((PM×IDMATRIX)+AVMATRIX)
[19] ⍝'THE NUMBER OF MATRIX TRANSITION'
[20] N←1
[21] GO:Y←(DIMENSION,DIMENSION)⍴Y
[22] YNEW←TRANSMATRIX×Y
[23] YNEW←+⌿(⍉YNEW)
[24] YNEW←(1,DIMENSION)⍴YNEW
[25] Y←⍉YNEW
[26] COUNT←N-1
[27] N←COUNT
[28] →(COUNT>0)/GO
      ∇

      ∇ CUMKILL
[1]  ⍝*******************************************
[2]  ⍝** GIVES CUMULATIVE PROBABILITY OF KILL FOR  ****
[3]  ⍝** A TARGET THAT HAS BEEN ENGAGED WITH MORE  ****
[4]  ⍝***      THAN ONE BURST     ********
[5]  ⍝*******************************************
[6]  Q←⍉(((DATA+1),(⍴RANGES))⍴P)
[7]  Q←1-Q
[8]  Q←×\Q
[9]  Q←,(⍉Q)
[10] Q←1-Q
      ∇

      ∇ R←AB UNIF N;A;B
[1]  ⍝*******************************************
[2]  ⍝* GENERATES N UNIFORM RANDOM NUMBERS  **********
[3]  ⍝***  ON INTERVAL(A,B) *********************
[4]  ⍝*******************************************
[5]  A←AB[1]
[6]  B←AB[2]
[7]  R←A+(B-A)×(÷2147483647)×?N⍴2147483646
```

```
      ∇

      ∇ R←N UNIRAND B
[1]   ⍝**********************************************
[2]   ⍝** GENERATES N PSEUDO UNIFORM RANDOM NUMBERS ***
[3]   ⍝**********************************************
[4]    R←(B[1]-B[2])+((N?100000000)×2×B[2])÷100000000
      ∇

      ∇ Z←N NORRAND P;S;I;T
[1]   ⍝****************************************
[2]   ⍝*** GENERATES N NORMAL RANDOM NUMBERS *****
[3]   ⍝****************************************
[4]    P←2↑P,1
[5]    Z←(N)⍴0
[6]    I←1
[7]   F10:T←2 UNIRAND 0.5 0.5
[8]    T←(2×T)-1
[9]    S←(T[1]*2)+T[2]*2
[10]   →F10×⍳S≥1
[11]   Z[I]←P[1]+P[2]×(T[1]×((¯2×⍟S)÷S)*0.5)
[12]   →F10×⍳(I←I+1)≤N
      ∇
```

CORRECTIONS MADE TO KEELING'S PROGRAM IN :

Subroutine AIRCRAFT (36, 37, 38)

Subroutine MISSVECTOR (9)

Subroutine MISSDIST (5, 30, 31, 37)

Subroutine PROBHIT (6)

APPENDIX B
APL COMPUTER PROGRAM

1. NEW PROGRAM

```
     ∇ DRIVER;COUNT;I
    ⍝*************************************************
    ⍝*******       DATA  INPUT       ******************
    ⍝*************************************************
    ⍝ 'HOW MANY DATA POINTS DO YOU WANT PER RANGE?'
     DATA←25
     DATA←DATA-1
    ⍝ 'WHAT IS THE GUN FIRING RATE IN ROUNDS PER MINUTE?'
     FIRERATE←600
    ⍝ 'WHAT IS THE MUZZLE VELOCITY?'
     V0←1005
    ⍝ 'WHAT IS THE BURST SIZE?'
     BURST←5
    ⍝ 'WHAT IS THE ANGULAR DISPERSION IN RADIANS?'
     DISP←0.001
    ⍝ 'WHAT IS THE DRAG FACTOR  B=CD×V0*0.5?'
     B←0.163
    ⍝ 'WHAT ARE THE RANGES YOU WANT TO LOOK AT?'
     RANGES← 1900 1300 700 100
    ⍝ 'WHAT ARE THE AREAS OF THE AIRCRAFT?'
     AFRT←5
     ASIDE←30
     ATOP←50
     RL←5
     PK/H←0.25
    ⍝**************************************************
    ⍝*** PROGRAM EXECUTION AND DATA COLLECTION   *******
    ⍝***   P  IS PROBABILITY OF KILL          ******
    ⍝***   R SLANT IS RANGE  OF TARGET AT INTERCEPT  ***
    ⍝**************************************************
     R←⍳0
     P←⍳0
     I←1
     COUNT←⍴RANGES
    LOOP:RANGE←RANGES[I]
     GUN
     P←P,PKDATA
     R←R,RANGEI
     I←I+1
     →(I≤COUNT)/LOOP
    ⍝****************************************************
    ⍝** DATA CONTROL VECTOR USED FOR GRAPHS IN GRAFSTAT **
    ⍝****************************************************
    ⍝'PK='
    ⍝7 4 ⍕ 2 10 ⍴P
    ⍝ 'RANGE='
    ⍝ 5 0 ⍕ 4 10 ⍴R
    ⍝C←((DATA+1),(⍴RANGES))⍴RANGES
    ⍝C←,(⍉C)
     ∇

     ∇ GUN;I;VAC;BODY;TOP;SIDE;VRX;VRY;VRZ
    ⍝*************************************************
    ⍝******** SIMULATES AIR DEFENSE GUN BY CALLING *****
    ⍝********   APPROPRIATE SUBROUTINES  ********
    ⍝*************************************************
    RANGEI←⍳0
    I←0
```

```
[7]  PKDATA←⍳0
[8]  LOOP3:RANGECHEK
[9]   RADAR
[10]  AIRCRAFT
[11]  INTERCEPT
[12]  MISSVECTOR
[13]  PKVECTOR
[14]  BURSTKILL
[15]  RANGEI←RANGEI,RTGT
[16]  PKDATA←PKDATA,PK
[17]  I←I+1
[18]  →(I≤DATA)/LOOP3
     ∇

     ∇ RANGECHEK;R;O;D;MAXALT;MINALT
[1]  ⍝************************************************
[2]  ⍝**** DETERMINES A/C POSITION AT TIME OF FIRING **
[3]  ⍝************************************************
[4]  ⍝ 'WHAT IS THE MAXIMUM ALTITUDE IN METERS?'
[5]  MAXALT←1100
[6]  ⍝ 'WHAT IS THE MINIMUM ALTITUDE IN METERS?'
[7]  MINALT←90
[8]  R←RANGE
[9]  O← 0 1.5707963 UNIF 1
[10]  →(R>MAXALT)/LOOP1
[11]  ZF←(MINALT,R) UNIF 1
[12]  →LOOP2
[13] LOOP1:ZF←(MINALT,MAXALT) UNIF 1
[14] LOOP2:D←((R*2)-(ZF*2))*0.5
[15]  XF←D×(2○O)
[16]  YF←D×(1○O)
     ∇

     ∇ RADAR;S2;THETAA;R;DELRANGE;DTHETA;THETAE
[1]  ⍝**********************************************
[2]  ⍝****  DETERMINES RADAR LOCATED TARGET BASED  ***
[3]  ⍝*****  ON ERRORS INDUCED BY RADAR  *********
[4]  ⍝**********************************************
[5]  S2←0.0012+(2÷RANGE)
[6]  DELRANGE←1 NORRAND 0 6.2
[7]  DELRANGE←DELRANGE×0.1
[8]  DTHETA←2 NORRAND(0,S2)
[9]  DTHETA←DTHETA×0.1
[10]  R←RANGE+DELRANGE
[11]  THETAA← ¯1○(XF÷(((XF*2)+(YF*2))*0.5))
[12]  THETAA←THETAA
[13]  THETAE← ¯1○(ZF÷RANGE)
[14]  THETAE←THETAE
[15]  YR←R×(2○(DTHETA[2]+THETAE))×(2○(DTHETA[1]+THETAA))
[16]  XR←R×(2○(DTHETA[2]+THETAE))×(1○(DTHETA[1]+THETAA))
[17]  ZR←R×(1○(DTHETA[2]+THETAE))
     ∇

     ∇ AIRCRAFT;VMIN;AXMAX;AXMIN;AZMIN;AZMAX;D;O;VZ1
       ;VZ2;MINALT;MAXALT;AXY;AZ1;AZ2;O1
[1]  ⍝************************************************
[2]  ⍝*****  A/C PERFORMANCE DATA INPUTS  **********
[3]  ⍝************************************************
[4]  ⍝ 'WHAT IS THE MIN ALT THE A/C IS ALLOWED TO FLY?'
[5]  MINALT←90
[6]  ⍝ 'WHAT IS THE MAX ALT THE GUN WILL ENGAGE?'
[7]  MAXALT←1100
[8]  ⍝ 'WHAT IS THE MIN VELOCITY OF A/C IN METERS/SEC?'
[9]  VMIN←150
[10] ⍝ 'WHAT IS THE MAX VELOCITY OF A/C IN METERS/SEC?'
[11]  VMAX←220
[12] ⍝ 'MAX A/C ACCELERATION IN Z DIRECTION IN M/SEC2?'
[13]  AZMAX←49
```

```
⍝ 'MIN A/C ACCELERATION IN Z DIRECTION IN M/SEC2?'
 AZMIN←39.2
⍝ 'MAX ACCELERATION IN X OR Y DIRECTION IN M/SEC2?'
 AXY←0.4×AZMAX
⍝*************************************************
⍝**** DETERMINES A/C FLIGHT PATH MANEUVER  *****
⍝*****       AT TIME OF FIRING  ***********
⍝*************************************************
 VA←(VMAX,VMAX) UNIF 1
 O←0 6.2831853 UNIF 2
 VZ2←(((VA*2)÷2)*0.5)
 VZ1←((MINALT-ZF)÷(MAXALT-MINALT))×VZ2
 VZ←(VZ2,VZ2) UNIF 1
 D←((VA*2)-(VZ*2))*0.5
 VX←(D×(2○O[1]))
 VY←(D×(1○O[1]))
 AZ1←(MINALT-ZF)÷((MAXALT-MINALT)÷(AZMIN×0.6))
 AZ2←0.6×AZMAX
 AZ←(AZ2,AZ2) UNIF 1
 O1←0 6.2831853 UNIF 1
 AX←AXY×(2○O1)
 AY←AXY×(1○O1)
 VRZ←1 NORRAND(VZ,(0.1,VZ))
 VRX←1 NORRAND(VX,(0.1,VX))
 VRY←1 NORRAND(VY,(0.1,VY))
  ∇

  ∇ INTERCEPT;F1TN;BURSTIME;XF1;YF1;ZF1;RPROJ;N;FTN
⍝*************************************************
⍝** DETERMINES PREDICTED INTERCEPT POINT BASED  **
⍝***  ON LINEAR FIRE CONTROL COMPUTATIONS  ******
⍝***   AND THE NEWTON-RAPHSON TECHNIQUE *******
⍝*************************************************
 DELT←1÷(FIRERATE÷60)
 BURSTIME←BURST÷(FIRERATE÷60)
 N←⌊(BURST÷2)
 T0←N×DELT
 TAU←0
 LOOP1:XPI←XR+(VRX×T0)+(VRX×TAU)
 YPI←YR+(VRY×T0)+(VRY×TAU)
 ZPI←ZR+(VRZ×T0)+(VRZ×TAU)
 RTGT←((XPI*2)+(YPI*2)+(ZPI*2))*0.5
 RPROJ←(V0×TAU)÷((B×TAU)+1)
 FTN←RTGT-RPROJ
 F1TN←((XPI×VRX)+(YPI×VRY)+(ZPI×VRZ))÷RTGT
 F1TN←F1TN-(V0÷(1+(B×TAU)*2))
 →(RTGT<3000)/LOOP3
 AIRCRAFT
 LOOP3:→((|FTN)<0.0001)/LOOP2
 TAU←TAU-(FTN÷F1TN)
 →LOOP1
 LOOP2:→0
  ∇

  ∇ MISSVECTOR
⍝****************************************************
⍝** CALCULATES DISTANCE FROM A/C TO MEAN THEORETICAL *
⍝**  INTERCEPT POINT FOR EACH PROJECTILE     *******
⍝****************************************************
 APN←⍳0
 VIMPN←⍳0
 CPAN←⍳0
 N←1
 TAU0←¯0.1
 LOOP:MISSDIST
⍝****************************************************
⍝**  COLLECTS STATISTICS ON BIS, PRESENTED AREA, ***
⍝**     AND BURST VELOCITY. ****
⍝**  USED TO DETERMINE PKSS AND PK/BURST     *******
```

```
[15] ⍝*****************************************************
[16] CPAN←CPAN,CPA
[17] APDATA
[18] APN←APN,AP
[19] VIMPN←VIMPN,VIMP
[20] N←N+1
[21] →(N≤BURST)/LOOP
[22] →0
     ∇

     ∇ MISSDIST;A;F1;F2;F3;XA;YA;ZA;F1TN;F2TN;BT2;BT
      ;XAP;YAP;ZAP;T0
[1]  ⍝*********************************************
[2]  ⍝**** DETERMINES TIME AT WHICH CLOSEST POINT  **
[3]  ⍝*****    OF APPROACH OCCURS  ******
[4]  ⍝*********************************************
[5]  VPX←(XPI÷RTGT)×V0
[6]  VPY←(YPI÷RTGT)×V0
[7]  VPZ←(ZPI÷RTGT)×V0
[8]  LOOP1:BT←(B×TAU)+1
[9]  BT2←BT*2
[10] XP←(VPX×TAU)÷BT
[11] YP←(VPY×TAU)÷BT
[12] ZP←(VPZ×TAU)÷BT
[13] T0←(N-1)×DELT
[14] XA←XF+(VX×T0)+((AX÷2)×(T0*2))+((VX+(AX×T0))×TAU)
       +(AX  (TAU*2)  2)
[15] YA←YF+(VY×T0)+((AY÷2)×(T0*2))+((VY+(AY×T0))×TAU)
       +(AY  (TAU*2)  2)
[16] ZA←ZF+(VZ×T0)+((AZ÷2)×(T0*2))+((VZ+(AZ×T0))×TAU)
       +(AZ  (TAU*2)  2)
[17] XAP←XA-XP
[18] YAP←YA-YP
[19] ZAP←ZA-ZP
[20] F11←XAP×(VX+(AX×T0)+(AX×TAU)-(VPX÷BT2))
[21] F12←YAP×(VY+(AY×T0)+(AY×TAU)-(VPY÷BT2))
[22] F13←ZAP×(VZ+(AZ×T0)+(AZ×TAU)-(VPZ÷BT2))
[23] F14←((XAP*2)+(YAP*2)+(ZAP*2))*0.5
[24] F1TN←(F11+F12+F13)
[25] A←BT*3
[26] F21←((F11÷XAP)*2)+((AX+(2×VPX×B÷A))×XAP)
[27] F22←((F12÷YAP)*2)+((AY+(2×VPY×B÷A))×YAP)
[28] F23←((F13÷ZAP)*2)+((AZ+(2×VPZ÷B÷A))×ZAP)
[29] F2TN←F21+F22+F23
[30] →((TAU-TAU0)<0.0001)/LOOP2
[31] TAU0←TAU
[32] TAU←TAU-(F1TN÷F2TN)
[33] ⍝I←I+1
[34] ⍝→((LOOP2×⍳I)≥30)/LOOP2
[35] →LOOP1
[36] LOOP2:CPA←F14
[37] TAU0←0
[38] VP←V0÷(((B×TAU)+1)*2)
[39] RPROJ←(V0×TAU)÷((B×TAU)+1)
     ∇

     ∇ APDATA;VXIMP;VYIMP;VZIMP;AP1;AP2
[1]  ⍝***********************************************
[2]  ⍝**  COMPUTES PRESENTED AREA OF THE AIRCRAFT  **
[3]  ⍝***********************************************
[4]  VXIMP←VX-(VP×XP÷RPROJ)
[5]  VYIMP←VY-(VP×YP÷RPROJ)
[6]  VZIMP←VZ-(VP×ZP÷RPROJ)
[7]  VIMP←((VXIMP*2)+(VYIMP*2)+(VZIMP*2))*0.5
[8]  →(VIMP≤VP)/LOOP
[9]  AP1←ASIDE-((VIMP-VP)×(ASIDE-AFRT)÷(|(VA)))
[10] AP2←ATOP-((VIMP-VP)×(ATOP-AFRT)÷(|(VA)))
[11] →LOOP2
[12] LOOP:AP1←((VIMP-VP)×(ASIDE-AFRT)÷((|(VA))))+ASIDE
```

```
AP2←((VIMP-VP)×(ATOP-AFRT)÷((|(VA))))+ATOP
LOOP2:AP←(AP1,AP2) UNIF 1
∇

∇ PKVECTOR;I;N
⍝**************************************************
⍝** STORES PROBABILITY OF KILL FOR EACH ROUND **
⍝**************************************************
PKSSN←⍳0
N←⍴CPAN
I←1
LOOP:CPA←CPAN[I]
⍝**************************************************
⍝*** OPTIONAL SELECTION FOR TYPE OF FUZE **********
⍝*** FOR PROXIMITY FUZE, USE PROXFUZE    *********
⍝*** FOR CONTACT FUZE, USE CONTFUZE  **************
⍝***************************************************
PROXFUZE
⍝CONTFUZE
PKSSN←PKSSN,PKSS
I←I+1
→(I≤N)/LOOP
∇

∇ PROXFUZE;A;B;C;D;R;RL;R0;AL;PF
⍝************************************************
⍝** PROXIMITY FUZED WARHEAD STORES PROBABILITY **
⍝***  OF KILL FOR SINGLE SHOT **********
⍝************************************************
R←RANGE
PF←1
RL←5
R0←(1.2×RL)
A←(R0*2)×PF
B←(2×(DISP*2)×(R*2))+(R0*2)
C←A÷B
D←(¯1×(CPA*2))÷((2×(DISP*2)×(R*2))+(R0*2))
PKSS←C×(*D)
∇

∇ CONTFUZE;A;B;C;D;R
⍝**********************************************
⍝*** CONTACT FUZED WARHEAD CALCULATES ********
⍝***  PROBABILITY OF KILL FOR SINGLE SHOT *****
⍝**** AV=PK/H×AP, ASSUMED PK/H=0.25 **********
⍝**********************************************
R←RANGE
A←0.25×AP
B←((2×3.142)×(DISP*2)×(R*2))+(AP*2)
C←A÷B
D←(¯3.142×(CPA*2))÷B
PKSS←C×(*D)
∇

∇ BURSTKILL;AK;BK
⍝***********************************************
⍝*** PROBABILITY OF KILL GIVEN BURST ***********
⍝**********************************************
⍝AK←((⍴RANGES),(DATA+1))⍴PKSS
AK←PKSSN
BK←1-AK
CK←×/BK
EK←1-CK
PK←EK
⍝*************************************************
⍝**** PROBABILITY OF KILL ENCOUNTER **********
⍝****** ASSUMED PD×PF=1  *******************
⍝***** PK/E EQUAL TO PKBURST ******************
```

```
[14] ⍝*********************************************************
     ∇

     ∇ R←AB UNIF N;A;B
[1]  ⍝*********************************************************
[2]  ⍝* GENERATES N UNIFORM RANDOM NUMBERS  **********
[3]  ⍝***  ON INTERVAL(A,B)  ********************
[4]  ⍝*********************************************************
[5]   A←AB[1]
[6]   B←AB[2]
[7]   R←A+(B-A)×(÷2147483647)×?N⍴2147483646
     ∇

     ∇ R←N UNIRAND B
[1]  ⍝*********************************************************
[2]  ⍝** GENERATES N PSEUDO UNIFORM RANDOM NUMBERS ***
[3]  ⍝*********************************************************
[4]   R←(B[1]-B[2])+((N?100000000)×2×B[2])÷100000000
     ∇

     ∇ Z←N NORRAND P;S;I;T
[1]  ⍝**************************************************
[2]  ⍝*** GENERATES N NORMAL RANDOM NUMBERS *****
[3]  ⍝**************************************************
[4]   P←2↑P,1
[5]   Z←(N)⍴0
[6]   I←1
[7]  F10:T←2 UNIRAND 0.5 0.5
[8]   T←(2×T)-1
[9]   S←(T[1]*2)+T[2]*2
[10]  →F10×⍳S≥1
[11]  Z[I]←P[1]+P[2]×(T[1]×((¯2×⍟S)÷S)*0.5)
[12]  →F10×⍳(I←I+1)≤N
     ∇
```

APPENDIX C
SYMBOL NOTATION

SYMBOL	DEFINITION	UNITS
A_{frt}	area of aircraft front	meters^2
A_{top}	area of aircraft top	meters^2
A_{side}	area of aircraft side	meters^2
A_{max}	max aircraft acceleration	meters sec^2
A_{min}	min aircraft acceleration	meters sec^2
A_p	aircraft presented area	meters^2
A_v	aircraft vulnerable area	meters^2
A_{z1}, A_{z2}	range of A_z	meters sec^2
A_x, A_y, A_z	acceleration of aircraft	meters sec^2
C_d	coefficient of drag	1 meters
g	gravitational constant	meters sec^2
GD	gravity drop	meters
LM	lethality matrix	n a
P_H	probability of aircraft hit	n a
P_h	probability of component hit	n a
P_k	probability of kill	n a
P_{ke}	probability of kill engagement	n a
$P_{k\|h}$	probability of kill given hit	n a
R	slant range	meters
R_p	projectile range	meters
t	time	sec
V_a	aircraft total velocity	meters sec
V_{bust}	burst velocity	meters sec
V_0	muzzle velocity	meters sec

V_p	projectile velocity	meters sec
V_{px}, V_{py}, V_{pz}	projectile velocity component	meters sec
V_{rx}, V_{ry}, V_{rz}	radar aircraft velocity	meters sec
V_x, V_y, V_z	aircraft velocity components	meters sec
V_{z1}, V_{z2}	range of V_z	meters sec
W_i	uncorrected wind	meters sec
X_a, Y_a, Z_a	aircraft position at time t	meters
X_f, Y_f, Z_f	aircraft position at firing	meters
X_p, Y_p, Z_p	projectile location	meters
X_r, Y_r, Z_r	radar located target	meters
α_{dx}, α_{dy}	angular ballistic dispersion	radians
ΔX_p	projectile displacement	meters
ε_R	range error	meters
$\varepsilon_{\theta e}, \varepsilon_{\theta a}$	azimuth and elevation error	radian
μ_{bx}, μ_{by}	linear mean bias	meters
(ω_e, ω_a)	gun azimuth and elevation	milliradians
ρ	projectile linear range	meters
σ_{dx}, σ_{dy}	linear ballistic dispersion	meters
τ	projectile flight time	sec
θ_{bx}, θ_{by}	angular mean bias	radians
θ_e, θ_a	target azimuth and elevation	milliradians

APPENDIX D
EQUATIONS AND DATA

1. GOVERNING EQUATIONS

a. The Siacci Formula

This equation used here to determine the velocity of the projectile at a point in time is

$$V_p = V_0 / (C_d V_0^{1/2} \tau + 1)^2 \qquad \text{(eqn D.1)}$$

Where these two equation can be seen [Ref. 1].

- C_d is the coefficient of drag of the projectile.
- V_p is the velocity of the projectile.
- V_0 is the muzzle velocity.
- τ is the time of flight of the projectile.

b. Range of the Projectile

The formula for determine the range of the projectile at a point in time is taken as

$$R_g = V_0 \tau / (C_d V_0^{1/2} \tau + 1) \qquad \text{(eqn D.2)}$$

The derivation of these formula can be found in [Ref. 4]

c. Gravity Drop

$$GD = (-1/6) g \tau^2 (1 + 2(V_p / V_0)^{1/2}) \qquad \text{(eqn D.3)}$$

2. DATA

This study looked at 40 mm gun system, and firing high explosive round with both a contact fuze and a proximity fuze. The unclassified system characteristics were obtained from [Ref. 5]

All references to vulnerable area data for specific air threats to these guns are classified, therefore aircraft probability of kill given a hit and warhead lethal radius is entirely synthetic. Vulnerable area data for many different aircrafts and many different gun systems is readily available for personnel with appropriate clearances. The optimal angular ballistic dispersion was determined using the computer simulation and the coefficient of drag for the projectiles are assumed,the other data was from Jane's Weapon System. The aircraft's vulnerability measure r_l is assumed as discussed in Chapt 2, and A_V is determined by using the probability the aircraft is killed given the random hit on the aircraft $P_{K\ H}$ as discussed in Chapt 3.

TABLE 1
40 MM GUN CHARACTERISTICS

parameters	unit	data
firing rate	rds/min	600
muzzle velocity	m/sec	1005
coefficient of drag	1/sec	0.00515
maximum range	km	12
angular dispersion	mrad	1.0

APPENDIX E
EMPIRICAL COMPARISON OF MARGINAL DISTRIBUTIONS

1. TYPE OF FUZE

a. Proximity Fuzed Warhead

TABLE 2

PROXIMITY FUZED WARHEAD

PARAMETER VALUES: (100);(700);(1300);(1900)
POPULATIONS : RG14;RG13;RG12;RG11
WEIGHTS : 0
X LABEL : RANGE IN METERS
Y LABEL : PROBABILITY OF KILL

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
100	50	0.80613	0.1907	0.72854	8.6001E-1	9.4777E-1
700	50	0.2179	0.23669	0.021002	1.3836E-1	3.2233E-1
1300	50	0.051488	0.15441	0	9.2321E-13	7.8509E-3
1900	50	0.0094727	0.065348	0	0.0000E0	6.1062E-160

POOLED STANDARD DEVIATION ESTIMATE: 0.17357

b. Contact Fuzed Warhead

TABLE 3

CONTACT FUZED WARHEAD

PARAMETER VALUES: (100);(700);(1300);(1900)
POPULATIONS : CR14;CR13;CR12;CR11
WEIGHTS : 0
X LABEL : RANGE IN METERS
Y LABEL : PROBABILITY OF KILL

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
100	60	0.017203	0.0056864	0.013458	0.016796	1.9509E-2
700	60	0.0092551	0.0053786	0.0055805	0.010485	1.2478E-2
1300	60	0.003111	0.0055073	0	0.000038096	3.3590E-3
1900	60	0.00048311	0.002269	0	0	1.0408E-15

POOLED STANDARD DEVIATION ESTIMATE: 0.0049179

2. BALLISTIC DISPERSION

TABLE 4

BALLISTIC DISPERSION

```
PARAMETER VALUES: (0);(0.0005);(0.001);(0.0015);(0.002)
POPULATIONS      : AP1;AP2;AP3;AP4;AP5
WEIGHTS          : 0
X LABEL          : ANGULAR DISPERSION IN RADIANS
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
0	100	0.18605	0.31466	0	2.7032E-4	0.23817
0.0005	100	0.17568	0.28455	0	5.7949E-6	0.27246
0.001	100	0.18497	0.31487	0	6.9311E-5	0.231
0.0015	100	0.1703	0.2776	0	1.9134E-5	0.23033
0.002	100	0.16013	0.28406	0	3.8859E-6	0.13654

POOLED STANDARD DEVIATION ESTIMATE: 0.29559

3. RANGE EVALUATION

TABLE 5

RANGE EVALUATION

```
PARAMETER VALUES: (100);(700);(1300);(1900)
POPULATIONS      : RG14;RG13;RG12;RG11
WEIGHTS          : 0
X LABEL          : RANGE IN METERS
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
100	50	0.80613	0.1907	0.72854	8.6001E-1	9.4777E-1
700	50	0.2179	0.23669	0.021002	1.3836E-1	3.2233E-1
1300	50	0.046666	0.15413	0	2.0463E-9	2.1222E-3
1900	50	0.0094727	0.065348	0	0.0000E0	6.1062E-16

POOLED STANDARD DEVIATION ESTIMATE: 0.16975

4. EFFECTS OF TARGET MANEUVERS

a. Maneuvering Target

TABLE 6

MANEUVERING TARGET

```
PARAMETER VALUES: (100);(700);(1300);(1900)
POPULATIONS      : RG14;RG13;RG12;RG11
WEIGHTS          : 0
X LABEL          : RANGE IN METERS
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
100	50	0.80613	0.1907	0.72854	8.6001E-1	9.4777E-1
700	50	0.2179	0.23669	0.021002	1.3836E-1	3.2233E-1
1300	50	0.051488	0.15441	0	9.2321E-13	7.8509E-3
1900	50	0.0094727	0.065348	0	0.0000E0	6.1062E-16

POOLED STANDARD DEVIATION ESTIMATE: 0.17357

b. Non-maneuvering Target

TABLE 7

NON-MANEUVERING TARGET

```
PARAMETER VALUES: (100);(700);(1300);(1900)
POPULATIONS      : NOM1;NOM2;NOM3;NOM4
WEIGHTS          : 0
X LABEL          : RANGE IN METRES
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
100	50	0.97438	0.009994	9.6751E-1	0.97509	0.98113
700	50	0.46539	0.25853	2.2858E-1	0.41486	0.6943
1300	50	0.16126	0.27431	6.3071E-11	0.0013258	0.28581
1900	50	0.012354	0.075884	0.0000E0	0	0

POOLED STANDARD DEVIATION ESTIMATE: 0.19231

5. EFFECTS OF RADAR ERROR

TABLE 8

RADAR ERROR

```
PARAMETER VALUES: (0);(50);(100);(150)
POPULATIONS      : RD21;RD22;RD23;RD24
WEIGHTS          : 0
X LABEL          : PERCENT RADAR ERROR
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
0	60	0.3018	0.41684	1.4979E-12	0.014793	0.804
50	60	0.2246	0.33077	0.0000E0	0.0000149	0.44578
100	60	0.13527	0.25116	1.3878E-17	0.00014144	0.12566
150	60	0.083055	0.1728	0.0000E0	0.0017942	0.078821

POOLED STANDARD DEVIATION ESTIMATE: 0.29038

6. EFFECTS OF MUZZLE VELOCITY AND RATE OF FIRE

a. Muzzle Velocity

TABLE 9

MUZZLE VELOCITY

```
PARAMETER VALUES: (1000);(1500);(2000);(2500)
POPULATIONS      : MV1;MV2;MV3;MV4
WEIGHTS          : 0
X LABEL          : MUZZLE VELOCITY(METERS/SEC)
Y LABEL          : PROBABILITY OF KILL
```

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
1000	60	0.11223	0.23417	0	0.000029386	0.055264
1500	60	0.24198	0.29665	0.0022309	0.1129	0.39962
2000	60	0.35197	0.27194	0.09949	0.31493	0.57484
2500	60	0.51254	0.2572	0.28252	0.5894	0.70919

POOLED STANDARD DEVIATION ESTIMATE: 0.26596

b. **Rate of Fire**

TABLE 10

RATE OF FIRE

PARAMETER VALUES: (400);(600);(800);(1000)
POPULATIONS : FR21;FR22;FR23;FR24
WEIGHTS : 0
X LABEL : FIRING RATE(RDS/MIN)
Y LABEL : PROBABILITY OF KILL

POPULATION NUMBER	NO. OF POINTS	Y MEAN	Y STD DEV	PERCENTILES 0.25	0.5	0.75
400	60	0.1618	0.25813	0.00038014	0.021753	0.19313
600	60	0.21132	0.27047	0.00079402	0.058486	0.39343
800	60	0.30698	0.30679	0.0028055	0.23851	0.55131
1000	60	0.40055	0.32489	0.039921	0.38058	0.74919

POOLED STANDARD DEVIATION ESTIMATE: 0.29132

LIST OF REFERENCES

1. Keeling, J. C., *Determining Air Defense Gun Performance Using Computer Simulations*, Masters Thesis, Naval Postgraduate School, Monterey, CA, Sept 1986.

2. Ball, R. E., *The Fundamentals of Aircraft Combat Survivability Analysis and Design*, AIAA Education Series, 1985.

3. NAVORD OP3000 Vol 2, 1st Revision, *Weapon Systems Fundamentals*, Commander Naval Ordnance Systems Command, Jan 1971.

4. McShane, E. J., *Exterior Ballistics*, The University of Denver Press, Denver, CO, 1953.

5. *Jane's Weapon Systems 1986-1987*, Jane's Publishing Inc., N Y, N Y.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Department Chairman, Code 67 Dept. of Aeronautics Engineering Naval Postgraduate School Monterey, CA 93943-5000	1
4.	Prof. Robert E. Ball, Code 67BP Dept. of Aeronautics Engineering Naval Postgraduate School Monterey, CA 93943-5000	2
5.	Commander (Attn: Mr John Morrow, Code 338) Naval Weapons Center China Lake, CA 93555-6001	1
6.	Prof. Richard D. Wood, Code 67WR Dept. of Aeronautics Engineering Naval Postgraduate School Monterey, CA 93943-5000	1
7.	Jung, Hyun Dae. BuORD of DCNO, Headquarters of R.O.K. Navy Singildong, Youngdeongpoku, Seoul 102-00 Republic of Korea	10
8.	Director, Kim, Jung Hwa. BuORD of DCNO, Headquarters of R.O.K. Navy, Singildong, Youngdeongpoku, Seoul, 102-00 Republic of Korea	2
9.	Naval Academy Library Angokdong, Jinhae City, Gyungnam 602-00 Republic of Korea	1
10.	Naval Staff Collage Jehwangsandong, Jinhae City, Gyungnam 602-00 Republic of Korea	1

11. Commander of N.O.A.D | 1
Logistic Supply Center. Hyundong.
Jinhae City, Gyungnam 602-00
Republic of Korea

12. Commander of BuORD. | 1
Naval Shipyard. Hyundong.
Jinhae City, Gyungnam 602-00
Republic of Korea

13. CDR & Mrs. John C. Connell, Jr | 1
8010 Orange Plank Dr.
Springfield, VA 22153

14. LT D. P. Roane | 1
P.O. Box 360
Dutton, VA 23050

15. LCDR Youn, Duck Sang | 1
SMC #1254 NPS
Monterey, CA 93943

16. LCDR Shin, Dong Ryong | 1
SMC #1732 NPS
Monterey, CA 93943

17. LT Sur, Joo No | 1
SMC #1655 NPS
Monterey, CA 93943

END

DATE

FILMED

APRIL

1988

DTIC