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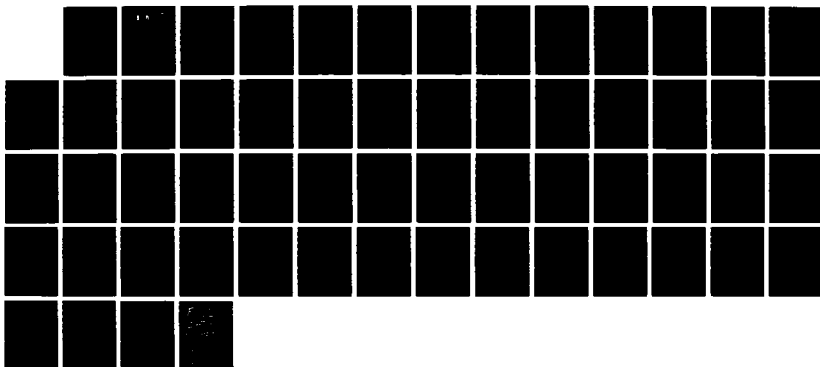
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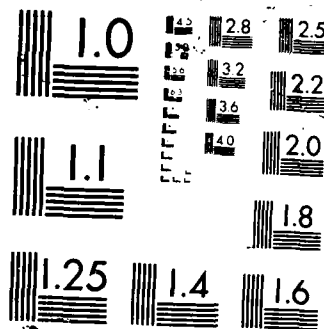
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This contract supported the Center For Multivariate Analysis which included support for two core researchers and several visiting researchers. A total of 160 technical reports were issued; of these 48 have been published, 38 have been accepted for publication, and 74 have been submitted for publication. Some important contribution were made to several areas of multivariate analysis, with applications in manufacturing technology, signal processing, automation, expert systems, pattern recognition, and machine intelligence.

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MULTIVARIATE ANALYSIS AND ITS APPLICATIONS
Final Report Under the Contract F49260-85-0008

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P.R. Krishnaiah and C.R. Rao
Principal Investigators



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This is the final report on the research work done during the 3-year period October 1, 1984 to September 30, 1987 under the contract F49260-85-0008 sponsored by the Air Force Office of Scientific Research.

Research work was carried out by the principal investigators, P.R. Krishnaiah and C.R. Rao and visitors to the Center for Multivariate Analysis.

Unfortunately, Professor Krishnaiah passed away on August 1, 1987.

During the three year period of the grant, altogether 160 technical reports were issued; of these 48 have already been published, 38 have been accepted for publication and are due to appear and 74 have been submitted for publication and are under consideration.

This report is divided into five parts as follows.

	Page
Part 1: Some High Lights of the Research Work Done	4
Part 2: List of Technical Reports and Abstracts	7
Part 3: List of Technical Reports already Published	44
Part 4: List of Technical Reports Accepted for Publication and due to Appear	49
Part 5: List of Technical Reports Submitted for Publication.	52

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November 20, 1987

Part 1. SOME HIGH LIGHTS OF THE RESEARCH WORK DONE

During the 3-year period, some important contributions were made to several areas of multivariate analysis. They will be specially useful in solving some problems of interest to the Department of Defense. For example, they have applications in manufacturing technology, signal processing, automation, expert systems, pattern recognition, machine intelligence etc.. A brief description of some of the significant contributions is given below.

1.1 SIGNAL PROCESSING

When a message is received, it may be pure noise or it may be a specified or an unknown signal mixed with noise. The problems that arise are detection and extraction of signals from a received message. Satisfactory solutions are obtained in the following situations.

1. A test is developed to decide whether a received message contains a specified signal (including the case of no signal) when the noise distribution is Gaussian, elliptically symmetric and so on. The test is based on an estimated covariance matrix for noise, and its power depends on the degrees of freedom of the estimated covariance matrix. Some guide lines are provided for the choice of degrees of freedom to ensure a desired level for the power for a specified signal.
2. A model for a received message which has been extensively studied is of the form $x(t) = A s(t) + n(t)$ where $n(t)$ is $p \times 1$ noise vector, $s(t)$ is $q \times 1$ signal vector and $A = [A(\phi_1), \dots, A(\phi_q)]$ is $p \times q$ matrix whose columns are functions of unknown parameters. Under the above model, methods are developed for estimating the number of signals and also the signal parameters. The problem of estimation of the number of signals is treated within the framework of model selection procedures using some new information theoretic criteria. The problems are solved under white and colored noise distributions for the errors. The new methods are more general and flexible than those that are currently in use.
3. Another model in the area of signal processing is of the form
$$x(t) = a_1 \exp(i\omega_1 t) + \dots + a_q \exp(i\omega_q t) + n(t), \quad t=1, \dots, N$$
involving multiple sinusoids. The parameters a_i , ω_i and q , the number of signals, are unknown. The usual non-linear least squares theory and tests involve heavy computations. A new method of estimation called EVLP (equivariation linear prediction) has been proposed to estimate the number of signals, amplitudes and frequencies of signals. The estimates obtained by EVLP have nice properties, are easy to compute and are more efficient than some of the available methods.
4. In the same context as in (2) and (3) above, and in a more general situation

of superimposed exponential signals, some non-parametric procedures have been proposed to estimate the number of signals. They are based on general information theoretic considerations.

5. Alternative criteria based on Loo (leave one out) method have also been developed for the estimation of number of signals. This method is easy to implement and is useful in small samples.

1.2 DENSITY ESTIMATION

The discriminant function in a classification problem is based on the ratio of probability densities in two alternative populations. In practice one has to estimate these densities from observed samples from each population. In a parametric situation, the estimation of the density function is done through the estimation of the unknown parameters. In a non-parametric situation there are several methods available in the literature on density estimation in the univariate case. Special methods have been developed in the multivariate cases when we have binary data and multidimensional directional data. The latter is useful when we want to discriminate using only the information on shape (ratios of measurements).

1.3 CHANGE POINT PROBLEMS

Problems of detection of change points arise in quality control programs. For example, in continuous production it is important to find out when a deterioration in the quality of a product takes place. They also arise in problems of edge detection. Studies have been made on estimating the number and location of change points. The strong consistency of the proposed procedure is established.

1.4 ANALYSIS OF CONTINGENCY TABLES

The well known likelihood ratio and chi-square tests for independence of attributes in contingency tables are not efficient when the categories of each attribute are known to be ordinal in nature and the alternative hypothesis is specified to be a particular type of dependence.

Some extreme point methods have been introduced in analyzing problems of this type. These methods are useful in obtaining precise expressions for the size and power of alternative tests. As a consequence, it is much easier to select a test depending on a particular problem.

1.5 GROWTH CURVE MODELS

The problem of predicting a future measurement on an individual given the past measurements has been studied using a growth curve model for the measurements. The efficiencies of different methods of prediction, such as direct regression, inverse regression, extrapolating a growth curve fitted to an individual's past measurements, empirical Bayes, factor analytic regression etc., are examined by cross-validation or leave-one-out technique. A number of data reduction methods are developed and analyses based on them are discussed. A new method known as calibrated prediction is developed.

1.6 MULTIVARIATE CALIBRATION PROBLEMS

The problem of predicting response variables using explanatory variables is well known and this is accomplished by computing the regression of the former on the latter. But situations arise where it is of interest to determine the values of the explanatory variables knowing the values of the response variables. Such a setup is called calibration. The multivariate calibration problem can be described theoretically through the equation

$$\mathbf{y} = \mathbf{\alpha} + \mathbf{B}\mathbf{x} + \mathbf{\varepsilon}$$

where $\mathbf{y}$ is the vector of response variables, $\mathbf{x}$ is the vector of explanatory variables, $\mathbf{B}$ is the matrix of regression coefficients and $\mathbf{\alpha}$ is constant vector. Methods have been developed under the condition that $\mathbf{\varepsilon} \sim N_p(0, \Sigma)$, i.e., $\mathbf{\varepsilon}$ is distributed as p -variate normal with mean zero and covariance matrix Σ , for estimating $\mathbf{x}$ given $\mathbf{y}$ and for selecting the best subset of $\mathbf{y}$ for predicting $\mathbf{x}$.

1.7 TIME SERIES ANALYSIS

New methods have been developed to estimate the order (moving average and autoregressive terms) of an ARMA process based on time series data. The building of an ARMA model applicable to a given time series data is viewed as a search for a parsimoniously parameterized linear filter that transforms the observed time series into white noise, i.e., a sequence of uncorrelated random variables. Model detection criteria are used to test for white noise, which automatically assigns a significance level for the test, which depends on the sample size and the model detection criterion used such as the AIC, BIC and so on. Extensive simulation results have shown the superiority of the

proposed method over the existing ones. The method is applicable to both univariate and multivariate ARMA processes.

Part 2: LIST OF TECHNICAL REPORTS AND ABSTRACTS

All the Technical Reports were written with partial or complete support under contract F49620-85-0008 with the Air Force Office of Scientific Research during the period October 1, 1984 - September 30, 1987

1. Bai, Z.D., Limiting Properties of Large System of Random Linear Equations. Technical Report #84-41, Center for Multivariate Analysis, October 1984.

S. Geman and Chi R. Hwang proposed a kind of algebraic system of equations and proved the law of large numbers for its solution. In this paper, the conditions to ensure these results are significantly weakened for the law of large numbers. Also, the central limit theorem is shown. For both the law of large numbers and the central limit theorem, the only needed assumption is that the random variables have finite second moment.

2. Bai, Z. D., Yin, Y. Q., and Krishnaiah, P. R. On Limiting Empirical Distribution Function of the Eigenvalues of a Multivariate F Matrix. Technical Report #84-42, Center for Multivariate Analysis, October 1984.

In this paper, the authors derived an explicit expression for the limit of the empirical distribution function of a central multivariate F matrix when the number of variables and degrees of freedom tend to infinity in certain fashion. This distribution is useful in deriving the limiting distributions of certain test statistics which arise in multivariate analysis of variance, canonical correlation analysis and tests for the equality of two covariance matrices.

3. Bai, Z. D. The Strong Consistency of Error Probability Estimates in NN Discrimination. Technical Report #84-43, Center for Multivariate Analysis, October 1984.

Since 1971, T. J. Wagner introduced the error probability estimate $\hat{R}_n$ in NN discrimination rule and proved the weak convergency of this estimate. In this paper, we show the exponential convergence rate of this estimate. Hence, the strong consistency of $\hat{R}_n$ holds automatically.

4. Bai, Z. D., Krishnaiah, P. R., and Yin, Y. Q. On Limit of the Largest Eigenvalue of the Large Dimensional Sample Covariance Matrix. Technical Report #84-44, Center for Multivariate Analysis, October, 1984.

In this paper, the authors showed that the largest eigenvalue of the sample covariance matrix tends to a limit under certain conditions when both the number of variables and the sample size tend to infinity. The above result is proved under the mild restriction that the fourth moment of the elements of the sample sums of squares and cross products (SP) matrix exist.

5. Fang, Zhaoben. A Note about the Strong Convergence of the Nonparametric Estimation of a Regression Function. Technical Report #84-45, Center for Multivariate Analysis, September 1984.

Consider the regression model $Y_i^* = g(x_i^*) + e_i^*$, $i=1,2,\dots,n$. x_i^* 's are

unordered design variables, g unknown function defined on $[0,1]$. $\{e_i^*\}$ i.i.d r.v with mean 0 and finite moment of order $p > 1$. The asymptotic behavior of estimator g_n are studied.

6. Bai, Z. D., and Yin, Y. Q. Limiting Behavior of the Norm of Products of Random Matrices and Two Problems of Geman-Hwang. Technical Report #84-46, Center for Multivariate Analysis, November 1984.

Let $W_n = (w_{ij})$ $i, j = 1, 2, \dots, n$ be a matrix of iid entries with mean zero, variance σ^2 and finite fourth moment. The first result of this paper is that

$$\lim_{n \rightarrow \infty} \left\| \frac{W_n}{\sqrt{n}} \right\|^k \leq (1+k)\sigma^k, \text{ a.s. where } \|\bullet\| \text{ denotes the operator norm in } L_2.$$

As a consequence of this result we have that $\lim_{n \rightarrow \infty} SR\left(\frac{W_n}{\sqrt{n}}\right) \leq \sigma$, a.s., where

SR stands for spectral radius. Applying the first result, we solved two open problems of Geman and Hwang concerning the asymptotic normality of the solution to large systems of linear equations and that of differential equations.

7. Rao, C. Radhakrishna. The Inefficiency of Least Squares: Extensions of Kantorovich Inequality. Technical Report #84-47, Center for Multivariate Analysis, November 1984.

Four different measures of inefficiency of the simple least squares estimator in the general Gauss-Markoff model are considered. Previous work on the bounds to some of these measures is briefly reviewed and new bounds are obtained for a particular measure.

8. Bai, Z. D., Krishnaiah, P. R., and Yin, Y. Q. On Limiting Spectral Distribution of Product of Two Random Matrices when the Underlying Distribution is Isotropic. Technical Report #84-48, Center for Multivariate Analysis, November 1984.

Let $S = XX'$ be distributed independent of a nonnegative symmetric random matrix T , where $X = [x_1, \dots, x_n]$: $p \times n$ and $x_1, \dots, x_n$ is a sample from an isotropic population and the second moments of the norm $|x_i|$, ($i = 1, 2, \dots, n$), exist. In this paper, the authors proved that the limit of the spectral distribution of ST/n exists.

9. Bai, Z. D. A Note on Asymptotic Joint Distribution of the Eigenvalues of a Noncentral Multivariate F Matrix. Technical Report #84-49, Center for Multivariate Analysis, November 1984.

In P. L. Hsu (1941), the proof of the basic Lemma 3 is based on Lemma 1 which is wrong. The aim of this note is to correct the proof of Lemma 3, consequently, to ensure the main theorem in P. L. Hsu (1941).

10. Yin, Y. Q. Limiting Spectral Distribution for a Class of Random Matrices. Technical Report #84-50, Center for Multivariate Analysis, December 1984.

Let $X = \{X_{ij} : i, j = 1, 2, \dots\}$ be an infinite dimensional random matrix, T_p

be a $p \times p$ nonnegative definite random matrix independent of X , for $p = 1, 2, \dots$. Suppose $\frac{1}{p} \text{tr } T_p^k \rightarrow H_k$ a.s. as $p \rightarrow \infty$ for $k = 1, 2, \dots$, and $\sum H_{2k}^{-1/2k} = \infty$. Then the spectral distribution of

$$A_p = \frac{1}{n} X_p X_p^T T_p$$

where $X_p = [X_{ij} : i = 1, \dots, p; j = 1, \dots, n]$, tends to a nonrandom limit distribution as $p \rightarrow \infty, n \rightarrow \infty$ but $p/n \rightarrow y > 0$, under the mild conditions that X_{ij} are i.i.d. and $EX_{11}^2 > \infty$.

11. Stoffer, D. S. Bootstrapping the Kalman Filter. Technical Report #84-51, Center for Multivariate Analysis, December 1984.

The bootstrap is proposed as a method for estimating the precision of forecasts and estimates of parameters of the Kalman Filter model. It is shown that when the system and the filter is in steady state the bootstrap applied to the Gaussian innovations yields asymptotically consistent standard errors. That the bootstrap works well with moderate sample sizes and supplies robustness against departures from normality is substantiated by empirical evidence.

12. Burbea, Jacob. Informative Geometry of Probability Spaces. Technical Report #84-52, Center for Multivariate Analysis, December 1984.

The paper is concerned with the geometrical properties that are induced by the local information contents and structures of the parameter space of probability distributions. Of particular interest in this investigation is the Rao distance which is the geodesic distance induced by the differential metric associated with the Fisher information matrix of the parameter space. Moreover, following Efron, Dawid and Amari, some affine connections are introduced into the informative geometry of parameter space and thereby elucidating the role of the curvature in statistical studies. In addition, closed form expressions of the Rao distances for certain families of probability distributions are given and discussed.

13. Bai, Z. D., Krishnaiah, P. R., and Liang, W. Q. On Asymptotic Joint Distribution of the Eigenvalues of the Noncentral MANOVA Matrix for Nonnormal Populations. Technical Report #84-53, Center for Multivariate Analysis, December 1984.

In this paper, the authors derived asymptotic joint distribution of the eigenvalues of the multivariate analysis of variance matrix in the noncentral case when the underlying distribution is not multivariate normal.

14. Giri, N., and Sinha, B. K. Robust Tests of Mean Vector in Symmetrical Multivariate Distributions. Technical Report #85-01, Center for Multivariate Analysis, January 1985.

Let $X = (X_{11}, \dots, X_{1p}, \dots, X_{n1}, \dots, X_{np})' = (X_{1j}, \dots, X_{nj})'$ be a $n \times p$ random matrix with probability density function

$$f_X(x) = \sum_{i=1}^m c_i^2 q_i(x \Sigma_i^{-1}(x - \xi_i \mu)'(x - \xi_i \mu))$$

where $x \in \chi = \{x: n \times p \text{ matrix} \mid \text{rank of } x = p\}$, $\mu = (\mu_1, \dots, \mu_p)' \in R^p$, $e = (1, \dots, 1)'$ n -vector and $\Sigma > 0$ (positive definite). Assume that $q \in Q = \{q: [0, \infty), \rightarrow [0, \infty), \text{convex}\}$ and $n > p$ so that $X'X > 0$ with probability one. It is proved that for testing $H_0: \mu = Q$ versus the alternative $H_1: \mu \neq Q$, the Hotelling's T^2 -test is locally minimax and for testing $H_0: \mu_{(1)} = Q$ versus the alternative $H_1: \mu_{(1)} \neq Q$, the appropriate Hotelling's T^2 -test is both UMPI and locally minimax. In the second case $\mu_{(1)} = (\mu_1, \dots, \mu_{p_1})'$, $p_1 < p$, and $(\mu_{p_1+1}, \dots, \mu_p)$, Σ are unknown. The above results generalize those of Giri and Kiefer (Ann. Math. Statist., 1964) under the assumption $q \sim \text{normal}$. As a technical tool, Wijsman's representation theorem is used.

15. Bueno Vanderlei C. A Model in Negative Dependence Using Stochastic Order. Technical Report #85-02, Center for Multivariate Analysis, February 1985.

A model gives rise to the NDS conditions from Block, Savits and Shaked. The main theorem is: Let $X_1, \dots, X_n$ be independent and identically distributed random variables with continuous distribution F . Let

$$(Y_1, \dots, Y_n) = [(X_1, \dots, X_n) \mid X_{(k)} \leq z]$$

Then $(Y_1, \dots, Y_n)$ is Negative Dependent through Stochastic Order (NDS).

The NDS condition implies the NUOD and NLOD condition of negative dependence that enable us to get bounds in joint probabilities.

The conditioning event considered occurs naturally in Reliability Theory as the time to system failure for k -out-of- n system.

The conditioning in $\{g(X_{(k)}) \leq \mu\}$ where g is increasing and left continuous function, also define a NDS vector. However conditioning in a general coherent system doesn't rise a NDS vector.

16. Khattree, Ravindra, and Yin, Y. Q. An Inequality and Its Application to the Truncated Distributions. Technical Report #85-03, Center for Multivariate Analysis, February 1985.

An inequality is proved and its interpretation is given. Using the inequality, it is shown, under some mild conditions, that for the univariate truncated distributions, the variance of the truncated distribution increases with the value of the truncation point.

17. Das, Rita and Sinha, Bimal K. Detection of Multivariate Outliers with Dispersion Slippage in Elliptically Symmetric Distributions. Technical Report #85-04, Center for Multivariate Analysis, March 1985.

An extension of Ferguson's (Fourth Berkeley Symposium on Probability and Mathematical Statistics, 1961, Volume 1) univariate normal results for detection of outliers with variance slippage is made to the multivariate elliptically symmetric case with dispersion slippage. The locally optimum test statistic we derive possesses all three robustness properties: optimality, null and nonnull (Kariya and Sinha, Ann. Statist., 1985). As a technical

tool, Wijsman's representation theorem (Fifth Berkeley Symposium on Probability and Mathematical Statistics, 1967, Volume 1) is used.

18. Rao, C. Radhakrishna. Evolution of Data Collection Censuses, Sample Surveys, and Design of Experiments. Technical Report #85-05, Center for Multivariate Analysis, March 1985.

Statistics as we understand and practice it today has a short history but a long antiquity. Originally the word statistics meant data or information relating to a State, and gradually it acquired a wider meaning to cover numerical data in any field of enquiry and also analyses performed on data to draw inferences and to take decisions. In early days, attempts were made to make a complete enumeration when information on people and resources of a country was required. This was called Census. But with increase in demand for extensive and timely data by governments, individuals and private enterprises, the method of sample surveys involving the collection of data only on a small subset of the population and making estimates for the whole population was developed. This introduced flexibility, speed and economy in acquiring information of sufficient accuracy for practical purposes. Simultaneously, methods of generating data through well designed experiments was developed with the twin objectives of providing valid estimates of differences between treatments with maximum precision for given cost. Design of experiments also introduced great simplicity in the analysis and interpretation of data. Although census, sample surveys and design of experiments were introduced for separate purposes, now-a-days investigations in all fields of enquiry use a combination of the principles of all these three methods of data collection. New developments in data collection are computer assisted telephone enquiry (CATI) and use of satellites in taking pictures of earth for estimating average and yield of crops through pattern recognition techniques.

19. Mathew, Thomas. On the Characterization of Nonnegatively Estimable Linear Combinations of Variance Components. Technical Report #85-06, Center for Multivariate Analysis, March 1985.

It is shown that, by a reparametrization, the problem of estimating a linear combination of variance components can be reduced to that of estimating a single variance component. Such a reduction is used to obtain some characterizations of nonnegatively estimable linear combinations of variance components. Characterization of nonnegative estimability using MINQUE is also discussed.

20. Langberg, Naftali A., Stoffer, David S. Moving Average Models with Bivariate Exponential and Geometric Distributions. Technical Report #85-07, Center for Multivariate Analysis, March 1985.

Two classes of finite and infinite moving average sequences of bivariate random vectors are considered. The first class has bivariate exponential marginals while the second class has bivariate geometric marginals. The theory of positive dependence is used to show that in various cases the two classes consist of associated random variables. Association is then applied to establish moment inequalities and to obtain approximations to some joint probabilities of the related bivariate point processes.

21. Rao, C. Radhakrishna. Differential Metrics in Probability Spaces Based on Entropy and Divergence Measures. Technical Report #85-08, Center for Multivariate Analysis, April 1985.

In this paper we discuss some general methods of metrizing probability spaces through the introduction of a quadratic differential metric in the parameter manifold of a set of probability distributions. These methods extend the investigation made in Rao (1945) where the Fisher information matrix was used to construct the metric, and the geodesic distance was suggested as a measure of dissimilarity between probability distributions.

The basic approach in the present paper is first to construct a divergence or a dissimilarity measure between any two probability distributions, and use it to derive a differential metric by considering two distributions whose characterizing parameters are close to each other. One measure of divergence considered is the Jensen difference based on an entropy functional as defined in Rao (1982a). Another is the f-divergence measure studied by Csiszar (1967). The latter class leads to the differential metric based on the Fisher information matrix. The geodesic distances based on this metric computed by various authors are listed.

22. Krishnaiah, P. R., and Sarkar, Shakuntala. Principal Component Analysis Under Correlated Multivariate Regression Equations Model. Technical Report #85-09, Center for Multivariate Analysis, April 1985.

In this paper, the authors consider the problem of testing for the equality of the last few eigenvalues of the covariance matrix under correlated multivariate regression equations (CMRE) model. Asymptotic distributions of various test statistics are derived when the underlying distribution is multivariate normal. Some of the distribution theory is extended to the case when the underlying distribution is elliptically symmetric.

23. Lau, Ka-Sing, and Gu, Hua-Min. A Note on an Integrated Cauchy Functional Equation. Technical Report #85-10, Center for Multivariate Analysis, April 1985.

In characterizing the semistable law, Shimizu reduced the problem into solving the equation

$$H(x) = \int_0^\infty H(x+y)d(\mu-\nu)(y), \quad x \geq 0$$

where μ and ν are given positive measures on $[0, \infty)$. In this note, we obtain a simple proof and show that some of his conditions can be weakened.

24. Alzaid, Abdulhamid A., Rao, C. Radhakrishna and Shanbhag, D. N. An Extension of Spitzer's Integral Representation Theorem with an Application. Technical Report #85-11, Center for Multivariate Analysis.

Using a new approach, an extended version of Spitzer's integral representation for stationary measures of a discrete branching process is obtained. This result is used to provide a complete solution to a problem in damage models satisfying a generalized Rao-Rubin condition.

25. Singh, Radhey S. and Ahmad, Manzoor. Modified Nonparametric Kernel

Estimates of a Regression Function and Their Consistencies with Rates. Technical Report #85-12, Center for Multivariate Analysis, April 1985.

Two sets of modified kernel estimates of a regression function are proposed: one when a bound on the regression function is known and the other when nothing of this sort is at hand. Explicit bounds on the mean square errors of the estimators are obtained. Pointwise as well as uniform consistency in mean square and consistency in probability of the estimators are proved. Speed of convergence in each case is investigated.

26. Alzaid, Abdulhamid A., Rao, C. Radhakrishna and Shanbhag, D. N. An Application of the Perron-Frobenius Theorem to a Damage Model Problem. Technical Report #85-13, Center for Multivariate Analysis, April 1985.

Using the Perron-Frobenius theorem, it is established that if (X,Y) is a random vector of non-negative integer valued components such that $Y \leq X$ almost surely and two modified Rao-Rubin conditions hold, then under some mild assumptions the distribution of (X,Y) is uniquely determined by the conditional distribution of Y given X . This result extends the recent unpublished work of Shanbhag and Taillie (1979) on damage models.

27. Krishnaiah, P. R., Lin, J. and Wang, L. Inference on the Ranks of the Canonical Correlation Matrices for Elliptically Symmetric Populations. Technical Report #85-14, Center for Multivariate Analysis, April 1985.

In this paper, the authors considered the likelihood ratio tests and some other tests for the ranks of the canonical correlation matrices when the underlying distributions are real and complex elliptically symmetric distributions. Also asymptotic joint distributions of the eigenvalues of the sample canonical correlation matrices are derived under the assumptions mentioned above regarding the underlying distributions. Finally, applications of tests for the rank of the complex canonical correlation matrix in the area of time series in the frequency domain are discussed.

28. Bai, Z. D. Exponential Bound for Error Probability in NN-Discrimination. Technical Report #85-15, Center for Multivariate Analysis, April 1985.

Let $(\theta_1, X_1), \dots, (\theta_n, X_n)$ be n simple samples drawn from the population (θ, X) which is a $\{1, 2, \dots, s\} \times \mathbb{R}^d$ -valued random vector. Suppose that X is known. Let X'_n be the nearest one among $X_1, \dots, X_n$, from X , in the sense of Euclidean norm or l_∞ -norm, and let θ_n be the θ -value paired with X'_n . The posterior error probability is defined by $L_n = P(\theta'_n \neq \theta | \{(\theta_1, X_1), \dots, (\theta_n, X_n)\})$. It is well known that $EL_n = P(\theta'_n \neq \theta)$ has always a limit R . In this paper it is shown that for any $\epsilon > 0$ there exists constant $C < \infty$ and $b > 0$ such that $P(|L_n - R| \geq \epsilon) \leq ce^{-bn}$, under the only assumption that the marginal distribution of X is nonatomic.

29. Alzaid, Abdulhamid A., Rao, C. Radhakrishna, and Shanbhag, D. N. Characterization of Discrete Probability Distributions by Partial Independence. Technical Report #85-16, Center for Multivariate Analysis, May 1985.

If X and Y are random variables such that $P(X \geq Y) = 1$ and the conditional

distribution of Y given X is binomial, then Moran (1952) showed that Y and $(X-Y)$ are independent if X is Poisson. We extend Moran's result to a more general type of conditional distribution of Y given X , using only partial independence of Y and $X-Y$. This provides a generalization of a recent result of Janardhan and Rao (1982) on the characterization of generalized Polya-Eggenberger distribution. A variant of Moran's theorem is proved which generalizes the results of Patil and Seshadri (1964) on the characterization of the distribution of a random variable X based on some conditions on the conditional distribution of Y given X and the independence of Y and $X-Y$.

30. Yin, Y. Q. and Bai, Z. D. Spectra for Large Dimensional Random Matrices. Technical Report #85-17, Center for Multivariate Analysis, May 1985.

In this paper, the authors reviewed some recent developments in the area of large dimensional random matrices.

31. Hanumara, R. Choudary. An Alternate Derivation of the Distribution of the Conditioned Signal-To-Noise Ratio. Technical Report #85-18, Center for Multivariate Analysis, May 1985.

A result in multiple regression analysis is used to derive the probability distribution of the conditioned signal-to-noise ratio.

32. Alzaid, Abdulhamid A., Rao, C. Radhakrishna and Shanbhag, D. N. A Note on the Integrated Cauchy Functional Equation. Technical Report #85-19, Center for Multivariate Analysis, May 1985.

New proof is given using de Finetti's theorem on infinite sequence of exchangeable random variables for the solution of the integrated Cauchy functional equation studied by Lau and Rao (1982) in the continuous case and by Shanbhag (1977) in the discrete case.

33. Fang, C., and Krishnaiah, P. R. On Asymptotic Distribution of the Test Statistic for the Mean of the Non-Isotropic Principal Component. Technical Report #85-20, Center for Multivariate Analysis, May 1985.

In this paper, the authors derived the large sample distribution of the t statistic based upon the observations on the first principal component instead of the original variables. It is shown that the above statistic is distributed asymptotically as Student's t distribution.

34. Ahmad, Manzoor, and Singh, Radhey S. Role of Auxiliary Variate and Additional Data in Density Estimation. Technical Report #85-21, Center for Multivariate Analysis, May 1985.

Some new estimators of a univariate p.d.f. $f(y)$ of a random variable Y , based on a set of observations taken from a bivariate joint density $\beta(x,y)$ of Y and a suitably chosen concomitant variable X , have been investigated. Asymptotic unbiasedness, mean square consistency, asymptotic normality and rates of convergence have been established. A related problem of estimation of a conditional density has also been studied.

35. Alzaid, Abdulhamid A., Rao, C. Radhakrishna, and Shanbhag, D. N. A Variant of Shanbhag's Lemma Arising out of a Modified Rao-Rubin

Condition. Technical Report #85-22, Center for Multivariate Analysis, May 1985.

A modified Rao-Rubin condition for damage models gives rise to a recurrence relation which is somewhat different from that considered by Shanbhag (1977). A complete solution to the new recurrence relation is obtained and its applications are indicated.

36. Ahmad Manzoor. New Measures of Diversity. Technical Report #85-23, Center for Multivariate Analysis, June 1985.

The problem of measuring diversity within populations and dissimilarity or similarity between populations has been extensively treated in the literature. In this context a general procedure called Analysis of Diversity has been outlined and examined by C. R. Rao in a series of papers.

In this paper we propose three new measures of diversity and study related inference problems. Denote by S^k the simplex $S^k = \{\pi = (\pi_1, \dots, \pi_k)', \pi_j \geq 0, \sum \pi_j = 1\}$. Then the proposed measures are of the form:

$H_m(\pi) = 1 - a_m \sum_j \pi_j \phi_m(\pi_j)$, $m=1,2,3$ where $\phi_1(x) = (1+k^{-1}x)^{-\gamma}$, $\gamma \geq 0$, $\phi_2(x) = (2-k^{-1}x)^{-1}$, $\gamma \geq 0$, $\phi_3(x) = (a_3 + (1-x)^{-1})^{-1}$, $0 < \gamma \leq 1$, and the a 's are suitable normalizing constants. Estimation of $H_m(\pi)$, derivation of the penalty function and cross entropy and the problem of testing independence have been treated. Asymptotic distributions of relevant test statistics are indicated.

37. Ahmad, Manzoor, and Singh, R. S. Double Sampling in Estimation of A Ratio. Technical Report #85-24, Center for Multivariate Analysis, June 1985.

Problems connected with the estimation of the ratio of the means of a finite bivariate population have been considered in this report. The usual estimator of the ratio, based on the means of a bivariate simple random sample drawn without replacement (s.r.s. (w.o.r.)), has been compared with estimators based on alternative double sampling design. Under this design a very large s.r.s. (w.o.r.) is drawn for measuring only one of the variables and a subsample (s.r.s.(w.o.r.)) is drawn out of the first phase units for measuring the other variable. Efficiency and bias comparisons have been made by subjecting each of the competitors to the same budgetary constraint. It turns out that deviation from the usual set-up sometimes leads to better sampling 'strategies'.

38. Mathew, Thomas. Hermitian and Nonnegativity Preserving Subspaces. Technical Report #85-25, Center for Multivariate Analysis, July 1985.

The concepts of hermitian preserving and nonnegativity preserving subspaces of complex square matrices are introduced. Characterizations of such subspaces are obtained and applications are discussed.

39. Baiqi, Miao, and Lincheng, Zhao. Non-uniform Bounds of Normal Approximation for Finite-population U-statistics. Technical Report #85-26, Center for Multivariate Analysis, July 1985.

Let A_N be a population with N balls bearing numbers $a_{N1}, \dots, a_{NN}$ respectively. Draw n balls from A_N randomly without replacement, and denote the numbers appearing on these n balls by $X_1, \dots, X_n$. Suppose that $\phi_N(x, y)$ be a Borel-measurable function, symmetric in x and y . Set

$$U_n = \binom{n}{2}^{-1} \sum_{1 \leq j < k \leq n} \phi_N(X_j, X_k), \quad \theta_N = E\phi_N(X_1, X_2),$$

$$g(X_1) = E[\phi_N(X_1, X_2) | X_1]$$

$\sigma_g^2 = \text{Var}\{g(X_1)\}$. In this paper we established that, if there exists fixed constants λ_1 and λ_2 such that $0 < \lambda_1 \leq n/N < \lambda_2 < 1$, then it is valid for all positive integer n and real x that

$$|P\left(\frac{\sqrt{N}(U_n - \theta_N)}{2\sqrt{1-n/N}\sigma_g} \leq x\right) - \Phi(x)| \leq Cn^{-1/2}\sigma_g^{-3}E|\phi_N(X_1, X_2)|^3(1+|x|)^{-3}$$

where $\Phi(x)$ is the standard normal distribution function, and C is an absolute constant depending solely on λ_1 and λ_2 .

40. Bimal K. Sinha and Malay Ghosh. Inadmissibility of the Best Equivariant Estimators of the Variance-Covariance Matrix and the Generalized Variance Under Entropy Loss Technical Report No. 85-27, Center for Multivariate Analysis, July 1985.

Based on a data matrix $X = (X_1, \dots, X_k)$: $p \times k$ with independent columns

$X_i \sim N_p(\xi_i, \Sigma)$, and an independent Wishart matrix S : $p \times p \sim W_p(n, \Sigma)$, estimators dominating the best equivariant estimators of Σ and $|\Sigma|$ are obtained under two types of entropy loss. For simultaneous estimation of the mean vector and the variance covariance matrix of a multinormal population, a suitable entropy loss is developed and testimators dominating the pair consisting of the sample mean vector and the best multiple of the sample Wishart matrix are derived. A technique of Sinha (Jour. Mult. Analysis, 1976) is heavily exploited.

41. Demetrios G. Kaffes, M. Bhaskara Rao, Thomas Mathew and K. Subramanyam. On The Matrix Convexity of the Moore-Penrose Inverse and Some Applications. Technical Report No. 85-28, Center for Multivariate Analysis, July 1985.

It is well known that if A and B are two positive definite matrices of the same order and $0 \leq \lambda \leq 1$, then

$$[\lambda A + (1-\lambda)B]^{-1} \leq \lambda A^{-1} + (1-\lambda)B^{-1}$$

It is easy to construct an example consisting of two positive semi-definite matrices for which the above inequality

$$[\lambda A + (1-\lambda)B]^+ \leq \lambda A^+ + (1-\lambda)B^+$$

for every $0 \leq \lambda \leq 1$. As an application, we give a sufficient condition under

which the inequality $(EA)^+ \leq E(A^+)$ is valid, where A is a square matrix of random variables which is almost surely positive semi-definite, generalizing the well-known result $(EA)^{-1} \leq EA^{-1}$ when A is almost surely positive definite.

42. Bai, Z. D., and Liang, W. Q. Strong Representation of Weak Convergence. Technical Report No. 85-29. Center for Multivariate Analysis, July 1985.

Let μ_n , $n = 1, 2, \dots$, and μ be a given sequence of probability measures each of which is defined on a complete separable metric space S_n and S respectively. Also, a sequence of measurable mappings ϕ_n from S_n into S is given. In this paper, it is proved that if $\mu_n \circ \phi_n^{-1}$ weakly converge to μ , then there is a probability space $(\Omega, \mathcal{F}, P)$, on which we can define a sequence of random elements X_n , from Ω into S_n , and a random element X , from Ω into S , such that μ_n is the distribution of X_n , μ is the distribution of X and $\lim_{n \rightarrow \infty} \Phi_n(X_n) = X$ pointwise. The result of Skorokhod (1956) is a special case of the result of this paper. Some applications in the area of random matrices, etc., are also given.

43. Lin-Cheng Zhao. An Inequality Concerning the Deviation Between Theoretical and Empirical Distributions. Technical Report No. 85-30, Center for Multivariate Analysis, August 1985.

In this paper the author established an inequality concerning the uniform deviation between theoretical and empirical distributions. An application in strong convergence of nearest neighbor density estimate is also discussed.

44. Fujikoshi, Y., Krishnaiah, P. R., and Schmidhammer, J. Effect of Additional Variables in Principal Component Analysis, Discriminant Analysis and Canonical Correlation Analysis. Technical Report No. 85-31, Center for Multivariate Analysis, August 1985.

In this paper, the authors derived asymptotic distributions of changes in certain functions of the eigenvalues of the sample covariance matrix, MANOVA matrix and canonical correlation matrix when some variables are added to the original sets of variables. The above results are useful in finding out as to whether the new variables give additional information.

45. Takeaki Kariya, and Bimal K. Sinha. Optimality Robustness of Tests in Two population Problems. Technical Report No. 85-32, Center For Multivariate Analysis, August 1985.

The problems of testing the equality of two location parameters (without the presence of scale parameters) and the equality of two scale parameters (with or without the presence of location parameters) based on two independent samples are considered. It is shown that, under some mild conditions on the underlying distributions, the standard optimum invariant tests derived under normality or exponentially continue to be optimum for a wide class of distributions. However, the tests are not null robust. As a technical tool, Waisman's (Fifth Berkeley Symposium, Vol. 1, 1967) representation theorem is used.

46. Kariya, T, Fujikoshi, Y. and Krishnaiah, P. R. On Tests for Selection of Variables and Independence Under Multivariate Regression Model. Technical Report No. 85- 33, Center for Multivariate Analysis, August 1985.

In this paper, the authors consider various procedures for testing the hypotheses of independence of two sets of variables and certain regression coefficients are zero under the classical multivariate regression model. Various properties of these procedures and the asymptotic distributions associated with these procedures are also considered.

47. Rao, Radhakrishna C., and Rao C. Veerendra. Computation of the Stationary Values of the Product of Two Raleigh Quotients. Technical Report No. 85-34, Center for Multivariate Analysis, September 1985.

A computational algorithm is developed for finding the stationary values of the function $x'Cx/(x'Ax)^{1/2}(x'Bx)^{1/2}$ where A and B are positive definite and C is a symmetric matrix. The square of the function under consideration is the product of two Raleigh coefficients $x'Cx/x'Ax$ and $x'Cx/x'Bx$. The general problem occurs in multivariate analysis in the computation of homologous canonical variates in studying relationships between two sets of homologous measurements. The special case with $C = I$ occurs in designing control systems with minimum norm feedback matrices.

48. Rosenblatt-Roth, M. The Relative Entropy of a Random Vector with Respect to Another Random Vector. Technical Report No. 85-35, Center for Multivariate Analysis October 1985.

In this paper, the author discusses some problems connected with the concept of entropy. Part of the paper is expository in nature and the remaining material deals with new results obtained by the author.

49. Krishnaiah, P. R., Lin, J., and Wang, L. Tests for the Dimensionality of the Regression Matrices when the Underlying Distributions are Elliptically Symmetric. Technical Report No. 85-36, Center for Multivariate Analysis, October 1985.

In this paper, the authors derive likelihood ratio tests for the dimensionality of the regression matrices for the cases when the joint distributions of the observations are real and complex elliptically symmetric. The authors also derive asymptotic distributions of the above test statistics for two situations. In the first situation, the joint distribution of the observations is elliptically symmetric whereas the second situation assumes that the observations are distributed independently as elliptically symmetric.

50. Zhao, L. C., Krishnaiah, P. R., and Bai, Z. D. On Detection of Number of Signals in Presence of White Noise. Technical Report No. 85-37, Center for Multivariate Analysis, October 1985.

In this paper, the authors propose procedures for the detection of the number of signals in presence of Gaussian white noise. The methods used fall within the framework of the model selection procedures using an information theoretic criterion. The strong consistency of the estimates of the number of signals, under different situations, is established. Extensions

of the results are also discussed to the case when the noise is not necessarily Gaussian.

51. Khatri, C. G., and Rao, C. Radhakrishna. Effects of Estimated Noise Covariance Matrix in Optimal Signal Detection. Technical Report No. 85-38, Center for Multivariate Analysis, October 1985.

There is loss of efficiency when an estimated noise covariance matrix is used in the place of the unknown true noise covariance matrix in the construction of the optimum filter for signal detection. In the case of detecting a single signal specified by a real or a complex vector, we investigate the extent of this loss by obtaining an exact confidence bound for the realized signal to noise ratio. We also give an estimate of this ratio which is useful in optimum selection of features. Some of these results are extended to the case of discrimination between a number of given signals.

52. Zhao, L. C., and Krishnaiah, P. R., and Bai, Z. D. On Detection of the Number of Signals When the Noise Covariance Matrix is Arbitrary. Technical Report No. 85-39, Center for Multivariate Analysis, October 1985.

In this paper, the authors proposed model selection methods for determination of the number of signals in presence of noise with arbitrary covariance matrix. This problem is related to finding the multiplicity of the smallest eigenvalue of $\Sigma_2 \Sigma_1^{-1}$, where $\Sigma_2 = \Gamma + \lambda \Sigma_1$, Σ_1 and Σ_2 are covariance matrices, λ is a scalar, and Γ is non-negative definite matrix and is not of full rank. Also, the authors proposed methods for determination of the multiplicities of various eigenvalues of $\Sigma_2 \Sigma_1^{-1}$. The methods used in these procedures are based upon certain information theoretic criteria. The strong consistency of these criteria is established in this paper.

53. Khatri, C. G., Rao, C. Radhakrishna, and Sun, Y. N. Tables for Obtaining Confidence Bounds for Realized Signal to Noise Ratio with An Estimated Discriminant Function. Technical Report No. 85-40, Center for Multivariate Analysis, November 1985.

Percentage points of a new distribution involving a confluent-hypergeometric distribution obtained by Khatri and Rao (1985) are tabulated. The use of the tabulated values in obtaining a lower confidence bound for the realized signal to noise ratio based on an estimated discriminant function for signal detection is explained.

54. Dailami, N., Subramanyam, K. and Rao, M. Bhaskara. Selecting the Best Population: A Decision Theoretic Approach: The Case of Pareto Distribution. Technical Report No. 85-41, Center for Multivariate Analysis, November 1985.

A decision theoretic approach is followed in selecting the best of k Pareto populations taking into account the cost of sampling and penalties for wrong decisions. Minimax sample sizes are determined under various types of penalty functions.

55. Dailami, N., Rao M. Bhaskara, and Subramanyam, K. On the Selection of

Best Gamma Population: Determination of Minimax Sample Sizes. Technical Report No. 85-42, Center for Multivariate Analysis, November 1985.

Selecting the best Gamma population from a given set of Gamma populations is treated from a decision theoretic point of view. Cost of sampling and penalties for wrong decision play a role in the determination of optimum common sample sizes. Minimax sample sizes are determined under two different penalty functions.

56. Zhao, L. C., Krishnaiah, P. R., and Bai, Z. D. Remarks on Certain Criteria for Detection of Number of Signals. Technical Report No. 85-43, Center for Multivariate Analysis, November 1985.

In this note, we derive the asymptotic distribution of logarithm of the likelihood ratio statistic for testing the hypothesis that the number of signals is equal to q against the alternative that it is equal to k (specified) for a special case. This distribution is not chi-square. The above statistic also arises (see Wax and Kailath (1985)) in studying consistency property of MDL and AIC criteria for detection of the number of signals.

57. Zhao, L. C. Exponential Bounds of Mean Error for the Nearest Neighbor Estimates of Regression Functions. Technical Report No. 85-44, Center for Multivariate Analysis, November 1985.

Let $(X, Y), (X_1, Y_1), \dots, (X_n, Y_n)$ be i.i.d. $R^r \times R$ -valued random vectors with $E|Y| < \infty$, and let $m_n(x)$ be a nearest neighbor estimate of the regression function $m(x) = E(Y|X=x)$. In this paper, we establish an exponential bound of the mean deviation between $m_n(x)$ and $m(x)$ given the training sample $Z^n = (X_1, Y_1, \dots, X_n, Y_n)$ under the conditions as weak as possible. This is a substantial improvement on Beck's Result.

58. Bai, Z. D., Krishnaiah, P. R., and Zhao, L. C. On Rates of Convergence of Efficient Detection Criteria in Signal Processing with With Noise. Technical Report No.85- 45, Center for Multivariate Analysis, November 1985.

Zhao, Krishnaiah and Bai[6] proposed certain information theoretic criteria for detection of the number of signals under an additive model with white noise when the noise variance is known or unknown. They showed that the above criteria are strongly consistent even when the underlying distribution is not necessarily Gaussian. In this paper the authors obtained upper bounds on the probabilities of correct detection.

59. Stoffer, David S. Walsh-Fourier Analysis of Discrete-Valued Time Series. Technical Report No. 85-46, Center for Multivariate Analysis, November 1985.

An approach to the analyses of discrete-valued time series is discussed. The analyses are accomplished in the spectral domain using the Walsh-Fourier transform which is based on Walsh functions. This approach will enable an investigator of discrete systems to analyze the data in terms of square-waveforms and sequency rather than sine-waves and frequency. We

develop a general signal-plus-noise type model for discrete-valued time series in which Walsh-Fourier spectral analysis is of interest. We consider the problems of detecting whether or not a common signal exists in repeated measures on discrete-valued time series and in discrete-valued processes collected in an experimental design. We show that these models may depend on unknown regression parameters and we develop consistent estimates of these parameters based on the finite Walsh-Fourier transform. Applications to certain Markov models are given, however, the methods presented also apply to non-Markov cases.

60. Khatri, C. G., and Rao, C. Radhakrishna. Test for a Specified Signal When The Noise Covariance Matrix is Unknown. Technical Report No.85-47, Center for Multivariate Analysis, November 1985.

In the univariate case it is well known that the one sided t test is uniformly most powerful for the null hypothesis against all one sided alternatives. Such a property does not easily extend to the multivariate case. In this paper, a test is derived for the hypothesis that the mean of a vector random variable is zero against specified alternatives, when the covariance matrix is unknown. The test depends on the given alternatives and is more powerful than Hotelling's T^2 . The results are derived both for real and complex vector observations and under normal and spherical distributions. The properties of the proposed tests are investigated in detail when a single alternative is specified.

61. Rao, M. Bhaskara, Krishnaiah, P. R., and Subramanyam, K. A Structure Theorem on Bivariate Positive Quadrant Dependent Distributions and Tests for Independence in Two-Way Contingency Tables. Technical Report No. 85-48, Center for Multivariate Analysis, December 1985.

In this paper, the set of all bivariate positive quadrant dependent distributions with fixed marginals is shown to be compact and convex. Extreme points of this convex set are enumerated in some specific examples. Applications are given in testing the hypothesis of independence against strict positive quadrant dependence in the context of ordinal contingency tables. Various procedures based upon certain functions of the eigenvalues of a random matrix are also proposed for testing for independence in two-way contingency table. The performance of some tests one of which is based on eigenvalues of a random matrix is compared.

62. Taniguchi, Masanobu. Validity of Edgeworth Expansions of Minimum Contrast Estimators for Gaussian ARMA Processes. Technical Report No. 85-49, Center for Multivariate Analysis, December 1985.

Let $\{X_t\}$ be a Gaussian ARMA process with spectral density $f_\theta(\lambda)$, where θ is an unknown parameter. To estimate θ we propose a minimum contrast estimation method which includes the maximum likelihood method and the quasi-maximum likelihood method as special cases. Let $\hat{\theta}_n$ be the minimum contrast estimator of θ . Then we derive the Edgeworth expansion of the distribution of $\hat{\theta}_n$ up to third order, and prove its validity. By this Edgeworth expansion we can see that this minimum contrast estimator is always second-order asymptotically efficient in the class of second-order

asymptotically median unbiased estimators. Also the third-order asymptotic comparisons among minimum contrast estimators will be discussed.

63. Zhao, L. C. Exponential Bounds of Mean Error for the Kernel Estimates of Regression Functions. Technical Report No. 85-50, Center for Multivariate Analysis, December 1985.

Let $(X, Y), (X_1, Y_1), \dots, (X_n, Y_n)$ be i.i.d. $R^r \times R$ -valued random vectors with $E|Y| \leq \infty$, and let $Q_n(x)$ be a kernel estimate of the regression function $Q(x) = E(Y|X = x)$. In this paper, we establish an exponential bound of the mean deviation between $Q_n(x)$ and $Q(x)$ given the training sample $Z^n = (X_1, Y_1, \dots, X_n, Y_n)$, under the conditions as weak as possible.

64. Rao, M. Bhaskara, Krishnaiah, P. R., and Subramanyam, K. Extreme Point Method in The Determination of the Structure of a Class of Bivariate Distributions and Some Applications to Contingency Tables. Technical Report No. 86-01, Center for Multivariate Analysis, January 1986.

A decomposition of the class of all bivariate positive quadrant dependent distributions into compact convex subsets is obtained. Extreme points of these compact convex subsets are investigated. A similar decomposition works out for the class of all bivariate distributions. Extreme points of the compact convex sets figuring in this decomposition are analyzed. An application to contingency tables is presented.

65. Stoffer, David S. Bootstrapping Nonlinear Least Squares Estimates in the Kalman Filter Model. Technical Report No. 86-02, Center for Multivariate Analysis, January 1986

The bootstrap is proposed as a method for estimating the precision of forecasts and maximum likelihood estimates of the transition parameters of the Kalman filter model when the estimates are obtained via Newton-Raphson. It is shown that when the system and the filter are in steady-state, the bootstrap applied to the Gaussian innovations yields asymptotically consistent standard errors. That the bootstrap works well with moderate sample sizes and supplies robustness against departures from normality is substantiated by empirical evidence.

66. Bai, Z. D., Krishnaiah, P. R., and Zhao, L. C. Signal Processing Using Model Selection Methods. Technical Report No. 86-03, Center for Multivariate Analysis, January 1986.

In this paper, the authors gave a review of some recent developments on multivariate statistical techniques for detection of the number of signals using eigenvalues. The main emphasis of the review is on EFFICIENT DETECTION CRITERION (EDC) procedures proposed by the authors recently. These procedures are strongly consistent.

67. Rao, C. R., Krishnaiah, P. R. ANODIV: Generalization of ANOVA Through Entropy and Cross Entropy Functions. Technical Report No. 86-04, Center for Multivariate Analysis, February 1986.

A unified approach is developed for measurement and analysis of diversity

in data with a factorial structure, i.e., classified by levels of a number of factors. It is shown that the total diversity in given data can be decomposed into several components each representing a source of diversity, by using any measure of diversity satisfying strong convexity properties. It is shown that the usual measures of diversity like Shannon's entropy have only limited convexity properties, besides being not applicable to quantitative data. A new measure of diversity called quadratic entropy is introduced and its applicability to both qualitative and quantitative data is discussed.

68. Taniguchi, M., Krishnaiah, P. R., and Chao, R. Normalizing Transformations of Some Statistics of Gaussian ARMA Processes. Technical Report No. 86-05, Center for Multivariate Analysis, February 1986.

In this paper, the authors investigate Edgeworth type expansions of certain transformations of some statistics of Gaussian ARMA processes. They also investigated transformations which will make the second order part of the Edgeworth expansions vanish. Some numerical studies are made and they show that the above transformations give better approximations than the usual approximation.

69. Wu, Yuehua, On Strongly Consistent Estimates of Regression Coefficients When the Errors are not Independently and Identically Distributed, Technical Report # 86-06, Center for Multivariate Analysis, March 1986.

In this paper, the author proposes two methods of estimation of the regression coefficients when the errors are not disturbed identically and independently and are of nonzero mean. The estimates proved in this paper are shown to be strongly consistent and mean square consistent.

70. Chen, X.R. and Zhao, L. C., Almost sure L_1 -norm Convergence for Data-based Histogram Density Estimates, Technical Report # 86-07, Center for Multivariate Analysis, March 1986.

The main result of this paper is summarized in Theorem 1, which states that when certain conditions of a general nature are satisfied, the data-based histogram density estimator is strongly consistent in the sense that the mean absolute deviation of the estimator and the density function converges to zero almost surely for any density function, as the sample size increase to infinity.

71. Taniguchi, M. and Krishnaiah, P.R., Asymptotic Distributions of Functions of Eigenvalues of the sample Covariance Matrix and Canonical Correlation Matrix in Multivariate Time Series", Technical Report # 86-08, Center for Multivariate Analysis, March 1986.

Let $X(1), \dots, X(n)$ be $p \times 1$ random vectors with mean zero.

Put $S = \frac{1}{n} \sum_{t=1}^n X(t)X(t)'$. When $X(t)$, $(t=1, \dots, n)$ are distributed as i.i.d. $N(0, \Sigma)$, (i.e. usual multivariate analysis), many authors have investigated the asymptotic expansions for the distributions of various functions of the eigenvalues of S . In this paper we will extend the above results to the case when $\{X(t)\}$ is a Gaussian stationary process. Also we shall derive the

asymptotic expansions for certain functions of the sample canonical correlation multivariate time series. Applications of some of the results in signal processing are also discussed.

72. Pukkila, Tarmo M. and Rao, C. Radhakrishna, Pattern Recognition based on scale Invariant Discriminant Functions, Technical Report # 86-09, Center for Multivariate Analysis, April 1986.

Some probability models for classifying individuals as belonging to one of two or more populations using scale invariant discriminant functions are considered. The investigation is motivated by practical situations where the observed data on an individual are in the form of ratios of some basic measurements or measurements scaled by an unknown non-negative number. The probability models are obtained by considering a p -vector random variable X with a known distribution and deriving the distribution of the random vector $Y=[G(X)^{-1}]X$, where $G(X)$ is a non-negative measure of size such that $G(\lambda X)=\lambda G(X)$ for $\lambda>0$. Explicit expressions are obtained for the densities of what are called Angular Gaussian, Compositional Gaussian, Type 1 and Compositional Gaussian, type 2 distributions.

73. Krishnaiah, P.R., On Reduction of Dimensionality Under Multivariate Regression Canonical Correlation Models, Technical Report # 86-10, Center for Multivariate Analysis, April 1986.

In this paper, the author gives a review of the literature on various techniques for determination of the ranks of regression matrix and canonical correlation matrix. Also, methods of selection of important original variables under multivariate regression canonical correlation models are reviewed. The methods reviewed involve not only tests of hypothesis but also model selection methods based upon information theoretic criteria.

74. Naik, Dayanand N., Detection of Outliers in Multivariate Linear Regression Model, Technical Report # 86-11, Center for Multivariate Analysis, April 1986.

In this article we suggest multivariate kurtosis measure as a statistic for detection of outliers in a multivariate linear regression model. The statistics has some local optimal properties.

75. Subramanyam, K. and Rao, M. Bhaskara, Extreme Point Methods in the Study of Classes of Bivariate Distributions and Some Applications to Contingency Tables, Technical Report # 86-12, Center for Multivariate Analysis, April 1986.

The set of all bivariate positive quadrant distributions is neither compact nor convex. But the set of all bivariate positive quadrant distributions with fixed marginals is a convex set. These convex sets are compact in the case of discrete bivariate distributions if the marginals have finite support. A simple method to enumerate the extreme points of these convex sets is given. In the context of contingency tables for testing the null hypothesis independence against the alternative positive quadrant dependence one can use the method of extreme point analysis to compare the performance of various tests.

76. Bai, Z.D. and Fu, J.C., Likelihood Principal and Maximum Likelihood Estimator of Location Parameter for Cauchy Distribution, Technical Report # 86-13, Center for Multivariate Analysis, May 1986.

In the literature of point estimation, Cauchy distribution with location parameters was often cited as an example for the failure of maximum likelihood method and hence the failure of likelihood principle in general. Contrary to the above notion, we proved, even in this case that the likelihood equation has multiple roots, the likelihood estimator (the global maximum) remains an asymptotically optimal estimator in the Bahadur sense.

77. Bai, Z.D., Rao, C. Radhakrishna and Zhao, L.C., Kernel estimators of density function of directional data, Technical Report # 86-14, Center for Multivariate Analysis, May 1986.

Let X be a unit vector random variable taking values on a k -dimensional sphere Ω with probability density function $f(x)$. The problem considered is one of estimating $f(x)$ based on n independent observations $X_1, \dots, X_n$ on X . The proposed estimator is of the form

$$f_n(x) = (nh^{k-1})^{-1} C(h) \sum_{i=1}^n K[(1-x'X_i)/h^2], \quad x \in \Omega$$

where K is a kernel function defined on R_+ . Conditions are imposed on K and f to prove pointwise strong consistency, uniform strong consistency and strong L_1 -norm consistency of f_n as an estimator of f .

78. Pukkila, Tarmo M. and Krishnaiah, P.R., On the use of autoregressive order determination criteria in univariate white noise tests, Technical Report # 86-15, Center for Multivariate Analysis, June 1986.

In this paper new tests of white noise will be suggested. All of the proposed test are based on the application of known autoregressive order determinating criteria such as AIC, BIC, or on their modifications. Some of the proposed tests have, among other things the property that the significance level of a test approaches to zero as the number of observations increases. This on the other hand means that if this kind of a test is applied for the testing whether the residual series of a fitted model is white noise, the testing leads to a consistent order selection.

79. Pukkila, Tarmo M. and Krishnaiah, P.R., On the use of autoregressive order determination criteria in multivariate white noise tests, Technical Report # 86-16, Center for Multivariate Analysis, June 1986.

In this paper testing the hypothesis of multivariate white noise is seen as the selection of the order of a multivariate autoregressive model for the observed time series. Therefore multivariate white noise tests can be carried out by applying autoregressive order determination criteria such as AIC, BIC, etc. It is known that for example the BIC criterion estimates consistently the order of an autoregression. An order determination criterion with this property leads to a white noise test with the significance level approaching to zero as the number of observations increases. In this

paper the order of autoregressive moving average model is proposed to be determined by applying this kind of white noise test. Then the resulting model building procedure is nothing but a generalization of the procedure proposed by Box and Jenkins.

80. Bai, Z.D. and Yin, Y.Q., A convergence to the semicircle law, Technical Report # 86-17, Center for Multivariate Analysis, July 1986.

In this article, it is proved that the spectral distribution of the random matrix

$$A_p = \frac{1}{2\sqrt{np}}(X_p X_p' - nI_p)$$

tends to the semicircle law as $p \rightarrow \infty$, $n \rightarrow \infty$ and $p/n \rightarrow 0$. Here $X_p = [X_{ij}; 1 \leq i \leq p, 1 \leq j \leq n]$, X_{ij} 's are iid random variables with $EX_{11} = 0$, $EX_{11}^2 = 1$, $EX_{11}^4 < \infty$.

81. Bai, Z.D., Krishnaiah, P.R. and Yin, Y.Q., Inference on the Occurrence/Exposure Rate and Simple Rise Rate, Technical Report # 86-18, Center for Multivariate Analysis, August 1986.

In this paper, we study the asymptotic distributions of the functions of the occurrence/exposure rates of several groups of patients as well as Berry-Esseen bound on the distribution function of the occurrence/exposure rate. Asymptotic distributions of functions of the simple risk rates are also derived. The results are useful in not only medical research but also in the area of reliability.

82. Das, Rita and Sinha, Bimal K., Robust optimum invariant tests in one-way unbalanced and two-way balanced models, Technical Report # 86-19, Center for Multivariate Analysis, August 1986.

In one-way random effects unbalanced model the locally best invariant test for the equality of the treatment effects is derived. Surprisingly, this is different from the widely used familiar F-test. In the balanced case, however, the two tests coincide and represent the uniformly most powerful invariant test. For two-way random effects and mixed effects balanced model, the uniformly most powerful invariant test for the equality of the treatment effects is derived both with and without interaction, and shown to be equivalent to the usual F-test under fixed effect models. The optimum invariant tests derived shown not to depend on the assumption of normality. Different aspects of null, nonnull and optimality robustness of these tests [Kariya and Sinha, Annals of Statistics, 1985] are studied. In the unbalanced two-way models however unlike in the fixed effects model providing a UMPI test, both random and mixed effects models present a difficulty which is pointed out.

83. Das, Rita and Sinha, Bimal K., Robust optimum invariant tests in covariance structures useful in linear models, Technical Report # 86-20, Center for Multivariate Analysis, August 1986.

Necessary and sufficient conditions providing structure of V for the BLUE of estimable linear parametric functions and the LRT of a linear testable

hypothesis under $(\Sigma X\beta, \sigma^2 V)$ are well known in the literature (T. Mathew and P. Bhimasankaram, Sankhya (A), 1983, 221-225). In this paper we derive robust optimum invariant tests of such structures of V based on data generated for a fixed design matrix X . Aspects of null, nonnull and optimality robustness of the proposed tests are discussed.

84. Rao, C. Radhakrishna, Applications of multivariate analysis in signal detection: some recent work, Technical Report # 86-22, Center for Multivariate Analysis, August 1986.

The paper contains a review of recent research in the detection and estimation of signals. Both the cases of fixed and random signals are considered. The effects of using an estimated covariance matrix of the noise component in the place of the true one are studied. A new method known as the efficient detection criterion for estimating the number of signals is discussed.

85. Rao, C. Radhakrishna, Some recent results in signal detection, Technical Report # 86-23, Center for Multivariate Analysis, September 1986.

Some recent results on the detection and estimation of signals in the presence of noise are discussed. An exact confidence lower bound is obtained for the discriminatory power of an estimated linear discriminant function for signal detection. Information theoretic criteria are suggested for the estimation of number of signals. A new method is proposed for determining the number of signals and estimating them in exponential signal models.

86. Alzaid A., Lau, K.S., Rao, C. Radhakrishna and Shanbhag, D.N., Solution of Deny convolution equation restricted to half line via random approach, Technical Report # 86-24, Center for Multivariate Analysis, September 1986.

A general solution of the Deny convolution equation restricted to half line is obtained using the concepts of random walk theory. The equation in question arises in several places in applied probability such as in queueing and storage theories and characterization problems of probability distributions. Some of the important applications are briefly discussed.

87. Khatree, Ravindra, On comparison of estimates of dispersion using generalized Pitman nearness criterion, Technical Report # 86-25, Center for Multivariate Analysis, September 1986.

In this paper, estimates of dispersion matrix and its functions are compared based on generalized Pitman nearness criterion. Various loss functions are considered for the purpose. Locally superior estimates are defined. Comparison of these estimates are made with other standard ones. It is shown that within certain classes, defined in the paper, these are the best estimators in generalized Pitman nearness sense.

88. Bai, Z. D. A note on Feller's proof on a law of iterated logarithm, Technical Report # 86-26, Center for Multivariate Analysis, October 1986.

In 1946, Feller proposed and proved a famous law of iterated logarithm.

But unfortunately his proof was found to be incorrect, although the main result was true. In this paper the author gives a new proof of the main result due to Feller.

89. Nishii, R. and Krishnaiah, P.R., On the moments of classical estimates of explanatory variables under a multivariate calibration model. Technical Report # 86-27. Center for Multivariate Analysis, October 1986.

In the area of regression analysis, the problem of predicting the response variable using explanatory variables received considerable attention. But situations arise when it is of interest to predict the explanatory variable using information on the response variables.

90. Yin, Y.Q. and Krishnaiah, P.R., On some nonparametric methods for detection on the number of signals, Technical Report # 86-28, Center for Multivariate Analysis, October 1986.

In this paper, the authors considered a general information theoretic criteria for detection of the number of signals under additive model. The strong consistency of this procedure as well as a bound on the probability of wrong detection are established. Applications of this method to detection of the number of signals when the signals are coherent is also discussed.

91. Bai, Z.D., Silverstein, Jack W. and Yin, Y.Q. A note on the largest eigenvalue of a large dimensional sample covariance matrix. Technical Report # 86-29. Center for Multivariate Analysis, October 1986.

Let $\{v_{ij}; i, j = 1, 2, \dots\}$ be a family of i.i.d. random variables with $E(v_{11}^4) = \infty$. For positive integers p, n with $p = p(n)$ and $p/n \rightarrow y > 0$ as $n \rightarrow \infty$, let $M_n = (1/n)V_n V_n^T$ where $V_n = (v_{ij})_{\substack{1 \leq i \leq p \\ 1 \leq j \leq n}}$ and let $\lambda_{\max}(n)$ denote the largest eigenvalue of M_n . It is shown that $\lim_n \lambda_{\max}(n) = \infty$ a.s. This result verifies the boundedness of $E(v_{11}^4)$ to be the weakest condition known to assure the almost sure convergence of $\lambda_{\max}(n)$ for a class of sample covariance matrices.

92. Kariya, T. and Bimal K. Sinha, and Giri, N.C., Robustness of t-test. Technical Report # 86-31, Center for Multivariate Analysis, October 1986.

In this paper we establish the optimality robustness of Student's t-test mainly without invariance. This generalizes some well-known results of Lehmann and Stein (1949), and Kariya and Eaton (1977).

93. Das, Rita and Sinha, Bimal K., Robust optimum invariant tests for random MANOVA models. Technical Report # 86-32, Center for Multivariate Analysis, October 1986.

Consider the canonical form MANOVA setup with $X: n \times p = (X_1' X_2' X_3')' = (M_1' M_2' 0)' + E$, $X_i: n_i \times p$, $i=1,2,3$, $M_i: n_i \times p$, $i=1,2$, $n_1 + n_2 + n_3 = n$, $n_3 \geq p$, where E is a random error matrix with location 0 and unknown scale matrix $\Sigma > 0$ (p.d.). Assume, unlike in the usual sense, that M_1 is random with location 0 and scale matrix $\sigma_1^2 \Sigma$, σ_1^2 , Σ being unknown. For testing $H_0: \sigma_1^2 = 0$ versus $H_1: \sigma_1^2 > 0$ under a left orthogonally invariant

distribution of X , it is shown that when either $n_2 = 0$ or M_2 fixed if $n_2 > 0$ the trace test of Pillai (1955) is UMPI if $\min(n_1, p) = 1$ and LBI if $\min(n_1, p) > 1$. The test is null, nonnull and optimality robust (Kariya and Sinha(1985)). However, such a result does not hold if $n_2 > 0$ and M_2 random.

94. Bai, Z.D., Krishnaiah, P.R., Rao, C. Radhakrishna, Reddy, P.S., Sun, Y.N. and Zhao, L.C., Reconstruction of the left ventricle from two orthogonal projections, Technical Report # 86-33, Center for Multivariate Analysis, November 1986.

A new method for reconstruction of the shape of the left ventricle from the biplane angiocardiograms is proposed. This approach utilizes a pair of orthogonal X-ray projection images. The shape of the ventricle is reconstructed by dividing these projection images into parallel slices and then being processed slice by slice stepwise. Each corresponding pair of slices from two one-dimensional projection profiles which are used to reconstruct a cross section of the ventricle.

95. Mathew, Thomas and Sinha, Bimal Kumar, Optimum tests for fixed effects and variance components in balanced models, Technical Report # 86-34, Center for Multivariate Analysis, November 1986.

For any ANOVA model with balanced data involving both fixed and random effects, UMPU and UMPI tests are derived for the significance of a fixed effect or a variance component, under the assumption of normality of random effects. The tests coincide with usual F-tests. Robustness of the UMPI test against suitable deviations from normality is established.

96. Sinha, Bimal K., Inadmissibility of the best equivariant estimators of the variance-covariance matrix, the precision matrix, and the generalized variance: A survey, Technical Report # 86-35, Center for Multivariate Analysis, November 1986.

Based on a data matrix $X = (X_1, \dots, X_k)$: $p \times k$ with independent columns

$X_i \sim N_p(\xi_i, \Sigma)$, and an independent $p \times p$ Wishart matrix $S \sim W_p(n, \Sigma)$, procedures to obtain estimators dominating the best equivariant estimators of Σ, Σ^{-1} and $|\Sigma|$ under various loss functions are reviewed.

97. Sinha, Bimal K., and Sen, Pranab Kumar, Nonparametric estimation of the generalized variance, Technical Report # 86-36, Center for Multivariate Analysis, November 1986.

For multivariate distributions with finite second order moments, a nonparametric symmetric, unbiased estimator of the generalized variance is considered and it is shown to be (nonparametric) optimal for the class of distributions having finite fourth order moments. A jackknife version of the sample generalized variance is also considered as a contender; it is computationally more convenient and asymptotically equivalent to the former. It is also shown that the second estimator performs quite well (in large sample) relative to the optimal normal theory estimators under several loss functions.

98. Bai, Z.D., Krishnaiah, P.R. and Zhao, L.C., On simultaneous estimation of the number of signals and frequencies under a model with multiple sinusoids, Technical Report # 86-37. Center for Multivariate Analysis, December 1986.

In this paper, the authors considered the problem of estimation of the frequencies and the number of signals under a signal processing model with multiple sinusoids. The frequencies are estimated with eigenvariation linear prediction method. The number of signals is estimated with an information theoretic criterion. The strong consistency of the estimates of the frequencies and the number of signals is also established. Also, a modification of forward backward linear prediction method is suggested to yield consistent estimators of the frequencies.

99. Bai, Z.D., Krishnaiah, P.R. and Zhao, L.C., On rate of convergence of equivariation linear prediction estimates of the number of signals and frequencies of multiple sinusoids, Technical Report # 86-38. Center for Multivariate Analysis, December 1986.

In a companion paper, the authors (see Bai, Krishnaiah and Zhao(1986)) considered the problem of estimation of the number of signals and the frequencies of these signals under a signal processing model with multiple sinusoids. The number of signals was estimated by using an information theoretic criterion. They have also established the strong consistency of their estimates. In this paper, we established the rates of convergence of the above estimates of the number of signals and frequencies.

100. Khatri, C.G., Rao, C. Radhakrishna, Schaafsma, W., Steememan, A.G.M. and Van Vark G.N. Inference about the performance of Fisher's linear discriminant function with application to signal detection, Technical Report # 86-39. Center for Multivariate Analysis, December 1986.

We consider the problem of assessing the discriminatory power of a sample based discriminant function for classifying a p -vector observation as arising from one or the other of two alternative distributions (populations). An important application of this problem arises in signal detection where a decision has to be taken whether a received message in the form of p -vector observation is pure noise or signal plus noise. When the signal to be detected is a known vector δ and the noise variance-covariance matrix is a known matrix Σ , the classification is done through Fisher's linear discriminant function which has the maximum discriminatory power. If δ or Σ or both are unknown we estimate the LDF by substituting for δ and Σ their estimates based on previous correctly classified samples. However, there is loss in discriminatory power in using an estimated LDF, and the paper focuses on the estimation or prediction of the actual discriminatory power.

The discriminatory power of any linear discriminant function (LDF) is measured by what is known as signal to noise ratio, which is the difference in expected values of the LDF under the two hypotheses expressed in standard deviation units. In order to estimate signal to noise ratio we use parametric and leave one out methods.

101. Rao, C. Radhakrishna, Prediction of future observations in growth curve

models, Technical Report # 86-40, Center for Multivariate Analysis, December 1986.

The problem of predicting a future measurement on an individual given the past measurements is discussed under non-parametric and parametric growth models. The efficiencies of different methods of prediction are assessed by cross-validation (CV) or leave-one-out (LOO) technique in each of three data sets and the results are compared. Under non-parametric models, direct and inverse regression methods of prediction are described and their relative advantages and disadvantages are discussed. Under parametric models polynomial and factor analytic type growth curves are considered. Bayesian and empirical Bayesian methods are used to deal with unknown parameters.

A general finding is that much of the information for forecasting is contained in the immediate past few observations or a few summary statistics based on past data. A number of data reduction methods are suggested and analyses based on them are described. The usefulness of the LOO technique in model selection is demonstrated.

102. Tauxe, W.N. , Klein, Hervert A. ,Bagchi, Ansuman ,Krishnaiah, P.R. Kundu, Debasis and Tepe, Paul, Clinical evaluation of the filtration fraction: a multivariate statistical analysis, Technical Report # 86-41, Center for Multivariate Analysis, December 1986.

Using multivariate statistical technique, we have separated glomerular filtration fraction (FF) data into various clinical groups. In general population referred for renal function evaluation, we found that the FF did not vary with glomerular filtration rate(GFR), but varied inversely with ERPF. A group of hypertensive patients was identified to lie significantly above the FF/ERPF regression line for the whole group. Another group of two-kidneyed normal subjects was identified to lie below this regression line. FF of a group of one-kidneyed normally functioning regression subjects was found also to lie similarly disposed below this regression line but to have significantly higher FF than the two-kidneyed normals. Coefficients of ellipses that defined 99% confidence regions for the loci of the means of these groups were derived.

103. Nishii, R. , Bai, Z.D. ,Krishnaiah, P.R., Strong consistency of certain information theoretic criteria for model selection in calibration, discriminant analysis and canonical correlation analysis, Technical Report # 86-42 , Center for Multivariate Analysis, December 1986.

In this paper, the authors show that the criteria for model selection based upon efficient detection (ED) criteria are consistent for certain problems in multivariate calibration, discriminant analysis and canonical correlation analysis. These results will be proved under mild conditions on the underlying distribution.

104. Subramanyam, K and Rao, M. Bhaskara, Analysis of total positivity of order two in $2 \times n$ ordinal contingency tables, Technical Report # 86-43 , Center for Multivariate Analysis, December 1986.

The set of all bivariate probability distributions with support contained in

$\{(i,j); i=1,2 \text{ and } j=1,2, \dots, n\}$ which are totally positive of order two is shown to be a convex set under some conditions on one of the marginal distributions. The extreme points of the compact convex set are explicitly enumerated. Using the structure of this convex set, we show that the power function of any test for testing the hypothesis of independence against the hypothesis of strict total positivity of order two in $2 \times n$ ordinal contingency tables has a simple form in terms of the extreme points. A numerical illustration is provided.

105. Shanbhag, D.N. and Kotz, S., Some new approaches to multivariate probability distributions, Technical Report # 86-44, Center for Multivariate Analysis, December 1986.

In this paper we extend and generalize to the multivariate set-up our earlier investigation related to expected remaining life functions and general hazard measures including representations and stability theorems for arbitrary probability distributions in terms of these concepts.

106. Niroomand Chapeh, H.A. and Rao, M. Bhaskara, On the least squares estimator in moving average models of order one, Technical Report # 86-45, Center for Multivariate Analysis, December 1986.

A simple expression is derived in this paper for the error sum of squares in the context of moving average models of order one. A computer program is developed to estimate the parameter of a moving average model of order one based on the method of least squares.

107. Niroomand Chapeh, H.A., Rao, M. Bhaskara, A new method of estimation in a moving average model of order one, Technical Report # 86-46, December 1986.

The exact likelihood of the data coming from a moving average model of order one is complicated. In this paper, we propose a method of estimation of the parameters of a moving average model of order one based on the approximate likelihood of the data and on the simulation of a pair of random variables. Some comparisons were made of this with some well known methods for moderate sample sizes. A computer program is appended which is helpful in using this method.

108. Krishnaiah, P.R., Miao, B.Q. and Zhao, L.C., On detection of change points using mean vector, Technical Report # 86-47, Center for Multivariate Analysis, December 1986.

In this paper, the authors consider the problem of change points within the framework of model selection procedures using information theoretic criteria. The authors proposed procedures for estimation of the locations of change points and the number of change points. The strong consistency of these procedures is also established. Also, the problem of change points is discussed within the framework of the simultaneous test procedures.

109. Chen, X.R., Krishnaiah, P.R. and Liang, W.Q., Estimation of multivariate binary density using orthonormal functions, Technical Report # 86-48, Center for Multivariate Analysis, December 1986.

In this paper, the authors studied certain properties of the estimate of Liang and Krishnaiah (*J. Multivariate Anal.*, 16, 162-172) for multivariate binary density. An alternative shrinkage estimate is also obtained. The above results are generalized to general orthonormal systems.

110. Chen, X.R. and Krishnaiah, P.R., Test of linearity in general regression models, Technical Report # 86-49, Center for Multivariate Analysis, December 1986.

Let $(\underline{X}_i, Y_i)$, $i=1, \dots, n$, be iid. samples of $(\underline{X}, Y)$. This paper proposes a method for testing the linearity of the regression function $E(Y|\underline{x} = \underline{x})$. The asymptotic distribution (under null hypothesis) and the asymptotic power of the test are determined. Also, consistency of the test is proved under mild conditions.

111. Chen, X.R. and Krishnaiah, P.R., Estimation and testing in truncated and non truncated linear median-regression models, Technical Report # 86-50, Center for Multivariate Analysis, December 1986.

Suppose $(X_i, \bar{Y}_i)$, $i=1, \dots, n$ are iid. samples of $(X, \bar{Y})$. Instead of $\bar{Y}_i$, we can only observe $Y_i = \max(\bar{Y}_i, 0)$. Denote by $m(x)$ the median-regression function of $\bar{Y}$ with respect to X . This paper discuss the estimation of $m(x)$ when it is assumed that $m(x) = \alpha + \beta'x$ for some α, β . The consistency and asymptotic normality of the estimators (of α and β) are established. Also, a method is given to test the linearity of the regression function $m(x)$.

112. Nishii, Ryuei, Maximum likelihood principle and model selection when the true model is unspecified, Technical Report # 87-01, Center for Multivariate Analysis, February 1987.

Suppose independent observations come from an unspecified distribution. Then we consider the maximum likelihood based on a specified parametric family by which we can approximate the true distribution well. We examine the asymptotic properties of the Quasi likelihood estimate and of the quasi-maximum likelihood. These results will be applied to model selection problem.

113. Nishii, Ryuei, Identities and inequalities for the expected values of random matrices and Loewner's Theory, Technical Report # 87-02, Center for Multivariate Analysis, March 1987.

We consider the identity for the expected values of the quadratic forms of random matrices. As a special case the identity yields the matrix-valued inequality of the Schwarz type. The most important application is for Loewner's theory in the context of the operator-monotone function, and many inequalities based on the concavity or convexity of the matrix-valued functions are derived. Further generalization of positive semi-definite matrices will be made.

114. Bai, Z. D., Krishnaiah, P.R. and Zhao, L.C., Multivariate components of covariance model in unbalanced case, Technical Report # 87-03, Center for Multivariate Analysis, March 1987.

In this paper we are interested in the determination of the rank of the random effect covariance matrix in an unbalanced components of covariance model, and obtain some strongly consistent estimates of the rank based on eigenstructure methods.

115. Chen, X., Necessary and Sufficient Conditions for the Existence of Consistent Estimates in One-dimensional Linear Models with Truncated Error Distribution, Technical Report #87-04, Center for Multivariate Analysis, March 1987.

Consider the linear regression model with one independent variable x : $Y_i = \alpha + \beta x_i + e_i$, $i = 1, \dots, n$, where α, β are unknown parameters, $\{x_i\}$ is a sequence of known real numbers, and $\{e_i\}$ is a sequence of iid. random errors, with $Ee_i = 0$, $0 < \text{Var } e_i = \sigma^2 < \infty$, σ^2 unknown. Suppose that the probability density $f(x)$ of e_i vanishes outside a bounded interval $[\sigma_1, \sigma_2]$, and on $[\sigma_1, \sigma_2]$, $f(x)$ is positive and satisfies the Lipschitz condition, it is shown that the necessary and sufficient condition for the existence of a consistent estimate of α, β is $x_n/H_n \rightarrow 0$ and $H_n \rightarrow 0$ respectively, where $x_n = 1/n \sum_{i=1}^n x_i$ and $H_n = \sum_{i=1}^n |x_i - x_n|$. The case in which $\text{med}(e_i) = 0$ is also considered, resulting in the same necessary and sufficient conditions.

116. Bai, Z.D. and Yin, Y.Q. Necessary and Sufficient Conditions for Almost Sure Convergence of the Largest Eigenvalue of Wigner Matrix, Technical Report #87-05, Center for Multivariate Analysis, April 1987.

Let $W = (X_{ij}; 1 \leq i, j < \infty)$ be an infinite matrix. Suppose W is symmetric, entries on the diagonal are iid, entries off the diagonal are iid, and they are independent. Then it is proved that the necessary and sufficient conditions for $\lambda_{\max}(1/\sqrt{n} W_n) \rightarrow a$, a.s., are (1) $E(X_{11}^2) < \infty$, (2) $EX_{12}^4 < \infty$, (3) $EX_{12} \leq 0$, (4) $a = 2\sigma$, $\sigma^2 = EX_{12}^2$.

117. Bai, Z.D., Chen, X.R., Necessary and sufficient conditions for the convergence of integrated and mean integrated P -th order error of the kernel density estimates, Technical Report # 87-06, Center for Multivariate Analysis, April 1987.

Let $f_n(x) = (nh_n^d)^{-1} \sum_{i=1}^n K((x-X_i)/h_n)$ be the kernel estimate of a d -dimensional density f , where the kernel function K is a probability density on R^d , $h_n > 0$ is a constant, and $X_1, X_2, \dots, X_n$ are iid. samples drawn from the population with density f . Let $p \geq 1$ and define $m_{np} = \int |f_n(x) - f(x)|^p dx$, $M_{np} = E(m_{np})$. It is shown that the necessary and sufficient condition for any one of the following two assertions $\lim_{n \rightarrow \infty} m_{np} = 0$, a.s., $\lim_{n \rightarrow \infty} M_{np} = 0$, is that the four conditions: $\int f(x) dx < \infty$, $\int K^2(x) dx < \infty$, $h_n \rightarrow 0$, $nh_n^d \rightarrow \infty$ hold simultaneously.

118. Chen, X.R. and Zhao, L.C., Necessary and sufficient conditions for the convergence of integrated and mean integrated r -th order error of histogram density estimates, Technical Report #87-07, Center for Multivariate Analysis, April 1987.

Suppose $X_1, \dots, X_n$ are samples drawn from a population with density function f , and $f_n(x) = f_n(x; X_1, \dots, X_n)$ is an estimate of $f(x)$. Denote by $m_{nr} = \int |f_n(x) - f(x)|^r dx$ and $M_{nr} = E(m_{nr})$ the integrated r -th Order Error and Mean Integrated r -th Order Error of f_n for some $r \geq 1$ (when $r=2$, they are the familiar and widely studied ISE and MISE). In this paper the same necessary and sufficient condition for $\lim_{n \rightarrow \infty} M_{nr} = 0$ and $\lim_{n \rightarrow \infty} m_{nr} = 0$ a.s. is obtained when $f_n(x)$ is the ordinary histogram estimator.

119. Bai, Z.D., Krishnaiah, P.R., Rao, C.R., Sun, Y.N. and Zhao, L.C., Reconstruction of the shape and size of objects from two orthogonal projections. Technical Report # 87-08, Center for Multivariate Analysis, April 1987.

Suppose that V is a family of regular regions in R^d ($d \geq 2$) with a common constructive profile (CP), i.e., the projection along x_d -axis. In this paper, the authors establish, under certain restrictions on the equal divisor (ED), the one-to-one correspondence between V and the family Θ of the induced testing profiles i.e., the projection along x_1 -axis of the element in V . This proves that, under certain restrictions, a regular region in R^d is continuous function of its profiles (CP and TP), which theoretically guarantees the justification of the reconstruction of the left ventricle given in a previous paper of ours.

120. Krishnaiah, P.R. and Miao, B.Q., Control charts when the observations are correlated. Technical Report # 87-09, Center for Multivariate Analysis, May 1987.

Traditionally, control charts are based on independently normal samples, but in practice it so happens that the samples are dependent. In this review, dependent samples are considered as ARMA time series. Also, multidimensional time series samples are discussed.

121. Rao, C. Radhakrishna, Estimation in linear models with mixed effects: a unified theory. Technical Report # 87-10, Center for Multivariate Analysis, May 1987

A unified theory without any assumptions on the ranks of the matrices involved is developed for the estimation of a linear function of unknown parameters (β) and hypothetical variables (ξ) in a mixed Gauss-Markoff linear model, $Y = X\beta + U\xi + \epsilon$. The expressions for the estimates and the mean square errors of the estimates depend on the elements of an inverse partitioned matrix as in the case of the Gauss-Markov model with fixed effects developed earlier by the author [Rao(1971)].

The general theory is applied to several special problems that arise in social and biological sciences. In particular, explicit expressions are obtained for simultaneous estimation or prediction in several similar and dissimilar linear models with or without concomitant variables and in special univariate and multivariate linear models in genetic selection.

122. Krishnaiah, P.R. and Miao, B.Q., Detecting and interval estimation about a slope change point. Technical Report # 87-11, Center for Multivariate Analysis, June 1987

In this paper, the authors consider the problem of change points using Gaussian process. The distribution of the statistic to estimate a change point constructed in this paper can be approximated by the first type of extremal distribution. Based on this, detection and interval estimation of a change point in various situation are discussed.

123. Bai, Z.D. , Krishnaiah P.R. and Zhao, L.C., On the detection of arrival estimation, Technical Report # 87-12, Center for Multivariate Analysis, June 1987.

The estimation of arrival direction is important task in signal processing and has recently received considerable attention in the literature. In this paper, the authors proposed a method to estimate the direction of arrival and proved the strong consistency of the estimates for both cases in presence of white noise and colored noise.

124. Subramanyam K. and Rao, M.B. , On the extreme points of the set of all $2 \times n$ bivariate positive quadrant dependent distributions with fixed marginals and some applications , Technical Report # 87-13, Center for Multivariate Analysis, June 1987.

The set of all bivariate distributions with support contained in $\{(i,j); i=1,2 \text{ and } j=1,2, \dots, n\}$ which are positive quadrant dependent is a convex set. In this paper, an algebraic method is presented for the enumeration of all extreme points of this convex set. Certain measure of dependence, including Kendall's tau, are shown to be affine functions on this convex set. This property of being affine helps us to evaluate the asymptotic power of tests based on these measures of dependence for testing the hypothesis of independence against strict positive quadrant dependence.

125. Rao, C. Radhakrishna , Strategies of data analysis, Technical Report # 87-14, Center for Multivariate Analysis, June 1987

Styles in statistical analysis change over time while the object of "extracting all the information from data" or "summarization and exposure" remains the same. Statistics has not yet aged into a stable discipline with complete agreement on foundations. Certain methods become popular at one time and are replaced in course of time by others which look more fashionable. In spite of the controversies, the statistical methodology and fields of applications are expanding. The computer and more specifically the availability of graphic facilities have had a greater impact on data analysis. It may be of interest to briefly review some historical developments in data analysis.

126. Wu, Yuehua , Strong consistency of estimation of number of regression variables when the errors are independent and their expectations are not equal to each other , Technical Report # 87-15, Center for Multivariate Analysis, June 1987

Consider the linear regression model $y_i = x_i' \beta + \epsilon_i, i=1,2, \dots, n$, where $\{x_i\}$ is a sequence of known p vectors, β is a vector of regression coefficients, $\{\epsilon_i\}$ is a sequence of random errors. It is of interest to test the hypothesis $H_k: \beta_{k+1} = \dots = \beta_p = 0, k=0, 1, \dots, p$. We do not assume that the

random errors are identically distributed and have zero means, since it is sometimes unrealistic. As a compensation for this relaxation, we assume the errors have a common bounded support $[a_1, a_2]$. Under certain conditions, we obtain the strongly consistent estimate of the number k for which $\beta_k \neq 0$ and $\beta_{k+1} = \dots = \beta_p = 0$, by using the information theoretical criteria.

127. Bai, Z.D., Krishnaiah, P.R. and Zhao, L.C., On the asymptotic joint distributions of the eigenvalues of random matrices which arise under components of covariance model, Technical Report # 87-16, Center for Multivariate Analysis, June 1987.

In this paper, the authors derived asymptotic joint distributions of the eigenvalues of some random matrices which arise under components of covariance model.

128. Bai, Z.D., Chen, X.R., Krishnaiah, P.R., Wu, Y.H. and Zhao, L.C., Strong consistency of maximum likelihood parameter estimation of superimposed exponential signals in noise, Technical Report # 87-17, Center for Multivariate Analysis, June 1987.

Consider the model of multiple superimposed exponential signals in additive Gaussian noise

$$Y_j(t) = \sum_{i=1}^p s_{ij} \lambda_i^t + e_j(t), \quad t=0, 1, \dots, n-1, \quad j=1, \dots, N$$

where N is fixed and $n \rightarrow \infty$, $\lambda_i = \exp(\sqrt{-1} \omega_i)$, $\omega_i \in [0, 2\pi)$, $i=1, \dots, p$, s_{ij} are unknown parameters and p is known. Further, $e_j(t) = e_{j1}(t) + \sqrt{-1} e_{j2}(t)$, and $e_{j1}(t), e_{j2}(t)$, $t=0, 1, 2, \dots$, $j=1, \dots, N$, are mutually independent and identically distributed real random variables with a common distribution $N(0, \sigma^2/2)$, $0 < \sigma^2 < \infty$, σ^2 is unknown. It is shown that if $\omega_i \neq \omega_j$ when $i \neq j$ and $\sum_{j=1}^N |s_{ij}| > 0$ for $i=1, \dots, p$, then the Maximum Likelihood Estimate $(\hat{\lambda}_1, \dots, \hat{\lambda}_p)$ is strongly consistent. Moreover, it is shown that $\hat{\lambda}_i$ converges to λ_i with an exponential rate.

129. Wu, Y.H., Strong consistency and exponential rate of the minimum L_1 -norm estimate in linear regression models, Technical Report # 87-18, Center for Multivariate Analysis, June 1987.

Let $y_i = \alpha + x_i' \beta + e_i$, $i=1, \dots, n, \dots$, be a linear regression model where $\{x_i\}$ is a sequence of experimental points, i.e., known p -vectors, $\{e_i\}$ is a sequence of independent random errors, with $\text{med}(e_i)=0$, $i=1, 2, \dots$. Define the minimum L_1 -norm estimate of $(\alpha, \beta)'$, by $(\hat{\alpha}_n, \hat{\beta}_n)'$, to be chosen such that

$$\sum_{i=1}^n |y_i - \alpha - x_i' \beta| = \min_{(\alpha, \beta)} \sum_{i=1}^n |y_i - \alpha - x_i' \beta|$$

Under quite general conditions on $\{x_i\}$ and $\{e_i\}$ the strong consistency of

the minimum L_1 -norm estimate is established. Further, under an additional condition on $\{x_i\}$, it is also proved that for any given $\varepsilon > 0$, there exist constant $C > 0$ not depending on n , such that

$$P\{\|\hat{\alpha}_n - \alpha\|^2 + \|\hat{\beta}_n - \beta\|^2 \geq \varepsilon^2\} \leq \exp\{-Cn\}$$

, for large n .

130. Bai, Z.D. Chen, X.R. Krishnaiah, P.R. and Zhao, L.C. , Asymptotic property of the EVLP estimation for superimposed exponential signals in noise. Technical Report # 87-19, Center for Multivariate Analysis, July 1987.

This paper studies the model of superimposed exponential signals in noise:

$$Y_j(t) = \sum_{k=1}^p a_{kj} \lambda_k^t + e_j(t), \quad t=0, 1, \dots, n-1, \quad j=1, \dots, N$$

where $\lambda_1, \dots, \lambda_q$ are unknown complex parameters with module 1, $\lambda_{q+1}, \dots, \lambda_p$ are unknown complex parameters with module less than 1, $\lambda_1, \dots, \lambda_p$ are assumed distinct, p assumed known and q unknown. $a_{kj}, k=1, \dots, p, j=1, \dots, N$ are unknown complex parameters. $e_j(t), t=0, 1, \dots, n-1, j=1, \dots, N$, are iid complex random noise variables such that

$$Ee_1(0), E|e_1(0)|^2 = \delta^2, 0 < \delta^2 < \infty, E|e_1(0)|^4 < \infty$$

and σ^2 is unknown. This paper gives:

- A strong consistent estimate of q ;
- Strong consistent estimates of $\lambda_1, \dots, \lambda_q, \delta^2$ and $|a_{kj}|, k \leq q$;
- Limiting distributions for some of these estimates;
- A proof of non-existence of consistent estimates for λ_k and $a_{kj}, k > q$;
- A discussion of the case that $N \rightarrow \infty$.

131. Burbea Jacob and Oller, Jose M. , The information metric for univariate linear elliptic models , Technical Report # 87-20, Center for Multivariate Analysis, June 1987

The information metric associated with a univariate linear elliptic family is shown to be. Essentially, the Poincare' hyperbolic metric on a half-space whose geodesic Rao distance is an increasing hyperbolic function of a modified Mahalanobis distance. This result enables us to construct new statistical tests and to recover earlier results as special cases.

132. Subramanyam K. Rao, M. Bhaskara , Analysis of regression dependence in $2 \times n$ bivariate distributions and some applications in contingency tables , Technical Report # 87-21, Center for Multivariate Analysis, July 1987

The notion of positive regression dependence among discrete bivariate distributions is examined vis-a-vis other well-known notions of dependence. The extreme points of the set of all bivariate $2 \times n$ distributions in which Y is regression dependent on X are enumerated. Some applications to testing of hypotheses are given.

133. Krishnaiah P.R., Miao, B.Q. and Zhao, L.C., Local likelihood method in the problems related to change points , Technical Report # 87-22, Center for Multivariate Analysis, June 1987.

In this paper, the so-called local likelihood method is suggested for solving the change point problems when the data are distributed as multivariate normal. The detection procedures proposed not only provide strongly consistent estimates for the number and location of the change points, but also simplify significantly the computation.

134. Bai, Z.D. , Krishnaiah, P.R. and Zhao, L.C., Variable selection in logistic regression, Technical Report # 87-23, Center for Multivariate Analysis, July 1987.

In many situations, we are interested in selection of important variables which are adequate for prediction under a logistic regression model. In this paper, some selection procedures based on the information theoretic criteria are proposed, and these procedures are proved to be strongly consistent.

135. Chen, X.R and Yuehua, Wu , Strong consistency of M-estimates for the linear model , Technical Report # 87-24, Center for Multivariate Analysis, July 1987.

This article defines the M-estimate for the linear model directly from the minimization problem

$$\sum_{i=1}^n \rho(y_i - \alpha - \beta'X_i) = \min.$$

Suppose that $(X_1, Y_1), \dots, (X_n, Y_n), \dots$ are i.i.d. observations of a random vector (X, Y) , where Y is one-dimensional and X may be multidimensional. It is shown that the M-estimates $\hat{\alpha}_n, \hat{\beta}_n$ defined in this manner converge with probability one to α_0, β_0 respectively ((α_0, β_0') is the true parameter) as $n \rightarrow \infty$, under very general conditions on the function ρ and the distribution of (X, Y) .

136. Chen, Xiru , Testing and interval estimation in a change-point model allowing at most one change , Technical Report # 87-25, Center for Multivariate Analysis, July 1987.

This paper considers the simplest model of change-point in which at most one change in the mean may occur. Results include:

- Introduction of a test for the null hypothesis that "no change in the mean occurs", and the limit distribution of the test-statistic.
- Approximate calculation of the power of the test.
- Interval estimation of the position of change.
- Point estimation of the jump at the point of change and its asymptotic distribution.
- Evaluation of the bias of the MLE of error variance.

137. Rao, C. Radhakrishna, and Shanbhag, D.N., Further Extensions of the

Choquet-Deny and Deny Theorems With Applications In Characterization Theory, Technical Report #87-26, Center for Multivariate Analysis, August 1987.

Davies and Shanbhag (1987) identified, under some mild conditions, the solution to the convolution equation $H(x) = \int_S H(x+y)\mu(dy)$, $x \in S$ where S is a Polish Abelian semigroup with zero element, $H: S \rightarrow R_+$, a nonnegative continuous function and μ is a measure. A variant of the result in the case where H is bounded and μ is a certain bounded signed measure is obtained. This provides a generalized version of the Choquet-Deny theorem where μ is considered to be a probability measure and S to be a group. The stability of the solution to the above equation is examined by adding an error term. Further, solutions to simultaneous equations of the above type are considered. The results of this paper generalize those obtained by Gu and Lau (1984), Lau and Rao (1982), Ramachandran et al (1987), Shimuzu (1978) and others for $S = R$ or R_+ .

138. Yin, Y.Q. Detection of the Number, Locations and Magnitudes of Jumps, Technical Report #87-27, Center for Multivariate Analysis, August 1987.

Consider a signal $x(t) = f(t) + w(t)$, $0 \leq t \leq 1$. Here the noise $w(t)$ is an independent process, and $f(t)$ is a function with only finitely many jumps, satisfies a Lipschitz' condition between any two consecutive jumps. This paper gives an algorithm to determine the number, locations and magnitudes of the jumps of $f(t)$. The consistency and speeds of convergence are obtained.

139. Yin, Y.Q. An Effective Method to Estimate the Number and Locations of Jumps, Technical Report #87-28, Center for Multivariate Analysis, August 1987.

Let $x(t) = f(t) + w(t)$ be a signal, where the noise $w(t)$ is an independent process. Suppose $f(t)$ has only finitely many jumps, satisfies a Lipschitz' condition between each two consecutive jumps. This paper gives a very simple algorithm to determine the number and locations of the jumps. The consistency is established, the speed of convergence is obtained.

140. Zhao, L.C. and Chen, X.R. Normal Approximation For Finite-population U-statistics. Technical Report #87-29, Center for Multivariate Analysis, August 1987.

In this article, the general central limit theorem and Berry-Esseen bounds for finite-population U-statistics with degree m are established under very weak conditions. The results achieve a substantial improvement of those in Zhao and Chen (1987).

141. Zhao, L.C., Krishnamah, P.R. and Chen, X.R., Almost Sure L_p -norm Convergence For Data based Histogram Density Estimates, Technical Report #87-30, Center for Multivariate Analysis, August 1987.

Let $X_1, \dots, X_n$ be n i.i.d. samples drawn from a d -dimensional distribution with density f . Partition the space R^d into a union of disjoint intervals $\{I_l = I_l(X_1, \dots, X_n)\}$ with the form

$$I_i = \{x = (x^{(1)}, \dots, x^{(d)}) : -\infty < a_{li} \leq x^{(i)} \leq b_{li} < \infty, i=1, \dots, d\}$$

Define the data-based histogram estimate of $f(x)$ based on this partition as $f_n(x)$ = The number of $X_1, \dots, X_n$ falling into I_i / n times the volume of I_i , for $x \in I_i$, $i=1, 2, \dots$. For given constant $r \geq 1$ we obtain the sufficient condition for $\lim_{n \rightarrow \infty} \int_R |f_n(x) - f(x)|^r dx = 0$. The results give substantial improvements upon the existing results.

142. Shen, W.H. and Sinha, B.K. Admissible Bayes Tests For Structural Relationship, Technical Report #87-31, Center for Multivariate Analysis, September 1987.

It is an open problem to construct a test for structural relationship among the mean vectors of several multivariate normal populations with known but unequal covariance matrices. In this paper, a class of admissible Bayes tests for the above problem is derived. As a byproduct, in the special case of known and equal covariance matrices, the likelihood ratio test of Rao(1973) is shown to be admissible Bayes.

143. Subramanyam, K. Some Comments On Positive Quadrant Dependence In Higher Dimensions, Technical Report #87-32, Center for Multivariate Analysis, September 1987.

An extreme point analysis has been performed on two natural definitions of positive quadrant dependence of three random variable. This analysis helps us to understand how much these two notions of dependence are different.

144. Krishnaiah, P.R., Miao, B.Q. and Wong, H. Multidimensional Control Chart For Correlated Data, Technical Report #87-33, Center for Multivariate Analysis, June 1987.

The authors discuss the problem of multivariate control charts by considering observations as time series.

145. Ghosh, M. and Sinha, B.K. Empirical and Hierarchical Bayes Competitors of Preliminary Test Estimators In Two Sample Problems, Technical Report #87-34, Center for Multivariate Analysis, September 1987.

Suppose in a laboratory, say Laboratory I, a certain instrument is designed to measure several characteristics, and a number of vector-valued measurements is recorded. Our objectives to estimate the unknown population mean. It is known, however, that a similar instrument is used in another laboratory, say Laboratory II for the same purpose, and a number of observations is recorded from the second instrument. It is also suspected that the two population means are equal, in which case, observations recorded in Laboratory II can possibly be used effectively together with those in Laboratory I for estimating the population mean of the first instrument. Thus, the question that naturally arises is whether one should use the sample mean from Laboratory I or the pooled mean from the two laboratories.

In problems of this type what is normally sought is a compromise estimator

which leans more towards the pooled sample mean when the null hypothesis of the equality of the two population mean is accepted, and towards the sample mean from Laboratory I which when such a hypothesis is rejected.

In this paper, we propose instead an empirical Bayes(EB) estimator which achieves the intended compromise. Such an EB estimator is quite often a weighted average of the pooled mean and the first sample mean. Unlike a subjective Bayes estimator, the EB estimator is quite robust (with respect to its frequentist or Bayesian risk) against a wide class of priors.

146. Bai, Z.D., Chen, X.R., Wu, Y. and Zhao, L.C. Asymptotic Normality of Minimum L_1 -norm Estimates In Linear Models. Technical Report #87-35, Center for Multivariate Analysis, September 1987.

Consider the standard linear model $Y_i = x_i' \beta + e_i$, $i = 1, \dots, n, \dots$ where $x_1, x_2, \dots$ are assumed to be known p - vectors, β the unknown p -vector of regression coefficients, and $e_1, e_2, \dots$ the independent random error sequence each having a median zero. Define the Minimum L_1 -Norm estimator $\hat{\beta}_n$ as the solution of the minimization problem $\sum_{i=1}^n |Y_i - x_i' \hat{\beta}_n| = \inf\{\sum_{i=1}^n |Y_i - x_i' \beta| : \beta \in R^p\}$. It is proved in this paper that $\hat{\beta}_n$ is asymptotically normal under very weak conditions. In particular, the condition imposed on $\{x_i\}$ is exactly the same which ensures the asymptotic normality of Least Squares estimate: $\lim_{n \rightarrow \infty} \max_{1 \leq i \leq n} x_i (\sum_{j=1}^n x_j x_j')^{-1} x_i = 0$.

147. Bhandary M. Robust M-estimation of Dispersion Matrix With a Structure. Technical Report #87-36, Center for Multivariate Analysis, September 1987.

Marona (1976) considered the problem of robust M-estimation of the dispersion matrix Σ which is arbitrary. In this paper a procedure for robust M- estimation of the dispersion matrix of the form $\Gamma + \sigma^2 I_p$, where Γ is n.n.d. of rank $q(<p)$ and $\sigma^2 > 0$ are unknowns, is given. The structure of the above covariance matrix appears in the area of signal processing. The procedure is an algorithm which involves iterative solution of the covariance matrix. This procedure takes care of outliers in the data and in fact reduces the effect of outliers. Simulation result shows that the iteration converges after some steps. The consistency and asymptotic normality of the estimator follows from Huber's (1967) result.

148. Bhandary M. Detection of Outliers in Signal Processing and Detection of Number of Signals in Presence of Outliers. Technical Report #87-37, Center for Multivariate Analysis, September 1987.

Likelihood ratio tests for mean-slippage outliers(s) and Roy's (1953) union-intersection principle for dispersion-slippage outliers(s) are applied in the signal processing data. Procedures to get approximate critical values of the tests are given. Then likelihood ratio tests and intuitive tests are suggested for mean-slippage and dispersion-slippage outliers respectively to test the rank of Γ for the covariance matrix $(\Gamma + \sigma^2 I)$ in the presence of outliers.

Finally the estimate of rank of Γ is obtained through information criterion developed by Zhao, Krishnaiah and Bai (1986a,b) and through modified information criterion proposed by the author.

149. Bhandary M. Detection of the Number of Signals in the Presence of White-noise in Elliptically Symmetric Distribution. Technical Report #87-38, Center for Multivariate Analysis, September 1987.

A new method for the detection of the number of signals in the white-noise case when the underlying distribution is elliptically symmetric is proposed. This new method suggests an algorithm for the maximum likelihood estimation of parameters which involves iterative procedure. The estimate of parameters reduce the effect of outliers. Finally the estimate of number of signals is obtained through modified information criterion proposed by the author and the estimator is strongly consistent.

150. Bhandary M. Detection of the Number of Defective Components of Outlier Observations in Signal Processing Data. Technical Report #87-39, Center for Multivariate Analysis, September 1987.

Methods of detection of number of defective components in the outlier observations in signal processing data have been suggested. The problem is solved for both mean-slippage and dispersion-slippage outliers by Roy's (1953) and by modified information criterion proposed by the author. The estimators are strongly consistent.

151. Taniguchi, M., Zhao, L.C., Krishnaiah, P.R. and Bai, Z.D. Statistical Analysis of Dyadic Stationary Processes. Technical Report #87-40, Center for Multivariate Analysis, September 1987.

In this paper we consider a multiple dyadic stationary process with the Walsh spectral density matrix $f_{\Theta}(\lambda)$, where Θ is an unknown parameter vector. We define a quasi-maximum likelihood estimator $\hat{\Theta}$ of Θ and give the asymptotic distribution of $\hat{\Theta}$ under appropriate conditions. Then we propose an information criterion which determines the order of the model, and show that this criterion gives the consistent order estimate. As for a finite order dyadic autoregressive model, we propose a simpler order determination criterion, and discuss its asymptotic properties in detail. This criterion gives strong consistent order estimate. Also detections of signals for dyadic stationary processes will be discussed. In Section 6 we discuss testing whether an unknown parameter Θ satisfies a linear restriction. Then we give the asymptotic distribution of the likelihood ratio criterion under the null hypothesis.

152. Bai, Z.D., Krishnaiah, P.R. and Zhao, L.C. On Multiplicities of the Eigenvalues of Components of Covariance With Applications In Signal Processing. Technical Report #87-41, Center for Multivariate Analysis, September 1987.

In this paper, the authors investigated procedures for determination of the multiplicities of the eigenvalues of the covariance matrix of the random effects vector under multivariate random effects model.

**PART 3: LIST OF PAPERS WRITTEN WITH PARTIAL
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PUBLISHED PAPERS

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