

AD-A190 026

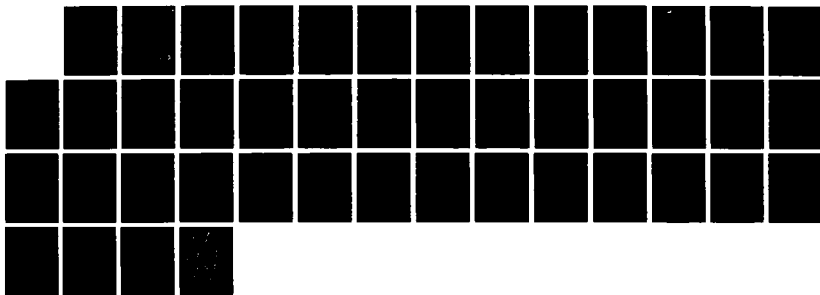
COMPUTATIONAL SCIENCES(U) ATLANTA UNIV GA K B BOTA
NOV 87 ARO-23145. 2-MA-H DARG29-85-G-0109

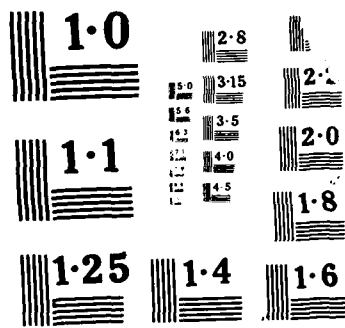
1/1

UNCLASSIFIED

F/G 12/1

ML





UNCLASSIFIED

MASTER COPY

FOR REPRODUCTION PURPOSES

16

AD-A190 826

REPORT DOCUMENTATION PAGE

Unclassified		1b. RESTRICTIVE MARKINGS		DTIC FILE COPY	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT			
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		Approved for public release; distribution unlimited.			
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S) AR2 23145.2-MA-H			
6a. NAME OF PERFORMING ORGANIZATION Atlanta University	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office			
6c. ADDRESS (City, State, and ZIP Code) Atlanta, Georgia 30314		7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U. S. Army Research Office	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER DAAG29-85-G-0109			
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211		10. SOURCE OF FUNDING NUMBERS			
		PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Computational Sciences					
12. PERSONAL AUTHOR(S) Kofi B. Bota					
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM 7/1/85 TO 9/30/87	14. DATE OF REPORT (Year, Month, Day) November 1987		15. PAGE COUNT 40	
16. SUPPLEMENTARY NOTATION The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.					
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)			
FIELD	GROUP	SUB-GROUP			
		Computational Sciences, Stochastic Differential Equations, Differential Equations, Integral Equations, Optimization Problems			
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
→ The research results are contained in the eleven papers published during the course of the research. Abstracts of these papers are contained in the final report of this document. → See next page (A)					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT		21. ABSTRACT SECURITY CLASSIFICATION			
☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS		Unclassified			
22a. NAME OF RESPONSIBLE INDIVIDUAL		22b. TELEPHONE (Include Area Code)		22c. OFFICE SYMBOL	

DTIC
ELECTE
FEB 02 1988
S H D

-A-

FINAL REPORT

1. ARO PROPOSAL NUMBER: 23145-MA-H
2. PERIOD COVERED BY REPORT: July 1, 1985 - September 30, 1987
3. TITLE OF PROPOSAL: Computational Sciences
4. CONTRACT OR GRANT NUMBER: DAAG29-85-G-0109
5. NAME OF INSTITUTION: Atlanta University
6. AUTHOR OF REPORT: Kofi B. Bota → Contents
7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING GRANT PERIOD, INCLUDING JOURNAL REFERENCES:

G.S. Ladde, and M. Sambandham, "Numerical Solutions to Stochastic Difference Equations;" to appear in *VII International Conference Volume on Trends in Nonlinear Analysis and Application* (ed. V. Lakshmikantham).

M. Sambandham, and N. Medhin, "Approximate Solution of Random Differential Equations;" to appear in *Conference Volume of the International Conference in Differential Equations*, University of Alabama, Birmingham, AL.

N. Medhin, M. Sambandham, and C. K. Zoltani, "Numerical Solution to a System of Random Volterra Integral Equations;" to appear in the *IVth Army Research Conference Volume in Applied Mathematics*, pp 123-142.

N. Medhin, "Optimal Processes Governed by Integral Equations with Unilateral Constraint;" to appear in *Journal of Mathematical Analysis and Applications*, Vol. 128, 1988.

M. Sambandham, V. Thangaraj, and K.B. Bota, "Numerical Solution of Random Love's Integral Equation;" submitted to the *Journal of Mathematical Analysis and Applications*.

N. Medhin, and M. Sambandham, "Mixed Min-Max Optimization Problem with Restrictions;" to appear in *Applied Mathematics and Computation*.

N. Medhin, "A Mixed Min-Max Control Problem Governed by Integral Equations;" to appear in *Journal of Mathematics and Computer Application*.

QUALITY
INSPECTED
2

GRA&I ☒
AB ☐
anced ☐
cation

ution/

Availability Codes

Dist	Avail and/or Special
A-1	

M. Sambandham, and N. Medhin, "Approximate Solution of Random Differential Equation;" to appear in *Lecture Notes in Mathematics*, Springer-Verlag (ed. Knowles and Saito).

N. Medhin, M. Sambandham, and C. K. Zoltani, "Numerical Solution to a System of Random Volterra Integral Equations I: Successive Approximation Method;" submitted to *International Journal of Mathematics and Computer Application*.

M. Sambandham, T. S. Srivatsan, and K.B. Bota, "A Method for the Numerical Solution of Random Cauchy Singular Integral Equation;" to appear in the *International Journal of Mathematics and Computer Applications*.

M. Sambandham, and K.B. Bota, "Approximation to Autoregressive Model with Stochastic Coefficients," *Third SIAM Conference on Discrete Mathematics*, May 14-16, p. A25, Clemson University.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

K.B. Bota, R. Mickens, M. Sambandham, and N. Medhin

Degrees Awarded: 2 - M.S. in Mathematics

TOKEN PERFORMANCE AND LOCAL AREA NETWORKS

by

P. Jackson
Department of Economics
Atlanta University
Atlanta, GA 30314

N. Medhin^{*}
Department of Mathematics and Computer Sciences
Atlanta University
Atlanta, GA 30314

M. Sambandham^{*}
Department of Mathematics and Computer Sciences
Atlanta University/Morehouse College
Atlanta, GA 30314

^{*} Supported by U. S. Army Research Contract No. DAAG29-85-G-0109

NUMERICAL SOLUTION OF A SYSTEM OF RANDOM VOLTERRA INTEGRAL

EQUATIONS I: SUCCESSIVE APPROXIMATION METHOD*

N. MEDHIN

Department of Mathematics and Computer Science
Atlanta University
Atlanta, GA 30314

M. SAMBANDHAM

Center for Computational Sciences
Atlanta University and
Department of Mathematics
Morehouse College
Atlanta, GA 30314

Abstract

In this article we discuss successive approximation method for a system of random Volterra integral equations. An example is presented to implement the theory. Kolmogorov-Smirnov test is used to fit a distribution of the solutions.

1. Introduction

The study of random Volterra integral equations and their applications play an important role in the area of probabilistic analysis. One of the main reasons is that integral equations are suitable for numerical treatment. For a recent survey of approximate solution of random integral equations we refer to Bharucha-Reid and Christensen [4]. For the numerical treatment of random integral equations we refer to Bharucha-Reid [3], Becus [2], Christensen and Bharucha-Reid [5], Lax [8-10], Tsokes and Padgett [14]. For a detailed survey of analytical and numerical methods

*Supported by U.S. Army Research Contract no. DAAG29-85-G-0109.

APPROXIMATE SOLUTION OF RANDOM
DIFFERENTIAL EQUATION*

M. Sambandham
Department of Mathematics
Morehouse College/Atlanta University
Atlanta, GA 30314

and

Negash Medhin
Department of Mathematics and Computer Science
Atlanta University
Atlanta, GA 30314

Abstract

Chebyshev method for solving random differential equation is presented. The convergence of the random coefficients of the Chebyshev series is established. Statistical properties of the random coefficients are discussed.

1. INTRODUCTION

In recent years, increasing interest in the numerical solution of random differential equations has led to the progressive development of several numerical methods. A large number of papers have appeared in the literature containing approximate solutions of random differential equations. For Newton's method, successive approximations, perturbation methods, method of moments, finite element methods and other methods on the approximate solution of random equations we refer to Bharucha-Reid [1]. Numerical methods of random polynomials can be found in Bharucha-Reid and Sambandham [2]. For a short and elegant note on several of these methods we also refer to Lax [9, 10]. Some analytical and numerical estimates on error estimates of stochastic differential equations are presented in Ladde et al. [6-8]. For other interesting numerical techniques we refer to Boyce [3] and [11]. Numerical treatment of Ito equations can be found in Klauder and Petersen [4], Rumelin [12] and Taley [15]. We notice that most of these numerical methods are successful

*Research supported by U.S. Army Research Office, Grant No. DAAG 29-85-G-0109.

*A Mixed Min-Max Control Problem
Governed by Integral Equations

Negash G. Medhin
Department of Mathematics and Computer Science
Atlanta University
Atlanta, GA 30314

A mixed min-max control problem is considered using relaxed controls. Mixed min-max problems governed by differential equations have been considered using Dubovitskii and Milyutin Theory and Convex Analysis [6]. Here we use relaxed controls and penalization to deal with a process governed by integral equations. We use a technique employed by us in dealing with control problems governed by integral equations [2].

*Research supported by U.S. Army Research Office

Grant Number: DAAG29-85-G-0109

NUMERICAL SOLUTIONS TO STOCHASTIC DIFFERENCE EQUATIONS*

G. S. Ladde
Department of Mathematics
University of Texas at Arlington
Arlington, TX

M. Sambandham
Center for Computational Sciences
Atlanta University and
Department of Mathematics
Morehouse College
Atlanta, GA

ABSTRACT

Statistical properties of the numerical solutions of random difference equations are estimated. By an application of variation of constant formula, error estimates between random solutions and smooth solutions (deterministic solution) are discussed.

1. INTRODUCTION

Numerical solutions of mathematical models of dynamical systems in applied mathematics demand a fair knowledge of stochastic difference equations. That is stochastic difference equations where some of the variables can change stochastically in time. More recently Ladde and Sambandham [5] developed several random difference inequalities which are very powerful tools to study stability properties of stochastic difference equations. By an application of variation of constant formula, error estimates between stochastic systems and the respective deterministic (smooth) systems are discussed in [6]. These estimates provide statistical properties of the upper bound for the error estimates. For related results we refer to Fai Ma and Caughey [2,3,4], Mann and Wald [7], Deller [1].

We organize our article as follows. In Section 2 we present the analytical upper bounds for error estimates. In Section 3 we include a few discussions on the numerical solutions. In addition, we have presented a few tables and figures to illustrate the behavior of the error between the mean of the random solutions and deterministic solutions. We

*Research supported by U.S. Army Research Office Grant Numbers
DAAG 29-85-G-0109 and DAAG 29-84-G-0060.

NUMERICAL SOLUTION TO A SYSTEM OF
RANDOM VOLTERRA INTEGRAL EQUATIONS*

N. Medhin¹, M. Sambandham² and C. K. Zoltani³

Abstract

In this article we present a brief summary of the numerical solution of a system of random Volterra integral equations. The methods we use are (i) Newton's method and (ii) successive approximation method. Based on the simulation, we discuss the mean and variance of the solution of a system of random Volterra integral equations.

*Supported by U.S. Army Research Contract No. DAAG29-85-G-0109.

¹Department of Mathematics and Computer Science, Atlanta University, Atlanta, GA 30314.

²Center for Computational Sciences, Atlanta University, Atlanta, GA 30314; Department of Mathematics, Morehouse College, Atlanta, GA 30314.

³Ignition and Combustion Branch, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland 21005.



PARIS - LA VILLETTE
Cité des Sciences et de l'Industrie

INSTRUCTIONS

(For Contributed Papers and Poster Presentations)

1. Please use an elite typing element and carbon ribbon, if available.
2. Begin typing the abstract title, text, and author information directly below the heading captions. Type flush left, single-spaced within the space designated. Do not indent. (See enclosed sample.)
3. Abstract title should be underlined. Capitalize the first letter of all words, except articles, prepositions and conjunctions.
4. Abstract should not exceed 100 words. Please state the problems to be addressed, their relevance, the methodology and results. References, if necessary, should be in the body of the abstract. Formulas should be kept to a minimum—please, no vertical fractions, multiple subscripts, or handwritten symbols. Abstracts must be submitted on this form and will be printed as received. Errors in the text are the author's responsibility.
5. For two or more authors of the same affiliation and address, type the latter only once, directly below the authors' names.
6. You must type your abstract directly on this form. Abstracts that are cut and pasted onto this form will be returned.

**IMPORTANT: ABSTRACTS MUST
BE MAILED UNFOLDED.**

PLEASE ANSWER THE FOLLOWING:

1. Your preference for presentation.
Check one:
(See reverse side for explanation)
☒ Prefer Standard Presentation.
☐ Prefer Poster Presentation.
2. Visual Equipment
Standard Presentation
☒ Overhead Projector
☐ 2"x2" Slide Projector (35mm)
☐ None
Poster Presentation
☐ Corkboard
☐ Easel for Flip Chart
☐ None
3. Subject Classification
(Choose one or two from list on reverse side)
65 60

GAS FLOWS GENERATED BY SOLID-PROPELLANT
BURNING UNDER RANDOM ENVIRONMENT.

In the literature it is customary to assume that the local mass source term, momentum source term and energy source term to be deterministic. In this paper we consider some of these source terms to be random. We assume that the flow generated by gasification of the solid propellant material is one dimensional and unsteady. By an application of random choice method, we discuss the statistical properties of all field variables.

M. SAMBANDHAM
Center for Computational Sciences
Atlanta University
Atlanta, GA 30314
and Department of Mathematics
Morehouse College
Atlanta, GA 30314

K. B. BOTA
Department of Physics
Atlanta University
Atlanta, GA 30314

Deadline for Submission of Abstract

NOVEMBER 28th, 1986

(PLEASE COMPLETE PAGE 2)

Optimal Processes Governed by Integral Equations
with Unilateral Constraint

Negash G. Medhin*

An optimal process with unilateral constraint is considered using relaxed controls. Existence of optimal control is automatic. Further, problems with linear and certain quadratic cost functions with linear integral and state constraints are such that the controls are actually ordinary, i.e., bounded measurable functions. In addition, if the control set is a convex polyhedron the optimal controls take their values in the vertices of the polyhedron. Control processes governed by integral equations have been considered in [2]. However, in [2] existence is not shown for the general problem and optimality conditions are not available for the problem when state constraints are present. Furthermore, the work in [2] has some errors as pointed out in [3]. Finally we point out the basic idea in our approach in this paper is the same as in [6].

*Associate Professor of Mathematics, Atlanta University, Atlanta, GA 30314. Research supported by U.S. Army Research Office Grant #DAAG-29-85-G-0109.

*A Mixed Min-Max Control Problem
Governed by Integral Equations

Negash G. Medhin
Department of Mathematics and Computer Science
Atlanta University
Atlanta, GA 30314

A mixed min-max control problem is considered using relaxed controls. Mixed min-max problems governed by differential equations have been considered using Dubovitskii and Milyutin Theory and Convex Analysis [6]. Here we use relaxed controls and penalization to deal with a process governed by integral equations. We use a technique employed by us in dealing with control problems governed by integral equations [2].

*Research supported by U.S. Army Research Office

Grant Number: DAAG29-85-G-0109

Mixed Min-Max Optimization Problem with Restrictions *

Negash G. Medhin
Department of Mathematics
and Computer Science
Atlanta University
Atlanta, GA 30314

M. Sambandham
Department of Mathematics
and Computer Science
Atlanta University and
Morehouse College
Atlanta, GA 30314

Introduction

We consider a min-max optimization problem using relaxed controls. Such problems have been considered using Dubovitskii and Milyutin Theory and convex analysis [5]. We reformulate the problem where a unilateral constraint is added. We deal with the reformulated problem using penalization [2], [3]. Our procedure produces more optimality conditions than [5] and also more insight into [5]. In the special case where the controls appear linearly and the control sets are convex polyhedra the relaxed controls are ordinary controls taking values in the vertices of the polyhedra. Finally, we present examples where we demonstrate how the optimal controls and trajectories could be calculated and some that are numerically worked out.

KEY WORDS: Relaxed controls, unilateral constraint,
penalization

* Supported by U.S. Army Research Contract No. DAAG29-85-G-0109.

NUMERICAL SOLUTION TO A SYSTEM OF
RANDOM VOLTERRA INTEGRAL EQUATIONS*

N. Medhin¹, M. Sambandham² and C. K. Zoltani³

Abstract

In this article we present a brief summary of the numerical solution of a system of random Volterra integral equations. The methods we use are (i) Newton's method and (ii) successive approximation method. Based on the simulation, we discuss the mean and variance of the solution of a system of random Volterra integral equations.

*Supported by U.S. Army Research Contract No. DAAG29-85-G-0109.

¹Department of Mathematics and Computer Science, Atlanta University, Atlanta, GA 30314.

²Center for Computational Sciences, Atlanta University, Atlanta, GA 30314; Department of Mathematics, Morehouse College, Atlanta, GA 30314.

³Ignition and Combustion Branch, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland 21005.

APPROXIMATE SOLUTION OF RANDOM
DIFFERENTIAL EQUATION*

M. Sambandham
Department of Mathematics
Morehouse College/Atlanta University
Atlanta, GA 30314

and

Negash Medhin
Department of Mathematics and Computer Science
Atlanta University
Atlanta, GA 30314

Abstract

Chebyshev method for solving random differential equation is presented. The convergence of the random coefficients of the Chebyshev series is established. Statistical properties of the random coefficients are discussed.

1. INTRODUCTION

In recent years, increasing interest in the numerical solution of random differential equations has led to the progressive development of several numerical methods. A large number of papers have appeared in the literature containing approximate solutions of random differential equations. For Newton's method, successive approximations, perturbation methods, method of moments, finite element methods and other methods on the approximate solution of random equations we refer to Bharucha-Reid [1]. Numerical methods of random polynomials can be found in Bharucha-Reid and Sambandham [2]. For a short and elegant note on several of these methods we also refer to Lax [9, 10]. Some analytical and numerical estimates on error estimates of stochastic differential equations are presented in Ladde et al. [6-8]. For other interesting numerical techniques we refer to Boyce [3] and [11]. Numerical treatment of Ito equations can be found in Klauder and Petersen [4], Rumelin [12] and Taley [15]. We notice that most of these numerical methods are successful

*Research supported by U.S. Army Research Office, Grant No. DAAG 29-85-G-0109.

TOKEN PERFORMANCE AND LOCAL AREA NETWORKS

by

P. Jackson
Department of Economics
Atlanta University
Atlanta, GA 30314

N. Medhin^{*}
Department of Mathematics and Computer Sciences
Atlanta University
Atlanta, GA 30314

M. Sambandham^{*}
Department of Mathematics and Computer Sciences
Atlanta University/Morehouse College
Atlanta, GA 30314

^{*} Supported by U. S. Army Research Contract No. DAAG29-85-G-0109

A METHOD FOR THE NUMERICAL SOLUTION OF
RANDOM CAUCHY SINGULAR INTEGRAL EQUATION

M. Sambandham
Department of Mathematics
Atlanta University and
Morehouse College
Atlanta, GA 30314

T. S. Srivatsan
Georgia Institute of
Technology
Atlanta, GA 30332

and

K. B. Bota
Department of Physics
Atlanta University
Atlanta, GA 30314

ABSTRACT

The solution of several elasticity problems, and particularly crack problems, can be reduced to the solution of one-dimensional singular integral equations with a Cauchy-type kernel. In this paper, we present a method for the numerical solution of a random singular integral equation of the Cauchy-type. To illustrate this method, it is applied to the random singular integral equation that arises in the problem of periodic array of straight cracks inside an infinite isotropic elastic medium and subjected to random pressure distribution along the crack edges. The statistical properties of the random solution are evaluated numerically, and used to determine the stress intensity factor at the crack tips. Results of this study highlight the advantage of using this method to solve a random Cauchy-type singular

*Research supported by U.S. Army Research Office under Grant numbers DAAG 29-85-G-0109 and National Science Foundation Grant number PRM-8215949.

integral equation. The method is applied to the crack problem subjected to different forcing functions.

1. INTRODUCTION

In recent years, an unprecedented widespread interest in the numerical solution of singular integral equations of the Cauchy-type has resulted in the development of several numerical methods [1-5] besides stimulating a mathematical interest in their numerical analysis. Since the pioneering work by Erdogan [1] in 1969 and subsequently by Erdogan and co-workers [3], a large number of papers have appeared in the literature concerning singular integral equations that arise in several fields of mathematical physics and engineering. The solution methods developed have aimed at reducing the singular integral equation to a system of linear algebraic equations which can be easily solved to give an approximate expression for the solution of a singular integral equation or solutions in the case of a system of singular integral equations.

Several of the numerical methods developed over the years were found to have disadvantages due to their inherent complexity, low degree of accuracy and limitations as regards the classes of singular integral equations to which they were applicable. It is known that numerical solutions of Fredholm integral equations by reduction to a system of linear equations could be extended to the case of Cauchy singular integral

equations (CSIEs), that is to the case when the kernels of the Fredholm integral equations have Cauchy-type singularities. Since Cauchy-type singular integral equations on closed contours have been treated elsewhere [7, 8], in this paper we confine our attention to equations on intervals (a,b) of the real line, which may be either finite or infinite. In particular, we consider linear equations since the theory for these is fairly complete and well developed. The most frequently used technique for the numerical solution of linear singular integral equations consists in approximating the integral terms by using an approximate numerical integration rule and applying the singular integral equation at appropriately selected collocation points [2-4, 9]. One determination of the unknown function at the abscissas used, the function is approximated along the whole integration interval [10].

A wide body of literature pertaining to the numerical solution of singular integral equations of the deterministic type is succinctly summarized by Ergodan, et al. [3], Golberg [11], and Theocaris [6]. In referring to singular integral equations, Erdogan, Golberg and Theocaris in their articles imply Cauchy-type singular integrals and Cauchy-type singular integral equations. We refer to Bharucha-Reid and Bharucha-Reid and Christensen [12, 13] for surveys of the different analytical methods for the solution of random integral equations. Most recently, Christensen and Bharucha-Reid [14] developed numerical solution procedures for Fredholm equations

with random kernels, random right hand side or both and for Fredholm equations with random degenerate kernels [15]. The Cauchy-type singular integral equations which arise frequently in the fields of aerodynamics, hydrodynamics, solid mechanics, elasticity and fracture mechanics, are closely related to Fredholm-type equations. There exists a pragmatic need to develop numerical techniques in order to be able to study the characteristics of the random singular integral equations. The rationale for this study was to extend the concept for the solution of random Fredholm integral equations [14-16], to the solution of random singular integral equations of the Cauchy-type that arise in crack problems in the classical theory of elasticity. In particular, we consider the problem of an array of periodic cracks inside an infinite isotropic elastic medium and subjected to random pressure distribution along the crack edges, i.e., the case of a random forcing function. The statistical properties of the solution are evaluated and the results obtained for the random equation are compared with those of the deterministic equation. Further we also discuss the distribution of the stress intensity factor for different sample sizes.

2. SINGULAR INTEGRAL EQUATIONS

Consider the singular integral equation of the form

$$(2.1) \quad \frac{1}{\pi} \int_{-1}^1 \frac{\phi(t)}{t-x} dt + \int_{-1}^1 k(x,t) \phi(t) dt = f(x), \quad -1 < x < 1,$$

where $k(x,t)$ is a regular kernel. $\phi(t)$ is the unknown function proportional to the crack tip opening displacement and $f(x)$ is the known function representing pressure distribution along the crack.

Let $g(t) = \phi(t)/z(t)$, where $z(t) = (1-t^2)^{-1/2}$ and

$$(2.2) \quad \int_{-1}^1 \phi(t) dt = 0.$$

We consider (2.1) together with (2.2) and replace $g(t)$ by a new unknown function $G(x)$ defined by the first integral of (2.1), that is,

$$(2.3) \quad G(x) = \frac{1}{\pi} \int_{-1}^1 (1-t^2)^{-1/2} \frac{g(t)}{t-x} dx, \quad -1 < x < 1.$$

Since $g(t)$ satisfies (2.2), we can express $g(t)$ in terms of $G(x)$ by

$$(2.4) \quad g(t) = \frac{1}{\pi} \int_{-1}^1 (1-y^2)^{1/2} \frac{G(y)}{t-y} dy, \quad -1 \leq t \leq 1.$$

By an application of (2.3) and (2.4), we can write (2.1) in the form

$$(2.5) \quad G(x) + \int_{-1}^1 (1-y^2)^{1/2} K(y,x) G(y) dy = f(x) \quad -1 < x < 1,$$

where

$$(2.6) \quad K(y,x) = \frac{1}{\pi} \int_{-1}^1 (1-t^2)^{-1/2} \frac{K(t,x)}{t-y} dt$$

We remark that (2.5) is another form of (2.1). The equation (2.5) is Fredholm equation of second type and its numerical solution can be found by quadrature method.

The probabilistic analogue of (2.5) is the random equation. We call an equation random if certain components such as coefficients, kernels, nonhomogeneous terms or forcing functions and initial and or boundary data are random functions. The random version of (2.5) is of the form

$$(2.7) \quad G(x, \omega) + \int_{-1}^1 (1-y^2)^{1/2} K(y, x, \omega) G(y, \omega) dy = f(x, \omega) \quad -1 < x < 1$$

where $K(y, x, \omega)$ is the random kernel and $f(x, \omega)$ is the random forcing term. The parameter ω is an element of a given probability space (Ω, A, P) . Here $G(x, \omega)$ is the random function referred to as the solution or output. For numerical solution of (2.7) we refer to [14, 15, 16].

In this article we consider a periodic array of cracks along a straight line in an infinite isotropic elastic medium under plane strain or generalized plane stress. The length of each crack is assumed to be '2a' and the period of the array is 'b'. For the case of a constant compressive loading distribution along the crack edges, Ioakimidis [17] found the singular integral equation to be

$$(2.8) \quad \frac{a}{b} \int_{-1}^1 (1-t^2)^{-1/2} \cot\left(\frac{\pi a(t-x)}{b}\right) g(t) dt = f(x) \quad -1 < x < 1$$

which is of the form (2.1) with

$$(2.9) \quad k(t, x) = \frac{a}{b} \cot\left(\frac{\pi a(t-x)}{b}\right) - \frac{1}{\pi(t-x)},$$

where cracks are assumed to be loaded by constant compressive

loading distribution $f(x)$ along both crack edges. If the loading distribution $f(x)$ is random, then (2.8) is a random singular integral equation. If we denote random loading distribution by $f(x, \omega)$ then the random version of (2.8) is

$$(2.10) \quad \frac{a}{b} \int_{-1}^1 (1-t^2)^{-1/2} \cot\left(\frac{\pi a(t-x)}{b}\right) g(t, \omega) dt = f(x, \omega) \quad -1 < x < 1$$

Equation (2.10) together with (2.2) can be solved numerically by methods suggested in [12-17].

3. NUMERICAL TREATMENT

In this section we discuss the numerical solution of (2.10). We assume the ratio $a/b = .4$ and the successive cracks lie close to each other. We denote the value of the stress intensity factor by $K(\pm 1, \omega)$ at the crack tips $t = \pm 1$. It is given by

$$(3.1) \quad K(\pm 1, \omega) = \pm g(\pm 1, \omega) \\ \approx \frac{1}{N} \sum_{k=1}^N (1 \pm x_k) G(x_k, \omega).$$

where x_k are suitable nodes

$$x_k = \cos(k-0.5) \frac{\pi}{N}$$

which we use to solve (2.7). The unknown random function $g(x, \omega)$ in (2.10) is evaluated at different pressure distributions along the crack edges, namely,

Example 3.1. $f(x, \omega) \in N(1, \sigma^2)$

Example 3.2. $f(x, \omega) \in N(e^x, \sigma^2)$

Example 3.3. $f(x, \omega) \in N(e^x, \sigma^2 e^{2x})$

A sample of 5000 standard normal random numbers are generated from IMSL routines (GGNML). The resulting random numbers are used to generate the random input functions, namely, the random pressure distributions in the manner described above. For each random input function, equation (2.7) and hence equation (2.10) is solved numerically to obtain the random output function, and the stress intensity factor. At the crack tips, $K(\pm 1, \omega)$ are computed based on the relationship in equation (3.1).

4. RESULTS AND DISCUSSION

Accurate estimates of the stress intensity factor at the crack tips under more realistic pressure distributions is essential for the reliable prediction of fatigue crack growth rates in structural members. Several analytical techniques are presently available to obtain solutions to random Cauchy-type singular integral equations that arise in crack problems. Most of these numerical techniques for obtaining the stress intensity factors for crack and elasticity problems have been concerned with pressure distributions that are purely deterministic in nature.

In dealing with random equations, the main objectives to be concerned with are: (1) determination of the statistical properties of the random solution such as its expectation,

variance and higher moments; (2) to establish the relationship between the expected solution of the random equation and the solution of the deterministic equation or the mean solution [18]; (3) determining the distribution of the solutions, and (4) discussing the limiting properties of the solutions. Of particular interest is the relationship between the statistical properties of the solution and the statistical properties of the random input function introduced into the equation. Table I-V summarize the statistical properties of $K(\pm 1, \omega)$.

The distribution of stress intensity factor $K(\pm 1, \omega)$ for the functions $f(x, \omega)$ are shown in Figures 1-30. These figures show that as the value of N increases, the distribution of $K(\pm 1, \omega)$ is approximately normal with $E[K(\pm 1, \omega)]$ being the solution of the mean equation. It is observed that as the value of N increases, the distribution of the stress intensity factor $K(\pm 1, \omega)$ is normal as long as the forcing function $f(x, \omega)$ is normal.

5. CONCLUSIONS

1. The foregoing analysis demonstrates a fairly simple yet useful method for the solution of random Cauchy-type singular integral equation that arises in a crack problem in the classical theory of elasticity. The solution of the random equation helps in the accurate evaluation of the stress intensity factor at the crack

tips under more realistic input or forcing functions.

2. The distributions of the stress intensity factors $K(\pm 1, \omega)$ are approximately normal when the forcing functions are normal.
3. The results of this study highlight the influence of the nature of the random input function on the stress intensity factor.

TABLE I

$f(x, \omega) \in N(1, 0.02^2)$			
Sample Size	N	Mean of $K(1, \omega)$	Variance of $K(1, \omega)$
500	2	1.634664	0.001065
	3	1.575876	0.000896
	5	1.563470	0.000866
	7	1.566512	0.000969
	9	1.566098	0.000910
1000	2	1.635569	0.001079
	3	1.575410	0.000960
	5	1.563799	0.000931
	7	1.567005	0.001003
	9	1.565704	0.000940
2000	2	1.636609	0.001098
	3	1.575890	0.000962
	5	1.564721	0.000965
	7	1.566515	0.000993
	9	1.564965	0.000987
3000	2	1.637003	0.001088
	3	1.575030	0.000962
	5	1.564731	0.000977
	7	1.566254	0.000990
	9	1.564981	0.000974
4000	2	1.636695	0.001094
	3	1.575152	0.000984
	5	1.565015	0.000977
	7	1.566710	0.000985
	9	1.564917	0.000983
5000	2	1.636726	0.001089
	3	1.575127	0.000969
	5	1.564924	0.000989
	7	1.566634	0.000969
	9	1.564916	0.000983

TABLE II

$f(x, \omega) \in N(e^x, 0.02^2)$			
Sample Size	N	Mean of $K(1, \omega)$	Variance of $K(1, \omega)$
500	2	2.567233	0.000985
	3	2.548983	0.000967
	5	2.538348	0.000880
	7	2.539721	0.000997
	9	2.539926	0.001007
1000	2	2.566281	0.001026
	3	2.548836	0.001010
	5	2.537966	0.000897
	7	2.540643	0.000933
	9	2.539931	0.000971
2000	2	2.566361	0.001063
	3	2.548886	0.001033
	5	2.538958	0.000955
	7	2.540021	0.000937
	9	2.539534	0.000992
3000	2	2.566199	0.001082
	3	2.549063	0.001023
	5	2.539601	0.000987
	7	2.539973	0.000944
	9	2.539349	0.000992
4000	2	2.565732	0.001077
	3	2.548748	0.001026
	5	2.539587	0.000976
	7	2.539796	0.000949
	9	2.539454	0.001007
5000	2	2.565745	0.001069
	3	2.548838	0.001015
	5	2.539710	0.000975
	7	2.539714	0.000955
	9	2.539338	0.001008

TABLE III

$f(x, \omega) \in N(e^x, 0.02^2 e^{2x})$			
Sample Size	N	Mean of $K(1, \omega)$	Variance of $K(1, \omega)$
500	2	2.565219	0.002773
	3	2.544691	0.002617
	5	2.539554	0.002960
	7	2.541145	0.002661
	7	2.542362	0.002784
1000	2	2.565728	0.002727
	3	2.546564	0.002680
	5	2.541254	0.002728
	7	2.541461	0.002704
	9	2.542656	0.002535
2000	2	2.565767	0.002784
	3	2.548370	0.002618
	5	2.539482	0.002647
	7	2.540158	0.002643
	9	2.540600	0.002528
3000	2	2.565145	0.002694
	3	2.547493	0.002589
	5	2.539537	0.002630
	7	2.539658	0.002603
	9	2.541302	0.002609
4000	2	2.565149	0.002677
	3	2.547726	0.002611
	5	2.539973	0.002641
	7	2.538959	0.002602
	9	2.540734	0.002614
5000	2	2.564995	0.002644
	3	2.547745	0.002592
	5	2.539966	0.002628
	7	2.539020	0.002585
	9	2.540420	0.002630

TABLE IV

$f(x, \omega) \in N(e^{-x}, 0.02^2)$			
Sample Size	N	Mean of $K(-1, \omega)$	Variance of $K(-1, \omega)$
500	2	1.562386	0.000996
	3	1.478523	0.001107
	5	1.465161	0.000979
	7	1.466776	0.001004
	9	1.469071	0.000901
1000	2	1.563262	0.001020
	3	1.479762	0.001065
	5	1.465880	0.001015
	7	1.466557	0.000990
	9	1.468161	0.000939
2000	2	1.563492	0.001015
	3	1.479095	0.000998
	5	1.466543	0.001003
	7	1.466125	0.000990
	9	1.468237	0.000972
3000	2	1.562768	0.001029
	3	1.478868	0.000993
	5	1.467130	0.000992
	7	1.466727	0.000992
	9	1.468084	0.000990
4000	2	1.562527	0.001035
	3	1.478661	0.000985
	5	1.467698	0.000993
	7	1.466606	0.000992
	9	1.467634	0.000986
5000	2	1.562371	0.001046
	3	1.478330	0.000980
	5	1.467851	0.000987
	7	1.466590	0.000985
	9	1.467648	0.000978

TABLE V

$f(x, \omega) \in N(e^{-x}, 0.02^2 e^{-2x})$			
Sample Size	N	Mean of $K(-1, \omega)$	Variance of $K(-1, \omega)$
500	2	1.563375	0.000957
	3	1.480107	0.000776
	5	1.468341	0.000749
	7	1.467878	0.000831
	9	1.467650	0.000818
1000	2	1.562688	0.000943
	3	1.479829	0.000833
	5	1.467521	0.000833
	7	1.466508	0.000835
	9	1.468472	0.000826
2000	2	1.561791	0.000963
	3	1.479141	0.000852
	5	1.467389	0.000858
	7	1.467030	0.000867
	9	1.467370	0.000843
3000	2	1.561813	0.000973
	3	1.478927	0.000861
	5	1.467764	0.000881
	7	1.466812	0.000870
	9	1.467162	0.000861
4000	2	1.562072	0.000976
	3	1.478724	0.000876
	5	1.467863	0.000880
	7	1.467345	0.000872
	9	1.467029	0.000851
5000	2	1.561947	0.000975
	3	1.478948	0.000869
	5	1.467711	0.000880
	7	1.467042	0.000875
	9	1.466858	0.000842

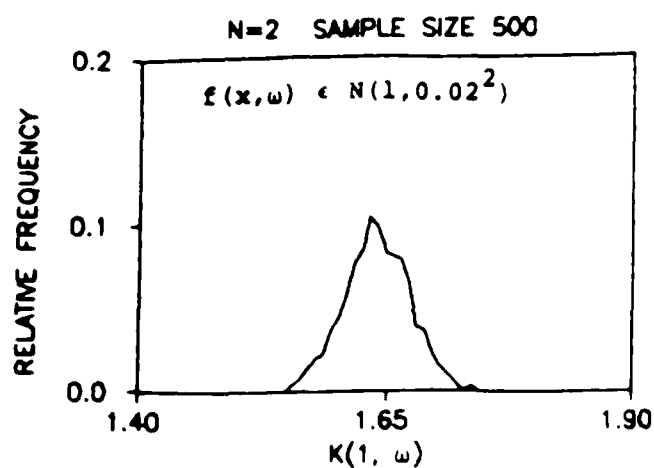


Fig. 1.

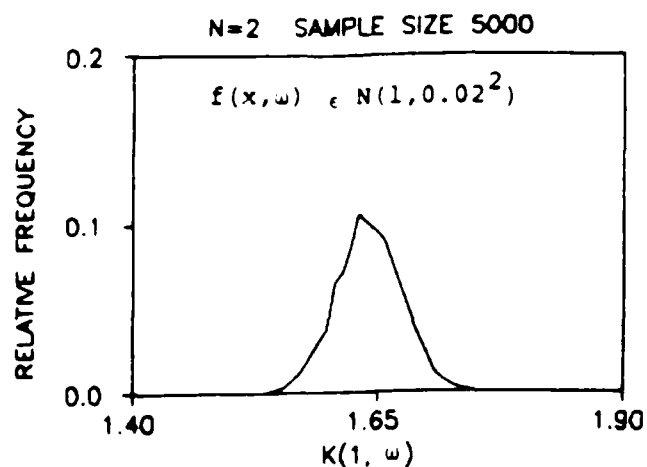


Fig. 2.

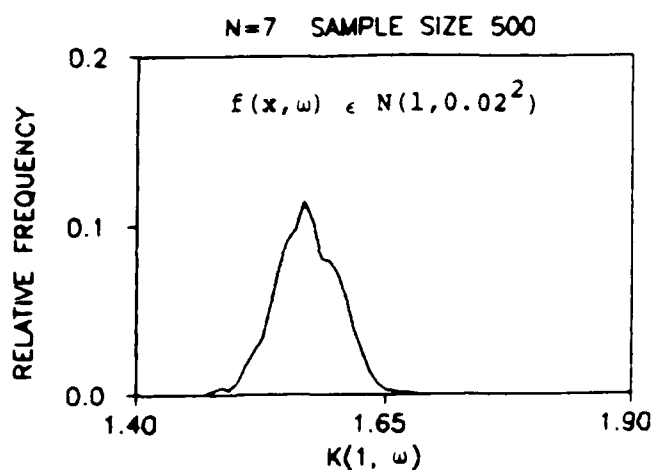


Fig. 3.

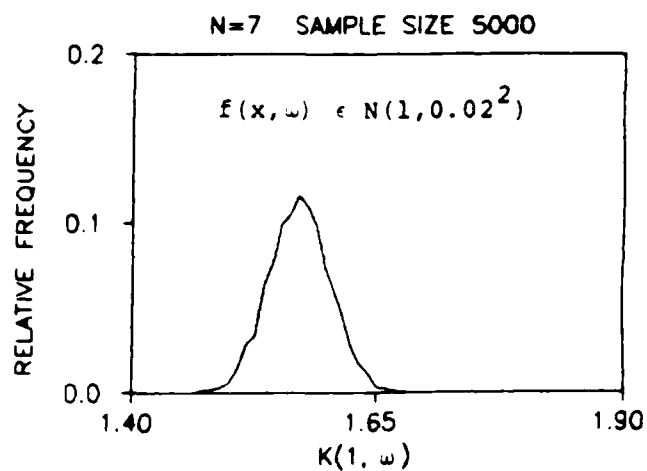


Fig. 4.

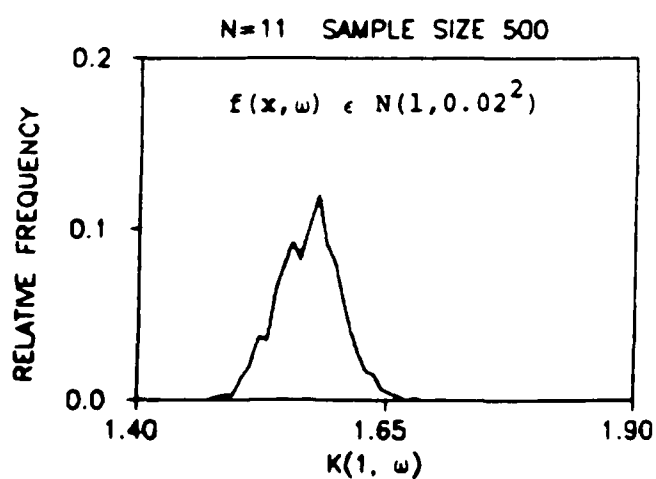


Fig. 5.

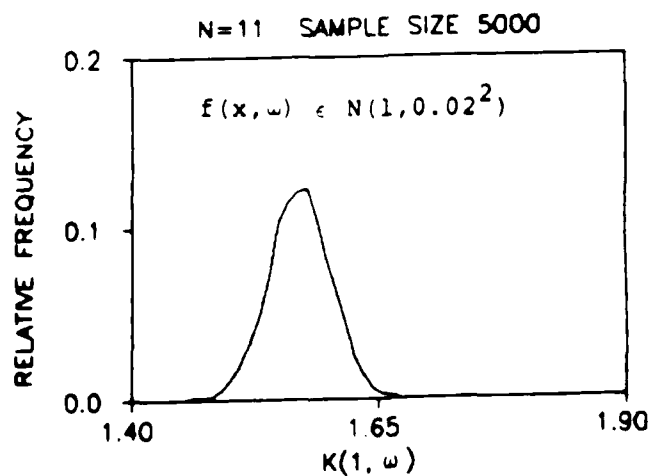


Fig. 6.

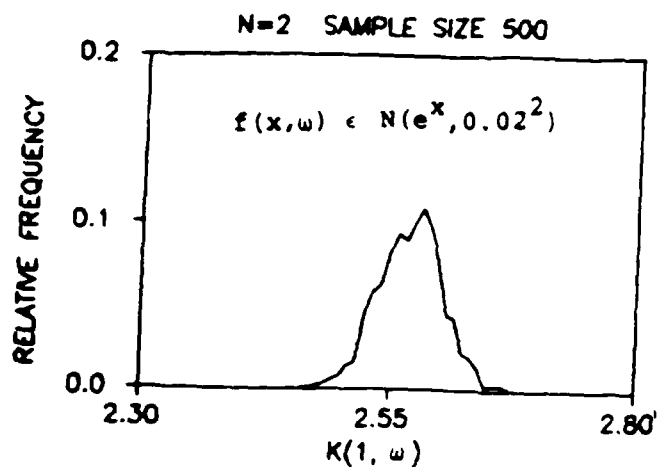


Fig. 7.

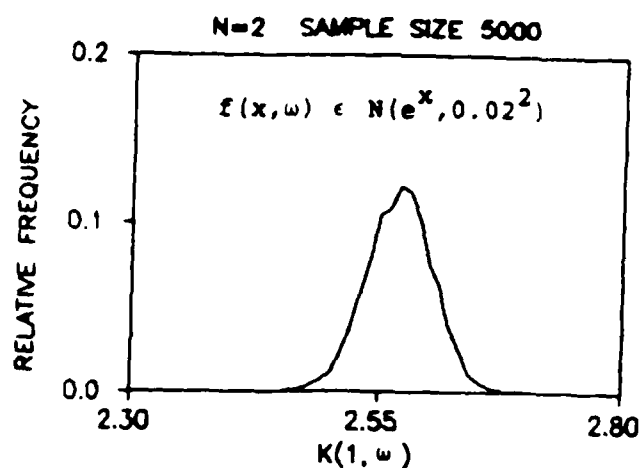


Fig. 8.

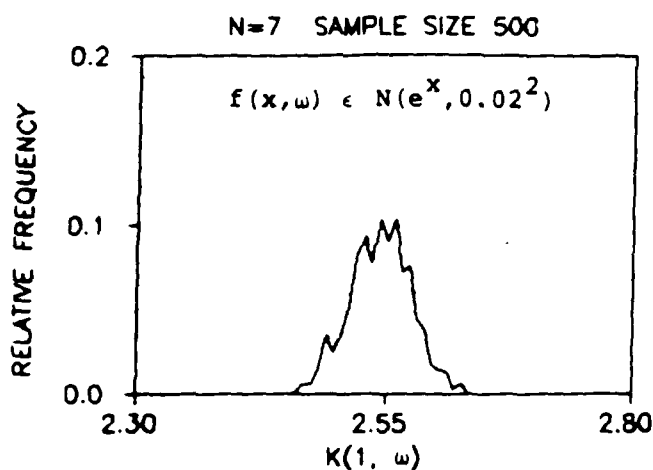


Fig. 9.

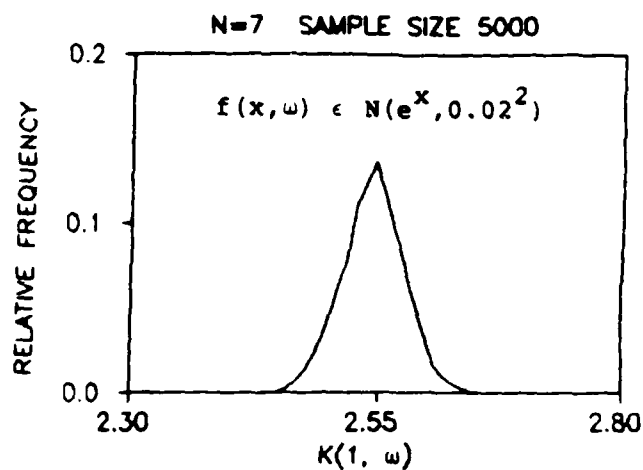


Fig. 10.

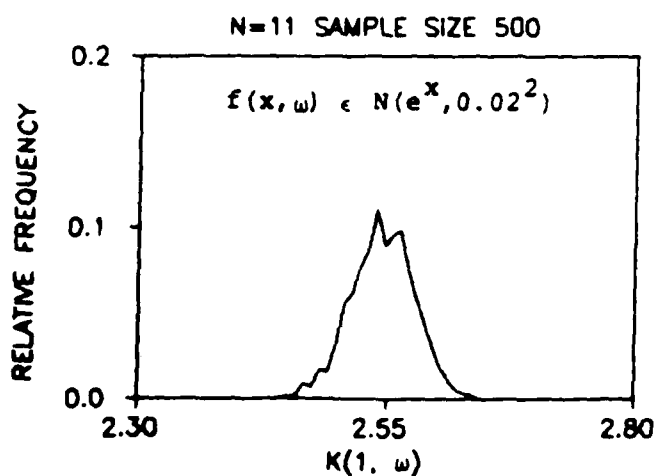


Fig. 11.

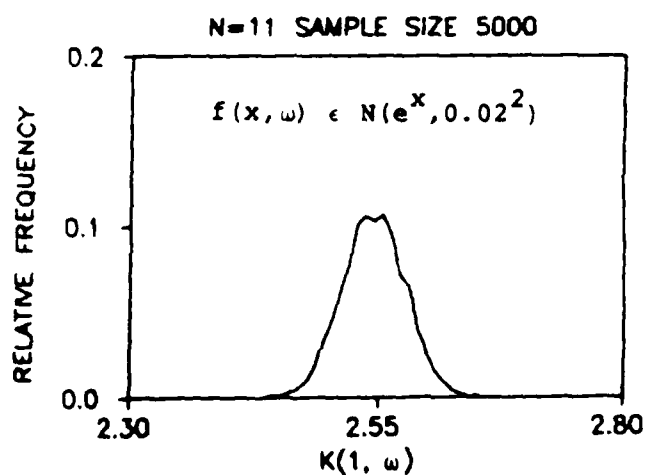


Fig. 12.

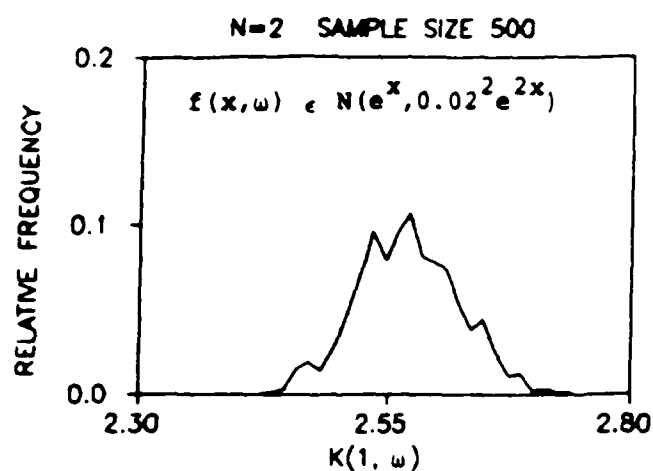


Fig. 13.

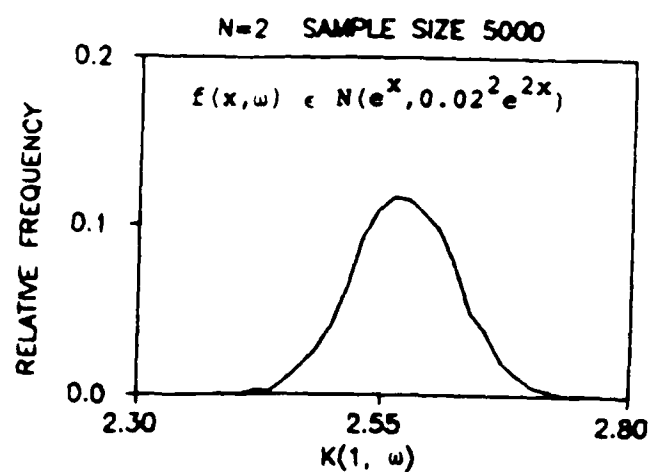


Fig. 14.

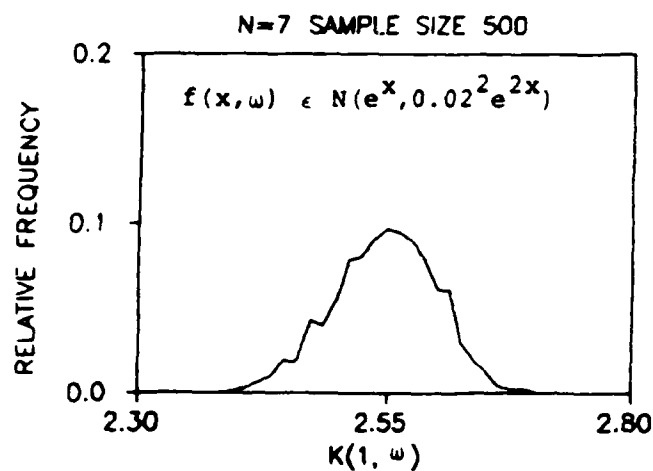


Fig. 15.

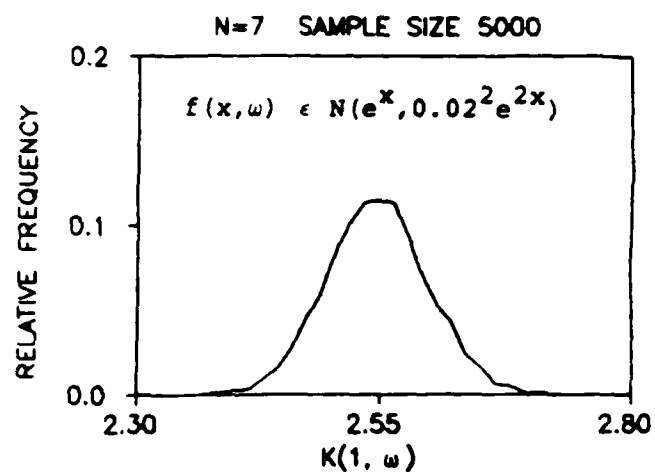


Fig. 16.

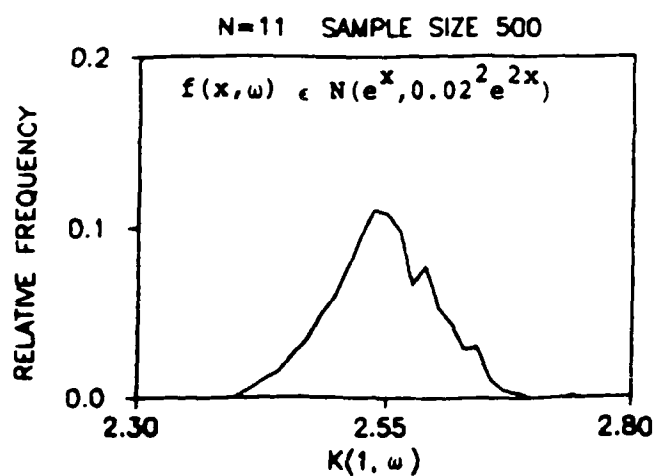


Fig. 17.

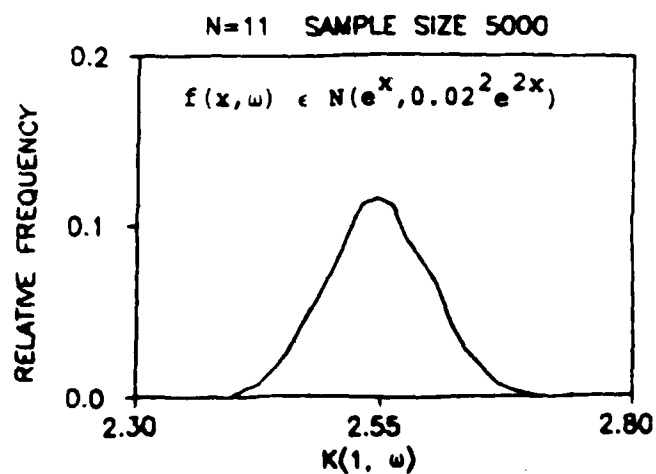


Fig. 18.

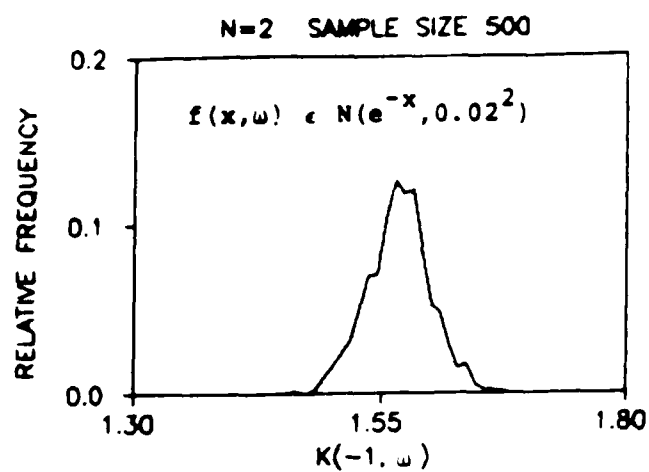


Fig. 19.

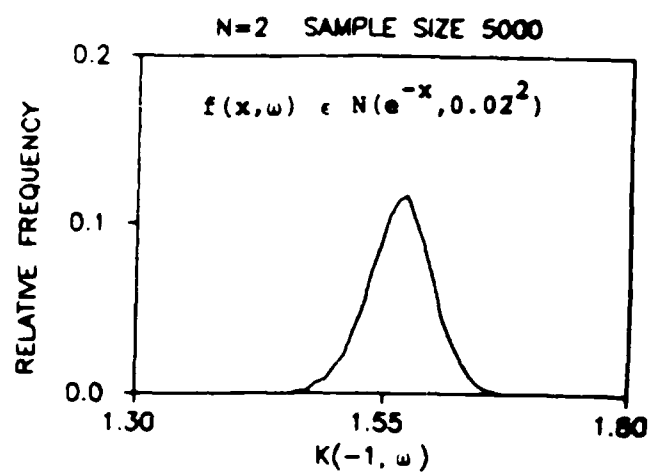


Fig. 20.

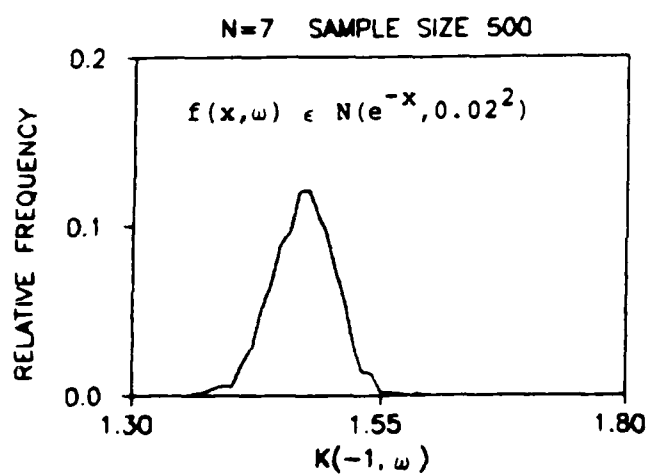


Fig. 21.

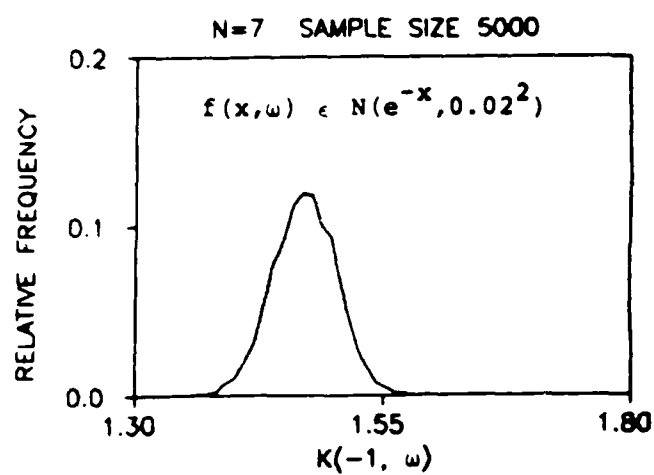


Fig. 22.

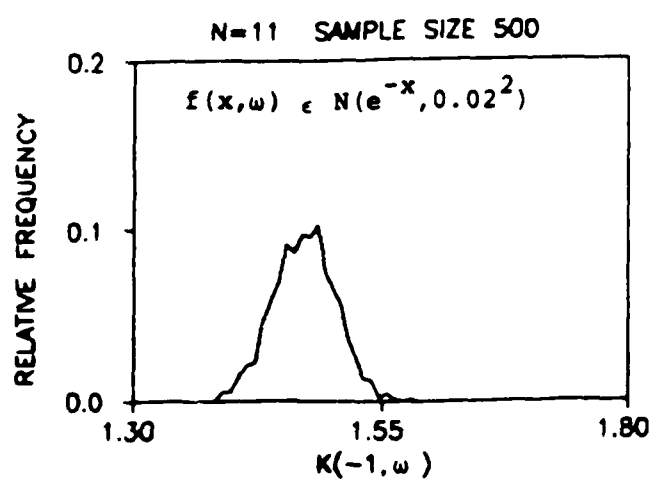


Fig. 23.

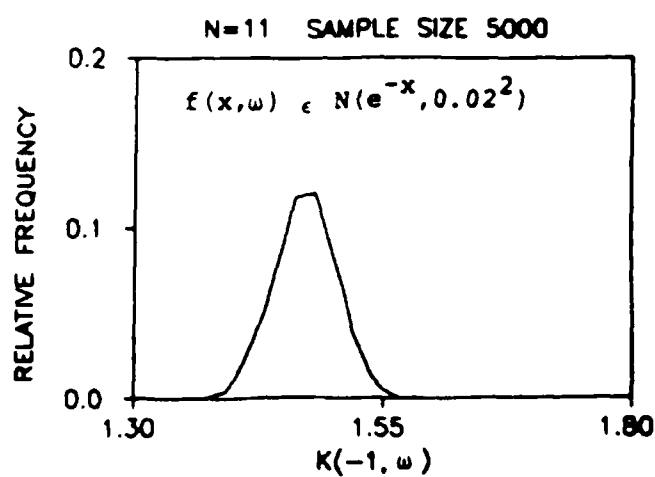


Fig. 24.

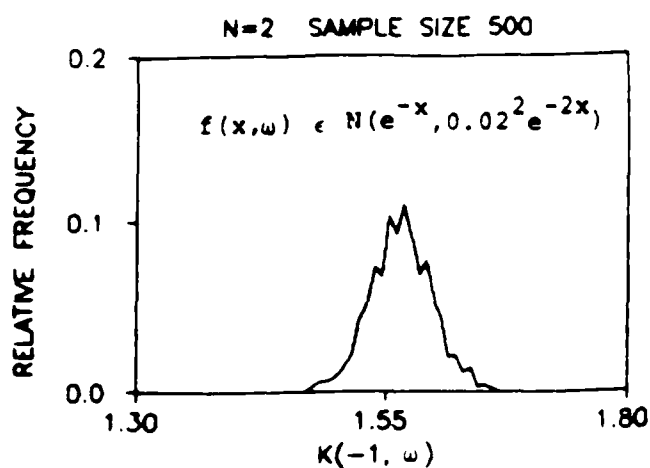


Fig. 25.

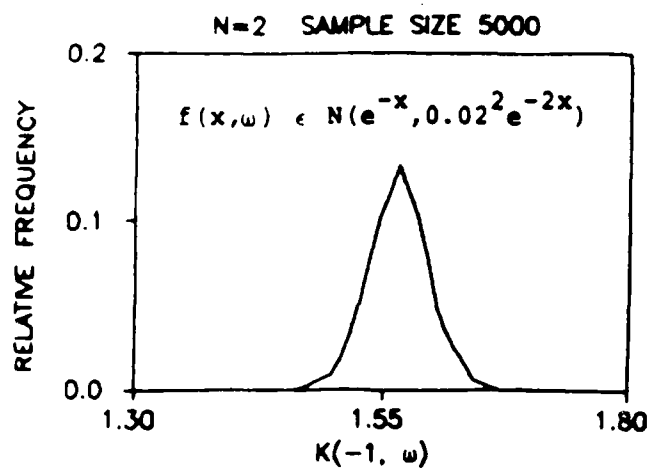


Fig. 26.

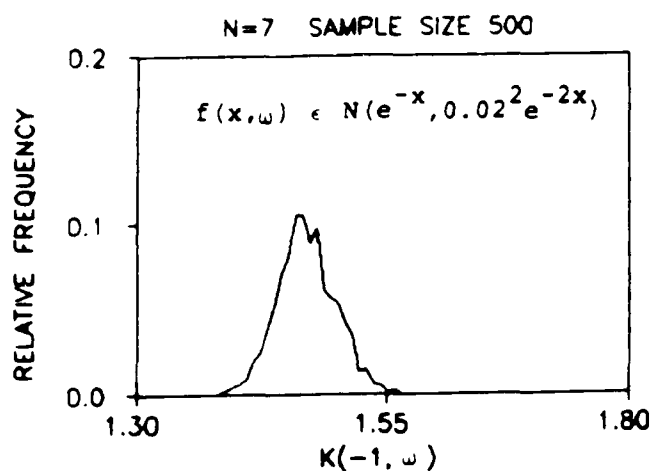


Fig. 27.

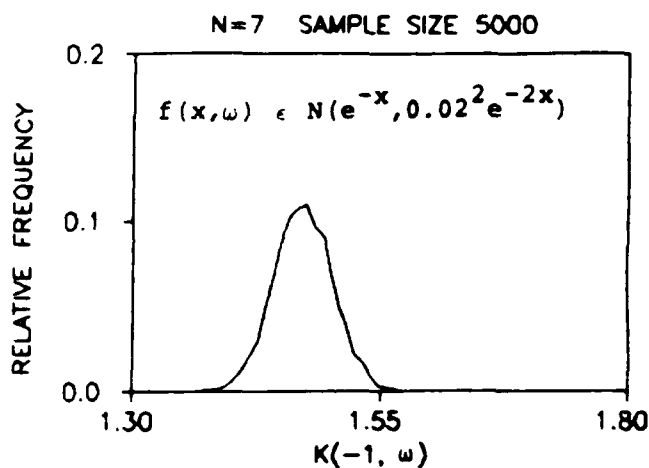


Fig. 28.

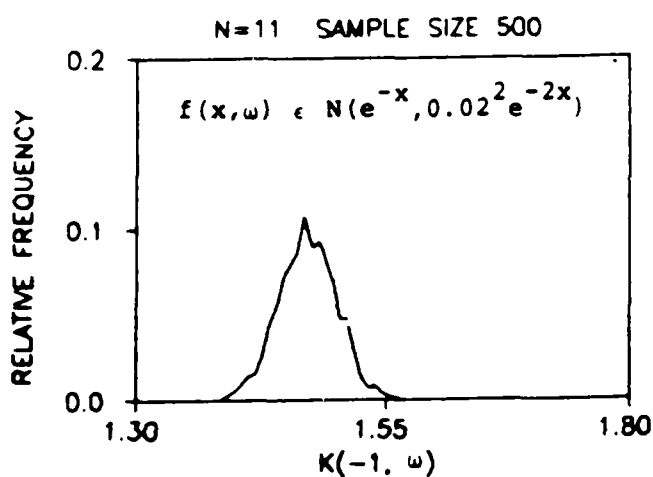


Fig. 29.

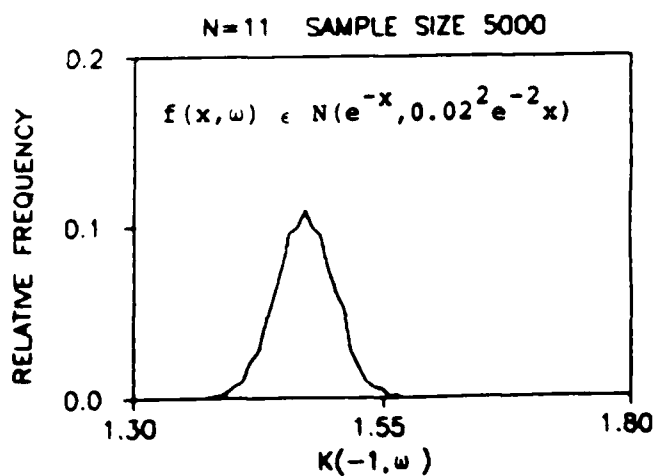


Fig. 30.

REFERENCES

1. Erdogan, F.: SIAM Journal of Applied Mathematics, Vol. 12, 1969, pp. 1041-1059.
2. Erdogan, F. and Gupta, G. D.: Quarterly Applied Math., Vol. 30, 1972, pp. 525-534.
3. Erdogan, F., Gupta, G. D. and Cook, T. S.: in Methods of Analysis and Solution to Crack Problems, Leyden, 1973, pp. 368-425.
4. Theocaris, P. S. and Ioakimidis, N. I.: Quarterly Applied Math., Vol. 34, 1976.
5. Majumdar, S. R.: Journal of Mathematics and Physics Science, Vol. 7, 1973, pp. 415-439.
6. Theocaris, N. I.: Serdica, Vol. 2, 1976, pp. 252-276.
7. Prossdorf, S.: Applicable Analysis, Vol. 7, 1977, pp. 259-270.
8. Prossdorf, S. and Liberman, B.: Dokl. Akad. Nauk., USSR, 226, 1976, pp. 516-519.
9. Theocaris, P. S. and Ioakimidis, N. I.: Trans. of the Academy of Athens, Vol. 40 (No. 1), 1977, pp. 1-39.
10. Krenk, S.: Quarterly of Applied Mathematics, Vol. 32, 1975, pp. 479-484.
11. Golberg, M. A.: Journal of Integral Equations, Vol. 5, 1983, p. 330.
12. Bharucha-Reid, A. T. and Christensen, M. J.: Mathematics and Computers in Simulation, Vol. 26, 1984, pp. 321-328.
13. Bharucha-Reid, A. T.: Approximate Solution of Random Equations, North Holland, New York, 1979.
14. Christensen, M. J. and Bharucha-Reid, A. T.: Journal of Integral Equations, Vol. 3, 1981, pp. 217-229.
15. Christensen, M. J. and Bharucha-Reid, A. T.: Journal of Integral Equations, Vol. 3, 1981, pp. 333-344.
16. Sambandham, M. and Bharucha-Reid, A. T.: Stochastic Analysis and Applications, Vol. 3, 1985, pp. 467-484.

17. Ioakimidis, N. I. and Theocaris, P. S.: Acta. Mechanica.
Vol. 28, 1977, pp. 239-254.
18. Bharucha-Reid, A. T.: Random Integral Equations,
Academic Press, New York, 1972, pp. 98-133.
19. International Mathematical and Statistical Libraries,
9th edition, 1986.

**NUMERICAL SOLUTION OF RANDOM LOVE'S
INTEGRAL EQUATION***

M. Sambandham

**Department of Mathematics
Morehouse College/Atlanta University
Atlanta, Georgia 30314**

V. Thangaraj

**Ramanujan Institute for Advanced Study in Mathematics
University of Madras Chepauk
Madras India**

and

K. B. Bota ,

**Department of Physics
Atlanta University
Atlanta, Georgia 30314**

***Research supported by U.S. Army Research Office**

Grant Number: DAAG 29-85-G-0109

ABSTRACT

In this paper, we present a method for the numerical solution of random Love's integral equation. The technique involves a Chebyshev series approximation, the coefficients of which are the solutions of a system of random linear equations. The statistical properties of the random solution are evaluated numerically and used to determine the distribution of the random coefficients. We express these distributions in histograms.

Abstracts: Contributed Papers and Poster Presentations

algebras A_L , L a line of τ . The combinatorial properties of the $(0,1)$ - matrices (x_i) which span these algebras are transferred to a polynomial ring $R = k[x_1, \dots, x_n]/I$, I a certain homogeneous ideal. R is noetherian of Krull dimension $n+1$ in which certain statements about the prime ideals of R are equivalent to statements about the collineations of τ . The Krull intersection theorem provides an interesting topology for R when τ admits no collineation.

John L. Hayden
Bowling Green State University
Department of Mathematics
Bowling Green, OH 43403

also discussed. We illustrate our analytical techniques through computer algorithms and numerical results. These results will highlight the error involved if one uses deterministic AR models instead of stochastic AR models.

M. SAMBANDHAM
Department of Mathematics
Morehouse College/ Atlanta University
Atlanta GA 30314

K.B. BOTA
Department of Physics
Atlanta University
Atlanta GA 30314

#98 (C/P Session 1; Wed. 12:00NOON)

A Family of Algorithms for the Graph Bisection Problem

The Graph Bisection Problem, GBP, is the problem of partitioning the vertex set of a given graph into two subsets of the same size (within one element) so that the number of edges cut is minimized.

We present a family of algorithms for GBP that implements the idea of a restricted branch and bound method.

It is shown that for every constant $\lambda > 0$, there is an algorithm with running time of $O(c^n)$ ($c < 2$), which performs as an exact algorithm on the set of graphs with n vertices and $5\lambda n$ edges.

We present the results of computer simulations with the algorithms and discuss several conjectures which are based on these results.

Mark K. Goldberg
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, New York 12180-3590

#90 (C/P Session 13; Fri. 5:00PM)

Discriminative Optimization

The extensions to the simplex algorithm required by Goal Programming (GP) inspire a further generalization on a broader class of mixed binary problems with a structure among goal functions.

The Discriminative Optimization (DO) is defined in its generality and the score of optimization methods like; Set-up cost, Goal Tree Programming, Aggregative Goal Programming, Linear Goal Programming, etc. are shown to be particular cases of the same scheme. Correspondingly, simplex-based solution algorithms for those problems, and even for the Linear Programming, can be obtained as implementations of the general solution process.

Dr. Pawel Radzikowski
Seton Hall University
School of Business
South Orange, NJ 07079

#99 (C/P Session 8; Thu. 12:00NOON)

APPROXIMATION TO AUTOREGRESSIVE MODEL WITH STOCHASTIC COEFFICIENTS

An autoregressive (AR) model with stochastic coefficients is generally modelled by deterministic AR model where stochastic coefficients are replaced by the mean of the stochastic coefficients. It has been proved that this approximation is restrictive and not applicable in general. In this article we suggest suitable improved approximations which are less restrictive and more 'closer' to the exact value. Some applications of power spectral estimation algorithm with stochastic coefficients are

#91 (C/P Session 15; Fri. 5:00PM)

A Model for Cannibalism

Many species of insects and fish practice cannibalism, presumably due to selective pressure toward this behavior. This paper presents a system of non-linear ODE's which reflects population dynamics in an insect species whose members pass through a sequence of stages, and in which later stages cannibalize earlier stages. Solution trajectories must converge when the number of stages is four or less. We conjecture that convergence always obtains but, when

THIRD SIAM CONFERENCE ON DISCRETE MATHEMATICS, MAY 14-16, 1986, CLEMSON UNIVERSITY

ABSTRACT
APPLIED MATHEMATICS

SEKO, JOSEPH T.

M.S. ATLANTA UNIVERSITY, 1986

THE USE OF LAGRANGE MULTIPLIERS AND KUHN-TUCKER'S
THEORY IN OPTIMIZATION PROBLEMS

Advisor: Professor Negash Medhin

Thesis dated November, 1986

Lagrange multipliers, penalty methods, and Kuhn-Tucker's theory are some important mathematical tools used in optimization problems. These tools are discussed so that one can appreciate the current areas of optimization research. Moreover, since extensive research work exists for linear optimization problems, only nonlinear applications are discussed.

ABSTRACT
MATHEMATICS

EPHRAIM, DANIEL

B.S., MOREHOUSE COLLEGE, 1985

NUMERICAL SOLUTION OF LINEAR INTEGRAL EQUATIONS
WITH RANDOM FORCING TERMS

Advisor: M. Sambandham

Thesis dated July, 1987

In Chapter one of this report we define Fredholm integral equations of the second kind, Volterra integral equations of the second kind and differentiate between the two of them and explain why integral equations are important. In Chapter two we discuss numerical procedures to integral equations. The equations we used in this report are of two types: (1) Fredholm equations and (2) Volterra equations. The methods we used for Fredholm equations are: (i) Simpson's rule, (ii) Trapezoidal rule, (iii) Weddle's rule, (iv) the Collocation method, and (iv) the Galerkin method.

For Volterra equations we used the successive approximation method with (i) Simpson's rule, (ii) Trapezoidal rule and (iii) Weddle's rule to evaluate the integrals.

In both Fredholm and Volterra integral equations we have the forcing term to be random. Our simulation results are presented in tables and graphs.

END

DATE

FILMED

5-88

DTIC