

AD-A192 491

EDGE PROCESS MODELS FOR REGULAR AND IRREGULAR PIXELS

1/1

(U) STANFORD UNIV CA DEPT OF STATISTICS

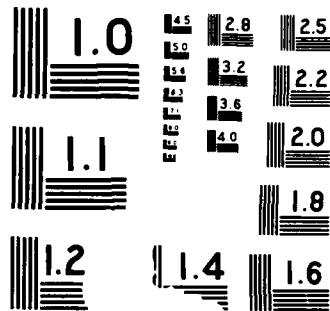
T C BROWN ET AL MAR 87 TR-267 NSF-DMS86-00235

UNCLASSIFIED

F/G 12/2

NL





AD-A192 491

EDGE PROCESS MODELS FOR REGULAR AND IRREGULAR PIXELS

BY

TIMOTHY C. BROWN AND B. W. SILVERMAN

TECHNICAL REPORT NO. 267

MARCH 1937

PREPARED UNDER THE AUSPICES

OF

NATIONAL SCIENCE FOUNDATION GRANT DMS86-00235

DEPARTMENT OF STATISTICS

STANFORD UNIVERSITY

STANFORD, CALIFORNIA

DTIC
ELECTE

MAR 11 1988

D



88 3 09 116

EDGE PROCESS MODELS FOR REGULAR AND IRREGULAR PIXELS

BY

TIMOTHY C. BROWN AND B. W. SILVERMAN

TECHNICAL REPORT NO. 267

MARCH 1987

PREPARED UNDER THE AUSPICES

OF

NATIONAL SCIENCE FOUNDATION GRANT DMS86-00235

ALSO PREPARED UNDER NATIONAL SCIENCE FOUNDATION GRANT MCS81-09584
AND ISSUED AS TECHNICAL REPORT NO. 16, DEPARTMENT OF STATISTICS,
STANFORD UNIVERSITY.

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

Accession For	
NTIS GUMI	☒
DTIC 1AD	☐
Unannounced	☐
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	



Edge Process Models for Regular and Irregular Pixels

Timothy C. Brown

University of Western Australia

and

B. W. Silverman

University of Bath, United Kingdom

The financial support of the U. K. Science and Engineering Research Council, the European Research Office of the U. S. Army and the National Science Foundation is gratefully acknowledged. The authors warmly thank R. Sibson for his stimulating remarks on the use of the prior log probability as a penalty function. This work was commenced while TCB was visiting the University of Bath and completed while BWS was visiting Stanford University.

1. Introduction

Geman and Geman (1984) discussed a methodology for pixel image restoration which depended on the idea of modelling the image by a Markov random field. A key feature of their approach was the possible placing of "edge elements" at "line sites" between pixels of the image.

In Geman and Geman's approach, a prior distribution for the image is constructed by first constructing a prior Gibbs distribution for the process of edge elements and then specifying the prior for the pixels themselves conditional on the edge process. In the specification of the pixel process, contiguous pixels separated by a line site at which an edge element is actually present are not considered as neighbours, and so are allowed to have quite different grey levels without incurring any penalty in the prior likelihood.

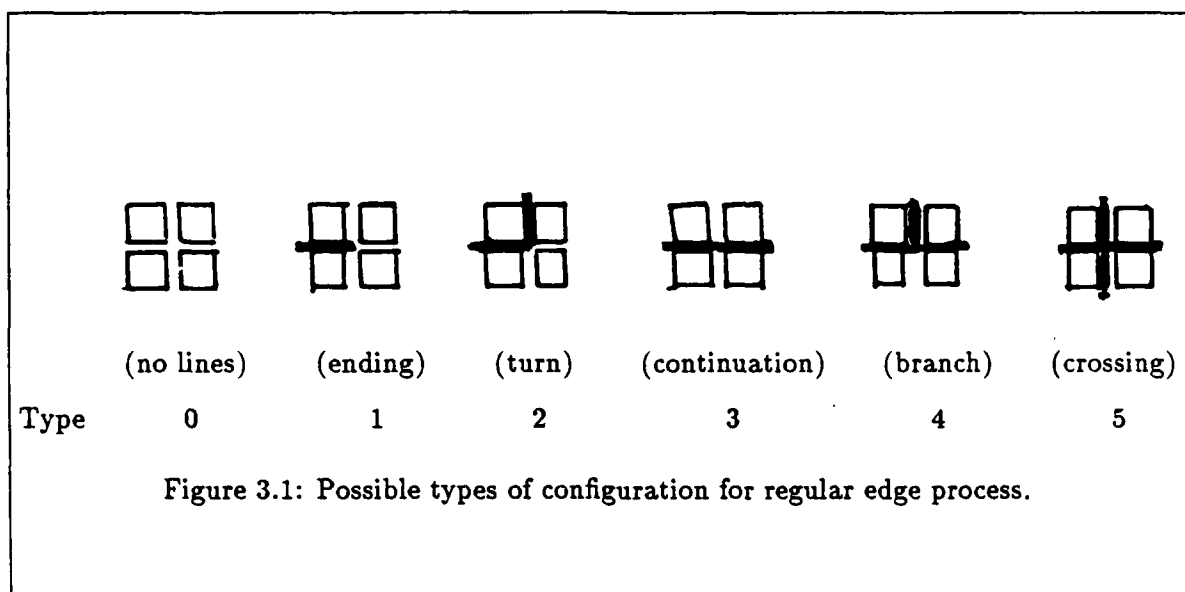
The edge process idea corresponds to the notion that the image is segmented into regions over each of which its behaviour is relatively homogeneous, or at least is not subject to abrupt changes; from one region to another, however, large differences in behaviour are possible. The changes in behaviour may relate either to overall grey level or to more subtle properties such as texture.

In this report we shall focus attention on the specification of the edge process, and show how various geometrical insights suggest how the prior Gibbs distribution should be constructed. Our discussion will suggest relative costs for possible configurations somewhat different from those proposed by Geman and Geman (1984). In addition ^{the}our scheme will provide methods for dealing with rectangular and irregular pixel patterns.

The present report is unashamedly speculative and theoretical. Practical implementation and investigation of the ideas presented here is in progress and will be reported subsequently.

2. The Gibbs Log Likelihood as a Penalty Function

The Gibbs distribution approach constructs a prior likelihood for the edge process by first defining a set of *cliques* of line sites. Each clique C consists of a small set of sites; in the Geman and Geman paper the cliques are the collections of four line sites with a common vertex. The



The costs ascribed to these configurations by Geman and Geman (1984) are given in Table 3.1.

Table 3.1: Costs of the configurations of Figure 3.1 used by Geman and Geman (1984)

Type of configuration	0	1	2	3	4	5
Cost V	0	2.7	1.8	0.9	1.8	2.7

We shall write v_i for the cost of a configuration of type i , and explore the consequences of various choices of v_i .

$I(\alpha) = c(0)/c(\frac{\pi}{4}) = 1/(\alpha\sqrt{2})$. To deal with $\frac{1}{2} < \alpha < 1$, define $\theta_0 = \tan^{-1}(2\alpha - 1)$ and rewrite

$$\begin{aligned} c(\theta) &= h^{-1} v_3 \sec \theta_0 \{ \cos \theta_0 \cos \theta + \sin \theta_0 \sin \theta \} \\ &= h^{-1} v_3 \sec \theta_0 \cos(\theta - \theta_0). \end{aligned} \quad (3.2)$$

Since for $\frac{1}{2} < \alpha < 1$ we have $0 < \theta_0 < \frac{\pi}{4}$, it follows that, for α in this range, $c(\theta)$ has a maximum at θ_0 and that $I(\alpha) = \max \{ \sec \theta_0, \sec(\frac{\pi}{4} - \theta_0) \}$. Hence $I(\alpha)$ is minimized by setting $\theta_0 = \frac{\pi}{8}$.

The minimum value $\sec \frac{\pi}{8} = (4 - 2\sqrt{2})^{1/2} \approx 1.082$. Thus it follows that the minimax score $I(\alpha)$ is optimized by setting $2\alpha - 1 = \tan \frac{\pi}{8}$, which implies that $\alpha = \frac{1}{2} (1 + \tan \frac{\pi}{8}) = 1/\sqrt{2}$. If this value of α is used, then lines parallel to the lattice directions or those at 45° to these directions will cost the same amount per unit length, while the most expensive lines will be those at $22\frac{1}{2}^\circ$ to the axis directions, which will cost about 8% more. It is interesting to note that the Geman-Geman value $\alpha = 2$ yields $I(\alpha) = 2\sqrt{2} \approx 2.83$, a much larger value.

It can also be shown, by somewhat tedious algebra, that $\alpha = 1/\sqrt{2}$ also minimizes other criteria of variability of $c(\theta)$, for example the coefficient of variation of $c(\theta)$ with θ uniformly distributed over $[0, \frac{\pi}{4}]$.

The arguments of this section make it possible to settle on a charge for configurations of types 2 and 3. Suppose it is intended to penalize boundaries in the underlying picture by an amount β per unit length. In an ideal world we would like to choose v_2 and v_3 in (3.1) to ensure that $c(\theta) = \beta$ for all θ . As we have seen, this cannot be attained exactly for all θ , but setting $v_2/v_3 = 2^{-1/2}$ will minimize the variability of $c(\theta)$ as θ varies. Having settled the ratio v_2/v_3 , it is natural to choose v_3 to ensure that $(2\pi)^{-1} \int_0^{2\pi} c(\theta) d\theta = \beta$. By simple algebra, from (3.2),

$$\begin{aligned} (2\pi)^{-1} \int_0^{2\pi} c(\theta) d\theta &= 4\pi^{-1} \int_0^{\pi/4} h^{-1} v_3 \sec(\frac{\pi}{8}) \cos(\theta - \frac{\pi}{8}) d\theta \\ &= 8\pi^{-1} h^{-1} v_3 \tan(\frac{\pi}{8}) = v_3 h^{-1} k^{-1} \end{aligned}$$

where the constant $k = \frac{1}{8}\pi / \tan(\frac{\pi}{8}) \approx 0.948$.

It follows that setting $v_3 = k\beta h$ and $v_2 = 2^{-1/2} k\beta h$ will ensure that, while $c(\theta)/\beta$ is only exactly 1 for certain values of θ , it will be the case that $c(\theta)/\beta$ lies between 0.948 and 1.027 for all θ and furthermore that the average value of $c(\theta)$ over (uniformly distributed) θ is precisely β .

Let n_b be the number of branches and n_c the number of crossings. It is immediate that

$$n_v = n_b + n_c. \quad (3.3)$$

In order to count the number of boundary sections, notice that three boundary sections meet at each branch and four at each crossing. Thus the number of ends of boundary sections is $3n_b + 4n_c$, and since each boundary section has two ends, we have

$$n_e = \frac{3}{2}n_b + 2n_c. \quad (3.4)$$

Substituting (3.3) and (3.4) into (3.5) yields

$$n_f = 1 + \frac{1}{2}n_b + n_c. \quad (3.5)$$

Formula (3.5) gives a natural price to be charged for branches and crossings. If it is desired to penalize an amount ρ for each region in the pattern, then one should charge $\frac{1}{2}\rho$ for each branch point and ρ for each crossing. Of course, if the edge configuration gives rise to regions that are not simply connected, then the number of regions will no longer be given by (3.5), and the charge $(\frac{1}{2}n_b + n_c)\rho$ will have to be considered in its own right as a penalty for the complexity of the pattern.

3.3. Endings

A pattern made up of disjoint regions cannot, of course, have a configuration of edges containing any endings at all. Therefore the philosophy that we are adopting would naturally lead to an infinite charge for configurations of type 1 in Figure 1. However to set any penalty value to infinity leads to algorithmic difficulties in using the model in practice, because it yields a prior model for the edge process under which some configurations have probability zero. This violates the condition of positive probability for all configurations under which the theory and practice of Markov random fields is developed; see, for example, Geman and Geman (1984, Section 4). In any case, it seems excessively dogmatic to exclude certain configurations completely, since there may be good physical reasons for a boundary to peter out in the middle of a region. Therefore an approach that is likely to be more satisfactory is to ascribe a cost λ to each "loose end" in the boundary pattern, where λ is set to a relatively large value. Precisely how large

3.5. Incomplete cliques

At the edge of the pattern, there will not be four line sites meeting at each vertex. A great advantage of our methodology is tht it makes it possible very easily to find the appropriate charge for the possible configurations that arise. Consider first the case where the window



Figure 3.3: A simple configuration at the edge of the window.

is locally aligned with the pixel direction, as in Figure 3.3. The vertex A will be associated with a clique containing exactly one line site; if the edge shown in Figure 3.3 is present then the point A will be a branch, and so the appropriate charge is $\frac{1}{2}\rho$, while if the edge is absent the appropriate charge is 0.

A rarer kind of incomplete clique arises as in Figure 3.4.



Figure 3.4: Incomplete cliques comprising three pixels.

The case where just a single edge is present will, as before, count as a branch point and so should be charged $\frac{1}{2}\rho$, while the configuration containing both edges contributes two boundary section ends and so should be charged ρ , provided the pixels numbered 1 and 3 are not to be regarded as neighbours.

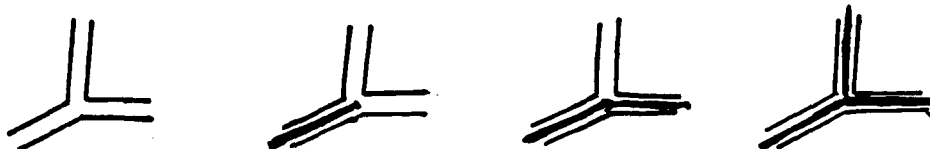


Figure 4.2: Configurations for a vertex of degree 3.

Although, in contrast with the case of square pixels, there are fewer types of configuration to consider, the irregularity of the pixels means that it is no longer necessarily the case that all configurations of a particular type should attract the same penalty.

The first stage in the assignment of costs to various configurations is to use the same arguments as in the square lattice case to deal with configurations of types 0, 1 and 3, which are charged 0, λ and $\frac{1}{2}\rho$ respectively. It remains to ascribe costs to "continuation" configurations. In order to do this, construct a dual edge pattern by placing a point in each cell of the original pixel array, and joining points if their corresponding pixels have some boundary in common. The vertices of the dual array can, in principle, be placed anywhere in their corresponding pixels, but in practice they will have a natural position. For example if the pixels are constructed as the Voronoi polygons of a point process then the points of the process will themselves be the vertices of the dual array.

Our assumption that exactly three pixels meet at each vertex of the original tessellation implies that the dual edge pattern will be a triangulation of the plane. In the case of the square pixel array the cost of "continuation" configurations was determined by considering a pattern with a single long straight edge, suitably discretized to fit the pixel pattern. In the more general case, it is no longer quite so clear how this discretization should be performed. One natural way to proceed is to prescribe that an edge segment will be present in the edge process if and only if the corresponding dual edge is intersected by the straight line boundary. We assume, if

derive possible ways of penalizing for the "continuation" configuration bc given by the presence of the edges dual to b and c and the absence of the edge dual to a . These costs will be based on the general idea that boundaries should cost an amount β per unit length; for notational simplicity we shall assume henceforth that $\beta = 1$, and note that the costs obtained should be multiplied by β in the general case.

Let ℓ be a random line in the plane, random in a sense that will be made precise below. Let ℓ_T be the length of the intersection of ℓ with the triangle T . Then our first possible cost for the continuation configuration bc is

$$V_1 = E(\ell_T \mid \ell \text{ intersects } b \text{ and } c).$$

The motivation for this definition is clear. Summing ℓ_T over all triangles T gives the length of the line ℓ , neglecting end effects, and, when deciding how much to charge for the configuration bc , we can only take note of information given by the current clique. Hence, by standard statistical theory, the natural estimator of ℓ_T is the posterior expectation V_1 of ℓ_T given all the information available.

A second possible cost is given in a slightly less transparent way. Let ℓ_a be the projected length of the side a on the line ℓ ; this length is to be counted as negative if, as in Figure 4.4, the half of ℓ that intersects b and c makes an angle of more than $\frac{\pi}{2}$ with a . The second proposed cost is

$$V_2 = E\left(\frac{1}{2}\ell_a \mid \ell \text{ intersects } b \text{ and } c\right).$$

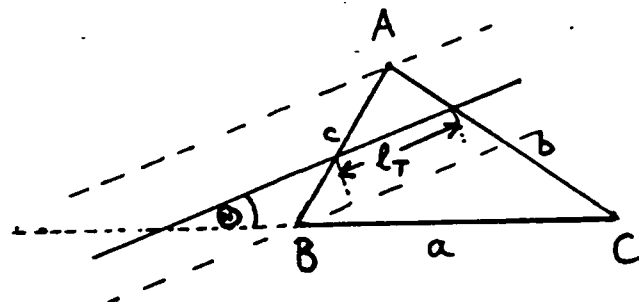


Figure 4.5: A random line intersecting the dual triangle.

For such θ , ℓ will intersect c and b if and only if it intersects c . The set of lines at orientation θ that intersect c make up a strip of width $c \sin(B - \theta)$ and so we have

$$f(\theta) \propto c \sin(B - \theta) \quad \text{for } 0 < \theta < B.$$

For $-C < \theta < 0$, a similar argument yields

$$f(\theta) \propto b \sin(C + \theta) \quad \text{for } 0 < -\theta < C.$$

To calculate the constant of proportionality, we note that

$$\begin{aligned} \int_0^B c \sin(B - \theta) d\theta + \int_{-C}^0 b \sin(C + \theta) d\theta \\ &= c \int_0^B \sin \phi d\phi + b \int_0^C \sin \phi d\phi \\ &= b + c - c \cos B - b \cos C \\ &= b + c - a, \end{aligned}$$

and hence we have

$$f(\theta) = \begin{cases} c \sin(B - \theta) / (b + c - a) & 0 < \theta < B \\ b \sin(C + \theta) / (b + c - a) & -C < \theta < 0 \\ 0 & \text{otherwise.} \end{cases}$$

To calculate V_1 , consider first $\theta > 0$. Given that $\Theta = \theta$ and that ℓ intersects c and b , the expected value of ℓ_T is half its value when $\Theta = \theta$ and ℓ passes through B . This length is, by

expressed. Given that $\Theta = \theta$, we have $\ell_1 = a \cos \theta$, and hence

$$\begin{aligned} V_2 &= \frac{1}{2}a \int_{-C}^B \cos \theta f(\theta) d\theta \\ &= (b + c - a)^{-1} \left\{ \int_0^B \frac{1}{2}ac \cos \theta \sin(B - \theta) d\theta + \int_0^C \frac{1}{2}ab \cos \theta' \sin(C - \theta') d\theta' \right\}. \end{aligned} \quad (4.3)$$

The first integral in (4.3) is equal to

$$\int_0^B \frac{1}{4}ac \{ \sin B + \sin(B - 2\theta) \} d\theta = \frac{1}{4}ac B \sin B = \frac{1}{2}\Delta B$$

where Δ is the area of the triangle T , and hence

$$V_2 = \frac{1}{2}(B + C)\Delta / (b + c - a) \quad (4.4)$$

Thus it is clear that the formula for V_2 is very simple and more appealing than that for V_1 .

5. Regular Arrays Revisited

In the last section we defined two different ways of obtaining penalties for continuation configurations. One of these was based on the length of the *intersection* of a region in the dual triangulation with a random line, and the other on the length of *projection* of such a region on a random line. It turned out that the projection penalty gave a much more elegant result. In this section, we shall apply the intersection and projection ideas to the regular square lattice considered earlier.

Our aim is to obtain costs for the "turn" and "continuation" configurations as illustrated in Figure 3.1. The dual of the square lattice is itself a square lattice, and the part of the dual corresponding to a clique is a single square of side h as in Figure 5.1.

will satisfy

$$f_1(\theta) = (2 - \sqrt{2})^{-1} \sin\left(\frac{\pi}{4} - |\theta|\right), \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

using simple calculus to find the constant of proportionality. The intersection length s is equal to $h \sec \theta$, and hence the expected intersection length is

$$\begin{aligned} \int_{-\pi/4}^{\pi/4} h \sec \theta f_1(\theta) d\theta &= \int_0^{\pi/4} (1 - \tfrac{1}{2}\sqrt{2})^{-1} \sec \theta \sin\left(\frac{\pi}{4} - \theta\right) d\theta \\ &= (\sqrt{2} - 1)^{-1} \int_0^{\pi/4} (1 - \tan \theta) d\theta = (\sqrt{2} - 1)^{-1} [\theta - \log \sec \theta]_0^{\pi/4} \\ &= (\sqrt{2} - 1)^{-1} \left(\frac{\pi}{4} - \tfrac{1}{2} \log 2\right). \end{aligned}$$

Thus the "intersection" penalty for a configuration of type 3 in Figure 3.1 would be

$$\left(\frac{\pi}{4} - \tfrac{1}{2} \log 2\right) h$$

$$h/(\sqrt{2} - 1) \approx 1.06 h.$$

To find the "projection" penalty for such a configuration, note that the appropriate generalization of the projection argument given in Section 4 is to take as penalty $\frac{1}{2}$ (projection of AB and DC) because both AB and DC will be edges of the irregular strip formed by the union of those dual pixels intersected by ℓ . Both AB and DC have projection length $h \cos \theta$ on ℓ , and so the "projection" penalty for a configuration of type 3 will be

$$\begin{aligned} \int_{-\pi/4}^{\pi/4} h \cos \theta f_1(\theta) d\theta &= (1 - \tfrac{1}{2}\sqrt{2})^{-1} h \int_0^{\pi/4} \cos \theta \sin\left(\frac{\pi}{4} - \theta\right) d\theta \\ &= (2 - \sqrt{2})^{-1} h \int_0^{\pi/4} \{\sin \tfrac{\pi}{4} - \sin(2\theta - \tfrac{\pi}{4})\} d\theta \\ &= \tfrac{1}{8} \pi h / (\sqrt{2} - 1) = kh \approx 0.95 h \end{aligned}$$

where k is defined exactly as in Section 3.

To find the penalties for "turn" configurations, the work of Section 4 can be used almost directly, by noticing that both the "intersection" and "projection" penalties will be the same as those obtained there, for the case of a line crossing the two short sides of an isosceles right-angled triangle. Thus we set $a = h\sqrt{2}$, $b = c = h$, $B = C = \frac{\pi}{4}$ and $A = \frac{\pi}{2}$ in the formulas (4.2) and (4.4).

We obtain as the intersection penalty for the turn configuration $\frac{1}{2} h \log 2 / (2 - \sqrt{2}) \approx 0.59 h$ and for the projection penalty $\frac{1}{8} \pi h / (2\sqrt{2}) = 2^{-1/2} kh \approx 0.67 h$. It is noteworthy that

give as the cost of a "continuation" as shown in Figure 6.1 the quantity

$$\begin{aligned}
 & h_1 \int_0^{\theta_0} \cos \theta \sin(\theta_0 - \theta) d\theta \bigg/ \int_0^{\theta_0} \sin(\theta - \theta_0) d\theta \\
 &= \frac{1}{2} h_1 \int_0^{\theta_0} \{\sin \theta_0 + \sin(\theta_0 - 2\theta)\} d\theta \bigg/ \int_0^{\theta_0} \sin \theta' d\theta' \\
 &= \frac{1}{2} h_1 \theta_0 \sin \theta_0 / (1 - \cos \theta_0). \tag{6.1}
 \end{aligned}$$

The other type of continuation, consisting of two edges of length h_2 , will cost an amount obtained by substituting h_2 for h_1 and $\pi - \theta_0$ for θ_0 in (6.1), viz. $\frac{1}{2} h_2 (\pi - \theta_0) \cos \theta_0 / (1 - \sin \theta_0)$.

The general idea of evolving penalties for continuation configurations based on a conditional expected projection length can of course be extended to more general polygons in the dual tessellation. The advantage of the projection approach is that consistent penalties can be written down for cliques of different kinds that appear in different parts of the same pattern. Thus the circular pixel grid shown in Figure 4.1 contains some vertices of degree 3, which can be dealt with using the formulas of Section 4, and some of degree 4 whose dual polygons in the dual tessellation are, for all practical purposes, rectangles – which are treated in this section.

END

DATE

FILMED

6-88

DTIC