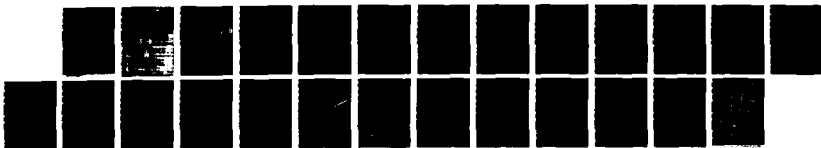
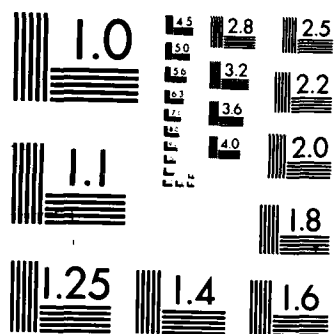


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EFFICIENCY OPTIMIZATION FOR FEL OSCILLATORS

A. Serbeto, B. Levush and T. M. Antonsen, Jr.

Plasma Preprint UMLPF #88-031

December 1987

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EFFICIENCY OPTIMIZATION FOR FEL OSCILLATORS

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ABSTRACT

Using a normalized set of nonlinear equations, which describe an FEL ^{free electron laser} oscillator, the efficiency of energy extraction from the electron beam to the radiation can be optimized. The optimum values of this efficiency are presented for an (a) untapered and for a (b) tapered wiggler FEL oscillator.

I. INTRODUCTION

The generation of a high-power coherent radiation using stimulated emission from an intense relativistic electron beam moving in a static undulating magnetic field has received considerable interest lately. The device based on this effect, known as a free electron laser, has the great advantage of being readily tunable over a wide frequency range, from submillimeter to optical wavelengths, by varying the electron beam energy. Many applications can be found for an FEL, for example, electron cyclotron resonance heating (ECRH), diagnostics, isotope separation, and radar.^{1,2} In all these applications high efficiency is desirable. As is well known, to have a high efficiency free electron laser one must appropriately taper the wiggler parameters.³⁻⁵ Such tapering enhances dramatically the energy transferred from the relativistic electron beam to the coherent electromagnetic wave.

The goal of this paper is to present calculations that enable one to design an FEL oscillator in such a way that it will operate with the maximum efficiency. Our approach is complementary to the usual procedure of attempting to simulate a particular experiment with a high degree of accuracy, including as many effects as possible. Instead, we will make as many approximations as possible (some better than others) in order to reduce the complexity and dimension of parameter space. In addition, using a normalized set of equations to describe the FEL interaction the number of the parameters can be further reduced. In the nontapered case there are only two, one is related to the initial beam energy and the second is the normalized beat wave amplitude. For a tapered FEL the number of parameters is, of course, greater. In fact, one must specify

two free functions; one describing the profile of the wiggler strength, and the other the profile of the wiggler period. In this paper these functions will be represented in terms of simple functions characterized by a few parameters. Other simplifications include neglecting transverse mode structure and self-fields, and adopting the low gain limit. While the determination of the transverse mode structure is important in particular devices one can argue that in an "optimal" device all electrons should experience the same radiation field, and thus will gain or lose energy as in a one-dimensional model. The problem of self-fields is somewhat different. Preliminary calculations have shown that inclusion of self fields puts strong limitations on the rate of taper that can be effective in an FEL.⁵ We will defer the consideration of self-fields to a future publication. Consideration of the high gain limit adds an extra parameter, namely the beam current, so we will focus on the simpler problem of the low gain regime where the field profile is specified. We divided this paper in the following way. In Sec. II we describe the basic equations which describe the dynamics of the system and taper design. In Sec. III the numerical simulation and results are presented and discussed. Finally, in Sec. IV we draw the conclusion of our calculations and point out some future studies and simulations.

II. BASIC EQUATIONS

In an FEL the motion of the electrons is affected by the combined wiggler magnetic field

$$\vec{A}_{wx} = \hat{x} A_{w0}(z) \cos \int^z k_w(z') dz' \quad (1)$$

and radiation field

$$\vec{A}_s = \hat{x} A_{s0} e^{i(k_z z - \omega t)} + \text{c.c.} \quad (2)$$

which propagates along the z-direction. With the given fields, the dynamics of the electrons can be described by the standard one-dimensional pendulum equation¹

$$\frac{d\psi}{dz} = \delta\gamma \frac{\omega}{c} \frac{1 + (1/2)\kappa^2}{(\gamma_R \beta_z)^3}, \quad (3a)$$

$$\frac{d}{dz} (\gamma_R + \delta\gamma) = - \frac{\partial}{\partial \psi} \frac{\omega \kappa}{c (\gamma_R \beta_z)} \left[\frac{q A_{s0}}{mc^2} e^{i\psi} + \frac{q A_{s0}^*}{mc^2} e^{-i\psi} \right], \quad (3b)$$

where

$$\kappa(z) = \frac{q A_{w0}}{mc^2}$$

is the wiggler parameter, β_z is the longitudinal velocity (normalized to the speed of light) of the relativistic electron beam, $\gamma_R(z)$ is the resonant value of γ which is required to keep the particle motion in phase with the ponderomotive wave. The variation of the particle energy

$$\delta\gamma = \gamma - \gamma_R(z)$$

represents a small deviation of the particle energy from the resonant energy, while the phase of a particle in the beat wave potential is given by

$$\psi = \int^z k_w(z') dz' + k_s z - \omega t .$$

The profile for the resonant energy γ_R is selected in the following way

$$\gamma_R(z) = \gamma_R(0) - \eta_b [\gamma_R(0) - 1] (z/L)^\rho , \quad (4)$$

where L is the interaction length and $\gamma_R(0)$ is the value of γ_R at the entrance.

The parameter η_b gives the total change in γ_R in the interaction length and is defined such that if all particles remained trapped in the beat wave the resulting efficiency of energy extraction would be η_b . The parameter ρ describes the profile of the deceleration along the interaction length. If $\rho = 1$ the rate of deceleration is constant. In practice we will find that higher values of ρ are desirable because these give a weaker deceleration at the entrance of the interaction region than at the exit. Such a profile of deceleration is required for an oscillator that is to start from noise.

To simplify and reduce the number of parameters in Eqs. (3a) and (3b) we will make a number of approximations. First, we will assume that

the axial velocity β_z of a particle is independent of axial position in the interaction region and equal to its value at injection. Further, we will assume that the variation of γ_R in the interaction region is also weak and we will consider γ_R to be independent of z (and equal to its value at injection) except in Eq. (3b) where it is explicitly differentiated. These approximations are valid for devices with small efficiencies $\eta_b \ll 1$. Furthermore, we will assume that the wiggler parameter κ is small so that $(1/2)\kappa^2$ can be neglected compared with 1 in Eq. (3a). While these approximations are not always justified and indeed not needed for a numerical integration of Eqs. (3a) and (3b) they allow us to recast Eqs. (3a) and (3b) in the following form

$$\frac{d\psi}{d\xi} = \frac{\partial H}{\partial P}, \quad (5a)$$

$$\frac{dP}{d\xi} = -\frac{\partial H}{\partial \psi}, \quad (5b)$$

where the normalized distance ξ is given by,

$$\xi = z/L,$$

and

$$H = \frac{1}{2} P^2 - \Omega_b^2 \sin\psi + \psi\alpha$$

is the interaction Hamiltonian. The normalized "momentum" is given by

$$P = \delta\gamma \frac{\omega L}{c(\gamma_R \beta_z)^3}. \quad (6)$$

The normalized beat wave amplitude is

$$\Omega_b^2 = \left(\frac{\omega}{c} L\right)^2 \frac{a_{w0}(\xi) a_{s0}}{(\gamma_R \beta_z)^4} \quad (7)$$

and normalized deceleration term due to tapering is

$$\alpha = \left(\frac{\omega}{c} L\right) \frac{1}{(\gamma_R \beta_z)^3} \frac{\partial \gamma_R}{\partial z} . \quad (8)$$

Roughly speaking, $\Omega_b/2\pi$ is the number of synchrotron oscillations a particle will experience in the interaction region. Using the profile for $\gamma_R(z)$ given in Eq. (4), we obtain

$$\alpha = - P_b \rho \xi^{\rho-1} , \quad (9)$$

where

$$P_b = \eta_b \left(\frac{\omega}{c} L\right) \frac{(\gamma_R - 1)}{(\gamma_R \beta_z)^3} , \quad (10)$$

represents the normalized "bucket" efficiency.

Equations (5a) and (5b) are integrated numerically for an ensemble of electrons whose initial phases are distributed uniformly between 0 and 2π , and whose initial momenta are all equal to P_{inj} . The normalized efficiency is determined from the average of the momentum for the exiting electrons and is given by

$$\Delta P = P_{inj} + P_b - \langle P(1) \rangle , \quad (11)$$

where the angular bracket's indicate an average over the initial phases.

The actual efficiency is determined from Eqs. (6) and (11),

$$\eta = \left(\frac{c}{L\omega}\right) \frac{(\gamma_R \beta_z)^3}{\gamma_R - 1} \Delta P . \quad (12)$$

There still remains one free function to specify which is the profile of the wiggler strength. In our model in which we specify independently the profile of the wiggler strength and resonant energy the profile of the wiggler strength only enters explicitly the coupling coefficient between the radiation and the particles. However, it must be remembered that in practice different profiles of wiggler strength will require different profiles of wiggler period in order to maintain the prescribed profile of resonant energy.

To incorporate the effect of a spatially varying coupling coefficient we specify the profile of the beat wave amplitude to be

$$\Omega_b^2 = \Omega_{b0}^2 f(\xi) , \quad (13)$$

where Ω_{b0}^2 is the amplitude defined by Eq. (7) with the nominal value of a_{w0} , and $f(\xi)$ is a profile function describing variations of a_{w0} over the interaction length. The profile we choose is given by

$$f(\xi) = \begin{cases} \xi/\xi_0 , & 0 \leq \xi < \xi_0 \\ 1 , & \xi_0 \leq \xi \end{cases} . \quad (14)$$

This profile, which provides a weaker ponderomotive wave at the entrance than at the exit allows particles to become more deeply trapped in the ponderomotive wave when it reaches maximum amplitude. This effect will be illustrated later. As we can see, the number of parameters on which

the normalized efficiency depends is five, namely

$$\Delta P = \Delta P(P_{inj}, \Omega_{b0}^2, P_b, \rho, \xi_0) .$$

The last three parameters characterize the tapering design. The parameters P_b and ρ are related to the deceleration of the particles trapped in the ponderomotive potential well. These parameters can be controlled and adjusted by changing the wave period of the wiggler field or the strength of the wiggler magnetic field. The parameter ξ_0 controls the distance at which the height of the ponderomotive well reaches its maximum value.

In the following sections we find numerically the maximum value of ΔP with respect to P_{inj} and plot it as a function of Ω_b^2 for given values of P_b , ρ , and ξ_0 .

III. SIMULATION AND RESULTS

A. Untapered FEL

In the untapered case the efficiency is a function of two parameters, i.e., $\Delta P = \Delta P(P_{inj}, \Omega_{b0}^2)$. Level curves of ΔP in the P_{inj}, Ω_b^2 plane appear in Fig. (1) for $\Omega_b^2 < 45$, $P_{inj} < 15$. A maximum occurs at $P_{inj} = 5.14$, $\Omega_b^2 = 18.0$ where $\Delta P = 5.5$. Additional local maximum appear at $\Omega_b^2 = 35$ and $P_{inj} = 10.29$ and $\Omega_b^2 = 25$, and $P_{inj} = 11.5$. Further increasing the range of Ω_b^2 and P_{inj} would reveal additional maxima. Figure (2) shows level curves of $\Omega_b^2/\Delta P$ which is proportional to the beam current required to maintain energy balance at a particular value of Ω_b^2 and P_{inj} . A plot of ΔP maximized over P_{inj} for a given value of Ω_b^2 appears in Fig. 3. From this curve, we observe the two values of Ω_{b0}^2 for which the extracted energy from the beam is maximum. The two maximum values are almost equal. Also, we note that the behavior of the maximized ΔP has a cusp at $\Omega_{b0}^2 \approx 26$. For this value of Ω_{b0}^2 the value of energy extracted ΔP as a function of P_{inj} has a much broader maximum. This is clearly shown in the level curves of Fig. (1).

B. Tapered FEL

As we already discussed, in the tapered FEL the energy extracted ΔP depends on five parameters, $P_{inj}, \Omega_{b0}^2, P_b, \rho$, and ξ_0 . For this case the parameters $\Omega_{b0}^2, \rho, \xi_0$, and P_b will be considered given, and ΔP will be maximized over values of P_{inj} .

Figures 4a and 4b show the behavior of the energy extracted for a

tapered wiggler with a constant wiggler parameter $\xi_0 = 0$ in Eq. (13) and for a value of $\rho = 1$ and $\rho = 1.5$. As can be seen in Fig. (4a) where a constant rate of taper is present the normalized efficiency is low until a sufficiently large amplitude is imposed to trap a significant fraction of particles. Clearly, an oscillator with this type of efficiency vs. field strength curve would not be able to start from noise and attain high values of signal field. This problem is resolved in Fig. (4b) where the profile in the gradient of the resonant energy is tapered ($\rho = 1.5$). Here the efficiency vs. field strength curves are such that an oscillator could start from noise. The physical reason for the improved performance is that electrons feel a gradually increasing deceleration force as they propagate through the interaction region. Whereas, for the case of Fig. (4a) electrons feel a constant deceleration gradient. Thus, at low signal field strengths in the case of the tapered gradient (4b) electrons are trapped at the entrance and partially decelerated before becoming detrapped while in the case of the constant gradient the ponderomotive wave is not strong enough to trap any electrons.

The overall efficiency of trapping can be gauged by comparing the normalized efficiency with the value of P_b . If all particles were trapped these two numbers would be roughly equal. Thus, we can see from Fig. (4b) that trapping efficiencies between 45 and 70% are realized at the strongest fields.

The trapping efficiencies can be enhanced considerably by tapering the coupling coefficient. In Figs. (5a) and (5b) we have plotted normalized efficiency vs. field strength for tapered wigglers with $\rho = 1.5$ (5a) and $\rho = 2.0$ (5b) and the profile of the wiggler strength given by Eqs. (13) and (14) with $\xi_0 = 0.3$. The reason for the observed

improvement is as follows. When particles enter the interaction region they feel a relatively weak ponderomotive wave compared with what they will feel throughout most of the interaction region. Thus, the momentum of these particles is modified by a small amount compared with the ultimate size of the ponderomotive well. This small amount, however, is adequate to cause the particles to become bunched in phase as they propagate further into the interaction region. Once they are bunched in phase they can be trapped at the bottom of the well of a larger ponderomotive field. This effect is also illustrated in Fig. 6. The effect is analogous to the operation of a klystron where the particles are bunched in a field free region.

Figure 7 shows the effect of varying the exponent ρ in the profile of the resonant energy for different values of "bucket" efficiency. It appears that a profile with $\rho = 2$ gives the optimum efficiency (we have verified that at higher values of ρ the efficiency degrades).

IV. CONCLUSION

In the case of an untapered FEL oscillator using a normalized set of nonlinear equations we produced curves of equal energy extraction ΔP in the P_{inj}, Ω_b^2 plane. Optimizing ΔP with the respect of P_{inj} , allowed us to plot the optimum value of ΔP versus Ω_b^2 .

In the case of a tapered FEL oscillator we have considered the effect of variation of the profile of wiggler strength and resonant energy on the achievable efficiencies in low gain free electron laser oscillators. The optimum profile of resonant energy appears to be one which is quadratic in distance down the interaction length ($\rho = 2$) although nearly the same results can be achieved with profile that is somewhat closer to linear ($\rho = 1.5$). Such tapering of the resonant energy is necessary if the oscillator is to start from noise.

Secondly, we have found that the trapping efficiency can be enhanced if the strength of the coupling coefficient (wiggler strength) is tapered in such a way that particles are prebunched at the entrance and then trapped further down the interaction region. The universal plots in Figs. (3) - (5) allow one to estimate the maximum achievable efficiency and corresponding tapering profiles for an interaction length of a given number of synchrotron oscillations ($\Omega_{b0}/2\pi$).

ACKNOWLEDGMENT

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FIGURE CAPTIONS

Fig. 1. The equal energy extraction ΔP curves in the P_{inj}, Ω_b^2 plane. P_{inj} is the initial normalized momentum and Ω_b^2 the normalized value of the beat-wave amplitude. The curve 1 corresponds to $\Delta P = 0$, the curve 2 corresponds to $\Delta P = 5.5$. The difference between the neighboring level curves is 0.5.

Fig. 2. The level curves of $\hat{I} = \Omega_b^2 / \Delta P$, in the P_{inj}, Ω_b^2 plane. $\hat{I}$ is proportional to the beam current required to maintain energy balance at a particular value Ω_b^2 and P_{inj} . The numbers on the curves indicate the value of $\hat{I}$.

Fig. 3. Optimum ΔP vs beat wave amplitude for an untapered wiggler.

Fig. 4. ΔP versus Ω_b^2 curves for a tapered wiggler with $\xi_0 = 0$, (a) $\rho = 1$, (b) $\rho = 1.5$, and $P_b = 200 \rightarrow 1000$.

Fig. 5. Similar to Fig. 4, with $\xi_0 = 0.3$, (a) $\rho = 1.5$, (b) $\rho = 2$ and $P_b = 200 \rightarrow 1000$.

Fig. 6. Comparison in ξ_0 for $\rho = 1.5$ for a tapered wiggler with (a) $P_b = 200$, (b) $P_b = 400$, (c) $P_b = 600$, (d) $P_b = 800$, and (e) $P_b = 1000$.

Fig. 7. Comparison in ρ between efficiency curve for a tapered wiggler with $\xi_0 = 0.3$ and (a) $p_b = 200$, (b) $p_b = 400$, (c) $p_b = 600$, (d) $p_b = 800$ and (e) $p_b = 1000$.

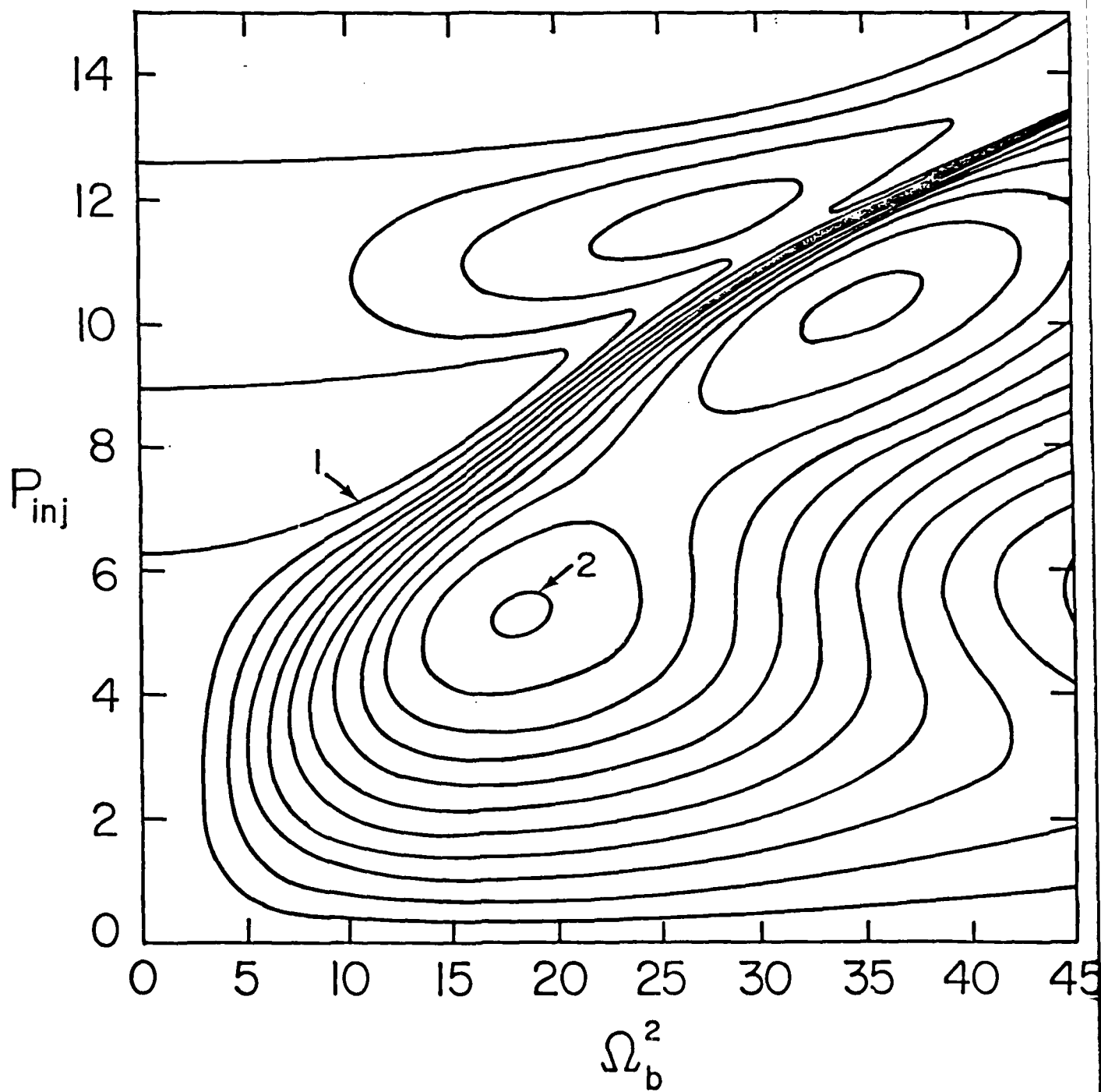


fig. 1

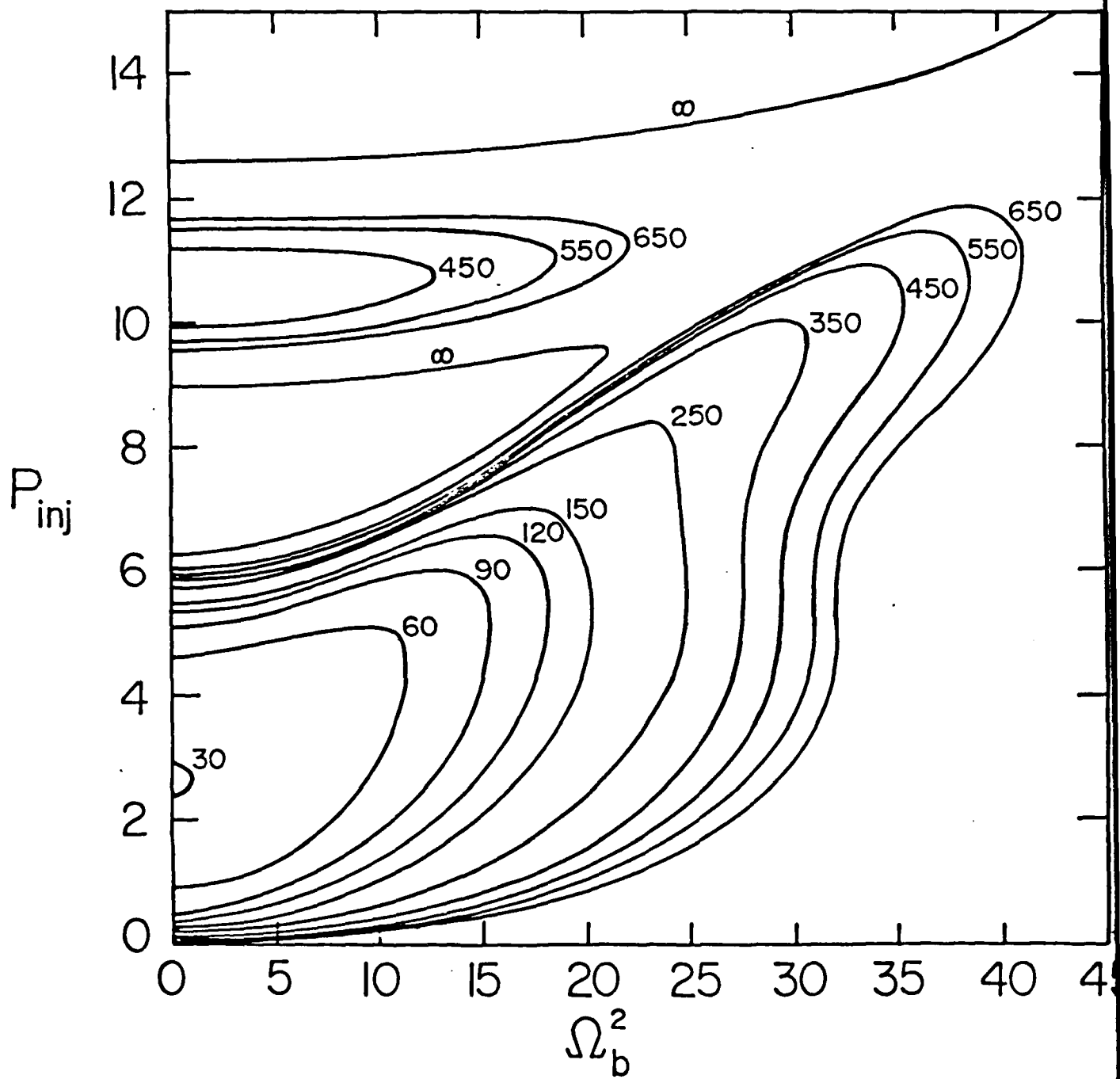


fig. 2

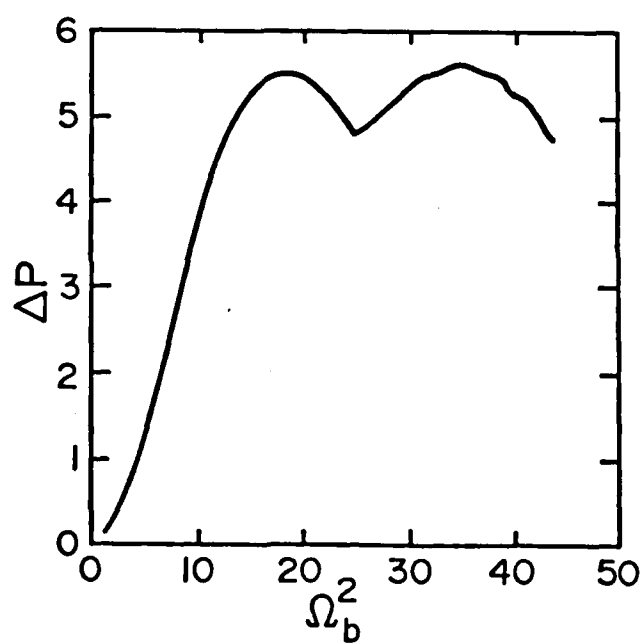


fig. 3

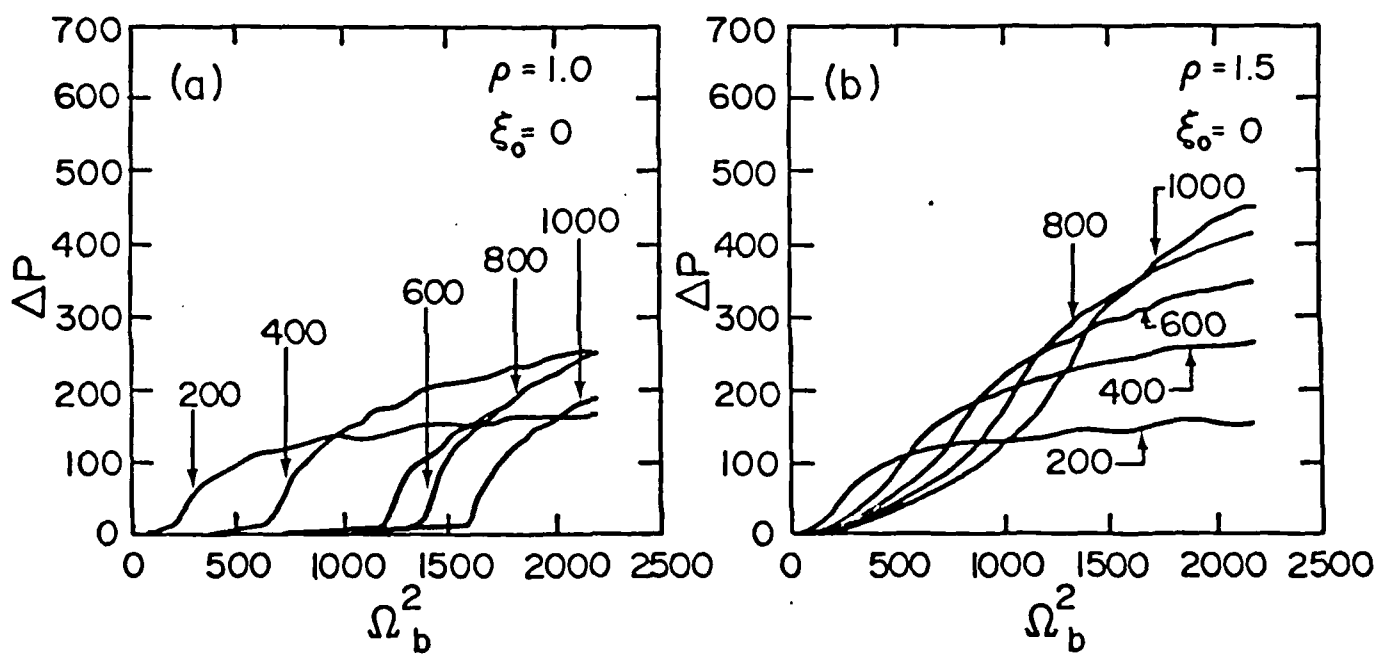


fig. 4

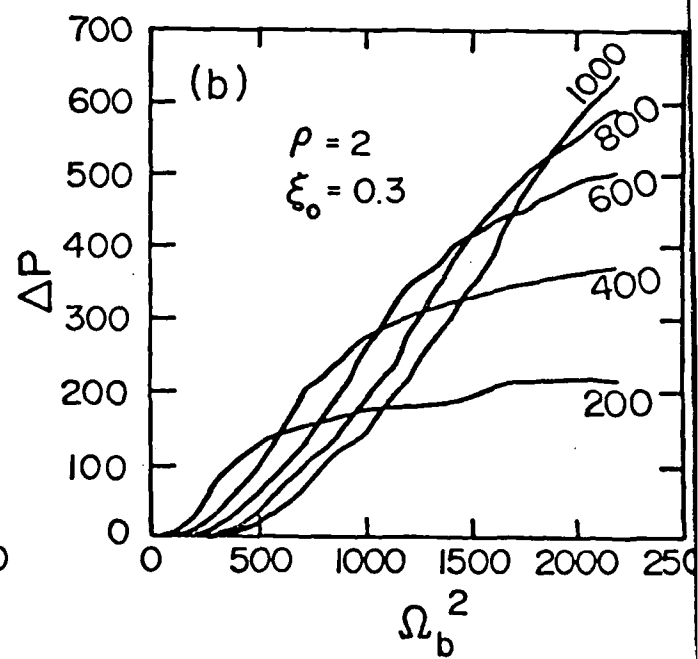
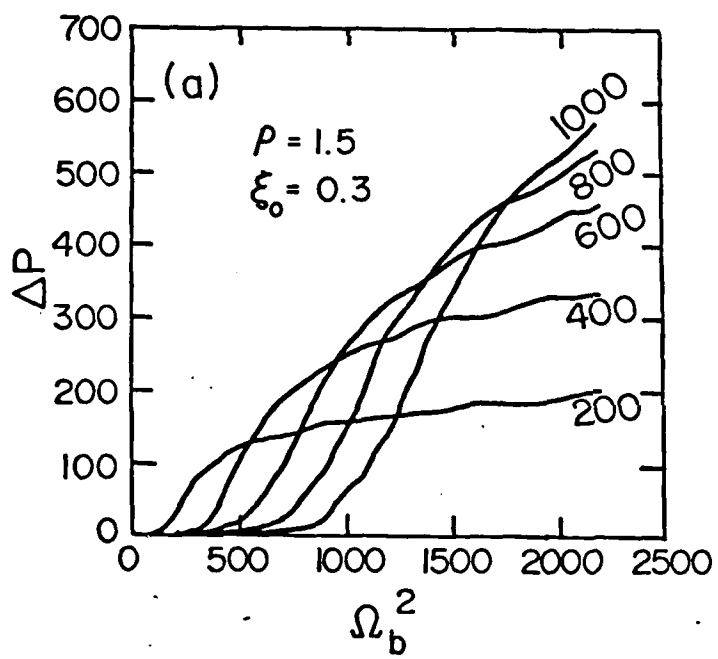


Fig. 5

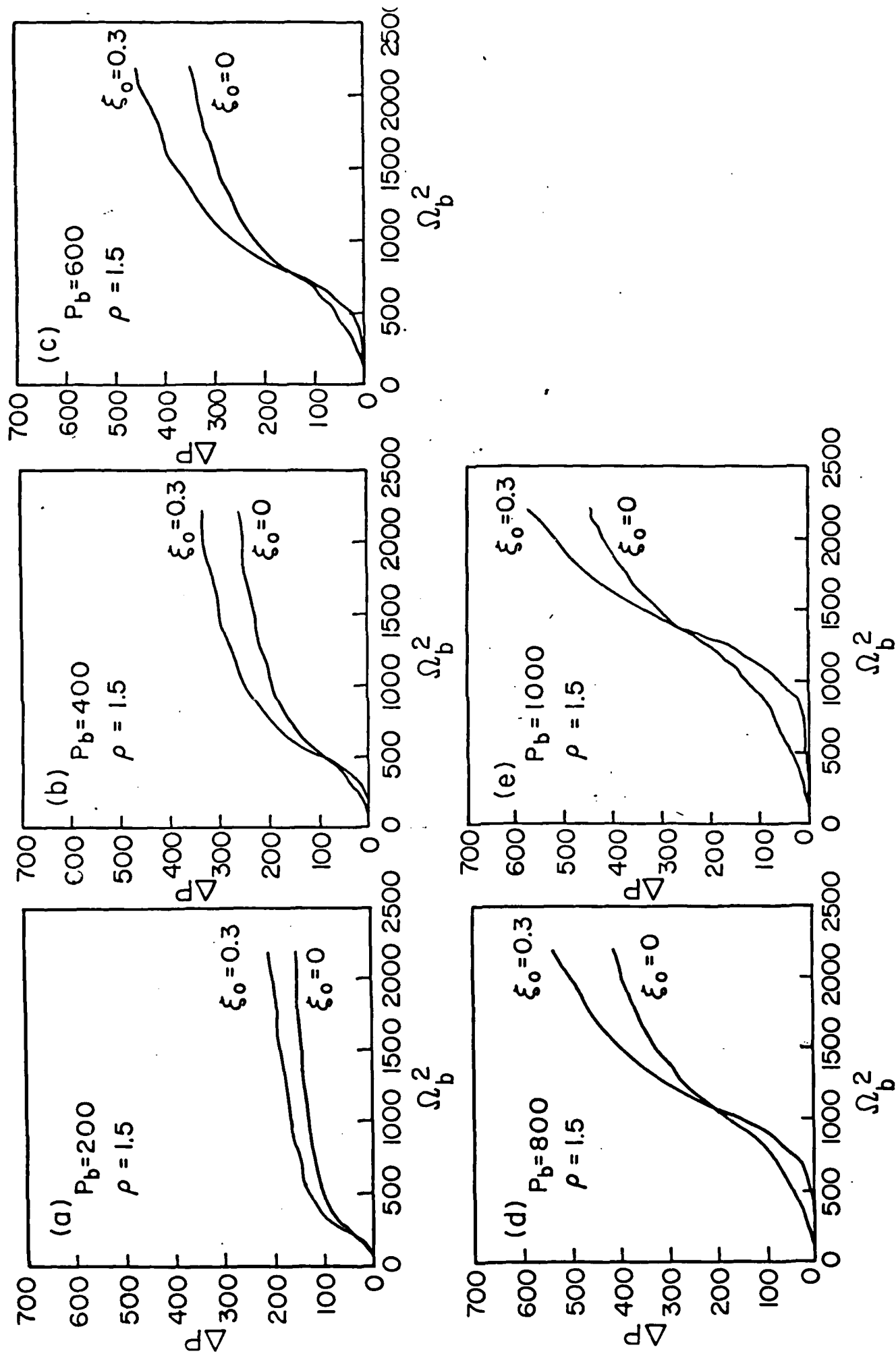


fig. 6

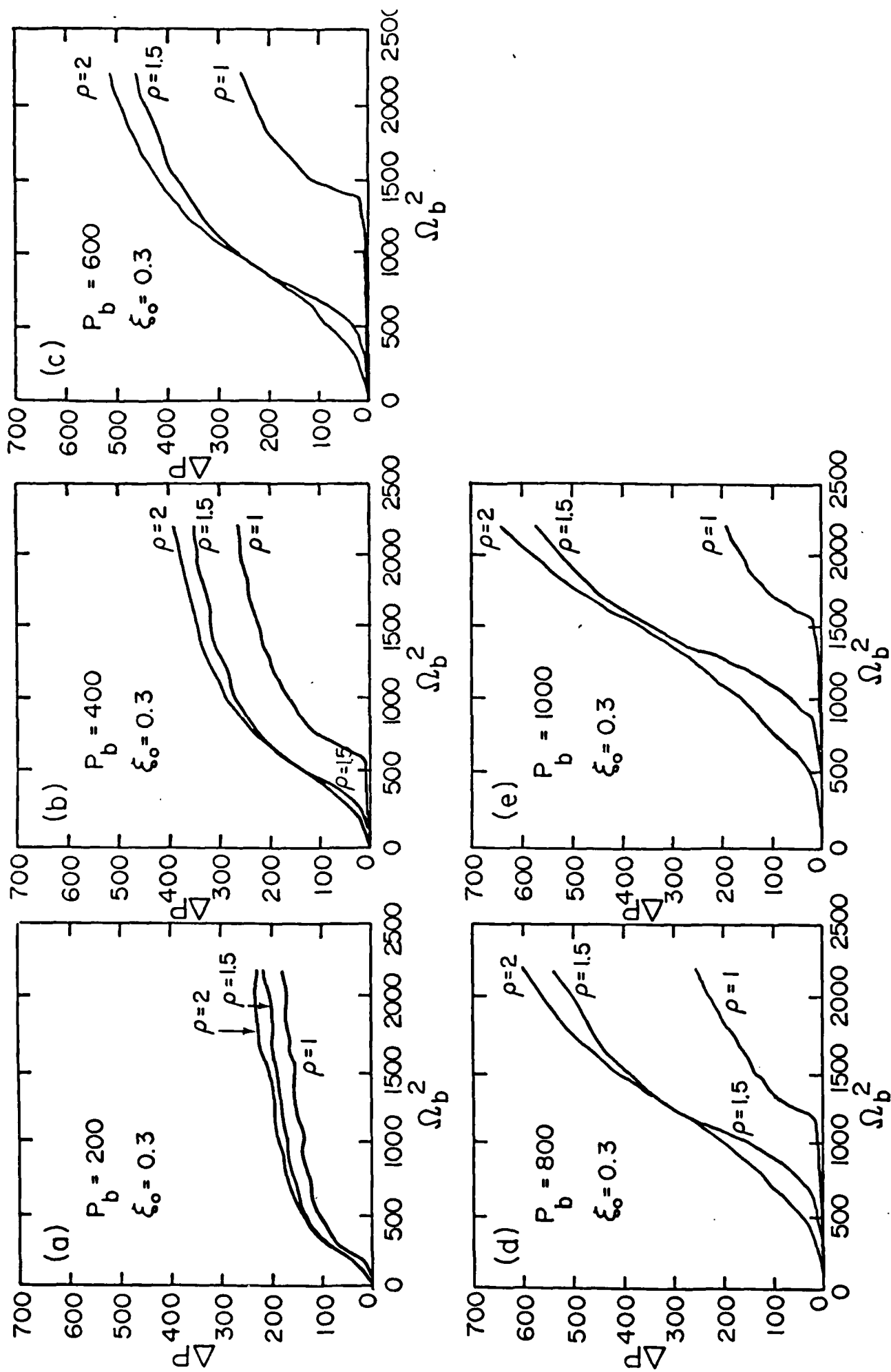


fig. 7

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