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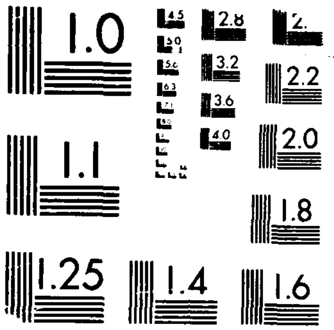

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OFF-BOUNDARY APPROACH TO THE BOUNDARY ELEMENT METHOD

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N00014-85-K-0401

February 15, 1988

NU-SML-TR-88-2

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Abstract

An alternative to the boundary element method for external domains is proposed, whereby the elements are located on the boundary, but the points of observation are taken inside the boundary. The modification removes the non-integrable singularities from the domain of integration. It also provides a simple way of avoiding the ill-conditioning that occurs at fictitious eigenfrequencies. The off-boundary BEM is applied to scattering of a plane, time-harmonic, longitudinal wave by a spherical cavity in an unbounded linearly elastic, isotropic, homogeneous solid. Results obtained by the off-boundary approach are compared with exact results and with results obtained by the conventional BEM approach. The off-boundary approach produces excellent results with less effort than the conventional BEM.

Introduction

This paper is concerned with scattering of time-harmonic waves by compact inhomogeneities in an elastic solid. In the calculation procedure a boundary integral equation is solved numerically by the boundary element method (BEM). An alternative to the usual boundary element method is employed. The alternative approach is applicable to cavities or fixed rigid bodies of general shape located in linearly elastic, isotropic, homogeneous solids, but the specific results reported herein are for spherical cavities. The alternative approach eliminates the cumbersome singularities associated with the usual approach to BEM, and it also eliminates the problems encountered at the so called fictitious eigenfrequencies.

First a brief presentation of the usual approach to solving these problems using BEM is presented. This section relies heavily on references to recent works. Then a discussion of common difficulties, including fictitious eigenfrequencies, is given - again relying heavily on available references. The fictitious eigenfrequencies coincide with the eigenfrequencies of a conjugate solid body, shaped like the cavity. For a sphere the frequency equation is presented and the relevant frequencies have been computed. Next the alternative BEM approach is described and numerical results obtained using this procedure on spherical cavities are presented for cases both near and away from frequencies at which difficulties are experienced with the usual method. Finally, advantages of the new approach are summarized.

Exact analytical solutions for scattering of harmonic waves by spherical scatterers have been studied extensively for problems in acoustics, electromagnetic scattering [1], and scattering of elastic waves [2]-[4]. These exact solutions are useful for a variety of well known applications. Recently, the exact solutions for elastodynamic scattering have been used as benchmarks for developing numerical procedures and computer programs for solving scattering problems with more complicated geometries [5]-[8]. These recent applications include both time [9] and frequency domain formulations [10].

Summary of the Usual Approach

The scatterer, the incident wave and the scattered field are shown in Fig. 1. Note that the surface of the scatterer is denoted by S and the regions inside and outside the scatterer by D_i and D_e, respectively. The incident field is time-harmonic, but the factor $\exp(-i\omega t)$, where ω is the circular frequency, is being omitted.

Reference [10] gives a detailed exposition of the way BEM is typically applied to elastodynamic boundary value problems. The description in reference [10] is applicable for both cavities and inclusions. For the present purpose we will confine our interest to cavities, but the new approach is also relevant for fixed rigid inclusions.

In the usual manner, the total displacement field is expressed as the sum of the incident and the scattered fields:

$$\underset{\sim}{u}(\underset{\sim}{x}) = \underset{\sim}{u}^{i}(\underset{\sim}{x}) + \underset{\sim}{u}^{s}(\underset{\sim}{x}) \qquad (1)$$

The integral representation for the total displacement field is based on the use of the basic singular elastodynamic displacement solution, which may be written as

$$U_{ij}(\underset{\sim}{x},\underset{\sim}{y}) = \frac{1}{4\pi\mu}\left[\frac{e^{ik_T r}}{r}\delta_{ij} + \frac{1}{k_T^2}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\left\{\frac{e^{ik_T r}}{r} - \frac{e^{ik_L r}}{r}\right\}\right] , \qquad (2)$$

where

$$r = |\underset{\sim}{x} - \underset{\sim}{y}| \qquad (3)$$

$$k_L = \omega/c_L \quad , \quad c_L^2 = (\lambda + 2\mu)/\rho \qquad (4a,b)$$

$$k_T = \omega/c_T \quad , \quad c_T^2 = \mu/\rho \qquad (5a,b)$$

The expression given by Eq.(2) represents the displacement at position $\underset{\sim}{x}$ in the x_i direction due to a unit point load applied at $\underset{\sim}{x} = \underset{\sim}{y}$ in the x_j-direction,

Now let us consider scattering by a cavity whose surface is free of tractions. By the use of Eq.(2), the integral representation for the total field may be written as

$$\int_S T_{ij}(\underset{\sim}{x},\underset{\sim}{y})u_j(\underset{\sim}{y})dS(\underset{\sim}{y}) + u_i^I(\underset{\sim}{x}) = \begin{cases} 0 \quad , \quad \underset{\sim}{x} \in D^i & \text{(6a)} \\ u_i(\underset{\sim}{x}) \quad , \quad \underset{\sim}{x} \in D^e & \text{(6b)} \end{cases} ,$$

where

$$T_{ij}(\underset{\sim}{x},\underset{\sim}{y}) = - \left[-\lambda \frac{\partial}{\partial x_m} U_{im}\delta_{jk} + \mu \frac{\partial}{\partial x_k} U_{ij} + \mu \frac{\partial}{\partial x_j} U_{ik} \right] n_k(\underset{\sim}{y}) \qquad (7)$$

In Eq.(7), $\underset{\sim}{n}(\underset{\sim}{y})$ is the unit outward normal from D_e.

Equations (6a) and (6b) are called integral <u>representations</u> because the source points, $\underset{\sim}{y}$, lie on the surface of the scatterer, while the field points, $\underset{\sim}{x}$, lie either inside or outside S. To obtain an integral equation, the field point is also taken to lie on S. The result is

$$C_{ij}(\underset{\sim}{x})u_j(\underset{\sim}{x}) = - \int_S T_{ij}(\underset{\sim}{x},\underset{\sim}{y})u_j(\underset{\sim}{y})dS_y + u_i^I(\underset{\sim}{x}), \quad \underset{\sim}{x} \in S \quad , \qquad (8)$$

where

$$C_{ij}(\underset{\sim}{x}) = \lim_{\epsilon \to 0} \int_{S(\underset{\sim}{x},\epsilon)} T_{ij}(\underset{\sim}{x},\underset{\sim}{y})dS_y \ . \qquad (9)$$

Here $S(\underset{\sim}{x},\epsilon)$ is the part of the surface of the sphere of radius ϵ, contained in D^e and centered at $\underset{\sim}{x}$. The BEM now solves the integral equation (8) by discretizing S into elements (also called surface patches), assuming a shape function for $\underset{\sim}{u}(\underset{\sim}{x})$ on each element, and numerically evaluating the integrals over each surface element. By sequentially moving the point of observation, $\underset{\sim}{x}$, to lie on each surface element, a set of linear algebraic equations is obtained and solved for the surface displacements. Once the surface displacements are known, they may be substituted into a discretized version of equation (6b) to obtain the field at any point in D^e.

The principal difficulty associated with the above procedure is that $T_{ij}(\underset{\sim}{x},\underset{\sim}{y})$ is extremely singular and, in fact, non-integrable, when $\underset{\sim}{x}=\underset{\sim}{y}$. Again, reference [10] gives a detailed exposition of the singularities and some techniques to circumvent them. Another difficulty typically encountered with this method is the enormous amount of computer time required if the frequency of the harmonic incident wave is high. Convergence at high frequency requires very fine meshes, so the costs of building and solving the set of algebraic equations rises exponentially. Nonetheless, this approach has been successfully applied to a wide variety of scattering problems and is currently increasing in popularity.

Discussion of Fictitious Eigenfrequencies

Still another difficulty associated with the usual BEM approach is that the solution of the integral equation is non-unique at certain frequencies [7]. These frequencies coincide with the eigenfrequencies of the interior problem with boundary conditions of zero displacement (the Dirichlet problem) [12]. This non-uniqueness is a result of the method of solution only. The physical problem does have a unique solution, hence the term fictitious eigenfrequencies.

At these frequencies the boundary integral equations are ill-conditioned but some approaches have been suggested to circumvent the difficulties, and obtain numerical solutions [7],[11]-[16]. One of the more popular approaches [7] involves formulating the BEM equations for both the exterior Neumann (zero tractions) and the interior Dirichlet (zero displacements) problems. The full set of equations for the exterior Neumann problem are then "constrained" by including equations from the interior Dirichlet problem so that a set of overdetermined equations results, and the least squares method or a similar technique can be applied to obtain solutions. Some of the other schemes first address the non-uniqueness of the integral equation before applying BEM techniques [12].

As noted above, the fictitious eigenfrequencies for the cavity problem coincide with the eigenfrequencies of the corresponding solid body, shaped like the cavity, but whose external surface is under zero displacement conditions. For a solid sphere the latter eigenfrequencies can easily be calculated, and hence we can obtain the fictitious eigenfrequencies for scattering by a spherical cavity.

Figure 2 shows the system of spherical coordinates that will be used. For the scattering problem the fields are axially symmetric, and hence we also consider axially symmetric vibrations of the solid sphere, i.e., $u_\phi = 0$. Using the notation of Ref.[3], expressions for the radial and polar displacements of any mode are written as

$$U_r^{(n)} = \frac{1}{r}\left[C_n \epsilon_{71}^{(1)} + D_n \epsilon_{72}^{(1)}\right] P_n(\cos\theta) \qquad (10)$$

$$U_\theta^{(n)} = \frac{1}{r}\left[C_n\epsilon_{81}^{(1)} + D_n\epsilon_{82}^{(1)}\right]\frac{dP_n(\cos\theta)}{d\theta} \tag{11}$$

where

$$\epsilon_{71}^{(1)}(k_L r) = nj_n(k_L r) - k_L r\; j_{n+1}(k_L r) \tag{12}$$

$$\epsilon_{72}^{(1)}(k_T r) = n(n+1)j_n(k_T r) \tag{13}$$

$$\epsilon_{81}^{(1)}(k_L r) = j_n(k_L r) \tag{14}$$

$$\epsilon_{82}^{(1)}(k_T r) = (n+1)j_n(k_T r) - k_T r\; j_{n+1}(k_T r) \tag{15}$$

Here k_L and k_T are defined by Eqs.(4a) and (5a), $j_n(\ )$ are spherical Bessel functions of the first kind of order n, $P_n(\cos\theta)$ are Legendre Polynomials of order n, and C_n and D_n are constants.

The case of spherical symmetry corresponds to n = 0. The component $U_\theta^{(o)}$ then vanishes identically, and Eq.(10) reduces to

$$U_r^{(o)} = C_o k_L \frac{\sin(k_L r) - (k_L r)\cos(k_L r)}{(k_L r)^2} \tag{16}$$

The condition $U_n^{(o)} = 0$ at r = a then yields the frequency equation

$$\tan(k_L r) = k_L r \tag{17}$$

The solutions to Eq.(17) are listed in Table I.

For $n \neq 0$ the conditions $U_r^{(n)} = 0$ and $U_\theta^{(n)} = 0$ at $r = a$ yield two homogeneous equations for the constants C_n and D_n. The condition that the determinant of the coefficients must vanish yields the frequency equation in the form

$$\epsilon_{71}^{(1)}(k_L a)\epsilon_{82}^{(1)}(k_T a) - \epsilon_{72}^{(1)}(k_T a)\epsilon_{81}^{(1)}(k_L a) = 0 \tag{18}$$

For a specific value of n, Eq.(18) has an infinite number of roots. For two values of Poisson's ratio, ν, and for $n = 1$ and $n = 2$, the first five roots are also listed in Table I. It is noted that the first spherically symmetric mode ($n = 0$) does not produce the lowest eigenfrequency. At least four lower eigenfrequencies occur, as indicated in Table I.

<u>Alternative to the Usual Approach</u>

Equations (6a) and (6b) are general, they can be used, in principle, for cavities of any shape. If the point of observation, $\underset{\sim}{x}$, is taken inside the cavity instead of on the surface of the cavity, then equation (6a) is applicable. Now the surface of the cavity is discretized and shape functions for $\underset{\sim}{u}(\underset{\sim}{x})$ over each element are assumed, exactly as before. As before, we select as many points of observation as there are surface elements, but now all the observation points are inside the cavity. By this procedure we again generate a set of linear algebraic equations which can be solved to give the set of surface displacements exactly as in the usual approach. In fact the two methods should give the same results. One advantage is that the integrals are not singular now since $\underset{\sim}{x} \neq \underset{\sim}{y}$ for $\underset{\sim}{x} \in D^i$.

This technique can also be used to calculate solutions at the fictitious eigenfrequencies. The set of equations generated by this method will differ from those generated by the usual method and will yield good results without the use of the least squares method or the interior Dirichlet BEM equation. In principle the points $\underset{\sim}{x}$ can be chosen arbitrarily, but there are indications that the observation points should not be selected too far from the surface of the cavity so that the dominance of the diagonal terms is retained. This method can also be used to easily generate the additional equations used in a least squares approach, as will be demonstrated in the next section.

Comparison of Results and Discussion

To verify the results of the alternative method, several cases of plane longitudinal-waves-incidence on a spherical cavity in an unbounded elastic solid have been considered. Figure 2 shows the geometry. First a frequency for which the usual BEM approach yields a solution that can be compared to an available exact solution, has been considered. The surface displacements and the backscattered field are shown in Fig. 3 and Fig. 4, respectively. The solutions by both the alternative and the usual BEM compare well with the exact solution (exact results borrowed from Ref.[5]).

Next a problem where the usual BEM approach fails was considered. Figures 5-7 show the radial surface displacements as calculated for three closely spaced non-dimensional frequencies by the same BEM programs as used for the results of Figs. 3 and 4. These frequencies are all close to the first eigenfrequency for $\nu = 0.41$ and $n = 1$ shown in Table I. The exact solution is not shown in Figs. 5-7, but the results presented in Fig. 5 indicate large changes in the solution obtained by the usual BEM approach for very minor changes in the non-dimensional frequency, a behavior which is typical near fictitious eigenfrequencies. The surface displacements for the same three non-dimensional frequencies as calculated by two modified approaches are shown in Figures 6 and 7. In Fig. 6 all of the observation points were moved inside of the cavity; an equal number of observation points and surface elements were used. In Fig.7 all but one of the observation points were taken on the surface, at the center of each element. One additional observation point was taken at the center of the cavity and was used to generate an overdetermined system of equations which was then solved using the least squares method. The agreement between Figs. 6 and 7 can be further improved by using more elements for the calculations.

The precise location of the observation points inside the cavity is not critical, but ill-conditioning can result if the points are chosen to lie too far away from the surface. For all of the cases presented in this paper using the alternative approach, the distance from the center of the cavity was 0.9a, and the points were located on lines joining the center of the elements and the center of the cavity. The surface displacement, $\underset{\sim}{u}(\underset{\sim}{x})$ was assumed constant over each element. Cases with the observation points further away from the surface (closer to the center) have also been successfully worked out, but the limits of the approach have not been tested.

Unfortunately the new technique will not work for scattering from cracks because there is no D^i for the observation point to be located in. For crack problems techniques similar to those in Refs.[10],[16]-[17] are recommended.

Summary and Conclusions

The straightforward alternative to the usual BEM discussed in this paper eliminates two substantial difficulties typically associated with solving wave scattering problems using the BEM. By taking the points of observation inside of the cavity (or fixed rigid body) and following the usual discretization and integration procedures employed in the BEM, both the singularities of the integrands and the difficulties associated with fictitious eigenfrequencies are eliminated. The precise location of the points inside the cavity is not critical and hence adjustment of their position to avoid ill-conditioning is possible. There are indications that the observation points should not be located too far from the surface of the cavity, but the maximum distance that can be tolerated may vary from problem to problem.

The alternative procedure produced very satisfactory results when applied to spherical cavities in unbounded elastic media, even when the usual BEM failed to give satisfactory results.

Acknowledgment

The work reported here was carried out in the course of research sponsored by the Office of Naval Research under Contract N00014-85-K-0401 with Northwestern University. A Grant from Cray Research Inc. for access to the Pittsburgh Supercomputer Center is also gratefully acknowledged.

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Table I: Eigenfrequencies for axially symmetric vibration modes of a solid sphere of radius a, with zero displacement conditions at r = a.

$\nu = 0.25$		1	2	3	4	5
n=0	$k_L a$	4.493	7.725	10.904	14.066	17.220
	$k_T a$	7.782	13.380	18.886	24.363	29.826
n=1	$k_L a$	2.303	3.581	5.345	5.962	7.215
	$k_T a$	3.989	6.202	9.257	10.326	12.497
n=2	$k_L a$	3.334	4.466	6.160	7.269	8.077
	$k_T a$	5.775	7.735	10.669	12.590	13.990

$\nu = 0.41$						
n=0	$k_L a$	4.493	7.725	10.904	14.066	17.220
	$k_T a$	7.782	13.380	18.886	24.363	29.826
n=1	$k_L a$	1.914	2.564	3.657	4.870	5.900
	$k_T a$	4.901	6.564	9.363	12.469	15.106
n=2	$k_L a$	2.571	3.452	4.285	5.446	6.650
	$k_T a$	6.583	8.838	10.971	13.943	17.027

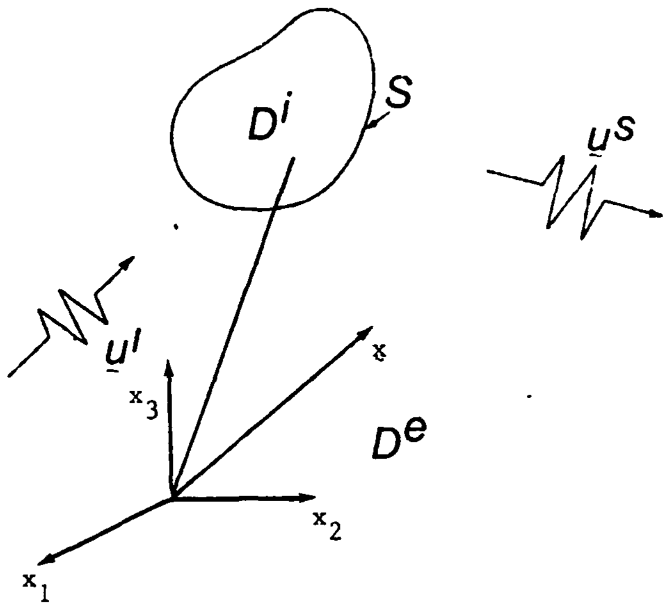

Fig. 1 Incident field, scatterer and scattered field

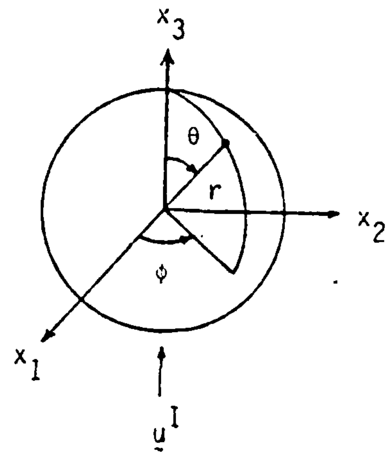

Fig. 2 Spherical geometry

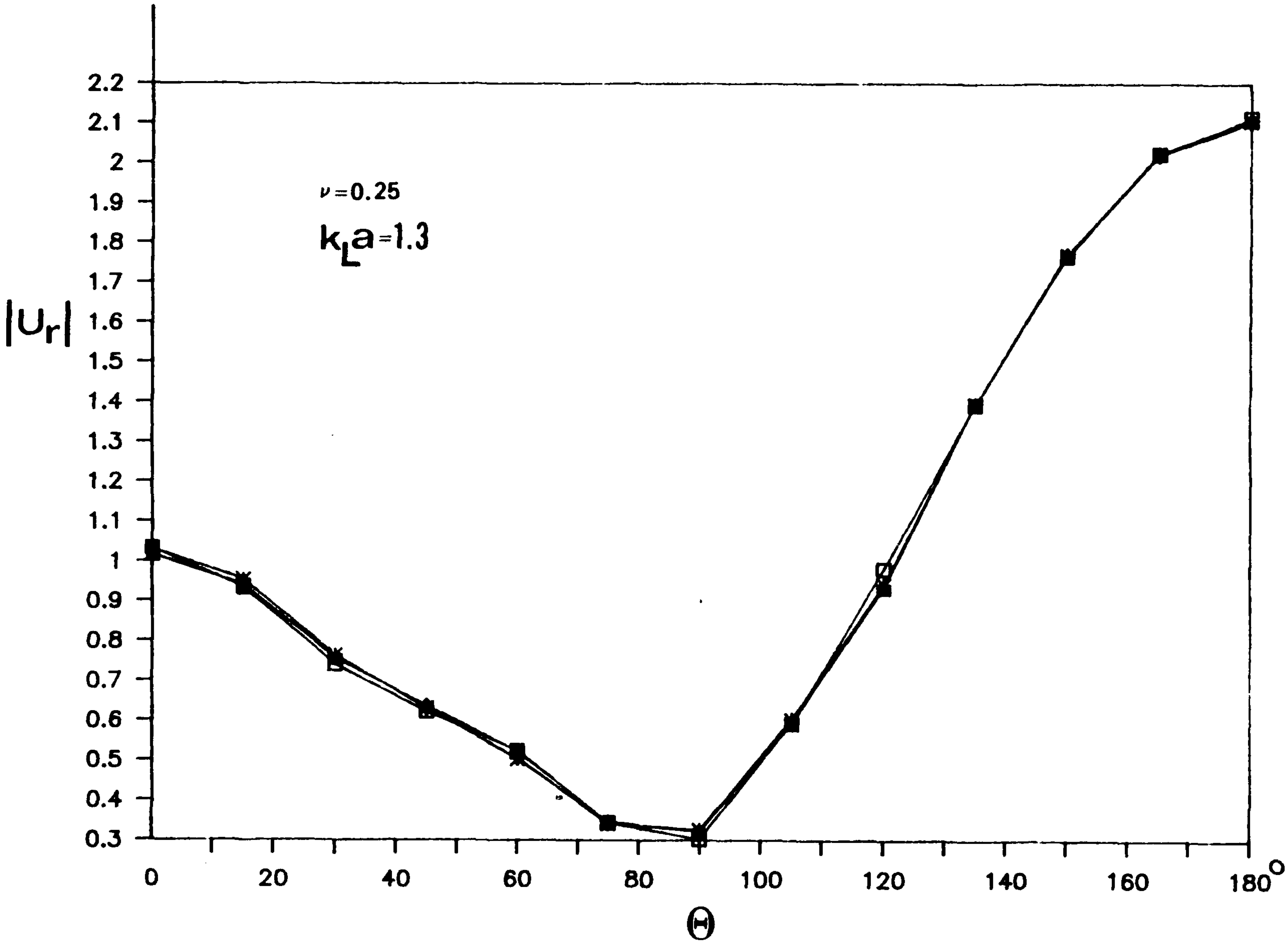

Fig. 3 Radial surface displacements for $k_L a = 1.3$: □ exact, + usual BEM, ○ alternative method, ◇ alternative method with least squares

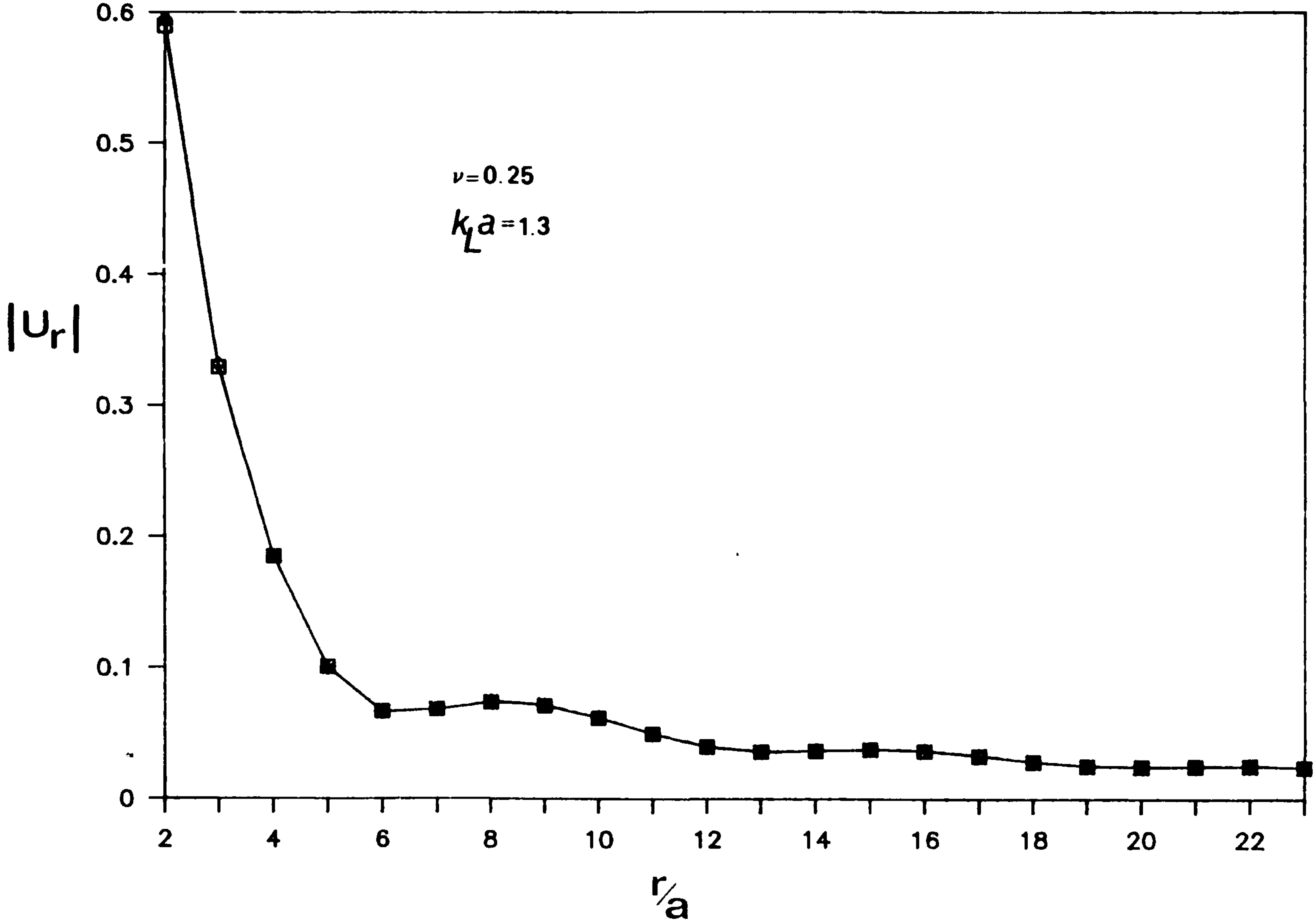

Fig. 4 Backscattered radial displacement for $k_L a = 1.3$: + usual BEM, O alternative method, ◇ alternative method with least squares

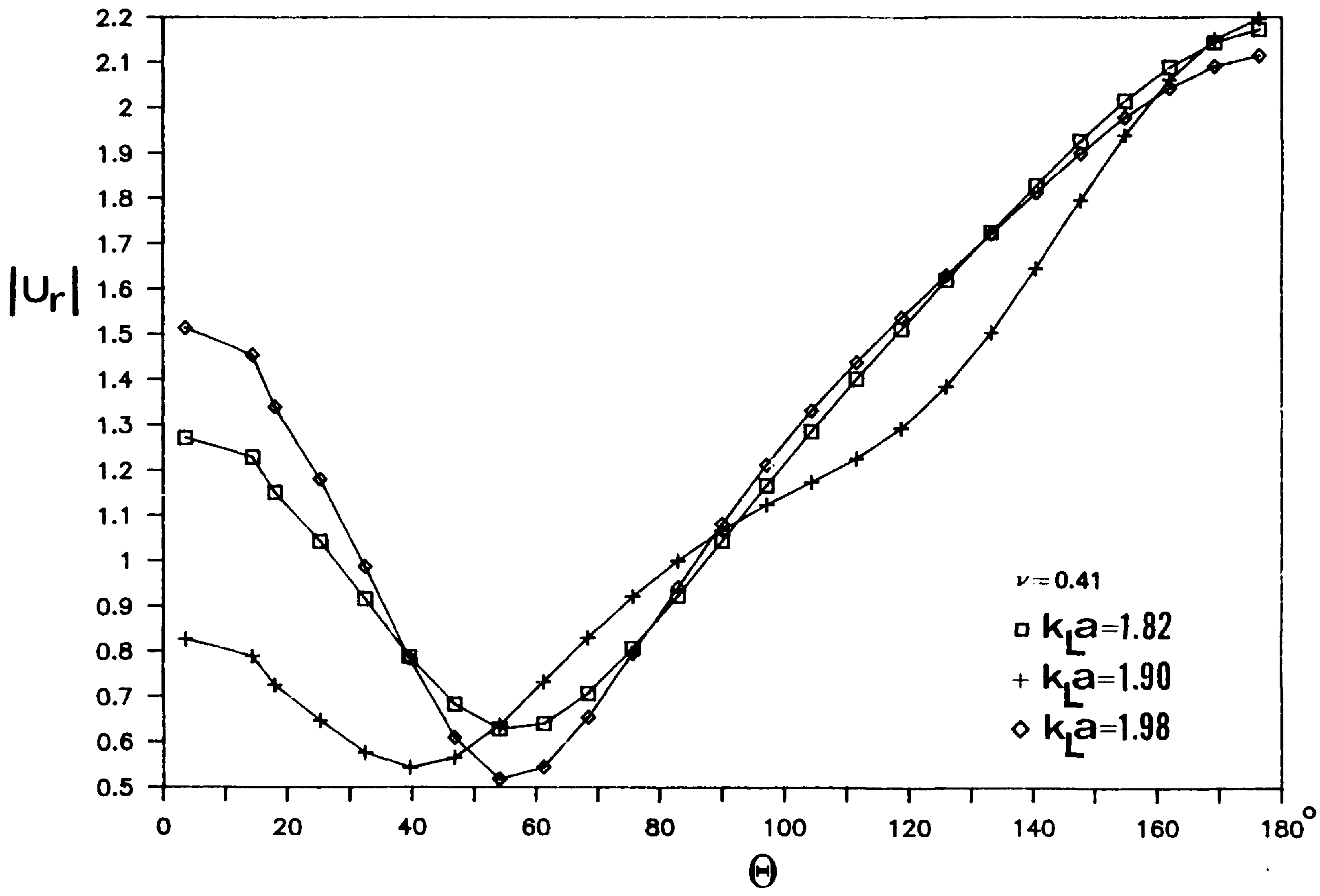

Fig. 5 Radial surface displacement using conventional BEM for three frequencies near an eigenfrequency

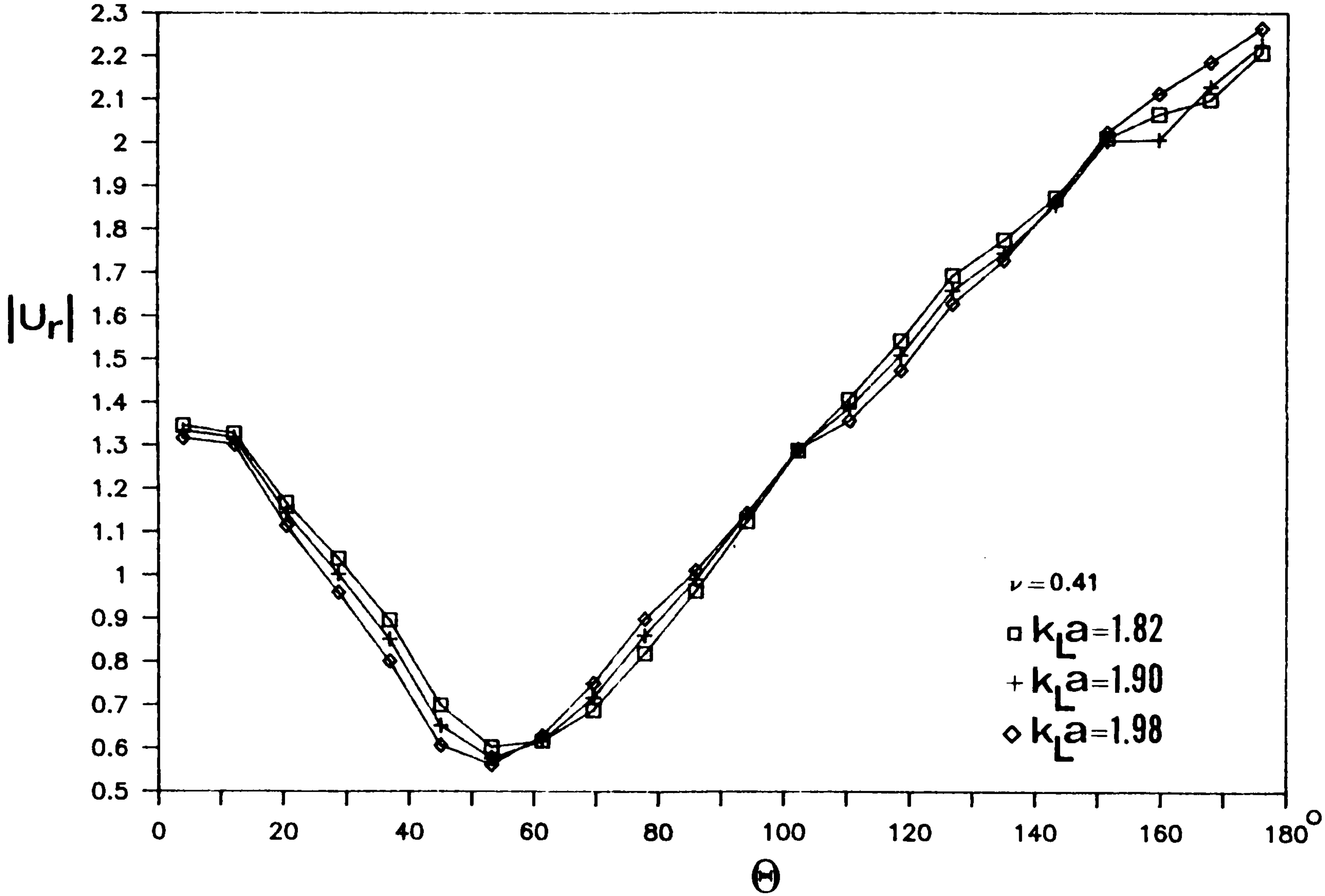

Fig. 6 Radial surface displacement using alternative method for same three frequencies as Fig. 5 (near eigenfrequency)

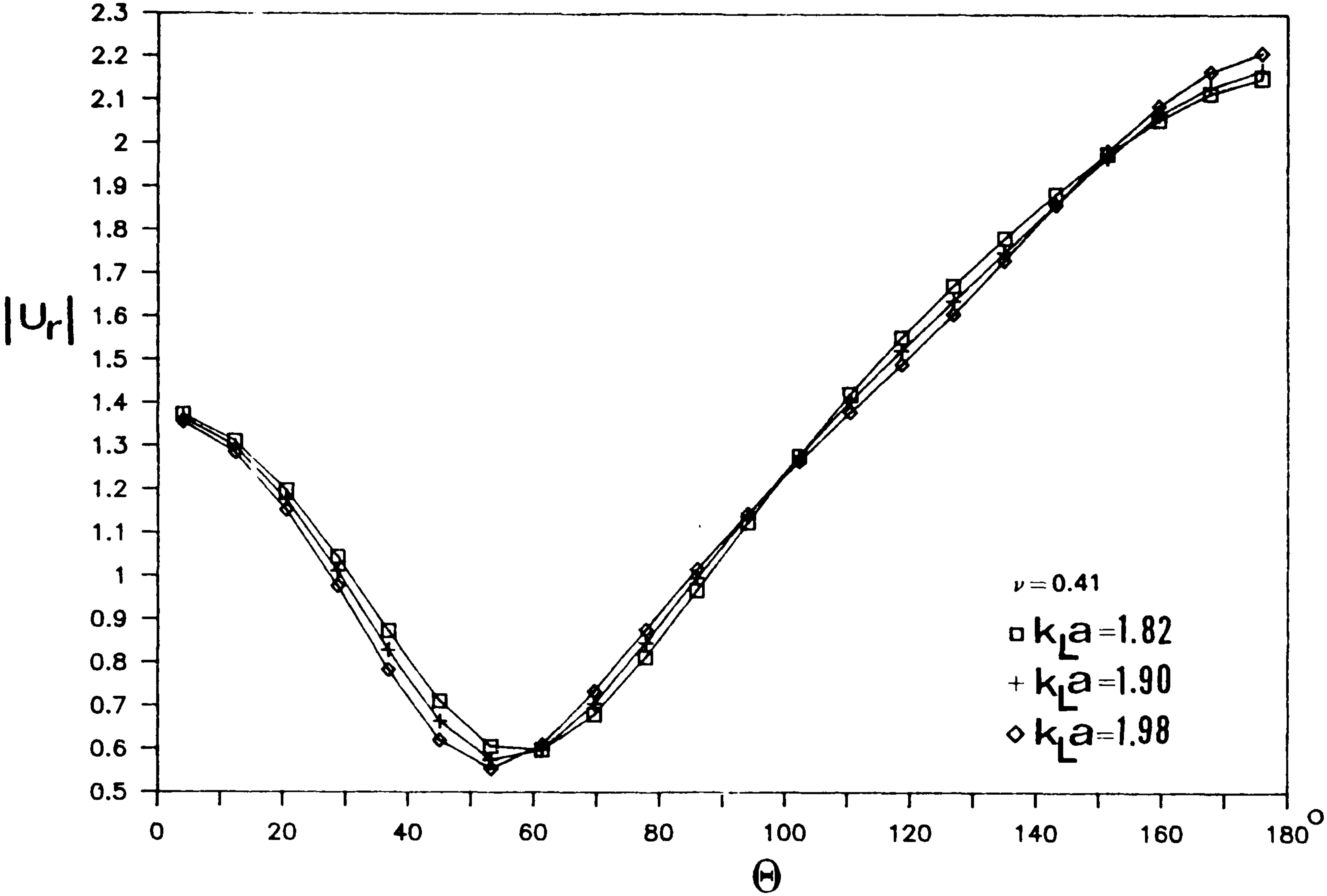

Fig. 7 Radial surface displacement using alternative least squares method for same three frequencies as Fig. 5 (near eigenfrequency)

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1. REPORT NUMBER: NU-SML-TR-88-2	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Off-Boundary Approach to the Boundary Element Method	5. TYPE OF REPORT & PERIOD COVERED: Interim
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): J.D. Achenbach, G.E. Kechter and Y.-L. Xu	8. CONTRACT OR GRANT NUMBER(s): N00014-85-0401
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Northwestern University, Evanston, IL 60208	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Structural Mechanics Department, Department of the Navy, Arlington, VA 22217	12. REPORT DATE: February 15, 1988
	13. NUMBER OF PAGES: 23
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION, DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This paper will appear in Computer Methods in Applied Mechanics and Engineering.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

boundary element method
scattering
cavity
elastic solids

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An alternative to the boundary element method for external domains is proposed, whereby the elements are located on the boundary, but the points of observation are taken inside the boundary. The modification removes the non-integrable singularities from the domain of integration. It also provides a simple way of avoiding the ill-conditioning that occurs at fictitious eigen-frequencies. The off-boundary BEM is applied to scattering of a plane, time-harmonic, longitudinal wave by a spherical cavity in an unbounded linearly, elastic, isotropic, homogeneous solid. Results obtained by the off-boundary

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) approach are compared with exact results and with results obtained by the conventional BEM approach. The off-boundary approach produces excellent results with less effort than the conventional BEM.

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