



2

DTIC FILE COPY

MULTI-ECHELON INVENTORY SYSTEMS
WITH LATERAL SUPPLY:
A TECHNICAL NOTE

Report AF701R1

AD-A193 202

DTIC
ELECTE
MAR 15 1988
S D

January 1988

Craig C. Sherbrooke

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

Prepared pursuant to Department of Defense Contract MDA903-85-C-0139.
The views expressed here are those of the Logistics Management Institute at
the time of issue but not necessarily those of the Department of Defense.
Permission to quote or reproduce any part must - except for Government
purposes - be obtained from the Logistics Management Institute.

LOGISTICS MANAGEMENT INSTITUTE
6400 Goldsboro Road
Bethesda, Maryland 20817-5886

88 3 14 046

Executive Summary**MULTI-ECHELON INVENTORY SYSTEMS
WITH LATERAL SUPPLY: A TECHNICAL NOTE**

Exact analytic solutions to many logistics problems can be obtained only by making unrealistic simplifying assumptions. An alternative is to construct efficient computational techniques that provide close approximations to the solutions of the real problems over a large range of simulated situations. *The author chose* ~~We have chosen~~ to take that approach in this analysis of a long-standing and difficult problem

arc developed ~~We develop~~ approximations to estimate the expected backorders in a multi-echelon system in which lateral supply actions between bases are allowed when a backorder occurs. These approximations are easy to compute, and the average absolute error is less than 3 percent when items are depot-reparable, even when bases are dissimilar; the average error is nearly 5 percent, however, and the technique is less convincing when items are base-reparable.

The average number of units of an item that are in repair and resupply is referred to as the pipeline. The benefit of lateral supply is negligible unless the spares in the system exceed the pipeline, and lateral supply is rarely needed when spares are more than 1.5 to 2 times the pipeline. However, for intermediate spares values, lateral supply provides large improvements in both absolute and percentage terms. Backorder reductions of 30 to 50 percent are not uncommon, and a 72-percent reduction was observed in one case. ←

Additional results include:

- Lateral supply has a greater impact when the demand rates are low.
- In the base-reparable case, the lateral supply time should be one-fourth or less of the base repair time; otherwise, lateral supply actions may degrade performance.
- In the base-reparable case where bases are identical, there is a kind of resonance effect; lateral supply has a much greater impact when the number of spares divided by the number of bases leaves a remainder of about 0.5. In one low-demand case, the backorders were 30 percent less than they would have been if the remainder had been 0.

CONTENTS

	<u>Page</u>
Executive Summary	iii
Chapter 1. Introduction	1
Chapter 2. Simulation Description	3
Chapter 3. The Interpolation Function	5
Chapter 4. Numerical Results	7
Depot-Reparable-Only Items	7
Base-Reparable Items	17
Number of Lateral Shipments	18
Chapter 5. Conclusion	31
References	33



Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution	
Availability Codes	
Dist	Avail and/or Special
A-1	

CHAPTER 1

INTRODUCTION

There has been substantial interest recently in the topic of lateral supply between bases. Logistics personnel in the U.S. Air Force have become more interested as the capabilities to make lateral shipments have increased, because of physical capabilities such as the European Distribution System and a similar system in the Pacific.

On the theoretical side, there have been several recent efforts. Slay described an approach to solving this problem at a Multi-Echelon Inventory Conference that has apparently stimulated research by several others [1]. One such paper by Lee [2] was published recently. Unfortunately, Lee makes simplifying assumptions that limit the applicability of the paper. For example, he assumes that if a backorder is needed and no base has stock on the shelf, that backorder will not be satisfied by a lateral action at a later point in time when the stock on hand at some base becomes positive. These "delayed lateral" transactions are explicitly considered in Slay's description of the problem. Moreover, our simulations of the problem show that these delayed lateral actions are an important component; they can exceed the immediate lateral actions when system stock is quite low. Our simulation has been used to test the accuracy of Slay's approximations. On a small sample of six cases, the average error was about 17 percent; the Slay upper bounds were closer, but the error was still nearly 8 percent. Another limitation of both the Slay and Lee approaches is that bases must be similar (Lee considers clusters of similar bases, with lateral actions made within a cluster). Slay considers the case in which some of the items can be repaired at the bases; Lee does not.

Our objective is to find an accurate approximation of the expected backorders under lateral supply that is:

- Easy to compute
- Easy to understand
- Capable of handling dissimilar bases

- Capable of handling cases with some base repair
- Able to determine whether lateral supply is useful in a particular situation.

The last characteristic refers to the fact that when the amount of stock is so low that there are always a number of backorders, lateral supply is of little help; similarly, when stock is very high, lateral actions are rarely needed. It is desirable that we be able to determine quickly when either condition prevails, even before computing the approximation.

We had some success with a new approximation, particularly when all bases are similar. The computation relates the steady-state probabilities at a random time, t , to the probabilities after a constant lateral supply time has elapsed (when all lateral actions placed before t and none placed after t would have arrived); the computation was quite involved, however.

Culosi of SRA Corporation [3] noticed that in the Slay report, bounds on the backorders under lateral supply were easily obtained using standard multi-echelon models such as VARI-METRIC [4, 5]: a lower bound if all bases were combined into a single base; an upper bound from the VARI-METRIC solution with no lateral supply. He used some of the Slay relationships to develop interpolation formulas between the bounds.

We have adopted the Culosi idea of using interpolation between the lower and upper bounds, but with totally different formulas. We have run a large number of simulations to check the accuracy of the formulas, and the results have been very encouraging. Furthermore, the computation is fast and accurate even for different types of bases.

CHAPTER 2

SIMULATION DESCRIPTION

A few comments about our simulation are in order since it is used as the standard for comparing the lateral approximations. We consider a single item and a system consisting of several bases at which demands occur and a supporting depot. We use the METRIC [5] assumptions, the most significant of which is that no queuing occurs for repair (the infinite channel queue assumption). Palm's theorem shows that the results depend only on the mean of the repair distribution. The simulation can be run with either constant or exponential repair times, and the same results are obtained. However, since the confidence intervals around the simulated backorders tend to be shorter for constant repair times, they were used in most cases. We assume that the status of all units (in transit between specific sites, in repair at a specific site, or on the shelf at a specific site) is known at any point. Furthermore, we assume that we know when each repair will be completed and when each shipment will arrive. Demand at each base is assumed to be Poisson distributed with a known, constant mean (although the rates may vary from one base to another). The percent of demand that is base-reparable is known, and the balance is assumed to be depot-reparable. Base and depot repair times are known.

A lateral supply is made whenever a demand at a base causes a backorder (i.e., stock on hand is zero and a customer has an unfilled demand) and a spare on hand at some other base can be lateralled to arrive at the base with the backorder before an item already in transit from a depot or completing base repair. In other words, a lateral supply is not made if the laterally supplied spare would arrive after the backorder had been satisfied.

A delayed lateral supply may be made when a backorder cannot be satisfied by an immediate lateral supply because no base has stock but some base receives stock at a later point in time and can laterally supply the deficient base before a spare can be provided by another source.

Note that the concept of stock level has little relevance in a lateral supply system such as we have described. With perfect information about the status and

location of items, decisions on where to ship from and to are made on the basis of expected need.

When all bases are similar and lateral shipment times are less than the shipment time from a depot to a base, any item completing depot repair should be sent to some base immediately. The appropriate base is the one with the smallest value for (stock on hand) + (in-transit time to it from another base, a lateral, or depot) + (in-base repair, assumed to be no longer than the depot-to-base shipping time) – backorders.

When bases are different, we subtract from the above quantity the expected demand at each base over some depot planning horizon. By simulation we determined that a good depot planning horizon is about twice the length of the depot-to-base order and ship time.

The other decision rule selects the base from which to make a lateral shipment. We compute the same quantity as above over a base-planning horizon and ship from the base with the largest value (provided that the on-hand stock is positive). Again simulation suggests that a planning horizon about 1.5 times as long as the lateral supply time is appropriate.

CHAPTER 3

THE INTERPOLATION FUNCTION

The interpolation function should give values between 0 and 1 that can be applied to the difference between the upper and lower bounds for the expected system backorders at a random point in time. A logical choice is the exponential function of the form:

$$F = 1 - \exp(-AT) \qquad 0 \leq T < S \qquad \text{[Eq. 3-1]}$$

where:

- F = interpolation value
- T = lateral supply time (assumed to be a fixed constant that is the same between any two bases)
- A = parameter.

With this formulation, $T=0$ results in $F=0$, which is appropriate because the several bases have become a single base with expected backorders (EBOs) equal to the lower bound. An infinite value for T corresponds to no lateral supply for which a value of $F=1$, corresponding to the upper bound, is appropriate. We will restrict the upper value for T to the order and ship time, S , because we are not really interested in lateral supply times that exceed S .

Another rationale for the exponential is that most objective functions in inventory theory, such as backorders, fill rates, and availability, are approximately exponential. It turns out that if the value of A is selected appropriately, Equation 3-1 gives values of F as a function of T that agree with the simulation. In other words, the exponential is a good model for the change in F as a function of T .

We must now find a method of estimating A . After reviewing a large number of simulations, we concluded that:

- A is independent of the number of bases.

- A decreases with the order and ship time.
- A decreases with the depot repair time.
- A increases with the demand rate per base.

The following empirical relationship was derived for A using regression, where all coefficients are highly statistically significant:¹

$$A = 0.279 \left(\frac{8}{S} \right)^{0.486} \left(\frac{30}{R} \right)^{0.127} \left(\frac{D}{0.22} \right)^{0.356} \quad [\text{Eq. 3-2}]$$

where:

S = average order and ship time (days)

R = average depot repair time (days)

D = average single-base daily demand rate.

Thus, when S=8, R=30, and D=0.22, the value of A=0.279. More generally, for bases of different types the average order and ship time is the demand-weighted average time; the average daily demand rate is the average of the base demand rates.

¹The probability that each coefficient is actually 0 is less than 0.00005.

CHAPTER 4

NUMERICAL RESULTS

We examined a number of cases in which parameters reflected U.S. Air Force experience. Our numbers and the approximate mean and standard deviation for the Air Force are shown in Table 4-1.

TABLE 4-1
SCENARIOS EXAMINED

Parameter values	Air Force	
	Mean	Standard deviation
Number of bases (2, 4, 5, 10, and 20)	----	----
Order and ship time (8, 16, and 30 days)	16	4
Depot repair time (30, 60, and 120 days)	60	30
Daily demand rate (0.055, 0.11, 0.22, 0.44, 0.88, and combinations of dissimilar bases)	0.04	0.30
Base repair percent (0, 50, and 100)	----	----
Base repair time (4, 8, and 16 days)	7	4

Source: Air Force Recoverable Consumption Item Requirements System (D041). Statistics derived from several D041 data bases for the period 1984 - 1986.

Our data source did not give the mean and standard deviation for the number of bases or the percent of items that are base-reparable, but our parameter choices cover the range of interest. The distributions for the remaining four parameters tend to be skewed with long right-hand tails. Consequently, we have not chosen parameter values symmetrically around the means.

DEPOT-REPARABLE-ONLY ITEMS

The 40 cases in which all failures were repaired at a depot is discussed first. The case of depot repair is particularly important because the time to obtain

resupply from the depot can be fairly long, particularly if there are no spares on the shelf when the request is received at depot. In contrast, if the item is base-reparable and a backorder exists (an aircraft is down for the item), base management can sometimes expedite the repair and preclude the need for lateral supply.

The results are presented in Table 4-2 for 14 scenarios. Each set of results is preceded by a heading with parameters characterizing the scenario. Each scenario has one or more cases summarized in the body of the table and determined by the base and depot stock levels and the lateral resupply time. For each case, the table includes minimum EBO (the lower bound corresponding to treating all activity as occurring at a single base), maximum EBO (from VARI-METRIC with no lateral resupply), the associated fill rates, and the estimated EBO using the value of A determined from Equation 3-2. These quantities were all computed analytically; the final section of the table contains the simulation results.

For example, Case 1.1 in Table 4-2 indicates that the estimated backorders from the value of $A = 0.457$ in the heading give estimated backorders of 10.795 when the lateral supply time is 2 days. The simulation showed that the mean value for EBO was 10.824 with a 95-percent confidence interval from 10.464 to 11.184. The percent error was $100 \times (10.795 - 10.824)/10.824 = -0.3$. The absolute error is also shown for reasons discussed below. This is followed by the average number of units in the lateral pipeline (immediate and delayed). Note that the allocation of stock between depot and bases in each case is optimal as determined by VARI-METRIC.

The scenarios in Table 4-2 are presented in order of decreasing demand. For a given demand level, the scenarios are in order of decreasing number of bases. At the end of the table, a few scenarios with dissimilar bases are presented.

Overall, the 40 cases shown in Table 4-2 have 24 positive and 16 negative errors, with the average positive error being 2.9 percent and the average negative error being -2.1 percent. More important, the average absolute error is 2.6 percent. In fact, the results are probably even more accurate because the true value is obtained by simulation and is known only within statistical confidence limits. Only 3 of the 40 estimates fall outside of the 95-percent confidence limits, which is very impressive although it should be remembered that the four coefficients in Equation 3-2 were obtained using regression on the 40 cases. Thus, some

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS

<i>Scenario 1</i>		<i>Scenario 2</i>			
Number of bases	= 5	Number of bases	=	20	
Daily demand rate	= 0.88	Daily demand rate	=	0.22	
Base repair percent	= 0	Base repair percent	=	0	
Depot daily demand rate	= 4.4	Depot daily demand rate	=	4.4	
Order and ship time	= 8 days	Order and ship time	=	8 days	
Depot repair time	= 30 days	Depot repair time	=	30 days	
Depot repair pipeline	= 167.2	Depot repair pipeline	=	167.2	
Value of A from regression	= 0.457	Value of A from regression	=	0.279	
	Scenario 1				Scenario 2
	Case 1.1	Case 1.2	Case 1.3	Case 1.4	Case 2.1
Analytic Results					
Total stock	160	180	200	200	200
Depot stock level	120	125	135	135	120
Base stock level	8	11	13	13	4
Lateral supply time (days)	2	2	2	4	2
Minimum EBO	9.504	1.137	0.031	0.031	0.031
Fill rate (percent)	30.5	84.8	99.4	99.4	99.4
Maximum EBO	11.659	2.931	0.377	0.377	3.427
Fill rate (percent)	44.7	70.9	88.7	88.7	77.9
Estimated EBO	10.795	2.211	0.238	0.321	1.483
Simulation Results					
EBO mean	10.824	2.217	0.237	0.306	1.404
EBO lower confidence limit	10.464	2.077	0.210	0.283	1.364
EBO upper confidence limit	11.184	2.357	0.264	0.329	1.444
Lateral supply pipeline	0.902	0.521	0.058	0.017	0.875
Interpolation value F	0.612	0.602	0.596	0.796	0.404
"True" A	0.474	0.461	0.454	0.397	0.259
Percent error	-0.3	-0.3	0.4	4.9	5.6 ^a
Absolute error	0.029	0.006	0.001	0.015	0.079

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

Scenario 3		Scenario 4			
Number of bases	= 10	Number of bases	= 5		
Daily demand rate	= 0.22	Daily demand rate	= 0.22		
Base repair percent	= 0	Base repair percent	= 0		
Depot daily demand rate	= 2.2	Depot daily demand rate	= 1.1		
Order and ship time	= 8 days	Order and ship time	= 30 days		
Depot repair time	= 30 days	Depot repair time	= 120 days		
Depot repair pipeline	= 83.6	Depot repair pipeline	= 165.0		
Value of A from regression	= 0.279	Value of A from regression	= 0.123		
	Scenario 3				Scenario 4
	Case 3.1	Case 3.2	Case 3.3	Case 3.4	Case 4.1
Analytic Results					
Total stock	100	100	100	120	180
Depot stock level	60	60	60	70	125
Base stock level	4	4	4	5	11
Lateral supply time (days)	1	2	4	2	4
Minimum EBO	0.156	0.156	0.156	0.001	0.808
Fill rate (percent)	96.5	96.5	96.5	100.0	88.5
Maximum EBO	2.013	2.013	2.013	0.234	2.342
Fill rate (percent)	77.9	77.9	77.9	91.7	73.5
Estimated EBO	0.608	0.950	1.404	0.100	1.404
Simulation Results					
EBO mean	0.598	0.934	1.421	0.100	1.423
EBO lower confidence limit	0.554	0.894	1.355	0.095	1.243
EBO upper confidence limit	0.642	0.974	1.487	0.105	1.603
Lateral supply pipeline	0.000	0.548	0.459	0.059	0.365
Interpolation value F	0.238	0.419	0.681	0.426	0.401
"True" A	0.272	0.272	0.286	0.277	0.128
Percent error	1.7	1.7	-1.2	0.5	-1.3
Absolute error	0.010	0.016	0.017	0.000	0.019

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

Scenario 5							
	Scenario 5						
	Case 5.1	Case 5.2	Case 5.3	Case 5.4	Case 5.5	Case 5.6	Case 5.7
Number of bases = 10 Daily demand rate = 0.22 Base repair percent = 0 Depot daily demand rate = 2.2 Order and ship time = 16 days Depot repair time = 30 days Depot repair pipeline = 101.2 Value of A from regression = 0.199							
Analytic Results							
Total stock	104	104	110	120	120	140	140
Depot stock level	64	64	60	60	60	70	70
Base stock level	4	4	5	6	6	7	7
Lateral supply time (days)	2	4	4	2	4	2	4
Minimum EBO	2.782	2.782	1.095	0.142	0.142	0.001	0.001
Fill rate (percent)	63.4	63.4	82.3	97.0	97.0	100.0	100.0
Maximum EBO	7.798	7.798	5.341	2.781	2.781	0.595	0.595
Fill rate (percent)	61.8	61.8	68.3	78.0	78.0	91.0	91.0
Estimated EBO	4.431	5.537	3.427	1.009	1.591	0.196	0.327
Simulation Results							
EBO mean	4.560	5.746	3.460	0.926	1.570	0.168	0.311
EBO lower confidence limit	4.358	5.535	3.350	0.887	1.500	0.159	0.291
EBO upper confidence limit	4.762	5.955	3.570	0.965	1.640	0.177	0.331
Lateral supply pipeline	1.441	2.125	1.570	0.583	0.780	0.109	0.000
Interpolation value F	0.354	0.591	0.557	0.297	0.541	0.282	0.522
"True" A	0.219	0.223	0.204	0.176	0.195	0.165	0.185
Percent error	-2.8	-3.6	-1.0	9.0 ^a	1.3	16.6 ^a	5.1
Absolute error	0.129	0.208	0.033	0.083	0.021	0.028	0.016

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

<i>Scenario 6</i>		<i>Scenario 7</i>			
Number of bases	= 10	Number of bases	= 2		
Daily demand rate	= 0.22	Daily demand rate	= 0.22		
Base repair percent	= 0	Base repair percent	= 0		
Depot daily demand rate	= 2.2	Depot daily demand rate	= 0.44		
Order and ship time	= 8 days	Order and ship time	= 8 days		
Depot repair time	= 60 days	Depot repair time	= 30 days		
Depot repair pipeline	= 149.6	Depot repair pipeline	= 16.72		
Value of A from regression	= 0.255	Value of A from regression	= 0.279		
	<i>Scenario 6</i>				<i>Scenario 7</i>
	<i>Case 6.1</i>	<i>Case 6.2</i>	<i>Case 6.3</i>	<i>Case 6.4</i>	<i>Case 7.1</i>
Analytic Results					
Total stock	140	170	190	190	20
Depot stock level	120	130	140	140	12
Base stock level	2	4	5	5	4
Lateral supply time (days)	2	4	2	4	2
Minimum EBO	11.069	0.269	0.002	0.002	0.521
Fill rate (percent)	23.0	95.4	99.9	99.9	82.4
Maximum EBO	13.461	1.820	0.258	0.258	0.758
Fill rate (percent)	47.0	75.2	87.7	87.7	77.9
Estimated EBO	12.026	1.262	0.105	0.166	0.623
Simulation Results					
EBO mean	12.020	1.240	0.114	0.159	0.624
EBO lower confidence limit	11.670	1.165	0.091	0.140	0.589
EBO upper confidence limit	12.370	1.315	0.117	0.178	0.659
Lateral supply pipeline	0.908	0.460	0.053	0.041	0.072
Interpolation value F	0.398	0.626	0.397	0.612	0.434
"True" A	0.253	0.246	0.253	0.237	0.284
Percent error	0.0	1.7	0.8	4.6	-0.2
Absolute error	0.006	0.022	0.001	0.007	0.001

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

<i>Scenario 8</i>						
Number of bases	=	5				
Daily demand rate	=	0.22				
Base repair percent	=	0				
Depot daily demand rate	=	1.1				
Order and ship time	=	8 days				
Depot repair time	=	30 days				
Depot repair pipeline	=	41.8				
Value of A from regression	=	0.279				
	<i>Scenario 8</i>					
	Case 8.1	Case 8.2	Case 8.3	Case 8.4	Case 8.5	Case 8.6
<i>Analytic Results</i>						
Total stock	30	40	40	45	45	50
Depot stock level	25	30	30	30	30	30
Base stock level	1	2	2	3	3	4
Lateral supply time (days)	2	2	8	2	8	2
Minimum EBO	11.863	3.556	3.556	1.313	1.313	0.349
Fill rate (percent)	3.5	43.0	43.0	72.2	72.2	90.8
Maximum EBO	12.218	4.893	4.893	2.599	2.599	1.267
Fill rate (percent)	27.1	54.2	54.2	67.1	67.1	77.9
Estimated EBO	12.015	4.128	4.750	1.863	2.461	0.742
<i>Simulation Results</i>						
EBO mean	12.066	4.160	4.760	1.838	2.360	0.724
EBO lower confidence limit	11.888	4.060	4.600	1.708	2.230	0.684
EBO upper confidence limit	12.244	4.260	4.860	1.968	2.490	0.764
Lateral supply pipeline	0.117	0.469	0.000	0.440	0.215	0.280
Interpolation value F	0.573	0.452	0.878	0.408	0.814	0.408
"True" A	0.425	0.301	0.263	0.262	0.210	0.262
Percent error	-0.4	-0.8	0.4	1.4	4.3	2.4
Absolute error	0.051	0.032	0.020	0.025	0.101	0.018

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

<i>Scenario 9</i>		<i>Scenario 10</i>	
Number of bases	= 5	Number of bases	= 5
Daily demand rate	= 0.22	Daily demand rate	= 0.055
Base repair percent	= 0	Base repair percent	= 0
Depot daily demand rate	= 1.1	Depot daily demand rate	= 0.275
Order and ship time	= 16 days	Order and ship time	= 8 days
Depot repair time	= 120 days	Depot repair time	= 30 days
Depot repair pipeline	= 149.6	Depot repair pipeline	= 10.45
Value of A from regression	= 0.167	Value of A from regression	= 0.170
	Scenario 9		Scenario 10
	Case 9.1	Case 9.2	Case 10.1
<i>Analytic Results</i>			
Total stock	160	180	15
Depot stock level	130	135	10
Base stock level	6	9	1
Lateral supply time (days)	4	2	2
Minimum EBO	1.381	0.033	0.147
Fill rate (percent)	81.4	99.3	93.4
Maximum EBO	2.648	0.216	0.668
Fill rate (percent)	69.0	88.3	84.1
Estimated EBO	1.999	0.085	0.297
<i>Simulation Results</i>			
EBO mean	2.050	0.090	0.291
EBO lower confidence limit	1.858	0.061	0.265
EBO upper confidence limit	2.242	0.119	0.317
Lateral supply pipeline	0.396	0.038	0.144
Interpolation value F	0.528	0.311	0.277
"True" A	0.188	0.186	0.162
Percent error	-2.5	-5.5	2.2
Absolute error	0.051	0.005	0.006

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

<i>Scenario 11</i>		<i>Scenario 12</i>		
Number of bases	= 5 each of Types 1 & 2	Number of bases	= 5	
Daily demand rate	= 0.22; 0.11	Daily demand rate	= 0.055	
Base repair percent	= 0	Base repair percent	= 0	
Depot daily demand rate	= 1.65	Depot daily demand rate	= 0.275	
Order and ship time	= 8 days	Order and ship time	= 16 days	
Depot repair time	= 30 days	Depot repair time	= 60 days	
Depot repair pipeline	= 62.7	Depot repair pipeline	= 20.9	
Value of A from regression	= 0.252	Value of A from regression	= 0.111	
	<i>Scenario 11</i>			<i>Scenario 12</i>
	<i>Case 11.1</i>	<i>Case 11.2</i>	<i>Case 11.3</i>	<i>Case 12.1</i>
<i>Analytic Results</i>				
Total stock	60	80	90	25
Depot stock level	45	50	50	15
Base stock level - Type 1	2	4	5	2
Base stock level - Type 2	1	2	3	----
Lateral supply time (days)	2	2	2	2
Minimum EBO	4.665	0.054	0.001	0.496
Fill rate (percent)	39.8	98.5	100.0	84.3
Maximum EBO	7.351	1.213	0.375	1.341
Fill rate (percent)	56.0	80.9	90.1	75.6
Estimated EBO	5.728	0.513	0.149	0.665
<i>Simulation Results</i>				
EBO mean	5.800	0.507	0.155	0.651
EBO lower confidence limit	5.660	0.487	0.149	0.578
EBO upper confidence limit	5.940	0.527	0.161	0.724
Lateral supply pipeline	1.023	0.344	0.098	0.157
Interpolation value F	0.423	0.391	0.411	0.184
"True" A	0.275	0.248	0.265	0.101
Percent error	-1.2	1.1	-3.7	2.1
Absolute error	0.072	0.006	0.006	0.014

TABLE 4-2

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - DEPOT-REPARABLE-ONLY ITEMS (Continued)

<i>Scenario 13</i>		<i>Scenario 14</i>	
Number of bases	= 2 each of Types 1 & 2	Number of bases	= 5 of Type 1 10 of Type 2
Daily demand rate	= 0.44; 0.11	Daily demand rate	= 0.44; 0.11
Base repair percent	= 0	Base repair percent	= 0
Depot daily demand rate	= 1.1	Depot daily demand rate	= 3.3
Order and ship time	= 8 days	Order and ship time	= 16 days
Depot repair time	= 30 days	Depot repair time	= 120 days
Depot repair pipeline	= 41.8	Depot repair pipeline	= 448.8
Value of A from regression	= 0.302	Value of A from regression	= 0.167

	<i>Scenario 13</i>				<i>Scenario 14</i>
	<i>Case 13.1</i>	<i>Case 13.2</i>	<i>Case 13.3</i>	<i>Case 13.4</i>	<i>Case 14.1</i>
<i>Analytic Results</i>					
Total stock	36	44	52	52	480
Depot stock level	24	30	34	34	395
Base stock level - Type 1	5	6	7	7	11
Base stock level - Type 2	1	1	2	2	3
Lateral supply time (days)	2	2	2	4	4
Minimum EBO	6.408	1.644	0.187	0.187	0.696
Fill rate (percent)	20.9	67.0	94.7	94.7	93.2
Maximum EBO	7.089	2.656	0.654	0.654	4.504
Fill rate (percent)	39.8	66.3	83.3	83.3	70.9
Estimated EBO	6.717	2.103	0.399	0.515	2.552
<i>Simulation Results</i>					
EBO mean	6.690	2.080	0.398	0.526	2.740
EBO lower confidence limit	6.550	2.000	0.370	0.497	2.416
EBO upper confidence limit	6.830	2.160	0.426	0.555	3.064
Lateral supply pipeline	0.275	0.322	0.137	0.000	1.469
Interpolation value F	0.414	0.431	0.452	0.726	0.537
"True" A	0.267	0.282	0.301	0.323	0.192
Percent error	0.4	1.1	0.2	-2.2	-6.8
Absolute error	0.027	0.023	0.001	0.011	0.188

degradation can be expected for new cases or parameter values that are very different from those in Table 4-2.

Even though the average absolute error is only 2.6 percent, the size of the maximum error is also of significant interest. In our set of 40 cases, the largest errors were 16.6 percent, 9.0 percent, and 6.8 percent. However, since these are percentage errors, they tend to be large when the true value (in the denominator) is small. The corresponding absolute errors were 0.028, 0.083, and 0.188 backorders. The first two errors are small, e.g., a backorder somewhere in the system for 1 day in every 36 (or 1 day in every 12) on the average. The third error of 0.188 is fairly large, but since this lies within the 95-percent confidence limits of the simulation, we cannot be sure that the error is really that large.

In summary, the results for depot-reparable-only items are very good. The simplicity of the calculation suggests that it should be useful.

BASE-REPARABLE ITEMS

Results for items that are fully or partially base-reparable, presented in Table 4-3, are less satisfactory. The value of A varies within a group of cases sharing the same parameters even when the lateral supply time is held constant. This variation can be seen most vividly in the results of Table 4-3, along with a new resonance phenomenon that did not appear for depot-reparable-only items. The latter refers to the fact that the interpolation value F (and A) is much larger (0.657) when the number of bases divides equally into the number of spares and is smallest (0.306 and 0.460) when the remainder is about 0.5. This is because lateral supply gives an even bigger improvement over VARI-METRIC, which has no lateral supply, when the spares do not divide equally.

Instead of modeling A as before, we have used a regression equation for the interpolation value F itself. The following formula was derived empirically and is much less satisfactory than that for depot-reparable-only items. Although the coefficients are all highly statistically significant,² more are needed:

$$F = 1.510 T^{-0.658} R^{-0.490} (1 + Z)^{3.100} D^{0.198} \quad [\text{Eq. 4-1}]$$

²The probability that each coefficient is actually 0 is less than 0.00005.

where Z is used to model the resonance phenomenon and was set to:

$$Z = \left(\frac{0.02}{D^{0.5}} \right) \left(\frac{2}{\text{Bases}} \right)^{0.36} \quad [\text{Eq. 4-2}]$$

The results for 40 cases with items that are fully or partially base-reparable are less good. Twenty-three cases have positive errors averaging 4.2 percent; the 17 negative errors averaged -5.1 percent; and the average absolute error is 4.6 percent. In more than half of the cases, the errors are outside the 95-percent confidence limits. The maximum error is 14.3 percent, and one absolute error is quite large as well (0.609). The values of A, as given by Equation 3-1, are not used in this computation, but the "true" values are shown in Table 4-3 to indicate their variability; when they exceed 0.999, four asterisks are shown.

An important finding in this work is that when the lateral supply time exceeds about one-fourth of the base repair time, lateral supply provides essentially no improvement. In some cases it degrades performance (F greater than 1.0), as shown by the cases in Table 4-4. The errors are very large, but they are irrelevant since our recommendation is that lateral supply should be avoided when long lateral times will occur.

NUMBER OF LATERAL SHIPMENTS

It is desirable to estimate the number of lateral shipments (both immediate and delayed) because of the cost associated with them. We used the same approximation approach as above with regression on the depot-reparable-only cases for which the independent variables were estimated backorders under lateral supply, the pipeline stock divided by the total stock in the system, and the lateral supply time divided by the order and ship time. The dependent variable was the average number of units in the lateral pipeline (immediate plus delayed), shown in Tables 4-2, 4-3, and 4-4. Unfortunately, the regression did not give satisfactory results (the average absolute error was 14 percent) and we have not presented them.

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - BASE-REPARABLE ITEMS

<i>Scenario 1</i>		<i>Scenario 2</i>	
Number of bases	= 5 each of Types 1 & 2	Number of bases	= 1 each of Types 1 & 2
Daily demand rate	= 0.88	Daily demand rate	= 0.88
Base repair percent	= 100	Base repair percent	= 100
Base repair time	= 8 days	Base repair time	= 8 days
Base repair pipeline	= 70.4	Base repair pipeline	= 14.08

	Scenario 1			Scenario 2		
	Case 1.1	Case 1.2	Case 1.3	Case 2.1	Case 2.2	Case 2.3
Analytic Results						
Total stock	80	85	90	16	17	18
Depot stock level	0	0	0	0	0	0
Base stock level - Type 1	8	9	9	8	9	9
Base stock level - Type 2	8	8	9	8	8	9
Lateral supply time (days)	2	2	2	2	2	2
Minimum EBO	0.564	0.164	0.038	0.750	0.499	0.321
Fill rate (percent)	88.4	96.1	99.0	74.9	82.1	87.8
Maximum EBO	6.579	5.198	3.818	1.316	1.040	0.764
Fill rate (percent)	72.4	77.5	82.6	72.4	77.5	82.6
Estimated EBO	5.797	4.231	3.326	1.257	0.923	0.717
Simulation Results						
EBO mean	5.188	4.009	3.071	1.243	0.945	0.712
EBO lower confidence limit	5.135	3.963	3.035	1.214	0.920	0.692
EBO upper confidence limit	5.241	4.055	3.107	1.272	0.970	0.732
Lateral supply pipeline	2.234	1.642	1.193	0.287	0.241	0.191
Interpolation value F	0.769	0.764	0.802	0.871	0.825	0.884
"True" A	0.732	0.721	0.811	****b	0.871	****b
Percent error	11.7 ^a	5.5 ^a	8.3 ^a	1.1	-2.3	0.7
Absolute error	0.609	0.222	0.255	0.014	0.022	0.005

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

^b Exceeds 0.999

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE ITEMS (Continued)

<i>Scenario 3</i>				
Number of bases	=	4 of Type 1 1 of Type 2 5 of Type 3		
Daily demand rate	=	0.22		
Base repair percent	=	100		
Base repair time	=	8 days		
Base repair pipeline	=	17.6		
	<i>Scenario 3</i>			
	Case 3.1	Case 3.2	Case 3.3	Case 3.4
<i>Analytic Results</i>				
Total stock	20	24	25	30
Depot stock level	0	0	0	0
Base stock level – Type 1	2	3	3	3
Base stock level – Type 2	2	2	3	3
Base stock level – Type 3	2	2	2	3
Lateral supply time (days)	2	2	2	2
Minimum EBO	0.765	0.145	0.089	0.005
Fill rate (percent)	76.2	94.4	96.4	99.8
Maximum EBO	4.069	3.034	2.775	1.482
Fill rate (percent)	74.1	80.4	81.9	89.8
Estimated EBO	3.030	1.905	1.677	1.018
<i>Simulation Results</i>				
EBO mean	3.018	1.876	1.672	0.985
EBO lower confidence limit	2.986	1.858	1.649	0.970
EBO upper confidence limit	3.050	1.894	1.695	1.000
Lateral supply pipeline	1.927	1.359	1.231	0.677
Interpolation value F	0.682	0.599	0.589	0.664
"True" A	0.573	0.457	0.445	0.545
Percent error	0.4	1.6 ^a	0.3	3.3 ^a
Absolute error	0.012	0.029	0.005	0.033

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - BASE-REPARABLE ITEMS (Continued)

<i>Scenario 4</i>						
Number of bases		= 2 of Type 1 1 of Type 2 2 of Type 3				
Daily demand rate		= 0.22				
Base repair percent		= 100				
Base repair time		= 16 days				
Base repair pipeline		= 17.60				
	<i>Scenario 4</i>					
	Case 4.1	Case 4.2	Case 4.3	Case 4.4	Case 4.5	Case 4.6
<i>Analytic Results</i>						
Total stock	20	22	25	20	22	25
Depot stock level	0	0	0	0	0	0
Base stock level - Type 1	4	5	5	4	5	5
Base stock level - Type 2	4	4	5	4	4	5
Base stock level - Type 3	4	4	5	4	4	5
Lateral supply time (days)	2	2	2	4	4	4
Minimum EBO	0.765	0.353	0.089	0.765	0.353	0.089
Fill rate (percent)	76.2	87.6	96.4	76.2	87.6	96.4
Maximum EBO	2.664	2.108	1.273	2.664	2.108	1.273
Fill rate (percent)	72.2	77.5	85.5	72.2	77.5	85.5
Estimated EBO	1.712	1.104	0.679	2.258	1.539	1.020
<i>Simulation Results</i>						
EBO mean	1.601	1.081	0.615	2.216	1.612	1.002
EBO lower confidence limit	1.555	1.048	0.595	2.163	1.571	0.974
EBO upper confidence limit	1.647	1.114	0.635	2.269	1.653	1.030
Lateral supply pipeline	0.727	0.620	0.425	1.073	0.857	0.555
Interpolation value F	0.440	0.415	0.445	0.764	0.718	0.771
"True" A	0.290	0.268	0.294	0.361	0.316	0.369
Percent error	6.9 ^a	2.2	10.3 ^a	1.9	-4.5 ^a	1.7
Absolute error	0.111	0.023	0.064	0.042	0.073	0.018

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - BASE-REPARABLE ITEMS (Continued)

<i>Scenario 5</i>						
Number of bases = 1 each of Types 1 - 5 Daily demand rate = 0.22 Base repair percent = 100 Base repair time = 4 days Base repair pipeline = 4.4						
	<i>Scenario 5</i>					
	Case 5.1	Case 5.2	Case 5.3	Case 5.4	Case 5.5	Case 5.6
Analytic Results						
Total stock	5	6	7	8	9	10
Depot stock level	0	0	0	0	0	0
Base stock level - Type 1	1	2	2	2	2	2
Base stock level - Type 2	1	1	2	2	2	2
Base stock level - Type 3	1	1	1	2	2	2
Base stock level - Type 4	1	1	1	1	2	2
Base stock level - Type 5	1	1	1	1	1	2
Lateral supply time (days)	1	1	1	1	1	1
Minimum EBO	0.574	0.294	0.138	0.059	0.024	0.009
Fill rate (percent)	72.0	84.4	92.1	96.4	98.5	99.4
Maximum EBO	1.474	1.254	1.033	0.813	0.593	0.373
Fill rate (percent)	78.0	81.2	84.4	87.6	90.8	94.0
Estimated EBO	1.135	0.849	0.617	0.463	0.353	0.235
Simulation Results						
EBO mean	1.169	0.848	0.619	0.457	0.343	0.260
EBO lower confidence limit	1.153	0.835	0.609	0.449	0.336	0.255
EBO upper confidence limit	1.185	0.861	0.629	0.465	0.350	0.265
Lateral supply pipeline	0.559	0.512	0.439	0.355	0.276	0.215
Interpolation value F	0.661	0.577	0.537	0.527	0.561	0.690
"True" A	****b	0.861	0.770	0.750	0.823	****b
Percent error	-2.9 ^a	0.1	-0.2	1.3	2.8 ^a	-9.4 ^a
Absolute error	0.034	0.001	0.002	0.006	0.010	0.025

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

^b Exceeds 0.999

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE ITEMS (Continued)

Scenario 6			
	Number of bases	=	1 each of Types 1 & 2
	Daily demand rate	=	0.22
	Base repair percent	=	100
	Base repair time	=	8 days
	Base repair pipeline	=	3.52
	Scenario 6		
	Case 6.1	Case 6.2	Case 6.3
Analytic Results			
Total stock	4	5	6
Depot stock level	0	0	0
Base stock level – Type 1	2	3	3
Base stock level – Type 2	2	2	3
Lateral supply time (days)	2	2	2
Minimum EBO	0.533	0.255	0.109
Fill rate (percent)	72.2	85.5	93.3
Maximum EBO	0.814	0.555	0.296
Fill rate (percent)	74.1	81.9	89.8
Estimated EBO	0.737	0.422	0.245
Simulation Results			
EBO mean	0.753	0.445	0.255
EBO lower confidence limit	0.736	0.432	0.246
EBO upper confidence limit	0.770	0.458	0.264
Lateral supply pipeline	0.197	0.160	0.115
Interpolation value F	0.784	0.634	0.779
"True" A	0.765	0.502	0.754
Percent error	- 2.2	- 5.2 ^a	- 3.9 ^a
Absolute error	0.016	0.023	0.010

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE ITEMS (Continued)

<i>Scenario 7</i>				
Number of bases	=	5 of Type 1 1 of Type 2 2 each of Types 3 & 4		
Daily demand rate	=	0.055		
Base repair percent	=	100		
Base repair time	=	8 days		
Base repair pipeline	=	4.4		
	<i>Scenario 7</i>			
	Case 7.1	Case 7.2	Case 7.3	Case 7.4
<i>Analytic Results</i>				
Total stock	5	6	8	10
Depot stock level	0	0	0	0
Base stock level – Type 1	1	1	1	1
Base stock level – Type 2	0	1	1	1
Base stock level – Type 3	0	0	1	1
Base stock level – Type 4	0	0	0	1
Lateral supply time (days)	2	2	2	2
Minimum EBO	0.574	0.294	0.059	0.009
Fill rate (percent)	72.0	84.4	96.4	99.4
Maximum EBO	2.620	2.264	1.552	0.840
Fill rate (percent)	78.6	81.4	87.1	92.7
Estimated EBO	1.426	1.166	0.804	0.474
<i>Simulation Results</i>				
EBO mean	1.358	1.063	0.707	0.490
EBO lower confidence limit	1.334	1.044	0.695	0.481
EBO upper confidence limit	1.382	1.082	0.719	0.499
Lateral supply pipeline	0.757	0.765	1.020	0.412
Interpolation value F	0.383	0.390	0.434	0.579
“True” A	0.241	0.247	0.284	0.432
Percent error	5.0 ^a	9.7 ^a	13.7 ^a	– 3.2 ^a
Absolute error	0.068	0.103	0.097	0.016

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE ITEMS (Continued)

<i>Scenario 8</i>				<i>Scenario 9</i>		
Number of bases	=	3 each of Types 1 – 3		Number of bases	=	1 each of Types 1 & 2
Daily demand rate	=	0.88; 0.11; 0.11		Daily demand rate	=	0.055
Base repair percent	=	100		Base repair percent	=	100
Base repair time	=	8 days		Base repair time	=	8 days
Base repair pipeline	=	26.4		Base repair pipeline	=	0.88
	Scenario 8			Scenario 9		
	Case 8.1	Case 8.2	Case 8.3	Case 9.1	Case 9.2	Case 9.3
Analytic Results						
Total stock	30	33	36	1	2	3
Depot stock level	0	0	0	0	0	0
Base stock level – Type 1	8	9	9	1	1	2
Base stock level – Type 2	1	1	2	0	1	1
Base stock level – Type 3	1	1	1	----	----	----
Lateral supply time (days)	2	2	2	2	2	2
Minimum EBO	0.764	0.275	0.082	0.295	0.075	0.015
Fill rate (percent)	79.1	91.3	97.0	78.0	94.0	98.8
Maximum EBO	3.742	2.914	2.253	0.524	0.168	0.095
Fill rate (percent)	73.5	81.7	83.3	78.6	92.7	95.9
Estimated EBO	2.822	2.099	1.706	0.379	0.133	0.045
Simulation Results						
EBO mean	2.967	2.194	1.667	0.365	0.136	0.052
EBO lower confidence limit	2.923	2.161	1.641	0.355	0.130	0.049
EBO upper confidence limit	3.011	2.227	1.693	0.375	0.142	0.055
Lateral supply pipeline	1.470	1.136	0.844	0.101	0.071	0.036
Interpolation value F	0.740	0.727	0.730	0.306	0.657	0.460
"True" A	0.673	0.649	0.654	0.183	0.535	0.308
Percent error	-4.9 ^a	-4.3 ^a	2.3 ^a	3.8 ^a	-2.3	-14.3 ^a
Absolute error	0.145	0.095	0.039	0.014	0.003	0.007

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-3

EXPECTED BACKORDERS UNDER LATERAL SUPPLY - BASE-REPARABLE ITEMS (Continued)

<i>Scenario 10</i>					
	Number of bases	=	10		
	Daily demand rate	=	0.22		
	Base repair percent	=	50		
	Depot daily demand rate	=	1.1		
	Order and ship time	=	8 days		
	Base repair time	=	30 days		
	Depot repair time	=	30 days		
	Base repair pipeline	=	50.6		
<i>Scenario 10</i>					
	Case 10.1	Case 10.2	Case 10.3	Case 10.4	Case 10.5
<i>Analytic Results</i>					
Total stock	60	65	70	80	60
Depot stock level	30	35	30	30	30
Base stock level	3	3	4	5	3
Lateral supply time (days)	2	2	2	2	4
Minimum EBO	0.344	0.074	0.012	0.000	0.344
Fill rate (percent)	91.5	97.9	99.6	100.0	91.5
Maximum EBO	2.959	1.969	1.164	0.410	2.959
Fill rate (percent)	75.3	82.2	86.0	92.8	75.3
Estimated EBO	1.517	0.807	0.529	0.184	2.195
<i>Simulation Results</i>					
EBO mean	1.616	0.913	0.530	0.181	2.410
EBO lower confidence limit	1.576	0.877	0.511	0.176	2.359
EBO upper confidence limit	1.656	0.943	0.549	0.186	2.461
Lateral supply pipeline	0.976	0.617	0.355	0.106	1.084
Interpolation value F	0.487	0.441	0.450	0.441	0.790
"True" A	0.333	0.291	0.299	0.291	0.390
Percent error	-6.1 ^a	-11.3 ^a	-0.2	1.6	-8.9 ^a
Absolute error	0.099	0.103	0.001	0.003	0.215

^a The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-4

**EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE-ONLY ITEMS –
LONG LATERAL TIMES**

<i>Scenario 1</i>		<i>Scenario 2</i>	
Number of bases	= 10	Number of bases	= 4 of Type 1 1 of Type 2 5 of Type 3
Daily demand rate	= 0.88	Daily demand rate	= 0.22
Base repair percent	= 100	Base repair percent	= 100
Base repair time	= 8 days	Base repair time	= 8 days
Base repair pipeline	= 70.4	Base repair pipeline	= 17.6
	Scenario 1		Scenario 2
	Case 1.1	Case 1.2	Case 2.1
<i>Analytic Results</i>			
Total stock	80	90	24
Depot stock level	0	0	0
Base stock level – Type 1	8	9	3
Base stock level – Type 2	----	----	2
Base stock level – Type 3	----	----	2
Lateral supply time (days)	4	4	4
Minimum EBO	0.564	0.038	0.145
Fill rate (percent)	88.4	99.0	94.4
Maximum EBO	6.579	3.818	3.034
Fill rate (percent)	72.4	82.6	80.4
Estimated EBO	8.821	5.226	2.923
<i>Simulation Results</i>			
EBO mean	6.782	3.930	2.862
EBO lower confidence limit	6.718	3.882	2.832
EBO upper confidence limit	6.846	3.978	2.892
Lateral supply pipeline	0.820	0.307	1.447
Interpolation value F	1.034	1.030	0.940
"True" A	*****a	*****a	0.705
Percent error	30.1 ^b	33.0 ^b	2.1 ^b
Absolute error	2.039	1.296	0.061

^a Exceeds 0.999

^b The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-4

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE-ONLY ITEMS –
LONG LATERAL TIMES (Continued)

<i>Scenario 3</i>				
Number of bases	=	2 of Type 1 1 of Type 2 2 of Type 3		
Daily demand rate	=	0.22		
Base repair percent	=	100		
Base repair time	=	16 days		
Base repair pipeline	=	17.6		
<i>Scenario 3</i>				
	Case 3.1	Case 3.2	Case 3.3	Case 3.4
<i>Analytic Results</i>				
Total stock	20	23	25	20
Depot stock level	0	0	0	0
Base stock level – Type 1	4	5	5	4
Base stock level – Type 2	4	5	5	4
Base stock level – Type 3	4	4	5	4
Lateral supply time (days)	6	6	6	8
Minimum EBO	0.765	0.229	0.089	0.765
Fill rate (percent)	76.2	91.5	96.4	76.2
Maximum EBO	2.664	1.829	1.273	2.664
Fill rate (percent)	72.2	80.2	85.5	72.2
Estimated EBO	2.715	1.641	1.304	3.121
<i>Simulation Results</i>				
EBO mean	2.645	1.702	1.257	2.884
EBO lower confidence limit	2.588	1.660	1.223	2.823
EBO upper confidence limit	2.702	1.744	1.291	2.945
Lateral supply pipeline	1.122	0.711	0.486	0.931
Interpolation value F	0.990	0.920	0.987	1.116
"True" A	0.764	0.422	0.721	****a
Percent error	2.6 ^b	-3.6 ^b	3.8 ^b	8.2 ^b
Absolute error	0.070	0.061	0.047	0.237

^a Exceeds 0.999

^b The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

TABLE 4-4

EXPECTED BACKORDERS UNDER LATERAL SUPPLY – BASE-REPARABLE-ONLY ITEMS –
LONG LATERAL TIMES (Continued)

<i>Scenario 4</i>						
Number of bases = 1 each of Types 1 – 5 Daily demand rate = 0.22 Base repair percent = 100 Base repair time = 4 days Base repair pipeline = 4.4						
	<i>Scenario 4</i>					
	Case 4.1	Case 4.2	Case 4.3	Case 4.4	Case 4.5	Case 4.6
Analytic Results						
Total stock	5	6	7	8	9	10
Depot stock level	0	0	0	0	0	0
Base stock level – Type 1	1	2	2	2	2	2
Base stock level – Type 2	1	1	2	2	2	2
Base stock level – Type 3	1	1	1	2	2	2
Base stock level – Type 4	1	1	1	1	2	2
Base stock level – Type 5	1	1	1	1	1	2
Lateral supply time (days)	2	2	2	2	2	2
Minimum EBO	0.574	0.294	0.138	0.059	0.024	0.009
Fill rate (percent)	72.0	84.4	92.1	96.4	98.5	99.4
Maximum EBO	1.474	1.254	1.003	0.813	0.593	0.373
Fill rate (percent)	78.0	81.2	84.4	87.6	90.8	94.0
Estimated EBO	1.458	1.169	0.895	0.696	0.543	0.367
Simulation Results						
EBO mean	1.607	1.237	0.942	0.719	0.546	0.408
EBO lower confidence limit	1.587	1.220	0.928	0.707	0.536	0.400
EBO upper confidence limit	1.627	1.254	0.956	0.731	0.556	0.416
Lateral supply pipeline	0.870	0.761	0.612	0.470	0.350	0.249
Interpolation value F	1.148	0.983	0.898	0.875	0.917	1.096
"True" A	*****	*****	*****	*****	*****	*****
Percent error	-9.2 ^b	-5.5 ^b	-5.0 ^b	-3.2 ^b	-0.5	-10.1 ^b
Absolute error	0.149	0.068	0.047	0.023	0.003	0.041

^a Exceeds 0.999

^b The estimated EBO are outside the 95-percent confidence interval for the simulated EBO

CHAPTER 5

CONCLUSION

We have demonstrated simple computational models that provide estimates for the expected backorders when lateral supply between bases is allowed. The estimates have an absolute average error of less than 3 percent for depot-reparable-only items. For fully or partially base-reparable items, the average error is about 5 percent and the regression requires additional parameters. Fortunately, the depot-reparable situation is more likely to be important.

Lateral supply can reduce backorders significantly. In the cases presented, reductions of 30 to 50 percent are not uncommon, with a 72-percent reduction observed in one case.

REFERENCES

- [1] LMI Report AF501-2. *Lateral Resupply in a Multi-Echelon Inventory System*. Slay, F. Michael. Apr 1986.
- [2] Lee, Hau L. "A Multi-Echelon Inventory Model for Repairable Items with Emergency Lateral Transshipments." *Management Science*, Vol. 33, No. 10 (1987).
- [3] SRA Corporation. *The Logistics Information Systems Analysis*, Phase IIB Final Report. *Analysis of Stock Control and Distribution System*, Vol. 2. 16 Jan 1987.
- [4] LMI Working Note AF301-3. *VARI-METRIC: An Approach to Modeling Multi-Echelon Resupply when the Demand Process is Poisson with a Gamma Prior*. Slay, F. Michael. Mar 1984.
- [5] Sherbrooke, Craig C. "VARI-METRIC: Improved Approximations for Multi-Indenture, Multi-Echelon Availability Models." *Operations Research*, Vol. 34, No. 2 (1986).

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified		1b RESTRICTIVE MARKINGS A193 202	
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION / AVAILABILITY OF REPORT "A" Approved for public release; distribution unlimited.	
2b DECLASSIFICATION / DOWNGRADING SCHEDULE			
4 PERFORMING ORGANIZATION REPORT NUMBER(S) LMI-AF70121		5 MONITORING ORGANIZATION REPORT NUMBER(S)	
6a NAME OF PERFORMING ORGANIZATION Logistics Management Institute	6b OFFICE SYMBOL (if applicable)	7a NAME OF MONITORING ORGANIZATION	
6c ADDRESS (City, State, and ZIP Code) 6400 Goldsboro Road Bethesda, Maryland 20817-5886		7b ADDRESS (City, State, and ZIP Code)	
8a NAME OF FUNDING / SPONSORING ORGANIZATION HQ USAF	8b OFFICE SYMBOL (if applicable) LEXY	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER MDA903-85-C-0139	
8c ADDRESS (City, State, and ZIP Code) The Pentagon, Room 4B283 Washington, D.C. 20330		10 SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO	PROJECT NO
		TASK NO	WORK UNIT ACCESSION NO
11 TITLE (Include Security Classification) Multi-Echelon Inventory Systems with Lateral Supply: A Technical Note			
12 PERSONAL AUTHOR(S) Craig C. Sherbrooke			
13a TYPE OF REPORT Permanent	13b TIME COVERED FROM _____ TO _____	14 DATE OF REPORT (Year, Month, Day) January 1988	15 PAGE COUNT 40
16 SUPPLEMENTARY NOTATION			
17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
19 ABSTRACT (Continue on reverse if necessary and identify by block number) We describe an interpolation model to estimate the effect of lateral supply in a multi-echelon supply system. Calculations of expected backorders using the interpolation agree closely with simulated results for depot-reparable-only items. The technique is not so successful with base-reparable items.			
20 DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION	
22a NAME OF RESPONSIBLE INDIVIDUAL		22b TELEPHONE (Include Area Code)	22c OFFICE SYMBOL

END

DATE

FILMED

DTIC

JULY 88