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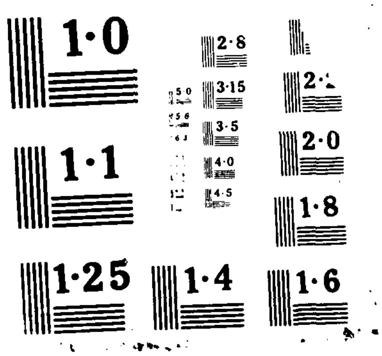
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Analysis of Transverse Strength of Composites Using Percolation Theory

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As the fiber content is increased in order to increase the tensile strength of a unidirectional metal matrix composite, the transverse strength decreases rapidly. This is caused by a combination of low transverse fiber strength and weak fiber/matrix bond strength. Percolation theory, which has been used to describe phenomena that involve phase transitions and that can be modeled as a number of particles interacting with a random medium, has been used here to show that low transverse fracture strengths may be expected as the fiber content of a unidirectional composite is increased. This increase results in connected chains of fibers, weakly bonded to each other and to the matrix. The expected strength is lower in this case than is predicted by a simple regular-array-of-holes model.

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FIGURES

I. INTRODUCTION

Much effort has been devoted to the development of metal matrix composites (MMCs) for advanced applications that utilize the advantages of strength, stiffness, light weight, and elevated temperature capability of MMCs. One difficulty in the application of these composites is poor transverse strength. For unidirectional carbon fiber MMCs, transverse strength can be one to two orders of magnitude lower than the strength in the longitudinal direction, depending on the amount of fiber reinforcement present. Metal matrix composites have been studied in the past to attempt to improve their transverse strength properties without degrading other strength and physical properties, and to identify particular failure mechanisms. Their low strength is due in part to the low transverse tensile strength of the carbon fibers and the poor metal/fiber bond strength.[1, 2]

Several simple models have been advanced to predict the low transverse strength of composites. In one model,[3] uniform diameter fibers are placed in a square array. The fiber volume fraction, $V_f = \pi/4 \cdot d^2/a^2$, is calculated from the distance between neighboring fiber centers, a, and the fiber diameter, d. In this model, at a volume fraction of $\pi/4$ (= 0.785), the fibers are in contact with each other. If we assume that the fiber/fiber and fiber/matrix interfaces have zero strength, then the composite also has zero transverse strength at this volume fraction.

Percolation theory[4] is used here to show that lower transverse strength in a unidirectional composite can occur at a lower fiber volume fraction when the fibers are distributed randomly rather than regularly on a simple square array, as described above. This approach can show why the strength predicted from a regular array of fibers[3] is higher than for a random arrangement of filaments. A larger fiber concentration can be maintained in a regular array without having fibers in contact with each other, while in random packing, fibers can come in contact at lower volume fractions and may even link together to form a continuous chain across the sample, causing a larger reduction in strength.

II. ANALYSIS

The linking of weak parallel fibers in a strong matrix was modeled by considering a square array of sites, each of which is occupied either by fiber, with probability p, or by matrix, with probability 1-p. The fiber phase is taken to have zero strength. If a connected chain of adjacent filaments is found across the array horizontally, the composite has zero strength. Two fibers are assumed to be connected if they are adjacent vertically or horizontally, but not diagonally. Figure 1 shows several fibers on a square array, illustrating connected and nonconnected fibers. This represents a simple percolation problem on a square lattice in two dimensions.

A computer program to calculate the probability for percolation as a function of area fraction was written. It generates random arrangements of "conducting cells" at a given area fraction and attempts to find a connected path. The approach is to model the composite as a collection of nonconducting and conducting cells and to calculate whether the entire array is conducting or not in the horizontal direction. This can be done by solving Laplace's equation numerically on the array. The conducting cells are initialized to zero potential, and then the cells at the left edge are set to a fixed constant potential. Laplace's equation is solved by using an iterative relaxation method.[5] Table 1 shows sample results for a 4×4 array, with p ranging from 0.4 to 0.7. The boundary condition at the right-hand edge is that of an insulating boundary, so that the right edge should eventually rise to the same potential as the left edge, if a continuous path exists. A path may be detected by generating an array, applying a potential on the left edge, calculating several iterations of the numerical solution to the potential problem, and checking whether the potential at the right edge differs from zero. The iterations give a diffusive-like behavior as the charge from the source at the left meanders through the conducting regions of the array.

Table 1. Sample Results for 4 × 4 Array

Site Probability	No. Fracture	Fracture Probability[a]	Avg. No. Fibers	Std. Dev.[b]
0.4	1	20%	5.6	2.1
0.5	2	40%	7.8	1.6
0.6	3	60%	9.8	1.3
0.7	4	80%	10.6	1.5

[a] Indicates number of fractures to number of trials.
[b] Indicates standard deviation for mean number of fibers over each set of trials.

A simple example of the relaxation approach[5] to the problem is incorporated in a FORTRAN 77 program. Various array sizes were used. The relaxation method consists of scanning from left to right across the array and setting the potential of each conducting element to the average value of its four nearest neighbors (in two dimensions):

$$V(i,j) = \frac{V(i+1,j) + V(i-1,j) + V(i,j+1) + V(i,j-1)}{4}$$

which is the numerical equivalent of Laplace's equation in two dimensions

$$\nabla^2 V = 0$$

To account for the insulating (matrix) cells, the potential of the nearest neighbor is set equal to that of the cell being calculated if the neighbor is an insulator. If the neighbor would lie one row or column beyond the array boundary, it is also treated as an insulator. In other words, if a nearest neighbor cell, $A(k,l)$, where $k = i \pm 1$, $l = j \pm 1$, is occupied by matrix (nonconductor) or would lie outside the array boundary, then $V(k,l) = V(i,j)$. This effectively eliminates that cell from contributing a conducting path.

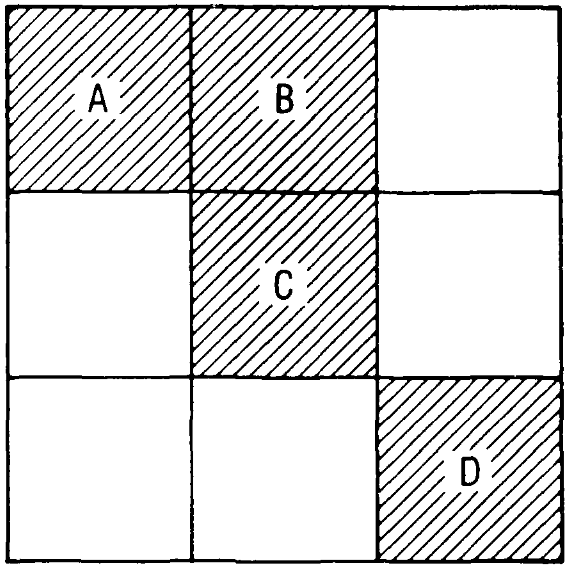


Fig. 1. Examples of Three Connected Cells (A,B,C) and Unconnected Cell (D)

III. RESULTS

Figure 2 shows an example of a 10 × 10 array with p = 0.6 of a site being a fiber. Areas of nonzero potential are shaded. The diffusion of potential from the source into the array is seen by the increase of potential in the cells which are connected by a continuous path to the left edge. Several successive iterations of the solution for the potential are shown, illustrating how the solution improves with each iteration. For comparison, the critical volume fraction for a square array of round fibers is about 0.785,[3] at which point all fibers are touching.

Further demonstration by this model of the relation of fiber site occupation probability, p (equal to the average fiber volume fraction), to the conductivity of the array can be studied by calculating a number of arrays and calculating the percentage of arrays which conduct as a function of p. This a means of obtaining an ensemble average probability of percolation through the array of conducting cells.

Figure 3 shows the calculated percolation probability for several runs with different array sizes. Two features are apparent. Firstly, the range of values of p over which the percolation probability changes from almost zero to almost unity is fairly small. In other words the onset of percolation is fairly sudden as p is increased. The range also appears to decrease as the array size increases. Secondly, the midpoint value of p where the percolation probability is 1/2 appears to be relatively independent of the array size. Thus with increasing array size, the onset of percolation as p is increased is more sudden near a certain critical value.

This simple model shows that the volume fraction of fiber at which the presence of a complete transverse fracture through the composite becomes highly likely becomes more sharply defined with increasing array size. This is an example of the classic percolation problem for conductivity in the bulk limit of a mixture of conducting and nonconducting particles, where a critical percolation probability, p_c, is found to exist for a number of different kinds of arrays.[4]

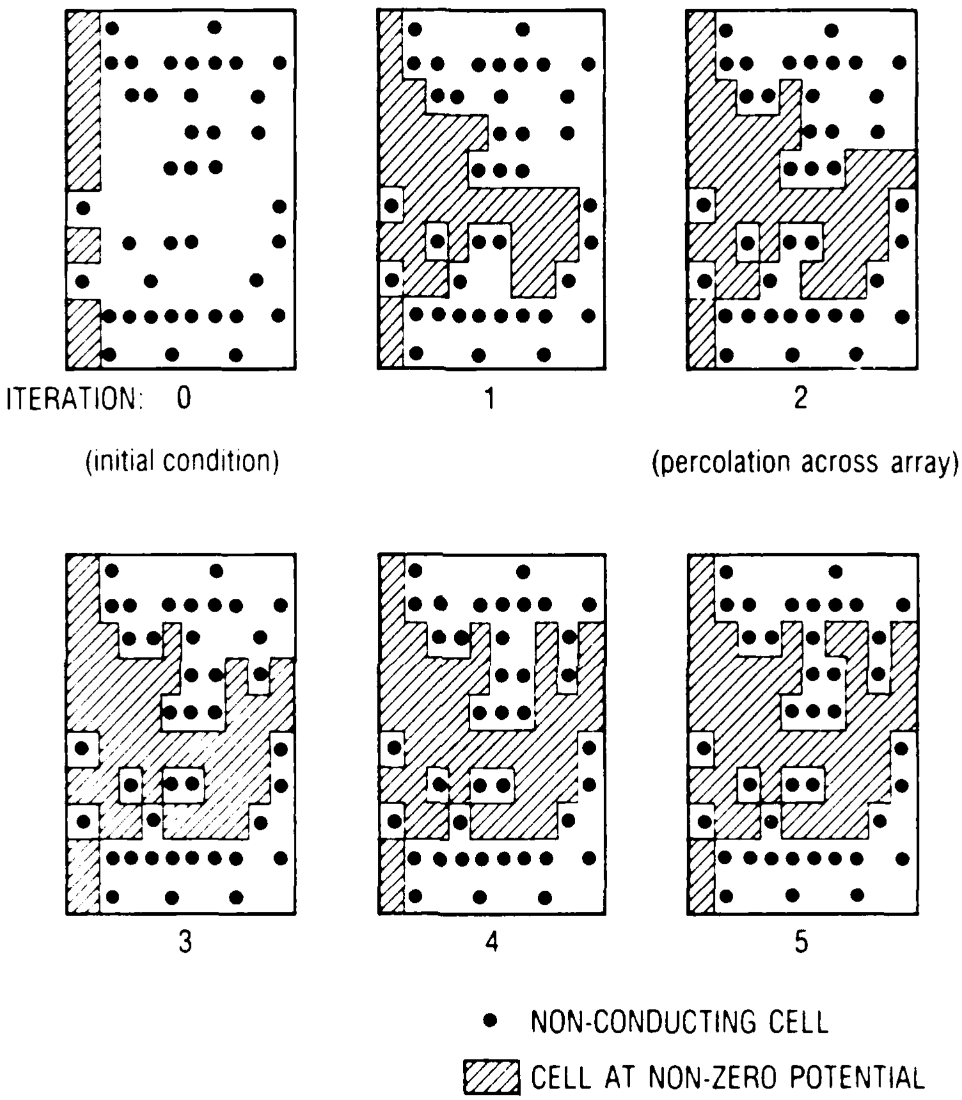
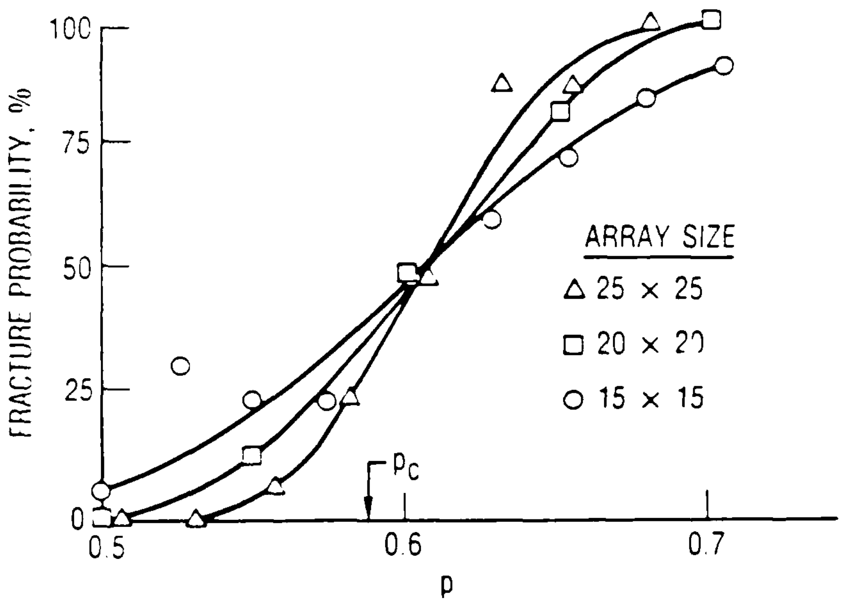


Fig. 2. Illustration of Typical Progression of Potential Through Connected Regions by Relaxation Method on 10 × 10 Square Array. Percolation detected on second iteration. (Nonzero potential appears on right side of array.)



p_c *(exact)* = 0.5928...
(Rosso et al., PRL 9 1985)

- FIND SQUARE LATTICE RESULT IN REASONABLE AGREEMENT WITH EXACT RESULT
- $v_f \approx 0.59 \times$ FILLING FRACTION $= 0.59 \times \pi/4 \approx 46$ vol/o

Fig. 3. Fracture Probability vs Fiber Site Occuption Probability

IV. DISCUSSION

Figure 4 shows experimental data for the transverse tensile strength of MMCs as a function of fiber volume fraction. In comparison, the strengths are somewhat lower than indicated for the uniform square array of holes discussed earlier. A rough extrapolation of the data suggests zero transverse strength occurs at around 50 vol% fiber. This is consistent with the simple percolation model. Thus the model indicates that the random distribution of fibers is an important determinant of the transverse strength.

Note that in the present square array geometry, the fiber volume percent is equal to the fiber site occupation fraction. This is expressed as the lattice filling factor, f, being unity.[4] Thus, we find the critical percolation probability is about 0.59 on the square lattice, giving a critical volume fraction of 0.59 for square fibers on a square lattice; these fibers have a maximum theoretical packing density of 100 vol%. This is not the case for round fibers on a square lattice, where the filling factor f = 0.785, in which case the critical volume fraction would be about (0.785)(0.59) = 0.46, which again is roughly comparable with the observed value.

An approximately "dimensionally invariant" quantity for percolation, (fp_c), has been postulated by Scher and Zallen.[6] It was found that this product tends toward a constant value for different lattices of the same dimensionality, having values of approximately 0.45 and 0.15 in two and three dimensions, respectively. The assumption that this "invariant" will also apply to an amorphous structure allows an estimate to be made of the critical volume fraction. It was found experimentally that for a three dimensional system of conductive particles dispersed in a ceramic, the onset of conductivity occurs at about 15 vol%, in agreement with the prediction.[6] Metal matrix composites can provide a two dimensional test of this invariant. Percolation leading to zero mechanical strength should occur at about 45 vol%, given the simplifying assumptions on fiber/matrix interface strength noted earlier. This does appear to be in agreement with experimental observations, since transverse strength falls drastically around 40 vol% fiber.[1]

Further work will be required to extend the model to predict strength values at fiber fractions below the percolation threshold.

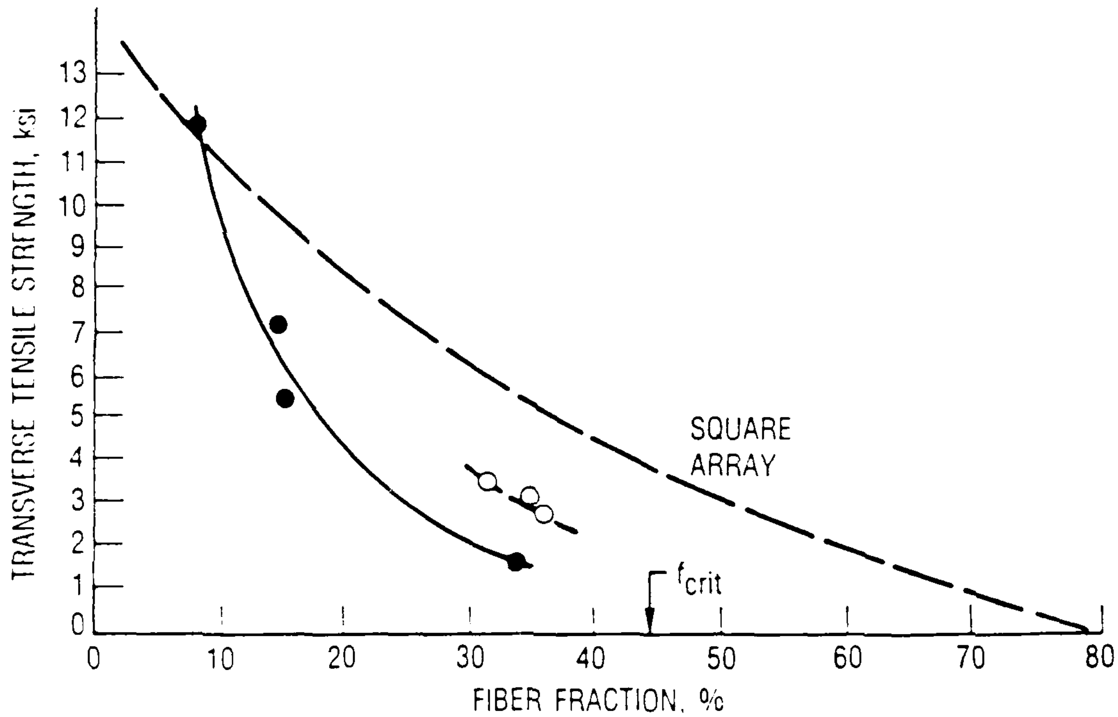


Fig. 4. Effect of Fiber Fraction and Fiber Arrangement on Transverse Tensile Strength of Various Graphite-Aluminum Composites

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