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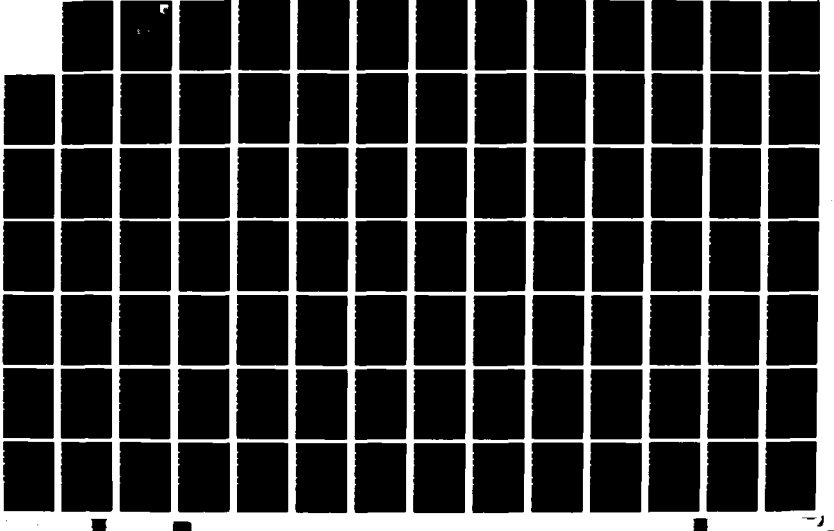

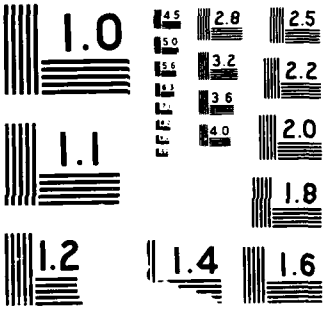
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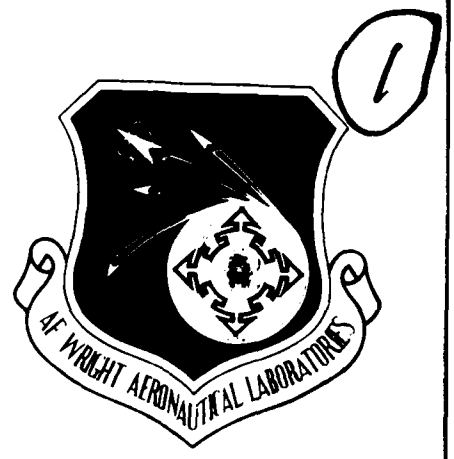

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ON BROADBAND MATCHING OF MULTIPORT ELECTRICAL NETWORKS WITH APPLICATIONS

Wai-Kai Chen
University of Illinois at Chicago
Department of Electrical Engineering and Computer Science
P.O. Box 4348
Chicago, IL 60680

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JAMES B.Y. TSUI, Proj Engr
ESM Technology Gp, EW Division
Avionics Laboratory

PAUL S. HADORN, PhD, Chief
Passive ECM Br, EW Division
Avionics Laboratory

FOR THE COMMANDER

JOHN E. TEHAN, Actg Chief
Electronic Warfare Division
Avionics Laboratory

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The objective of the research is to solve several problems of fundamental importance in the design of broadband matching networks and multiplexers. More specifically, we investigate the following:

1) Obtain a set of coefficient constraints that are necessary and sufficient for a real rational matrix to be the scattering matrix of a passive n-port network normalizing to the n prescribed non-Foster positive-real impedances.

2) Derive a set of coefficient constraints that are necessary and sufficient for the existence of a lossless reciprocal n-port network, which, when terminated in the n given frequency-dependent loads, realizes the prescribed transducer power-gain characteristics among its ports.

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3) Study the relationships between the broadband matching of many ports and the design theory of multiplexers and apply the known results in the theory of broadband matching to minimize the interaction effects of the individual channels.

FOREWORD

This is the final technical report submitted to the Department of the Air Force, Aeronautical Systems Division/PMREC, Wright-Patterson AFB, Ohio, by University of Illinois at Chicago, Department of Electrical Engineering and Computer Science, Chicago, Illinois, under contract entitled, "Broadband Matching of Multiport Electrical Networks with Applications," F33615-84-K-1556/P00004. The personnel supported by the contract included the following:

Dr. Wai-Kai Chen, Principal Investigator

Dr. Wolfgang-M. Boerner, Professor

Mr. Zhao-Ming Wang, Visiting Scholar and Associate Professor

Mr. Yi-Sheng Zhu, Visiting Scholar and Associate Professor

Mr. Bing-Yuen Foo, Ph.D. Research Assistant

Mr. Matthias Walther, M.S. Research Assistant

Mr. Jonas Okeke, M.S. Research Assistant

Mrs. Victoria Zapatka, Project Coordinator

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Section 1

INTRODUCTION

A fundamental problem in the design of communication systems is to realize a lossless coupling network between a given source and a given load so that the transfer of power from the source to the load is maximized over a prescribed frequency band of interest. We refer to this operation as impedance matching or equalization, and the resulting coupling network as matching network or equalizer.

The matching problem was initiated by Bode [2] in the study of coupling networks for a class of very useful but restricted load impedance composed of the parallel combination of a capacitor and a resistor. He established the fundamental gain-bandwidth limitation for this class of equalizers, but did not go further to investigate the additional limitation imposed on the lossless equalizers by the load. Fano [24] extended Bode's work, and brilliantly solved the impedance matching problem between a resistive generator and an arbitrary passive load, in its full generality. The key idea underlying Fano's solution is that of replacing the load impedance by its Darlington equivalent. The results on physical realizability are then transformed into a set of integral relations involving the logarithm of the magnitude of the reflection coefficient. Nevertheless, Fano's approach, although very ingenious and elegant, suffers two main drawbacks, both from a practical and theoretical viewpoint. First of all, the replacement of the load impedance by its Darlington equivalent leads to the complications of translating certain of its properties into structural properties of its associated Darlington. Secondly, since Darlington's theorem is valid only for passive impedance, the technique cannot be extended to the design of general

active equalizers. With the advent of solid-state technology and its apparently unending proliferation of new devices such as the IMPATT diode and the GaAs FET, this consideration can no longer be ignored. Based on the principle of complex normalization of a scattering matrix [56,59], Youla [58] developed a new theory that completely circumvents the two main objections encountered in Fano's work. Since Youla's method deals directly with the load, it is much simpler to apply. In fact, his theory was generalized by Chan and Kuh [6] to include the situation where the load impedance is active.

Both matching theories indicate the many directions of extensions. Since Fano and Youla's equivalent of a source system is represented by a single resistance, the restrictions imposed by the complex source system are not present in their work. Thus, a clear extension is to allow frequency-dependent source impedance as depicted in Fig. 1.1. This was recently solved by Chien [21], Chen and Satyanarayana [19,46] and Chen [16], who derived necessary and sufficient conditions for the physical realizability of the matching network in terms of the parameters of the known load and source impedances and the desired transducer power-gain characteristic. Chien's conditions guarantee the

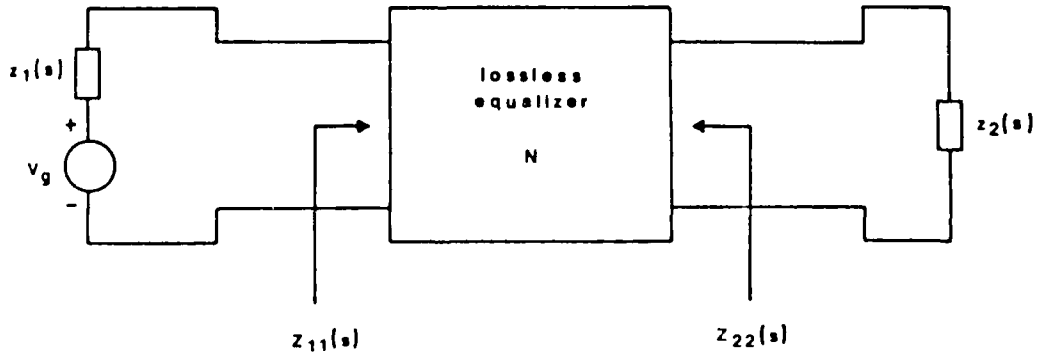

Fig. 1.1. Schematic of Broadband Matching between Arbitrary Source and Load Impedances.

physical realizability of the scattering parameters so constructed, thus insuring the existence of the desired matching network. The actual realization of the equalizer is accomplished by realizing its scattering matrix, using standard techniques developed elsewhere. Chen and Satyanarayana's result [19,46], on the other hand, provides a direct means of constructing the equalizer. Specifically, they demonstrated that with appropriate choice of some regular all-pass functions, the equalizer back-end impedance is first realized as the input impedance of a lossless two-port network terminated in a 1-ohm resistor. By replacing the 1-ohm resistor by the source impedance, we obtain the desired matching network. Needless to say, the broadband matching problem has been discussed and elaborated upon by many workers [4, 7-15, 18, 30, 34-36, 48]. The extension of the theory to frequency-dependent source and active load is very recent and is given by Chen and Tsai [20].

The present research is a direct extension and generalization of the work reported in the foregoing. The major point of departure arises from the difference in the number of accessible ports of the matching networks. Here we investigated the broadband matching problem of many ports. The significance of the work is that it enables us to study lossy two-port networks as well as to design channel multiplexers. For example, a lossy two-port matching network containing n-2 resistors can be considered as a lossless n-port matching network with n-2 of its ports terminated in the resistors.

1.1 Existence of normalized scattering matrix

Referring to the n-port network N of Fig. 1.2, let [14,56,59]

$$\underset{\sim}{S}(s) = [S_{ij}] \tag{1.1}$$

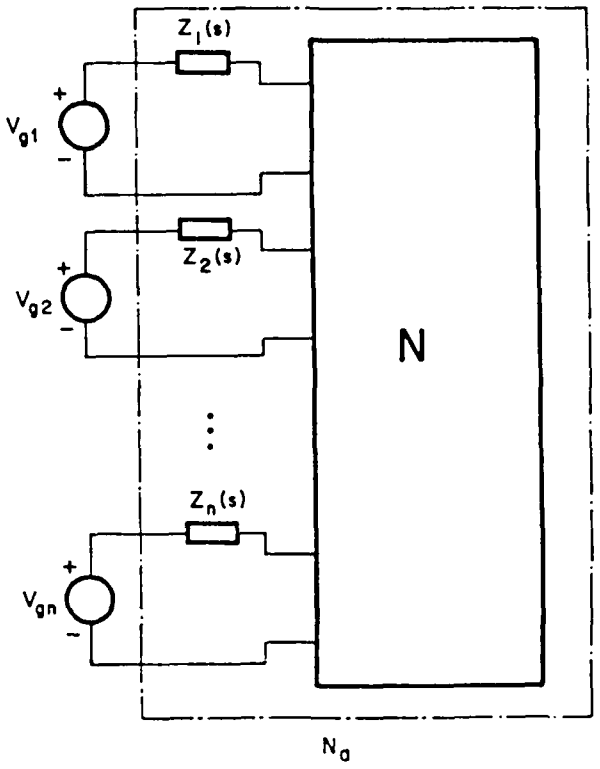

Fig. 1.2. Schematic of General Broadband Matching of an n-port Network.

be its scattering matrix normalized to the reference impedance matrix

$$\underset{\sim}{z}(s) = \text{diag}\,[z_1(s),\ z_2(s),\ \ldots,\ z_n(s)], \tag{1.2}$$

each of its elements being a non-Foster impedance. Write

$$\underset{\sim}{S}_*(s) = \underset{\sim}{S}'(-s), \tag{1.3}$$

where the prime denotes the matrix transpose. The para-hermitian part of $\underset{\sim}{z}(s)$ is defined as

$$\underset{\sim}{r}(s) = \tfrac{1}{2}[z(s) + z_*(s)]. \tag{1.4}$$

Factoring each diagonal element of $\underset{\sim}{r}(s)$ as the product $h_k(s)h_{k*}(s)$

such that $h_k(s)$ and $h_{k*}^{-1}(s)$ are analytic in the open RHS (right-half of the complex-frequency plane), where $h_{k*}(s) = h_k(-s)$, and express $\underset{\sim}{r}(s)$ accordingly

$$\underset{\sim}{r}(s) = \underset{\sim}{h}(s)\underset{\sim}{h}_*(s), \tag{1.5}$$

where

$$\underset{\sim}{h}(s) = \text{diag } [h_1(s), h_2(s), \ldots, h_n(s)]. \tag{1.6}$$

Then we can show that

$$\underset{\sim}{S}(s) = \underset{\sim}{h}(s)\underset{\sim}{h}_*^{-1}(s) - 2\underset{\sim}{h}(s)\underset{\sim}{Y}_a(s)\underset{\sim}{h}(s), \tag{1.7}$$

where $\underset{\sim}{Y}_a(s)$ is the admittance matrix for the augmented n-port network N_a as shown in Fig. 1.2.

The scattering matrix $\underset{\sim}{S}(s)$ thus defined has the following remarkable attributes:

(i) $\underset{\sim}{S}(s)$ is rational.

(ii) $\underset{\sim}{S}(s)$ is analytic in the closed RHS.

(iii) $\underset{\sim}{1}_n - \underset{\sim}{S}^*(s)\underset{\sim}{S}(s)$ is hermitian and nonnegative-definite for all s in the closed RHS, where $\underset{\sim}{S}^*(s)$ denotes the transpose conjugate of $\underset{\sim}{S}(s)$ and $\underset{\sim}{1}_n$ the identity matrix of order n.

(iv) $\underset{\sim}{S}(s)$ is a real for real s, i.e., $\bar{\underset{\sim}{S}}(s) = \underset{\sim}{S}(\bar{s})$.

(v) If N is lossless, $\underset{\sim}{S}(s)$ is para-unitary, i.e., $\underset{\sim}{S}_*(s)\underset{\sim}{S}(s) = \underset{\sim}{1}_n$.

(vi) If N is reciprocal, $\underset{\sim}{S}(s)$ is symmetric, i.e., $\underset{\sim}{S}'(s) = \underset{\sim}{S}(s)$.

(vii) The transducer power gain $G_{ik}(\omega^2)$ from port i to port k under the matched condition is given by

$$G_{ik}(\omega^2) = |S_{ki}(j\omega)|^2. \tag{1.8}$$

These properties of $\underset{\sim}{S}(s)$ are all necessarily true, but they are not sufficient to guarantee that a matrix with these properties can be realized as the scattering matrix of some physical passive n-port

network normalizing to the reference impedances $z_1(s)$, $z_2(s)$, ..., $z_n(s)$. The necessary and sufficient conditions for the existence of such a matrix is given below.

Theorem 1.1: (Wohlers [54]) The necessary and sufficient conditions that an $n \times n$ rational matrix be the scattering matrix of a lumped and passive n-port network normalized to the n non-Foster positive-real impedances $z_1(s)$, $z_2(s)$, ..., $z_n(s)$ are that:

(i) $\underset{\sim}{1}_n - \underset{\sim}{S}^*(j\omega)\underset{\sim}{S}(j\omega)$ be nonnegative-definite for all real ω;

(ii) the matrix defined by the relation

$$\underset{\sim}{Y}_a(s) = \tfrac{1}{2}\underset{\sim}{h}^{-1}(s)[\underset{\sim}{h}(s)\underset{\sim}{h}_*^{-1}(s) - \underset{\sim}{S}(s)]\underset{\sim}{h}^{-1}(s) \tag{1.9}$$

be analytic in the open RHS, where $\underset{\sim}{h}(s)$ is a factorization of the para-hermitian part $\underset{\sim}{r}(s) = \underset{\sim}{h}(s)\underset{\sim}{h}_*(s)$ of the reference impedance matrix $\underset{\sim}{z}(s) = \text{diag}\,[z_1(s), z_2(s), \ldots, z_n(s)]$ such that $h(s)$ and $h_*^{-1}(s)$ are analytic in the open RHS;

(iii) either

(a) $\det\{\underset{\sim}{1}_n - [\underset{\sim}{z}(s) - \underset{\sim}{1}_n]\underset{\sim}{Y}_a(s)\} \neq 0$ in the open RHS, or

(b) the matrix defined by the relation

$$\underset{\sim}{M}(s) = \{\underset{\sim}{1}_n - [\underset{\sim}{z}(s) - \underset{\sim}{1}_n]\underset{\sim}{Y}_a(s)\}[\underset{\sim}{z}(s) + \underset{\sim}{1}_n] \tag{1.10}$$

have at most simple poles on the real-frequency axis with nonnegative-definite residue matrix.

In addition, if the network is reciprocal, the matrix $\underset{\sim}{S}(s)$ must be symmetric. This result is computationally very difficult to apply. Nevertheless, the theorem is of fundamental importance in its own right. It forms the basis of the approach to the solution of broad-band matching between two frequency-dependent impedances as reported by Chien [21] and Chen and Satyanarayana [19,46]. It is also essential

in the formulation of the compatibility problem of two positive real impedances, as demonstrated by Wohlers [54] and Ho and Balabanian [33]. We propose to investigate and develop alternate and more tractable conditions that would alter our viewpoint on this and many other problems. The conditions will be formulated in terms of the coefficients in the Laurent series expansions of the functions derived from the reference impedances $z_k(s)$. An outline of this approach will now be described.

Refer to the n-port network N of Fig. 1.2. Let

$$\underset{\sim}{S}^I(s) = \left[S^I_{ij}\right] \tag{1.11}$$

be its current-based scattering matrix. Then we have [14]

$$\underset{\sim}{S}(s) = \underset{\sim}{h}(s)\underset{\sim}{S}^I(s)\underset{\sim}{h}_*^{-1}(s). \tag{1.12}$$

If $Z_{kk}(s)$ denotes the impedance looking into port k with all other ports being terminated in their reference impedances, then from (1.12) we have

$$S_{kk}(s) = \frac{h_k(s)}{h_{k*}(s)} \cdot \frac{Z_{kk}(s) - z_{k*}(s)}{Z_{kk}(s) + z_k(s)} . \tag{1.13}$$

We recognize that $h_k(s)h_{k*}^{-1}(s)$ is a real regular all-pass function, whose poles include all the open LHS (left-half of the complex-frequency plane) poles of the reference impedance $z_k(s)$. Thus, it can be written as the product of the real regular all-pass function

$$A_k(s) = \prod_{i=1}^{w} \frac{s - s_{ki}}{s + s_{ki}} \tag{1.14}$$

defined by the open RHS poles s_{ki} $(i = 1, 2, \ldots, w)$ of $z_k(-s)$ and another real regular all-pass function η_k, that is

$$h_k(s)/h_k(-s) = \eta_k(s)A_k(s). \tag{1.15}$$

Now observe that since

$$S^I_{kk}(s) = \frac{Z_{kk}(s) - z_k(-s)}{Z_{kk}(s) + z_k(s)}\ , \tag{1.16}$$

the open RHS poles of $S^I_{kk}(s)$ are precisely those of $z_k(-s)$, the function defined by

$$\rho_k(s) = A_k(s)S^I_{kk}(s) \tag{1.17}$$

is analytic in the closed RHS. In other words, $\rho_k(s)$ is bounded-real. Write

$$\underset{\sim}{Y}_a(s) = [y_{ija}]. \tag{1.18}$$

Then from (1.9) we have

$$2y_{ija}(s) = -S_{ij}(s)/h_i(s)h_j(s) \tag{1.19}$$

for $i \neq j$, and

$$\begin{aligned} y_{jja}(s) &= \frac{1}{2h_j(s)h_j(-s)}\left[1 - h_j(-s)S_{jj}(s)/h_j(s)\right] \\ &= \frac{1}{2r_j(s)A_j(s)}\left[A_j(s) - A_j(s)S^I_{jj}(s)\right] \\ &= \frac{A_j(s) - \rho_j(s)}{F_j(s)}\ , \end{aligned} \tag{1.20}$$

where

$$F_j(s) = 2r_j(s)A_j(s). \tag{1.21}$$

Following Youla [58], we call a closed RHS zero of multiplicity m of the function $r_k(s)/z_k(s)$ as a zero of transmission of order m of the impedance $z_k(s)$, and divide the zeros of transmission s_{0k} of $z_k(s)$, written as $s_{0k} = \sigma_{0k} + j\omega_{0k}$, into the following four mutually exclusive classes:

Class I zero: $\sigma_{0k} > 0$,

Class II zero: $\sigma_{0k} = 0$ and $z_k(s_{0k}) = 0$,

Class III zero: $\sigma_{0k} = 0$ and $0 < |z_k(s_{0k})| < \infty$,

Class IV zero: $\sigma_{0k} = 0$ and $|z_k(s_{0k})| = \infty$,

The notion of an inherent restriction on $\underset{\sim}{S}(s)$ emerges immediately from (1.19) and (1.20). Since $\underset{\sim}{S}(s)$ is analytic in the closed RHS, the closed RHS poles of $\underset{\sim}{Y}_a(s)$ can occur only at the zeros of transmission of the reference impedances $z_k(s)$. In other words, if $\underset{\sim}{Y}_a(s)$ is the admittance matrix of a physical n-port network, then the real-frequency axis poles of $y_{ija}(s)$, $i \neq j$, can occur only at the real-frequency-axis zeros of $h_i(s)$ and $h_j(s)$ and $S_{ij}(s)$ must contain all the open RHS zeros of $h_i(s)$ and $h_j(s)$, to at least the same multiplicity; and every zero of transmission of $z_j(s)$ must also be a zero of $A_j(s) - \rho_j(s)$. Stated differently, regardless of the choice of the n-port network N in Fig. 1.2, there exist points s_{0j} in the closed RHS, dictated solely by the choice of the normalizing impedances $z_j(s)$, such that

$$A_j(s_{0j}) = \rho_j(s_{0j}), \qquad j = 1, 2, \ldots, n. \tag{1.22}$$

Recall that $A_j(s)$ are completely specified via (1.14) by the open RHS poles of $z_j(-s)$.

These restrictions are most conveniently and compactly formulated in terms of the coefficients in the Laurent series expansions of the functions $A_j(s)$, $\rho_j(s)$, $F_j(s)$ and $S_{ij}(s)/h_i(s)h_j(s)$ about a zero of transmission s_{0k} of $z_k(s)$:

$$A_j(s) = \sum_{x=0}^{\infty} a_{xj}(s - s_{0k})^x, \tag{1.23}$$

$$\rho_j(s) = \sum_{x=0}^{\infty} \rho_{xj}(s - s_{0k})^x, \tag{1.24}$$

$$F_j(s) = \sum_{x=0}^{\infty} f_{xj}(s - s_{0k})^x, \tag{1.25}$$

$$S_{ij}(s)/h_i(s)h_j(s) = \sum_{x=0}^{\infty} q_{xj}(s - s_{0k})^x, \tag{1.26}$$

where $i,j = 1, 2, \ldots, n$. For example, at a Class I zero of transmission s_{0k} of order m, for $y_{jja}(s)$ to be analytic in the open RHS, $A_j(s) - \rho_j(s)$ must vanish at s_{0k} to at least the same multiplicity, yielding

$$a_{xj} = \rho_{xj}, \qquad x = 0, 1, 2, \ldots, m-1. \tag{1.27}$$

At other classes of zero of transmission, similar coefficient constraints can be obtained. However, they are much more complicated than those suggested above. We will derive a set of coefficient constraints that are both necessary and sufficient for a real rational matrix to be the scattering matrix of a lumped passive n-port network normalizing to the n prescribed non-Foster positive-real impedances $z_1(s)$, $z_2(s)$, $\ldots$, $z_n(s)$.

1.2 General matching theory of many ports

Having successfully developed an alternate and more tractable set of conditions that chracterize the normalized scattering matrix, we shall now apply it to study the general matching problem of many ports. To be definitive we shall consider the following problem. Given n arbitrary non-Foster positive real rational functions $z_1(s)$, $z_2(s)$, ..., $z_n(s)$ as the internal impedance of the generator and the load impedances and given a set of real rational functions $G_{ij}(\omega^2)$, $i \neq j$, bounded by unity for all real ω, as the transducer power gain from port i to port j, the problem is to determine conditions under which there exists a lossless n-port network, which, when terminated in these impedances, will realize the prescribed set of $G_{ij}(\omega^2)$. The approach will be described below.

Refer to the n-port network of Fig. 1.2. Let $\underset{\sim}{S}(s)$ of (1.1) be its normalized scattering matrix. Assume that the n-port network N is reciprocal and lossless. Then in view of the complex normalization concept discussed in the foregoing, $\underset{\sim}{S}(s)$ is symmetric and para-unitary and imbeds the given set of transducer power gain characteristics $G_{ij}(\omega^2)$. Hence, we shall first construct the most general symmetric para-unitary matrix S(s) from the given set of $G_{ij}(\omega^2)$. Then from physical realizability conditions obtained earlier on $\underset{\sim}{S}(s)$, we can introduce constraints on $G_{ij}(\omega^2)$ such that $\underset{\sim}{S}(s)$ is indeed physically realizable. Note that not all the transducer power-gain characteristics $G_{ij}(\omega)$ can be specified independently; only n - 1 of them can be pre-assigned by the designer.

The symmetric and para-unitary conditions on $\underset{\sim}{S}(s)$,

$$\underset{\sim}{S}(s) = \underset{\sim}{S}'(s), \tag{1.28}$$

$$\underset{\sim}{S}_*(s)\underset{\sim}{S}(s) = \underset{\sim}{1}_n, \tag{1.29}$$

yield, in expanded form,

$$S_{ij}(s) = S_{ji}(s), \qquad i, j = 1, 2, \ldots, n, \tag{1.30}$$

$$\sum_{j=1}^{n} S_{ji*}(s) S_{jk}(s) = \delta_{ik}, \tag{1.31}$$

where δ_{ik} is the Kronecker delta and i, k = 1, 2, ..., n. Moreover, we have

$$|S_{ik}(j\omega)|^2 = G_{ki}(\omega^2), \qquad i \neq k. \tag{1.32}$$

On the real-frequency axis, (1.31) and (1.32) become

$$|S_{ii}(j\omega)|^2 = 1 - \sum_{\substack{k=1 \\ k\neq i}}^{n} G_{ik}(\omega^2), \qquad i = 1, 2, \ldots, n. \tag{1.33}$$

Hence, the gain functions $G_{ij}(\omega^2)$ determine uniquely the magnitude of $S_{ij}(s)$ on the real-frequency axis. The phase of these functions, which cannot be determined from $G_{ij}(\omega^2)$, may be represented by real regular all-pass functions. Appealing to the theory of analytic continuation, let $\hat{\rho}_{ij}$ be the minimum-phase factorizations of $G_{ji}(-s^2)$, $i \neq j$, or

$$1 - \sum_{\substack{k=1 \\ k\neq i}}^{n} G_{ik}(-s^2), \qquad i = j. \tag{1.34}$$

Then the most general solution of (1.32) and (1.33) that is analytic in the closed RHS is given by

$$S_{ij}(s) = \hat{n}_{ij}(s)\hat{\rho}_{ij}(s), \qquad i, j = 1, 2, \ldots, n. \tag{1.35}$$

This together with $S_{ij}(s) = S_{ji}(s)$ gives the most general form of the scattering parameter representation of a lossless reciprocal n-port network normalizing to $z_1(s)$, $z_2(s)$, ..., $z_n(s)$ which imbeds the given

set of transducer power gain characteristics $G_{ij}(\omega^2)$, $i \neq j$. However, not every representation of this form is physically realizable, and the solution to the n-port matching problem reduces to the question of the existence of a set of real regular all-pass functions $\hat{\eta}_{ij}(s)$ so that the matrix $\underset{\sim}{S}(s)$ is realizable. Thus, by examining the coefficient constraints imposed on $\underset{\sim}{S}(s)$, we can arrive at a complete solution to the matching problem of many ports. These result in a set of coefficient conditions that are necessary and sufficient for the existence of a lossless reciprocal n-port network, which, when terminated in the given load impedances, will realize the prescribed transducer power-gain characteristics.

1.3 General design theory for multiplexers

As a consequence to the above solution of the broadband matching problem of many ports, we can design a lossless reciprocal n-port network that matches a frequency-dependent source and n - 1 frequency-dependent loads with a priori apportionment of average power over a preassigned band of frequencies. The situation occurs in typical antenna applications, where a single generator drives n - 1 frequency sensitive loads. The constraints imposed on the transducer power-gain characteristics $G_{ik}(\omega^2) = |S_{ki}(j\omega)|^2$ are expressed in terms of the maximum allowable voltage standing wave ratios

$$\frac{1 + |S_{ii}(j\omega)|}{1 - |S_{ii}(j\omega)|} \tag{1.36}$$

In this case, the synthesis of the n-port network must be carried out with prescribed bounds on the $|S_{ii}(j\omega)|$. The preceding solution also allows the study of the effect of loss on the gain-bandwidth product

of a two-port matching network, because a lossy two-port matching network can be viewed as an n-port with n - 2 of its ports being terminated in resistors.

A multiplexer is a device that splits a single channel carrying many frequencies into a number of separate channels carrying narrower bands of frequencies. It might at first appear that the design of a multiplexer could easily be accomplished by designing the band-pass filters using any of the many known techniques, and then connecting these band-pass filters in parallel. Such a procedure does not work well because of the undesirable interaction between the filters, which could result in very poor performance. To avoid this difficulty, many techniques have been proposed to eliminate such undesirable interaction effects [43-45].

An important practical configuration for the multiplexers makes use of the constant-resistance filters. Filters of this kind, when all designed for the same terminating resistance, can be cascaded as shown in Fig. 1.3 to form a multiplexer which in theory completely avoids the filter interaction effects mentioned above. Each filter provides the proper termination for its neighbor, so that to the extent that there are no residual voltage standing wave ratios and manufacturing

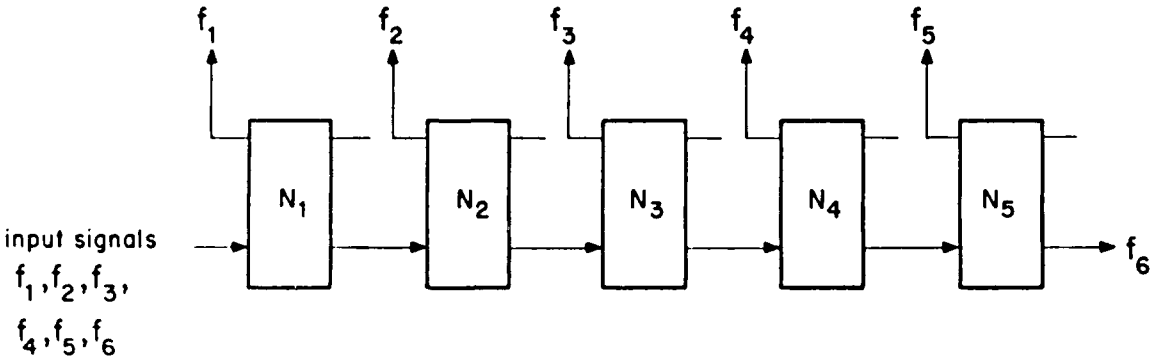

Fig. 1.3. Constant-Resistance Filters Used for Multiplexing.

imperfections, the structure is reflectionless. However, in reality, each filter will generally have some parasitic voltage standing wave ratio, which will affect the system significantly if many filters are to be cascaded. Thus, to design a good multiplexer we must consider frequency-dependent load. Instead of using constant-resistance filters, we investigated the notion of constant-impedance filters, which, when connected in cascade to form a multiplexer, will take into account of the parasitic effects of the elements. This formulation is closely related to the broadband matching of two frequency-dependent impedances.

Another common configuration [44] for the multiplexers is shown in Fig. 1.4 where n specially designed band-pass filters are connected in

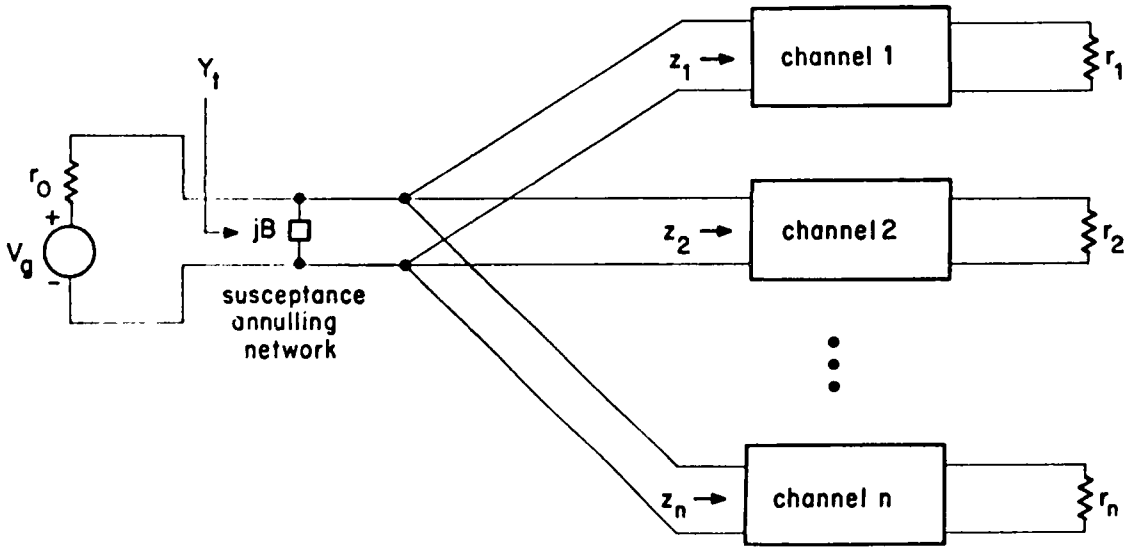

Fig. 1.4. A Parallel-Connected Multiplexer.

parallel. A susceptance-annulling network is added in shunt to help provide a nearly constant total input admittance Y_t which approximates the source conductance $1/r_0$ across the operating band of the multiplexer. Figure 1.5 shows the analogous situation of a series-connected multiplexer.

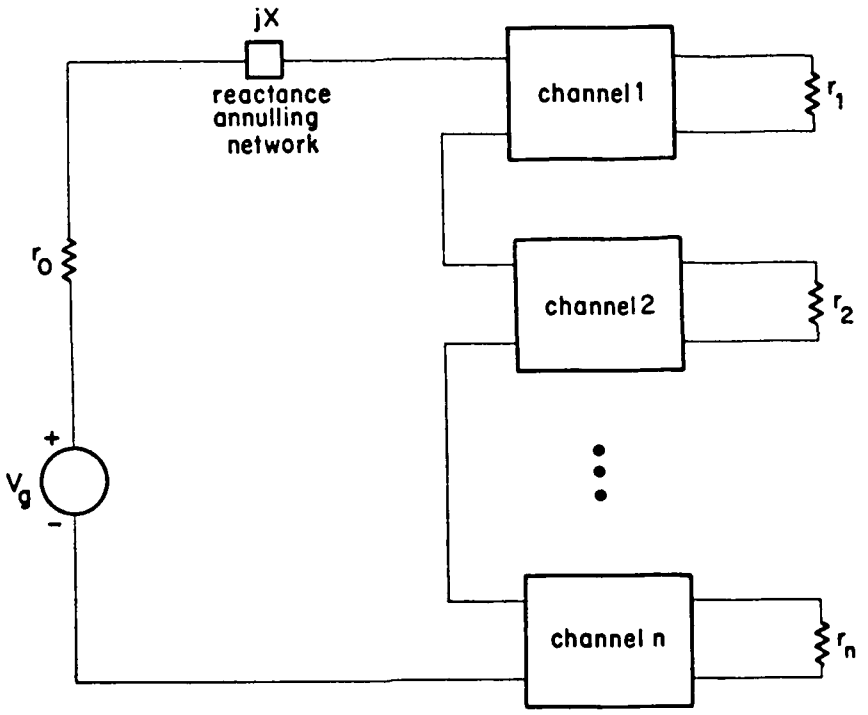

Fig. 1.5. A Series-Connected Multiplexer.

Since the series case is the exact dual of the shunt case, the same principles apply to both. Therefore, out attention will be confined exclusively to the shunt case.

Each channel in the parallel configuration of Fig. 1.4 is a specially designed band-pass filter. When these band-pass filters are connected in parallel as in Fig. 1.4, they can be represented by their input impedances z_i. Instead of using the annulling network, the problem may be viewed as broadband matching of frequency-dependent loads and a resistive source. In the present research, we studied the relationships between the broadband matching of multiport networks and the design theory of multiplexers, and applied th known results in the theory of broadband matching to minimize the interaction effects of the individual channels.

Section 2

ON COMPLEX NORMALIZED SCATTERING MATRIX AND ITS APPLICATION TO BROADBAND MATCHING OF MULTIPORT NETWORKS

2.1 Introduction

The necessary and sufficient conditions that an $n \times n$ matrix be the scattering matrix of a lumped, lossless n-port normalizing to n non-Foster positive-real impedances are presented in matrix form in terms of coefficients of the Laurent series of the parameters. Also, a method is introduced for constructing a generalized scattering matrix of a lossless reciprocal n-port equalizer from the preassigned transducer power-gain characteristics and a set of n passive terminations.

The significance of this work is that it provides a means for testing the realizability of an $n \times n$ scattering matrix by dealing with the coefficients of the Laurent series expansions of the parameters. An example of a three-port network is given to show the procedure for constructing a scattering matrix possessing the paraunitary property, and achieving the preassigned transducer power-gain characteristics when it is terminated in the given impedances.

This section studies the broadband matching problem of multiport networks. The first part presents the necessary and sufficient conditions under which a rational $n \times n$ matrix $\underset{\sim}{S}(s)$ be the scattering matrix of a lumped, lossless n-port equalizer terminated in a set of positive real impedances. The results are shown in matrix form in terms of the coefficients of the Laurent series expansions of the parameters. These conditions are directly applicable to the realization of a lossless reciprocal n-port equalizer from a given matrix $\underset{\sim}{S}(s)$ and a given set of positive real impedances. The second part

introduces a method for constructing a generalized scattering matrix $\underset{\sim}{S}(s)$ of a lossless reciprocal n-port equalizer from the preassigned transducer power-gain characteristics and a set of n passive impedances, which satisfies the para-unitary condition. By imposing the physical realizability requirements for a scattering matrix, necessary and sufficient conditions of a lossless reciprocal n-port equalizer terminated in n passive impedances satisfying the preassigned transducer power-gain characteristics are established.

In an earlier study of the broadband matching problem, Youla [58] developed a new theory of broadband matching an arbitrary load to a resistive generator based on the principle of complex normalization. Wohlers [54] offered an existence theorem of an $n \times n$ scattering matrix of a lumped, passive n-port normalized to n non-Foster positive-real impedances. Chien [21] derived the necessary and sufficient conditions for the realizability of a two-port network having preassigned transducer power-gain characteristics by constructing a scattering matrix. Chen [16] presented a unified theory from the point of view of impedance compatibility. This section develops the realizability conditions for a lossless multiport as an extension of a double-port matching problem.

In paragraph 2.2, we give preliminary considerations. In paragraph 2.3, the necessary and sufficient conditions that a rational $n \times n$ matrix be a scattering matrix are derived. In paragraph 2.4, a method for constructing a generalized scattering matrix is introduced and realizability conditions of the lossless reciprocal n-port equalizer are discussed. Paragraph 2.5 presents an example to show the design procedure.

2.2 Preliminary considerations

Consider the n-port network N of Fig. 2.1 with n non-Foster positive-real terminating impedances $z_i(s)$ $(i = 1,2,\ldots,n)$. Write[†]

$$\underset{\sim}{z}(s) = \text{diag}\,[z_1(s),\ z_2(s),\ \ldots,\ z_n(s)] \tag{2.1}$$

The scattering matrix normalized to the reference impedance matrix $\underset{\sim}{z}(s)$ is written as

$$\underset{\sim}{S}(s) = [S_{ij}(s)]_{n\times n} \tag{2.2}$$

The even part of $z_i(s)$ $(i = 1,2,\ldots,n)$ is

$$r_i(s) = \tfrac{1}{2}[z_i(s) + z_i(-s)] = h_i(s)h_i(-s) \tag{2.3}$$

where the factorization is to be performed so that $h_i(s)$ and $h_i^{-1}(-s)$

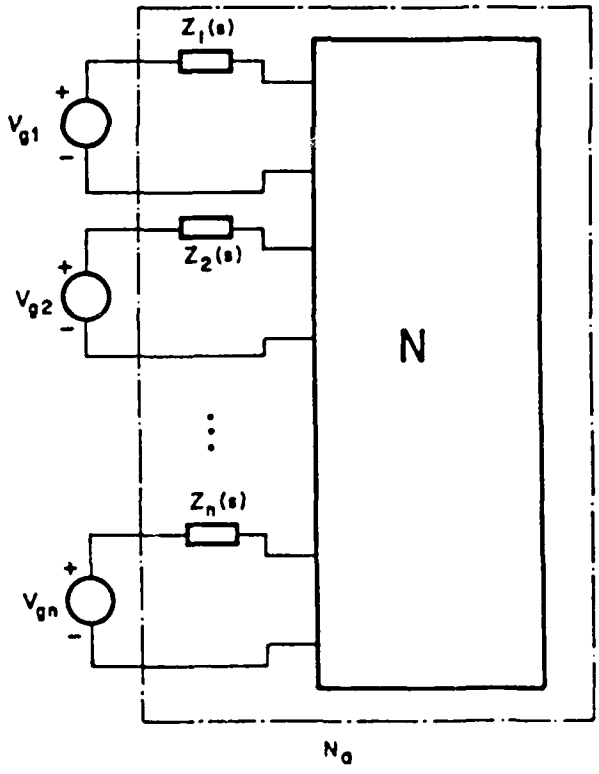

Fig. 2.1. Schematic Diagram of an n-port Network N and the Augmented n-port Network N_a.

[†] diag denotes diagonal matrix

analytic in the open RHS. $\underset{\sim}{H}(s)$ is written as

$$\underset{\sim}{H}(s) = \text{diag}\,[h_1(s),\ h_2(s),\ \ldots,\ h_n(s)] \tag{2.4}$$

Define two regular all-pass functions A(s) and B(s) as

$$A_i(s) = \prod_{j=1}^{u_i} \frac{s-a_j}{s+a_j} \qquad \text{Re } a_j \geq 0 \qquad i = 1,\ 2,\ \ldots,\ n \tag{2.5a}$$

$$B_i(s) = \prod_{j=1}^{v_i} \frac{s-b_j}{s+b_j} \qquad \text{Re } b_j \geq 0 \qquad i = 1,\ 2,\ \ldots,\ n \tag{2.5b}$$

where a_j $(j=1,2,\ldots,u_i)$ are the open RHS poles of $z_i(-s)$ and b_j $(j=1,2,\ldots,v_i)$ are the open RHS zeros of $r_i(s)$. Thus,

$$\frac{h_i(s)}{h_i(-s)} = A_i(s)B_i(s) \tag{2.6}$$

Define

$$F_i(s) = 2r_i(s)A_i(s) \qquad i = 1,\ 2,\ \ldots,\ n \tag{2.7}$$

Definition 2.1. The closed RHS zeros of multiplicity k_i of the function $\frac{r_i(s)}{z_i(s)}$ are the <u>zeros of transmission of order k_i</u> of $z_i(s)$. Furthermore, a zero of transmission s_0 of $z_i(s)$ is said to be a

Class I zero if Re $s_0 > 0$

Class II zero if $s_0 = j\omega_0$ and $z_i(j\omega_0) = 0$

Class III zero if $s_0 = j\omega_0$ and $0 < |z_i(j\omega_0)| < \infty$

Class IV zero if $s_0 = j\omega_0$ and $|z_i(j\omega_0)| = \infty$

Definition 2.2. Zeros of transmission s_0 of $z_i(s)$ $(i=1,2,\ldots,n)$ are called the <u>normalization zero</u> of the n-port network N.

Definition 2.3. Let $\underset{\sim}{Z}_N(s)$ be the open-circuit impedance matrix of an n-port network and $\underset{\sim}{z}(s)$ the reference impedance matrix. The Augmentation admittance matrix $\underset{\sim}{Y}_a(s)$ is defined as

$$\underset{\sim}{Y}_a(s) = [Y_{ij}(s)] = [\underset{\sim}{Z}_N(s) + \underset{\sim}{z}(s)]^{-1} \tag{2.8}$$

As indicated in [14], the normalized reflection coefficient at port i, $S_{ii}(s)$ can be expressed as

$$S_{ii}(s) = B_i(s)\rho_i(s) \qquad i = 1, ?, \ldots, n \tag{2.9}$$

The matrix $\underset{\sim}{Y}_a(s)$ is constructed by using[†]

$$\underset{\sim}{Y}_a(s) = \tfrac{1}{2}\underset{\sim}{H}^{-1}(s)[\underset{\sim}{H}(s)\underset{\sim}{H}_*^{-1}(s) - \underset{\sim}{S}(s)]\underset{\sim}{H}^{-1}(s) \tag{2.10}$$

Invoking (2.7) and (2.9), gives

$$\underset{\sim}{Y}_a(s) = \begin{bmatrix} \dfrac{A_1(s)-\rho_1(s)}{F_1(s)} & -\dfrac{S_{12}(s)}{2h_1(s)h_2(s)} & \cdots & -\dfrac{S_{1n}(s)}{2h_1(s)h_n(s)} \\ \vdots & \vdots & & \vdots \\ -\dfrac{S_{n1}(s)}{2h_n(s)h_1(s)} & -\dfrac{S_{n2}(s)}{2h_n(s)h_2(s)} & \cdots & \dfrac{A_n(s)-\rho_n(s)}{F_n(s)} \end{bmatrix} \tag{2.11}$$

$A_i(s)$, $\rho_i(s)$, and $F_i(s)$ $(i = 1,2,\ldots,n)$ are expanded in the Laurent series about a zero of transmission s_0. Thus

$$A_i(s) = \sum_{x=0}^{\infty} A_{xi}(s-s_0)^x \tag{2.12}$$

$$\rho_i(s) = \sum_{x=0}^{\infty} \rho_{xi}(s-s_0)^x \tag{2.13}$$

[†] $\underset{\sim}{H}_*(s) = \underset{\sim}{H}(-s)$

$$F_i(s) = \sum_{x=0}^{\infty} F_{xi}(s-s_0)^x \tag{2.14}$$

If $s_0 = \infty$, then $(s-s_0)$ is replaced by $\frac{1}{s}$ in each of (2.12) and (2.14)

The augmentation admittance matrix $\underset{\sim}{Y}_a(s)$ can be expanded as

$$\underset{\sim}{Y}_a(s) = \sum_{x=-1}^{\infty} \underset{\sim}{Q}_x(u)(s-s_0)^x \tag{2.15a}$$

where

$$\underset{\sim}{Q}_x(u) = [q_{x,ij}(u)] \tag{2.15b}$$

and u refers to either class II, III or IV.

Denote the residue matrix of impedances $z_i(s)$ at a Class IV zero of transmission $s_0 = j\omega_0$ by

$$\underset{\sim}{A}_{-1} = \text{diag}\,[a_{-11}, a_{-12}, \ldots, a_{-1n}] \tag{2.16}$$

The necessary and sufficient conditions for the normalization impedances $z_1(s)$, $z_2(s)$, ..., $z_n(s)$ and the $n \times n$ matrix $\underset{\sim}{S}(s)$ to be compatible are given in the following theorem which was first presented by Wohlers [54].

Theorem 2.1 (Existence Theorem)

The necessary and sufficient conditions that an $n \times n$ rational matrix be the scattering matrix of a lumped, passive n-port normalized to n non-Foster, positive-real impedances are:

1) $\underset{\sim}{Q} = \underset{\sim}{U}_n - \underset{\sim}{S}^*(j\omega)\underset{\sim}{S}(j\omega)^{\dagger}$ be nonnegative definite for all ω.

2) $\underset{\sim}{Y}_a = \frac{1}{2}\underset{\sim}{H}^{-1}(\underset{\sim}{H}\underset{\sim}{H}_*^{-1} - \underset{\sim}{S})\underset{\sim}{H}^{-1}$ be analytic in the open RHS, and,

either,

$^{\dagger}\underset{\sim}{U}_n$ is the $n \times n$ identity matrix. $\underset{\sim}{S}^*(j\omega) = \underset{\sim}{S}'(-j\omega)$ and prime denotes the transpose of a matrix.

3a) $\det [\underset{\sim}{U}_n - (\underset{\sim}{z} - \underset{\sim}{U}_n)\underset{\sim}{Y}_a] \neq 0$ [†] in the open RHS, or,

3b) the matrix $[\underset{\sim}{U}_n - (\underset{\sim}{z} - \underset{\sim}{U}_n)\underset{\sim}{Y}_a](\underset{\sim}{z} + \underset{\sim}{U}_n)$ have simple poles on the real-frequency axis, and the matrix formed with residues at each of these poles be nonnegative definite.

2.3 Main result

In this portion, we present a generalized theorem for a multiport matching network in which the coefficient constraints are expressed in matrix form in terms of the Laurent series expansions.

Theorem 2.2

Given a real rational $n \times n$ matrix $\underset{\sim}{S}(s)$ and n non-Foster positive-real functions $z_1(s)$, $z_2(s)$, ..., and $z_n(s)$, the necessary and sufficient conditions for the matrix $\underset{\sim}{S}(s)$ to be the scattering matrix of a lumped, lossless, n-port network normalizing to the impedances $z_1(s)$, $z_2(s)$, ..., and $z_n(s)$ are

1) $\underset{\sim}{S}^*(j\omega)\underset{\sim}{S}(j\omega) = \underset{\sim}{U}_n$ for all ω (2.17)

2) $\underset{\sim}{Y}_a(s) = \tfrac{1}{2}\underset{\sim}{H}^{-1}(s)\,[\underset{\sim}{H}(s)\underset{\sim}{H}^{-1}(-s) - \underset{\sim}{S}(s)]\underset{\sim}{H}^{-1}(s)$

be analytic in the closed RHS and its off-diagonal elements Y_{ij} $(i,j = 1,2,\ldots,n \quad i \neq j)$ have at most simple poles at the $j\omega$-axis normalization zeros.

3) $\rho_i(s)$ satisfy the coefficient constraints

$$A_{xi} = \rho_{xi} \qquad x = 0, 1, \ldots, m_i - 1 \qquad i = 1, 2, \ldots, n \tag{2.18}$$

where

$$m_i = \begin{cases} k_i & \text{for Class I, II, or IV} \\ k_i - 1 & \text{for Class III} \end{cases}$$

[†] det denotes the determinant of a matrix.

4) at every jω-axis normalization zero $s_0 = j\omega_0$,

(i) If $|z_i(j\omega_0)| < \infty \quad i = 1,2,\ldots,n$ then[†]

$$\underset{\sim}{Q}_{-1}(u) \geq 0 \tag{2.19a}$$

where u refers to Class II or III and

$$q_{-1,ii}(uu) = \frac{A_{m_i i} - \rho_{m_i i}}{F_{(m_i+1)i}} \quad i = 1,2,\ldots,n \tag{2.19b}$$

(ii) If $|z_i(j\omega_0)| = \infty \quad i = 1,2,\ldots,n$ then

(a) $\underset{\sim}{Q}_{-1}(u) = \underset{\sim}{Q}_0(u) = \underset{\sim}{Q}_n$ and (2.20)

(b) $[\underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_1(u)] \geq 0$ (2.21a)

where u refers to Class IV and

$$q_{1,ii}(IV) = \frac{A_{m_i i} - \rho_{m_i i}}{F_{(m_i-1)i}} \quad i = 1,2,\ldots,n \tag{2.21b}$$

(iii) If $|z_\ell(j\omega_0)| = \infty \quad \ell = 1,2,\ldots,k$ and

$|z_p(j\omega_0)| < \infty \quad p = k+1, k+2, \ldots, n$ then

(a) $\underset{\sim}{Q}_{-1,\ell p} = \underset{\sim}{Q}$[††] (2.22)

(b) $\underset{\sim}{Q}_{-1,\ell\ell'}(IV) = \underset{\sim}{Q}_{0,\ell\ell'}(IV) = \underset{\sim}{Q} \quad \ell,\ell' = 1,2,\ldots,k$ (2.23)

(c)
$$\begin{array}{c} \\ k \\ n-k \end{array} \begin{array}{c} \begin{array}{cc} k & n-k \end{array} \\ \left[\begin{array}{c|c} \underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_{1,\ell\ell'}(IV) & -\underset{\sim}{Q}_{0,\ell p} \\ \hline \underset{\sim}{Q}_{0,\ell p} & \underset{\sim}{Q}_{-1,pp'}(u) \end{array}\right] \end{array} \geq 0 \tag{2.24}$$

where u refers to Class II or III.

[†] $\underset{\sim}{Q}_n$ denotes the $n \times n$ zero matrix and ≥ 0 denotes the nonnegative definiteness.

[††] $\underset{\sim}{Q}_{-1,\ell p}$ denotes the submatrix of $\underset{\sim}{Q}_{-1}$ with subscripts ℓ and p.

The proof is given in Section 2.6. By applying Theorem 2.2, we can test to see whether or not a given $n \times n$ matrix is the scattering matrix of a lossless n-port network terminated in n given passive impedances. In many practical situations, the design of a lossless reciprocal equalizer to match out n arbitrary passive impedances is required. The design procedure is divided into three steps. First, we construct a scattering matrix in its most general form. Second, we apply Theorem 2.2 to determine the realizability of the scattering matrix. Finally, the desired matching network is obtained by realizing this scattering matrix, applying any of the known multiport synthesis techniques [39].

2.4 Construction method of scattering matrix

In this portion, we study the construction of a scattering matrix in its most general form for a lossless reciprocal n-port. The scattering matrix of a lossless n-port must have para-unitary property. Hence, we need to construct a para-unitary scattering matrix within arbitrary all-pass functions based on the preassigned transducer power-gain characteristics and n terminations. Using this general form of the scattering matrix, Theorem 2.2 gives the necessary and sufficient conditions for the existence of a lossless, reciprocal n-port equalizer terminated in n passive impedances satisfying the preassigned transducer power-gain characteristics.

From the para-unitary property of a lossless, reciprocal n-port,

$$\underset{\sim}{S}(s)\underset{\sim}{S}_{*}(s) = \underset{\sim}{U}_{n} \tag{2.25}$$

we obtain

$$S_{ii}(s)S_{ii*}(s) + \sum_{\substack{j=1 \\ j\neq i}}^{n} S_{ij}(s)S_{ij*}(s) = 1 \tag{2.26}$$

$$\sum_{k=1}^{n} S_{ik}(s)S_{kj*}(s) = 0 \qquad \begin{matrix} i = 1, 2, \ldots, n-1 \\ j = i+1, \ldots, n \end{matrix} \tag{2.27}$$

where $S_{ij} = S_{ji}$. From the given transducer power-gain characteristic $G(\omega^2)$, we have

$$G_{ij}(\omega^2) = |S_{ji}(j\omega)|^2 \tag{2.28a}$$

where

$$0 \leq \sum_{\substack{j=1 \\ j\neq 1}}^{n} G_{ij}(\omega^2) \leq 1 \qquad \begin{matrix} 0 \leq \omega < \infty \text{ and} \\ i = 1, 2, \ldots, n \end{matrix} \tag{2.28b}$$

Factorizing $G_{ij}(-s^2)$ obtains

$$G_{ij}(-s^2) = \frac{P_{ij}(s)P_{ij*}(s)}{Q_{ij}(s)Q_{ij*}(s)} \tag{2.29}$$

where $Q_{ij}(s)$ and $P_{ij}(s)$ are Hurwitz polynomials. Then

$$S_{ij}(s)S_{ij*}(s) = \frac{P_{ij}(s)P_{ij*}(s)}{Q_{ij}(s)Q_{ij*}(s)} \tag{2.30}$$

From (2.26)

$$S_{ii}(s)S_{ii*}(s) = 1 - \sum_{\substack{j=1 \\ j\neq 1}}^{n} G_{ij}(-s^2) = \frac{U_i(s)U_{i*}(s)}{Q_i(s)Q_{i*}(s)} \tag{2.31}$$

where $U_i(s)$ is a Hurwitz polynomial. According to [14],

$$S_{ii}(s) = \theta_{ii}(s)B_i(s)\rho_{im}(s) = \theta_{ii}(s)s_{ii}(s) \tag{2.32}$$

$$S_{ij}(s) = \theta_{ij}(s)B_i(s)B_j(s)s_{ijm}(s) = \theta_{ij}(s)s_{ij}(s) \tag{2.33}$$

where $\theta_{ii}(s)$ and $\theta_{ij}(s)$ are arbitrary all-pass functions, $\rho_{im}(s)$ and $s_{ijm}(s)$ are minimum-phase solutions of (2.26) and (2.27) respectively, i.e.,

$$\rho_{im}(s) = \frac{U_i(s)}{Q_i(s)} \tag{2.34}$$

$$s_{ijm}(s) = \frac{P_{ij}(s)}{Q_{ij}(s)} \tag{2.35}$$

and

$$s_{ii}(s) = B_i(s)\rho_{im}(s) \tag{2.36}$$

$$s_{ij}(s) = \left[B_i(s)B_j(s)\right]^{\frac{1}{2}} s_{ijm}(s) \tag{2.37}$$

It is clear that $S_{ii}(s)$ and $S_{ij}(s)$ so constructed have the desired magnitudes on the $j\omega$-axis and satisfy (2.26); i.e., $|S_{ii}(j\omega)|^2 = 1 - \Sigma G_{ij}(\omega^2)$. As for the other conditions in (2.27) generated by the off-diagonal equations of $\underset{\sim}{S}\underset{\sim}{S}_* = \underset{\sim}{U}$, the constraints on $\theta_{ii}(s)$ and $\theta_{ij}(s)$ must be identified.

We claim that the relation between $\theta_{ii}(s)$ and $\theta_{ij}(s)$ is

$$\theta_{ii}(s)\theta_{jj}(s) = \theta_{ij}^2(s) \qquad i,j = 1, 2, \ldots, n \quad \text{and} \quad i \neq j \tag{2.38}$$

written as

$$\theta_{ij}(s) = \zeta_i(s)\zeta_j(s) \tag{2.39}$$

$$\theta_{ii}(s) = \zeta_i^2(s) \tag{2.40}$$

Substituting (2.39) and (2.40) into (2.27)

$$\sum_{k=1}^{n} S_{ik}S_{kj*} = \sum_{k=1}^{n} \zeta_i\zeta_k s_{ik}\zeta_{j*}\zeta_{k*}s_{kj*} = 0 \tag{2.41}$$

we obtain

$$\sum_{k=1}^{n} s_{ik} s_{kj*} = 0 \qquad \begin{matrix} i = 1, 2, \ldots, n-1 \\ j = i+1, \ldots, n \end{matrix} \qquad (2.42)$$

Note that $\zeta_k \zeta_{k*} = 1$. $\zeta_i \zeta_{j*}$ can cancel out because they are independent of the index k. For fixed i and j, as k varies, equation (2.27) remains valid. Since it is valid for any i and j, we conclude that multiplication of regular all-pass functions constrinted by (2.38) does not affect the para-unitary property. These all-pass functions only affect the phases of the scattering parameters.

2.5 Example

It is desired to equalize two series RL loads to a generator with a parallel RC internal impedance and to achieve the second order *Butterworth transducer* power gain. The specifications are shown in Fig. 2.2 with $R_1 = 1\ \Omega$, $C_1 = 1$ F, $R_2 = R_3 = 2\ \Omega$, and $L_2 = L_3 = 1$ H.

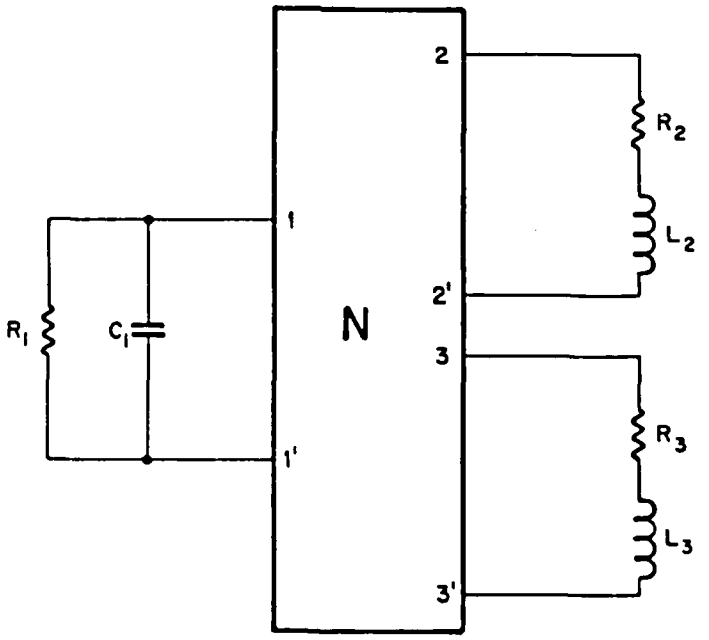

Fig. 2.2. An Illustrative Example of a Three-Port Network.

Given

$$z_1(s) = \frac{1}{s+1} \tag{2.43}$$

$$z_2(s) = z_3(s) = s+2 \tag{2.44}$$

and

$$G_{12}(\omega^2) = G_{13}(\omega^2) = \frac{k^2}{1+\omega^4} \tag{2.45}$$

We wish to construct a scattering matrix $\underset{\sim}{S}(s)$ terminated in z_1, z_2, and z_3, achieving the given G_{12} and G_{13}.

Solution: Write

$$G_{12}(-s^2) = G_{13}(-s^2) = \frac{k^2}{1+s^4} \tag{2.46}$$

Factorizing G_{12} and G_{13}, we obtain the minimum-phase solutions of (2.46) as

$$s_{12m} = s_{13m} = \frac{k}{s^2 + \sqrt{2}s + 1} \tag{2.47}$$

We write $G_{23}(-s^2)$ in a general form

$$G_{23}(-s^2) = \frac{k_{23}^2 (s^2+c)}{(s^4+1)(s^4+a's^2+b')} \tag{2.48}$$

where $c=-4$, a' and b' are to be determined. Factorizing $G_{23}(-s^2)$, we get

$$s_{23m} = \frac{k_{23}(s+2)}{(s^2 + \sqrt{2}s + 1)(s^2+as+b)} \tag{2.49}$$

Observe that $z_1(s)$ has a Class II zero of transmission of order

one at $s_0=\infty$ and $z_2(s)$ and $z_3(s)$ each has a Class IV zero of transmission of order one at $s_0=\infty$. Next, we find reflection coefficient $\rho_1(s)$.

The minimum phase solution of (2.26) with i=1 is

$$s_{11m} = \epsilon_{11} t^2 \frac{\left(\frac{s}{t}\right) + \sqrt{2}\left(\frac{s}{t}\right) + 1}{s^2 + \sqrt{2}s + 1} = \epsilon_{11}[1 - \sqrt{2}(1-t)\frac{1}{s} + \ldots] \quad (2.50)$$

where $t = (1-2k^2)^{1/4}$. Introducing the all-pass function

$$\theta_{11}(s) = \prod_i \frac{s-\alpha_i}{s+\alpha_i} = 1 - \frac{2}{s}\sum_i \alpha_i + \ldots \quad (2.51)$$

The reflection coefficient ρ_1 is

$$\rho_1(s) = \theta_{11}(s)s_{11m}(s) = \epsilon_{11}\{1 - [2\sum_i \alpha_i + \sqrt{2}(1-t)]\frac{1}{s} + \ldots\} \quad (2.52)$$

Obviously, $\rho_1(s)$ satisfies the constraint $A_{xi} = \rho_{xi}$ (x=0) of Theorem 2.2 with $\epsilon_{11} = +1$. The requirement from condition (4.iii.c)

$$q_{-1,11}(II) = \frac{A_{11} - \rho_{11}}{F_{21}} \geq 0 \quad (2.53)$$

becomes

$$1 - \sum_i \alpha_i - \frac{1-t}{\sqrt{2}} \geq 0 \quad (2.54)$$

To maximize k, set $\sum_i \alpha_i = 0$, i.e., $\theta_{11} = 1$ and t=0, then

$$k = \frac{\sqrt{2}}{2} \quad (2.55)$$

$$\rho_1 = \frac{s^2}{s^2 + \sqrt{2} + 1} \quad (2.56)$$

$$S_{11}(s) = \epsilon_{11} B_1 \rho_1 = -\frac{s^2}{s^2 + \sqrt{2}s + 1} \tag{2.57}$$

To compute $S_{23m}(s)$, we apply the following formula [1]

$$\pm(df_{12*}f_{13*}f_{23*} + d*f_{12}f_{13}f_{23}) = f_{12}f_{12*}f_{13}f_{13*} + f_{12}f_{12*}f_{23}f_{23*} + f_{13}f_{13*}f_{23}f_{23*} \tag{2.58}$$

where $S_{ij} = \frac{f_{ij}}{d}$ and d is the common denominator of all S_{ij} $(i,j = 1,2,3)$

$$S_{23m} = \epsilon_{23} \frac{s + 2}{(s^2 + \sqrt{2}s + 1)[2(4 - \sqrt{2})s^2 + 2(2\sqrt{2} - 1)s + 4)} \tag{2.59}$$

The minimum phase solution of

$$S_{22m} = \epsilon_{22} \frac{2(4 - \sqrt{2})s^4 + s(2\sqrt{2} - 1)s^3 + 4s^2 + s + 2}{(s^2 + \sqrt{2}s + 1)[2(4 - \sqrt{2})s^2 + (2\sqrt{2} - 1)s + 4]} \tag{2.60}$$

By applying condition (3) of Theorem 2.2 at $s_0 = \infty$,

$$A_{02} = \rho_{02} \tag{2.61}$$

we obtain $\epsilon_{22} = +1$ and $\rho_{02} = 1$. Furthermore $\epsilon_{12} = \epsilon_{23} = -1$.

Thus, the scattering matrix can be formed according to (2.32) and (2.33), where θ_{ii} and θ_{ij} $(i,j = 1,2,3)$ satisfy the relation $\theta_{ii}\theta_{jj} = \theta_{ij}^2$.

From condition (4.iii.c), we must have

$$q_{-1,22}(IV) = \frac{A_{12} - \rho_{12}}{F_{02}} \leq \frac{1}{a_{-12}} \tag{2.62}$$

Let

$$\theta_{22} = \prod_i \frac{s-\beta_i}{s+\beta_i} \tag{2.63}$$

Then (2.62) becomes

$$\frac{2\sum_i \beta_i + \sqrt{2}}{4} \leq 1 \tag{2.64}$$

To satisfy (2.64), set $\sum_i \beta_i = 0$, i.e., $\theta_{22} = 1$. Similarly $\theta_{33} = 1$. Therefore, the regular all-pass functions of the off-diagonal elements S_{ij} $(i \neq j)$ are also determined. $\theta_{ij} = 1$ $(i,j = 1,2,3 \quad i \neq j)$.

Based on the above analyses, we construct the required para-unitary scattering matrix as

$$\underset{\sim}{S}(s) = \frac{1}{s^2+\sqrt{2}s+1} \begin{bmatrix} -s^2 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{2(4-\sqrt{2})s^4+2(2\sqrt{2}-1)s^3+4s^2+s+2}{2(4-\sqrt{2})s^2+2(2\sqrt{2}-1)s+4} & -\frac{s+2}{2(4-\sqrt{2})s^2+2(2\sqrt{2}-1)s+4} \\ \frac{\sqrt{2}}{2} & -\frac{s+2}{2(4-\sqrt{2})s^2+2(2\sqrt{2}-1)s+4} & \frac{2(4-\sqrt{2})s^4+2(2\sqrt{2}-1)s^3+4s^2+s+2}{2(4-\sqrt{2})s^2+2(2\sqrt{2}-1)s+4} \end{bmatrix} \tag{2.65}$$

The corresponding augmentation matrix $\underset{\sim}{Y}_a(s)$ is

$$\underset{\sim}{Y}_a(s) = \frac{1}{4(s^2 + \sqrt{2}s + 1)} \times$$

$$\begin{bmatrix} 2(2-\sqrt{2})s^3+2s^2+2\sqrt{2}s+1 & -(s+1) & -(s+1) \\ -(s+1) & \dfrac{(4\sqrt{2}-1)s^3+(8-2\sqrt{2})s^2+(4\sqrt{2}-\frac{3}{2})s+1}{(4-\sqrt{2})s^2+(2\sqrt{2}-1)s+2} & \dfrac{s+2}{2[(4-\sqrt{2})s^2+(2\sqrt{2}-1)s+2]} \\ -(s+1) & \dfrac{s+2}{2[(4-\sqrt{2})s^2+(2\sqrt{2}-1)s+2]} & \dfrac{(4\sqrt{2}-1)s^3+(8-2\sqrt{2})s^2+(4\sqrt{2}-\frac{3}{2})s+1}{(4-\sqrt{2})s^2+(2\sqrt{2}-1)s+2} \end{bmatrix} \quad (2.$$

Clearly, $\underset{\sim}{Y}_a(s)$ is analytic in the RHS and all the off-diagonal elements $Y_{ij}(s)$ have no poles on the $j\omega$-axis. Condition (2) of Theorem 2.2 is satisfied. Thecalculation also shows that conditions (4.iii.a) and (4.iii.b) are fulfilled.

$$q_{-1,12} = q_{-1,13} = 0$$

$$\underset{\sim}{Q}_{-1,\ell\ell'} = \underset{\sim}{Q}_{0,\ell\ell'} = 0 \qquad \ell,\ell' = 2,3$$

Finally, condition (4.iii.c) can be written, in this particular case, as

$$\left[\begin{array}{c|cc} q_{-1,11} & -q_{0,12} & -q_{0,13} \\ \hline q_{0,12} & \dfrac{1}{a_{-12}} - q_{1,22} & -q_{1,23} \\ q_{0,13} & -q_{1,23} & \dfrac{1}{a_{-13}} - q_{1,33} \end{array}\right] \geq 0 \qquad (2.67a)$$

or

$$\left[\begin{array}{c|cc} \frac{2-\sqrt{2}}{2} & 0 & 0 \\ \hline 0 & 1-\frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & 1-\frac{\sqrt{2}}{4} \end{array}\right] \geq 0 \qquad (2.67b)$$

This matrix is nonnegative definite and condition (4.iii.c) is satisfied. Thus, the scattering matrix constructed in (2.65) is indeed realizable which achieves the preassigned transducer power gain when the 3-port network is terminated in $\underset{\sim}{z}(s)$.

2.6 Proof of Theorem 2.2

We prove Theorem 2.2 via Wohlers existence theorem and consider the following four cases:

Case I: $z_i(s)$ $(i = 1,2,\ldots,n)$ have only Class I zero of transmission s_0

Case II: $|z_i(j\omega_0)| < \infty \quad i = 1,2,\ldots,n$

Case III: $|z_i(j\omega_0)| = \infty \quad i = 1,2,\ldots,n$

Case IV: $|z_\ell(j\omega_0)| = \infty, \quad \ell = 1,2,\ldots,k$ and $|z_p(j\omega_0)| < \infty$ $p = k+1, k+2, \ldots, n$

Since the theorem is valid for both reciprocal and nonreciprocal networks, we should keep the subscripts of all entries of the matrix distinguished. For example Y_{12} and Y_{21} are different in general.

Note that for those impedances which have zeros in the close RHS, they fall into case II where $|z_i(j\omega_0)| < \infty$. In this case all residues are zero.

In case I, at a class I zero of transmission s_0, constraint (2)

is the necessary and sufficient condition for $\underset{\sim}{Y}_a$ to be analytic in the open RHS and for the off-diagonal elements Y_{ij} $(i,j=1,2,\ldots,n$ $i\neq j)$ to have at most simple poles at the $j\omega$-axis normalization zeros. Constraint (3) ensures that Y_{ii} $(i=1,2,\ldots,n)$ have at most simple poles at the $j\omega$-axis normalization zeros. So the necessary and sufficient conditions for the matrix $\underset{\sim}{S}(s)$ to be a scattering matrix reduce to the first three statements of the theorem since

$$\underset{\sim}{M}(s) = \{\underset{\sim}{U}(s) - [\underset{\sim}{z}(s) - \underset{\sim}{U}(s)]\underset{\sim}{Y}_a(s)\}[\underset{\sim}{z}(s) + \underset{\sim}{U}(s)] \qquad (2.68)$$

is analytic on the $j\omega$-axis in this case.

Necessity. Suppose that $\underset{\sim}{S}(s)$ is the scattering matrix of a lossless, n-port network normalizing to n passive impedances having the preassigned transducer power-gain characteristics. For a lossless network, $\underset{\sim}{S}(s)$ is para-unitary. This shows that condition (1) is necessary. We next compute the augmentation admittance matrix $\underset{\sim}{Y}_a(s)$ from $\underset{\sim}{S}(s)$ and the loads. Since $\underset{\sim}{Y}_a(s)$ is positive real, there are at most simple poles on the $j\omega$-axis. Condition (2) indicates that $Y_{ij}(s)$ $(i\neq j)$ have at most simple poles on the $j\omega$-axis while condition (3) guarantees that $Y_{ii}(s)$ have at most simple poles on the $j\omega$-axis. These are part of Youla's constraints [58].

Next, we will show that condition (4) is necessary. From (2.68) we expand

$$\underset{\sim}{M}(s) = \begin{bmatrix} [1-Y_{11}(z_1-1)](z_1+1) & -Y_{12}(z_1-1)(z_2+1) & \cdots & -Y_{1n}(z_1-1)(z_n+1) \\ -Y_{21}(z_2-1)(z_1+1) & [1-Y_{22}(z_2-1)](z_2+1) & \cdots & -Y_{2n}(z_2-1)(z_n+1) \\ \vdots & \vdots & & \vdots \\ -Y_{n1}(z_n-1)(z_1+1) & -Y_{n2}(z_n-1)(z_2+1) & \cdots & [1-Y_{nn}(z_n-1)](z_n+1) \end{bmatrix} \qquad (2.69)$$

In case II, $|z_i(j\omega_0)| < \infty$ for all i. We need only show that $\underset{\sim}{Y}_a(s)$ has at most simple poles with a nonnegative residue matrix. This requires that M(s) have at most simple poles with a nonnegative residue matrix. Likewise, if $\underset{\sim}{M}(s)$ has simple poles with a nonnegative residue matrix then $\underset{\sim}{Y}_a(s)$ also has simple poles with a nonnegative residue matrix.

Since $\underset{\sim}{z}(s)$ are analytic on the jω-axis, the poles of $\underset{\sim}{Y}_a(s)$ on the jω-axis are contributed by zeros of the real part of $\underset{\sim}{z}(s)$. It is obvious from (2.69) that the poles of $\underset{\sim}{M}(s)$ on the jω-axis are contributed by those of $\underset{\sim}{Y}_a(s)$. Let the residue matrix of $\underset{\sim}{M}(s)$ be $\underset{\sim}{R}_M(s)$. We can write

$$\underset{\sim}{R}_M(j\omega_o) = \text{diag } (1-z_i) \cdot \underset{\sim}{Q}_{-1}(u) \cdot \text{diag } (1+z_i) \qquad i = 1,2,\ldots,n \tag{2.70}$$

$$\det \underset{\sim}{R}_M(j\omega) = \det \underset{\sim}{Q}_{-1}(u) \cdot [1-z_i^2(j\omega_o)] \tag{2.71}$$

where the residue matrix of $\underset{\sim}{Y}_a(s)$ is denoted by $\underset{\sim}{Q}_{-1}(u)$ and u refers to either class II or III. On the jω-axis, $z_i(j\omega_o)$ are pure imaginary and $1-z_i^2(j\omega_o)$ should be greater than zero. Therefore, because of nonnegative definite properties of $\underset{\sim}{R}_M(j\omega_o)$, $\underset{\sim}{Q}_{-1}(u)$ is also nonnegative definite at $j\omega_o$.

In case III, $|z_i(j\omega_0)| = \infty$ $i = 1,2,\ldots,n$. The open-circuit impedance matrix $\underset{\sim}{Z}_N(s)$ of n-port N can be expressed by the relation

$$\underset{\sim}{Z}_N = \underset{\sim}{Y}_a^{-1} - \underset{\sim}{z} \tag{2.72}$$

Let $\underset{\sim}{K}_N(j\omega_o)$ and $\underset{\sim}{K}''_a(j\omega_o)$ be the residue of $\underset{\sim}{Z}_N(s)$ and $\underset{\sim}{Y}_a^{-1}(s)$ at $j\omega_o$. Then

$$\underset{\sim}{K}_N(j\omega_0) = \underset{\sim}{K}''_a(j\omega_0) - \underset{\sim}{A}_{-1} \tag{2.73}$$

In order to have a positive real matrix $\underset{\sim}{Z}_N$, $\underset{\sim}{Y}_a^{-1}(s)$ must have simple poles at $j\omega_o$ with residue not less than those of $z_i(s)$. Therefore, we

must have $\underset{\sim}{Y}_a(j\omega_0) = \underset{\sim}{0}$, i.e.,

$$\underset{\sim}{Q}_{-1}(IV) = \underset{\sim}{Q}_0(IV) = \underset{\sim}{0}$$

Upon decomposition, we obtain

$$\det \underset{\sim}{R}_M(j\omega_0) = \det \underset{\sim}{A}_{-1} \cdot \det [\underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_1(IV)] \cdot \det \underset{\sim}{A}_{-1} \qquad (2.74)$$

The hermitian property together with the nonnegative definiteness of $\underset{\sim}{R}_M(j\omega_0)$ implies condition (4.ii.b).

In case IV, we relabel all the impedances, if necessary, so that $|z_\ell(j\omega_0)| = \infty \quad \ell = 1,2,\ldots,k$ and $|z_p(j\omega_0)| < \infty \quad p = k+1, k+2, \ldots, n$. The matrix $\underset{\sim}{M}(s)$ has a simple pole at $j\omega_0$ whose residue matrix is

$$R_M(j\omega_0) = \begin{array}{c} \\ k \\ n-k \end{array} \begin{array}{c} \begin{array}{cc} k & n-k \end{array} \\ \left[\begin{array}{c|c} \underset{\sim}{A}_{-1} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{U} - \underset{\sim}{z}(j\omega_0) \end{array}\right] \end{array} \begin{array}{c} \begin{array}{cc} k & n-k \end{array} \\ \left[\begin{array}{c|c} \underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_1 & -\underset{\sim}{Q}_0 \\ \hline \underset{\sim}{Q}_0 & \underset{\sim}{Q}_{-1} \end{array}\right] \end{array} \begin{array}{c} \begin{array}{cc} k & n-k \end{array} \\ \left[\begin{array}{c|c} \underset{\sim}{A}_{-1} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{z}(j\omega_0) + \underset{\sim}{U} \end{array}\right] \end{array} \qquad (2.75)$$

$$\det \underset{\sim}{R}_M(j\omega_0) = (\det \underset{\sim}{A}_{-1}) \cdot [\det \underset{\sim}{z}^2_{(k+1)n}(j\omega_0) + 1] \cdot \det \left[\begin{array}{c|c} \underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_1 & -\underset{\sim}{Q}_0 \\ \hline \underset{\sim}{Q}_0 & \underset{\sim}{Q}_{-1} \end{array}\right] \qquad (2.76)$$

By decomposition, the nonnegative definiteness of $\underset{\sim}{R}_M(j\omega_0)$ assures condition (4.iii.c).

For $\underset{\sim}{Y}_a^{-1}$ to have a simple pole with a nonnegative residue matrix of $j\omega_0$, it is necessary that $|\underset{\sim}{Y}_a(j\omega_0)| < \infty$, i.e.,

$$\underset{\sim}{Q}_{-1,\ell p} = \underset{\sim}{0}$$

Based on the same reason as in case III, condition (4.iii.b) requires

$$\underset{\sim}{Q}_{-1,\ell\ell'}(IV) = \underset{\sim}{Q}_{0,\ell\ell'}(IV) = \underset{\sim}{0} \qquad \ell,\ell' = 1,2,\ldots,k \ .$$

Sufficiency. Assume that conditions of Theorem 2.2 are satisfied. We show that conditions (1), (2) and (3b) of Theorem 2.1 are also satisfied. Therefore, matrix $\underset{\sim}{S}(s)$ is the scattering matrix of a lumped, passive n-port normalized to the given n non-Foster, positive-real impedances.

In case II, $|z_i(j\omega_0)| < \infty$ for all i. As discussed above, we have shown that if $\underset{\sim}{Y}_a(s)$ has simple poles with a nonnegative definite residue matrix, $\underset{\sim}{M}(s)$ also has simple poles with a nonnegative definite residue matrix. We need only focus on $\underset{\sim}{Y}_a(s)$, rather than $\underset{\sim}{M}(s)$, showing that requirements for $\underset{\sim}{M}(s)$ can be similarly imposed on $\underset{\sim}{Y}_a(s)$, i.e., if $\underset{\sim}{Y}_a(s)$ is satisfied by a given condition, $\underset{\sim}{M}(s)$ would also be satisfied.

From condition (2), $Y_{ij}(s)$ $(i,j=1,2,\ldots,n \quad i\neq j)$ have simple poles on the $j\omega$-axis. From condition (3), $Y_{ii}(s)$ have simple poles on the $j\omega$-axis. Also, from (2.70) the nonnegative definiteness of $\underset{\sim}{R}_M(j\omega_0)$ guarantees that $\underset{\sim}{Q}_{-1}(u)$ possess this property.

In case III, $|z_i(j\omega_0)| = \infty$ $i=1,2,\ldots,n$. Constraints (3) and (4.ii.a) require that $\underset{\sim}{Y}_a(j\omega_0) = \underset{\sim}{0}_n$. From (2.69), $\underset{\sim}{M}(s)$ has simple poles at $j\omega_0$ and the residue matrix becomes

$$\underset{\sim}{R}_M(j\omega_0) = \underset{\sim}{A}_{-1} \cdot [\underset{\sim}{A}_{-1}^{-1} - \underset{\sim}{Q}_1(IV)] \cdot \underset{\sim}{A}_{-1} \tag{2.77}$$

where

$$\underset{\sim}{Q}_1(IV) = [q_{1,ij}(IV)]$$

and

$$q_{1,ii}(IV) = \frac{A_{m_i i} - \rho_{m_i i}}{F_{(m_i-1)i}} \qquad i = 1, 2, \ldots, n$$

Since $\underset{\sim}{z}(s)$ is a positive real matrix, its residues a_{-1i} $(i=1,2,\ldots,n)$ are all real and positive. Therefore, condition (4.ii.b) assures that matrix $\underset{\sim}{R}_M(s)$ is nonnegative definite.

In case IV, constraints (4.iii.a) and (4.iii.b) guarantee that $\underset{\sim}{M}(s)$ has simple poles on the $j\omega$-axis. By appealing to (2.76), the nonnegative definite properties of $\underset{\sim}{R}_M(j\omega_0)$ are the result of constraint (4.iii.c).

2.7 Conclusion

Necessary and sufficient conditions were presented for a rational $n \times n$ matrix $\underset{\sim}{S}(s)$ to be the scattering matrix of a lossless n-port equalizer terminated in a set of positive-real impedances. The theorem is essentially valid for either a reciprocal or nonnreciprocal matrix. When the transducer power-gain characteristics are given, the method introduced can be used to construct a generalized symmetric scattering matrix such that the corresponding lossless reciprocal n-port equalizer achieves the preassigned transducer power-gain characteristics. A three-port illustrative example was given.

It is worth noting that the design problem for a lossless reciprocal n-port equalizer depends heavily on, first, solving for a scattering matrix possessing a para-unitary property and then seeking a set of regular all-pass factors to satisfy the realizability conditions.

By applying the multiport synthesis, the desired equalizer can be realized.

The main advantage of the proposed procedure lies in its simplicity in testing, since all the conditions are expressed in matrix form and in terms of the Laurent series coefficients.

Section 3

ON LOSSLESS RECIPROCAL AND NONRECIPROCAL MATCHING NETWORKS OF AN ACTIVE LOAD

3.1 Introduction

Section 3 presents a general matching theory between an arbitrary passive impedance and an active load impedance and extends the broadband matching theory to include both lossless reciprocal and nonreciprocal network. Application of the result to the design of nonreciprocal negative-resistance amplifier is given. The significance of the present approach is that the realization of the equalizer is accomplished by means of the driving-point synthesis based on the Darlington theory. The result enlarges the domain of realizable broadband matching networks.

An illustrative example is presented to show the design procedure and the conditions under which a lossless nonreciprocal networks is realized for an active load.

In studying the problem of broadband matching, Youla [58] developed a theory based on the principle of complex normalization. Chan and Kuh [6] generalized this theory to include both passive and active one-port load impedance. Ho and Balabanian [33] developed a technique for the synthesis of active and passive compatible impedances when the coupling networks is not necessarily reciprocal. In 1982, Chen and Satyanarayana [19] presented a method for realizing a lossless reciprocal equalizer which involves only the driving-point synthesis based on the Darlington theory. Chen and Tsai [20] then extended the above result to include the situation where the load impedance is active; however, only the reciprocal coupling networks were considered.

The present research is a direct extension and generalization of the previous works. Necessary and sufficient conditions are given for the existence of a lossless equalizer which, when operating between the given passive source impedance and the active load impedance, yields a preassigned transducer power-gain characteristic. The major point of departure arises from the realizability of the equalizer which may be a reciprocal or nonreciprocal network. Under certain conditions, the coupling network can only be realized as a nonreciprocal network. The use of nonreciprocal coupling networks will not only enlarge the domain of realizable networks, but may also lead to the simplification of their realizations. A design procedure is described and an example is given to illustrate the approach.

3.2 Preliminary considerations

Consider the lossless two-port network N of Fig. 3.1. The source impedance $z_1(s)$ is assumed to be passive and the load impedance $z_2(s)$ may be either passive or active. The driving-point impedances looking into the input and output ports when the output and input ports are

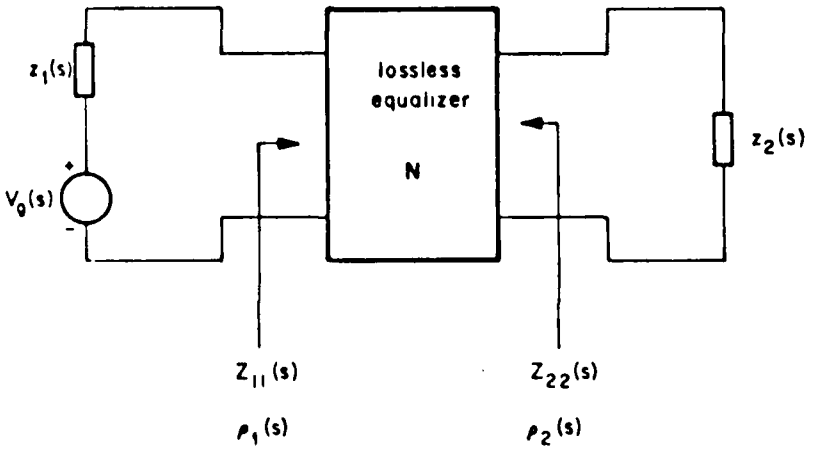

Fig. 3.1. Schematic of a Lossless Two-Port Network N with a Passive Source Impedance $z_1(s)$ and an Active Load $z_2(s)$.

terminated in $z_2(s)$ and $z_1(s)$ are denoted by $Z_{11}(s)$ and $Z_{22}(s)$, respectively. The symbol $\rho_1(s)$ is used to represent the reflection coefficient at the input port normalizing to $z_1(s)$, and $\rho_2(s)$ is the reflection coefficient at the output port with respect to $z_2(s)$. The even part of $z_i(s)$ $(i=1,2)$ is written as

$$r_i(s) = \text{Ev } z_i(s) = \tfrac{1}{2}[z_i(s) + z_i(-s)] = h_i(s)h_i(-s) \qquad (3.1)$$

where the factorization is to be performed so that $h_i(s)$ and $h_i^{-1}(-s)$ are analytic in the open RHS.† Let

$$\underset{\sim}{H}(s) = \text{diag } [h_1(s), h_2(s)] \qquad (3.2)$$

The augmentation admittance matrix $\underset{\sim}{Y}_a(s)$ is defined as

$$\underset{\sim}{Y}_a(s) = [Y_{ija}(s)] = [\underset{\sim}{Z}(s) + \underset{\sim}{z}(s)]^{-1} \qquad i,j=1,2 \qquad (3.3)$$

where $\underset{\sim}{Z}(s)$ is the open-circuit impedance matrix of the two-port network N and $\underset{\sim}{z}(s)$ is the reference impedance matrix

$$\underset{\sim}{z}(s) = \text{diag } [z_1(s), z_2(s)] \qquad (3.4)$$

Similarly, define

$$R_{22}(s) = \text{Ev } Z_{22}(s) = \tfrac{1}{2}[Z_{22}(s) + Z_{22}(-s)] = M_{22}(s)M_{22}(-s) \qquad (3.5)$$

where $M_{22}(s)$ and $M_{22}^{-1}(s)$ are analytic in the open RHS.

Denote by $A_i(s)$ $(i=1,2)$ the real regular all-pass function defined by the open RHS poles of $z_i(-s)$ and by $B_i(s)$ $(i=1,2)$ the real regular all-pass function defined by the open RHS zeros of $r_i(s)$. Thus,

$$\frac{h_i(s)}{h_i(-s)} = A_i(s)B_i(s) \qquad i=1,2 \qquad (3.6)$$

---

† RHS and LHS denote the right half and the left half of the s-plane, respectively.

Define

$$F_i(s) = 2\, r_i(s) A_i(s) \tag{3.7}$$

For a given impedance $z_i(s)$, a closed RHS zero s_{0i} of multiplicity k_i of the function $r_i(s)/z_i(s)$ is known as a <u>zero of transmission of order</u> k_i. The zeros of transmission $s_{0i} = \sigma_{0i} + j\omega_{0i}$ are divided into the following four mutual exclusive classes:

Class I: zero if Re $s_{0i} > 0$

Class II: zero if $s_{0i} = j\omega_{0i}$ and $z_i(j\omega_{0i}) = 0$

Class III: zero if $s_{0i} = j\omega_{0i}$ and $0 < |z_i(j\omega_{0i})| < \infty$

Class IV: zero if $s_{0i} = j\omega_{0i}$ and $|z_i(j\omega_{0i})| = \infty$

In this study, use will be made of the results derived by Youla [58] and Chan and Kuh [6] which are referred to as Theorem A-1 and Theorem A-2 (see Section 3.5). They are used to solve the matching problem between a resistive source and a passive and/or active load. By imposing various necessary constraints on the reflection coefficient $\rho_2(s)$, a set of sufficient conditions is obtained to guarantee that the back-end input impedance $Z_{22}(s)$ is a positive-real function. Thus it is realizable by a lossless two-port network terminating in a resistor.

The solution to a two-port matching problem with an active load can be described as follows: When a passive source impedance $z_1(s)$, an active load impedance $z_2(s)$, and a preassigned transducer power-gain characteristic $G(\omega^2)$ are given, the problem of impedance compatibility arises. $G(\omega^2)$ is expressed in terms of the reflection coefficient $\rho_2(j\omega)$ by

$$G(\omega^2) = 1 - |\rho_2(j\omega)|^2 \tag{3.8}$$

By applying Chan and Kuh's coefficient constraints, a realizable output

reflection coefficient $\rho_2(s)$ can be ascertained, from which the driving-point impedance $Z_{22}(s)$ looking into the output port is determined as

$$Z_{22}(s) = \frac{z_2(s) + z_2(-s)}{1 - \rho_2(s)} - z_2(s) \tag{3.9}$$

which is guaranteed to be positive real, while $Z_{22}(s) + z_2(s) \neq 0$ for Re $s \geq 0$.

Two impedances Z_{22} and z_1 are said to be <u>compatible</u> if $Z_{22}(s)$ can be realized as the input impedance of a lossless two-port network terminated in $z_1(s)$ as shown in Fig. 3.1. Set

$$\phi_1(s) = \frac{1 - Z_{22}(-s)}{1 + Z_{22}(s)} \cdot \frac{M_{22}(s)}{M_{22}(-s)} B_1(s)\theta(s) \tag{3.10}$$

$\phi_1(s)$ can be identified as the normalized reflection coefficient at the input port when the active load is replaced by a 1-ohm resistor. Denote by $Z_{10}(s)$ the driving-point impedance at the input port **with** 1-ohm termination at the output port. If and only if a real regular all-pass function $\theta(s)$ exists such that $\phi_1(s)$ is bounded real and satisfies Youla's coefficient constraints, $Z_{22}(s)$ is compatible with $z_1(s)$. As shown by Youla [58], the impedance $Z_{10}(s)$ as computed from

$$Z_{10}(s) = \frac{F_1(s)}{A_1(s) - \phi_1(s)} - z_1(s) \tag{3.11}$$

is positive real and is therefore realizable as the input impedance of a lossless two-port terminating in a 1-ohm resistor. The removal of the 1-ohm resistor yields the desired two-port network. Note that in certain situations, $Z_{10}(s)$ must be augmented [33] to ensure that the back-end impedance facing the 1-ohm resistor is $Z_{22}(s)$ as obtained in (3.9).

3.3 Main result

In this section, we consider the design of the nonreciprocal amplifier configuration of Fig. 3.2, where an ideal three-port circulator is used for isolation and the matching two-port network N may be either reciprocal or nonreciprocal but lossless.

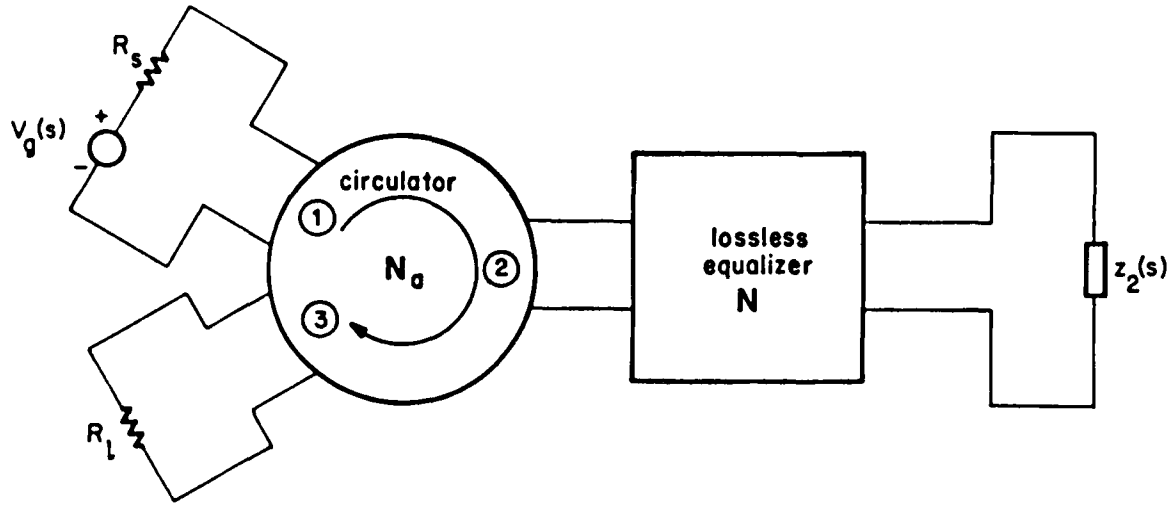

Fig. 3.2. Schematic of a Nonreciprocal Amplifier Configuration.

Theorem 3.1: Let $z_1(s)$ be a rational, non-Foster, positive-real function, and $z_2(s)$ be a rational, non-Foster, real function. Let $\rho_2(s)$ and $\phi_1(s)$ be real rational functions of s. Then the functions defined by the relations

$$Z_{22}(s) = \frac{z_2(s) + z_2(-s)}{1 - \rho_2(s)} - z_2(s) \tag{3.12}$$

$$Z_{10}(s) = \frac{F_1(s)}{A_1(s) - \phi_1(s)} - z_1(s) \tag{3.13}$$

are positive-real, $Z_{22}(s) + z_2(s) \neq 0$ for Re $s \geq 0$ except degenerate cases, and $Z_{10}(s)$ can be realized as a lossless reciprocal or nonreciprocal two-port network terminating in a 1-ohm resistor, the

output impedance $Z''_{22}(s)$ of which with input terminating in $z_1(s)$ can be made equal to $Z_{22}(s)$ if and only if the following conditions are satisfied:

(i) $\rho_2(s)$ satisfies the conditions of Theorem A-2.

(ii) There exists a real regular all-pass function $\theta(s)$ such that

$$\phi_1(s) = \frac{1 - Z_{22}(-s)}{1 + Z_{22}(s)} \cdot \frac{M_{22}(s)}{M_{22}(-s)} B_1(s)\theta(s) \tag{3.14}$$

is bounded real satisfying the coefficient constraints of Theorem A-1 where $M_{22}(s)M_{22}(-s) = \frac{1}{2}[Z_{22}(s) + Z_{22}(-s)]$ with $M_{22}(s)$ and $M_{22}^{-1}(s)$ being analytic in the open RHS and $A_1(s)$, $B_1(s)$ and $F_1(s)$ are specified by $z_1(s)$ as in (3.6) and (3.7) with $h_1(s)$ and $r_1(s)$ as defined in (3.1).

Proof. Necessity. Considered first is the general situation where network N may be reciprocal or nonreciprocal. Since condition (i) follows directly from Theorem A-2, we show that if the network N of Fig. 3.1 is the desired two-port lossless network, condition (ii) is true. Let

$$\underset{\sim}{S}(s) = [S_{ij}(s)] \qquad i,j = 1,2 \tag{3.15}$$

be the scattering matrix of N normalizing to the reference impedance matrix

$$\underset{\sim}{z}(s) = \text{diag } [z_1(s), 1] \tag{3.16}$$

The effect due to the nonreciprocity can be regarded as a phase shift represented by the multiplication by a real regular all-pass function. This gives

$$S_{21}(s) = S_{12}(s)\theta_{21}(s) \tag{3.17}$$

where $\theta_{21}(s)$ is a real regular all-pass function. Based on the para-

unitary property, we obtain the most general scattering matrix representation as

$$\underset{\sim}{S}(s) = \begin{bmatrix} \dfrac{1 - Z_{22}(-s)}{1 + Z_{22}(s)} \cdot \dfrac{M_{22}(s)}{M_{22}(-s)}\, \theta_{12}^2(s)\theta_{21}(s) & \dfrac{2M_{22}(s)}{Z_{22}(s) + 1}\, \theta_{12}(s) \\ \dfrac{2M_{22}(s)}{Z_{22}(s) + 1}\, \theta_{12}(s)\theta_{21}(s) & \dfrac{Z_{22}(s) - 1}{Z_{22}(s) + 1} \end{bmatrix} \tag{3.18}$$

where $\theta_{12}(s)$ is another real regular all-pass function.

The admittance matrix $\underset{\sim}{Y}_a(s)$ of the augmented two-port network N_a of Fig. 3.3 can be obtained by

$$\underset{\sim}{Y}_a(s) = \tfrac{1}{2}\underset{\sim}{H}^{-1}(s)[\underset{\sim}{H}(s)\underset{\sim}{H}^{-1}(-s) - \underset{\sim}{S}(s)]\, \underset{\sim}{H}^{-1}(s) = [Y_{ija}] \tag{3.19}$$

giving

$$Y_{12_a}(s) = -\frac{M_{22}(s)}{h_1(s)} \cdot \frac{\theta_{12}(s)}{Z_{22}(s) + 1} \tag{3.20}$$

For $Y_{12a}(s)$ to be analytic in the open RHS, $\theta_{12}(s)$ must contain all the open RHS zeros of $h_1(s)$ to at least the same order. Note that in the open RHS, zeros of $h_1(s)$, $r_1(s)$, and $B_1(s)$ coincide. Thus,

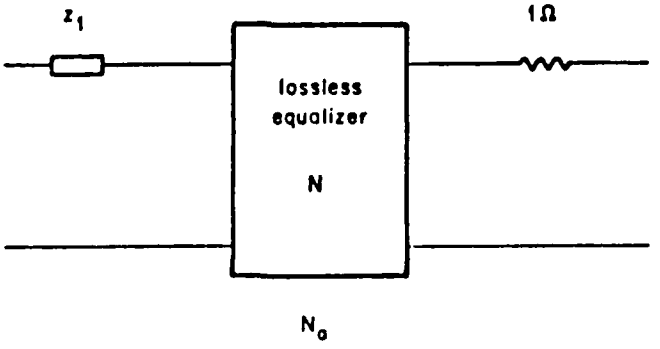

Fig. 3.3. The Augmented Two-Port Network N_a.

it is necessary that

$$\theta_{12}(s) = B_1(s)\theta_0(s) \tag{3.21}$$

where $\theta_0(s)$ is a real regular all-pass function.

The complex normalized reflection coefficient $S_{11}(s)$ is expressed as

$$S_{11}(s) = B_1(s)\theta_1(s) \tag{3.22}$$

where

$$\theta_1(s) = \frac{1 - Z_{22}(-s)}{1 + Z_{22}(s)} \cdot \frac{M_{22}(s)}{M_{22}(-s)} B_1(s)\theta_0^2(s)\theta_{21}(s) \tag{3.23}$$

is a bounded-real function satisfying Youla's coefficient constraints. Comparing (3.23) with (3.14), we can choose

$$\theta(s) = \theta_0^2(s)\theta_{21}(s) \tag{3.24}$$

If N is reciprocal, $\underset{\sim}{S}(s)$ is symmetric,

$$S_{12}(s) = S_{21}(s) \tag{3.25}$$

or

$$\theta_{21}(s) = 1 \tag{3.26}$$

Therefore, $\theta(s)$ is the perfect square of a real regular all-pass function, namely,

$$\theta(s) = \theta_0^2(s) \tag{3.27}$$

Sufficiency. Assume that conditions (i) and (ii) are satisfied. We show that a desired two-port network N exists. By Theorem A-1 and Theorem A-2, $Z_{10}(s)$ and $Z_{22}(s)$ as specified in (3.13) and (3.12), respectively, are positive-real functions. From these impedances, a two-port network is constructed as shown in Fig. 3.4. Our task is

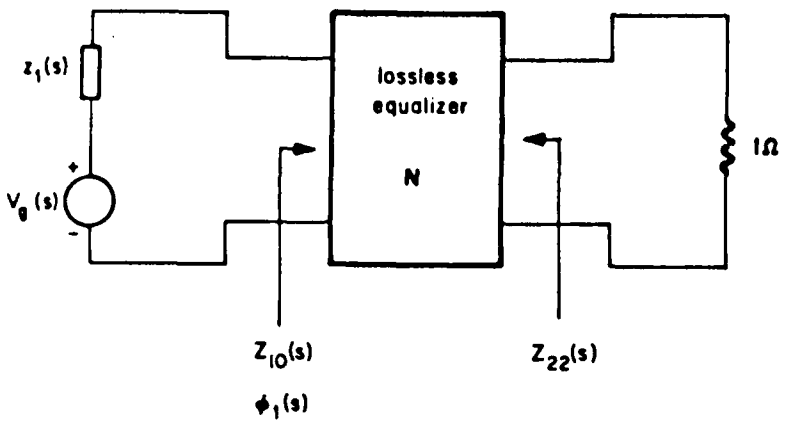

Fig. 3.4. Lossless Two-Port Network N Terminated in a Passive Source Impedance $z_1(s)$ and a 1-ohm Load.

to show that the output impedance $Z''_{22}(s)$ of this two-port network facing the 1-ohm resistor can be made equal to $Z_{22}(s)$ by augmenting $Z_{10}(s)$. For the constructed network, the scattering matrix is expressed as in

$$S_{11}(s) = \frac{h_1(s)}{h_1(-s)} \cdot \frac{Z_{10}(s) - z_1(-s)}{Z_{10}(s) + z_1(s)} \tag{3.28}$$

Based on the para-unitary property, $S_{21}(s)$ is obtained as

$$S_{21}(s) = \frac{2M_{10}(s)m_{11}(s)}{Z_{10}(s) + z_1(s)} \theta_{21}(s) \tag{3.29}$$

where

$$r_1(s) = Ev\ z_1(s) = m_{11}(s)m_{11}(-s) \tag{3.30}$$

with $m_{11}(s)$ and $m_{11}^{-1}(s)$ being analytic in the open RHS and θ_{21} is a real regular all-pass function. Under this factorization, all the LHS poles and zeros of $r_1(s)$ are contained in $m_{11}(s)$. In a similar way,

$$R_{10}(s) = \text{Ev}\ Z_{10}(s) = M_{10}(s)M_{10}(-s) \tag{3.31}$$

where $M_{10}(s)$ and $M_{10}^{-1}(s)$ are analytic in the open RHS.

$S_{12}(s)$ can be expressed as

$$S_{12}(s) = S_{21}(s)\theta_{12}(s) \tag{3.32}$$

where $\theta_{12}(s)$ is a real regular all-pass function. Then

$$S_{22}(s) = -\frac{M_{10}(s)}{M_{10}(-s)} \cdot \frac{h_1(-s)}{h_1(s)} \cdot \frac{m_{11}(s)}{m_{11}(-s)} \cdot \frac{Z_{10}(-s) - z_1(s)}{Z_{10}(s) + z_1(s)}\, \theta_{12}(s)\theta_{21}^2(s) \tag{3.33}$$

From (3.11), we obtain

$$R_{10}(s) = \frac{r_1(s)[1 - \phi_1(s)\phi_1(-s)]}{[A_1(s) - \phi_1(s)][A_1(-s) - \phi_1(-s)]} \tag{3.34}$$

Combining (3.31) and (3.34), $M_{10}(s)$ can be factored as

$$M_{10}(s) = \frac{h_1(s)}{A_1(s) - \phi_1(s)}\, \phi_{12m}(s) \tag{3.35}$$

where $\phi_{12m}(s)$ is the minimum-phase factorization of $1 - \phi_1(s)\phi_1(-s)$, because all its poles and zeros are in the open LHS. From (3.14), it is found to be

$$\phi_{12m}(s) = \frac{2M_{22}(s)}{1 + Z_{22}(s)} \tag{3.36}$$

Again, using formula (3.19) yields the (2,1)-element of $\underset{\sim}{Y}_a(s)$ as

$$Y_{21a}(s) = -\frac{M_{10}(s)m_{11}(s)}{h_1(s)[Z_{10}(s) + z_1(s)]}\, \theta_{12}(s)\theta_{21}(s) \tag{3.37}$$

Notice that $M_{10}(s)$ and $Z_{10}(s) + z_1(s)$ have no poles and zeros in the open RHS and

$$\frac{m_{11}(s)}{h_1(s)} = \frac{1}{B_1(s)} \tag{3.38}$$

Set

$$\theta_{12}(s)\theta_{21}(s) = B_1(s) \tag{3.39}$$

Combining (3.6), (3.7), (3.14), (3.35), (3.36) and (3.33) yields

$$S_{22}(s) = \frac{Z_{22}(s) - 1}{Z_{22}(s) + 1}\, \theta(-s)\theta_{12}(-s) \tag{3.40}$$

Denote a Hurwitz Polynomial P(s) by

$$P(s) = \prod_i (s + \beta_{0i}) \tag{3.41}$$

where β_{oi} is an arbitrary complex number and Re $\beta_{0i} > 0$. According to Ho and Balabanian [33], if $Z_{11}(s)$ is augmented by multiplying both the numerator and denominator by a predetermined auxiliary polynomial P(s), the resulting normalized reflection coefficient $S''_{22}(s)$ at the output port will be the product of $S_{22}(s)$ and $\alpha(s)$ where

$$\alpha(s) = \frac{P(-s)}{P(s)} \tag{3.42}$$

is a regular all-pass function.

In this case, $\alpha(s)$ can be chosen as

$$\alpha(s) = \theta(s)\theta_{12}(s) \tag{3.43}$$

Thus, a new realization can be obtained by augmenting $Z_{10}(s)$, and (3.40) becomes

$$S''_{22}(s) = \frac{Z_{22}(s) - 1}{Z_{22}(s) + 1} \tag{3.44}$$

which is the output reflection coefficient when the two-port network is terminated in a 1-ohm resistor.

In general, we can write

$$\theta(s) = \frac{g(-s)}{g(s)} \tag{3.45}$$

where

$$g(s) = \prod_i (s + \zeta_i)^{m_i} \tag{3.46}$$

and ζ_i is an arbitrary complex number, Re $\zeta_i > 0$, and m_i is a positive integer. If there exists an all-pass function $\theta(s)$ with all m_i being even, then either a reciprocal or nonreciprocal two-port network can be realized. If the all-pass function $\theta(s)$ can only be found with at least one odd m_i, the realization is nonreciprocal.

Corollary 3.1: In Theorem 3.1, if $\theta(s)$ can be found to be a real regular all-pass function, each of its factors being of even order, the lossless two-port network can be realized by either a reciprocal or nonreciprocal network; and if not, the realization can only be nonreciprocal.

3.4 Illustrative example

It is desired to design an optimum nonreciprocal negative-resistance amplifier of Fig. 3.2 with the tunnel diode as the load impedance, and to achieve the third-order, low-pass Chebyshev transducer power-gain characteristic having a 3-dB passband ripple $\varepsilon = 0.99763$ with cutoff frequency $\omega_c > 0.15$ rad/s, where $R_1 = 20\ \Omega$,

L_1 = 0.8 H, r = 0.5 Ω, L_t = 0.024 H, R = 1 Ω, and C = 1 F are shown in Fig. 3.5.

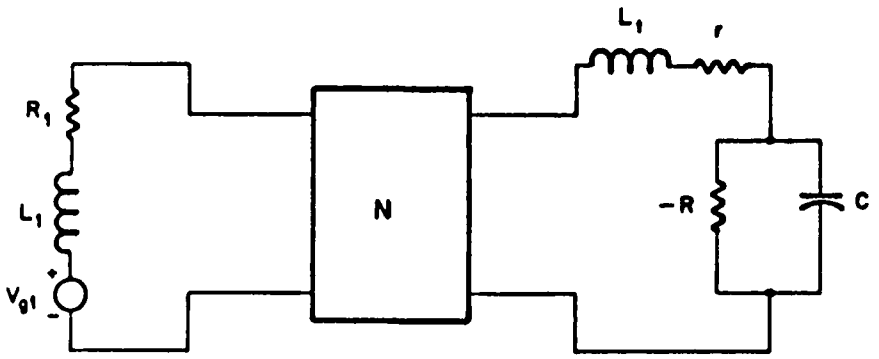

Fig. 3.5. A Lossless Two-Port Network to be Designed.

For the source impedance, we have

$$z_1(s) = 20 + 0.8s \tag{3.47}$$

$$r_1(s) = 20 \tag{3.48}$$

$$\frac{r_1(s)}{z_1(s)} = \frac{20}{20 + 0.8s} \tag{3.49}$$

$$A_1(s) = B_1(s) = 1 \tag{3.50}$$

$$F_1(s) = 2r_1(s)A_1(s) = 40 \tag{3.51}$$

For the load impedance, we have

$$z_2(s) = 0.5 + 0.024s + \frac{1}{s - 1} \tag{3.52}$$

$$r_2(s) = \frac{1}{2} \cdot \frac{s^2 + 1}{s^2 - 1} \tag{3.53}$$

Denote the zeros of $z_2(s) + z_2(-s)$ by

$$s_r = \pm j\omega_r = \pm j \tag{3.54}$$

giving

$$\omega_r = 1 \tag{3.55}$$

The third-order Chebyshev polynomial is

$$f(s) = 4s^3 - 3s \tag{3.56}$$

According to Chan and Kuh [6], we can choose the reflection coefficient $\rho_2(s)$ having the form

$$|\rho_2(j\omega)|^2 = 1 + \frac{K^2 - 1}{1 + \dfrac{\varepsilon^2\left[4\left(\dfrac{\omega}{\omega_c}\right)^3 - 3\left(\dfrac{\omega}{\omega_c}\right)\right]^2}{1 - \left(\dfrac{\omega}{\omega_r}\right)^2}} \tag{3.57}$$

$$= \frac{16\varepsilon^2\left(\dfrac{\omega}{\omega_c}\right)^6 - 24\varepsilon^2\left(\dfrac{\omega}{\omega_c}\right)^4 + \left[9\varepsilon^2 - \left(\dfrac{K\omega_c}{\omega_r}\right)^2\right]\left(\dfrac{\omega}{\omega_c}\right)^2 + K^2}{16\varepsilon^2\left(\dfrac{\omega}{\omega_c}\right)^6 - 24\varepsilon^2\left(\dfrac{\omega}{\omega_c}\right)^4 + \left[9\varepsilon^2 - \left(\dfrac{\omega_c}{\omega_c}\right)^2\right]\left(\dfrac{\omega}{\omega_c}\right)^2 + 1} \tag{3.58}$$

Write

$$\rho_2(s) = \rho_0(s)\rho_1(s)\rho_\lambda(s) \tag{3.59}$$

where $\rho_0(s)$ is the minimum-phase solution of (3.58) and $\rho_1(s)$ and $\rho_\lambda(s)$ are real regular all-pass functions. Set

$$\rho_0(s) = \frac{a_3\left(\dfrac{s}{\omega_c}\right)^3 + a_2\left(\dfrac{s}{\omega_c}\right)^2 + a_1\left(\dfrac{s}{\omega_c}\right) + a_0}{b_3\left(\dfrac{s}{\omega_c}\right)^3 + b_2\left(\dfrac{s}{\omega_c}\right)^2 + b_1\left(\dfrac{s}{\omega_c}\right) + b_0} \tag{3.60}$$

Thus,

$$|\rho_0(j\omega)|^2 = \frac{a_3^2\left(\dfrac{\omega}{\omega_c}\right)^6 + \left(a_2^2 - 2a_1a_3\right)\left(\dfrac{\omega}{\omega_c}\right)^4 + \left(a_1^2 - 2a_0a_1\right)\left(\dfrac{\omega}{\omega_c}\right)^2 + a_0^2}{b_3^2\left(\dfrac{\omega}{\omega_c}\right)^6 + \left(b_2^2 - 2b_1b_3\right)\left(\dfrac{\omega}{\omega_c}\right)^4 + \left(b_1^2 - 2b_0b_1\right)\left(\dfrac{\omega}{\omega_c}\right)^2 + b_0^2} \tag{3.61}$$

Equating (3.58) and (3.61) yields $a_3 = 1$ and

$$a_2^2 - 2a_1a_3 = -1.5 \tag{3.62a}$$

$$a_2^2 - 2a_0a_2 = 0.5625 - \frac{K^2\omega_c^2}{16\varepsilon^2\omega_r^2} \tag{3.62b}$$

$$a_0^2 = \frac{K^2}{16\varepsilon^2} \tag{3.62c}$$

$$b_3 = 1 \tag{3.63a}$$

$$b_2^2 - 2b_1b_3 = -1.5 \tag{3.63b}$$

$$b_1^2 - 2b_0b_2 = 0.5625 - \frac{\omega_c^2}{16\varepsilon^2\omega_r^2} \tag{3.63c}$$

$$b_0^2 = \frac{1}{16\varepsilon^2} \tag{3.63d}$$

Since $z_2(s)$ has a pole of order one at $s_p = 1$ where $z_2(-s)$ is regular, and a pole at the infinity, the following two conditions must be satisfied [6]:

$$\rho_2(1) = 0 \tag{3.64}$$

and

$$\rho_2(s)\Big|_{s\to\infty} = 1 - \frac{2r}{L_t}\ \frac{1}{s} + \ldots \tag{3.65}$$

In (3.59), $\rho_1(s)$ is specified by $s_p = 1$

$$\rho_1(s) = \frac{s-1}{s+1} \tag{3.66}$$

and ρ_λ can be written as

$$\rho_\lambda(s) = \prod_{i=1}^{2} \frac{s - \lambda_i}{s + \lambda_i} \tag{3.67}$$

where $\lambda_i = \sigma_i$ $(i = 1,2)$ are positive real numbers. Applying (3.62) and (3.63) yields

$$a_3 = b_3 = 1 \tag{3.68}$$

$$a_2 = \frac{1}{\omega_c}\left(2 + 2\sum_i \lambda_i - \frac{2r}{L_t}\right) + b_2 \tag{3.69}$$

In addition, the theoretical limitation on bandwidth derived in [6] gives the formula

$$\frac{\tanh^{-1}\left(\frac{\omega_c}{\omega_r}\right)}{\frac{\omega_c}{\omega_r}} = \frac{\omega_r \tan^{-1}\frac{1}{\omega_r} + \sum_i \frac{1}{2}\tan^{-1}\frac{2\omega_i\sigma_i}{\omega_r^2 - |\lambda_i|^2}}{1 - \frac{r}{L_t} + \sum_i \lambda_i} \tag{3.70}$$

or

$$\frac{1}{2\frac{\omega_c}{\omega_r}} \ln \frac{1 + \frac{\omega_c}{\omega_r}}{1 - \frac{\omega_c}{\omega_r}} = \frac{\omega_r \tan^{-1}\frac{1}{\omega_r} + \sum_i \frac{1}{2}\tan^{-1}\frac{2\omega_i\sigma_i}{\omega_r^2 - |\lambda_i|^2}}{1 - \frac{r}{L_t} + \sum_i \lambda_i} \tag{3.71}$$

which determines the range of ω_c.

In order to solve for a_j, b_j $(j = 0,1,2,3)$, ω_c, and λ_i $(i = 1,2)$, estimation for $\sum_i \lambda_i$ is made and iterative calculation is applied to (3.59), (3.62), (3.63), (3.64), (3.68) and (3.69). The results are

$$a_0 = 32.080112 \qquad b_0 = 0.250594$$

$$a_1 = 18.773009 \qquad b_1 = 0.926340$$

$$a_2 = 6.003834 \qquad b_2 = 0.593867 \tag{3.72}$$
$$a_3 = 1 \qquad b_3 = 1$$

$$\lambda_1 = 5.05490 \qquad \lambda_2 = 15.26533 \qquad \omega_c = 0.18 \tag{3.73}$$

Thus,

$$\rho_2(s) = \frac{171.467764s^3 + 185.303508s^2 + 104.294496s + 32.080112}{171.467764s^3 + 18.329228s^2 + 5.146328s + 0.250594} \cdot$$

$$\frac{s - 1}{s + 1} \cdot \frac{s - 5.05490}{s + 5.05490} \cdot \frac{s - 15.26533}{s + 15.26533} \tag{3.74}$$

$$1 - \rho_2(s) = \frac{7144.490118s^3 + 4242.349924s^2 + 5342.034112s + 2494.761901}{171.467764s^5 + 3502.593694s^3 + 13608.86314s^3 + 1519.199733s^2}$$

$$\frac{}{+ 402.207455s + 19.337015} \cdot \frac{s^2 + 1}{s + 1} \tag{3.75}$$

From (3.12), $Z_{22}(s)$ is obtained as

$$Z_{22}(s) = \frac{7889.050698s^2 + 4684.392941s + 1228.07667}{7144.490118s^3 + 4242.349924s^2 + 5342.034122s + 2494.760901} \tag{3.76}$$

Factoring $R_{22}(s)$ as indicated in (3.5) yields

$$\frac{M_{22}(s)}{M_{22}(-s)} = \frac{-7144.490118s^3 + 4242.349924s^2 - 5342.034122s + 2494.761901}{7144.490118s^3 + 4242.349924s^2 + 5342.034122s + 2494.761901}$$

$$\cdot \frac{7889.050698s^2 + 4684.392941s + 1228.07667}{7889.050698s^2 - 4684.392941s + 1228.07667} \tag{3.77}$$

As indicated in (3.5), $M_{22}(s)$ is identified so that $M_{22}(s)$ and

$M_{22}(-s)$ are analytic in the RHS. That this is always possible follows from $R_{22}(s)$. The problem essentially reduces to the existence of an all-pass function $\theta(s)$ having a certain property. Since $M_{22}(-s)$ contains all the RHS poles of $Z_{22}(-s)$ to the same order, $\theta(s)$ must be of the form

$$\theta(s) = \frac{7889.050698s^2 - 4684.392941s + 1228.07667}{7889.050698s^2 + 4684.392941s + 1228.07667}\,\theta_0(s) \qquad (3.78)$$

Furthermore, since the impedance $z_1(s)$ has a class IV zero of transmission of order 1 at $s=\infty$, the coefficient constraints [58] are

$$A_{01} = \phi_{01} \qquad (3.79)$$

and

$$\frac{F_{01}}{A_{11} - \phi_{11}} \geq a_{-1} \qquad (3.80)$$

where

$$Y_{ij}(s) = \sum_{x=0}^{\infty} Y_{xj}\left(\frac{1}{s}\right)^x \qquad (3.81)$$

denotes either $A_j(s)$, $\phi_j(s)$, or $F_j(s)$, and Y_{xj} $(j=1,2)$ are the coefficients of the Laurent series expansions of $Y_j(s)$ at $s=\infty$, and $a_{-1} = L_1 = 0.8$.

Obviously, we must have

$$\theta_0(s) = -1 \qquad (3.82)$$

Substituting (3.50), (3.76), and (3.77) into (3.14) yields

$$\phi_1(s) = \frac{7144.490118s^3 + 3646.700774s^2 + 657.641181s - 1226.685231}{7144.490118s^3 + 12131.40062s^2 + 10026.42706s + 3722.838571} \qquad (3.83)$$

Thus, from (3.13)

$$Z_{10}(s) = \frac{278991.8448s^3 + 308066.9992s^2 + 209721.7458s + 49923.0688}{8484.6998s^2 + 9368.7859s + 4949.5238} \qquad (3.84)$$

$Z_{10}(s)$ is realized as a lossless two-port network terminated in a 1-ohm resistor. When the 1-ohm resistor is replaced by the tunnel diode load, the complete network is shown in Fig. 3.6.

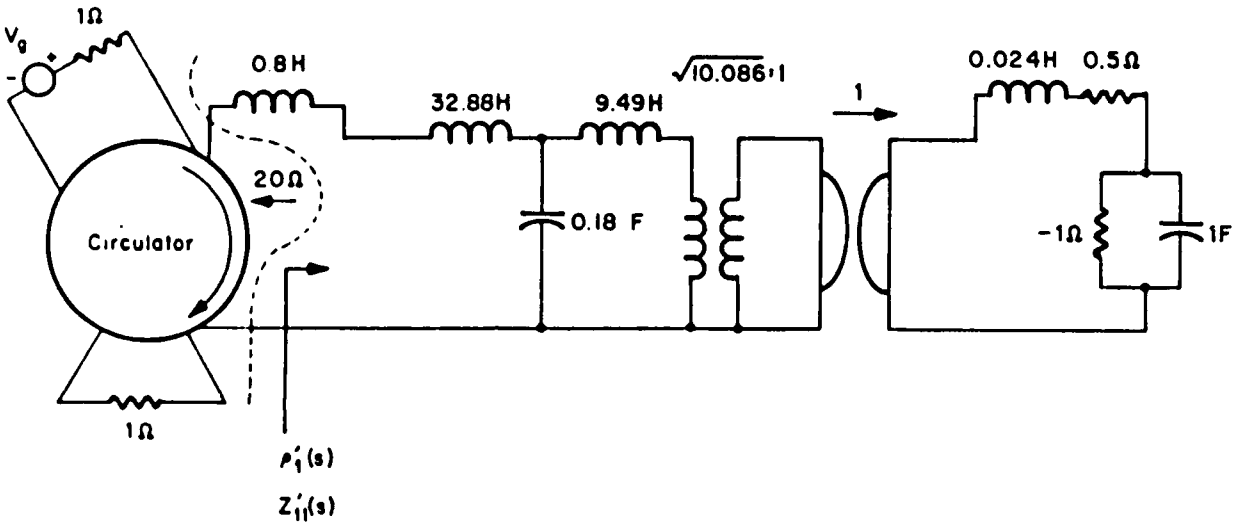

Fig. 3.6. An Example of a Complete Nonreciprocal Negative-Resistance Amplifier.

To verify the above design, the impedance $Z_{11}'(s)$ is calculated as

$$Z_{11}'(s) = \frac{1.386s^5 + 27.4838s^4 + 91.26886s^3 - 40.8118s^2 + 31.6722s - 10.086}{0.04115s^4 + 0.8160s^3 + 2.7029s^2 - 1.3458s + 0.5} \qquad (3.85$$

We find the reflection coefficient $\rho_1'(s)$ at the input to be

$$\rho_1'(s) = \frac{Z_{11}'(s) - 20}{Z_{11}'(s) + 20}$$

$$= \frac{1386s^5 + 26.6608s^4 + 74.9490s^3 - 94.8708s^2 + 58.5882s - 20.086}{1.386s^5 + 28.3068s^4 + 107.5882s^3 + 13.2472s^2 + 4.7562s - 0.086} \qquad (3.86)$$

giving

$$G(\omega^2) = |\rho_1'(j\omega)|^2 \tag{3.87}$$

The frequency response of the transducer power gain is plotted in Fig. 3.7.

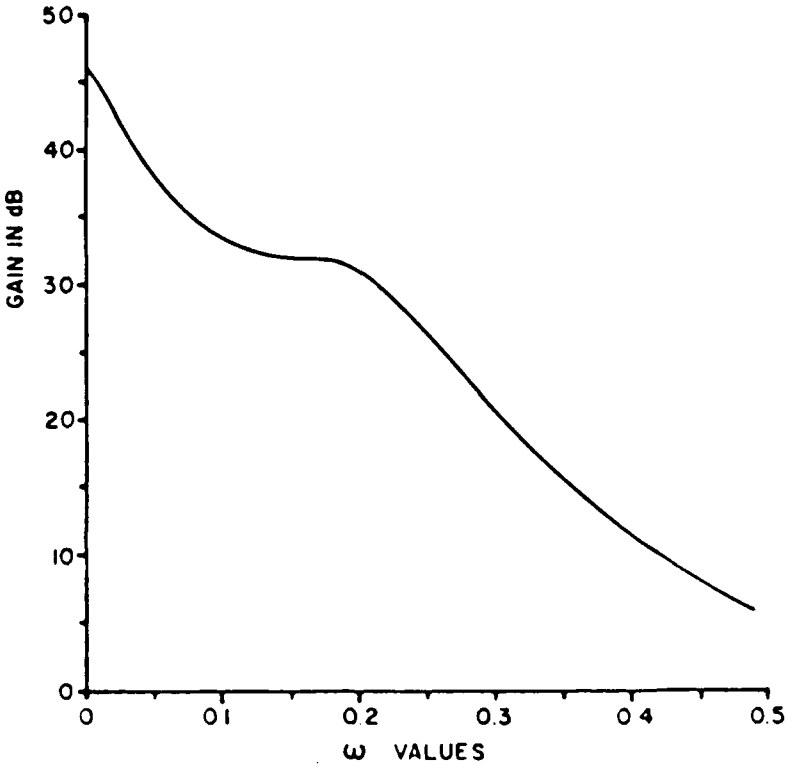

Fig. 3.7. Frequency Response of the Designed Amplifier.

3.5 Coefficient constraints

I. Youla's coefficient constraints

Youla's coefficient constraints $\rho_i(s)$ are stated in terms of the coefficients of the Laurent series expansions about the zeros of transmission $s_{0i} = \sigma_{0i} + j\omega_{0i}$ of the following functions:

$$A_i(s) = \sum_{x=0}^{\infty} A_{xi}(s-s_{0i})^x \tag{3.88}$$

$$F_i(s) = \sum_{x=0}^{\infty} F_{xi}(s-s_{0i})^x \tag{3.89}$$

$$\rho_i(s) = \sum_{x=0}^{\infty} \rho_{xi}(s-s_{0i})^x \tag{3.90}$$

Basic coefficient constraints on $\rho_i(s)$. For each zero of transmission s_{oi} of order k_i of $z_i(s)$, one of the following four sets of coefficient conditions must be satisfied, depending on the classification of s_{oi}:

(i) Class I: $A_{xi} = \rho_{xi}$ for $x = 0, 1, 2, \ldots, k_i-1$.

(ii) Class II: $A_{xi} = \rho_{xi}$ for $x = 0, 1, 2, \ldots, k_i-1$ and $(A_{k_i i} - \rho_{k_i i})/F_{(k_i+1)i} \geq 0$.

(iii) Class III: $A_{xi} = \rho_{xi}$ for $x = 0, 1, 2, \ldots, k_i-2$ and $[A_{(k_i-1)i} - \rho_{(k_i-1)i}]/F_{k_i i} \geq 0$.

(iv) Class IV: $A_{xi} = \rho_{xi}$ for $x = 0, 1, 2, \ldots, k_i-1$ and $F_{(k_i-1)i}/(A_{k_i i} - \rho_{k_i i}) \geq a_{-1i}$, the residue of $z_i(s)$ at the pole $j\omega_{oi}$.

The importance of these constraints is summarized in the following theorem first given by Youla [58].

Theorem A-1: Let $z_1(s)$ be a prescribed, rational, non-Foster positive-real function and $\rho_i(s)$ a real, rational function of the complex variable s. Then the function defined by

$$Z_{ii}(s) = \frac{F_i(s)}{A_i(s) - \rho_i(s)} - z_i(s), \qquad i = 1,2 \tag{3.91}$$

is positive real if and only if $\rho_i(s)$ is a bounded-real reflection coefficient satisfying the basic coefficient constraints.

II. Chan and Kuh's coefficient constraints

Refer again to the network of Fig. 3.1, where $z_2(s)$ is a given lumped, non-Foster impedance which can be either passive or active. The output current-basis complex reflection coefficient is defined by the relation

$$\rho_2(s) = \frac{Z_{22}(s) - z_2(-s)}{Z_{22}(s) + z_2(s)} \tag{3.92}$$

which yields

$$\rho_2(s) = \frac{Z_{22}(s)/z_2(s) - z_2(-s)/z_2(s)}{1 + Z_{22}(s)/z_2(s)} \tag{3.93}$$

$$1-\rho_2(s) = \frac{z_2(s) + z_2(-s)}{Z_{22}(s) + z_2(s)} = \frac{1 + z_2(-s)/z_2(s)}{1 + Z_{22}(s)/z_2(s)} \tag{3.94}$$

$$Z_{22}(s) = \frac{z_2(s) + z_2(-s)}{1 - \rho_2(s)} - z_2(s) = z_2(s)\,\frac{z_2(-s)/z_2(s) + \rho_2(s)}{1 - \rho_2(s)} \tag{3.95}$$

Denote by s_r a zero of $r_2(s)$ and by s_p a closed RHS poles of $z_2(s)$ and $z_2(-s)$. The Laurent series expansions around s_i (s_p, s_r, or other frequencies) for the following functions are given as follows:

$$z_2(s) = a_{-m}(s-s_i)^{-m} + a_{-m+1}(s-s_i)^{-m+1} + \ldots + a_0 + a_1(s-s_i) + \ldots \tag{3.96a}$$

$$z_2(-s) = b_{-n}(s-s_i)^{-n} + b_{-n+1}(s-s_i)^{-n+1} + \ldots + b_0 + b_1(s-s_i) + \ldots \tag{3.96b}$$

$$-z_2(-s)/z_2(s) = c_{-k}(s-s_i)^{-k} + c_{-k+1}(s-s_i)^{-k+1} + \ldots + c_0 + c_1(s-s_i) + \ldots \tag{3.96c}$$

$$\rho_2(s) = d_{-e}(s-s_i)^{-e} + d_{-e+1}(s-s_i)^{-e+1} + ... + d_0 + d_1(s-s_i) + ... \quad (3.96d)$$

$$Z_{22}(s) = k_{-1}(s-j\omega_i)^{-1} + k_0 + ... \quad (3.96e)$$

where k_{-1} is real and positive.

Theorem A-2: Let $z_2(s)$ be a given rational impedance which may be either active or passive but non-Foster. Then the impedance

$$Z_{22}(s) = \frac{z_2(s) + z_2(-s)}{1 - \rho_2(s)} - z_2(s) \quad (3.97)$$

is a positive-real function and $Z_{22}(s) + z_2(s) \neq 0$ for Re $s \geq 0$ except for degenerate cases, where $Z_{22}(s)$ and $z_2(s)$ have common poles and common zeros on the real-frequency axis, if and only if $\rho_2(s)$ satisfies the following conditions:

1a) In the open RHS, $1-\rho_2(s)$ is not zero except at a zero s_r of order $r_2(s)$, or possible at poles of $z_2(s)$. The latter case will be included in condition 3. In the former case,

$$d_0 = c_0 = 1 \quad (3.98a)$$

and if $z_2(s_r)$ is regular,

$$d_i = 0, \quad i = 1,2, ..., r-1 \quad (3.98b)$$

If s_r is a pole of $z_2(s)$ of order m,

$$d_i = c_i = 0, \quad i = 1,2, ..., r+m-1 \quad (3.98c)$$

and

$$d_i = c_i, \quad i = r+m, r+m+1, ..., r+2m-1 \quad (3.98d)$$

1b) On the real-frequency axis, the function $1-\rho_2(s)$ may have a first-order zero at $j\omega_0$, which is neither a zero of $r_2(s)$

nor a pole of $z_2(s)$. Then

$$\rho_2(s) = 1 + d_1(s - j\omega_0) + \ldots . \tag{3.99a}$$

where d_1 is real and

$$\text{Re } [z_2(j\omega_0)]/(-d_1) > 0 \tag{3.99b}$$

At a zero $j\omega_r$ of $r_2(s)$ of order r, $d_0 = c_0 = 1$; and if $z_2(j\omega_r)$ is regular,

$$d_i = 0, \quad i = 1,2, \ldots, r-1 \tag{3.100a}$$

and

$$d_{r-1} \neq 0 \quad \text{(if degenerate)} \tag{3.100b}$$

If $j\omega_r$ is a pole of $z_2(s)$ of order m, then

$$d_i = c_i = 0, \quad i = 1,2, \ldots, r+m-1 \tag{3.100c}$$

and

$$d_i = c_i, \quad i = r+m, r+m+1, \ldots, r+2m-1 \tag{3.100d}$$

and

$$d_{r+2m-1} = c_{r+2m-1} - k_{-1}c_{r+m}/a_{-m} \quad \text{(if degenerate)} \tag{3.100e}$$

where k_{-1} is real and positive.

2) On the real-frequency axis, $|\rho_2(j\omega)|$ satisfies the following:

$$|\rho_2(j\omega)| > 1, \quad \text{if Re } z_2(j\omega) < 0 \tag{3.101a}$$

$$|\rho_2(j\omega)| \leq 1, \quad \text{if Re } z_2(j\omega) \geq 0 \tag{3.101b}$$

3a) In the open RHS, $\rho_2(s)$ is analytic except at s_p, which is a pole of $z_2(s)$ of order m and is a pole of $z_2(-s)$ of order n, $(m,n \leq 0)$. Then

$$d_i = 0, \qquad i < m-n-1 \tag{3.102a}$$

and

$$d_i = c_i, \qquad i = m-n, m-n+1, \ldots, w \tag{3.102b}$$

where

$w = 2m-n+q$, if $m \geq n$, $c_{m-n} = 1$ and $c_{m-n+i} = 0$, $i = 1,2,\ldots,q$

$= m-n$, if $m \geq n$

$= 2m-n-1$, if $m < n$

and if $m = 0$, the second equation is not needed.

3b) On the real-frequency axis, $\rho_2(s)$ is analytic. At $s = j\omega_p$, which is a pole of $z_2(s)$ and $z_2(-s)$ of order m, and if $m > 1$, then

$$d_i = c_i, \qquad i = 0,1, \ldots, m-1 \tag{3.103a}$$

and

$$d_{m-1} = c_{m-1} + k_{-1}(1 - c_0)/a_{-m} \quad \text{(if degenerate)} \tag{3.103b}$$

and if $c_0 = 1$ and $c_i = 0$ for $i = 1, 2, \ldots, q$, then

$$d_i = c_i, \qquad i = 0, 1, 2, \ldots, m+q-1 \tag{3.103c}$$

$$d_{m+q} = c_{m+q} - k_{-1}c_{1+q}/a_{-m} \tag{3.103d}$$

and if $m = 1$, then

$$d_0 = (c_0 a_{-1} + k_{-1})/(a_{-1} + k_{-1}) \quad \text{(degenerate)} \tag{3.103e}$$

and if $m = 1$ and $c_0 = 1$, then $d_0 = c_0 = 1$ and

$$d_1 = a_{-1}c_1/(a_{-1} + k_{-1}) \tag{3.103f}$$

where k_{-1} is real and positive.

3.6 Conclusion

Microwave two-terminal semiconductor devices such as avalanche diodes, transferred electron devices, tunnel diodes, and varators are commonly found in modern communications systems. A circulator is usually used to separate the incoming signal from the amplified signal in constructing a reflection-type amplifier. For the tunnel diode amplifier problem, the lossless two-port network is expected to provide a complete mismatch.

In this study, the two-port broadband matching problem has been generalized to include both reciprocal and nonreciprocal networks with an active load. The realization heavily depends on the existence of a real regular all-pass function. If the all-pass function of even order does not exist, the two-port lossless network can only be realized by a nonreciprocal network.

Section 4

REALIZABILITY OF COMPATIBLE IMPEDANCES USING TRANSFORMERLESS LADDER TWO-PORT NETWORKS

4.1 Introduction

Section 4 studies the compatibility of two non-Foster positive-real impedances using a lossless reciprocal transformerless ladder two-port network. Illustrative examples are presented.

Two impedances are said to be compatible if one of them can be realized as the input impedance of a two-port coupling network terminated in the other. The problem was first discussed by Schoeffler [47] and then by Wohlers [54] and Ho and Balabanian [33]. Schoeffler [47] and Wohlers [54] studied the problem of compatibility between two passive impedances, the coupling network of which is assumed to be reciprocal. Ho and Balabanian [33] extended their results to the case where the impedances may be either passive or active and the coupling networks may be reciprocal or nonreciprocal. Recently Chen and Satyanarayana [19,46] simplified Wohlers' theorem and extended the compatibility problem to double impedance matching by studying the compatibility between the source and load impedances and, finally, Chen and Tsai [20] extended the problem to active load.

In this section we study the compatibility of two non-Foster positive-real impedances using a transformerless lossless ladder two-port network except the ideal transformers which are included in the discussion. As is well known, the ladder structure is important in that it is simple, easy to realize and has low sensitivity. The ladder networks have been studied for several decades by many authors [23,31,49]. In 1979, Fialkow [28] published his work on LC-R ladders without using transformers. He gave a sufficient

condition on the compatibility of two non-Foster positive-real impedances using a lossless ladder coupling two-port network (Theorem 13 in reference 28). For the present problem Fialkow, however, did not elaborate any further.

In the present report, we apply the broadband matching theory to this problem by studying the compatbility of two positive-real impedances using only transformerless LC ladder two-port networks. Illustrative examples are given.

4.2 Realizability of LC-R ladders

For an LC-R ladder we express its driving-point immittance as the ratio of two polynomials of their even and odd parts:

$$X(s) = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} \tag{4.1}$$

The rational immittance function X(s) is assumed to be of order n. Let

$$E(s^2) = m_1(s)m_2(s) - n_1(s)n_2(s) \tag{4.2}$$

The transfer polynomials are the polynomials defined by

$$T(s) = \sqrt{E(s^2)} = T'(s) \qquad \text{case A} \tag{4.3a}$$

$$T(s) = \sqrt{-E(s^2)} = sT'(s) \qquad \text{case B} \tag{4.3b}$$

Call the zeros of T(s) the transfer zeros. T(s) is considered to contain m finite zeros and (n-m) zeros at infinity [28]. Write

$$m_1(s) = m_1'(s^2) \qquad n_1(s) = sn_1'(s^2) \tag{4.4a}$$

$$m_2(s) = m_2'(s^2) \qquad n_2(s) = sn_2'(s^2) \tag{4.4b}$$

Fialkow [28] gave the following theorem:

Theorem 4.1. Let X(s) of (4.1) be the driving-point immittance of an LC-R ladder. Then each of the following polynomials in $q = s^2$,

$$T'(q), \quad m_1'(q) - \sqrt{R}\, T'(q), \quad \sqrt{R}\, m_2'(q) - T'(q) \qquad \text{case A} \qquad (4.5a)$$

$$T'(q), \quad n_1'(q) - \sqrt{R}\, T'(q), \quad \sqrt{R}\, n_2'(q) - T'(q) \qquad \text{case B} \qquad (4.5b)$$

has nonnegative coefficients.

4.3 The compatible impedances

We study the network of Fig. 4.1. Write

$$Z_1(s) = \frac{m_{11}(s) + n_{11}(s)}{m_{21}(s) + n_{21}(s)} \qquad (4.6)$$

$$Z_2(s) = \frac{m_{12}(s) + n_{12}(s)}{m_{22}(s) + n_{22}(s)} \qquad (4.7)$$

where m_{1i}, m_{2i} and n_{1i}, n_{2i} are even and odd parts of the numerator and denominator polynomials of $Z_i(s)$ $(i = 1,2)$. They are assumed to be relatively prime. Write

$$z_\ell(s) = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} \qquad (4.8)$$

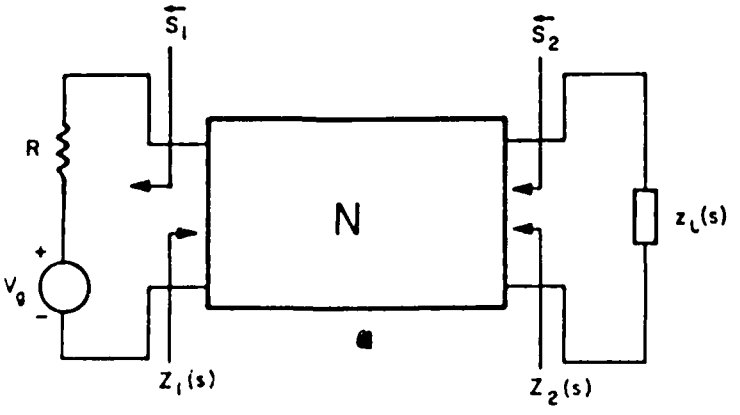

Fig. 4.1. Schematic for Studying the Compatible Impedances.

Then

$$E_1(s) = m_{11}(s)m_{21}(s) - n_{11}(s)n_{21}(s)$$

$$= E_2(s)E_\ell(s) \tag{4.9}$$

where

$$E_2(s) = m_{12}(s)m_{22}(s) - n_{12}(s)n_{22}(s) \tag{4.10}$$

$$E_\ell(s) = m_1(s)m_2(s) - n_1(s)n_2(s) \tag{4.11}$$

If we augment $Z_1(s)$ by the factor $(E+\emptyset)$ where E and $\emptyset$ are even and odd polynomials, then after augmentation

$$E_1(s) = [m_{11}(s)m_{21}(s) - n_{11}(s)n_{21}(s)][E^2 - \emptyset^2] \tag{4.12}$$

Since m_{i1} and n_{i1} $(i=1,2)$ are relatively prime, none of the zeros of $(E^2 - \emptyset^2]$ are purely imaginary [28]. Since $Z_2(s)$ is the driving-point impedance of an LC-R ladder one-port, its transfer zeros lie on the real-frequency axis. $Z_2(s)$ in (4.7) is unique. The augmentation of $Z_1(s)$ can only balance the transfer zeros of $z_\ell(s)$ in the open RHS for the network. If the transfer zeros of $Z_1(s)$ contain those of $z_\ell(s)$ in the open RHS, no augmentation is needed.

Theorem 4.2. Let $Z_1(s)$ and $z_\ell(s)$ be preassigned rational, non-Foster positive-real impedances as shown in (4.6) and (4.8). Let $Z_1(s)$, being an input impedance, be compatible with the terminated impedance $z_\ell(s)$ by an LC ladder two-port. Then

(1) The transfer zeros of $Z_1(s)$ contain those of $z_\ell(s)$

(2) $T'(q), \quad m'_{12}(q) - \sqrt{R}\, T'(q), \quad \sqrt{R}\, m'_{22}(q) - T'(q)$ case A (4.13a)

$T'(q), \quad n'_{12}(q) - \sqrt{R}\, T'(q), \quad \sqrt{R}\, n'_{22}(q) - T'(q)$ case B (4.13b)

have nonnegative coefficients where $q = s^2$, R is a positive constant, and

$$m'_{22} = \left. \frac{\begin{vmatrix} m_{11} & n_2 \\ n_{11} & m_2 \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}} \right|_{q=s^2}, \qquad n'_{12} = \left. \frac{\begin{vmatrix} m_1 & m_{11} \\ n_1 & n_{11} \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}} s^{-1} \right|_{q=s^2}$$

case A (4.14a)

$$m'_{12} = \left. \frac{\begin{vmatrix} m_{21} & n_1 \\ n_{21} & m_1 \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} \right|_{q=s^2}, \qquad n'_{22} = \left. \frac{\begin{vmatrix} m_2 & m_{21} \\ n_2 & n_{21} \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} s^{-1} \right|_{q=s^2}$$

and

$$m'_{22} = \left. \frac{\begin{vmatrix} m_{21} & n_2 \\ n_{21} & m_2 \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}} \right|_{q=s^2}, \qquad n'_{12} = \left. \frac{\begin{vmatrix} m_1 & m_{21} \\ n_1 & n_{21} \end{vmatrix}}{\begin{vmatrix} m_1 & n_1 \\ n_1 & m_2 \end{vmatrix}} s^{-1} \right|_{q=s^2}$$

case B (4.14b)

$$m'_{12} \left. \frac{\begin{vmatrix} m_{11} & n_1 \\ n_{11} & m_1 \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} \right|_{q=s^2}, \qquad n'_{22} = \left. \frac{\begin{vmatrix} m_2 & m_{11} \\ n_2 & n_{11} \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} s^{-1} \right|_{q=s^2}$$

$$T'(q) = [(m'_{22}m'_{12} - s^2 n'_{12}n'_{22})^{\frac{1}{2}}]_{q=s^2} \qquad \text{case A} \qquad (4.15a)$$

$$T'(q) = [s^{-1}(n'_{22}n'_{12} - s^2 m'_{12}m'_{22})^{\frac{1}{2}}]_{q=s^2} \qquad \text{case B} \qquad (4.15b)$$

An outline of a proof of Theorem 4.2 is given in Section 4.5.

4.4 Design procedure and illustrative examples

Given non-Foster positive-real impedances $Z_1(s)$ and $z_\ell(s)$, find a lossless, reciprocal transformerless ladder two-port coupling network so that when it is terminated in $z_\ell(s)$, the driving-point impedance is $Z_1(s)$.

We proceed the design with the following steps.

Step 1. Denote $Z_1(s)$ and $z_\ell(s)$ as in (4.6) and (4.8). From (4.10) find $E_2(s)$. If it is a perfect square, case A applies. If $-E_2(s)$ is a perfect square, case B applies.

Step 2. From (4.14a) and (4.14b) find m'_{i2} and n'_{i2} $(i=1,2)$ satisfying the conditions of Theorem 4.2, and find the constant R.

Step 3. From

$$m_{12}(s) = m'_{12}(s) \qquad n_{12}(s) = s\,n'_{12}(s) \qquad (4.16a)$$

$$m_{22}(s) = m'_{22}(s) \qquad n_{22}(s) = s\,n'_{22}(s) \qquad (4.16b)$$

find

$$Z_2(s) = \frac{m_{12}(s) + n_{12}(s)}{m_{22}(s) + n_{22}(s)} \qquad (4.17)$$

Step 4. Using $Z_2(s)$ as the driving-point impedance, synthesize a ladder two-port terminated in R. If $R \neq 1$, an ideal transformer is needed whose turns ratio is equal to $\sqrt{R}$.

We illustrate the above procedure by the following examples.

Example 4.1. Given

$$Z_1(s) = \frac{8s^2 + 12s + 7}{3(2s+3)} \tag{4.18}$$

and

$$z_\ell(s) = \frac{4}{s + 4} \tag{4.19}$$

find a lossless, reciprocal, transformerless ladder coupling two-port so that when $z_\ell(s)$ is terminated at one port, the driving-point impedance at another port is $Z_1(s)$.

First we compute

$$E_1(s) = (8s^2 + 7)9 - 72s^2 = 63 \tag{4.20a}$$

$$E_\ell(s) = 16 \tag{4.20b}$$

Thus, case A applies. From (4.18) and (4.19)

$$m_{11} = 8s^2 + 7 \qquad n_{11} = 12s \tag{4.21a}$$

$$m_{21} = 9 \qquad n_{21} = 6s \tag{4.21b}$$

$$m_1 = 4 \qquad n_1 = 0 \tag{4.22a}$$

$$m_2 = 4 \qquad n_2 = s \tag{4.22b}$$

Substituting (4.21) and (4.22) in (4.14a) yields

$$m'_{22} = \left. \frac{\begin{vmatrix} 8s^2+7 & s \\ 12s & 4 \end{vmatrix}}{\begin{vmatrix} 4 & s \\ 0 & 4 \end{vmatrix}} \right|_{q=s^2} = \frac{20q + 28}{16} \qquad (4.23a)$$

$$n'_{12} = \left. \frac{\begin{vmatrix} 4 & 8s^2+7 \\ 0 & 12s \end{vmatrix}}{\begin{vmatrix} 4 & s \\ 0 & 4 \end{vmatrix}} s^{-1} \right|_{q=s^2} = \frac{48}{16} \qquad (4.23b)$$

$$m'_{12} = \left. \frac{\begin{vmatrix} 9 & 0 \\ 6s & 4 \end{vmatrix}}{\begin{vmatrix} 4 & 0 \\ s & 4 \end{vmatrix}} \right|_{q=s^2} = \frac{36}{16} \qquad (4.23c)$$

$$n'_{22} = \left. \frac{\begin{vmatrix} 4 & 9 \\ s & 6s \end{vmatrix}}{\begin{vmatrix} 4 & 0 \\ s & 4 \end{vmatrix}} s^{-1} \right|_{q=s^2} = \frac{15}{16} \qquad (4.23d)$$

$$T'(q) = \left[\frac{20s^2 + 28}{16}\ \frac{36}{16} - s^2\ \frac{48}{16}\ \frac{15}{16}\right]^{\frac{1}{2}}_{q=s^2}$$

$$= \frac{3\sqrt{7}}{4} \tag{4.24}$$

From (4.13a) when

$$\frac{3}{\sqrt{7}} \geq \sqrt{R} \geq \frac{3}{\sqrt{7}}\ , \qquad \text{or} \qquad \sqrt{R} = \frac{3}{\sqrt{7}} \tag{4.25}$$

the conditions of Theorem 4.2 are satisfied. Substituting (4.23) in (4.17) in conjunction with (4.16) gives

$$Z_2(s) = \frac{48s + 36}{20s^2 + 15s + 28} \tag{4.26}$$

$Z_2(s)$ in (4.26) can be realized as an LC ladder two-port as shown in Fig. 4.2, where an ideal transformer is used to change the impedance level.

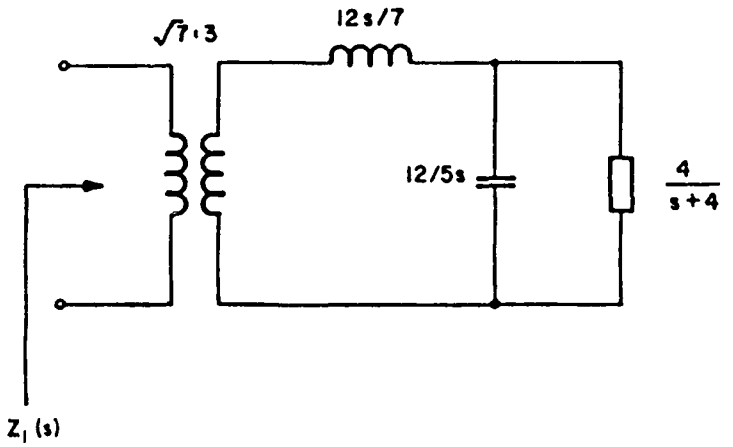

Fig. 4.3. A Ladder Realization of $Z_2(s)$.

Example 4.2. Given

$$Z_1(s) = \frac{8s^3 + 24s^2 + 12s + 16}{2s^4 + 6s^3 + 5s^2 + 8s} \tag{4.27}$$

and

$$z_\ell(s) = \frac{s + 0.5}{s + 2} \tag{4.28}$$

find an LC ladder coupling two-port so that $Z_1(s)$ is compatible with $z_\ell(s)$.

First compute

$$E_1(s^2) = (24s^2 + 16)(2s^4 + 5s^2) - (8s^3 + 12s)(6s^3 + 8s)$$

$$= 16s^2(s^2 - 1) \tag{4.29}$$

$$E_\ell(s^2) = 1 - s^2 \tag{4.30}$$

Thus, case B applies.

From (4.27) and (4.28), we obtain

$$m_{11} = 24s^2 + 16 \qquad n_{11} = 8s^3 + 12s \tag{4.31a}$$

$$m_{21} = 2s^4 + 5s^2 \qquad n_{21} = 6s^3 + 8s \tag{4.31b}$$

$$m_1 = 0.5 \qquad n_1 = s \tag{4.32a}$$

$$m_2 = 2 \qquad n_2 = s \tag{4.32b}$$

Substituting (4.31) and (4.32) in (4.14b) yields

$$m'_{22} = \frac{\begin{vmatrix} 2s^4+5s^2 & s \\ 6s^3+8s & 2 \end{vmatrix}}{1 - s^2}\Bigg|_{q=s^2} = \frac{-2s^4 + 2s^2}{1 - s^2} = 2q \qquad (4.33a)$$

$$m'_{12} = \frac{\begin{vmatrix} 24s^2+ 16 & s \\ 8s^3+12s & 0.5 \end{vmatrix}}{1 - s^2}\Bigg|_{q=s^2} = 8(1+q) \qquad (4.33b)$$

$$n'_{12} = \frac{\begin{vmatrix} 0.5 & 2s^4+5s^2 \\ s & 6s^3+8s \end{vmatrix}}{1 - s^2} s^{-1}\Bigg|_{q=s^2} = 2q + 4 \qquad (4.33c)$$

$$n'_{22} = \frac{\begin{vmatrix} 2 & 24s^2+16 \\ s & 8s^3+12s \end{vmatrix}}{1 - s^2} s^{-1}\Bigg|_{q=s^2} = 8 \qquad (4.33d)$$

From (4.13b) when

$$\frac{1}{4} \leq R \leq 1 \qquad (4.34)$$

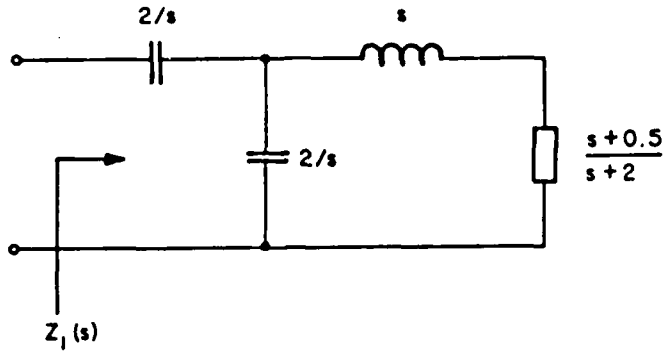

Fig. 4.3. A Ladder Realization of $Z_2(s)$ with R = 1 Ω.

the conditions of Theorem 4.2 are satisfied. Choosing R = 1 Ω, we obtain from (4.16) and (4.17)

$$Z_2(s) = \frac{s^3 + 4s^2 + 2s + 4}{s^2 + 4s} \tag{4.35}$$

Synthesizing $Z_2(s)$ yields the network shown in Fig. 4.3. If we choose R = ¼ Ω, we can obtain a coupling network of Fig. 4.4, where an ideal transformer is needed.

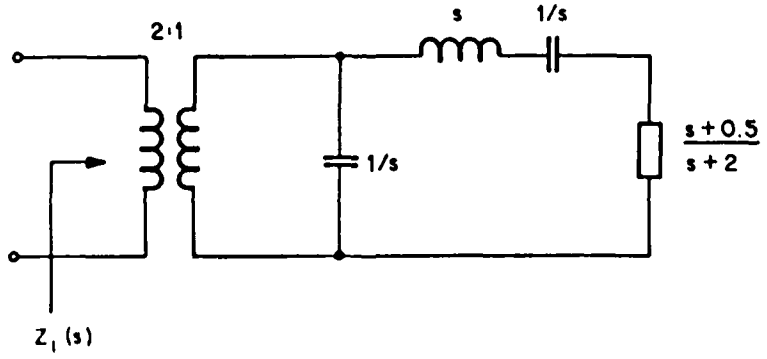

Fig. 4.4. A Ladder Realization of $Z_2(s)$ with R = ¼ Ω.

4.5 An outline of a proof of Theorem 4.2

For the network of Fig. 4.1, $Z_2(s)$ is the driving-point impedance of a lossless, reciprocal, transformerless ladder two-port network

terminated in a 1-ohm resistor at port 1:

$$Z_2 = \frac{z_{11}z_{22} - z_{12}^2 + z_{22}}{1 - z_{11}} \tag{4.36}$$

where z_{ij} $(i,j=1,2)$ are the open-circuit impedance parameters of N. Denote

$$z_{ij} = \frac{P_{ij}}{Q} \qquad (i,j = 1,2) \tag{4.37}$$

where P_{ij} $(i,j=1,2)$ and Q are relatively prime. Since z_{ij} are odd, Q is odd or even and P_{ij} are even or odd polynomials of s. Write

$$Z_2 = \frac{m_{12} + n_{12}}{m_{22} + n_{22}} \tag{4.38}$$

where m_{i2} and n_{i2} $(i=1,2)$ are even and odd polynomials, and are relatively prime. Let

$$E = m_{12}m_{22} - n_{12}n_{22} \tag{4.39}$$

Case A. Q in (4.37) is odd.

$$z_{12} = \frac{\sqrt{E}}{n_{22}} \tag{4.40}$$

$$T = \sqrt{E} \tag{4.41}$$

For R = 1 Ω, we obtain

$$\frac{Z_1(s) - 1}{Z_1(s) + 1} = \frac{z_\ell(s) - Z_2(-s)}{z_\ell(s) + Z_2(s)} \cdot \frac{m_{22} - n_{22}}{m_{22} + n_{22}} \tag{4.42}$$

Denote

$$Z_1 = \frac{m_{11} + n_{11}}{m_{21} + n_{21}} \tag{4.43}$$

From (4.42) and (4.43), we have

$$\frac{Z_1(s) - 1}{Z_1(s) + 1} = \frac{\dfrac{z_\ell m_{22} + n_{12}}{m_{12} + z_\ell n_{22}} - 1}{\dfrac{z_\ell m_{22} + n_{12}}{m_{12} + z_\ell n_{22}} + 1} \tag{4.44}$$

Since equation (4.44) holds for all s, it follows that

$$\frac{m_{11} + n_{11}}{m_{21} + n_{21}} = \frac{z_\ell m_{22} + n_{12}}{m_{12} + z_\ell n_{22}} \tag{4.45}$$

Write

$$z_\ell = \frac{m_1 + n_1}{m_2 + n_2} \tag{4.46}$$

where m_i and n_i $(i = 1,2)$ are even and odd polynomials. From (4.45) and (4.46) we have

$$m_{11} = m_{22}m_1 + n_{12}n_2 \tag{4.47a}$$

$$n_{11} = m_{22}n_1 + n_{12}m_2 \tag{4.47b}$$

and

$$m_{21} = m_{12}m_2 + n_{22}n_1 \tag{4.48a}$$

$$n_{21} = m_{12}n_2 + n_{22}m_1 \tag{4.48b}$$

From (4.47) and (4.48) we can solve for

$$m_{22} = \frac{\begin{vmatrix} m_{11} & n_2 \\ n_{11} & m_2 \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}}, \qquad n_{12} = \frac{\begin{vmatrix} m_1 & m_{11} \\ n_1 & n_{11} \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}} \tag{4.49a}$$

$$m_{12} = \frac{\begin{vmatrix} m_{21} & n_1 \\ n_{21} & m_1 \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}}, \qquad n_{22} = \frac{\begin{vmatrix} m_2 & m_{21} \\ n_2 & n_{21} \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} \tag{4.49b}$$

Using Theorem 4.1 we obtain condition 2 of Theorem 4.2 for case A.

Case B. Q in (4.37) is even.

$$z_{12} = \frac{\sqrt{-E}}{m_{22}} \tag{4.50}$$

$$T = \sqrt{-E} \tag{4.51}$$

From (4.50), T is an odd polynomial. Thus we have

$$\frac{Z_1(s) - 1}{Z_1(s) + 1} = \frac{Z_2(-s) - z_\ell(s)}{Z_2(s) + z_\ell(s)} \cdot \frac{m_{22} - n_{22}}{m_{22} + n_{22}} \tag{4.52}$$

From (4.52)

$$\frac{m_{11} + n_{11}}{m_{21} + n_{21}} = \frac{m_{12} + n_{22} z_\ell}{n_{12} + m_{22} z_\ell} \tag{4.53}$$

we obtain

$$m_{11} = m_{12}m_2 + n_{22}n_1 \tag{4.54a}$$

$$n_{11} = m_{12}n_2 + n_{22}m_1 \tag{4.54b}$$

and

$$m_{21} = m_{22}m_1 + n_{12}n_2 \tag{4.55a}$$

$$n_{21} = m_{22}n_1 + n_{12}m_2 \tag{4.55b}$$

which can be solved to yield

$$m_{22} = \frac{\begin{vmatrix} m_{21} & n_2 \\ n_{21} & m_2 \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}}, \qquad n_{12} = \frac{\begin{vmatrix} m_1 & m_{21} \\ n_1 & n_{21} \end{vmatrix}}{\begin{vmatrix} m_1 & n_2 \\ n_1 & m_2 \end{vmatrix}} \tag{4.56a}$$

$$m_{12} = \frac{\begin{vmatrix} m_{11} & n_1 \\ n_{11} & m_1 \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & n_1 \end{vmatrix}}, \qquad n_{22} = \frac{\begin{vmatrix} m_2 & m_{11} \\ n_2 & n_{11} \end{vmatrix}}{\begin{vmatrix} m_2 & n_1 \\ n_2 & m_1 \end{vmatrix}} \tag{4.56b}$$

Applying Theorem 4.1 we obtain condition 2 of Theorem 4.2 for case B.

4.6 Conclusions

In this section we presented a theorem on the impedance compatibility of two non-Foster positive-real impedances where the coupling

network is an LC ladder two-port, thus solving a problem posed by Fialkow [28].

A procedure for finding an LC ladder coupling network between two compatible impedances and two illustrative examples were given.

Section 5

A NEW DIPLEXER CONFIGURATION COMPOSED OF A THREE-PORT CIRCULATOR AND TWO RECIPROCAL TWO-PORT NETWORKS

5.1 Introduction

Section 5 presents a new diplexer configuration composed of an ideal three-port circulator and two reciprocal lossless two-port networks. Since the circulator provides the needed isolation, it becomes much easier to achieve the desired insertion loss at the crossover frequency as well as the frequency shaping. We show that the diplexer with the canonical Butterworth response can always be realized. Only one of the two-port is needed if the required insertion loss at the crossover frequency is 3 dB. The design of a diplexer composed of a circulator and two canonical Butterworth networks having low-pass and high-pass characteristics is discussed. In this case, the problem is simplified to that of realizing two Butterworth networks.

The design of a diplexer separating a frequency spectrum into two channels of signals is one of the basic problems in communications. The most popular configuration of a diplexer consists of a low-pass two-port network and a high-pass two-port network connected either in series or in parallel, as shown by many workers [37,40,42,61].

Because of the mutual interaction effect of the two-ports, the transducer power-gain characteristic of a diplexer is different from those of the individual two-ports. This makes the design of a diplexer very complicated. Recently Wang and Chen [51] presented a design approach where the diplexer is composed of two canonical Butterworth networks. Their problem is simplified to that of choosing

the order of the Butterworth response and the cutoff frequencies of the component two-ports.

In this section we present a nonreciprocal diplexer configuration composed of an ideal three-port circulator and two reciprocal lossless two-port networks, terminating in the source and loads as shown in Fig. 5.1. Since the three-port circulator provides the needed isolation,

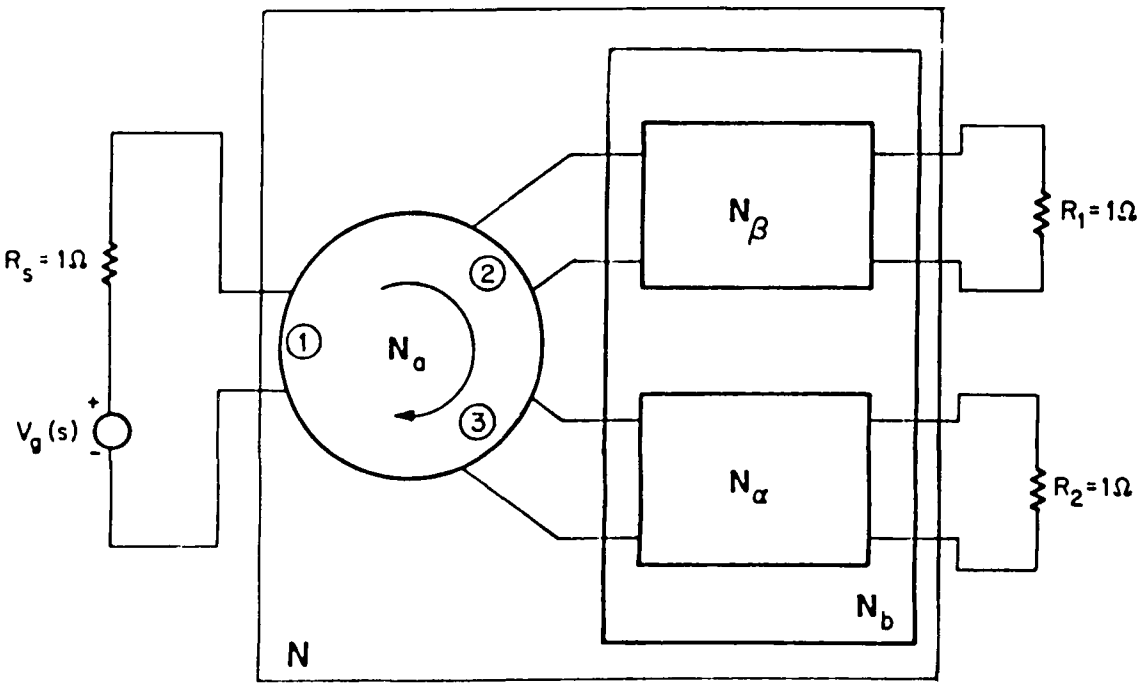

Fig. 5.1. A Diplexer N Composed of an Ideal Circulator and Two Reciprocal Lossless Two-Port Networks.

it is much easier to obtain a desired insertion loss at the crossover frequency as well as the frequency shaping. Having expressed the scattering parameters of the diplexer in terms of those of the component networks, we show that the diplexer with canonical Butterworth response can always be realized. The diplexer can be simplified to that shown in Fig. 5.2 if the required response is of Butterworth type with 3 dB insertion loss at the crossover frequency. In this case only

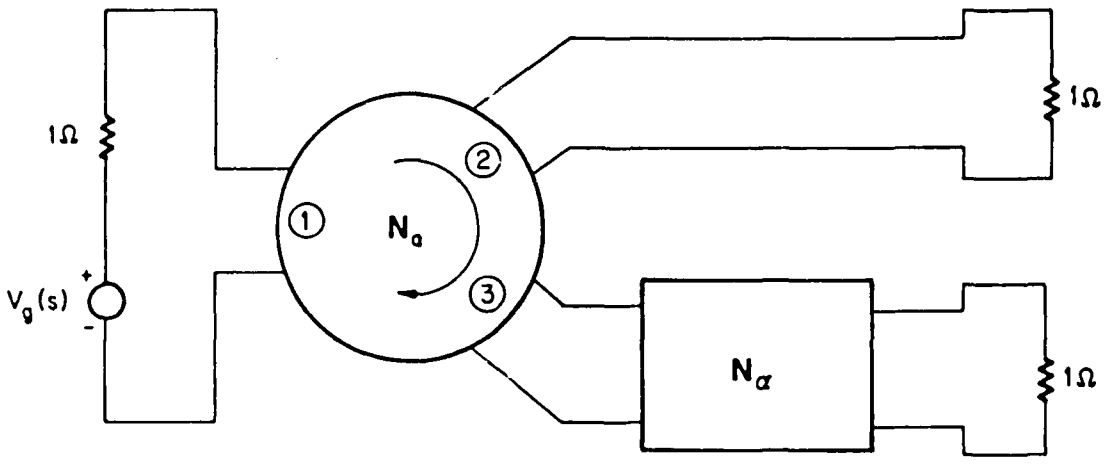

Fig. 5.2. The Diplexer Possesses the Butterworth Response with a 3 dB Insertion Loss at the Crossover Frequency.

one of the reciprocal two-port networks is needed.

Next we discuss the diplexer composed of a circulator and two canonical Butterworth networks having low-pass and high-pass characteristics. We show that the frequency response of the diplexer is only slightly different from the canonical Butterworth response. The problem reduces to that of choosing the parameters of the two Butterworth networks. Only one of the cutoff frequencies of the two ports needs to be adjusted to satisfy the specification of insertion loss at the edges of the pass bands. Computer programs for obtaining the final circuit configuration, the element values and the frequency response curves are available.

5.2 The scattering matrix of a nonreciprocal diplexer

The general configuration of a nonreciprocal diplexer shown in Fig. 5.1 can be viewed as an interconnection of a three-port circulator N_a and a reciprocal four-port network N_b, where N_b is formed by the two-port networks N_β and N_α. Without loss of generality we assume that

$R_1 = R_2 = R_s = 1\ \Omega$. The partitioned unit normalized scattering matrix $\underset{\sim}{S}_a(s)$ of the ideal circulator N_a can be expressed in the form [14]

$$\underset{\sim}{S}_a(s) = \begin{bmatrix} \underset{\sim}{S}_{11a} & \underset{\sim}{S}_{12a} \\ \underset{\sim}{S}_{21a} & \underset{\sim}{S}_{22a} \end{bmatrix} = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \end{array}\right] \tag{5.1}$$

Assume that the scattering matrices of the reciprocal two-port networks N_α and N_β normalizing to the 1-Ω resistance are given by

$$\underset{\sim}{S}_\alpha(s) = \begin{bmatrix} S_{11\alpha} & S_{12\alpha} \\ S_{21\alpha} & S_{22\alpha} \end{bmatrix} \tag{5.2}$$

$$\underset{\sim}{S}_\beta(s) = \begin{bmatrix} S_{11\beta} & S_{12\beta} \\ S_{21\beta} & S_{22\beta} \end{bmatrix} \tag{5.3}$$

The partitioned unit normalized scattering matrix of the four-port network N_b becomes

$$\underset{\sim}{S}_b(s) = \begin{bmatrix} \underset{\sim}{S}_{11b} & \underset{\sim}{S}_{12b} \\ \underset{\sim}{S}_{21b} & \underset{\sim}{S}_{22b} \end{bmatrix} = \left[\begin{array}{cc|cc} S_{11\beta} & 0 & S_{12\beta} & 0 \\ 0 & S_{11\alpha} & 0 & S_{12\beta} \\ \hline S_{21\beta} & 0 & S_{22\beta} & 0 \\ 0 & S_{21\alpha} & 0 & S_{22\alpha} \end{array}\right] \tag{5.4}$$

Let the partitioned scattering matrix of the composite three-port network N (the diplexer) normalizing to the 1-Ω resistance be

$$\underset{\sim}{S}(s) = \begin{bmatrix} \underset{\sim}{S}_{11} & \underset{\sim}{S}_{12} \\ \underset{\sim}{S}_{21} & \underset{\sim}{S}_{22} \end{bmatrix} = \left[\begin{array}{c|cc} S_{11} & S_{12} & S_{13} \\ \hline S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{array}\right] \tag{5.5}$$

where the subscripts 1 and 2 refer to the ports connected to the source and the loads, respectively. The interconnection formulas which relate the scattering matrix of the composite network N and those of the component networks N_a and N_b are given by [14]

$$\underset{\sim}{S}_{11}(s) = \underset{\sim}{S}_{11a} + \underset{\sim}{S}_{12a}(\underset{\sim}{U}_2 - \underset{\sim}{S}_{11b}\underset{\sim}{S}_{22a})^{-1}\underset{\sim}{S}_{11b}\underset{\sim}{S}_{21a} \tag{5.6}$$

$$\underset{\sim}{S}_{12}(s) = \underset{\sim}{S}_{12a}(\underset{\sim}{U}_2 - \underset{\sim}{S}_{11b}\underset{\sim}{S}_{22a})^{-1}\underset{\sim}{S}_{12b} \tag{5.7}$$

$$\underset{\sim}{S}_{21}(s) = \underset{\sim}{S}_{21b}(\underset{\sim}{U}_2 - \underset{\sim}{S}_{22a}\underset{\sim}{S}_{11b})^{-1}\underset{\sim}{S}_{21a} \tag{5.8}$$

$$\underset{\sim}{S}_{22}(s) = \underset{\sim}{S}_{22b} + \underset{\sim}{S}_{21b}(\underset{\sim}{U}_2 - \underset{\sim}{S}_{22a}\underset{\sim}{S}_{11b})^{-1}\underset{\sim}{S}_{22a}\underset{\sim}{S}_{12b} \tag{5.9}$$

where $\underset{\sim}{U}_2$ denotes the identity matrix of order 2. Substituting (5.1) and (5.4) in (5.8) yields

$$\underset{\sim}{S}_{21}(s) = \begin{bmatrix} S_{21\beta} & 0 \\ 0 & S_{21\alpha} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11\beta} & 0 \\ 0 & S_{11\alpha} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_{21\beta} & 0 \\ 0 & S_{21\alpha} \end{bmatrix} \begin{bmatrix} 1 & S_{11\alpha} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S_{11\alpha}\, S_{21\beta} \\ S_{21\alpha} \end{bmatrix} \tag{5.10}$$

As is well known, the jω-axis magnitude squared of the elements of $S_{21}(s)$ represent the transducer power gains from the source to the loads. Therefore, (5.10) can be rewritten as

$$\underset{\sim}{S}_{21}(s) = \begin{bmatrix} S_{21} \\ \\ S_{31} \end{bmatrix} = \begin{bmatrix} S_{11\alpha} & S_{21\beta} \\ \\ S_{21\alpha} & \end{bmatrix} \tag{5.11}$$

or

$$S_{21}(s) = S_{11\alpha}(s)S_{21\beta}(s) \tag{5.12}$$

$$S_{31}(s) = S_{21\alpha}(s) \tag{5.13}$$

The transducer power-gain characteristics of the diplexer become

$$G_{21}(\omega^2) = |S_{21}(j\omega)|^2 = |S_{11\alpha}(j\omega)|^2 \; |S_{21\beta}(j\omega)|^2 \tag{5.14}$$

$$G_{31}(\omega^2) = |S_{21\alpha}(j\omega)|^2 \tag{5.15}$$

We conclude from the above observations that the problem of designing a nonreciprocal diplexer to satisfy the specified transducer power-gain characteristics $G_{21}(\omega^2)$ and $G_{31}(\omega^2)$ reduces to that of designing two reciprocal lossless two-port networks N_α and N_β having the transducer power-gain characteristics:

$$|S_{21\alpha}(j\omega)|^2 = G_{31}(\omega^2) \tag{5.16}$$

and

$$|S_{21\beta}(j\omega)|^2 = \frac{G_{21}(\omega^2)}{|S_{11\alpha}(j\omega)|^2} \tag{5.17}$$

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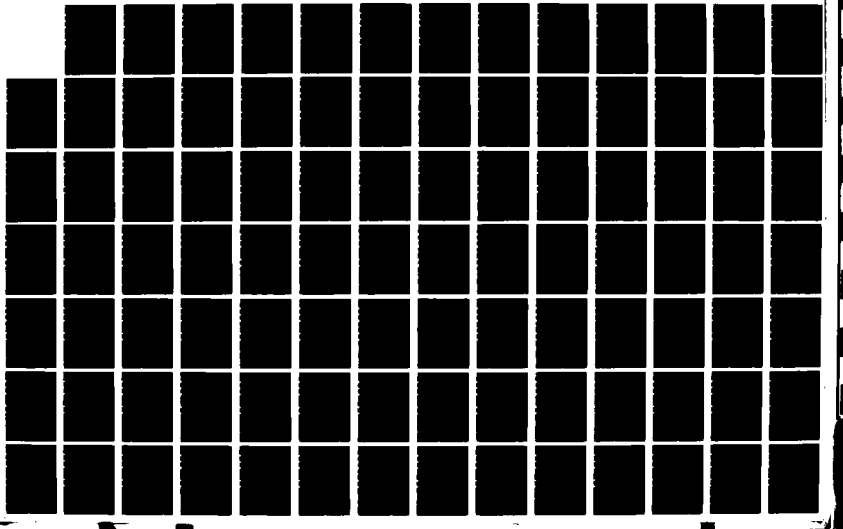

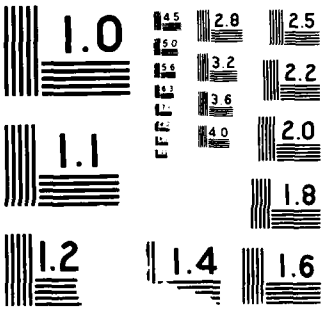
1.0
1.1
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1.6
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2.0
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2.5
2.8
3.2
3.6
4.0
4.5
5.0

From (5.12) and (5.13), the scattering parameters of N_α and N_β are found to be

$$S_{21\alpha}(s) = S_{31}(s) \tag{5.18}$$

$$S_{21\beta}(s) = \frac{S_{21}(s)}{S_{11\alpha}(s)} \tag{5.19}$$

5.3 Butterworth response

Consider a diplexer formed by the connection of an ideal circulator and two reciprocal two-port networks. Assume that the diplexer possesses the Butterworth response. The transducer power-gain characteristics from the source to the loads are given by

$$G_{i1}(\omega^2) = |S_{i1}(j\omega)|^2 = \frac{K_i}{1 + (\omega/\omega_c')^{2n}} \tag{5.20}$$

$$G_{j1}(\omega^2) = |S_{j1}(j\omega)|^2 = \frac{K_j(\omega/\omega_c'')^{2n}}{1 + (\omega/\omega_c'')^{2n}} \tag{5.21}$$

respectively, where i,j = 2 or 3, $i \neq j$. Equation (5.20) represents a low-pass Butterworth response, where ω_c' is the 3-dB radian bandwidth or radian cutoff frequency and $0 \leq K_i \leq 1$. Equation (5.21) is a high-pass Butterworth response with 3-dB cutoff frequency ω_c'' and $0 \leq K_j \leq 1$. Without loss of generality we assume $K_i = K_j = 1$. By appealing to the theorem on the uniqueness of analytic continuation of a complex variable function, we obtain for the low-pass response

$$G_{i1}(-s^2) = S_{i1}(s)S_{i1}(-s) = \frac{1}{1 + (-1)^n y^{2n}} = \frac{1}{q(y)q(-y)} \tag{5.22}$$

the minimum phase solution of which is

$$S_{i1}(s) = \frac{1}{q(y)} \tag{5.23}$$

where

$$y = s/\omega_c' \tag{5.24}$$

and q(y) is the Butterworth polynomial. Similarly, for the high-pass response

$$G_{j1}(-s^2) = S_{j1}(s)S_{ji}(-s) = \frac{z^{2n}}{1 + (-1)^n z^{2n}} = \frac{z^{2n}}{q(z)q(-z)} \tag{5.25}$$

the minimum phase solution of which is

$$S_{j1}(s) = \frac{z^n}{q(z)} \tag{5.26}$$

where

$$z = s/\omega_c'' \tag{5.27}$$

For simplicity, throughout the remainder of the paper we let the crossover frequency be 1. If the low-pass and high-pass responses are of the same order, it is straightforward to confirm that the condition to obtain a symmetrical characteristic with respect to the crossover frequency is given by [51]

$$\omega_c' = 1/\omega_c'' \tag{5.28}$$

For

$$\omega_c' = \omega_c'' = 1 \tag{5.29}$$

the diplexer gives a symmetrical characteristic with a 3-dB insertion loss in each channel at the crossover frequency.

Depending upon the distribution of the responses in both channels, two cases are considered:

Case 1. $G_{31}(\omega^2)$ is a low-pass Butterworth response and $G_{21}(\omega^2)$ is a high-pass response, i.e.

$$S_{31}(s) = \frac{1}{q(y)} \tag{5.30}$$

$$S_{21}(s) = \frac{z^n}{q(z)} \tag{5.31}$$

From (5.18), the scattering parameter $S_{21\alpha}(s)$ of the two-port network N_α is given by

$$S_{21\alpha}(s) = S_{31}(s) = \frac{1}{q(y)} \tag{5.32}$$

By appealing to the para-unitary property of the scattering matrix for a reciprocal lossless two-port network, we obtain

$$S_{11\alpha}(s) = \frac{y^n}{q(y)} \tag{5.33}$$

Substituting (5.31) and (5.32) in (5.19) and referring to (5.27) yield the scattering parameter of $S_{21\beta}(s)$ of N_β as

$$S_{21\beta}(s) = \frac{S_{21}(s)}{S_{11\alpha}(s)}$$

$$= \frac{z^n}{y^n} \frac{q(y)}{q(z)} = \omega_c'^{2n} \frac{q(y)}{q(z)} \tag{5.34}$$

From the above discussions, it is interesting to see that N_α can be realized as a canonical low-pass Butterworth network. To realize N_β, we derive its scattering parameter $S_{11\beta}(s)$ and the impedance $Z_{11\beta}(s)$

looking into the input port. Applying the para-unitary property of the scattering matrix yields

$$S_{11\beta}(s)S_{11\beta}(-s) = 1 - S_{21\beta}(s)S_{21\beta}(-s)$$

$$= 1 - \frac{\omega_c'^{4n}\, q(y)q(-y)}{q(z)q(-z)}$$

$$= \frac{1 - \omega_c'^{4n}}{q(z)q(-z)} = \frac{\beta^2}{q(z)q(-z)} \tag{5.35}$$

where

$$\beta = (1 - \omega_c'^{4n})^{\frac{1}{2}} \tag{5.36}$$

is a constant relating to the 3-dB cutoff frequency ω_c' and n. The minimum-phase decomposition of (5.35) is given by

$$S_{11\beta}(s) = \frac{\beta}{q(z)} \tag{5.37}$$

The input impedance $Z_{11\beta}(s)$ is found to be

$$Z_{11\beta}(s) = \frac{1 - S_{11\beta}(s)}{1 + S_{11\beta}(s)} = \frac{q(z) - \beta}{q(z) + \beta} \tag{5.38}$$

We will show that this impedance is a positive-real function. According to the Darlington theory [22], it can be realized as the input impedance of a passive, lossless reciprocal two-port network terminated in a nonnegative resistor. Therefore, both N_α and N_β are realizable.

As a special case if the diplexer requires a 3-dB insertion loss in each channel at the crossover frequency, the cutoff frequencies of the networks N_α and N_β are related by (5.29), implying that

$$y = z \tag{5.39}$$

and from (5.34)

$$S_{21\beta}(s) = 1 \tag{5.40}$$

The diplexer is simplified to that shown in Fig. 5.2. Therefore, only one Butterworth network is needed.

Case 2. $G_{31}(\omega^2)$ is a high-pass Butterworth response and $G_{21}(\omega^2)$ is a low-pass Butterworth response, i.e.

$$S_{31}(s) = \frac{z^n}{q(z)} \tag{5.41}$$

and

$$S_{21}(s) = \frac{1}{q(y)} \tag{5.42}$$

From (5.18) the scattering parameter $S_{21\alpha}(s)$ of N_α is given by

$$S_{21\alpha}(s) = S_{31}(s) = \frac{z^n}{q(z)} \tag{5.43}$$

Like case 1, by appealing to the para-unitary property of the scattering matrix, we obtain

$$S_{11\alpha}(s) = \frac{1}{q(z)} \tag{5.44}$$

Substituting (5.42) and (5.44) in (5.19) yield

$$S_{21\beta}(s) = \frac{S_{21}(s)}{S_{11\alpha}(s)} = \frac{1}{q(y)} \Big/ \frac{1}{q(z)}$$

$$= \frac{q(z)}{q(y)} \tag{5.45}$$

Thus, N_α can also be realized as a canonical high-pass Butterworth

network. To realize N_β, we express $S_{11\beta}(s)$ and $Z_{11\beta}(s)$, as follows:

$$S_{11\beta}(s)S_{11\beta}(-s) = 1 - S_{21\beta}(s)S_{21\beta}(-s)$$

$$= \frac{\beta^2 y^{2n}}{q(y)q(-y)} \tag{5.46}$$

the minimum phase solution of which is

$$S_{11\beta}(s) = \frac{\beta y^n}{q(y)} \tag{5.47}$$

The impedance looking into the input port is found to be

$$Z_{11\beta}(s) = \frac{1 - S_{11\beta}(s)}{1 + S_{11\beta}(s)} = \frac{q(y) - \beta y^n}{q(y) + \beta y^n} \tag{5.48}$$

We show that $Z_{11\beta}(s)$ is positive-real, so that N_β is always realizable.

If the diplexer requires a 3-dB insertion loss in each channel at the crossover frequency, we substitute (5.29) in (5.45) and obtain

$$S_{21\beta}(s) = 1 \tag{5.49}$$

The diplexer is simplified to that shown in Fig. 5.2, where N_α is a high-pass Butterworth network.

Example: Design a symmetrical diplexer having a second-order canonical Butterworth response with $\omega_c' = 0.7$, where ω_c' is the 3-dB cutoff frequency in the loss-pass channel.

We assume that N_α is a Butterworth low-pass network of order 2 and $\omega_c' = 0.7$. From (5.33)-(5.37) we obtain the following for network N_β:

$$S_{21\beta}(s) = \omega_c'^{2n} \frac{q(y)}{q(z)} = \omega_c'^{2n} \frac{y^2 + \sqrt{2}y + 1}{z^2 + \sqrt{2}z + 1} \tag{5.50}$$

where $y = s/\omega_c'$ and

$$z = s/\omega_c'' = s\omega_c' \tag{5.51}$$

$$\beta = (1 - \omega_c'^{4n})^{\frac{1}{2}} = 0.97075 \tag{5.52}$$

$$Z_{11\beta}(s) = \frac{q(z) - \beta}{q(z) + \beta} = \frac{z^2 + \sqrt{2}z + 1 - \beta}{z^2 + \sqrt{2}z + 1 + \beta} \tag{5.53}$$

The two-port N_α is realizable as a Butterworth low-pass network, whereas N_β can be realized by applying the technique proposed in [57]. The final circuit configuration and frequency characteristics are shown in Figs. 5.3 and 5.4, respectively. The insertion loss at the crossover frequency is found to be 5.37 dB.

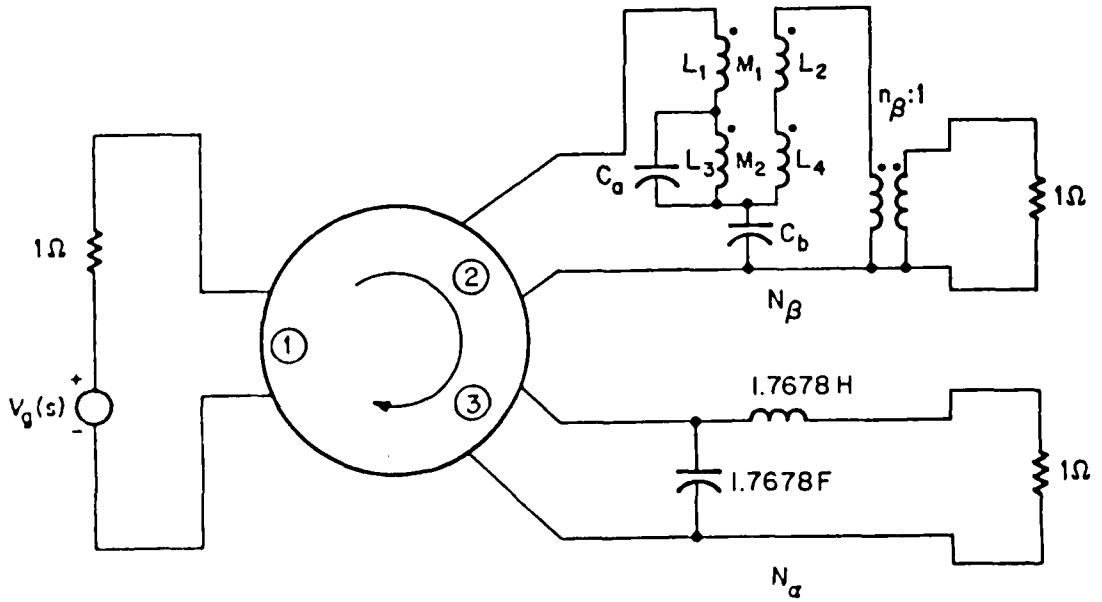


Fig. 5.3. The Circuit Configuration of Example 1, where N_α is a Butterworth Network. The Parameters of the Two-Port Network N_β are given as follows:
$L_1 = 0.6219$ H, $L_2 = 1.309\times10^{-3}$ H, $M_1 = 0.0285$ H, $L_3 = 0.0402$,
$L_4 = 0.02768$ H, $M_2 = -0.0334$ H, $C_a = 80.8$F, $C_b = 64.28$F, $n_\beta = 0.01484$

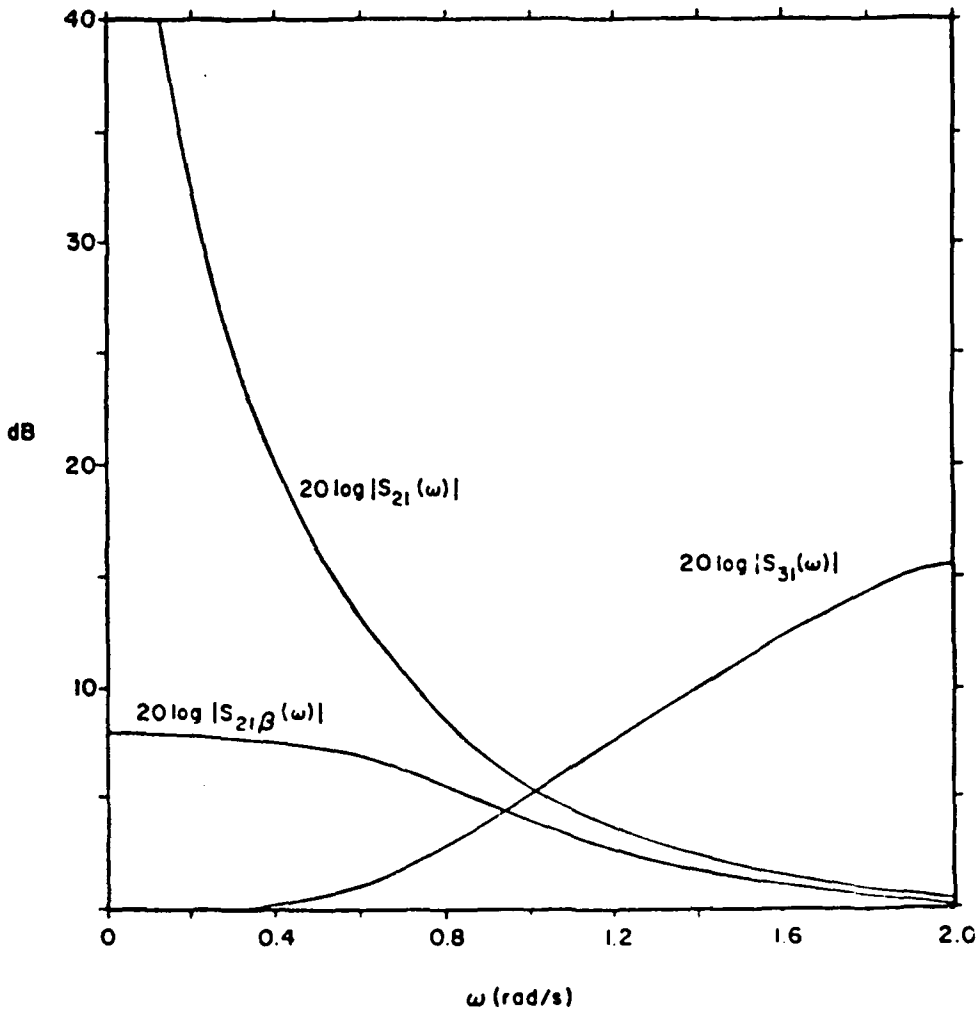


Fig. 5.4. The Frequency Response of the Diplexer Possessing the 2nd Butterworth Characteristic. The Insertion Loss at the Crossover Frequency $\omega = 1$ is 5.37 dB.

5.4 A diplexer composed of a circulator and the canonical Butterworth networks

The approach mentioned in the foregoing results shows a diplexer having the canonical Butterworth response in both channels. An example showed that the required characteristic for N_β is different from the canonical Butterworth response, and that the problem to design N_β is much more complicated except for the situation where a 3-dB insertion loss is required at the crossover frequency.

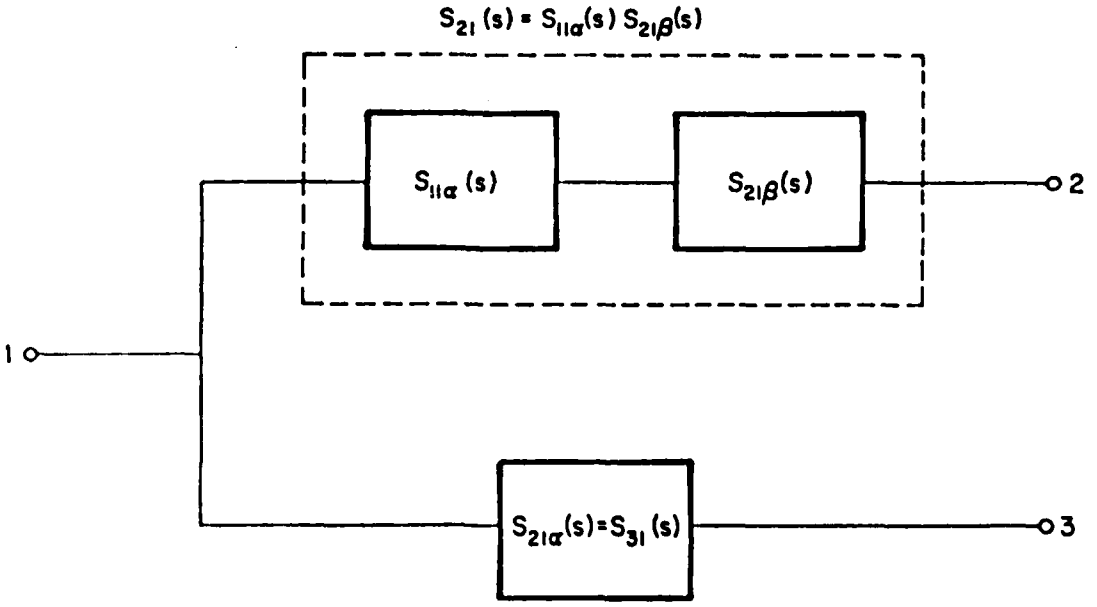


Fig. 5.5. An Equivalent Representation of Equations (5.18) and (5.19).

An equivalent representation of (5.18) and (5.19) is depicted in the block diagram of Fig. 5.5, where the existence of the block $S_{11\alpha}(s)$ requires the characteristic of N_β *to diverge from the* Butterworth response. Figure 5.6 shows the plots of $|S_{21}(j\omega)|$, $|S_{31}(j\omega)|$ and $|S_{11\alpha}(j\omega)|$ as functions of ω where $S_{21}(s)$ and $S_{31}(s)$ are required to possess the low-pass and high-pass Butterworth responses of order 4 with $\omega_c' = 0.7$ and $\omega_c'' = 1/\omega_c'$. We notice that the value of 20 log $|S_{11\alpha}(j\omega)|$ is much smaller than that of the Butterworth response 20 log $|S_{21}(j\omega)|$, so that in this channel the contribution of $S_{11\alpha}(s)$ is much smaller than that of $S_{21\beta}(s)$. Thus, if $S_{21\beta}(s)$ is required to possess the Butterworth response, the overall response $|S_{21}(j\omega)| = |S_{11\alpha}(j\omega)S_{21\beta}(j\omega)|$ plotted in Fig. 5.7 is usually close to the Butterworth response especially in the passband. The deviation becomes smaller as the order and/or the 3-dB cutoff frequency is increased.

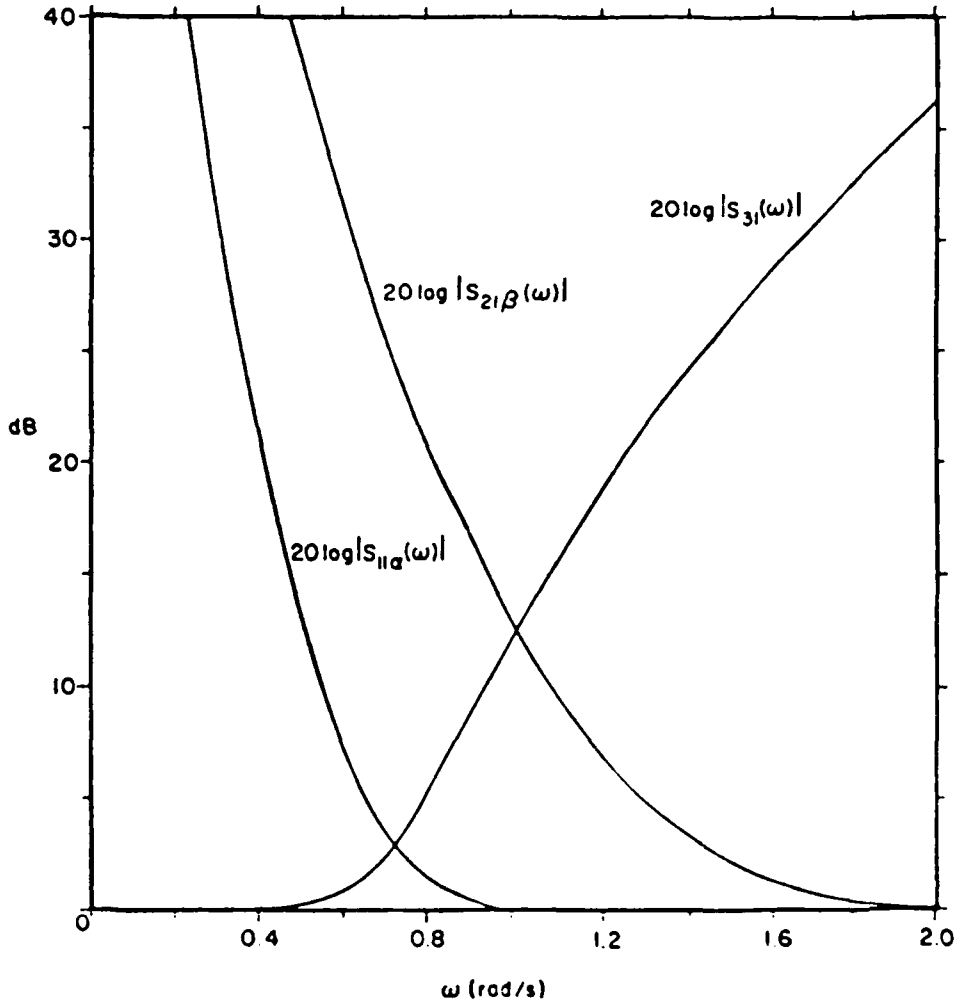


Fig. 5.6. The Frequency Response of a Diplexer Having Butterworth Characteristic of Order 4 with $\omega_c' = 0.7$ rad/s.

From a practical viewpoint, it is desirable to choose both N_α and N_β to be the Butterworth networks. The additional loss at the passband edge may be corrected by adjusting the cutoff frequency of N_β, where the original required cutoff frequencies are assumed to satisfy the symmetrical condition (5.27). To facilitate our discussion, two cases are considered:

Case 1. $G_{31}(\omega^2)$ is a low-pass Butterworth response and $G_{21}(\omega^2)$ is a high-pass response. The transducer power-gain characteristics

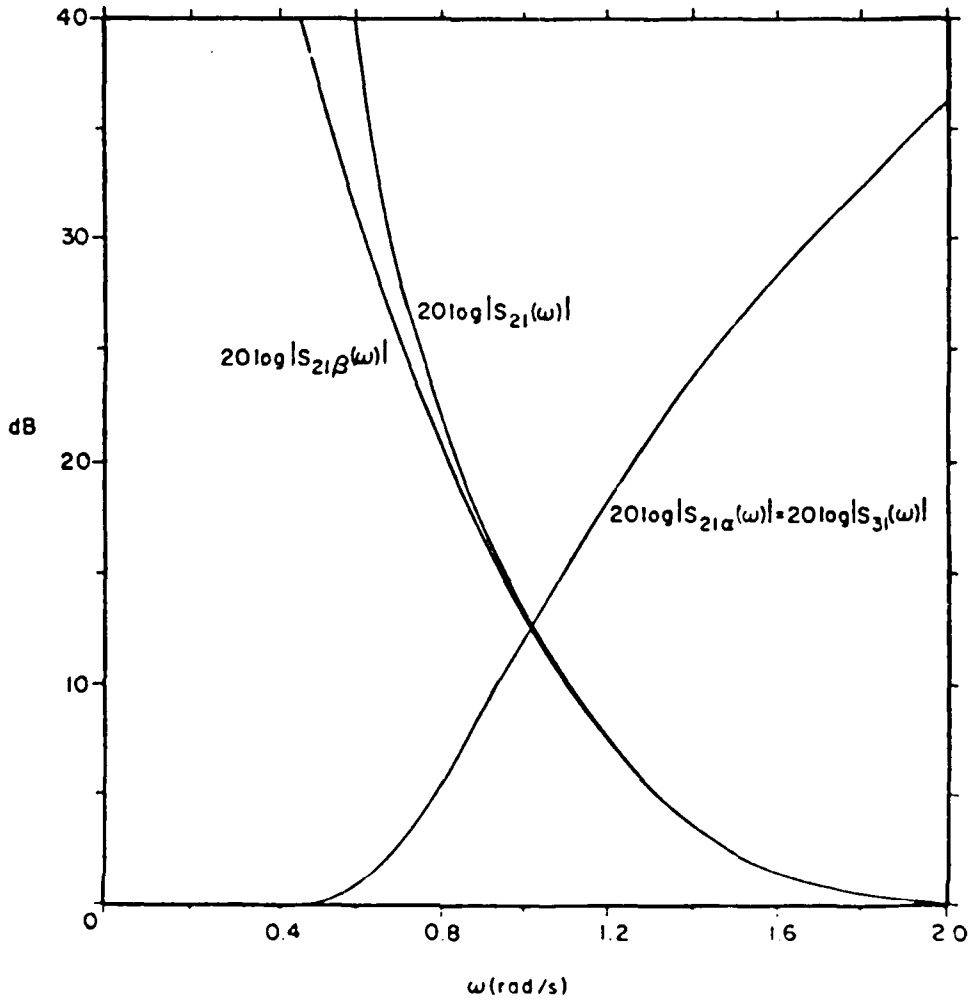


Fig. 5.7. The Frequency Response of a Diplexer, where N_α and N_β are Canonical Butterworth Networks of Order 4 with $\omega_c' = 0.7$ rad/s.

$G_{31}(\omega^2)$ and $G_{21}(\omega^2)$ are shown in (5.20) and (5.21), respectively. The additional power loss corresponding to the terms $S_{11\alpha}(j\omega)$ in

$$|S_{11\alpha}(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega_c}{\omega}\right)^{2n}} \tag{5.54}$$

Now suppose that we change the cutoff frequency of N_β from ω_c'' to $\omega_c'' + \delta\omega_c''$ to cancel the above additional loss. The required $\delta\omega_c''$ can be obtained by solving the following equation

$$|S'_{21\beta}(j\omega''_c)|^2 - |S_{21\beta}(j\omega''_c)|^2 = |S_{11\alpha}(j\omega''_c)|^2 \qquad (5.55)$$

where $|S'_{21\beta}(j\omega)|^2$ is the transducer power-gain characteristic of N_β with the adjusted cutoff frequency $\omega''_c + \delta\omega''_c$. The solution of (5.55) is given by

$$\omega''_c + \delta\omega''_c = \frac{1}{\omega'_c}\left(\frac{2}{1 + \omega'^{4n}_c} - 1\right)^{\frac{1}{2n}} \qquad (5.56)$$

Case 2. $G_{31}(\omega^2)$ is a high-pass Butterworth response and $G_{21}(\omega^2)$ is a high-pass response.

In this case, the cutoff frequency ω'_c of the low-pass network N_β need to be adjusted. Like case 1, we obtain

$$\omega'_c + \omega'_c = \omega'_c\left(\frac{2}{1 + \omega'^{4n}_c} - 1\right)^{-\frac{1}{2n}} \qquad (5.57)$$

Examples illustrating the above results are listed in Table 5.1. The circuit elements and frequency responses are plotted in Figs. 5.8 through 5.11 by the computer program DIPLX2.

Table 5.1. Frequency Characteristics of Diplexers of Various Orders.

	Order n	3-dB cutoff frequency of the diplexer		3-dB cutoff frequency of the two-ports		Insertion loss at the passband edge in dB		Insertion loss at ω = 1 rad/s dB		Circuits and response curves
		ω'_c	ω''_c	ω_{cl}	ω_{ch}	low-pass	high-pass	low-pass	high-pass	
1	3	0.7	1/0.7	0.7	1.4220	3.0103	3.0103	9.777	10.153	Figure 8 (a)(b)
2	4	0.7	1/0.7	0.70058	1/0.7	3.0103	3.0103	12.852	12.636	Figure 9 (a)(b)
3	5	0.7	1/0.7	0.7	1.4285	3.0103	3.0103	15.611	15.725	Figure 10 (a)(b)
4	5	0.8	1/0.8	0.8018	1/0.8	3.0103	3.0103	10.487	10.134	Figure 11 (a)(b)

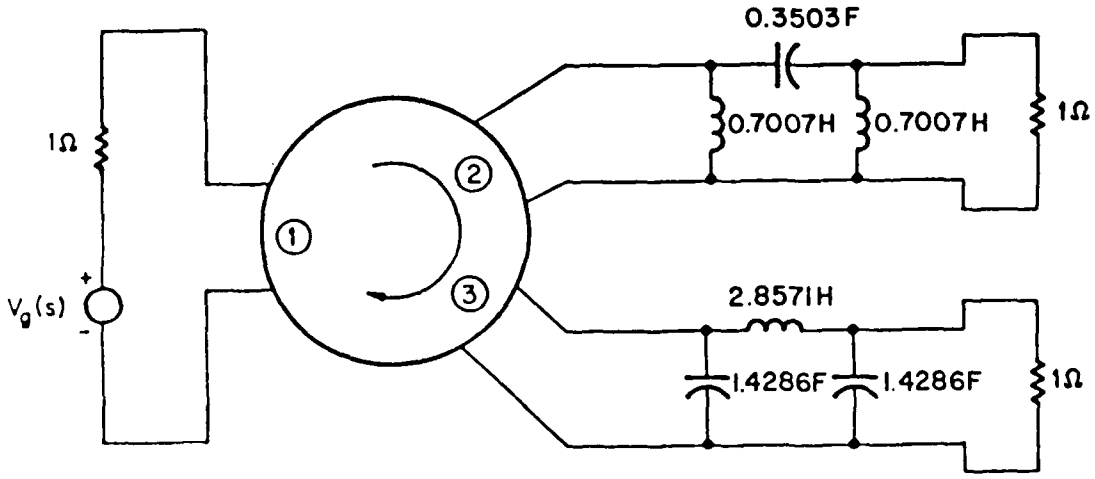


Fig. 5.8(a). A Diplexer Configuration Composed of a Low-Pass Butterworth Network of Order 3 with $\omega_c' = 0.7$ rad/s and a High-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c'' = 1.4272$ rad/s.

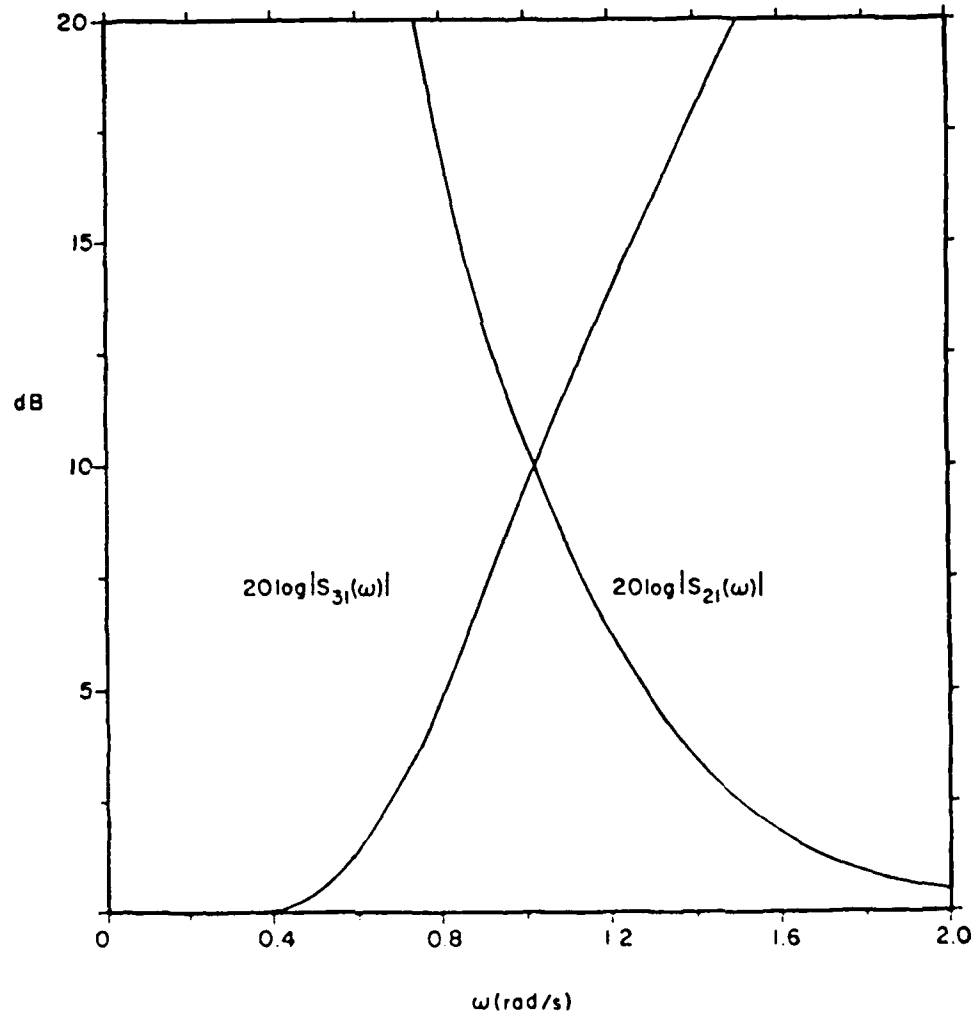


Fig. 5.8(b). The Frequency Response of a Diplexer, where N_α is a Low-Pass Butterworth Network of Order 3 with $\omega_c' = 0.7$ rad/s and N_β is a High-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c'' = 1.4272$.

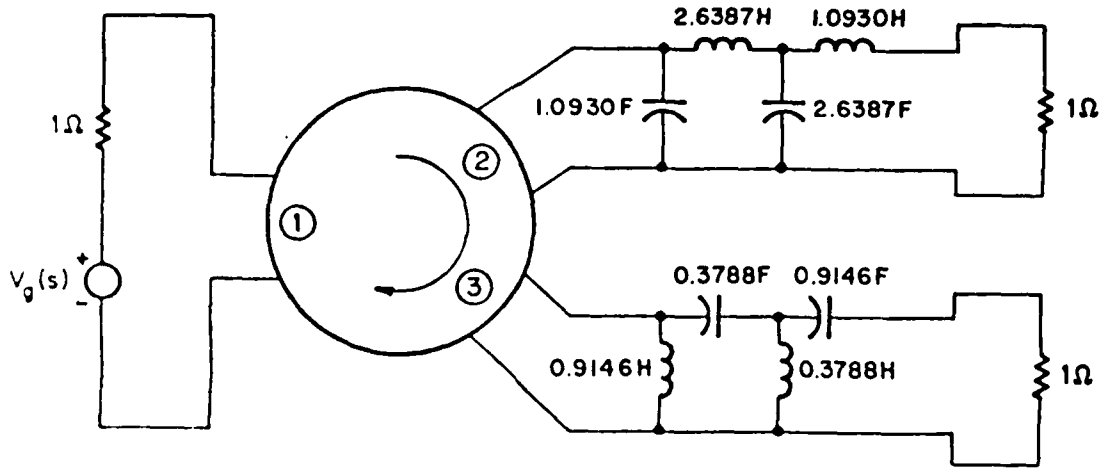


Fig. 5.9(a). A Diplexer Configuration Composed of a High-Pass Butterworth Network of Order 4 with $\omega_c'' = 1/0.7$ rad/s and a Low-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c' = 0.70025$ rad/s.

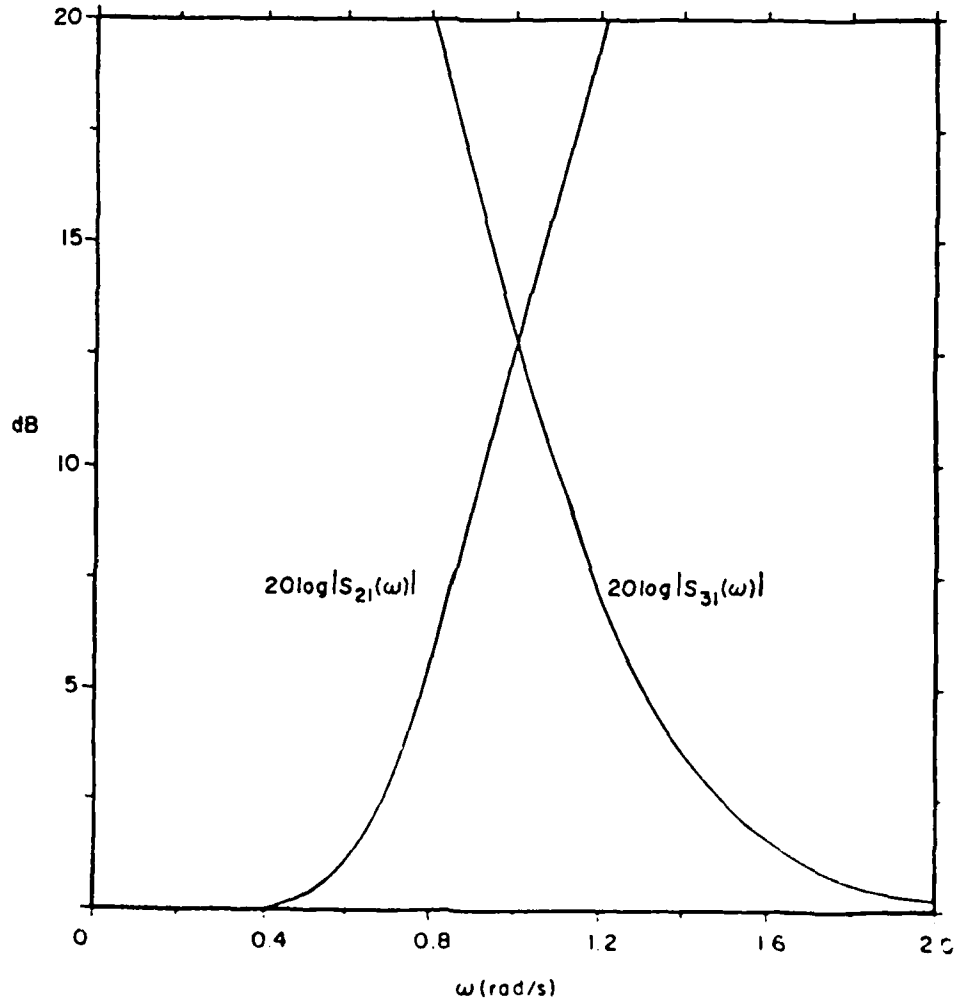


Fig. 5.9(b). The Frequency Response of a Diplexer, where N_α is a High-Pass Butterworth Network of Order 4 with $\omega_c'' = 1/0.7$ rad/s and N_β is a Low-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c' = 0.70025$ rad/s.

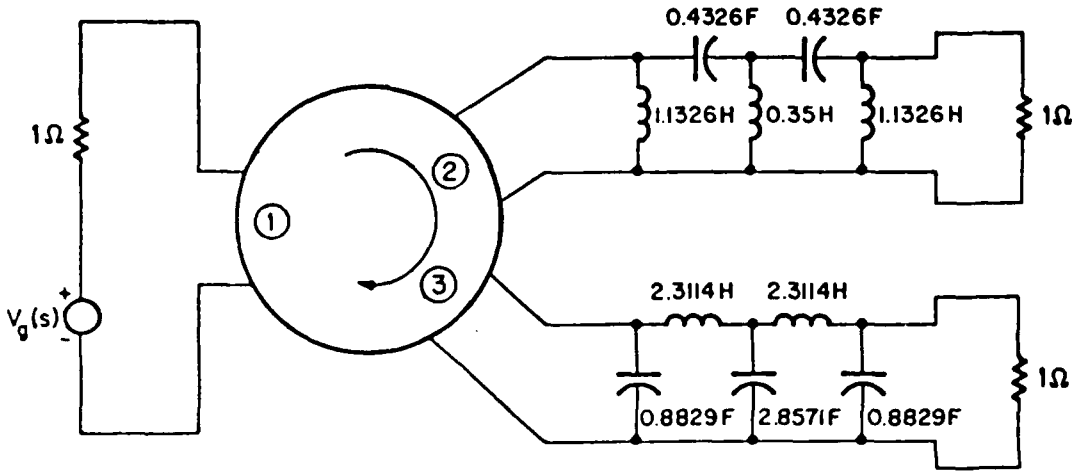


Fig. 5.10(a). A Diplexer Configuration Composed of a Low-Pass Butterworth Network of Order 5 with $\omega_c' = 0.7$ rad/s and a High-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c'' = 1.4285$ rad/s.

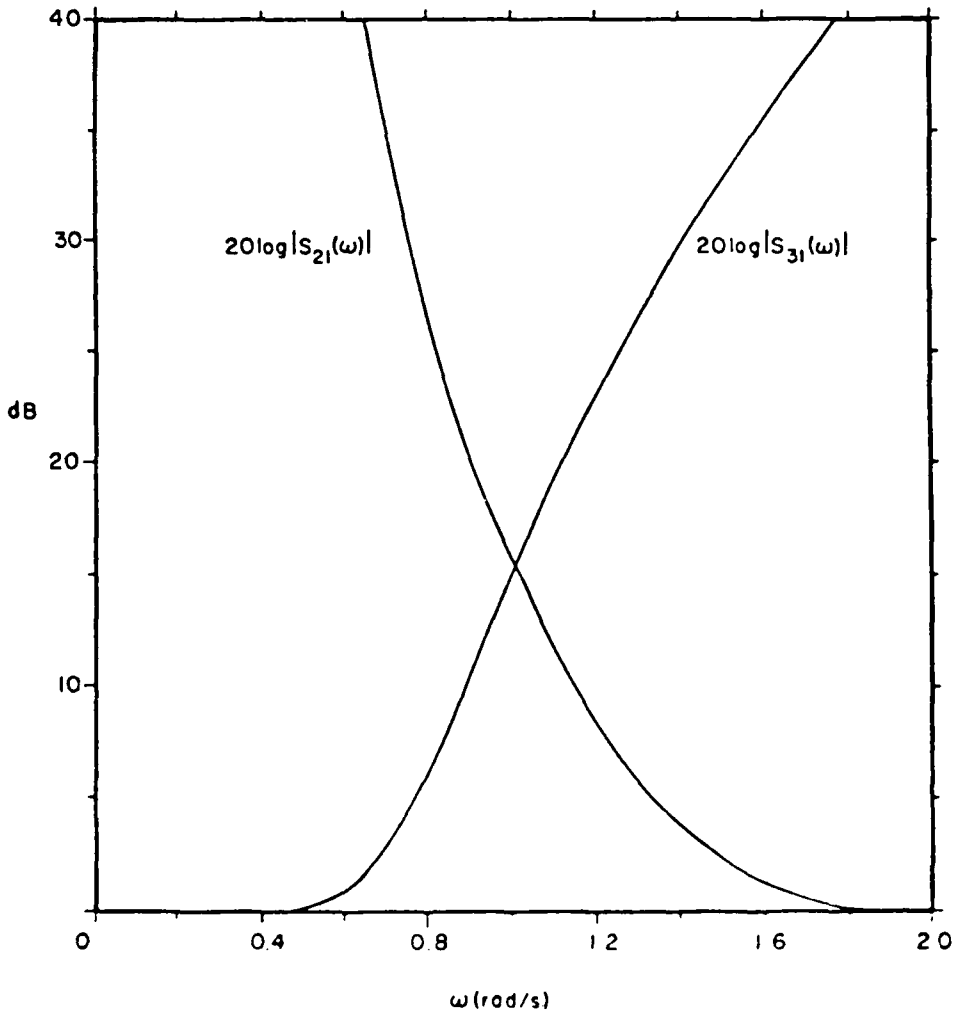


Fig. 5.10(b). The Frequency Response of a Diplexer, where N_α is a Low-Pass Butterworth Network of Order 5 with ω_c' 0.7 rad/s and N_β is a High-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c'' = 1.4285$ rad/s.

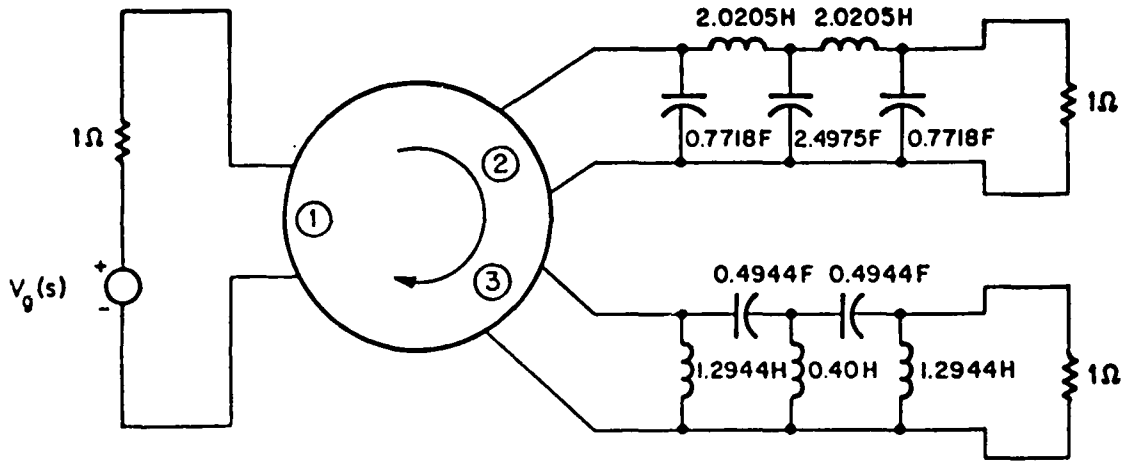


Fig. 5.11(a). A Diplexer Configuration Composed of a High-Pass Butterworth Network of Order 5 with $\omega_c'' = 1/0.8$ rad/s and a Low-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c' = 0.8008$ rad/s.

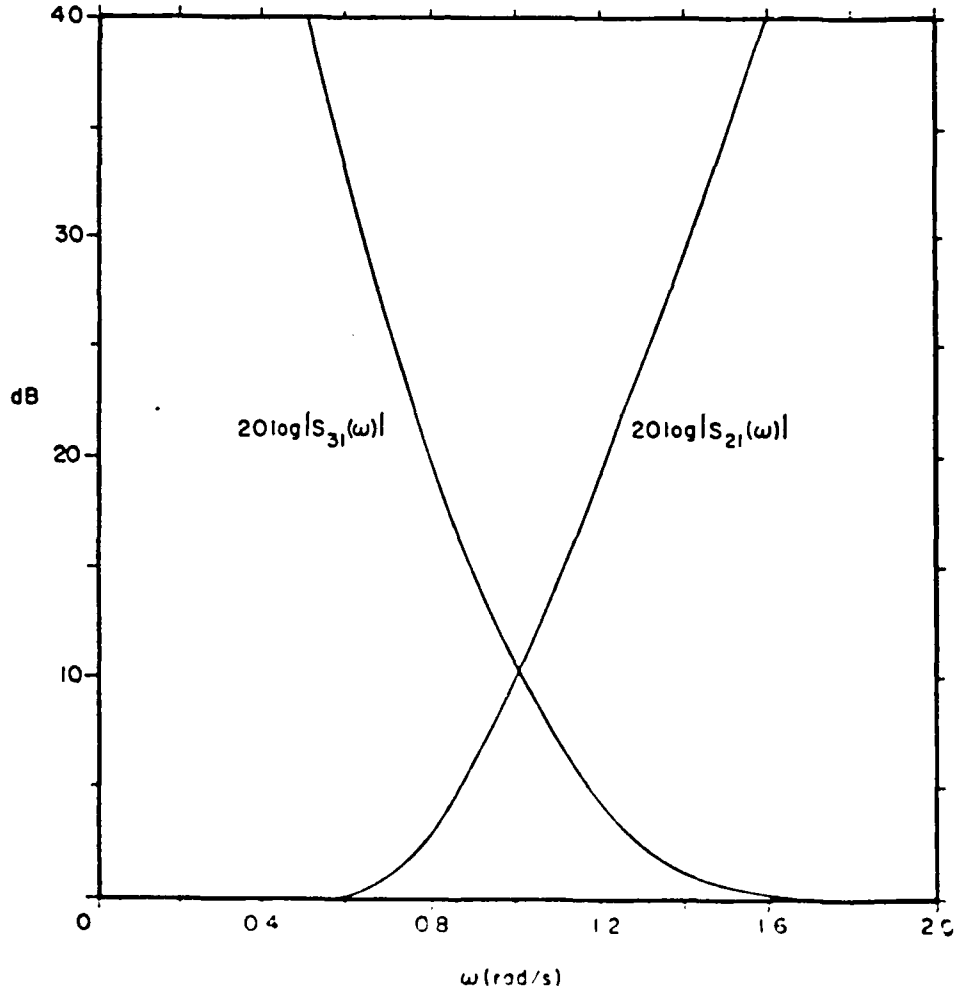


Fig. 5.11(b). The Frequency Response Curves of a Diplexer, where N_α is a High-Pass Butterworth Network of order 5 with $\omega_c'' = 1/0.8$ rad/s and N_β is a Low-Pass Butterworth Network of the Same Order with the Adjusted Cutoff Frequency $\omega_c' = 0.8008$ rad/s.

5.5 An outline of a proof of the positive-realness of $Z_{11\beta}(s)$

We show that the rational functions given (5.38) and (5.48) are positive real. Thus each can be realized as the input impedance of a passive, lossless reciprocal two-port network terminated in a non-negative resistor.

The following theorem presents the positive-real conditions of a rational function [14].

Theorem. A rational function represented in the form

$$f(s) = \frac{p_1(s)}{p_2(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} \tag{5.58}$$

where m_1, m_2 and n_1, n_2 are the even and odd parts of the polynomials $p_1(s)$ and $p_2(s)$, respectively, is positive real if and only if the following three conditions are satisfied:

(i) $f(s)$ is real when s is real.

(ii) $p_1(s) + p_2(s)$ is strictly Hurwitz.

(iii) $m_1(j\omega)m_2(j\omega) - n_1(j\omega)n_2(j\omega) \geq 0$ for all ω.

Consider function (5.37) rewritten as

$$Z_{11\beta}(s)\,\frac{q(s) - \beta}{q(s) + \beta} = \frac{m(s) + n(s) - \beta}{m(s) + n(s) + \beta} \tag{5.59}$$

where $q(s)$ is a Butterworth polynomial and $0 \leq \beta \leq 1$. If $\beta = 1$, the two-port network becomes a Butterworth network. The first condition is clearly satisfied. Since the tested function $p_1(s) + p_1(s)$ of (5.59) is the same as that of the canonical Butterworth network with $\beta = 1$ in (5.59), the second condition is satisfied. Finally to test condition (iii), we compute

$$m_1(s)m_2(s) - n_1(s)n_2(s) = m^2(s) - n^2(s) - \beta^2 \qquad (5.60)$$

For Butterworth network with $\beta = 1$ in (5.60), condition (iii) is satisfied. Thus, the function $Z_{11\beta}(s)$ is positive real.

We now consider the function $Z_{11\beta}(s)$ (5.48) rewritten as

$$Z_{11\beta}(s) = \frac{q(s) - \beta y^n}{q(s) + \beta y^n} = \frac{m(s) + n(s) - \beta y^n}{m(s) + n(s) + \beta y^n} \qquad (5.61)$$

Conditions (i) and (ii) are clearly satisfied. We next compute

$$m_1(s)m_2(s) - n_1(s)n_2(s) = m^2(s) - n^2(s) \pm (\beta y^n)^2 \qquad (5.62)$$

The plus sign corresponds to n odd and the minus to n even. Since condition (iii) is satisfied for Butterworth network with $\beta = 1$. The function $Z_{11\beta}(s)$ of (5.61) satisfies the third condition. Thus it is positive real.

5.6 Conclusion

In this section, we presented a new diplexer configuration composed of an ideal three-port circulator and two reciprocal lossless two-port networks. Having expressed the scattering parameters of a diplexer in terms of those of the component networks, we considered the design of a diplexer having Butterworth response. The diplexer having canonical Butterworth response is always realizable. In this case, only one of the two-port networks is needed if the insertion loss at the crossover frequency is 3-dB. In the case where both two-ports possess the canonical Butterworth response, the final response of the diplexer is slightly deviated from the canonical Butterworth characteristic. To keep the bandwidth or the insertion

loss specification at the passband edge, slight adjustment of the cut-off frequency of N_β is necessary. The final response is very close to the canonical Butterworth characteristic.

The approach presented here can be extended to the design of a diplexer having other types of responses, such as the Chebyshev or elliptic response. The above configuration can also be extended to the design of a multiplexer where an n-port circulator is used instead of a three-port circulator.

Section 6

A MULTIPLEXER CONFIGURATION COMPOSED OF A MULTI-PORT CIRCULATOR AND RECIPROCAL TWO-PORT NETWORKS

6.1 Introduction

Section 6 presents a nonreciprocal multiplexer configuration composed of an ideal multi-port circulator and a number of reciprocal two-port networks. Fundamental formulas which relate the scattering parameters of the multiplexer to those of the reciprocal two-port networks are derived. A multi-port circulator can always be realized as an interconnection of the three-port circulators. Design procedure for the proposed nonreciprocal multiplexer is given. A computer program MUPLX is available for computing the circuit element values as well as the final frequency response.

The design of a multiplexer separating a frequency spectrum into the individual channels of signals is one of the basic problems in communications. The most popular configuration of a multiplexer consists of the reciprocal two-port networks connected either in series or in parallel, as shown by many workers [37, 40, 42, 51, 61].

Recently Wang and Chen [52] presented a nonreciprocal diplexer configuration composed of an ideal three-port circulator and two reciprocal lossless two-port networks, terminating in the source and two loads. The configuration permits the specification of the insertion loss at the crossover frequency as well as the frequency shaping.

In section 6 we extend the above diplexer to an N-channel nonreciprocal multiplexer composed of an ideal (N+1)-port circulator and N reciprocal lossless two-port bandpass networks, terminating in the source and loads as shown in Fig. 6.1. Having expressed the scattering parameters of the multiplexer in terms of those of the component

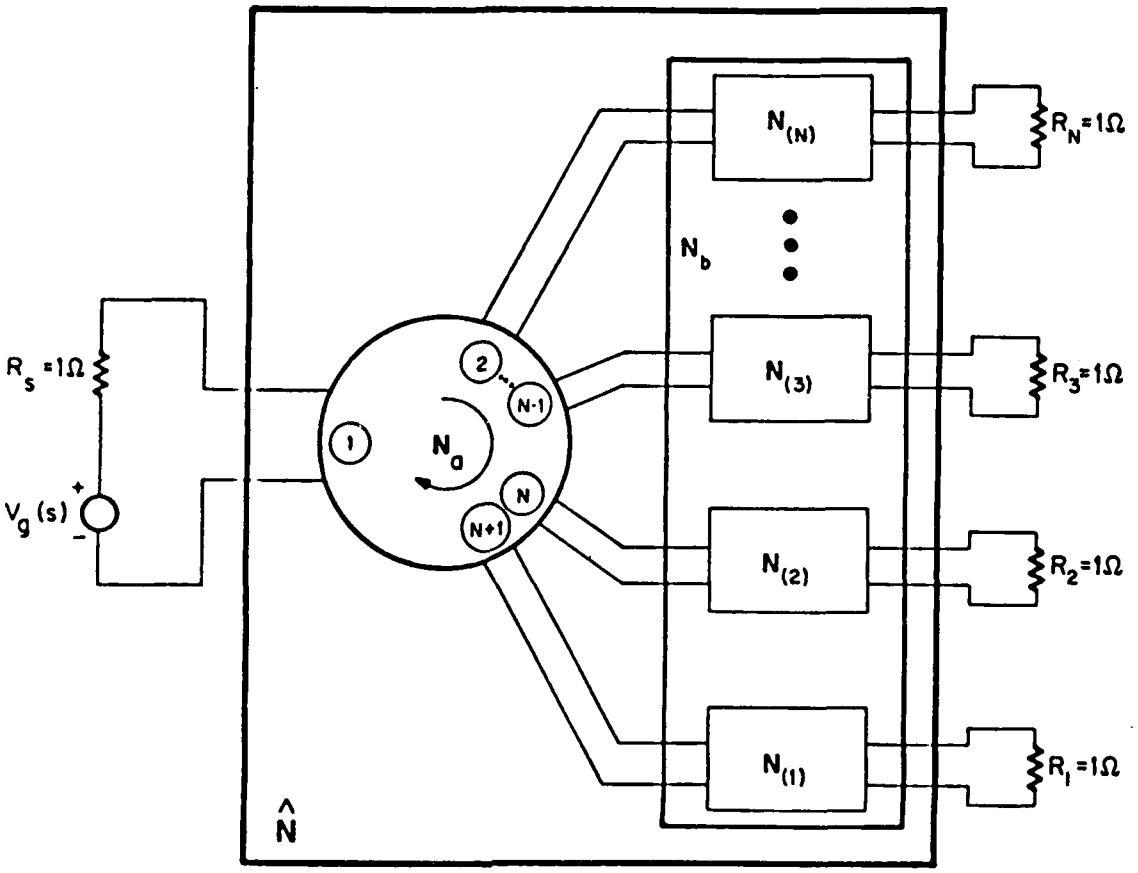


Fig. 6.1. A Nonreciprocal Multiplexer Configuration.

networks, we derive formulas which relate the scattering parameters of the multiplexer to those of the reciprocal two-port networks.

Another important problem is the realization of a multi-port circulator. An arbitrary multi-port circulator can be realized as an interconnection of the three-port circulators. Design procedure for the proposed nonreciprocal multiplexer and illustrative examples will be discussed in paragraph 6.4.

6.2 The scattering matrix of a nonreciprocal multiplexer

As an extension of a three-port circulator, the multi-port circulator is defined by the unit normalized scattering matrix

$$\underset{\sim}{S}(s) = \left[\begin{array}{c|c} \underset{\sim}{0} & \underset{\sim}{U}_{n-1} \\ \hline 1 & \underset{\sim}{0} \end{array}\right] \tag{6.1}$$

where n is the number of the ports, and $\underset{\sim}{U}_{n-1}$ denotes the identity matrix of order n-1.

The general configuration of a nonreciprocal multiplexer shown in Fig. 6.1 can be viewed as an interconnection of a multi-port circulator N_a and a reciprocal multi-port network N_b, where N_b is formed by the two-port networks. Without loss of generality we assume that the source resistance and the load reistances are 1 Ω. The unit-normalized scattering matrix (6.1) can be repartitioned in the form

$$\underset{\sim}{S}_a(s) = \begin{bmatrix} \underset{\sim}{S}_{11a} & \underset{\sim}{S}_{12a} \\ \underset{\sim}{S}_{21a} & \underset{\sim}{S}_{22a} \end{bmatrix} = \left[\begin{array}{c|ccccc} 0 & 1 & 0 & \cdots & 0 & 0 \\ \hline 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ \hline 1 & 0 & 0 & \cdots & 0 & 0 \end{array}\right] \tag{6.2}$$

where the port connecting to the source is numbered as port 1.

Consider a general N channel multiplexer of Fig. 6.1, where the multi-port circulator possesses N+1 ports. Assume that the scattering matrices of the reciprocal two-port networks $N_{(1)}$, $N_{(2)}$, ..., $N_{(N)}$ normalized to 1-Ω resistance are given by

$$\underset{\sim}{S}_{(i)}(s) = \begin{bmatrix} S_{11(i)} & S_{12(i)} \\ S_{21(s)} & S_{22(i)} \end{bmatrix}, \qquad i = 1, 2, \ldots, N. \tag{6.3}$$

The partitioned unit-normalized scattering matrix of the 2N-port reciprocal network N_b becomes

$$\underset{\sim}{S}_b(s) = \begin{bmatrix} \underset{\sim}{S}_{11b} & \underset{\sim}{S}_{12b} \\ \underset{\sim}{S}_{21b} & \underset{\sim}{S}_{22b} \end{bmatrix}$$

$$= \left[\begin{array}{cccc|cccc} S_{11(N)} & 0 & \cdots & 0 & S_{12(N)} & 0 & \cdots & 0 \\ 0 & S_{11(N-1)} & \cdots & 0 & 0 & S_{12(N-1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{11(1)} & 0 & 0 & \cdots & S_{12(1)} \\ \hline S_{21(N)} & 0 & \cdots & 0 & S_{22(N)} & 0 & \cdots & 0 \\ 0 & S_{21(N-1)} & \cdots & 0 & 0 & S_{22(N-1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{21(1)} & 0 & 0 & \cdots & S_{22(1)} \end{array}\right]$$

Let the partitioned scattering matrix of the composite (N+1)-port network N (the N-channel multiplexer) normalizing to the 1-Ω resistance be

$$\underset{\sim}{S}(s) = \begin{bmatrix} \underset{\sim}{S}_{11} & \underset{\sim}{S}_{12} \\ \underset{\sim}{S}_{21} & \underset{\sim}{S}_{22} \end{bmatrix} = \left[\begin{array}{c|ccc} S_{11} & S_{12} & \cdots & S_{1,N+1} \\ \hline S_{21} & S_{22} & \cdots & S_{2,N+1} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N+1,1} & S_{N+1,2} & \cdots & S_{N+1,N+1} \end{array}\right] \qquad (6.5)$$

The interconnection formulas which relate the scattering matrix of the composite (N+1)-port network N and those of the component networks N_a and N_b are given by [14]

$$\underset{\sim}{S}_{11}(s) = \underset{\sim}{S}_{11a} + \underset{\sim}{S}_{12a}(\underset{\sim}{U}_N - \underset{\sim}{S}_{11b}\underset{\sim}{S}_{22a})^{-1} \underset{\sim}{S}_{11b}\underset{\sim}{S}_{21a} \tag{6.6}$$

$$\underset{\sim}{S}_{12}(s) = \underset{\sim}{S}_{12a} + (\underset{\sim}{U}_N - \underset{\sim}{S}_{11b}\underset{\sim}{S}_{22b})^{-1} \underset{\sim}{S}_{12b} \tag{6.7}$$

$$\underset{\sim}{S}_{21}(s) = \underset{\sim}{S}_{21b}(\underset{\sim}{U}_N - \underset{\sim}{S}_{22a}\underset{\sim}{S}_{11b})^{-1} \underset{\sim}{S}_{21a} \tag{6.8}$$

$$\underset{\sim}{S}_{22}(s) = \underset{\sim}{S}_{22b} + \underset{\sim}{S}_{21b}(\underset{\sim}{U}_N - \underset{\sim}{S}_{22a}\underset{\sim}{S}_{11b})^{-1} \underset{\sim}{S}_{22a}\underset{\sim}{S}_{12b} \tag{6.9}$$

where $\underset{\sim}{U}_N$ denotes the identity matrix of order N. Substituting (6.2) and (6.4) in (6.8) yields

$$\underset{\sim}{S}_{21}(s) = \begin{bmatrix} S_{21} \\ S_{31} \\ \vdots \\ S_{N+1,1} \end{bmatrix} = \begin{bmatrix} S_{21(N)} & 0 & \cdots & 0 \\ 0 & S_{21(N-1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{21(1)} \end{bmatrix}$$

$$\cdot \left(\underset{\sim}{U}_N - \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{11(N)} & 0 & \cdots & 0 \\ 0 & 0 & S_{11(N-1)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & S_{11(1)} \end{bmatrix} \right)$$

$$\cdot \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S_{21(N)} & 0 & \cdots & 0 \\ 0 & S_{21(N-1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{21(1)} \end{bmatrix} \begin{bmatrix} S_{11(N-1)}S_{11(N-2)} & \cdots & S_{11(1)} \\ S_{11(N-2)}S_{11(N-1)} & \cdots & S_{11(1)} \\ \vdots \\ S_{11(1)} \end{bmatrix}$$

$$= \begin{bmatrix} S_{21(N)}S_{11(N-1)}S_{11(N-2)} & \cdots & S_{11(1)} \\ S_{21(N-1)}S_{11(N-2)}S_{11(N-3)} & \cdots & S_{11(1)} \\ \cdots & & \\ S_{21(2)}S_{11(1)} & & \\ S_{21(1)} & & \end{bmatrix} \tag{6.10}$$

To avoid the confusion in subscripts, we renumber the elements of $\underset{\sim}{S}_{21}(s)$ as follows:

$$\underset{\sim}{S}_{21}(s) = \begin{bmatrix} S_{21} \\ S_{31} \\ \vdots \\ S_{N1} \\ S_{N+1,1} \end{bmatrix} = \begin{bmatrix} S_{21M(N)} \\ S_{21M(N-1)} \\ \vdots \\ S_{21M(2)} \\ S_{21M(1)} \end{bmatrix} \tag{6.11}$$

where M stands for multiplexer and the numbers in the parentheses corresponding to the numberings of the reciprocal two-port networks. Thus (6.10) can be rewritten as

$$\underset{\sim}{S}_{21}(s) = \begin{bmatrix} S_{21M(N)} \\ S_{21M(N-1)} \\ \vdots \\ S_{21M(2)} \\ S_{21M(1)} \end{bmatrix} = \begin{bmatrix} S_{21(N)}S_{11(N-1)}S_{11(N-2)} & \cdots & S_{11(2)}S_{11(1)} \\ S_{21(N-1)}S_{11(N-2)}S_{11(N-3)} & \cdots & S_{11(1)} \\ \cdots & & \\ S_{21(2)}S_{11(1)} & & \\ S_{21(1)} & & \end{bmatrix} \tag{6.12}$$

The scattering parameter $S_{21M(i)}$ is obtained as

$$S_{21M(i)} = S_{21(i)}S_{11(i-1)}S_{11(i-2)} \cdots S_{11(1)}$$

$$= S_{21(i)} \prod_{k=1}^{i-1} S_{11(k)}, \qquad i = 1,2,\ldots,N \tag{6.13}$$

The transducer power-gain characteristics of the multiplexer become

$$G_{21M(i)}(\omega^2) = |S_{21M(i)}(j\omega)|^2 = |S_{21(i)}(j\omega)|^2 \cdot \prod_{k=1}^{i-1} |S_{11(k)}(j\omega)|^2 \tag{6.14}$$

$$i = 1, 2, \ldots, N$$

To obtain the required transducer power-gain characteristics, we calculate the scattering parameters $S_{21(i)}$ of $N_{(i)}$, $i = 1,2,\ldots,N$. From (6.13) and (6.4) they are found to be

$$S_{21(i)}(s) = \frac{S_{21M(i)}(s)}{\prod_{k=1}^{i-1} S_{11(k)}(s)}, \qquad i = 1,2,\ldots,N \tag{6.15}$$

Thus we reduce the problem of designing a nonreciprocal multiplexer to satisfy the specified transducer power-gain characteristics $G_{21M(i)}(\omega^2)$, $i = 1,2,\ldots,N$, to that of designing the reciprocal two-port networks $N_{(i)}$, $i = 1,2,\ldots,N$.

6.3 Realization of a multi-port circulator

In this section we show that an arbitrary multi-port circulator can be realized as an interconnection of the three-port circulators. Figures 6.2 and 6.3 are the examples of four-port and five-port circulators composed of two three-port circulators and three three-port circulators, respectively. The unit normalized scattering matrices

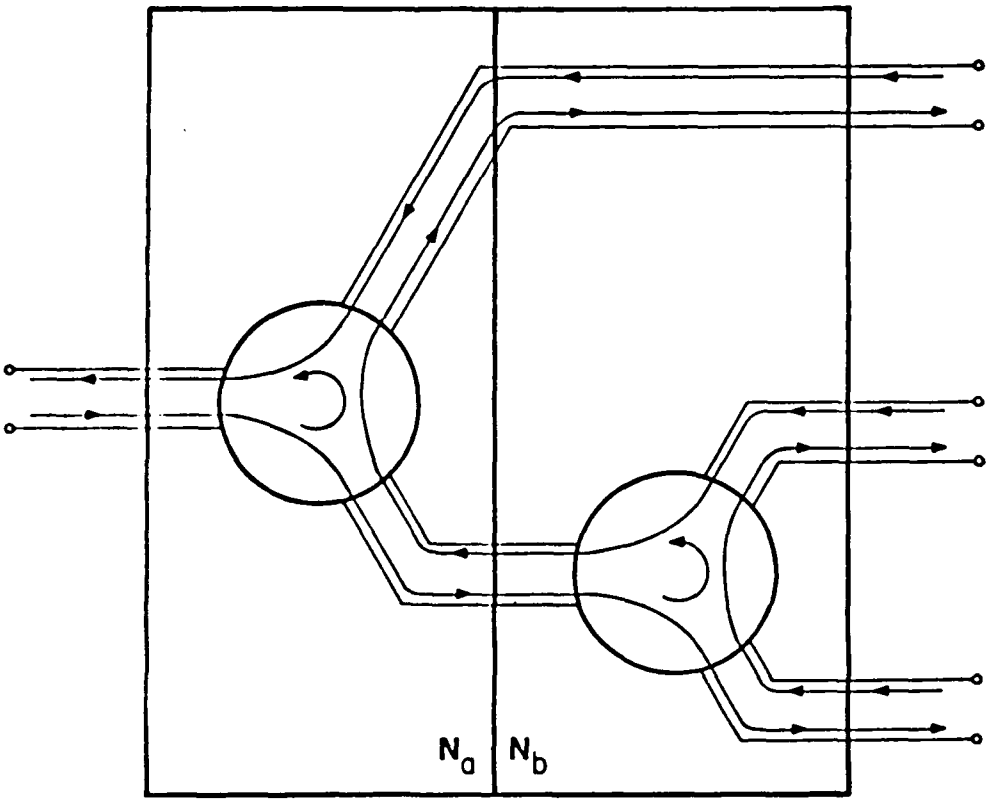


Fig. 6.2. Realization of a Four-Port Circulator.

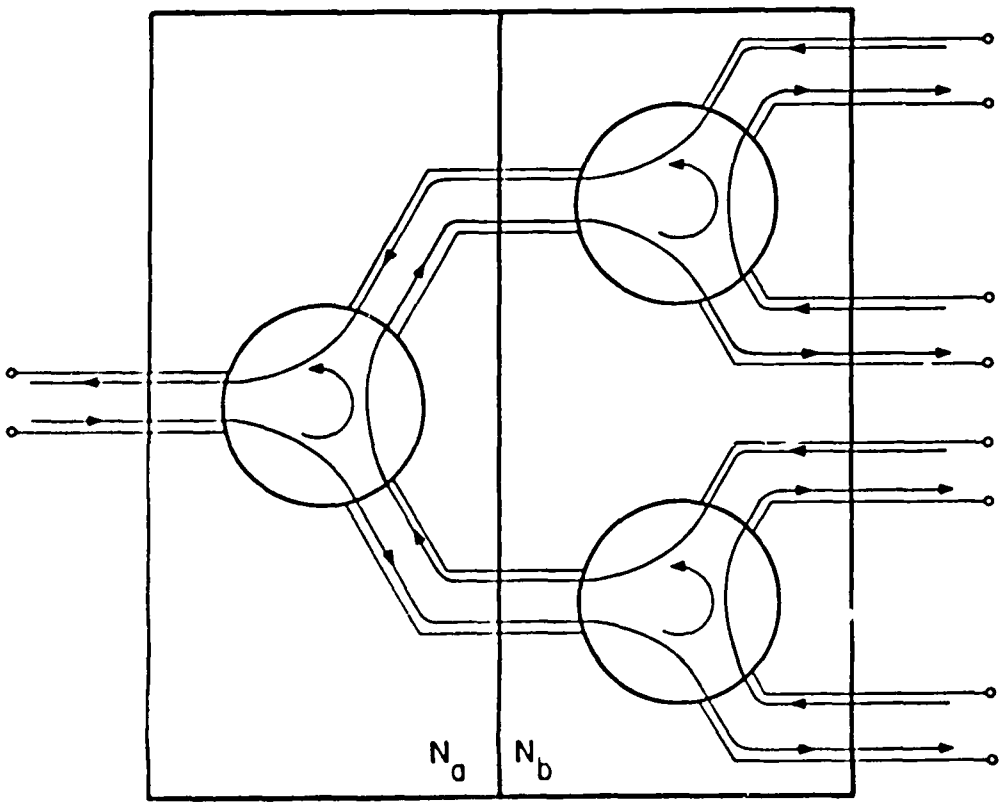


Fig. 6.3. Realization of a Five-Port Circulator.

of the composite multi-port circulators can be obtained by means of the formulas (6.6)-(6.9). For the circulator networks, these formulas reduce to

$$\underset{\sim}{S}_{11} = \underset{\sim}{0} \tag{6.16}$$

$$\underset{\sim}{S}_{12} = \underset{\sim}{S}_{12a}\underset{\sim}{S}_{12b} \tag{6.17}$$

$$\underset{\sim}{S}_{21} = \underset{\sim}{S}_{21b}\underset{\sim}{S}_{21a} \tag{6.18}$$

$$\underset{\sim}{S}_{22} = \underset{\sim}{S}_{22b} + \underset{\sim}{S}_{21b}\underset{\sim}{S}_{22a}\underset{\sim}{S}_{12b} \tag{6.19}$$

Now consider the network shown in Fig. 6.2. The partitioned scattering matrices of the three-port circulator N_a and the five-port network N_b are given by

$$\underset{\sim}{S}_a = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \end{array}\right] \tag{6.20}$$

$$\underset{\sim}{S}_b = \left[\begin{array}{cc|ccc} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right] \tag{6.21}$$

respectively. The unit-normalized scattering matrix of the resulting network can be readily obtained by using (6.16)-(6.19):

$$\underset{\sim}{S} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \left[\begin{array}{c|c} \underset{\sim}{0} & \underset{\sim}{U}_3 \\ \hline 1 & \underset{\sim}{0} \end{array}\right] \tag{6.22}$$

which is the scattering matrix of a four-port circulator.

To obtain the scattering matrix of Fig. 6.3, we use the partitioned scattering matrix of the six-port network N_b which is found to be

$$\underset{\sim}{S}_b = \left[\begin{array}{cc|cccc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \tag{6.23}$$

By (6.16)-(6.19), the scattering matrix of the five-port circulator of Fig. 6.3 is obtained as

$$\underset{\sim}{S} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \left[\begin{array}{c|c} \underset{\sim}{0} & \underset{\sim}{U}_4 \\ \hline 1 & \underset{\sim}{0} \end{array}\right] \tag{6.24}$$

We now interpret the above results from the power transmission characteristics of the circulator. As is well known, in a three-port circulator the wave entering port 1 is transmitted to port 3. The wave entering port 3 is transmitted to port 2, and the wave entering port 2 is transmitted to port 1. This is represented schematically by the arrows in Figs. 6.2 and 6.3, respectively. Their scattering matrices (6.22) and (6.24) are determined from the power transmission requirements of Figs. 6.2 and 6.3. The above results can be extended to the interconnection of the multi-port circulators. Figure 6.4 is an illustrative example in which the interconnection of two three-port

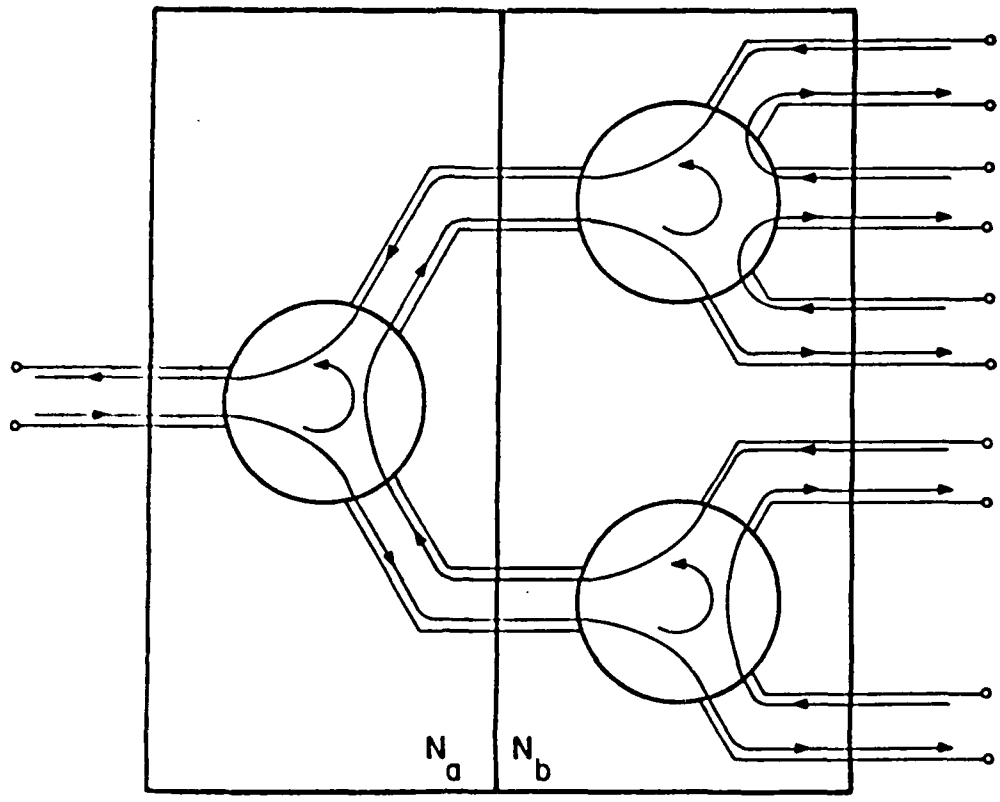


Fig. 6.4. Interconnection of Two Three-Port Circulators and a Four-Port Circulator.

circulators and a four-port circulator results in a six-port circulator with the following unit-normalized scattering matrix:

$$\underset{\sim}{S} = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{U}_5 \\ 1 & \underset{\sim}{0} \end{bmatrix} \tag{6.25}$$

6.4 Design procedure and illustrative examples

The approach mentioned in [52] for designing a diplexer can be used for the design of a multiplexer. However in the present case, all the reciprocal two-port networks are bandpass filters and the circuit structure is more complicated than that of the diplexer.

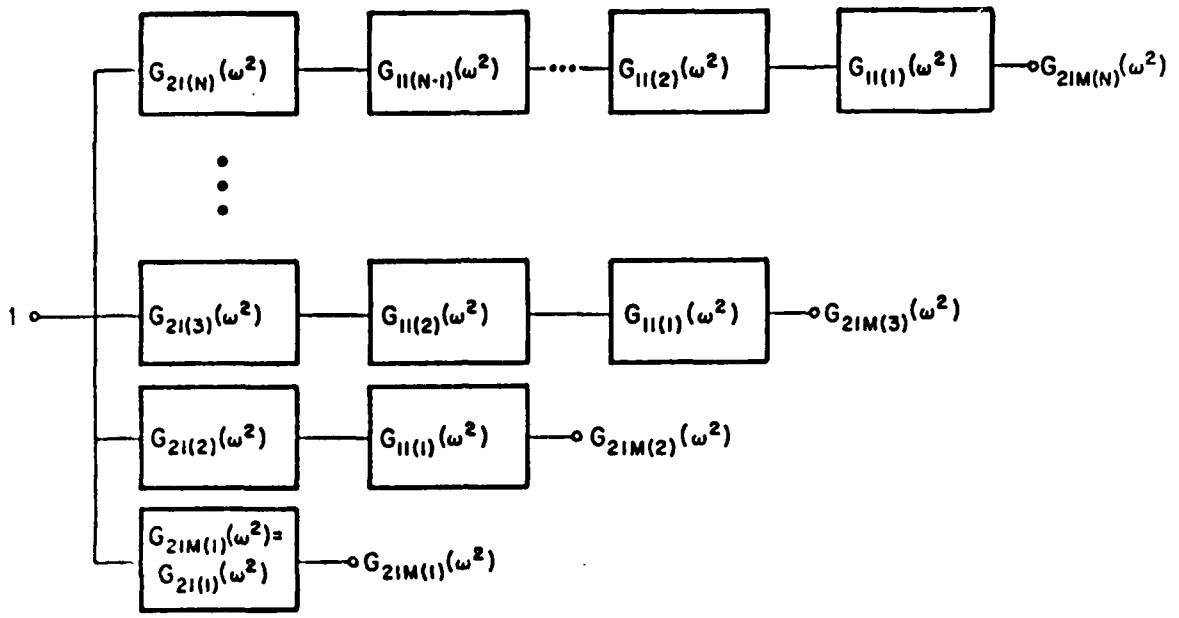


Fig. 6.5. An Equivalent Representation of Equation (6.13).

An equivalent representation of (6.13) is depicted in the block diagram of Fig. 6.5, where the existence of the block $S_{11(i)}(s)$, $i=1,2,\ldots,(N-1)$, causes the channel characteristics to deviate from those of the corresponding two-port networks. As an example, we consider a four-channel multiplexer where the four reciprocal two-port networks are Butterworth bandpass filters of order of 4. The plots of $|S_{21M(i)}(j\omega)|^2$, $i=1,2,3,4$, as a function of ω are shown in Fig. 6.6 and the plots of $|S_{21(4)}(j\omega)|^2$ and $\prod_{k=1}^{3} |S_{11(k)}(j\omega)|^2$ are presented in Fig. 6.7. We notice that the value of $\prod_{k=1}^{i-1} |S_{11(k)}(j\omega)|^2$ is much smaller than that of $|S_{21(i)}(j\omega)|^2$ in the passband of channel i, so that in the passband of channel i the contributions of $|S_{11(k)}(j\omega)|^2$, $k < i$ is much smaller than that of $|S_{21(i)}(j\omega)|^2$. Since the attenuation of the low-pass prototype filter yields an asymptotic slope of 6n dB/octave for either the Butterworth or the Chebyshev response, the contributions of $|S_{11(k)}(j\omega)|^2$, $k < i-1$, may be ignored when channel k is not adjacent to channel i. Equations (6.13) and (6.14) may be simplified to

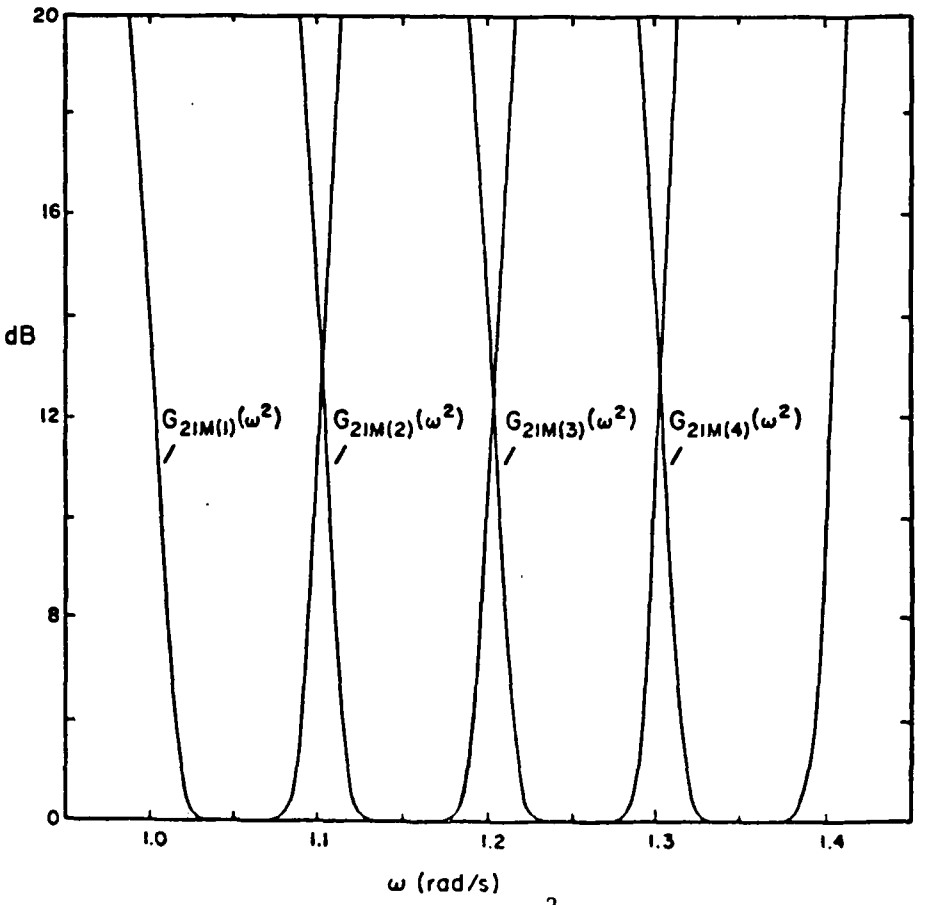


Fig. 6.6. The Plots of $|S_{21M(i)}(j\omega)|^2$ as a Function of ω for i=1,2,3,4.

$$S_{21M(i)} = S_{21(i)}S_{11(i-1)} \tag{6.26}$$

$$G_{21M(i)}(\omega^2) = |S_{21(i)}(j\omega)|^2 \; |S_{11(i-1)}(j\omega)|^2 \tag{6.27}$$

We illustrate the above procedure by the following numerical examples:

Example 6.1. We wish to design a four-channel multiplexer having 4th-order Butterworth response. The 3-dB edge frequencies for the channels are

Channel 1 $\omega_{11} = 1.01$, $\omega_{21} = 1.09$;

Chennel 2 $\omega_{12} = 1.11$, $\omega_{22} = 1.19$;

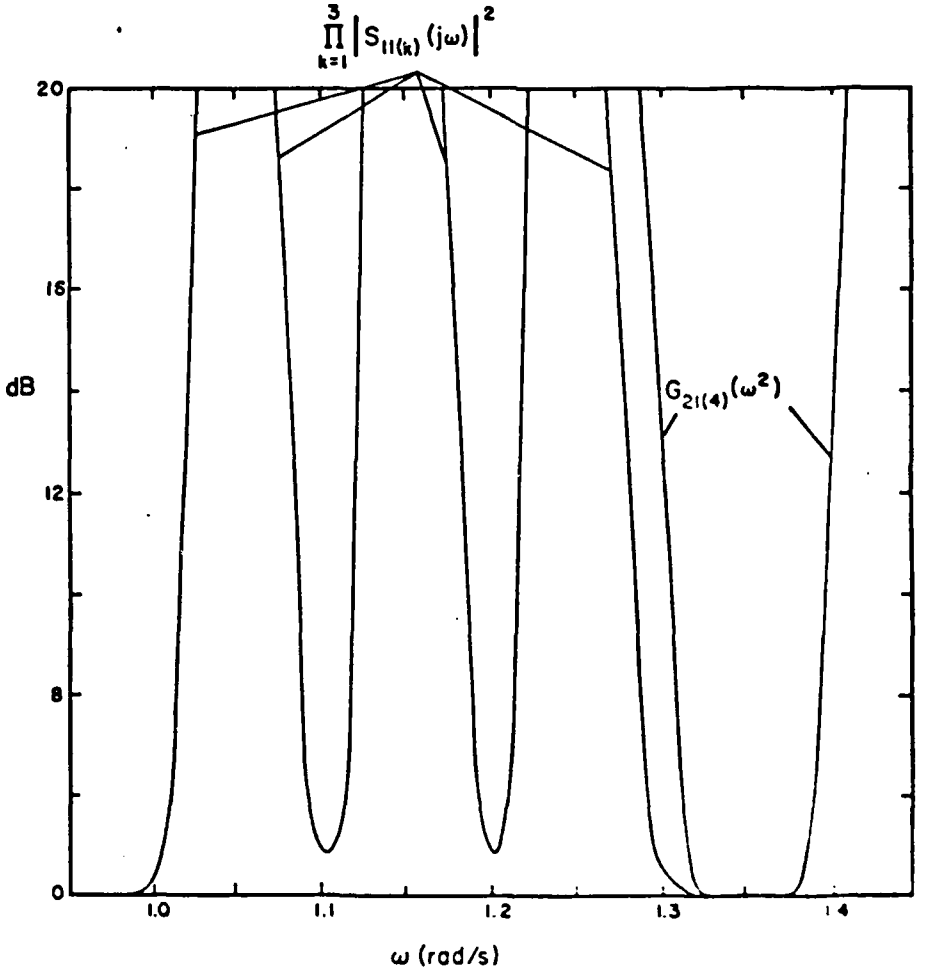


Fig. 6.7. The Plots of $|S_{21(4)}(j\omega)|^2$ and $\prod_{k=1}^{3} |S_{11(k)}(j\omega)|^2$ as a function of ω.

Channel 3 $\omega_{13} = 1.21$, $\omega_{23} = 1.29$;

Channel 4 $\omega_{14} = 1.31$, $\omega_{24} = 1.39$.

Suppose that we choose the canonical Butterworth networks to be the two-ports. Following the above procedure, we obtain the circuit and the element values shown in Fig. 6.8. The frequency response is presented in Fig. 6.6. The losses at the edge frequencies are given by

$G_{21M(1)}(\omega_{11}^2) = 3.01$ dB, $G_{21M(1)}(\omega_{21}^2) = 3.01$ dB;

$G_{21M(2)}(\omega_{11}^2) = 3.197$ dB, $G_{21M(2)}(\omega_{22}^2) = 3.01$ dB;

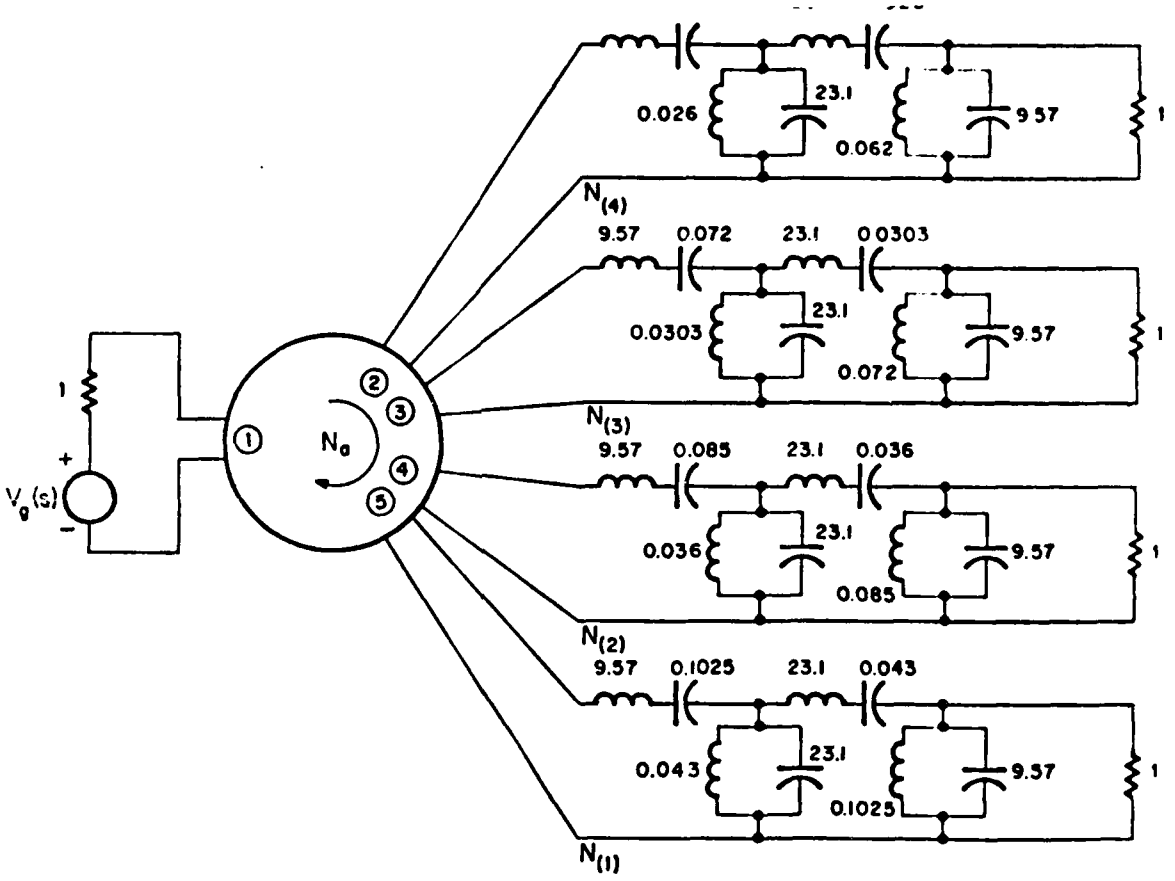


Fig. 6.8. A Four-Channel Multiplexer having Fourth-Order Butterworth response.

$$G_{21M(3)}(\omega_{13}^2) = 3.195 \text{ dB}, \qquad G_{21M(3)}(\omega_{23}^2) = 3.01 \text{ dB};$$

$$G_{21M(4)}(\omega_{14}^2) = 3.194 \text{ dB}, \qquad G_{21M(4)}(\omega_{24}^2) = 3.01 \text{ dB}.$$

The insertion losses at the crossover frequencies are about 11 dB.

Example 6.2. Repeat the problem considered in Example 6.1 with the 5th-order Butterworth response, using the canonical Butterworth two-port networks. The final circuit and all the element values are shown in Fig. 6.9. The losses at the edge frequencies are given by

$$G_{21M(1)}(\omega_{11}^2) = 3.01 \text{ dB}, \qquad G_{21M(1)}(\omega_{21}^2) = 3.01 \text{ dB};$$

$$G_{21M(2)}(\omega_{12}^2) = 3.09 \text{ dB}, \qquad G_{21M(2)}(\omega_{22}^2) = 3.01 \text{ dB};$$

$$G_{21M(3)}(\omega_{13}^2) = 3.09 \text{ dB}, \qquad G_{21M(3)}(\omega_{23}^2) = 3.01 \text{ dB};$$

$$G_{21M(4)}(\omega_{14}^2) = 3.09 \text{ dB}, \qquad G_{21M(4)}(\omega_{24}^2) = 3.01 \text{ dB}.$$

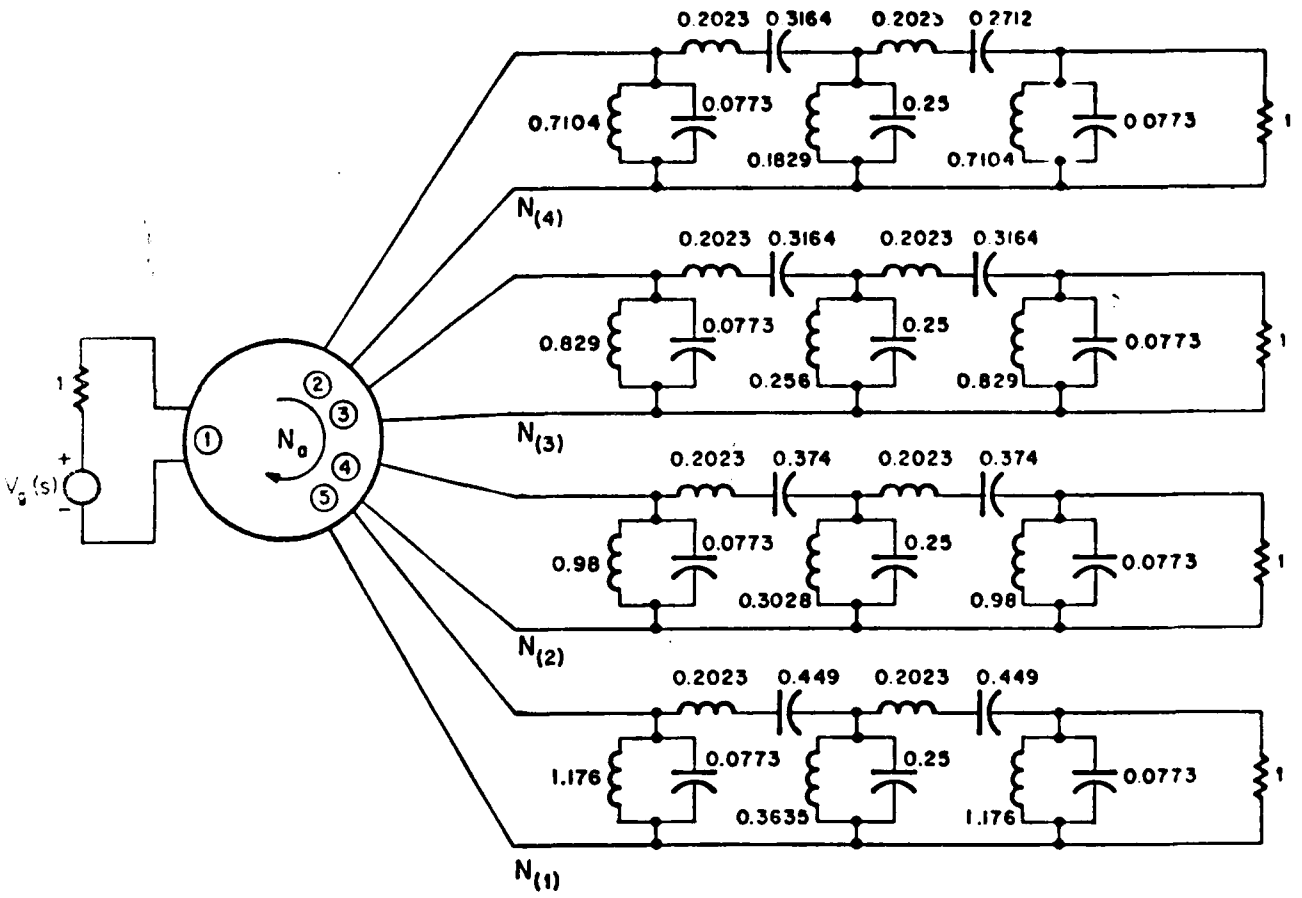


Fig. 6.9. A Four-Channel Multiplexer having Fifth-Order Butterworth Response.

The frequency response is presented in Fig. 6.10. The insertion losses at the crossover frequencies are about 13 dB.

6.5 Conclusion

In this section we presented a nonreciprocal multiplexer configuration composed of an ideal multi-port circulator and a number of reciprocal lossless two-port networks, terminating in the source and loads. Having expressed the scattering parameters of the multiplexer in terms of those of the component networks, we reduce the problem of designing a multiplexer to that of realizing the reciprocal two-port networks.

We also showed that a multi-port circulator can always be realized as an interconnection of three-port circulators. Design procedure for a nonreciprocal multiplexer and illustrative examples were given.

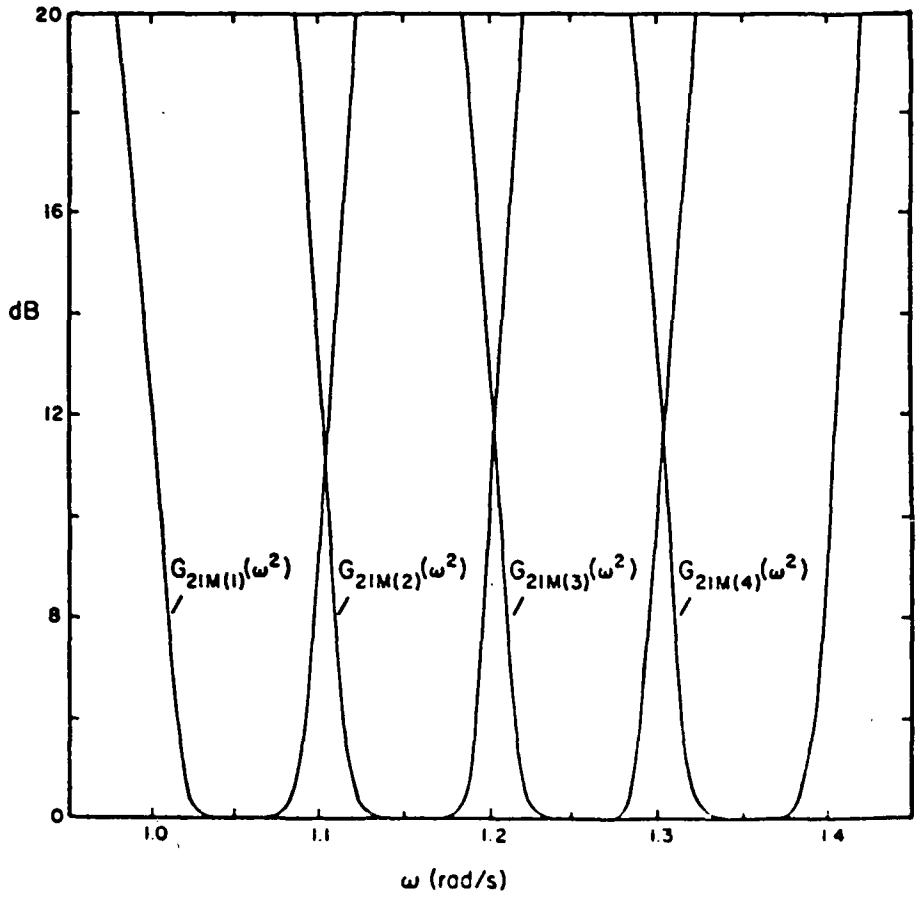


Fig. 6.10. The Frequency Response of a 5th-Order Butterworth Four-Channel Multiplexer.

Section 7

THE DESIGN OF A SYMMETRICAL DIPLEXER COMPOSED OF CANONICAL BUTTERWORTH TWO-PORT NETWORKS

7.1 Introduction

Section 7 presents a new approach to the design of a diplexer having given insertion loss at the crossover frequency. The diplexer consists of a low-pass two-port network and a high-pass two-port network connecting either in series or in parallel. Having expressed the transducer power gain of the diplexer in terms of the Butterworth polynomials, we show that the cut-off frequencies of the low-pass and high-pass networks must be symmetrical with respect to the crossover frequency in order to obtain a diplexer with symmetrical characteristic. The problem of designing a symmetrical diplexer is simplified to that of choosing the order of the Butterworth response and the cut-off frequency for the individual two-ports. A computer program DIPLX is available for computing the circuit elements as well as the final frequency response.

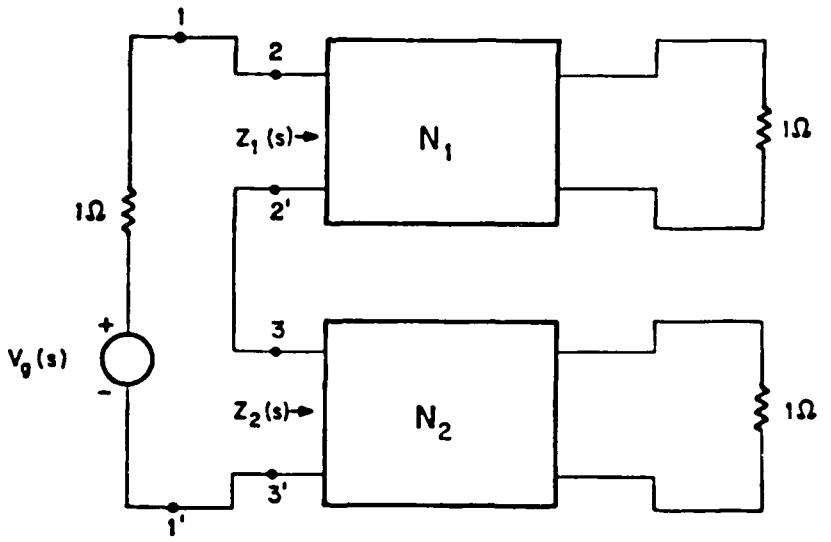


Fig. 7.1(a). The Series Configuration of a Diplexer.

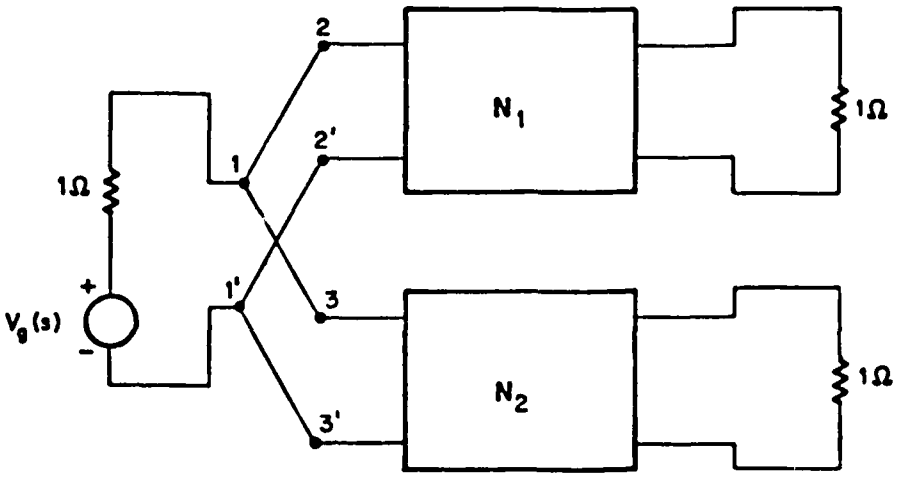


Fig. 7.1(b). The Parallel Configuration of a Diplexer.

The design of a diplexer separating a frequency spectrum into two channels of signals is one of the basic problems in communications. Figure 7.1 shows the most popular configuration of a diplexer composed of a low-pass two-port network and a high-pass two-port network connected either in series (Fig. 7.1a) or in parallel (Fig. 7.1b).

Because of the mutual interaction effect of the two two-ports, the transducer power-gain characteristic of a diplexer is different from the responses of the individual two-ports. The complexity of the interaction effect makes the design of a diplexer very complicated. Early design of a contiguous low-pass and high-pass diplexer gives only 3-dB insertion loss in each channel at the crossover frequency [40]. To obtain a sharper separation, other design approaches have been presented [37,42]. Recently Zhu and Chen [61] presented an analytic approach to obtain a given insertion loss which may be greater than 3-dB at the crossover frequency. The coefficients of the driving-point impedances of both two-ports can be determined by solving a number of nonlinear equations. Then the two-ports can be realized by the traditional approach. However, this approach requires

a sufficient knowledge in network theory, and the formulation and solution of simultaneous algebraic equations.

In this section, we study the design of a diplexer composed of two canonical Butterworth two-port networks having low-pass and high-pass characteristics. The problem can be simplified to that of choosing the order of the Butterworth response and determining the cut-off frequencies of the Butterworth two-port networks. To simplify our discussion, we assume that the characteristic of the diplexer is symmetrical with respect to the crossover frequency. This means that the order of the two-port networks must be the same. In paragraph 7.2, the transducer power gains of the diplexer will be expressed in terms of the Butterworth polynomials. To obtain a symmetrical characteristic, the cut-off frequency ω_c' of the low-pass two-port network and that ω_c'' of the high-pass two-port network should be chosen symmetrically with respect to the crossover frequency. Illustrative examples are presented in paragraph 7.3. Computer programs for obtaining the final network configuration, its element values, and the frequency response curve of its transducer power gain are available.

7.2 Transducer power-gain characteristics of a symmetrical diplexer

Consider a symmetrical diplexer formed by the connection of the lossless reciprocal two-port networks having low-pass and high-pass Butterworth characteristics. Assume that the low-pass two-port N_1 is described by its unit-normalized scattering parameters $S'_{ij}(s)$ $(i,j=1,2)$ and possesses the nth-order Butterworth transducer power-gain characteristic

$$G'(\omega^2) = |S'_{12}(j\omega)|^2 = \frac{k'}{1 + (\omega/\omega_c')^{2n}} \qquad (7.1)$$

where ω_c' is the 3-dB radian bandwidth or radian cut-off frequency, and $0 \leq k' \leq 1$. Without loss of generality we assume $k' = 1$. Appealing to the theorem on the uniqueness of analytic continuation of a complex variable function gives [14]

$$G'(-s^2) = S'_{12}(s)S'_{12}(-s) = \frac{1}{1 + (-1)^n y^{2n}} = \frac{1}{q(y)q(-y)} \tag{7.2}$$

where $y = s/\omega_c'$ and $q(s)$ is the Butterworth polynomial. Applying the para-unitary property of the scattering matrix of a lossless reciprocal two-port, which states that

$$S'_{11}(s)S'_{11}(-s) = 1 - G'(-s^2) = \frac{(-1)^n y^{2n}}{q(y)q(-y)} \tag{7.3}$$

we find the minimum-phase reflection coefficient to be

$$S'_{11}(s) = \pm \frac{y^n}{q(y)} \tag{7.4}$$

The impedance looking into the input port is given by [14]

$$Z_1(s) = \frac{1 + S'_{11}(s)}{1 - S'_{11}(s)} = \frac{q(y) \pm y^n}{q(y) \mp y^n} \tag{7.5}$$

the $\pm$ signs being determined in accordance with the circuit structures. The choice of a plus sign in (7.4) corresponds to the circuit structure shown in Fig. 7.2(b) which fits the diplexer of Fig. 7.1(b), and the choice of a minus sign to the structure of Fig. 7.2(a) which suits the need of Fig. 7.1(a).

Assume that the high-pass two-port N_2 is characterized by its scattering parameters $S''_{ij}(s)$ $(i,j = 1,2)$ and possesses the nth-order Butterworth transducer power-gain characteristic

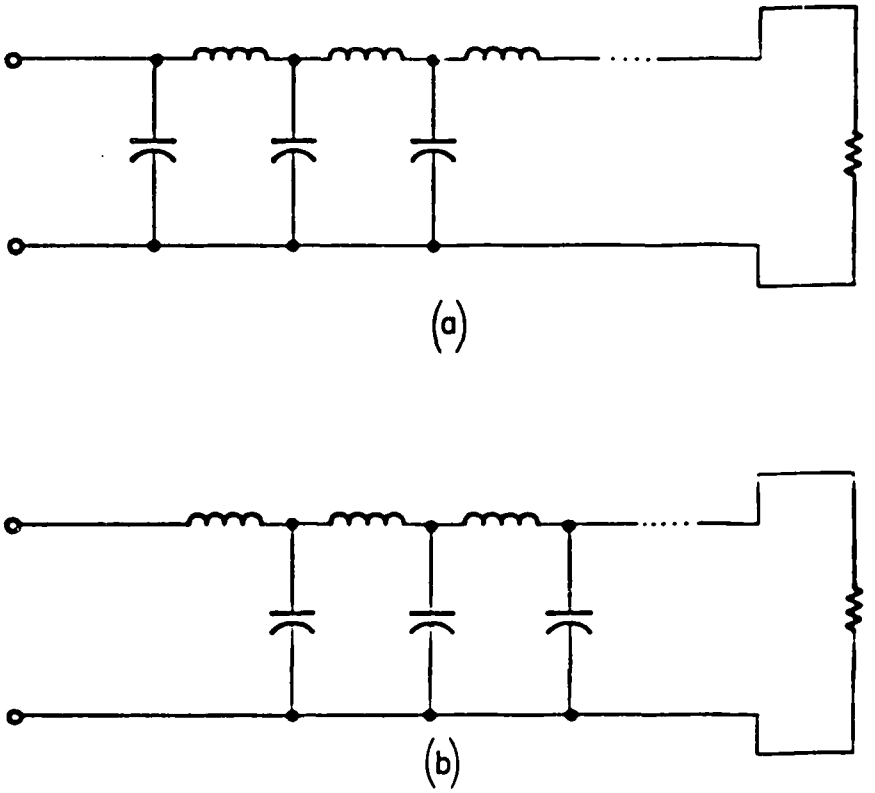


Fig. 7.2. Two Low-Pass LC Ladders Terminated in a Resistor.

$$G''(\omega^2) = |S''_{12}(j\omega)|^2 = \frac{k''(\omega/\omega''_c)^{2n}}{1 + (\omega/\omega''_c)^{2n}} \tag{7.6}$$

where ω''_c is the 3-dB cut-off frequency, $0 \leq k'' \leq 1$ and we again assume $k'' = 1$.

By appealing to the theorem on uniqueness of analytic continuation of a complex variable function and the para-unitary property of the scattering matrix, we obtain

$$G''(-s^2) = S''_{12}(s)S''_{12}(-s) = \frac{(-1)^n z^{2n}}{1 + (-1)^n z^{2n}} = \frac{(-1)^n z^{2n}}{q(z)q(-z)} \tag{7.7}$$

$$S''_{11}(s)S''_{11}(-s) = 1 - G''(-s^2) = \frac{1}{q(z)q(-z)} \tag{7.8}$$

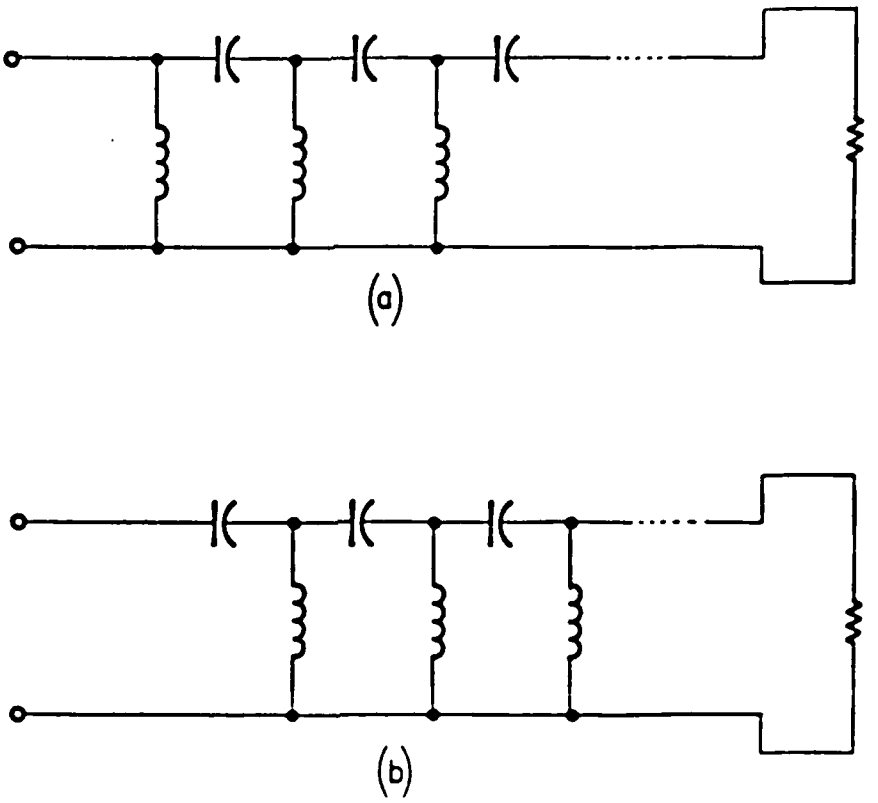


Fig. 7.3. Two High-Pass LC Ladders Terminated in a Resistor.

where $z = s/\omega_c''$. The minimum-phase reflection coefficient is found to be

$$S_{11}''(s) = \pm \frac{1}{q(z)} \tag{7.9}$$

The impedance looking into the input port becomes

$$Z_2(s) = \frac{1 + S_{11}''(s)}{1 - S_{11}''(s)} = \frac{q(z) \pm 1}{q(z) \mp 1} \tag{7.10}$$

The choice of a plus sign in (7.9) corresponds to the circuit structure shown in Fig. 7.3(b) which fits to the diplexer of Fig. 7.1(b), and the minus sign to the structure of Fig. 7.3(a) which suits the diplexer of Fig. 7.1(a)

We now derive expressions relating the scattering parameters $S_{ij}(s)$ $(i,j = 1,2,3)$ of the three-port diplexer shown in Fig. 7.1 in terms of

the Butterworth polynomials q(y) and q(z), so that the parameters of the individual two-ports may be determined according to the given specifications of the three-port diplexer at the crossover frequency or in the passband or both. For simplicity, throughout the remainder of this section, let the crossover frequency be 1.

In Fig. 7.1(a), the reflection coefficients $S_{ii}(s)$ $(i=1,2,3)$ normalized to the reference impedances 1, $Z_1(s)$ and $Z_2(s)$ at the ports 11', 22' and 33' can be related to $Z_1(s)$ and $Z_2(s)$ by [14]

$$S_{11}(s) = \frac{Z_1(s) + Z_2(s) - 1}{Z_1(s) + Z_2(s) + 1} \tag{7.11}$$

$$S_{22}(s) = \frac{h_1(s)}{h_1(-s)} \cdot \frac{Z_2(s) - Z_1(-s) + 1}{Z_1(s) + Z_2(s) + 1} \tag{7.12}$$

$$S_{33}(s) = \frac{h_2(s)}{h_2(-s)} \cdot \frac{Z_1(s) - Z_2(-s) + 1}{Z_1(s) + Z_2(s) + 1} \tag{7.13}$$

where

$$h_i(s)h_i(-s) = \tfrac{1}{2}[Z_i(s) + Z_i(-s)], \qquad i = 1,2 \tag{7.14}$$

and the factorization is to be performed, so that $h_i(s)$ and $h_i^{-1}(-s)$ are analytic in the open right-half of the s-plane. We recognize that $h_i(s)/h_i(-s)$ is a real regular all-pass function.

Since the diplexer is assumed to be lossless and reciprocal, its scattering matrix $\underset{\sim}{S}(s) = [S_{ij}(s)]$ normalized to the strictly passive impedances 1, $Z_1(s)$ and $Z_2(s)$ is para-unitary, i.e. [14]

$$\underset{\sim}{S}(s)\underset{\sim}{S}'(-s) = \underset{\sim}{S}(-s)\underset{\sim}{S}'(s) = U. \tag{7.15}$$

where the prime denotes the matrix transpose. As mentioned above,

the circuit structures of the low-pass and high-pass two-ports in Fig. 7.1(a) should be chosen as in Fig. 7.2(a) and Fig. 7.3(a), respectively. Thus, we choose the minus sign both in (7.4) and (7.9), and (7.5) and (7.10) become

$$Z_1(s) = \frac{q(y) - y^n}{q(y) + y^n} \tag{7.16}$$

$$Z_2(s) = \frac{q(z) - 1}{q(z) + 1} \tag{7.17}$$

From (7.11)-(7.17), we can ascertain $S_{12}(s)S_{12}(-s)$, $S_{13}(s)S_{13}(-s)$ and $S_{23}(s)S_{23}(-s)$ in terms of the Butterworth polynomials $q(y)$ and $q(z)$. After factorization, we obtain

$$S_{12}(s) = \theta_{12}(s) \frac{2[q(z) + 1]}{D(s)} \tag{7.18}$$

$$S_{13}(s) = \theta_{13}(s) \frac{2z^n[q(y) + y^n]}{D(s)} \tag{7.19}$$

$$S_{23}(s) = \theta_{23}(s) \frac{2z^n}{D(s)} \tag{7.20}$$

where

$$D(s) = 3q(y)q(z) + y^n[q(z) - 1] + q(y) \tag{7.21}$$

and

$$\theta_{12}(s) = \prod_i \frac{s - \sigma_i}{s + \sigma_i}, \qquad \text{Re } \sigma_i > 0 \tag{7.22}$$

$$\theta_{13}(s) = \prod_j \frac{s - \eta_j}{s + \eta_j}, \qquad \text{Re } \eta_j > 0 \tag{7.23}$$

$$\theta_{23}(s) = \prod_k \frac{s - \xi_k}{s + \xi_k}, \qquad \text{Re } \xi_k > 0 \tag{7.24}$$

are arbitrary real regular all-pass functions. The minimum-phase solutions can be written as

$$\hat{S}_{12}(s) = \frac{2[q(z) + 1]}{D(s)} \tag{7.25}$$

$$\hat{S}_{13}(s) = \frac{2z^n[q(y(+ y^n]}{D(s)} \tag{7.26}$$

$$\hat{S}_{23}(s) = \frac{2z^n}{D(s)} \tag{7.27}$$

The next problem is to choose the cut-off frequencies ω_c' and ω_c'', so that the diplexer has the symmetrical characteristic with respect to the crossover frequency. We assume that the low-pass and high-pass two-port networks are of the same order. To this end, we substitute s in (7.26) by 1/s and appeal to

$$q(1/x) = x^{-n}q(x) \tag{7.28}$$

obtaining

$$S_{13}(1/s) = \frac{2(\omega_c''s)^{-n}[(\omega_c's)^{-n}q(\omega_c's) + (\omega_c's)^{-n}]}{3(\omega_c's)^{-n}(\omega_c''s)^{-n}q(\omega_c's)q(\omega_c''s) + (\omega_c's)^{-n}[(\omega_c''s)^{-n}q(\omega_c''s)-1] + (\omega_c's)^{-n}q(\omega_c's)}$$

$$= \frac{2[q(\omega_c's) + 1]}{3q(\omega_c's)q(\omega_c''s) + (\omega''s)^n[q(\omega_c's)-1] + q(\omega''s)} \tag{7.29}$$

The symmetrical characteristic of the diplexer requires that

$$\hat{S}_{13}(1/s) = \hat{S}_{12}(s) \tag{7.30}$$

on the real-frequency axis. Substituting $y = s/\omega_c'$ and $z = s/\omega_c''$ into (7.25) gives

$$\hat{S}_{12}(s) = \frac{2[q(s/\omega_c'') + 1]}{3q(s/\omega_c')q(s/\omega_c'') + (s/\omega_c')^n[q(s/\omega_c'')-1] + q(s/\omega_c')} \tag{7.31}$$

By comparing (7.29) and (7.31), it is straightforward to confirm that if the cut-off frequencies of low-pass and high-pass two-port networks are related by

$$\omega_c'\omega_c'' = 1 \tag{7.32}$$

equation (7.30) will be satisfied and the diplexer is symmetrical.

We conclude from the above discussion that the problem in designing a symmetrical diplexer composed by a pair of low-pass and high-pass Butterworth two-port networks is equivalent to that of choosing the order n for both two-ports and their cut-off frequencies ω_c' and ω_c'' which are related by (7.32). Its transducer power-gain characteristic $|S_{12}(j\omega)|^2$ are plotted in Figs. 7.4 through 7.6 as a function of ω for various values of n and ω_c'.

Computer programs for solving the nonlinear equation (7.25) is available. The outputs are the circuit element values and the frequency response of the transducer power-gain characteristic. Because of the complexity of interaction effect between the two-port networks, checking the frequency response is necessary.

7.3 Illustrative examples

Examples illustrating the above approach are listed in Table 7.1. The first two examples are from Zhu and Chen's paper [61]. The final circuits and the transducer power-gain characteristics are shown in Figs. 7.7 through 7.12. The programs were run on the IBM 3801 system.

TABLE 7.1

Summary of Performance of Various Designed Symmetrical Diplexers.

	Insertion loss at $\omega = 1$	Order n	Final Results			
			ω_c'	ω_{3dB}	Execution time	Circuit and response curves
1	6.9 dB	3	0.71629	0.83	0.1	Fig. 7.7(a)(b)
2	12.75 dB	3	0.57852	0.62	0.07	Fig. 7.8(a)(b)
3	20 dB	3	0.44577	0.46	0.08	Fig. 7.9(a)(b)
4	20 dB	4	0.52471	0.52	0.09	Fig. 7.10(a)(b)
5	20 dB	5	0.56418	0.53	0.1	Fig. 7.11(a)(b)
6	40 dB	5	0.37882	0.37	0.09	Fig. 7.12(a)(b)

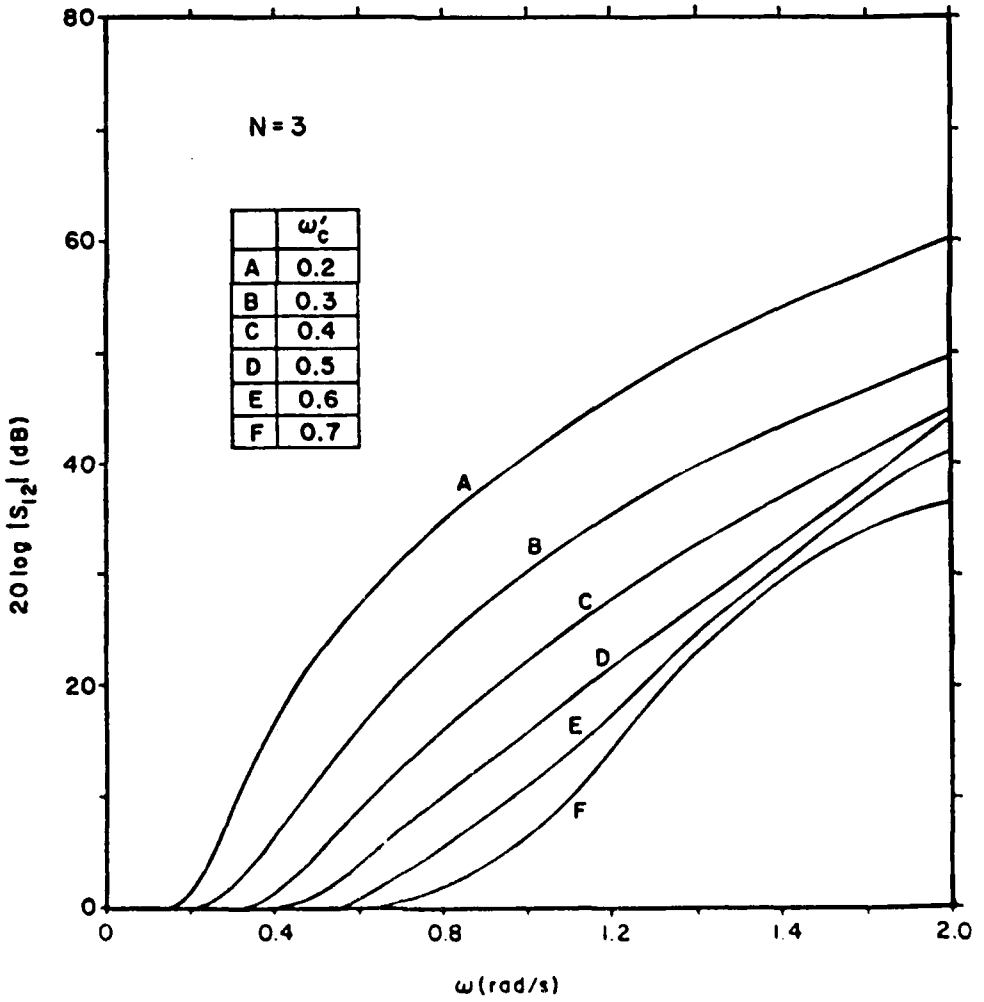


Fig. 7.4. The Third-Order Diplexer Transducer Power-Gain Characteristics.

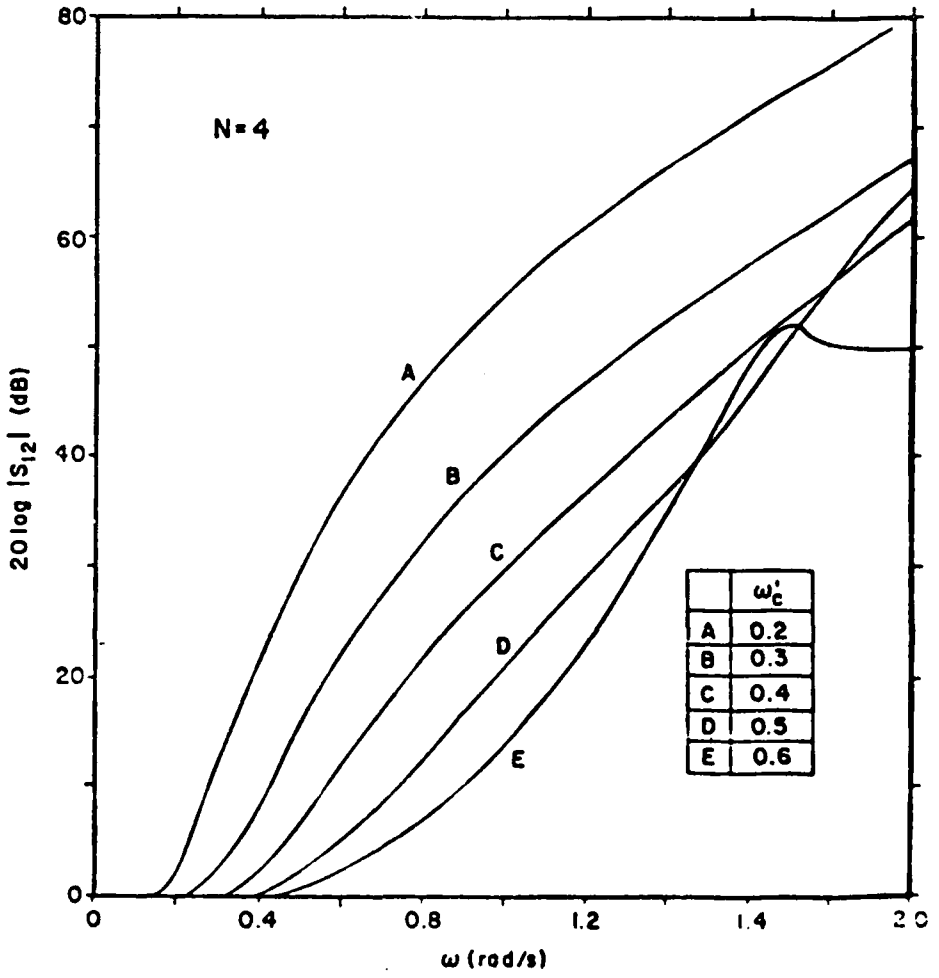


Fig. 7.5. The Fourth-Order Diplexer Transducer Power-Gain Characteristics.

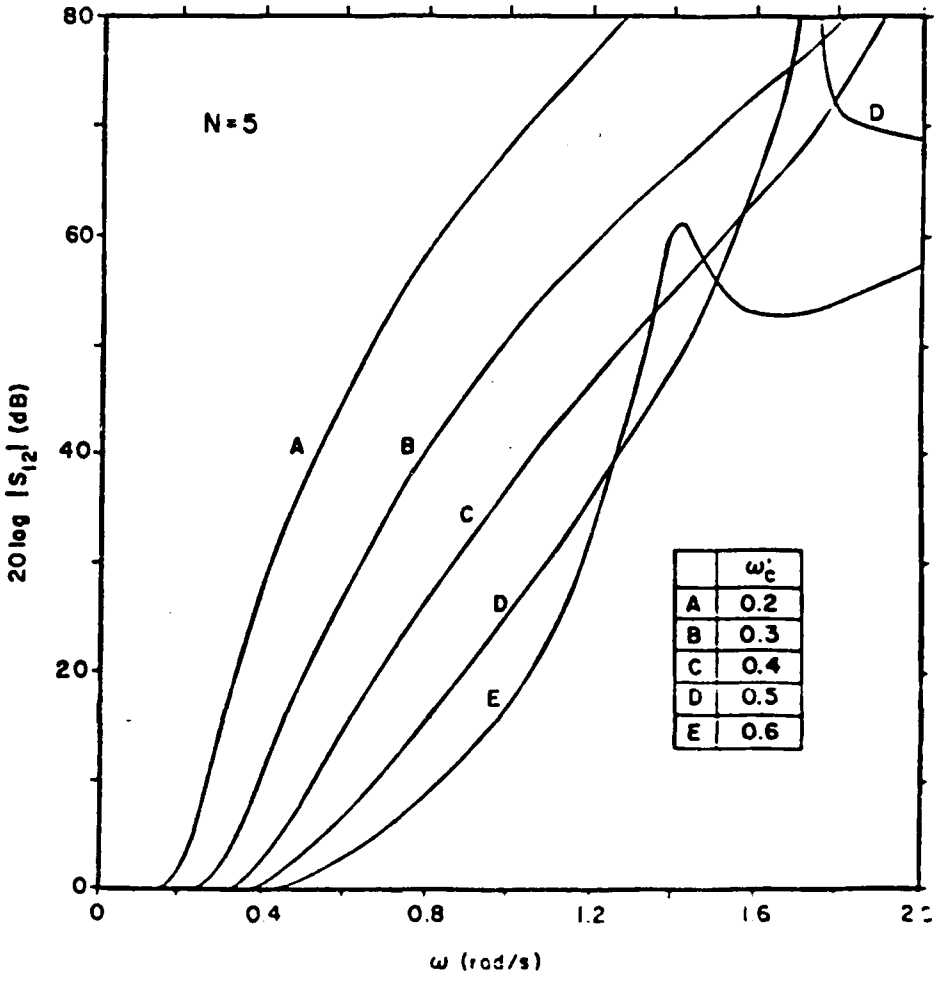


Fig. 7.6. The Fifth-Order Diplexer Transducer Power-Gain Characteristics.

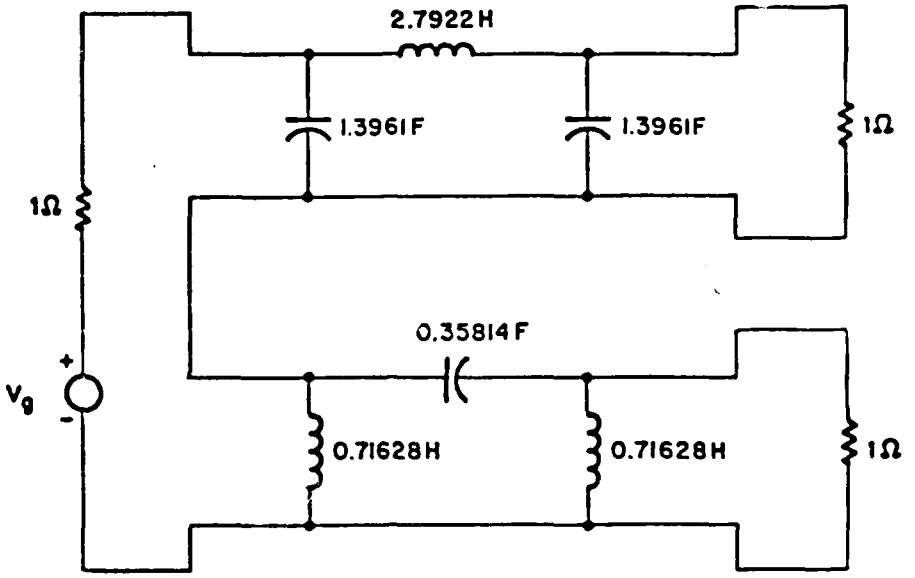


Fig. 7.7(a). A Third-Order Series Connected Diplexer Having 6.9-dB Insertion Loss at $\omega = 1$ rad/s.

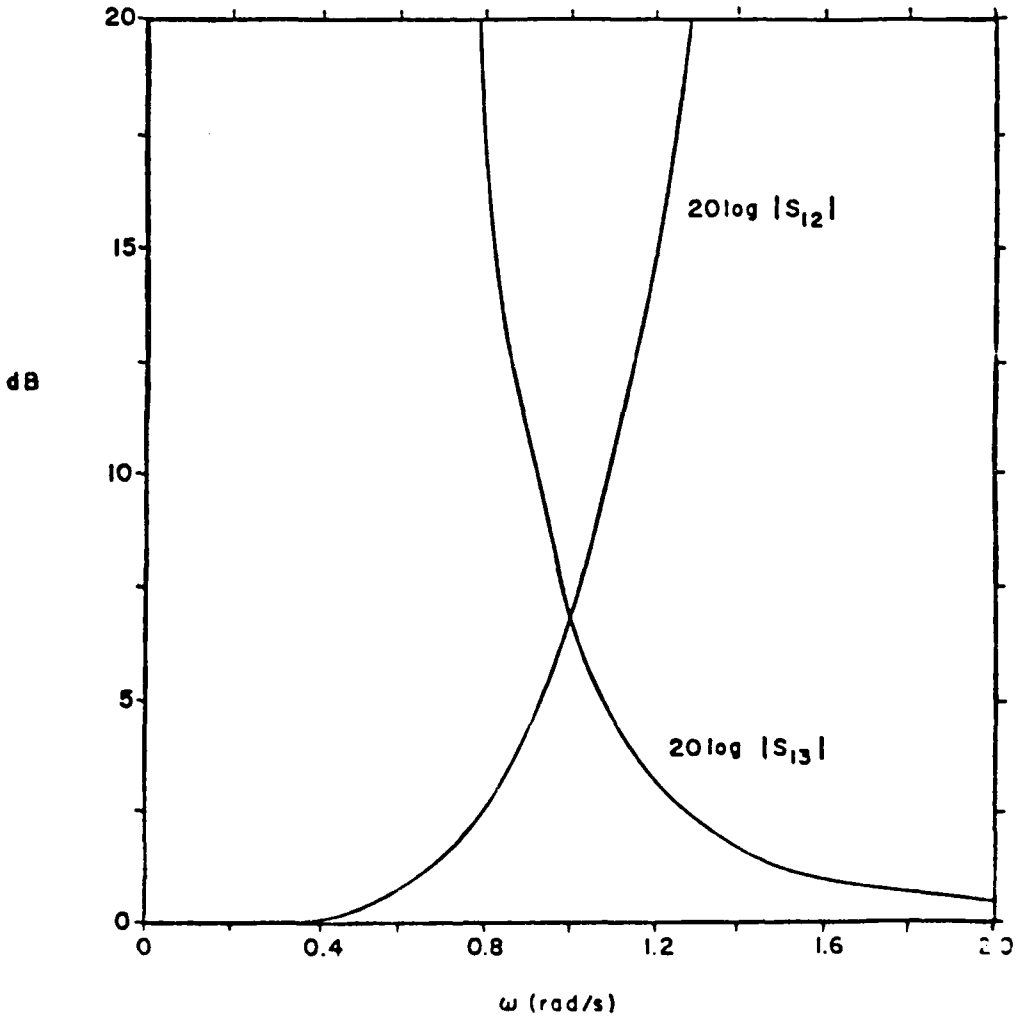


Fig. 7.7(b). The Frequency Response of the Diplexer of Fig. 7.7(a).

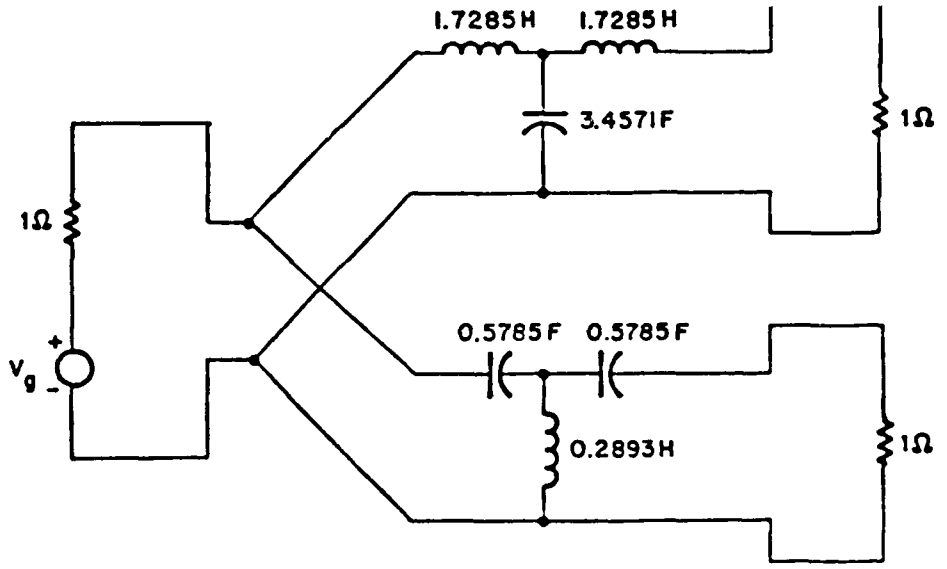


Fig. 7.8(a). A Third-Order Parallel-Connected Diplexer Having 12.75-dB Insertion Loss at $\omega = 1$ rad/s.

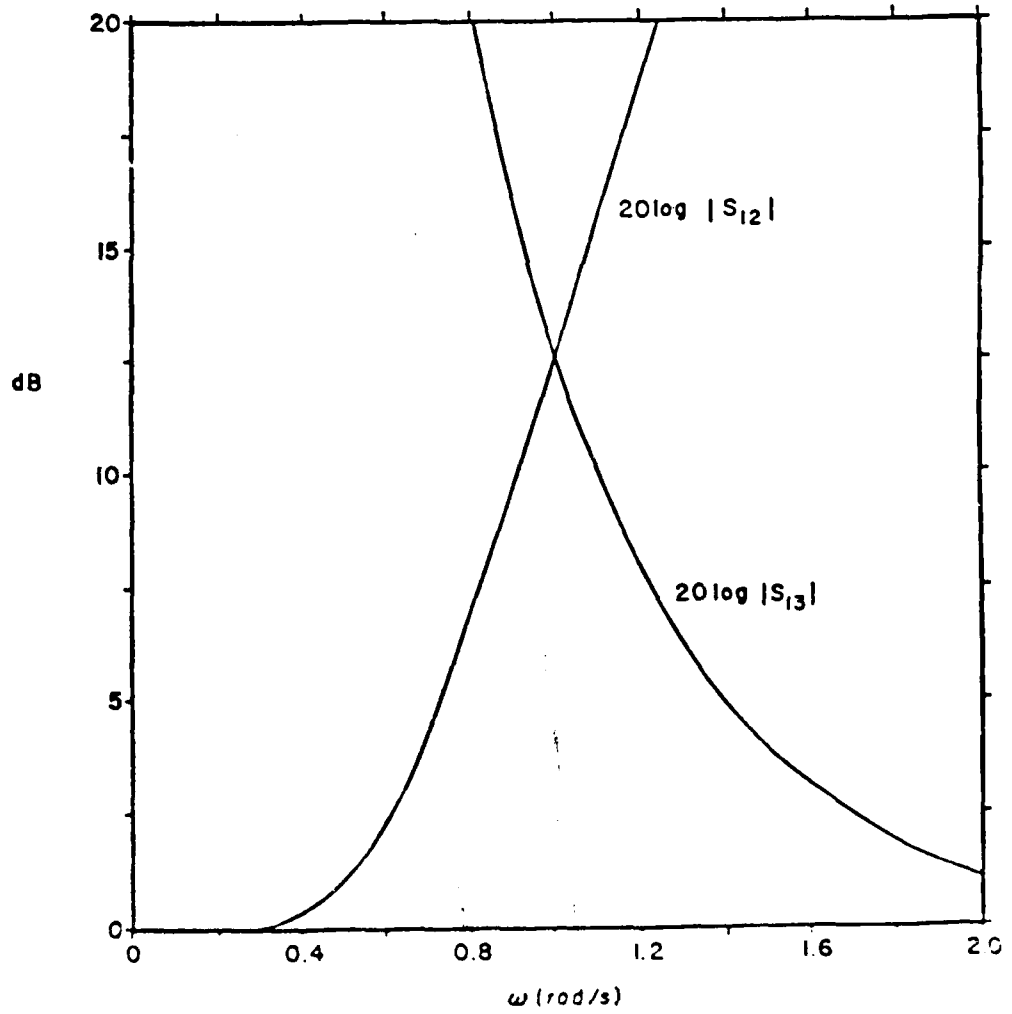


Fig. 7.8(b). The Frequency Response of the Diplexer of Fig. 7.8(a).

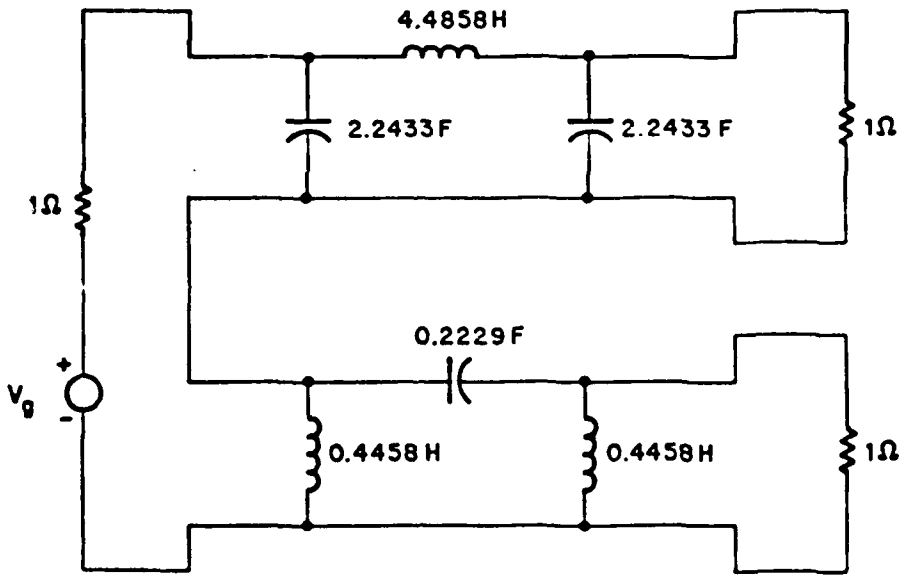


Fig. 7.9(a). A Third-Order Series-Connected Diplexer Having 20-dB Insertion Loss at $\omega = 1$ rad/s.

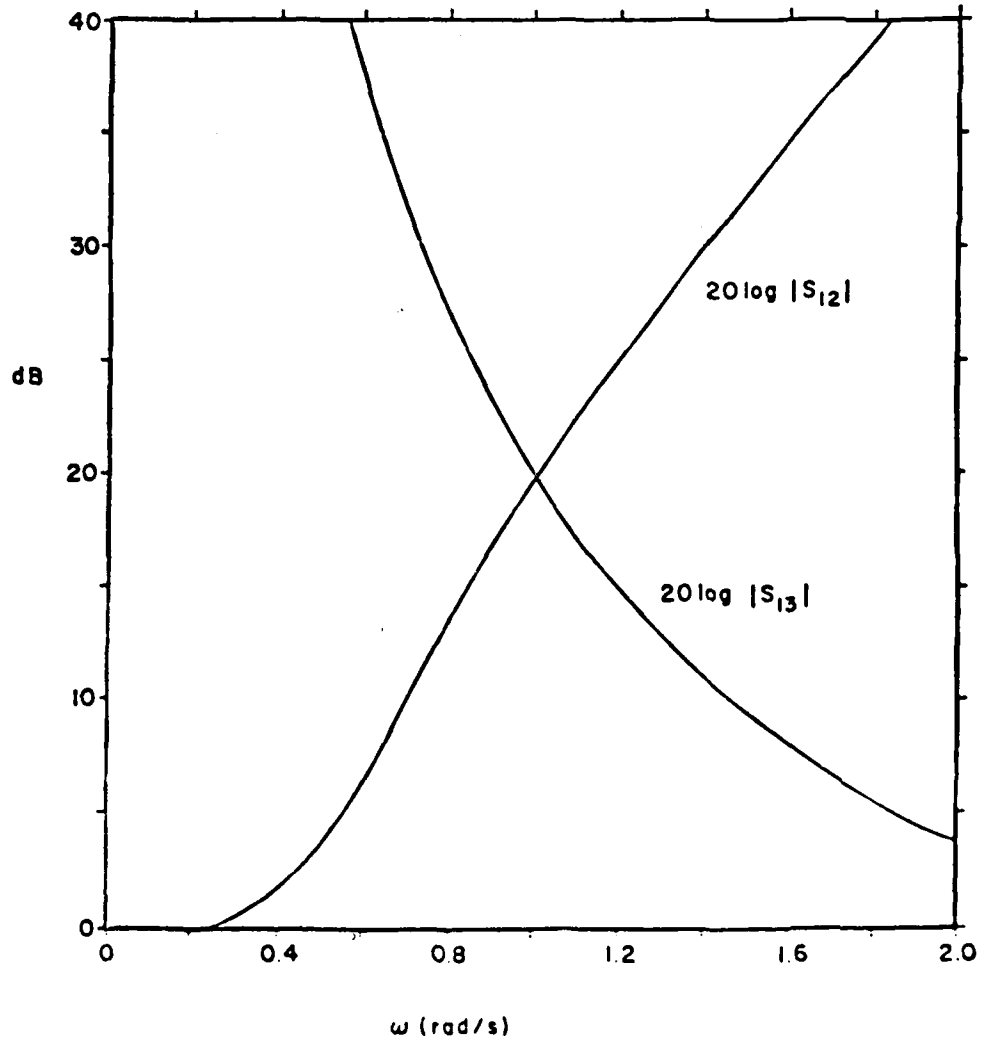


Fig. 7.9(b). The Frequency Response of the Diplexer of Fig. 7.9(a).

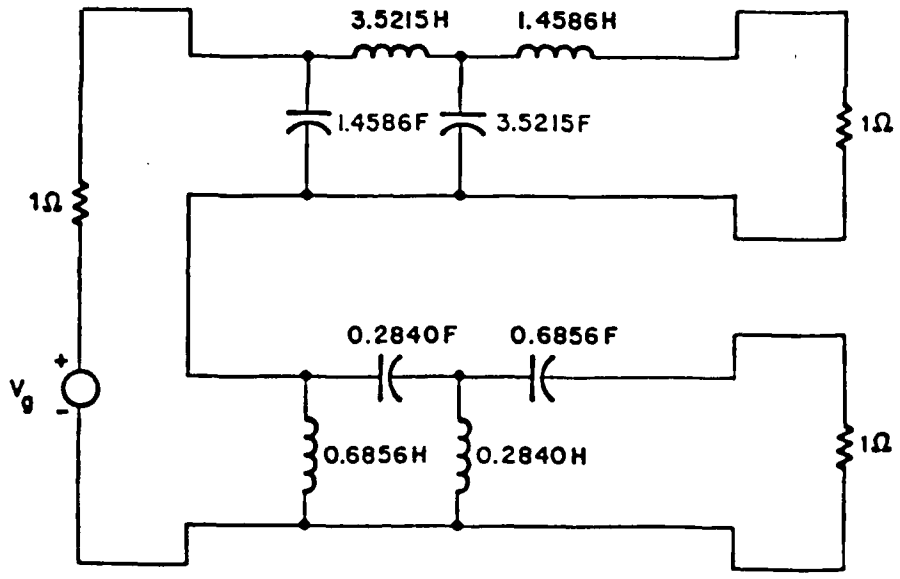


Fig. 7.10(a). A Fourth-Order Series-Connected Diplexer Having 20-dB Insertion Loss at $\omega = 1$ rad/s.

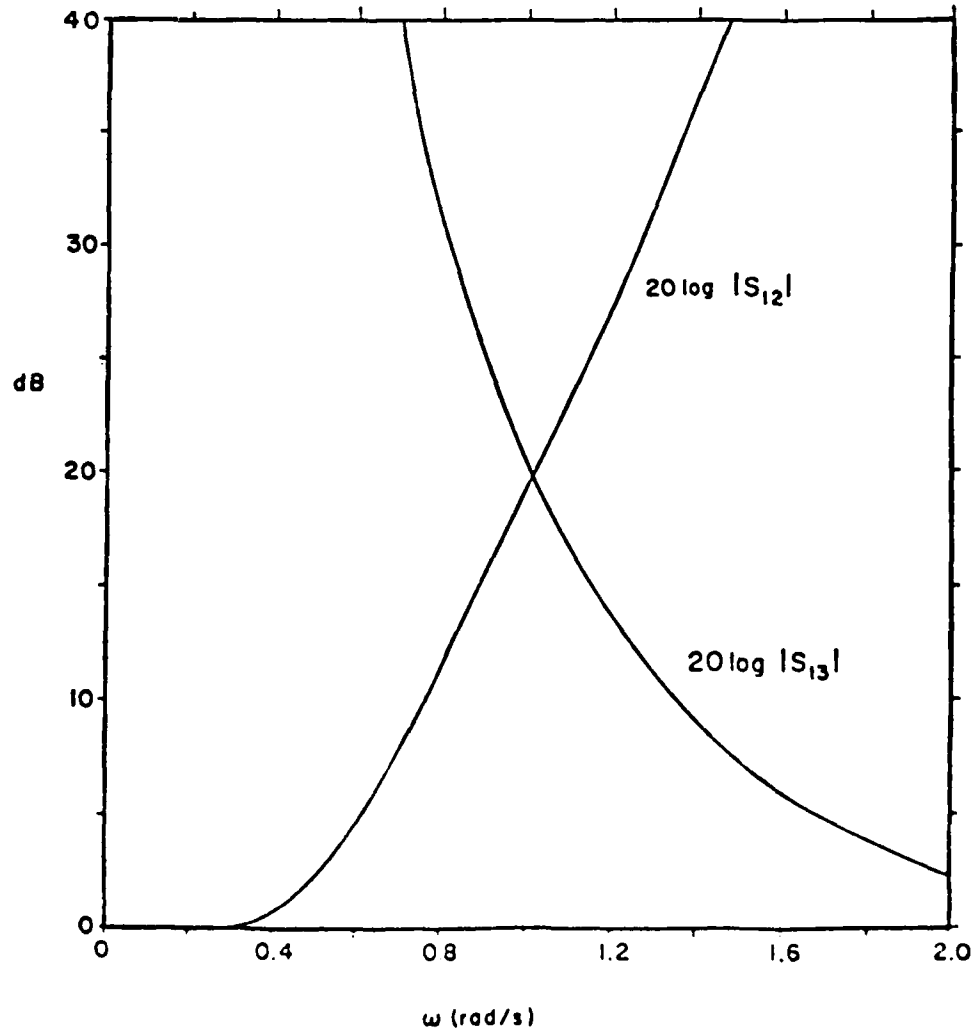


Fig. 7.10(b). The Frequency Response of the Diplexer of Fig. 7.10(a).

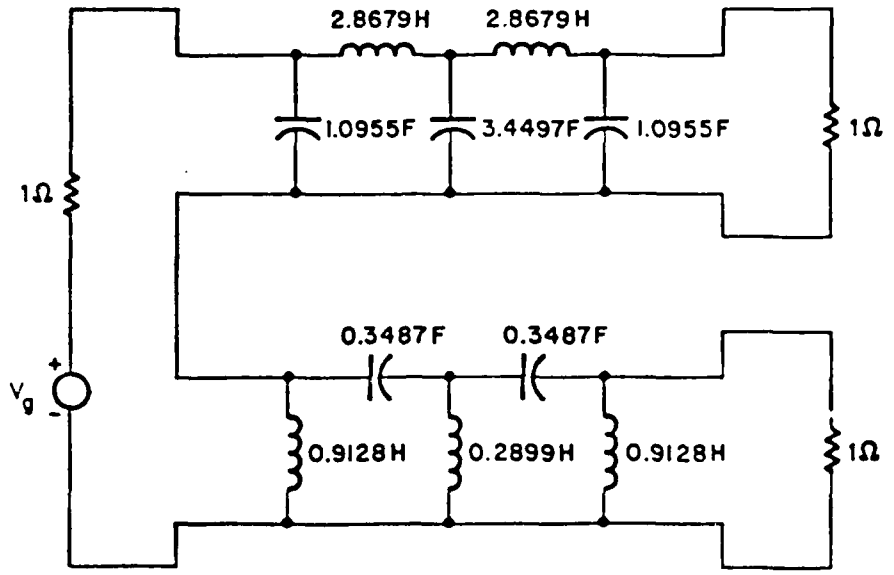


Fig. 7.11(a). A Fifth-Order Series-Connected Diplexer Having 20-dB Insertion Loss at $\omega = 1$ rad/s.

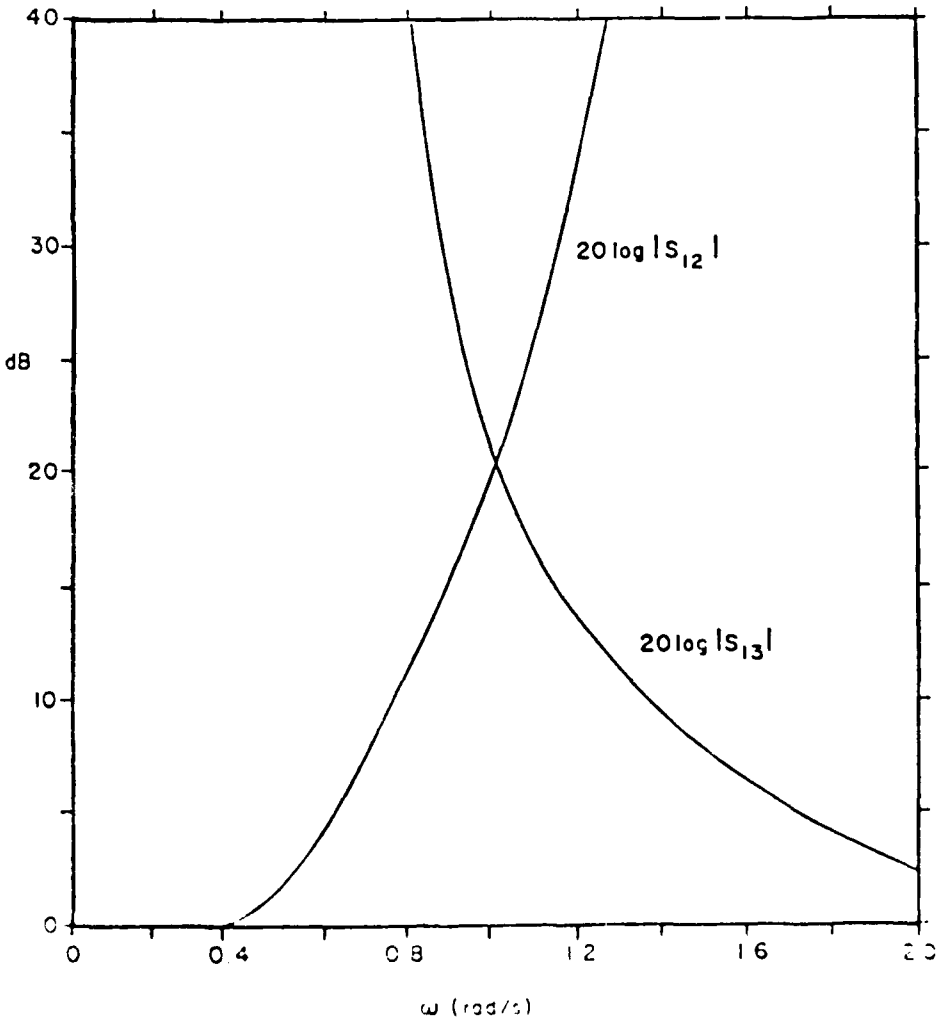


Fig. 7.11(b). The Frequency Response of the Diplexer of Fig. 7.11(a).

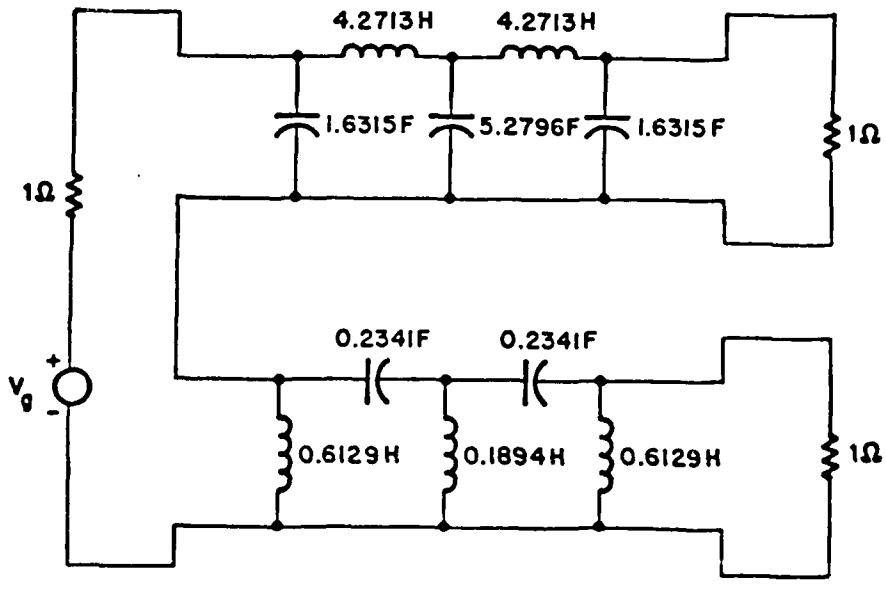


Fig. 7.12(a). A Fifth-Order Series-Connected Diplexer Having 40-dB Insertion Loss at $\omega = 1$ rad/s.

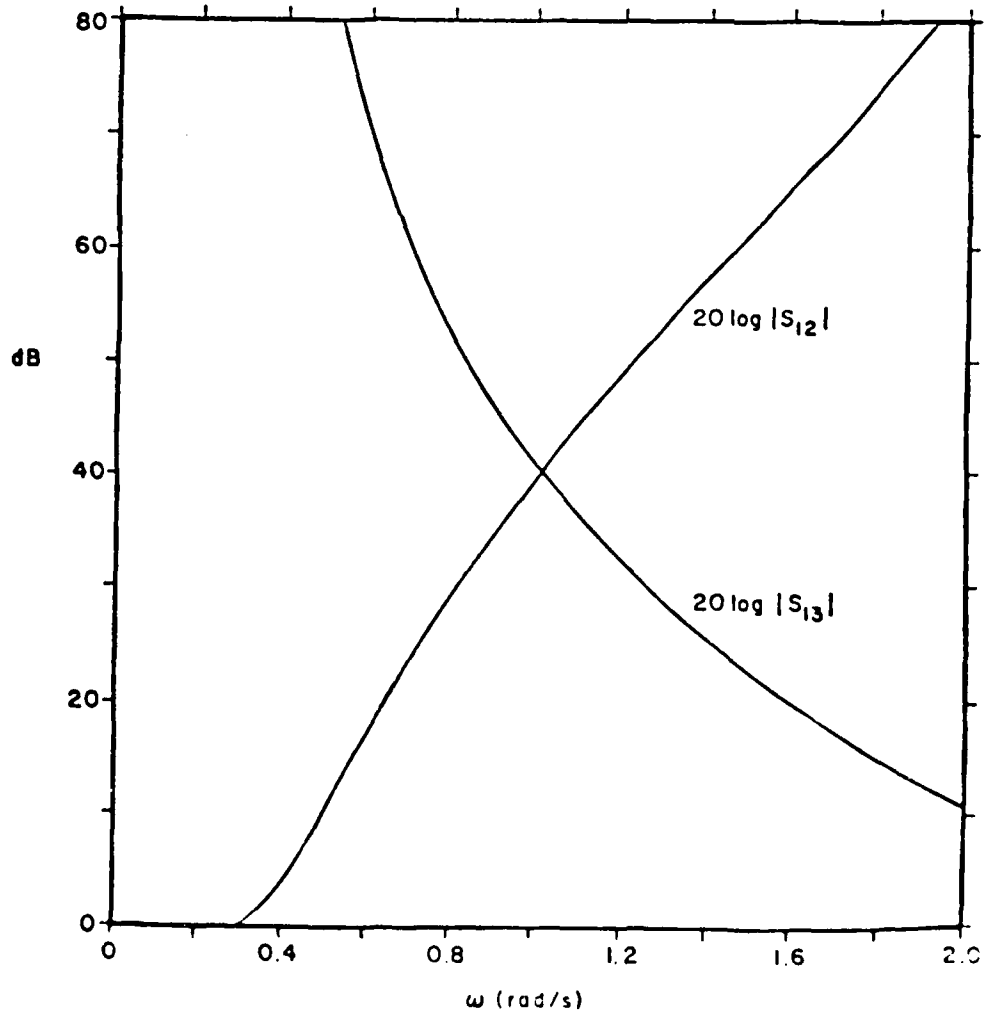


Fig. 7.12(b). The Frequency Response of the Diplexer of Fig. 7.12(a).

7.4 Conclusion

Because of the mutual interaction effect between the two-port networks, the design of a diplexer in general is very complicated. The diplexer presented in this section possesses the symmetrical frequency responses for the channels with respect to the crossover frequency, and can be obtained by connecting the two canonical Butterworth two-port networks either in series or in parallel. Having succeeded in expressing the transducer power gain of the diplexer in terms of Butterworth polynomials, the problem of designing a diplexer reduces to that of choosing the order of the Butterworth response, and the cut-off frequencies of the individual two-port networks. A computer program DIPLX is available that gives the required results as well as the frequency response of the transducer power-gain characteristic.

Section 8

ON THE DESIGN OF A DIPLEXER HAVING BUTTERWORTH CHARACTERISTICS

8.1 Introduction

A direct and analytic method is presented for the design of a diplexer having the maximally-flat characteristic at zero and infinite frequencies with any desired insertion loss at the crossover frequency. We first extend Carlin's theorem on reflection coefficients to n-port scattering matrix. We then construct the scattering matrix of a diplexer normalized to the 1-ohm resistance at the three ports, the elements of which are expressed in terms of the numerator and denominator polynomials of input impedances of the two-port filters. If the transmission coefficients possess the nth-order and mth-order Butterworth responses, where $2n \geq m$ and $2m \geq n$, a set of algebraic equations of second order can be obtained. An iterative procedure for their solution is proposed, thereby making the design of a diplexer having Butterworth responses by direct and analytic method possible. Two illustrative examples are given.

The design of a diplexer, which separates frequencies in certain ranges from a spectrum of signals, is a problem frequently encountered in communication engineering. The separation of the desired frequency bands can be accomplished by using filters connected either in parallel or in series. However, the design problem is much more complicated than it might at first seem, because by simply connecting two ordinary filters, interaction effects will certainly disrupt the performance of the system. Early design of the constant-impedance filter pair gives only 3-dB insertion loss in each channel at the crossover frequency [40]. In order to obtain a sharper separation, most of the practical designs

use the complementary techniques to modify the individual filters in the common port [37], or in the filters themselves [42].

The analytic technique for the design of a diplexer is of great importance, because the desired response can be directly obtained by using a minimum number of elements. As is well known, if the diplexer is lossless and reciprocal, its 3×3 scattering matrix is para-unitary [59,14].

Belevitch [1] first presented a method for the synthesis of a nonconstant-impedance filter pair. Neirynck and Carlin [38] gave a general solution to the construction of a scattering matrix of a three-port network, a practical example of which can be found in Carlin [3].

In the present section, a more direct method to constuct the scattering matrix of a diplexer consisting of a filter pair connected either in parallel or in series is given. The result can be considered as a special case of [38], but the expression of the scattering parameters in terms of the numerator and denominator polynomials of the input impedances of the filter pair is more meaningful and is much simplified. We first extend Carlin's theorem [5] on reflection coefficients to n-port scattering matrix and then apply the result to the design of a diplexer.

Another key problem is to determine these polynomials so that the transmission coefficients $S_{12}(s)$, $S_{13}(s)$ and $S_{23}(s)$ yield the desired responses. By requiring $S_{12}(s)$ and $S_{13}(s)$ possess the nth-order and mth-order maximally-flat response at zero and infinite frequencies, respectively, we obtain a set of nonlinear second-order equations. This solution process may be regarded as an optimization procedure and is equivalent to determining the exact locations of the complex zeros

of transmission. These equations greatly constrict the search range and can be solved by Newton iterative method [41], the initial value of which can be chosen in accordance with that suggested in paragraph 8.4. The resulting solution is guaranteed to be convergent.

8.2 The scattering matrix of a diplexer

Let us consider a three-port ideal transformer with unit turns ratios and terminated in $z_1(s)$, $z_2(s)$ and $z_3(s)$ at port 11', 22' and 33' as shown in Fig. 8.1, where $z_1(s)$, $z_2(s)$ and $z_3(s)$ are non-Foster

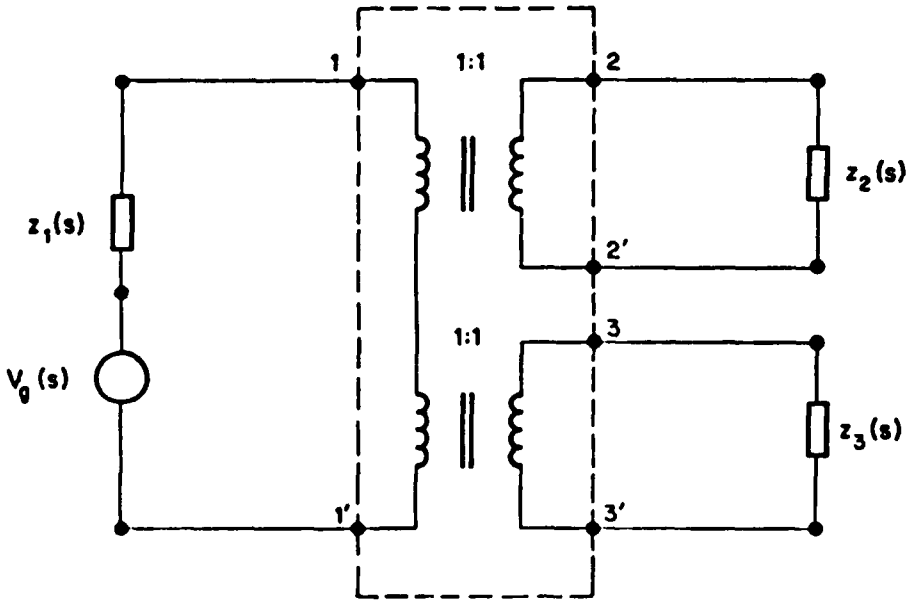


Fig. 8.1. An Ideal Transformer Terminated in $z_1(s)$, $z_2(s)$ and $z_3(s)$ at port 11', 22' and 33', respectively.

positive-real impedances. When $z_1(s) = 1$, and $z_2(s)$ and $z_3(s)$ are expressed by their Darlington equivalents, the ideal transformer and the two lossless reciprocal two-port networks constitute the diplexer as shown in Fig. 8.2.

In Fig. 8.1, we define

$$r_i(s) = \tfrac{1}{2}[z_i(s) + z_{i*}(s)] = h_i(s)h_{i*}(s), \qquad i = 1,2,3 \qquad (8.1)$$

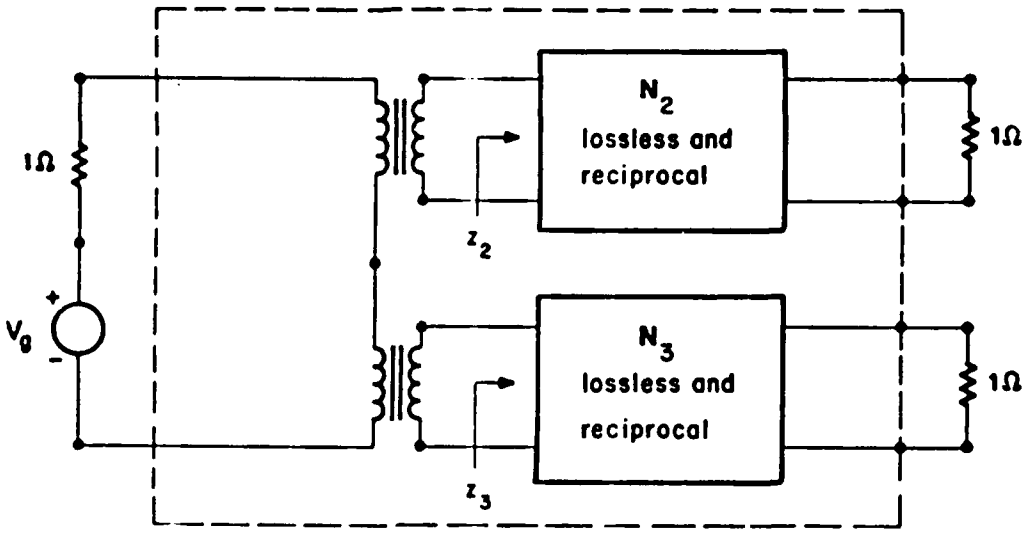


Fig. 8.2. The Ideal Transformer and Two Lossless Reciprocal Two-Port Networks N_2 and N_3 Constitute the Diplexer.

where

$$z_{i*}(s) = z_i(-s) \tag{8.2}$$

and the factorization is to be performed, so that $h_i(s)$ and $h_{i*}^{-1}(s)$ are analytic in the open right-half of the s-plane (RHS). Let $\hat{\underset{\sim}{S}}(s)$ be the scattering matrix of the three-port ideal transformer normalized to $z_1(s)$, $z_2(s)$ and $z_3(s)$. Then we have [14]

$$\hat{\underset{\sim}{S}}(s) = \begin{bmatrix} \dfrac{h_1}{h_{1*}} \cdot \dfrac{z_2+z_3-z_{1*}}{z_1+z_2+z_3} & \dfrac{2h_1h_2}{z_1+z_2+z_3} & \dfrac{2h_1h_3}{z_1+z_2+z_3} \\ \dfrac{2h_1h_2}{z_1+z_2+z_3} & \dfrac{h_2}{h_{2*}} \cdot \dfrac{z_1+z_3-z_{2*}}{z_1+z_2+z_3} & \dfrac{-2h_2h_3}{z_1+z_2+z_3} \\ \dfrac{2h_1h_3}{z_1+z_2+z_3} & \dfrac{-2h_2h_3}{z_2+z_2+z_3} & \dfrac{h_3}{h_{3*}} \cdot \dfrac{z_1+z_2-z_{3*}}{z_1+z_2+z_3} \end{bmatrix} \tag{8.3}$$

If $z_1(s) = 1\ \Omega$ and if the impedances $z_2(s)$ and $z_3(s)$ are written explicitly as

$$z_2(s) = \frac{p_2(s)}{q_2(s)} \tag{8.4}$$

$$z_3(s) = \frac{p_3(s)}{q_3(s)} \tag{8.5}$$

then we can state the following:

(i) $p_i(s)$ and $q_i(s)$ $(i=2,3)$ are polynomials with real coefficients.

(ii) $p_i(s) + q_i(s)$ is strictly Hurwitz.

(iii) $p_i(s)q_{i*}(s) + p_{i*}(s)q_i(s)$ is not identically zero.

(iv) $p_i(j\omega)q_{i*}(j\omega) + p_{i*}(j\omega)q_i(j\omega) \geq 0$ for all ω.

Since the matrix of (8.3) is the scattering matrix of the three-port ideal transformer normalizing to three non-Foster positive-real impedances, it is necessary that the matrix be rational and bounded-real and possess the para-unitary property. Write

$$W_i(s)W_{i*}(s) = \tfrac{1}{2}[p_i(s)q_{i*}(s) + p_{i*}(s)q_i(s)], \qquad i = 2,3 \tag{8.6}$$

The zeros of the polynomial $W_i(s)$ are restricted to the left-half of the s-plane (LHS), while the zeros of the polynomial $W_{i*}(s)$ are limited to RHS. The zeros on the imaginary axis are equally divided between $W_i(s)$ and $W_{i*}(s)$. Thus substituting (8.4), (8.5) and (8.6) with $z_1(s) = 1$ in (8.3) gives

$$\underset{\sim}{S}(s) = \begin{bmatrix} \dfrac{p_2q_3+p_3q_2-q_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{2*}}{W_2}\cdot\dfrac{2W_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{3*}}{W_3}\cdot\dfrac{2W_3q_2}{p_2q_3+p_3q_2+q_2q_3} \\ \dfrac{W_{2*}}{W_2}\cdot\dfrac{2W_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{2*}}{W_2}\cdot\dfrac{p_3q_{2*}+q_3q_{2*}-q_3p_{2*}}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{2*}W_{3*}}{W_2W_3}\cdot\dfrac{-2W_2W_3}{p_2q_3+p_3q_2+q_2q_3} \\ \dfrac{W_{3*}}{W_3}\cdot\dfrac{2W_3q_2}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{2*}W_{3*}}{W_2W_3}\cdot\dfrac{-2W_2W_3}{p_2q_3+p_3q_2+q_2q_3} & \dfrac{W_{3*}}{W_3}\cdot\dfrac{p_2q_{3*}-q_2p_{3*}+q_2q_{3*}}{p_2q_3+p_3q_2+q_2q_3} \end{bmatrix} \tag{8}$$

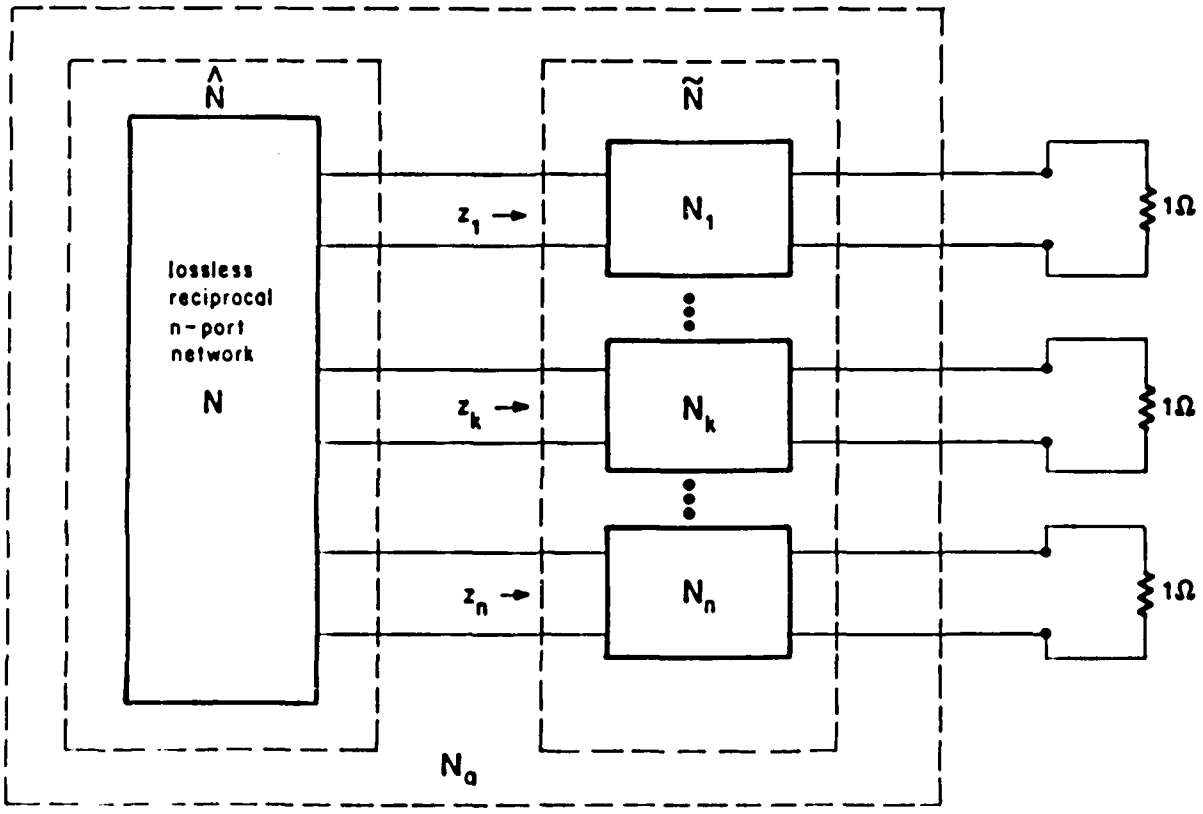


Fig. 8.3. An n-Port Network Terminated in n Non-Foster Positive-Real Impedances.

We next present a theorem which extends Carlin's result [5] to an n-port network.

Theorem 8.1. Let $\hat{\underset{\sim}{S}}(s) = [\hat{S}_{ij}]$ be the scattering matrix of a lossless reciprocal n-port network $\hat{N}$ of Fig. 8.3 normalized to n non-Foster positive-real impedances $z_k(s)$ $(k = 1,2,\ldots,n)$. Then the unit normalized scattering matrix of the augmented n-port network N_a of Fig. 8.3 is given by

$$\underset{\sim}{S}(s) = \begin{bmatrix} \pm \frac{W_1}{W_{1*}} \hat{S}_{11} & \left(\pm \frac{W_1 W_2}{W_{1*} W_{2*}}\right)^{1/2} \hat{S}_{12} & \cdots & \left(\pm \frac{W_1 W_n}{W_{1*} W_{n*}}\right)^{1/2} \hat{S}_{1n} \\ \left(\pm \frac{W_2 W_1}{W_{2*} W_{1*}}\right)^{1/2} \hat{S}_{21} & \pm \frac{W_2}{W_{2*}} \hat{S}_{22} & \cdots & \left(\pm \frac{W_2 W_n}{W_{2*} W_{n*}}\right)^{1/2} \hat{S}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \pm \left(\frac{W_n W_1}{W_{n*} W_{1*}}\right)^{1/2} \hat{S}_{n1} & \left(\pm \frac{W_n W_2}{W_{n*} W_{2*}}\right)^{1/2} \hat{S}_{n2} & \cdots & \pm \frac{W_n}{W_{n*}} \hat{S}_{nn} \end{bmatrix} \tag{8.8}$$

where

$$z_k(s) = \frac{p_k(s)}{q_k(s)}, \qquad k = 1,2,\ldots,n \tag{8.9}$$

$$W_k(s)W_{k*}(s) = \tfrac{1}{2}[p_{k*}(s)q_k(s) + p_k(s)q_{k*}(s)], \qquad k = 1,2,\ldots,n \tag{8.10}$$

and the factorization of $W_{k*}(s)$ is to be performed the same as that of (8.6).

The proof of the theorem follows paragraph 8.5, Conclusions.

Letting $n = 3$ in (8.8) and substituting the elements of (8.7) in (8.8) yield the unit normalized scattering matrix of the diplexer of Fig. 8.2 as

$$\underset{\sim}{S}(s) = \begin{bmatrix} \dfrac{p_2q_3+p_3q_2-q_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \left(\pm\dfrac{W_{2*}}{W_2}\right)^{\frac{1}{2}} \dfrac{2W_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \left(\pm\dfrac{W_{3*}}{W_3}\right)^{\frac{1}{2}} \dfrac{2W_3q_2}{p_2q_3+p_3q_2+q_2q_3} \\ \left(\pm\dfrac{W_{2*}}{W_2}\right)^{\frac{1}{2}} \dfrac{2W_2q_3}{p_2q_3+p_3q_2+q_2q_3} & \pm\dfrac{p_3q_{2*}+q_3q_{2*}-q_3p_{2*}}{p_2q_3+p_3q_2+q_2q_3} & \left(\pm\dfrac{W_{2*}W_{3*}}{W_2W_3}\right)^{\frac{1}{2}} \dfrac{-2W_2W_3}{p_2q_3+p_3q_2+q_2q_3} \\ \left(\pm\dfrac{W_{3*}}{W_3}\right)^{\frac{1}{2}} \dfrac{2W_3q_2}{p_2q_3+p_3q_2+q_2q_3} & \left(\pm\dfrac{W_{2*}W_{3*}}{W_2W_3}\right)^{\frac{1}{2}} \dfrac{-2W_2W_3}{p_2q_3+p_3q_2+q_2q_3} & \pm\dfrac{p_2q_{3*}-q_2p_{3*}+q_2q_{3*}}{p_2q_3+p_3q_2+q_2q_3} \end{bmatrix} \tag{8.11}$$

From (8.11), we arrive at the following conclusion:

1. $\underset{\sim}{S}(s)$ is rational and bounded-real and possesses the para-unitary property for the lossless, reciprocal, linear and lumped diplexer, so that the polynomial $p_2q_3+p_3q_2+q_2q_3$ is strictly Hurwitz.

2. $\left(\pm W_{k*}/W_k\right)$ is a complete square. When all zeros of W_kW_{k*} are on the $j\omega$-axis, $\left(\pm W_{k*}/W_k\right)^{\frac{1}{2}} = \pm 1$. When the minus sign inside the brackets is used, a minus is assigned to $S_{kk}(s)$.

3. By assumption, p_k/q_k is non-Foster and positive-real.

8.3 Butterworth response

By a diplexer having Butterworth response we mean that its transducer power-gain characteristics, say $|S_{12}(j\omega)|^2$ and $|S_{13}(j\omega)|^2$, possess the maximally-flat amplitude-frequency characteristics at the zero and infinite frequencies. Consider the transmission coefficients $S_{12}(s)$ and $S_{13}(s)$ of (8.11). Without loss of generality, from (8.10) $W_k(s)$ and $W_{k*}(s)$ can be expressed as

$$W_k(s) = s^\tau \left[\prod_i (s^2+\omega_i^2)\right] \left\{\prod_j [s+a_j)^2 + b_j^2]^2\right\} \tag{8.12a}$$

$$W_{k*}(s) = (-1)^\tau s^\tau \left[\prod_i (s^2+\omega_i^2)\right] \left\{\prod_j [(s-a_j)^2 + b_j^2]^2\right\} \tag{8.12b}$$

where ω_i, a_j and b_j are all real and positive; and τ, i and j are non-negative integers. If the filter networks of Fig. 8.2 are LC-R ladders, it is necessary that all the zeros of $W_k(s)$ be on imaginary axis [29]. In this case

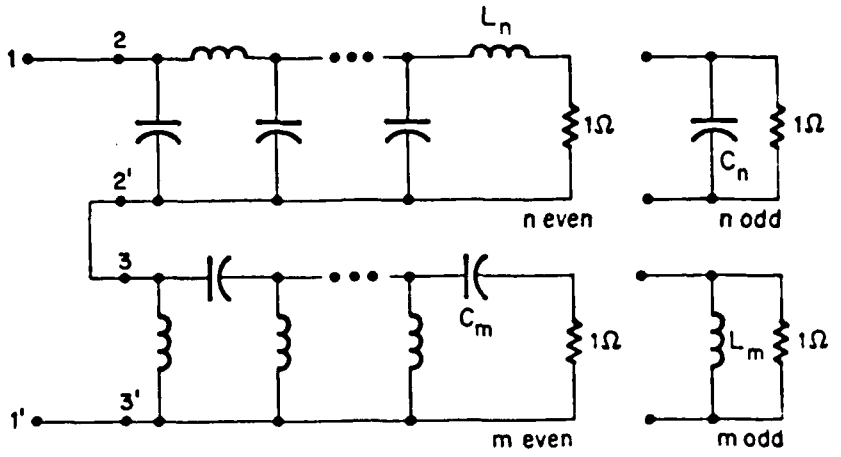
$$W_k(s) = s^\tau \prod_i (s^2+\omega_i^2) \tag{8.13}$$

Since $S_{12}(s)$ and $S_{13}(s)$ provide the maximally-flat characteristics and are devoid of zeros on the finite real-frequency axis, and since the finite zeros of $W_k(s)$ cannot be cancelled by the ones in the strictly Hurwitz polynomial $p_2q_3+p_3q_2+q_2q_2$, $W_2(s)$ and $W_3(s)$ can be written as

$$W_2(s) = 1 \tag{8.14}$$

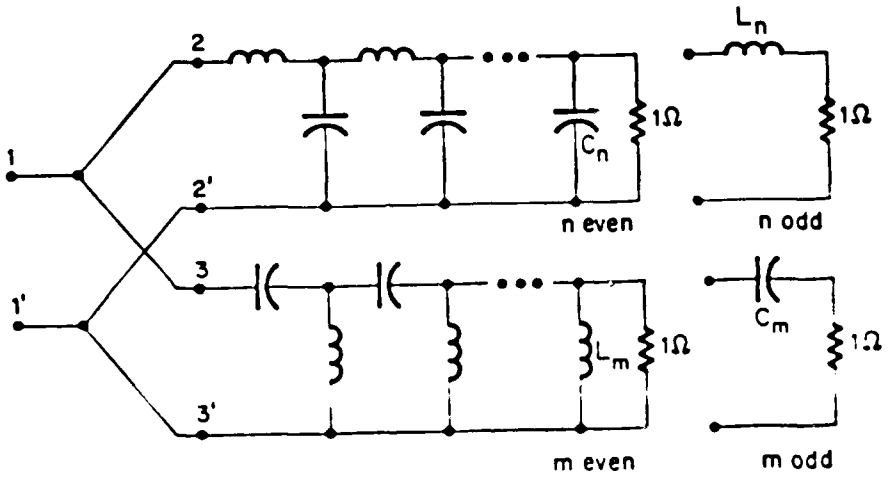
$$W_3(s) = s^\tau \tag{8.15}$$

The corresponding realizations are termed as the all-pole networks and their configurations are shown in Fig. 8.4a, from which $z_2(s)$ and $z_3(s)$ can be expressed as



(a)

Fig. 8.4(a). A Diplexer Composed of Two Ladder Connected in Series.



(b)

Fig. 8.4(b). A Diplexer Composed of Two Ladder Connected in Parallel.

$$z_2(s) = \frac{p_2(s)}{q_2(s)} = \frac{c_{n-1}s^{n-1} + c_{n-2}s^{n-2} + \ldots + c_1 s + 1}{\alpha_n s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1 s + 1} \qquad (8.16)$$

$$z_3(s) = \frac{p_3(s)}{q_3(s)} = \frac{s^m + d_{m-1}s^{m-1} + \ldots + d_1 s}{s^m + \beta_{m-1}s^{m-1} + \ldots + \beta_1 s + \beta_0} \qquad (8.17)$$

The dual expressions in terms of the input admittances $y_2(s)$ and $y_3(s)$ for the circuit of Fig. 8.4b can be written as

$$y_2(s) = \frac{c_{n-1}s^{n-1} + c_{n-2}s^{n-2} + \ldots + c_1 s + 1}{\alpha_n s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1 s + 1} \tag{8.18}$$

$$y_3(s) = \frac{s^m + d_{m-1}s^{m-1} + \ldots + d_1 s}{s^m + \beta_{m-1}s^{m-1} + \ldots + \beta_1 s + \beta_0} \tag{8.19}$$

Figure 8.4 shows that there are n+m element values to be determined, but there are 2m+2n-2 unknown coefficients in (8.16) and (8.17), or in (8.18) and (8.19). The m+n-2 constraints are obtained from (8.6), (8.14), (8.15), (8.16) and (8.17) with $\tau = m$ in (8.15), as follows:

$$\sum_{i=0}^{k_1=x} (-1)^i \alpha_i c_{k_1-i} = 0, \qquad x = 2, 4, \ldots, 2(n-1) \tag{8.20}$$

$$\sum_{j=0}^{k_2=y} (-1)^j \beta_j d_{k_2-j} = 0, \qquad y = 2, 4, \ldots, 2(m-1) \tag{8.21}$$

where $\alpha_0 = c_0 = 1$; $\alpha_i = 0$ for $i > n$; $c_{k_1-i} = 0$ for $k_1 - 1 \geq n$; and $\beta_m = d_m = 1$; $d_0 = 0$, $\beta_j = 0$ for $j > m$; and $d_{k_2-j} = 0$ for $k_2 - j > m$.

The additional flexibility in terms of the excessive n+m unknown coefficients can be used to require that the transmission coefficients possess the desired response. This is equivalent to assigning n zeros of $q_2(s)$ and m zeros of $q_3(s)$ anywhere in the open LHS, so that $S_{12}(s)$ and $S_{13}(s)$ give the maximally-flat response at zero and infinity frequencies. Substituting (8.14) and (8.15) with $\tau = m$ in (8.11) gives

$$S_{12}(s) = \frac{2q_3}{p_2 q_3 + p_3 q_2 + q_2 q_3} \tag{8.22}$$

$$S_{13}(s) = \frac{2s^m q_2}{p_2 q_3 + p_3 q_2 + q_2 q_3} \tag{8.23}$$

$$S_{23}(s) = \frac{-2s^m}{p_2q_3+p_3q_2+q_2q_3} \tag{8.24}$$

By combining (8.16), (8.17) and (8.22), we obtain

$$|S_{12}(j\omega)|^2 = \frac{B_0 + B_1\omega^2 + \ldots + B_m\omega^{2m}}{A_0 + A_1\omega^2 + \ldots + A_m\omega^{2m} + \ldots + A_{m+n}\omega^{2(m+n)}} \tag{8.25}$$

where

$$B_0 + B_1\omega^2 + \ldots + B_m\omega^{2m} = 4\, q_3(j\omega)q_{3*}(j\omega) \tag{8.26}$$

$$\begin{aligned} &A_0 + A_1\omega^2 + \ldots + A_m\omega^{2m} + \ldots + A_{m+n}\omega^{2(m+n)} \\ &\quad = [p_2(j\omega)q_3(j\omega) + p_3(j\omega)q_2(j\omega) + q_2(j\omega)q_3(j\omega)] \times \\ &\qquad [p_{2*}(j\omega)q_{3*}(j\omega) + p_{3*}(j\omega)q_{2*}(j\omega) + q_{2*}(j\omega)q_{3*}(j\omega)] \end{aligned} \tag{8.27}$$

If $B_i = A_i$, $i = 0, 1, \ldots, k$; $k \leq m$, then (8.25) can be written as

$$|S_{12}(j\omega)|^2 = \frac{1 + \dfrac{B_{k+1}\omega^{2(k+1)} + B_{k+2}\omega^{2(k+2)} + \ldots + B_m\omega^{2m}}{B_0 + B_1\omega^2 + \ldots + B_k\omega^{2k}}}{1 + \dfrac{A_{k+1}\omega^{2(k+1)} + A_{k+2}\omega^{2(k+2)} + \ldots + A_{m+n}\omega^{2(m+n)}}{B_0 + B_1\omega^2 + \ldots + B_k\omega^{2k}}} \tag{8.28}$$

As ω approaches zero, (8.28) can be approximated by

$$|S_{12}(j\omega)|^2 \approx \frac{1 + \omega^{2(k+1)}\phi_1(\omega^2)}{1 + \omega^{2(k+1)}f_1(\omega^2)} \approx 1 + \omega^{2(k+1)}(B_0' + B_1'\omega^2 + \ldots) \tag{8.29}$$

where $f_1(\omega^2)$ and $\phi(\omega^2)$ are polynomials in ω^2. In this case, $|S_{12}(j\omega)|^2$ is said to possess the (k+1)th-order maximally-flat response at the zero frequency. Likewise, $|S_{13}(j\omega)|^2$ can be expressed as

$$|S_{13}(j\omega)|^2 = \frac{C_m\omega^{2m} + C_{m+1}\omega^{2(m+1)} + \ldots + C_{m+n}\omega^{2(m+n)}}{A_0 + A_1\omega^2 + \ldots + A_m\omega^{2m} + \ldots + A_{m+n}\omega^{2(m+n)}} \quad (8.30)$$

where

$$C_m\omega^{2m} + C_{m+1}\omega^{2(m+1)} + \ldots + C_{m+n}\omega^{2(m+n)} = 4\,\omega^{2m} q_2(j\omega) q_{2*}(j\omega) \quad (8.31)$$

If $C_i = A_i$, $i = m+n, m+n-1, \ldots, m+n-h$, $h \leq n$, then we have

$$|S_{13}(j\omega)|^2 = \frac{1 + \dfrac{C_{m+n-h-1}\dfrac{1}{\omega^{2(h+1)}} + C_{m+n-h-2}\dfrac{1}{\omega^{2(h+2)}} + \ldots + C_m\dfrac{1}{\omega^{2n}}}{C_{m+n} + C_{m+n-1}\dfrac{1}{\omega^2} + \ldots + C_{m+n-h}\dfrac{1}{\omega^{2h}}}}{1 + \dfrac{A_{m+n-h-1}\dfrac{1}{\omega^{2(h+1)}} + A_{m+n-h-2}\dfrac{1}{\omega^{2(h+2)}} + \ldots + A_0\dfrac{1}{\omega^{2(m+n)}}}{C_{m+n} + C_{m+n-1}\dfrac{1}{\omega^2} + \ldots + C_{m+n-h}\dfrac{1}{\omega^{2h}}}} \quad (8.32)$$

As ω approaches infinity, (8.32) can be expressed as

$$|S_{13}(j\omega)|^2 \approx \frac{1 + \dfrac{1}{\omega^{2(h+1)}}\phi_2(1/\omega^2)}{1 + \dfrac{1}{\omega^{2(h+1)}} f_2(1/\omega^2)} \approx 1 + \frac{1}{\omega^{2(h+1)}}\left[C_0' + C_1'\frac{1}{\omega^2} + \ldots\right] \quad (8.33)$$

where $\phi_2(1/\omega^2)$ and $f_2(1/\omega^2)$ are polynomials in $1/\omega^2$ and $|S_{13}(j\omega)|^2$ is said to possess the $(h+1)$th-order Butterworth response at the infinite frequency.

Theorem 8.2. Of the diplexer configuration of Fig. 8.4, suppose that the transmission coefficients can be expressed as in (8.22), (8.23) and (8.24). Rewrite the reflection coefficient $S_{11}(s)$ according to that given in (8.11) as

$$S_{11}(s) = \frac{b_{m+n}s^{m+n} + \ldots + b_{m+n-h}s^{m+n-h} + \ldots + b_m s^m + \ldots + b_k s^k + \ldots + b_1 s + b_0}{a_{m+n}s^{m+n} + a_{m+n-1}s^{m+n-1} + \ldots + a_1 s + a_0} \quad (8.34)$$

where

$$p_2q_3+p_3q_2-q_2q_3 = b_{m+n}s^{m+n} + \ldots + b_{m+n-h}s^{m+n-h} + \ldots + b_m s^m + \ldots + b_k s^k + \ldots + b_1 s + b_0 \tag{8.35}$$

$$p_2q_3+p_3q_2+q_2q_3 = a_{m+n}s^{m+n} + a_{m+n}s^{m+n-1} + \ldots + a_1 s + a_0 \tag{8.36}$$

Then the following are true:

1) If $b_i = 0$, $i = 0, 1, 2, .., k$; $k < m$, then $|S_{12}(j\omega)|^2$ possesses (k+1)th-order maximally-flat response at the zero frequency.

2) If $b_j = 0$, $j = m+n, m+n-1, \ldots, m+n-h$, $h < n$, then $|S_{13}(j\omega)|^2$ possesses (h+1)th-order maximally-flat response at the infinite frequency.

3) If $b_i = 0$, $i = 0, 1, \ldots, k, m+n, m+n-1, \ldots, m+n-h$; $k < m$, $h < n$; then the diplexer possesses (k+1)th-order maximally-flat response at the zero frequency and (h+1)th-order maximally-flat response at the infinite frequency simultaneously.

4) Belevitch has shown in an unpublished note that $2m \geq n$ and $2n \geq m$ for obtaining the non-degenerate solution.

Proof: From (8.34) compute $|S_{11}(j\omega)|^2$ and from (8.22) and (8.23) compute $|S_{12}(j\omega)|^2$ and $|S_{13}(j\omega)|^2$. Expanding these functions about the zero frequency gives

$$|S_{11}(j\omega)|^2 = T_0^{(0)} + T_1^{(0)}\omega^2 + \ldots + T_k^{(0)}\omega^{2k} + \ldots \tag{8.37}$$

$$|S_{12}(j\omega)|^2 = U_0^{(0)} + U_1^{(0)}\omega^2 + \ldots + U_k^{(0)}\omega^{2k} + \ldots \tag{8.38}$$

$$|S_{13}(j\omega)|^2 = V_m^{(0)}\omega^{2m} + V_{m+1}^{(0)}\omega^{2(m+1)} + \ldots \tag{8.39}$$

If $b_i = 0$ in (8.34), $i = 0, 1, \ldots, k$; $k < m$, then we have

$$T_i^{(0)} = 0, \qquad i = 0, 1, \ldots, k; \quad k < m \tag{8.40}$$

By the para-unitary property of (8.11), we have

$$|S_{11}(j\omega)|^2 + |S_{12}(j\omega)|^2 + |S_{13}(j\omega)|^2 = 1 \tag{8.41}$$

Combining (8.37)-(8.40), with (8.41) gives

$$|S_{12}(j\omega)|^2 = 1 + U_{k+1}^{(0)}\omega^{2(k+1)} + U_{k+2}^{(0)}\omega^{2(k+2)} + \ldots \tag{8.42}$$

yielding a diplexer possessing (k+1)th-order maximally-flat response at the zero frequency.

Expanding $|S_{11}(j\omega)|^2$, $|S_{12}(j\omega)|^2$ and $|S_{13}(j\omega)|^2$ about the infinite frequency results in

$$|S_{11}(j\omega)|^2 = T_0^{(\infty)} + T_1^{(\infty)}\frac{1}{\omega^2} + \ldots + T_h^{(\infty)}\frac{1}{\omega^{2h}} + \ldots \tag{8.43}$$

$$|S_{12}(j\omega)|^2 = U_n^{(\infty)}\frac{1}{\omega^{2n}} + U_{n+1}^{(\infty)}\frac{1}{\omega^{2(n+1)}} + \ldots \tag{8.44}$$

$$|S_{13}(j\omega)|^2 = V_0^{(\infty)} + V_1^{(\infty)}\frac{1}{\omega^2} + \ldots + V_h^{(\infty)}\frac{1}{\omega^{2h}} + \ldots \tag{8.45}$$

If $b_j = 0$ in (8.34), $j = m+n, m+n-1, \ldots, m+n-h$; $h < n$, we obtain

$$T_i^{(\infty)} = 0, \qquad i = 0, 1, \ldots, h; \quad h < n \tag{8.46}$$

Combining (8.43)-(8.46), with (8.41) gives

$$|S_{13}(j\omega)|^2 = 1 + \frac{V_{h+1}^{(\infty)}}{\omega^{2(h+1)}} + \frac{V_{h+2}^{(\infty)}}{\omega^{2(h+2)}} + \ldots \tag{8.47}$$

and the resulting diplexer possesses the (h+1)th-order maximally-flat response at the infinite frequency.

If $b_i = 0$ in (8.34) for $i = 0, 1, \ldots, k, m+n, m+n-1, \ldots, m+n-h$, where $k < m$, $h < n$, combining (8.42) and (8.47) results in a diplexer

possessing the (k+1)th-order maximally-flat response at the zero frequency and the (h+1)th-order maximally-flat response at the infinite frequency simultaneously.

Finally, in order to obtain a non-degenerate solution, the restriction on respective degrees of the filter, $2m \geq n$ and $2n \geq m$ as given by Belevitch, has to be satisfied.

Corollary 8.1. In Theorem 8.2, if $b_i = 0$ for $i = 0, 1, \ldots, m+n$, then the diplexer possesses the ρth-order Butterworth response both at the zero frequency and the infinite frequency, where $\rho = \min(m, n)$. In this case, the diplexer degenerates into a constant-impedance filter pair.

Proof: In Theorem 8.2, if $b_i = 0$ for $i = 0, 1, \ldots, m+n$, then

$$p_2(s)q_3(s) + p_3(s)q_2(s) - q_2(s)q_3(s) = 0 \tag{8.48}$$

$$z_2(s) + z_3(s) \equiv 1 \tag{8.49}$$

Let $p(s)/q(s)$ be the impedance of $z(s)$, being either $z_2(s)$ or $z_3(s)$ of order ρ, then we have

$$S_{11}(s) = 0 \tag{8.50}$$

$$S_{12}(s) = \frac{1}{q(s)} \tag{8.51}$$

$$S_{13}(s) = \frac{s^\rho}{q(s)} \tag{8.52}$$

From (8.48)-(8.52), we obtain the following:

1) $q(s)$ is a ρth-order Butterworth polynomial.

2) Write

$$z_2(s) = \frac{c_{\rho-1}s^{\rho-1} + c_{\rho-2}s^{\rho-2} + \ldots + c_1 s + 1}{s^\rho + a_{\rho-1}s^{\rho-1} + a_{\rho-2}s^{\rho-2} + \ldots + a_1 s + 1} \tag{8.53}$$

$$z_3(s) = \frac{s^\rho + d_{\rho-1}s^{\rho-1} + d_{\rho-2}s^{\rho-2} + \ldots + d_2 s^2 + d_1 s}{s^\rho + \alpha_{\rho-1}s^{\rho-1} + \alpha_{\rho-2}s^{\rho-2} + \ldots + \alpha_1 s + 1} \tag{8.54}$$

then

$$c_i + d_i = \alpha_i \qquad i = 1, 2, \ldots, \rho-1 \tag{8.55}$$

3) From (8.20), (8.21), (8.53)-(8.55), we obtain

$$z_2(s) = z_3(1/s) \tag{8.56}$$

Using these properties, a constant resistance diplexer possessing the Butterworth response of any order can easily be designed. For example, let $\rho = 6$. We obtain

$$z_2(s) = \frac{0.64395s^5 + 2.48803s^4 + 4.57081s^3 + 4.97607s^2 + 3.21975s + 1}{s^6 + 3.86370s^5 + 7.46410s^4 + 9.14162s^3 + 7.46410s^2 + 3.86370s + 1} \tag{8.57}$$

$$z_2(s) = \frac{s^6 + 3.21975s^5 + 4.97607s^4 + 4.57081s^3 + 2.48803s^2 + 0.64395s}{s^6 + 3.86370s^5 + 7.46410s^4 + 9.14162s^3 + 7.46410s^2 + 3.86370s + 1} \tag{8.58}$$

from which the diplexer can be synthesized.

8.4 Illustrative examples

Newton Iteration Method

In paragraph 8.3, we can obtain m+n-2 equations from (8.20) and (8.21) and m+n-2 equations from Theorem 8.2 for the 2m+2n-2 unknown coefficients. The other two coefficients can be used to determine the crossover frequency and the loss at that frequency, where $2m \geq n$ and $2n \geq m$. We notice that all the equations are of second order and can be solved by the Newton iteration method of the improved Newton's method [41]. Finally the results must keep $p_2(s)/q_2(s)$ and $p_3(s)/q_3(s)$

positive-real, so that the matrix of (8.11) is bounded-real and possesses the para-unitary property. Write

$$\underset{\sim}{x} = [\underset{\sim}{\alpha}^T, \underset{\sim}{c}^T, \underset{\sim}{\beta}^T, \underset{\sim}{d}^T]^T \tag{8.59}$$

where

$$\underset{\sim}{\alpha} = [\alpha_n, \alpha_{n-1}, \ldots, \alpha_2, \alpha_1]^T \tag{8.60}$$

$$\underset{\sim}{c} = [c_{n-1}, c_{n-2}, \ldots, c_2, c_1]^T \tag{8.61}$$

$$\underset{\sim}{\beta} = [\beta_{m-1}, \beta_{m-2}, \ldots, \beta_1, \beta_0]^T \tag{8.62}$$

$$\underset{\sim}{d} = [d_{m-1}, d_{m-2}, \ldots, d_2, d_1]^T \tag{8.63}$$

and express

$$\underset{\sim}{f} = [f_1, f_2, \ldots, f_{2m+2n-4}, f_{2m+2n-3}, f_{2m+2n-2}]^T \tag{8.64}$$

where

$$f_1 = \sum_{i=0}^{2} (-1)^i \alpha_i c_{2-i} = 0 \tag{8.65a}$$

$$\vdots$$

$$f_{n-1} = \sum_{i=0}^{2(n-1)} (-1)^i \alpha_i c_{2(n-1)-i} = 0 \tag{8.65b}$$

$$f_n = \sum_{j=0}^{2} (-1)^j \beta_j d_{2-j} = 0 \tag{8.65c}$$

$$\vdots$$

$$f_{m+n-2} = \sum_{j=0}^{2(m-1)} (-1)^j \beta_j d_{2(m-1)-j} = 0 \tag{8.65d}$$

$$f_{m+n-2+k} = b_k = 0, \qquad k = 1, \ldots, m-1, m+1, \ldots, m+n-1 \tag{8.65e}$$

$$f_{2m+2n-3} = |S_{12}(j)|^2 - |S_{13}(j)|^2 = 0 \tag{8.65f}$$

$$f_{2m+2n-2} = |10 \log |S_{12}(j)|^2 - \alpha_{min} = 0 \qquad (8.65g)$$

where α_{min} is the desired loss in dB at $\omega = 1$ rad/s. Let $\underset{\sim}{x}_k$ be the kth-iteraction. Then

$$\underset{\sim}{x}_{k+1} = \underset{\sim}{x}_k - [\underset{\sim}{A}(\underset{\sim}{x}_k)]^{-1} f(\underset{\sim}{x}_k) \qquad (8.66)$$

where

$$\underset{\sim}{A}(\underset{\sim}{x}_k) = \begin{bmatrix} \frac{\partial f_1(\underset{\sim}{x}_k)}{\partial x_1}, & \frac{\partial f_2(\underset{\sim}{x}_k)}{\partial x_1}, & \ldots, & \frac{\partial f_{2(m+n-2)}(\underset{\sim}{x}_k)}{\partial x_1} \\ \frac{\partial f_1(\underset{\sim}{x}_k)}{\partial x_2}, & \frac{\partial f_2(\underset{\sim}{x}_k)}{\partial x_2}, & \ldots, & \frac{\partial f_{2(m+n-2)}(\underset{\sim}{x}_k)}{\partial x_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_1(\underset{\sim}{x}_k)}{\partial x_{2(m+n-1)}}, & \frac{\partial f_2(\underset{\sim}{x}_k)}{\partial x_{2(m+n-1)}}, & \ldots, & \frac{\partial f_{2(m+n-2)}(\underset{\sim}{x}_k)}{\partial x_{2(m+n-1)}} \end{bmatrix} \qquad (8.67)$$

In (8.65) some x_i of the (2m+2n-2)-dimensional vector $\underset{\sim}{x}$ can be simply expressed in terms of the others, and this can be used to reduce the dimension of the problem or make the selection of the initial value $\underset{\sim}{x}_0$ easier. But a careful choice is still important, without which the procedure may not converge and the design may fail.

The Choice of Initial Value $\underset{\sim}{x}_0$

Consider the diplexer given in Fig. 8.2. We notice that one filter is in the passband, while the other is in the stopband for most of the $j\omega$-axis frequencies. If N_2 is a low-pass structure and N_3 a high-pass structure and if the input impedance of one filter is nearly 1 ohm, while the other presents a small series reactance and a smaller resistance. Then we can choose the input impedances of the

two Butterworth filters with normalization frequencies ω_1 and ω_2, where $\omega_1 < 1$ and $\omega_2 > 1$, respectively, as our initial value $\underset{\sim}{x}_0$. When a symmetric response characteristic about $s = j$ is considered, we can use (8.56) and let $\omega_2 = 1/\omega_1$. For example, the input impedance of a 6th-order Butterworth lowpass filter is given by

$$z(s) = \frac{3.86370\left(\frac{s}{\omega_0}\right)^5 + 7.46410\left(\frac{s}{\omega_0}\right)^4 + 9.14162\left(\frac{s}{\omega_0}\right)^3 + 7.46410\left(\frac{s}{\omega_0}\right)^2 + 3.86370\left(\frac{s}{\omega_0}\right) + 1}{2\left(\frac{s}{\omega_0}\right)^6 + 3.86370\left(\frac{s}{\omega_0}\right)^5 + 7.46410\left(\frac{s}{\omega_0}\right)^4 + 9.14162\left(\frac{s}{\omega_0}\right)^3 + 7.46410\left(\frac{s}{\omega_0}\right)^2 + 3.86370\left(\frac{s}{\omega_0}\right) +} \tag{8.68}$$

Let $\omega_0 = 0.5$. Then

$$z(s) = \frac{123.6384s^5 + 119.4256s^4 + 73.13296s^3 + 29.85864s^2 + 7.7274s + 1}{128s^6 + 123.6384s^5 + 119.4256s^4 + 73.13296s^3 + 29.85864s^2 + 7.727s + 1} \tag{8.69}$$

The following sets of coefficients

$$\underset{\sim}{\alpha} = \begin{bmatrix} 128.0 \\ 123.6384 \\ 119.4256 \\ 73.13296 \\ 29.85864 \\ 7.7274 \end{bmatrix}, \qquad \underset{\sim}{c} = \begin{bmatrix} 123.6384 \\ 119.4256 \\ 73.13296 \\ 29.85864 \\ 7.7274 \end{bmatrix}, \tag{8.70}$$

can be used as the initial value $\underset{\sim}{x}_0$ of the iterative process for the 6th-order symmetrical diplexer with 3-dB bandwidth approximately at 0.5 and 2, respectively, where $z_2(s) = z_3(1/s)$. When the 3-dB bandwidth is close to 1, the value of a constant-impedance filter pair is given in (8.57) and (8.58) can be chosen as $\underset{\sim}{x}_0$.

Numerical Examples

Example 8.1. We wish to design a diplexer having the 2nd-order maximally-flat response at the zero frequency and the 3rd-order response at the infinite frequency.

Let $n = 3$ and $m = 2$. Then

$$z_2(s) = \frac{1.029768s^2 + 1.454091s + 1}{1.53622s^3 + 2.16923s^2 + 2.2s + 1} \tag{8.71}$$

$$z_3(s) = \frac{s^2 + 0.670326s}{s^2 + 1.340652s + 0.898673} \tag{8.72}$$

from which we compute the transmission coefficients

$$S_{12}(s) = \frac{s^2 + 1.340652s + 0.898673}{1.53622s^5 + 4.228766s^4 + 6.488742s^3 + 6.1242174s^2 + 3.3177315s + 0.898673} \tag{8.73}$$

$$S_{13}(s) = \frac{1.53622s^5 + 2.16923s^4 + 2.2s^3 + s^2}{1.53622s^5 + 4.228766s^4 + 6.488742s^3 + 6.1242174s^2 + 3.3177315s + 0.898673} \tag{8.74}$$

The resulting diplexer is shown in Fig. 8.5 and its amplitude-frequency response is plotted in Fig. 8.6.

Example 8.2. We wish to design a diplexer possessing the 6th-order symmetrical Butterworth response with 3-dB bandwidth close to 0.5 and 2, respectively.

Write

$$z_2(s) = \frac{c_5s^5 + c_4s^4 + c_3s^3 + c_2s^2 + c_1s + 1}{\alpha_6s^6 + \alpha_5s^5 + \alpha_4s^4 + \alpha_3s^3 + \alpha_2s^2 + \alpha_1s + 1} \tag{8.75}$$

$$z_3(s) = \frac{s^6 + d_5s^5 + d_4s^4 + d_3s^3 + d_2s^2 + d_1s}{s^6 + \beta_5s^5 + \beta_4s^4 + \beta_3s^3 + \beta_2s^2 + \beta_1s + \beta_0} \tag{8.76}$$

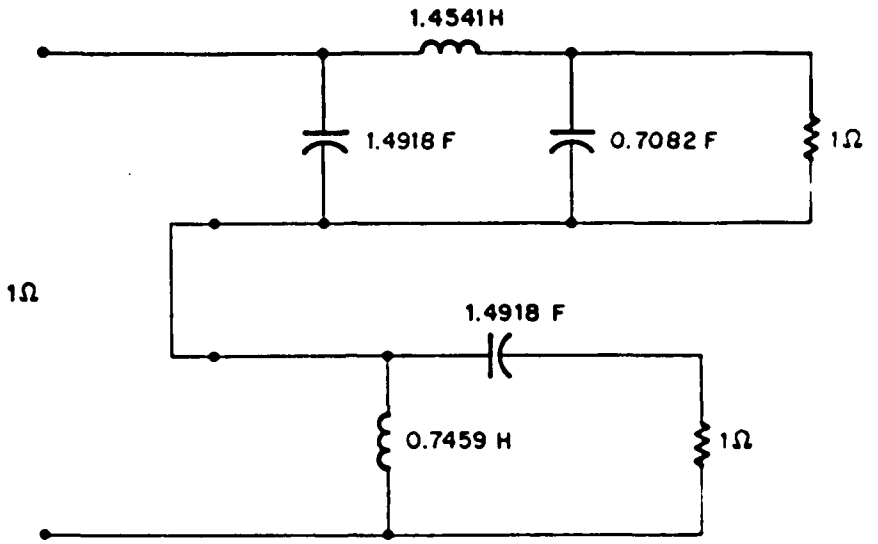


Fig. 8.5. A Diplexer Possessing the 3rd-Order Butterworth Response at the Infinite Frequency and the 2nd-Order Butterworth Response at the Zero Frequency with a 3-dB Insertion Loss at $\omega = 1$ rad/s.

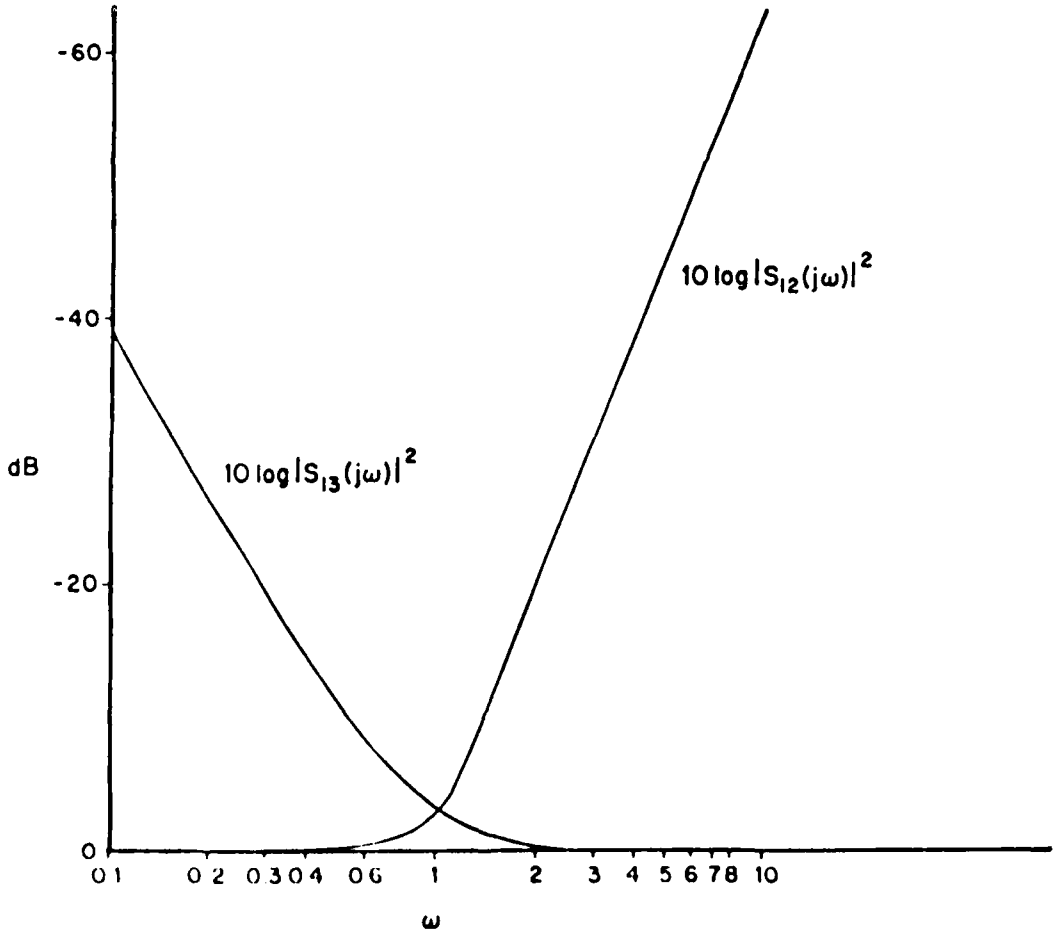


Fig. 8.6. The Frequency-Response Curve of the Diplexer Given in Fig. 8.3.

where $m = n = 6$. Let the crossover frequency be $s = j$. For the symmetrical Butterworth response, we require

$$z_2(s) = z_3(1/s) \tag{8.77}$$

and

$$z_3(s) = \frac{s^6 + c_1 s^5 + c_2 s^4 + c_3 s^3 + c_4 s^2 + c_5 s}{s^6 + \alpha_1 s^5 + \alpha_2 s^4 + \alpha_3 s^3 + \alpha_4 s^2 + \alpha_5 s + \alpha_6} \tag{8.78}$$

The initial approximation is shown in (8.70) and using (8.66) we obtain

$$\begin{aligned}
\alpha_6 &= 132.9572 \\
\alpha_5 &= 145.8101 & c_5 &= 76.699818 \\
\alpha_4 &= 125.868 & c_4 &= 84.114347 \\
\alpha_3 &= 73.9661 & c_3 &= 56.86165 \\
\alpha_2 &= 29.856355 & c_2 &= 25.398604 \\
\alpha_1 &= 7.7274 & c_1 &= 7.150524
\end{aligned} \tag{8.79}$$

The resulting network and its response are shown in Fig. 8.7 and Fig. 8.8, respectively. The transmission coefficients are found to be

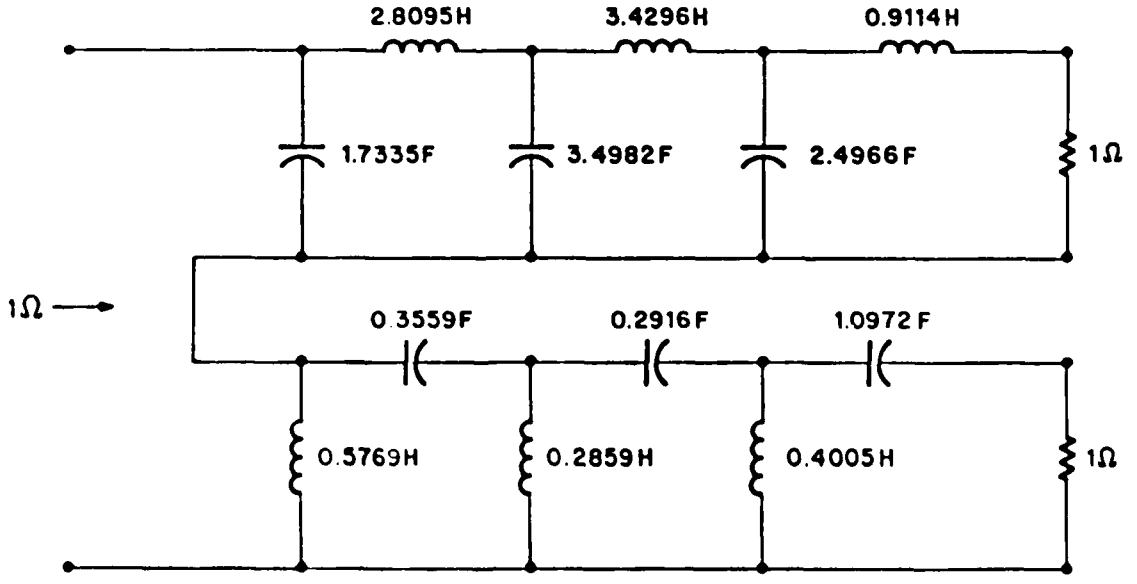


Fig. 8.7. A 6th-Order Symmetrical Butterworth Diplexer with Insertion Loss 32.8 dB at the Crossover Frequency $\omega = 1$ rad/s.

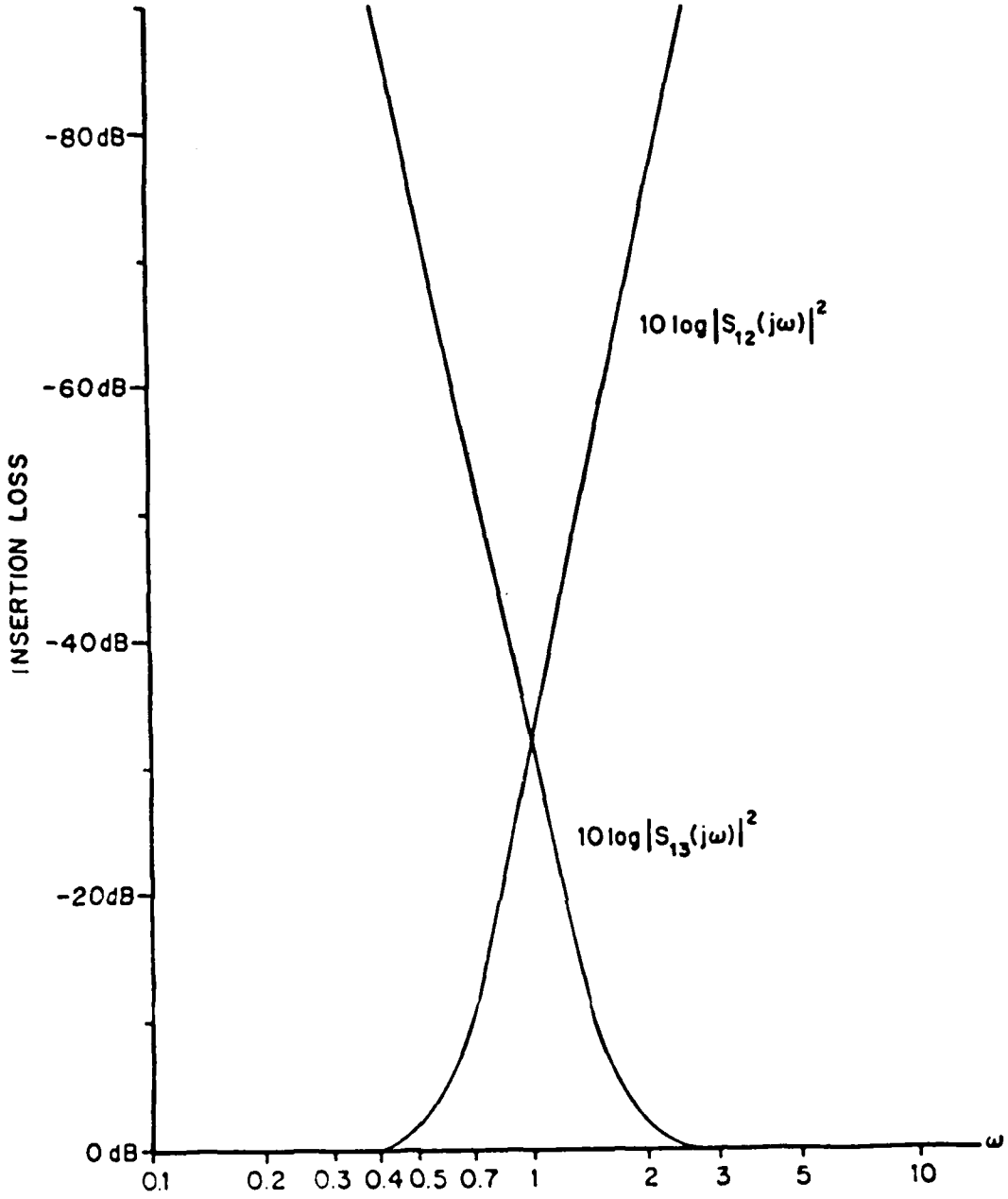


Fig. 8.8. The Frequency Response of the 6th-Order Symmetrical Butterworth Diplexer of Fig. 8 with Insertion Loss 32.8 dB at $\omega = 1$ rad/s.

$$S_{12}(s) = \frac{s^6 + 7.7274s^5 + 29.856355s^4 + 73.96661s^3 + 125.868s^2 + 145.8101s + 132.9572}{D(s)} \quad (8.80)$$

$$S_{13}(s) = \frac{132.9572s^{12} + 145.8101s^{11} + 125.868s^{10} + 7396661s^9 + 29.856355s^8 + 7.7274s^7 + s^6}{D(s)} \quad (8.81)$$

where

$$D(s) = 132.9572s^{12} + 1173.2236s^{11} + 5222.2183s^{10} + 15234.351s^{9}$$
$$+ 31879.521s^{8} + 49496.207s^{7} + 57393.406s^{6} + 49496.207s^{5}$$
$$+ 31879.521s^{4} + 15234.351s^{3} + 5222.2183s^{2} + 1173.2236s$$
$$+ 132.9572$$

8.5 Conclusion

The problem of designing a diplexer of multiplexer is equivalent to that of constructing a 3×3 or an n×n para-unitary matrix, whose transmission coefficients possess the desired frequency characteristics. Many techniques have been proposed in this regard. However, the process is still long and complicated. A direct method of writing the matrix of a diplexer is given, which extends Carlin's theorem to n-port networks. The problem of determining the elements of the scattering matrix, equivalently to locating the complex zeros of the transmission coefficients, reduces to that of solving a set of second-order nonlinear equations, which will constrict the search range and lead to the optimal solutions. Using Newton iterative method and choosing the initial values as suggested, the procedure converges rapidly, thereby making the design of a diplexer having Butterworth response by direct and analytic method possible.

Proof of Theorem 8.1. Consider the lossless reciprocal two-port N_k of Fig. 8.9. Let the input impedance of N_k be $z_k(s)$, when the output is terminated in 1 Ω. Write

$$z_k(s) = \frac{A_k(s) + B_k(s)}{C_k(s) + D_k(s)} \tag{8.82}$$

where $A_k(s)$, $D_k(s)$ are even polynomials and $B_k(s)$, $C_k(s)$ are odd, or vice versa; and $A_k(s)/\xi_k(s)$, $B_k(s)/\xi_k(s)$, $C_k(s)/\xi_k(s)$ and $D_k(s)/\xi_k(s)$

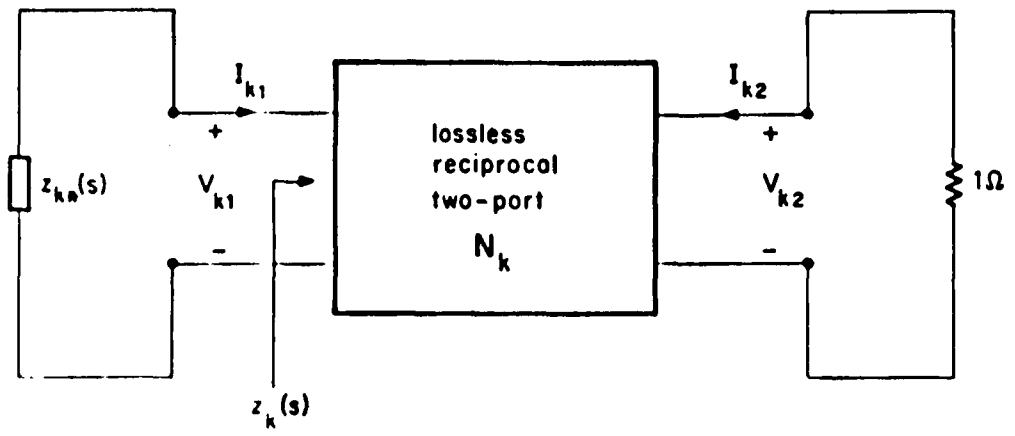


Fig. 8.9. A Lossless Reciprocal Two-Port Network Terminated in z_* and 1 Ω.

are the transmission parameters of N_k defined by the equations

$$V_{k1}(s) = \frac{A_k(s)}{\xi_k(s)} V_{k2}(s) - \frac{B_k(s)}{\xi_k(s)} I_{k2}(s) \tag{8.83}$$

$$I_{k1}(s) = \frac{C_k(s)}{\xi_k(s)} V_{k2}(s) - \frac{D_k(s)}{\xi_k(s)} I_{k2}(s) \tag{8.84}$$

When N_k is reciprocal, $A_k(s)D_k(s) - B_k(s)C_k(s)$ is a complete square [60], and we have the relation

$$\xi_k^2(s) = A_k(s)D_k(s) - B_k(s)C_k(s) \tag{8.85}$$

Substituting (8.82) in (8.10) in conjunction with (8.85) yields

$$W_k(s)W_{k*}(s) = \pm \xi_k^2(s) \tag{8.86}$$

In (8.86), if $A_k(s)$ and $D_k(s)$ are even, we choose the plus sign and if $A_k(s)$ and $D_k(s)$ are odd, we use the minus sign. The elements of the scattering matrix $\tilde{S}_k(s)$ of N_k normalizing to $z_{k*}(s)$ and 1 Ω are found to be

$$\tilde{S}_k(s) = \begin{bmatrix} 0 & \left(\pm \frac{W_k(s)}{W_{k*}(s)}\right)^{1/2} \\ \left(\pm \frac{W_k(s)}{W_{k*}(s)}\right)^{1/2} & 0 \end{bmatrix} \tag{8.87}$$

The block diagonal form of these n scattering matrices of (8.87) constitutes the 2n×2n scattering matrix of the 2n-port network $\tilde{N}$ of Fig. 8.3 normalizing to $z_{1*}(s)$, $z_{2*}(s)$, ..., $z_{n*}(s)$ and n 1-Ω resistances. Write the scattering matrix of the n-port N of Fig. 8.3 normalizing $z_1(s)$, $z_2(s)$, ..., $z_n(s)$ as

$$\hat{S}(s) = \begin{bmatrix} \hat{S}_{11}(s) & \hat{S}_{12}(s) & \ldots, & \hat{S}_{1n}(s) \\ \hat{S}_{21}(s) & \hat{S}_{22}(s) & \ldots, & \hat{S}_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{S}_{n1}(s) & \hat{S}_{n2}(s) & \ldots, & \hat{S}_{nn}(s) \end{bmatrix} \tag{8.88}$$

Using the above mentioned 2n×2n block diagonal matrix and (8.88) and applying the connection formula given in Chen [14], we obtain (8.8).

REFERENCES

[1] V. Belevitch, "Synthesis of nonconstant-impedance filter pairs," Philips J. Res., vol. 35, pp. 104-121, 1980.

[2] H.W. Bode, Network Analysis and Feedback Amplifier Design. Princeton, NJ, Van Nostrand, 1945.

[3] C.H. Carlin, "Approximation method for decomposable filter pairs," Int. J. Circuit Theory and Applications, vol. 10, pp. 123-138, 1982.

[4] H.J. Carlin, "A new approach to gain-bandwidth problems," IEEE Trans. Circuits and Systems, vol. CAS-24, pp. 170-175, April 1977.

[5] H.J. Carlin and B.S. Yarman, "The double matching problem: analytic and real frequency solutions," IEEE Trans. on Circuits and Systems, vol. CAS-30, pp. 15-28, 1983.

[6] Y.T. Chan and E.S. Kuh, "A general matching theory and its application to tunnel diode amplifiers," IEEE Trans. Circuit Theory, vol. CT-13, pp. 6-18, 1966.

[7] W.K. Chen, "On the minimum-phase reflection coefficient in broadband equalizers," Intl. J. Electronics, vol. 39, pp. 357-360, 1975.

[8] W.K. Chen, "Equalization of Darlington type-C load to give Chebyshev or elliptic transducer power-gain characteristics," Int. J. Electronics, vol. 39, pp. 667-680, 1975.

[9] W.K. Chen, "Design considerations of the impedance-matching networks having the elliptic gain response," Proc. of the 18th Midwest Symposium on Circuits and Systems. Concordia University, Montreal, Quebec, pp. 322-326, August 11-12, 1975.

[10] W.K. Chen, "Synthesis of Chebyshev and elliptic impedance-matching networks," Proc. of the 18th Midwest Symposium on Circuits and Systems, Concordia University, Montreal, Quebec, pp. 327-331, August 11-12, 1975.

[11] W.K. Chen, "Dual formulation of Youla's theory of broadband matching," Electronics Letters, vol. 12, pp. 337-338, 1976.

[12] W.K. Chen, "Explicit formulas for the synthesis of Chebyshev impedance-matching networks," Electronics Letters, vol. 12, pp. 412-413, 1976.

[13] W.K. Chen, "On the design of broadband elliptic impedance-matching networks," J. Franklin Inst., vol. 301, pp. 451-463, 1976.

[14] W.K. Chen, Theory and Design of Broadband Matching Networks. Pergamon Press, New York, 1976.

[15] W.K. Chen, "Explicit formulas for the synthesis of optimum broadband impedance-matching networks," IEEE Trans. Circuits and Systems, vol. CAS-24, pp. 157-169, April 1977.

[16] W.K. Chen, "Unified theory of broadband matching," J. Franklin Inst., vol. 310, pp. 287-301, 1980.

[17] W.K. Chen, "Mathematical theory of broadband matching of multiport networks," Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems, Phoenix, Arizona, pp. xxx-xxx, June 15-19, 1987.

[18] W.K. Chen and T. Chaisrakeo, "Explicit formulas for the synthesis of optimum band-pass Butterworth and Chebyshev impedance-matching networks," IEEE Trans. Circuits and Systems, vol. CAS-27, pp. 928-942, 1980.

[19] W.K. Chen and C. Satyanarayana, "General theory of broadband matching," Proc. IEE: Pt. G. Electronic Circuits and Systems, vol. 129, pp. 96-102, 1982.

[20] W.K. Chen and C.K. Tsai, "A general theory of broadband matching of an active load," Circuits, Systems, and Signal Processing, vol. 1, pp. 105-122, 1982.

[21] T.M. Chien, "A theory of broadband matching of a frequency-dependent generator and load - Part I: Theory," J. Franklin Inst., vol. 298, pp. 181-199, 1974.

[22] S. Darlington, "Synthesis of reactance 4-poles," J. Math. and Phys., vol. 18, pp. 257-353, September 1939.

[23] S. Darlington, "A history of network synthesis and filter theory for circuits composed of resistors, inductors, and capacitors," IEEE Trans. Circuits and Systems, vol. CAS-31, pp. 6-7, 1984.

[24] R.M. Fano, "Theoretical limitations on the broadband matching of arbitrary impedances," J. Franklin Inst., vol. 249, pp. 57-83 and 139-154, 1950.

[25] Z. Fei, "A generalized theory of broadband matching," Ph.D. dissertation, August 1985.

[26] Z. Fei and W.K. Chen, "On a complex normalized scattering matrix and its applications to broadband matching of multiport networks," J. Franklin Inst., vol. 320, nos. 3/4, pp. 175-190, September/October 1985.

[27] Z. Fei and W.K. Chen, "On lossless reciprocal and nonreciprocal matching networks of an active load," J. Franklin Inst., vol. 323, no. 2, pp. 235-252, February 1987.

[28] A.D. Fialkow, "Inductance, capacitance networks terminated in resistance," IEEE Trans. Circuits and Systems, vol. CAS-26, pp. 603-641, 1979.

[29] A.D. Fialkow, "Theorem on general LC-R ladder filters," IEEE Trans. on Circuits and Systems, vol. CAS-31, pp. 293-296, 1984.

[30] D.C. Fielder, "Broadband matching between load and source systems," IRE Trans. Circuit Theory, vol. CT-8, pp. 138-153, 1961.

[31] T. Fujisawa, "Realizability theorem for mid-series or mid-shunt low-pass ladders without mutual induction," IRE Trans. Circuit Theory, vol. CT-2, pp. 320-325, 1955.

[32] R. Gudipati, "Explicit formulas for element values of a lossless low-pass ladder network and their use in the design of broadband matching networks," M.S. thesis, August 1987.

[33] C.W. Ho and N. Balabanian, "Synthesis of active and passive compatible impedances," IEEE Trans. Circuit Theory, vol. CT-14, pp. 118-128, 1967.

[34] W.H. Ku, "Some results in the theory of optimum broad-band matching," IEEE Trans. Circuit Theory, vol. CT-17, pp. 420-423, 1970.

[35] R. Levy, "Explicit formulas for Chebyshev impedance-matching networks, filters and interstages," Proc. IEE (London), vol. 111, pp. 1099-1106, 1964.

[36] G.L. Matthaei, "Synthesis of Tchebycheff impedance-matching networks, filters and interstages," IRE Trans. Circuit Theory, vol. CT-3, pp. 163-172, 1956.

[37] G.L. Matthaei and E.G. Cristal, "Theory and design of diplexer and multiplexers," Advances in Microwaves, vol. 2, Academic Press, New York, 1967.

[38] J. Neirynck and C.H. Carlin, "Synthesis of the lossless reciprocal three-port based on a canonic form of its scattering matrix," IEEE Trans. on Circuits and Systems, vol. CAS-28, pp. 736-744, 1981.

[39] R.W. Newcomb, Linear Multiport Synthesis, McGraw-Hill, New York, 1966.

[40] E.L. Norton, "Constant resistance network with applications to filter groups," Bell Syst. Tech. J., vol. 16, pp. 178-193, April 1937.

[41] J.M. Ortega and W.C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, New York, 1970.

[42] J.D. Rhodes, "Direct design of symmetrical interacting bandpass channel diplexers," Inst. Elec. Eng. J. Microwaves, Opt. Acous., vol. 1, pp. 34-40, September 1976.

[43] J.D. Rhodes and S.A. Alseyab, "A design procedure for bandpass channel multiplexers connected at a common junction," IEEE Trans. Microwave Theory Tech., vol. MTT-28, pp. 246-253, 1980.

[44] J.D. Rhodes and R. Levy, "A generalized multiplexer theory," IEEE Trans. Microwave Theory Tech., vol. MTT-27, pp. 99-111, 1979.

[45] J.D. Rhodes and R. Levy, "Design of general manifold multiplexers," IEEE Trans. Microwave Theory Tech., vol. MTT-27, pp. 111-123, 1979.

[46] C. Satyanarayana and W.K. Chen, "Theory of broadband matching and the problem of compatible impedances," J. Franklin Inst., vol. 309, pp. 267-280, 1980.

[47] J.D. Schoeffler, "Impedance transformation using lossless network," IRE Trans. Circuit Theory, vol. CT-8, pp. 131-137, 1961.

[48] I. Smilen, "Broadband equalization of two-ports," Res. Rept. R-667-58, PIB-595, Polytechnic Institute of Brooklyn, NY, 1958.

[49] M.E. Van Valkenburg, Introduction to Modern Network Synthesis. Wiley, New York, pp. 297-322, 1964.

[50] J.L. Wan and W.K. Chen, "Realizability of compatible impedances using transformerless ladder two-port networks," J. Franklin Inst., vol. 322, nos. 5/6, pp. 273-283, November/December 1986.

[51] Z.M. Wang and W.K. Chen, "The design of a symmetrical diplexer composed of canonical Butterworth two-port networks," Tech. Rept. No. UIC-EECS-85-9, Dept. of Electrical Engineering and Computer Science, University of Illinois at Chicago, November 1985.

[52] Z.M. Wang and W.K. Chen, "A new diplexer configuration composed of a three-port circulator and two reciprocal two-port networks," Tech. Rept. No. UIC-EECS-86-2, Dept. of Electrical Engineering and Computer Science, University of Illinois at Chicago, February 1986.

[53] Z.M. Wang and W.K. Chen, "A multiplexer configuration composed of a multi-port circulator and reciprocal two-port networks," Dept. of Electrical Engineering and Computer Science, University of Illinois at Chicago, November 1986.

[54] M.R. Wohlers, "Complex normalization of scattering matrices and the problem of compatible impedances," IEEE Trans. Circuit Theory, vol. CT-12, pp. 528-535, 1965.

[55] E. Yasui, "Real-frequency approach to the design of a diplexer," Ph.D. dissertation, March 1988.

[56] D.C. Youla, "On scattering matrices normalized to complex port numbers," Proc. IRE, vol. 49, p. 1221, 1961.

[57] D.C. Youla, "A new theory of cascade synthesis," IRE Trans. Circuit Theory, vol. CT-8, pp. 244-260, March 1961.

[58] D.C. Youla, "A new theory of broadband matching," IEEE Trans. Circuit Theory, vol. CT-11, pp. 30-50, 1964.

[59] D.C. Youla, "An extension of the concept of scattering matrix," IEEE Trans. Circuit Theory, vol. CT-11, pp. 310-312, 1964.

[60] Y.S. Zhu and W.K. Chen, "Realizability of lossless reciprocal and nonreciprocal broadband matching networks," J. Franklin Inst., vol. 319, pp. 325-340, 1985.

[61] Y.S. Zhu and W.K. Chen, "On the design of a diplexer having Butterworth characteristics," Proceedings of the IEEE Int'l Symposium on Circuits and Systems, Kyoto, Japan, pp. 155-158, June 5-7, 1985. IEEE Publication No. 85CH2114-7.

[62] Y.S. Zhu and W.K. Chen, "An improved procedure for the design of a diplexer having Butterworth characteristics," Dept. of Electrical Engineering and Computer Science, University of Illinois at Chicago, May 1987.

BIBLIOGRAPHY

Papers

1. Z. Fei and W.K. Chen, "On a complex normalized scattering matrix and its applications to broadband matching of multiport networks," J. Franklin Inst., vol. 320, nos. 3/4, pp. 175-190, September/October 1985.

2. Y.S. Zhu and W.K. Chen, "On the design of a diplexer having Butterworth characteristics," Proc. of the IEEE Int'l Symposium on Circuits and Systems, Kyoto, Japan, pp. 155-158, June 5-7, 1985. IEEE Publication No. 85CH2114-7.

3. J.L. Wan and W.K. Chen, "Realizability of compatible impedances using transformerless ladder two-port networks," J. Franklin Inst., vol. 322, nos. 5/6, pp. 273-283, November/December 1986.

4. Z. Fei and W.K. Chen, "On lossless reciprocal and nonreciprocal matching networks of an active load," J. Franklin Inst., vol. 323, no. 2, pp. 235-252, February 1987.

5. W.K. Chen, "Mathematical theory of broadband matching of multiport networks," Proc. of the Int'l Symposium on the Mathematical Theory of Networks and Systems, Phoenix, Arizona, pp. xxx-xxx, June 15-19, 1987.

Technical Reports

1. Y.S. Zhu and W.K. Chen, "On the design of a diplexer having Butterworth characteristics," Dept. of EECS, September 1985.

2. Z.M. Wang and W.K. Chen, "A new diplexer configuration composed of a three-port circulator and two reciprocal two-port networks," Dept. of EECS, March 1986.

3. Z.M. Wang and W.K. Chen, "A multiplexer configuration composed of a multi-port circulator and reciprocal two-port networks," Dept. of EECS, November 1986.

4. Y.S. Zhu and W.K. Chen, "An improved procedure for the design of a diplexer having Butterworth characteristics," Dept. of EECS, May 1987.

Theses and Dissertations

1. Z. Fei, "A generalized theory of broadband matching," Ph.D. dissertation, August 1985.

2. R. Gudipati, "Explicit formulas for element values of a lossless low-pass ladder network and their use in the design of broadband matching networks," M.S. thesis, August 1987.

3. G.A.O. Mong, "Analysis and design of broadband microwave bipolar junction transistor (BJT) amplifiers," Ph.D. dissertation, March 1988.

4. E. Yasui, "Real-frequency approach to the design of a diplexer," Ph.D. dissertation, March 1988.

Appendix A: Program Package for the Design of Diplexers

Main programs: DIPLX2.

Subroutines: DPLXAJ, DPLXCH;
BTNK, BTCF;
POLY,FRQS;
PLOT1,CRT1,CRT2.

```
C
C      MAIN PROGRAM: DIPLX2.
C
       DOUBLE PRECISION OMEG(3),ALC(10),OMEGA,OMEGAC,OMGCL,OMGCH,G1,G2,
      1GA,OMGCH1
       DIMENSION  X(50),YG(50,6)
       READ, N,OMEGAC,ITYPE
       WRITE(6,15)N,OMEGAC,ITYPE
   15  FORMAT('1',5X,'N=',I3,10X,'OMEGAC=',D20.12,10X,'ITYPE=',I1//)
       CALL DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
       DO 80 I=1,50
       OMEGA=I*0.04
       CALL DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
       X(I)=OMEGA
       YG(I,1)=G1
       YG(I,2)=G2
       YG(I,3)=GA
   80  CONTINUE
       OMEG(1)=OMEGAC
       OMEG(2)=1.
       OMEG(3)=1./OMEGAC
       DO 82 K=1,3
       CALL DPLXCH(N,OMGCL,OMGCH,OMEG(K),G1,G2,GA,ITYPE)
   82  WRITE(6,85)OMEG(K),G1,G2,GA
   85  FORMAT(6X,'OMEGA=',D20.12,5X,'G1=',D20.12,5X,'G2=',D20.12,5X,'GA='
      1,D20.12)
       CALL BTNK (N,1.D0,1.D0,0.D0,OMGCL,ALC)
       CALL CRT1 (1.D0,ALC,N)
       OMGCH1=1./OMGCH
       CALL BTNK (N,1D0,1.D0,0.D0,OMGCH1,ALC)
       DO 86 I=1,N
   86  ALC(I)=1./ALC(I)
       CALL CRT2 (1.D0,ALC,N)
       CALL PLOT1(X,YG,50,0.25,2)
       STOP
       END
```

```
C       SUBROUTINE DPLXAJ
C
C       PURPOSE
C          THE PROGRAM IS TO DESGIN A DIPLEXER COMPOSED OF A THREE-PORT
C             CIRCULATOR AND TWO BUTTERWORTH NETWORKS AND TO DETERMINE
C             THE NECESSARY ADJUSTMENT OF THE 3 DB CUT-OFF FREQUENCY
C             IN EITHER LOW-PASS OR HIGH-PASS NETWORK.
C
C       USAGE
C         CALL DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
C
C         N      - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         OMEGAC - THE CROSSOVER FREQUENCY IN RADIANS PER SECOND.
C         OMGCL  - THE ADJUSTED FREQUENCY OF THE LOW-PASS NETWORK.
C         OMGCH  - THE ADJUSTED FREQUENCY OF THE HIGH-PASS NETWORK.
C         ITYPE  - 1, THE ADJUSTED NETWORK IS A HIGH-PASS NETWORK.
C                  2, THE ADJUSTED NETWORK IS A LOW-PASS NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,OMEGAC AND ITYPE.
C          THE OUTPUT  IS EITHER OMGCL OR OMGCH.
C
      SUBROUTINE DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
      DOUBLE PRECISION OMEGAC,OMGCL,OMGCH,OMGAJ
      OMGAJ=(2./(1.+ OMEGAC**(4*N)) -1.)**(1./(2*N))/OMEGAC
      GO TO (1,2),ITYPE
    1 OMGCH=OMGAJ
      OMGCL=OMEGAC
      WRITE(6,10)OMEGAC,OMGCL,OMGCH
   10 FORMAT('0',5X,'OMEGAC=',D20.12,5X,'OMGCL=',D20.12,5X,'OMGCH=',
     1D20.12//)
      GO TO 20
    2 OMGCL=1./OMGAJ
      OMGCH=1./OMEGAC
      WRITE(6,15)OMEGAC,OMGCL,OMGCH
   15 FORMAT('0',5X,'OMEGAC=',D20.12,5X,'OMGCL=',D20.12,5X,'OMGCH
     1D20.12//)
   20 RETURN
      END
```

```
C       SUBROUTINE DPLXCH
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF FREQUUENCY RESPONSE
C             OF A DIPLEXER COMPOSED OF A THREE-PORT CIRCULATOR AND
C             TWO BUTTERWORTH NETWORKS.
C
C       USAGE
C         CALL DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         OMGCL   - THE  CUT-OFF FREQUENCY OF THE LOW-PASS BUTTERWORTH
C                      NETWORK IN RADIANS PER SECOND.
C         OMGCH   - THE CUT-OFF FREQUENCY OF THE HIGH-PASS BUTTERWORTH
C                      NETWORK IN RADIANS PER SECOND.
C         G1      - THE TRANSDUCER POWER GAIN  OF THE LOW-PASS CHANNEL.
C         G2      - THE TRANSDUCER POWER GAIN OF THE HIGH-PASS CHANNEL.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT VALUES ARE N,OMGCL,OMGCH,OMEGA.
C          THE OUTPUT IS  G1 AND G2.
C
      SUBROUTINE DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
      DOUBLE PRECISION OMGCL,OMGCH,OMEGA,G1,G2,GA
      G1=10.*DLOG10(1.+(OMEGA/OMGCL)**(2*N))
      G2=10.*DLOG10(1.+(OMGCH/OMEGA)**(2*N))
      GO TO (1,2),ITYPE
    1 GA=10.*DLOG10(1.+(OMGCL/OMEGA )**(2*N))
      G2=G2+GA
      GO TO 20
    2 GA=10.*DLOG10(1.+(OMEGA /OMGCH)**(2*N))
      G1=G1+GA
   20 RETURN
      END
```

```
C
C       SUBROUTINE BTNK
C
C       PURPOSE
C          THE PROGRAM IS FOR THE CALCULATION OF THE ELEMENT VALUES OF
C             AN OPTIMUM BUTTERWORTH LOW-PASS LADDER NETWORK TERMINATED
C             IN A RESISTIVE GENERATOR WITH INTERNAL RESISTANCE R1 AND
C             A PARALLEL RC LOAD.
C
C       USAGE
C         CALL BTNK(N,R1,R,C,OMEGAC,ALC)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         R1      - THE INTERNAL RESISTANCE OF THE SOURCE.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH NETWORK IN
C                      RADIANS PER SECOND.
C         ALC     - THE ELEMENT VALUES OF THE BUTTERWORTH NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,R1,R,C AND OMEGAC.
C          THE OUTPUT IS ALC.
C
      SUBROUTINE BTNK(N,R1,R,C,OMEGAC,ALC)
      REAL*8 NT
      DOUBLE PRECISION R,R1,C,C1,OMEGAC,PI,XP,XQ,AK,B,DELTA,RA
     1,RB,RC,RD,RE,ALC(1)
      PI=3.1415926535898
      XP=R*C*OMEGAC
      XQ=2.*DSIN(PI/(2.*N))
      IF(XP.LT.XQ) GO TO 20
   10 DELTA=1.0-XQ/XP
      C1=C
      AK=1.-DELTA**(2*N)
      GO TO 30
   20 DELTA=0.
      AK=1.
      C1=XQ/(R*OMEGAC)
   30 B=DELTA**N
      NT=DSQRT(R1*(1.+B)/(R*(1.-B)))
      WRITE(6,50)DELTA,AK,NT
   50 FORMAT('1',2X,'DELTA= ',D20.12/3X,'AK=     ',D20.12/3X,'NT =',D23.1
     12/)
      ALC(1)=C1-C
      WRITE(6,60)ALC(1)
```

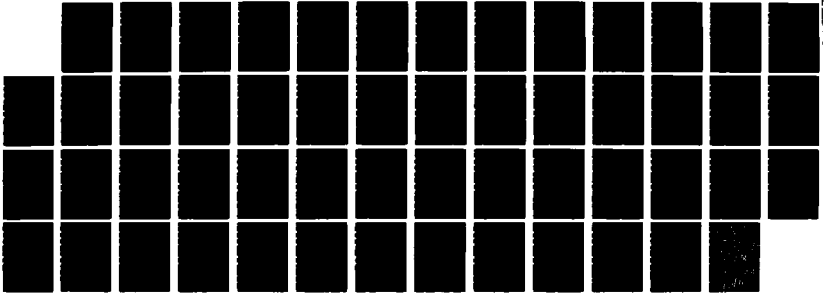
```
 60 FORMAT(3X,'C( 1)=',1X,D20.12)
    J=N/2
    DO 90 M=1,J
    RA=PI*(4*M-3)/(2.*N)
    RB=PI*(4*M-2)/(2.*N)
    RC=PI*(4*M-1)/(2.*N)
    MM=2*M
    ALC(MM)=4.*DSIN(RA)*DSIN(RC)/(C1*OMEGAC**2*(1.-2.*DCOS(RB)*
   4DELTA+DELTA**2))
    WRITE(6,65)MM,ALC(MM)
 65 FORMAT(3X,'L(',I2,')= ',D20.12)
    IF(MM-N) 70,90,90
 70 RD=PI*4*M/(2.*N)
    RE=PI*(4*M+1)/(2.*N)
    C1=4.*DSIN(RC)*DSIN(RE)/(ALC(MM)*OMEGAC**2*(1.-2.*DELTA*DCOS(RD)
   5+DELTA**2))
    MM1=MM+1
    ALC(MM1)=C1
    WRITE(6,80)MM1,ALC(MM1)
 80 FORMAT(3X,'C(',I2,')= ',D20.12)
 90 CONTINUE
100 RETURN
    END
```

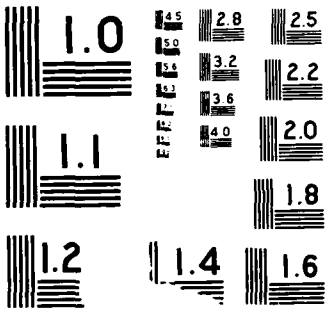
```
C
C       SUBROUTINE BTCF
C
C       PURPOSE
C          THE PROGRAM IS FOR THE COMPUTATION  OF THE COEFFICIENTS OF
C             THE BUTTERWORTH POLYNOMIAL OF  ORDER N.
C
C       USAGE
C         CALL BTCF(N,A)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         A       - THE VALUES OF THE BUTTERWORTH COEFFICIENTS.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT IS N.
C          THE OUTPUT IS A.
C
      SUBROUTINE BTCF(N,A)
      DOUBLE PRECISION A(1),AA,PI
      PI=3.1415926535898
      NN=N-1
      AA=1.
      A(1)=AA
      DO 10 IU=1,NN
      AA=AA*DCOS((IU-1)*PI/(2*N))/DSIN(IU*PI/(2*N))
      A(IU+1)=AA
   10 CONTINUE
      NN=N+1
      A(NN)=1.
      RETURN
      END
```

```
C       SUBROUTINE POLY
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE  VALUES OF
C             A COMPLEX   POLYNOMIAL AT A FIXED RADIAN COMPLEX
C             FREQUENCY Y.
C
C       USAGE
C         CALL POLY(N,A,Y,Q)
C
C         N       - THE ORDER OF THE POLYNOMIAL.
C         A       - THE POLYNOMIAL  COEFFICIENTS.
C         Y       - THE RADIAN COMPLEX FREQUENCY.
C         Q       - THE VALUE OF THE POLYNOMIAL EVALUATED AT Y.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,A AND Y.
C          THE OUTPUT  IS Q.
C
      SUBROUTINE POLY (N,A,Y,Q)
      COMPLEX*16 Y,Q
      DOUBLE PRECISION A(1)
      NN=N+1
      Q=0.
      DO 10 IU=1,NN
      Q=Q+A(IU)*Y**(IU-1)
   10 CONTINUE
      RETURN
      END
```

AD-A194 224 ON BROADBAND MATCHING OF MULTIPORT ELECTRICAL NETWORKS WITH APPLICATIONS (U) ILLINOIS UNIV AT CHICAGO CIRCLE DEPT OF ELECTRICAL ENGINEERIN W CHEN JAN 88 3/3

UNCLASSIFIED UIC-EECS-87-3 AFWAL-TR-87-1131 F/G 9/1 NL

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4.5
5.0
5.6

```
C       SUBROUTINE FRQS
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE MAGNITUDE RESPONSE
C             OF A SYMMETRICAL DIPLEXER COMPOSED OF TWO CANONICAL
C             BUTTERWORTH NETWORKS AT A FIXED RADIAN FREQUNCY OMEGA.
C
C       USAGE
C         CALL FRQS(N,A OMEGAC,OMEGA,G12,G13,ITYPE)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         A       - THE BUTTERWORTH COEFFICIENTS.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE LOWPASS BUTTERWORTH
C                      NETWORK IN RADIAN.
C         OMEGA   - THE RADIAN FREQUENCY.
C         G12     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 2 AT
C                      A FIXED FREQUENCY OMEGA.
C         G13     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 3 AT
C                      A FIXED FREQUENCY.
C         ITYPE   - 0, G12 AND G13 ARE IN RATIO.
C                 - 1, G12 AND G13 ARE IN DB.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          CALL POLY(N,A,Y,Q)
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS ARE N,A,OMEGAC AND OMEGA.
C          THE OUTPUTS ARE G12 AND G13.
C
      SUBROUTINE FRQS(N,A,OMEGAC,OMEGA,G12,G13,ITYPE)
      COMPLEX*16 Y,Y1,Y2,PP1,QQ1,PP2,QQ2,Q1,Q2,SD,S12,S13,Z1,Z2
      DOUBLE PRECISION OMEGA,OMEGAC,A(1),IMAG,G12,G13,REAL
      Y=DCMPLX(0.0D0 ,OMEGA)
      Y1=Y/OMEGAC
      CALL POLY(N,A,Y1,Q1)
      Y2=Y*OMEGAC
      CALL POLY (N,A,Y2,Q2)
      PP1=Q1-Y1**N
      QQ1=Q1+Y1**N
      PP2=Q2-1.
      QQ2=Q2+1.
      SD=PP1*QQ2+PP2*QQ1+QQ1*QQ2
      S12=2.*QQ2/SD
      S13=2.*(Y2**N)*QQ1/SD
      G12=CDABS(S12)
      G13=CDABS(S13)
      Z1=(Q1-Y1**N)/(Q1+Y1**N)
      Z2=(Q2-1.)/(Q2+1.)
      IF(ITYPE.NE.0) RETURN
      G12=-20.*DLOG10(G12)
      G13=-20.*DLOG10(G13)
      RETURN
      END
```

```
C       SUBROUTINE PLOT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PLOTTING ONE OR MORE CURVES
C             (UP TO SIX) ON ONE PLOT BY A LINE PRINTER.
C
C       USAGE
C         CALL PLOT1(X,Y,N,DY,M)
C
C         X       - A ONE-DIMENSIONAL ARRAY.
C         Y       - A TWO-DIMENSIONAL ARRAY.
C         N       - THE NUMBER OF POINTS FOR EACH CURVE TO BE PLOTTED.
C         DY      - THE SCALE FACTOR OF Y.
C                    DY WILL BE DETERMINED AUTOMATICLY IF SET DY=0.
C         M       - THE NUMEBER OF CURVES TO BE PLOTTED ON
C                    A SINGAL PLOT.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.

      SUBROUTINE PLOT1(X,Y,N,DY,M)
      INTEGER ROW(81),STAR,BLANK,POINT,S3,S4,S5,PLUS
      DATA STAR,BLANK,POINT,PLUS,S3,S4,S5,S6/1H*,1H ,1H.,1H+,1H-,1H$,1H:
     1,1H&/
      DIMENSION X(N),Y(N,M)
      YMIN=1.E10
      YMAX=-1.E-10
      DO 5 I=1,81
    5 ROW(I)=BLANK
      DO 10 J=1,M
      DO 10 I=1,N
      IF(Y(I,J).GT.YMAX) YMAX=Y(I,J)
   10 IF(Y(I,J).LT.YMIN) YMIN=Y(I,J)
      D=DY
      IF(DY.EQ.0.)      D=(YMAX-YMIN)/100.
      IF(YMAX.LT.0.OR .YMIN.GT.0.) GO TO 15
      NZ=-YMIN/D+1
      ROW(NZ)=POINT
   15 CONTINUE
      NN=N-4
      WRITE(6,20)
   20 FORMAT('1',5X,'X(I)',8X,'Y(I,1)')
      WRITE(6,25)
   25 FORMAT(37X,'+',8('---------+'))
      DO 90 L=1,NN,5
      DO 90 LL=1,5
      I=L+LL-1
      IF(I.NE.N)   GO TO 32
      DO 30 K=1,72,10
      ROW(K)=PLUS
```

```
      DO 30 KK=1,9
      K1=K+KK
   30 ROW(K1)=S3
      GO TO 34
   32 ROW(1)=S5
      ROW(81)=S5
      IF(LL.EQ.5) ROW(1)=PLUS
      IF(LL.EQ.5) ROW(81)=PLUS
   34 CONTINUE
      DO 50 J=1,M
      NY=(Y(I,J)-YMIN)/D+1.5
      IF(NY.GT.81) GO TO 50
      GO TO (41,42,43,44,45,46), J
   41 ROW(NY)=POINT
      GO TO 50
   42 ROW(NY)=STAR
      GO TO 50
   43 ROW(NY)=PLUS
      GO TO 50
   44 ROW(NY)=S3
      GO TO 50
   45 ROW(NY)=S4
      GO TO 50
   46 ROW(NY)=S5
   50 CONTINUE
      WRITE(6,60) X(I),Y(I,1),ROW
   60 FORMAT(1X,2E13.6,10X,81A1)
      DO 80 K=1,81
   80 ROW(K)=BLANK
   90 CONTINUE
      RETURN
      END
```

```
C       SUBROUTINE CRT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PRINTING OF THE CONFIGURATION
C             AND THE ELEMENT VALUES OF A LADDER  LOW-PASS
C             LOSSLESS NETWORK TERMINATED IN A RESISTANCE LOAD.
C
C       USAGE
C         CALL CRT1(R,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE ELEMENT VALUES OF THE LADDER NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUTS ARE R,ALC AND N.
C          THE OUTPUTS ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT1(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'C',11('-'),'|',11X,'C(',I2,')=',D20.12,3X,
     4'F'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'L',11X,'L(',I2,')=',D20.12,2X,
     5' H')
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     14X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

```
C       SUBROUTINE CRT2
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PRINTING OF THE CONFIGURATION AND
C             THE ELEMENT VALUES OF A LADDER  HIGH-PASS  LOSSLESS
C             NETWORK TERMINATED IN AN RESISTIVE LOAD.
C
C       USAGE
C         CALL CRT2(R,C,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE ELEMENT VALUES OF THE LADDER NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED.
C          THE INPUTS   ARE R,C,ALC AND N.
C          THE OUTPUT IS THE ELEMENT VALUES.
C
C
      SUBROUTINE CRT2(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'L',11('-'),'|',11X,'L(',I2,')=',D20.12,3X,
     4'H'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'C',11X,'C(',I2,')=',D20.12,2X,
     5' F'/)
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     1    4X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

Appendix B: Program Package for the Design of Multiplexers

Main programs: MUPLX .

Subroutines: MPLX, BTNK;
BPT, CRT1;
PLOT1.

```
C
C      MAIN PROGRAM
C
       DOUBLE PRECISION OMEGAE(10,2),OMEGA,OMEGA1,OMEGA2,OMEGAO(10)
      1G21(10),G11(10),G21M(10),DG21M(10),B(10),
      1R1,R,C,ALC(10),AL(10),AC(10)
       DIMENSION  X(50),YG(50,6),ZG(50,6),YG1(50,6),ZG1(50,6)
       READ, N,OMEGA1,OMEGA2
       WRITE(6,10)N,OMEGA1,OMEGA2
   10  FORMAT('1',5X,'N=',I3,10X,'OMEGA1=',D20.12,10X,'OMEGA2=',D20
       READ,M
       WRITE(6,15)M
   15  FORMAT(6X,'M=',I3//)
       DO 20 I=1,M
       READ, OMEGAE(I,1),OMEGAE(I,2)
   20  WRITE(6,25)I,OMEGAE(I,1),I,OMEGAE(I,2)
   25  FORMAT('0',5X,'OMEGAE(',I2,',1)=',D20.12,10X,'OMEGAE(',I2,',
      120.12)
       WRITE(6,30)
   30  FORMAT('1',4X,'OMEGA',8X,'G21M'/)
       DO 80 I=1,50
       OMEGA=OMEGA1+(OMEGA2-OMEGA1)*I/50.
       X(I)=OMEGA
       CALL MPLX(N,M,OMEGA,OMEGAE,G21M)
       WRITE(6,35)OMEGA,(G21M(J),J=1,M)
   35  FORMAT(1X,5(D12.5))
       DO 40 K=1,M
   40  YG(I,K)=G21M(K)
   80  CONTINUE
       CALL BTNK (N,1.D0,1.D0,0.D0,1.D0,ALC)
       CALL CRT1 (1.D0,ALC,N)
       DO 100 I=1,M
       WRITE(6,99) I
   99  FORMAT('0',8X,'M=',I2/)
       CALL BPT (N,OMEGAE(I,1),OMEGAE(I,2),ALC,AL,AC)
  100  CONTINUE
       CALL PLOT1(X,YG,50,0.25,M)
       STOP
       END
```

```
C       SUBROUTINE MPLX
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF FREQUUENCY RESPONSE
C             OF A MULTIPLEXER COMPOSED OF A MULTI-PORT CIRCULATOR
C             THE BUTTERWORTH NETWORKS.
C
C       USAGE
C         CALL MPLX(N,M,OMEGA,OMEGAE,G21M)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         M       - THE NUMBER OF CHANNELS.
C         OMEGA   - THE RADIAN FREQUENCY.
C         OMEGAE - THE EDGE FREQUENCIES  OF THE BAND-PASS RESPONSE.
C         DG21M   - THE TRANSDUCER POWER GAIN.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT VALUES ARE N,M, OMEGA,OMEGAE.
C          THE OUTPUT IS  G21M.
C
      SUBROUTINE MPLX(N,M,OMEGA,OMEGAE,G21M)
      DOUBLE PRECISION OMEGA,OMEGAB,OMEGAE(10,2),OMEGAO(10),G21(10
     1G11(10),DG21M(10),B(10),G21M(M)
      DO 10 I=1,M
      OMEGAO(I)=(OMEGAE(I,1)*OMEGAE(I,2))**0.5
      B(I)=OMEGAE(I,2)-OMEGAE(I,1)
      OMEGAB=(OMEGAO(I)/B(I))*(OMEGA/OMEGAO(I)-OMEGAO(I)/OMEGA)
      G21(I)=10.*DLOG10(1.+ OMEGAB**(2*N))
      G11(I)=10.*DLOG10(1.+(1./OMEGAB)**(2*N))
      IF(I.NE.1) GO TO 5
      G21M(I)=G21(I)
      DG21M(I)=G11(I)
      GO TO 10
    5 DG21M(I)=DG21M(I-1)+G11(I)
      G21M(I)=G21(I)+DG21M(I-1)
   10 CONTINUE
      RETURN
      END
```

```
C
C       SUBROUTINE BTNK
C
C       PURPOSE
C          THE PROGRAM IS FOR THE CALCULATION OF THE ELEMENT VALUES
C             AN OPTIMUM BUTTERWORTH LOW-PASS LADDER NETWORK TERMINA
C             IN A RESISTIVE GENERATOR WITH INTERNAL RESISTANCE R1 A
C             A PARALLEL RC LOAD.
C
C       USAGE
C         CALL BTNK(N,R1,R,C,OMEGAC,ALC)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         R1      - THE INTERNAL RESISTANCE OF THE SOURCE.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH NETWORK
C                      RADIANS PER SECOND.
C         ALC     - THE ELEMENT VALUES OF THE BUTTERWORTH NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,R1,R,C AND OMEGAC.,
C          THE OUTPUT IS ALC.
C
      SUBROUTINE BTNK(N,R1,R,C,OMEGAC,ALC)
      REAL*8 NT
      DOUBLE PRECISION R,R1,C,C1,OMEGAC,PI,XP,XQ,AK,B,DELTA,RA
     1,RB,RC,RD,RE,ALC(1)
      PI=3.1415926535898
      XP=R*C*OMEGAC
      XQ=2.*DSIN(PI/(2.*N))
      IF(XP.LT.XQ) GO TO 20
   10 DELTA=1.0-XQ/XP
      C1=C
      AK=1.-DELTA**(2*N)
      GO TO 30
   20 DELTA=0.
      AK=1.
      C1=XQ/(R*OMEGAC)
   30 B=DELTA**N
      NT=DSQRT(R1*(1.+B)/(R*(1.-B)))
      WRITE(6,50)DELTA,AK,NT
   50 FORMAT('1',2X,'DELTA= ',D20.12/3X,'AK=     ',D20.12/3X,'NT =',
     12/)
      ALC(1)=C1-C
      WRITE(6,60)ALC(1)
```

```
 60 FORMAT(3X,'C( 1)=',1X,D20.12)
    J=N/2
    DO 90 M=1,J
    RA=PI*(4*M-3)/(2.*N)
    RB=PI*(4*M-2)/(2.*N)
    RC=PI*(4*M-1)/(2.*N)
    MM=2*M
    ALC(MM)=4.*DSIN(RA)*DSIN(RC)/(C1*OMEGAC**2*(1.-2.*DCOS(RB)*
   4DELTA+DELTA**2))
    WRITE(6,65)MM,ALC(MM)
 65 FORMAT(3X,'L(',I2,')= ',D20.12)
    IF(MM-N) 70,90,90
 70 RD=PI*4*M/(2.*N)
    RE=PI*(4*M+1)/(2.*N)
    C1=4.*DSIN(RC)*DSIN(RE)/(ALC(MM)*OMEGAC**2*(1.-2.*DELTA*DCOS(
   5+DELTA**2))
    MM1=MM+1
    ALC(MM1)=C1
    WRITE(6,80)MM1,ALC(MM1)
 80 FORMAT(3X,'C(',I2,')= ',D20.12)
 90 CONTINUE
100 RETURN
    END
```

```
C
C       SUBROUTINE BPT
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE ELEMENT VALUES OF
C             A BANDPASS FILTER FROM THE LOWPASS FILTER.
C
C       USAGE
C         CALL BPT(N,OMEGA1,OMEGA2,ALC,AL,AC)
C
C         N      - THE ORDER OF THE FILTER.
C         OMEGA1 - THE EDGE FREQUENCY  OF THE BAND-PASS RESPONSE.
C         OMEGA2 - THE EDGE FREQUENCY  OF THE BAND-PASS RESPONSE.
C         ALC    - THE VALUES OF THE ELEMENTS OF LOW-PASS FILTER.
C         AL     - THE VALUES OF THE INDUCTANCES OF BANDPASS FILTER.
C         AC     - THE VALUES OF THE CAPACITANCES OF BANDPASS FILTER.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT VALUES ARE N,OMEGA1,OMEGA2 AND ALC.
C          THE OUTPUT IS  AL AND AC.
C
      SUBROUTINE BPT(N,OMEGA1,OMEGA2,ALC,AL,AC)
      DOUBLE PRECISION OMEGA1,OMEGA2,ALC(N),AL(N),AC(N),OMEGAO,B
      OMEGAO=(OMEGA1*OMEGA2)**0.5
      B=OMEGA2-OMEGA1
      DO 10 J=1,N,2
      AL(J)=ALC(J)/B
   10 AC(J)=B/(ALC(J)*OMEGAO**2)
      DO 20 J=2,N,2
      AC(J)=ALC(J)/B
   20 AL(J)=B/(ALC(J)*OMEGAO**2)
      DO 40 J=1,N
   40 WRITE(6,50) J,AL(J),J,AC(J)
   50 FORMAT(10X,'AL(',I2,')=',D20.12,5X,'AC(',I2,')=',D20.12)
      RETURN
      END
```

```
C
C       SUBROUTINE CRT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PRINTING OF THE CONFIGURATION
C             AND THE ELEMENT VALUES OF A LADDER  LOW-PASS
C             LOSSLESS NETWORK TERMINATED IN A RESISTANCE LOAD.
C
C       USAGE
C         CALL CRT1(R,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE ELEMENT VALUES OF THE LADDER NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUTS ARE R,ALC AND N.
C          THE OUTPUTS ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT1(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'C',11('-'),'|',11X,'C(',I2,')=',D20.12,3X,
     4'F'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'L',11X,'L(',I2,')=',D20.12,2X,
     5' H')
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     14X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

```
C       SUBROUTINE PLOT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PLOTTING ONE OR MORE CURVES
C             (UP TO SIX) ON ONE PLOT BY A LINE PRINTER.
C
C       USAGE
C         CALL PLOT1(X,Y,N,DY,M)
C
C         X       - A ONE-DIMENSIONAL ARRAY.
C         Y       - A TWO-DIMENSIONAL ARRAY.
C         N       - THE NUMBER OF POINTS FOR EACH CURVE TO BE PLOTTED
C         DY      - THE SCALE FACTOR OF Y.
C                   DY WILL BE DETERMINED AUTOMATICLY IF SET DY=0.
C         M       - THE NUMEBER OF CURVES TO BE PLOTTED ON
C                    A SINGAL PLOT.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.

      SUBROUTINE PLOT1(X,Y,N,DY,M)
      INTEGER ROW(81),STAR,BLANK,POINT,S3,S4,S5,PLUS
      DATA STAR,BLANK,POINT,PLUS,S3,S4,S5,S6/1H*,1H ,1H.,1H+,1H-,1H
     1,1H&/
      DIMENSION X(N),Y(N,M)
      YMIN=1.E10
      YMAX=-1.E-10
      DO 5 I=1,81
    5 ROW(I)=BLANK
      DO 10 J=1,M
      DO 10 I=1,N
      IF(Y(I,J).GT.YMAX) YMAX=Y(I,J)
   10 IF(Y(I,J).LT.YMIN) YMIN=Y(I,J)
      D=DY
      IF(DY.EQ.0.)      D=(YMAX-YMIN)/100.
      IF(YMAX.LT.0.OR .YMIN.GT.0.) GO TO 15
      NZ=-YMIN/D+1
      ROW(NZ)=POINT
   15 CONTINUE
      NN=N-4
      WRITE(6,20)
   20 FORMAT('1',5X,'X(I)',8X,'Y(I,1)')
      WRITE(6,25)
   25 FORMAT(37X,'+',8('---------+'))
      DO 90 L=1,NN,5
      DO 90 LL=1,5
      I=L+LL-1
      IF(I.NE.N)   GO TO 32
      DO 30 K=1,72,10
      ROW(K)=PLUS
```

```
      DO 30 KK=1,9
      K1=K+KK
   30 ROW(K1)=S3
      GO TO 34
   32 ROW(1)=S5
      ROW(81)=S5
      IF(LL.EQ.5) ROW(1)=PLUS
      IF(LL.EQ.5) ROW(81)=PLUS
   34 CONTINUE
      DO 50 J=1,M
      NY=(Y(I,J)-YMIN)/D+1.5
      IF(NY.GT.81) GO TO 50
      GO TO (41,42,43,44,45,46), J
   41 ROW(NY)=POINT
      GO TO 50
   42 ROW(NY)=STAR
      GO TO 50
   43 ROW(NY)=PLUS
      GO TO 50
   44 ROW(NY)=S3
      GO TO 50
   45 ROW(NY)=S4
      GO TO 50
   46 ROW(NY)=S5
   50 CONTINUE
      WRITE(6,60) X(I),Y(I,1),ROW
   60 FORMAT(1X,2E13.6,10X,81A1)
      DO 80 K=1,81
   80 ROW(K)=BLANK
   90 CONTINUE
      RETURN
      END
```

Appendix C: Program Package for the Design of Diplexers

Main programs: DIPLX.

Subroutines: DPLX;
BTNK, BTCF;
POLY,FRQS;
PLOT1,CRT1,CRT2.

```
C
C     MAIN PROGRAM: DIPLX.
C
      DOUBLE PRECISION OMEGA,OMEGAC,A(20),G12,G13,OMEGCO,ATCO,R,R1,
     1ALC(20)
      DIMENSION X(50),YG(50,2)
      READ, OMEGCO,ATCO,N,ITYPE
      WRITE(6,30) OMEGCO,ATCO
   30 FORMAT('1',4X,'OMEGCO=',D20.12,5X,'ATCO=',D20.12)
      CALL DPLX ( 1.0D00,ATCO,N,OMEGAC,A,ITYPE)
      CALL BTNK (N,1.D0,1.D0,0.D0,OMEGAC,ALC)
      CALL CRT1 (1.D0,ALC,N)
      DO 40 I=1,N
   40 ALC(I)=1./ALC(I)
      CALL CRT2 (1.D0,ALC,N)
      DO 90 I=1,50
      OMEGA=OMEGCO*0.04*I
      CALL FRQS(N,A,OMEGAC,OMEGA,G12,G13,ITYPE)
      X(I)=OMEGA
      YG(I,1)=G12
      YG(I,2)=G13
   90 CONTINUE
      CALL PLOT1 (X,YG,50,0.25,2)
      STOP
      END
```

```
C       SUBROUTINE DPLX
C
C       PURPOSE
C          THE PROGRAM IS TO DESIGN A SYMMETRICAL DIPLEXER COMPOSED OF
C             TWO BUTTERWORTH NETWORKS AND TO DETERMINE THE 3 DB CUT-
C             OFF RADIAN FREQUENCY OF  THE  LOW-PASS FILTER HAVING
C             THE PREASSIGNED  ATTENUATION AT CROSSOVER FREQUENCY.
C
C       USAGE
C         CALL DPLX(OMEGCO,ATCO,N,OMEGAC,ITYPE)
C
C         OMEGCO - THE CROSSOVER FREQUENCY IN RADIANS PER SECOND.
C         ATCO   - THE ATTENUATION AT THE CROSSOVER FREQUENCY.
C         N      - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         OMEGAC - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH  NETWORK IN
C                     RADIANS PER SECOND.
C         ITYPE  - 0, ATCO IS IN RATIO.
C                  1, ATCO IS IN DB.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          CALL BTCF(N,A)
C          CALL FRQS(N,A,OMEGAC,OMEGCO,G12,G13,ITYPE)
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE OMEGCO,ATCO,N AND ITYPE.
C          THE OUTPUT IS OMEGAC.
C
      SUBROUTINE DPLX(OMEGCO,ATCO,N,OMEGAC,A,ITYPE)
      DOUBLE PRECISION OMEGA1,OMEGCO,OMEGAC,AT1,ATCO,W1,W2,G12,G13,A(1)
     1,AA
      ALPHA=.618
      IF(N.EQ.0) N=2
   10 CALL BTCF(N,A)
      W1=0.
      W2=OMEGCO
    1 OMEGAC=W1+ALPHA*(W2-W1)
      CALL FRQS(N,A,OMEGAC,OMEGCO,G12,G13,ITYPE)
      IF(G12-ATCO) 15,5,2
    2 IF(G12-1.01*ATCO) 5,5,3
    3 W1=OMEGAC
      GO TO 1
   15 W2=OMEGAC
      GO TO 1
    5 WRITE(6,60)OMEGAC,G12
   60 FORMAT(5X,'OMEGAC=',D20.12,5X,'ATCO=',D20.12)
      RETURN
      END
```

```
C
C       SUBROUTINE BTNK
C
C       PURPOSE
C          THE PROGRAM IS FOR THE CALCULATION OF THE ELEMENT VALUES OF
C             AN OPTIMUM BUTTERWORTH LOW-PASS LADDER NETWORK TERMINATED
C             IN A RESISTIVE GENERATOR WITH INTERNAL RESISTANCE R1 AND
C             A PARALLEL RC LOAD.
C
C       USAGE
C         CALL BTNK(N,R1,R,C,OMEGAC,ALC)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         R1      - THE INTERNAL RESISTANCE OF THE SOURCE.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH NETWORK IN
C                      RADIANS PER SECOND.
C         ALC     - THE ELEMENT VALUES OF THE BUTTERWORTH NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,R1,R,C AND OMEGAC.
C          THE OUTPUT IS ALC.
C
      SUBROUTINE BTNK(N,R1,R,C,OMEGAC,ALC)
      REAL*8 NT
      DOUBLE PRECISION R,R1,C,C1,OMEGAC,PI,XP,XQ,AK,B,DELTA,RA
     1,RB,RC,RD,RE,ALC(1)
      PI=3.1415926535898
      XP=R*C*OMEGAC
      XQ=2.*DSIN(PI/(2.*N))
      IF(XP.LT.XQ) GO TO 20
   10 DELTA=1.0-XQ/XP
      C1=C
      AK=1.-DELTA**(2*N)
      GO TO 30
   20 DELTA=0.
      AK=1.
      C1=XQ/(R*OMEGAC)
   30 B=DELTA**N
      NT=DSQRT(R1*(1.+B)/(R*(1.-B)))
      WRITE(6,50)DELTA,AK,NT
   50 FORMAT('1',2X,'DELTA= ',D20.12/3X,'AK=     ',D20.12/3X,'NT =',D23.1
     12/)
      ALC(1)=C1-C
      WRITE(6,60)ALC(1)
```

```
 60 FORMAT(3X,'C( 1)=',1X,D20.12)
    J=N/2
    DO 90 M=1,J
    RA=PI*(4*M-3)/(2.*N)
    RB=PI*(4*M-2)/(2.*N)
    RC=PI*(4*M-1)/(2.*N)
    MM=2*M
    ALC(MM)=4.*DSIN(RA)*DSIN(RC)/(C1*OMEGAC**2*(1.-2.*DCOS(RB)*
   4DELTA+DELTA**2))
    WRITE(6,65)MM,ALC(MM)
 65 FORMAT(3X,'L(',I2,')= ',D20.12)
    IF(MM-N) 70,90,90
 70 RD=PI*4*M/(2.*N)
    RE=PI*(4*M+1)/(2.*N)
    C1=4.*DSIN(RC)*DSIN(RE)/(ALC(MM)*OMEGAC**2*(1.-2.*DELTA*DCOS(RD)
   5+DELTA**2))
    MM1=MM+1
    ALC(MM1)=C1
    WRITE(6,80)MM1,ALC(MM1)
 80 FORMAT(3X,'C(',I2,')= ',D20.12)
 90 CONTINUE
100 RETURN
    END
```

```
C
C       SUBROUTINE BTCF
C
C       PURPOSE
C          THE PROGRAM IS FOR THE COMPUTATION  OF THE COEFFICIENTS OF
C             THE BUTTERWORTH POLYNOMIAL OF  ORDER N.
C
C       USAGE
C         CALL BTCF(N,A)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         A       - THE VALUES OF THE BUTTERWORTH COEFFICIENTS.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT IS N.
C          THE OUTPUT IS A.
C
      SUBROUTINE BTCF(N,A)
      DOUBLE PRECISION A(1),AA,PI
      PI=3.1415926535898
      NN=N-1
      AA=1.
      A(1)=AA
      DO 10 IU=1,NN
      AA=AA*DCOS((IU-1)*PI/(2*N))/DSIN(IU*PI/(2*N))
      A(IU+1)=AA
   10 CONTINUE
      NN=N+1
      A(NN)=1.
      RETURN
      END
```

```
C       SUBROUTINE POLY
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE  VALUES OF
C             A COMPLEX   POLYNOMIAL AT A FIXED RADIAN COMPLEX
C             FREQUENCY Y.
C
C       USAGE
C         CALL POLY(N,A,Y,Q)
C
C         N       - THE ORDER OF THE POLYNOMIAL.
C         A       - THE POLYNOMIAL  COEFFICIENTS.
C         Y       - THE RADIAN COMPLEX FREQUENCY.
C         Q       - THE VALUE OF THE POLYNOMIAL EVALUATED AT Y.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS  ARE N,A AND Y.
C          THE OUTPUT  IS Q.
C
      SUBROUTINE POLY (N,A,Y,Q)
      COMPLEX*16 Y,Q
      DOUBLE PRECISION A(1)
      NN=N+1
      Q=0.
      DO 10 IU=1,NN
      Q=Q+A(IU)*Y**(IU-1)
   10 CONTINUE
      RETURN
      END
```

```
C       SUBROUTINE FRQS
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE MAGNITUDE RESPONSE
C             OF A SYMMETRICAL DIPLEXER COMPOSED OF TWO CANONICAL
C             BUTTERWORTH NETWORKS AT A FIXED RADIAN FREQUNCY OMEGA.
C
C       USAGE
C         CALL FRQS(N,A OMEGAC,OMEGA,G12,G13,ITYPE)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         A       - THE BUTTERWORTH COEFFICIENTS.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE LOWPASS BUTTERWORTH
C                      NETWORK IN RADIAN.
C         OMEGA   - THE RADIAN FREQUENCY.
C         G12     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 2 AT
C                      A FIXED FREQUENCY OMEGA.
C         G13     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 3 AT
C                      A FIXED FREQUENCY.
C         ITYPE   - 0, G12 AND G13 ARE IN RATIO.
C                 - 1, G12 AND G13 ARE IN DB.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          CALL POLY(N,A,Y,Q)
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUTS ARE N,A,OMEGAC AND OMEGA.
C          THE OUTPUTS ARE G12 AND G13.
C
      SUBROUTINE FRQS(N,A,OMEGAC,OMEGA,G12,G13,ITYPE)
      COMPLEX*16 Y,Y1,Y2,PP1,QQ1,PP2,QQ2,Q1,Q2,SD,S12,S13,Z1,Z2
      DOUBLE PRECISION OMEGA,OMEGAC,A(1),IMAG,G12,G13,REAL
      Y=DCMPLX(0.0D0 ,OMEGA)
      Y1=Y/OMEGAC
      CALL POLY(N,A,Y1,Q1)
      Y2=Y*OMEGAC
      CALL POLY (N,A,Y2,Q2)
      PP1=Q1-Y1**N
      QQ1=Q1+Y1**N
      PP2=Q2-1.
      QQ2=Q2+1.
      SD=PP1*QQ2+PP2*QQ1+QQ1*QQ2
      S12=2.*QQ2/SD
      S13=2.*(Y2**N)*QQ1/SD
      G12=CDABS(S12)
      G13=CDABS(S13)
      Z1=(Q1-Y1**N)/(Q1+Y1**N)
      Z2=(Q2-1.)/(Q2+1.)
      IF(ITYPE.NE.0) RETURN
      G12=-20.*DLOG10(G12)
      G13=-20.*DLOG10(G13)
      RETURN
      END
```

```
C       SUBROUTINE PLOT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PLOTTING ONE OR MORE CURVES
C             (UP TO SIX) ON ONE PLOT BY A LINE PRINTER.
C
C       USAGE
C         CALL PLOT1(X,Y,N,DY,M)
C
C         X       - A ONE-DIMENSIONAL ARRAY.
C         Y       - A TWO-DIMENSIONAL ARRAY.
C         N       - THE NUMBER OF POINTS FOR EACH CURVE TO BE PLOTTED.
C         DY      - THE SCALE FACTOR OF Y.
C                    DY WILL BE DETERMINED AUTOMATICLY IF SET DY=0.
C         M       - THE NUMEBER OF CURVES TO BE PLOTTED ON
C                    A SINGAL PLOT.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.

      SUBROUTINE PLOT1(X,Y,N,DY,M)
      INTEGER ROW(81),STAR,BLANK,POINT,S3,S4,S5,PLUS
      DATA STAR,BLANK,POINT,PLUS,S3,S4,S5,S6/1H*,1H ,1H.,1H+,1H-,1H$,1H:
     1,1H&/
      DIMENSION X(N),Y(N,M)
      YMIN=1.E10
      YMAX=-1.E-10
      DO 5 I=1,81
    5 ROW(I)=BLANK
      DO 10 J=1,M
      DO 10 I=1,N
      IF(Y(I,J).GT.YMAX) YMAX=Y(I,J)
   10 IF(Y(I,J).LT.YMIN) YMIN=Y(I,J)
      D=DY
      IF(DY.EQ.0.)      D=(YMAX-YMIN)/100.
      IF(YMAX.LT.0.OR .YMIN.GT.0.) GO TO 15
      NZ=-YMIN/D+1
      ROW(NZ)=POINT
   15 CONTINUE
      NN=N-4
      WRITE(6,20)
   20 FORMAT('1',5X,'X(I)',8X,'Y(I,1)')
      WRITE(6,25)
   25 FORMAT(37X,'+',8('---------+'))
      DO 90 L=1,NN,5
      DO 90 LL=1,5
      I=L+LL-1
      IF(I.NE.N)   GO TO 32
      DO 30 K=1,72,10
      ROW(K)=PLUS
```

```
      DO 30 KK=1,9
      K1=K+KK
   30 ROW(K1)=S3
      GO TO 34
   32 ROW(1)=S5
      ROW(81)=S5
      IF(LL.EQ.5) ROW(1)=PLUS
      IF(LL.EQ.5) ROW(81)=PLUS
   34 CONTINUE
      DO 50 J=1,M
      NY=(Y(I,J)-YMIN)/D+1.5
      IF(NY.GT.81) GO TO 50
      GO TO (41,42,43,44,45,46), J
   41 ROW(NY)=POINT
      GO TO 50
   42 ROW(NY)=STAR
      GO TO 50
   43 ROW(NY)=PLUS
      GO TO 50
   44 ROW(NY)=S3
      GO TO 50
   45 ROW(NY)=S4
      GO TO 50
   46 ROW(NY)=S5
   50 CONTINUE
      WRITE(6,60) X(I),Y(I,1),ROW
   60 FORMAT(1X,2E13.6,10X,81A1)
      DO 80 K=1,81
   80 ROW(K)=BLANK
   90 CONTINUE
      RETURN
      END
```

```
C       SUBROUTINE CRT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PRINTING OF THE CONFIGURATION
C             AND THE ELEMENT VALUES OF A LADDER  LOW-PASS
C             LOSSLESS NETWORK TERMINATED IN A RESISTANCE LOAD.
C
C       USAGE
C         CALL CRT1(R,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE ELEMENT VALUES OF THE LADDER NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUTS ARE R,ALC AND N.
C          THE OUTPUTS ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT1(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'C',11('-'),'|',11X,'C(',I2,')=',D20.12,3X,
     4'F'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'L',11X,'L(',I2,')=',D20.12,2X,
     5' H')
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     14X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

```
C       SUBROUTINE CRT2
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PRINTING OF THE CONFIGURATION AND
C             THE ELEMENT VALUES OF A LADDER  HIGH-PASS  LOSSLESS
C             NETWORK TERMINATED IN AN RESISTIVE LOAD.
C
C       USAGE
C         CALL CRT2(R,C,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE ELEMENT VALUES OF THE LADDER NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED.
C          THE INPUTS   ARE R,C,ALC AND N.
C          THE OUTPUT IS THE ELEMENT VALUES.
C
C
      SUBROUTINE CRT2(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'L',11('-'),'|',11X,'L(',I2,')=',D20.12,3X,
     4'H'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'C',11X,'C(',I2,')=',D20.12,2X,
     5' F'/)
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     1    4X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

APPENDIX D:

Program Package for the Design of Filters, Diplexers and Multiplexers

(Kept in Account U29459)

Apr. 17, 1986

```
C
C       SUBROUTINE DPLX
C
C       PURPOSE
C          THE PROGRAM IS TO DESGIN A SYMMETRICAL DIPLEXER COMPOSED OF
C             TWO BUTTERWORTH NETWORKS AND TO DETERMINE THE 3 DB CUT-
C             OFF RADIAN FREQUENCY OF THE LOW-PASS ONE FOR SATISFING
C             THE GIVEN ATTENUATION AT CROSSOVER FREQUENCY.
C
C       USAGE
C         CALL DPLX(OMEGCO,ATCO,N,OMEGAC,ITYPE)
C
C         OMEGCO - THE CROSSOVER FREQUENCY IN RADIANS PER SECOND.
C         ATCO   - THE ATTENUATION AT THE CROSSOVER FREQUENCY.
C         N      - THE ORDER OF BUTTERWORTH RESPONSE.
C         OMEGAC - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH  NETWORK IN
C                     RADIANS PER SECOND.
C         ITYPE  - 0, ATCO IS IN RATIO.
C                  1, ATCO IS IN DB.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          CALL BTCF(N,A)
C          CALL FRQS(N,A,OMEGAC,OMEGCO,G12,G13,ITYPE)
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE POMPUTATION.
C          THE INPUT DATUM ARE OMEGCO,ATCO,N,ITYPE.
C          THE OUTPUT DATUM ARE OMEGAC.
C
      SUBROUTINE DPLX(OMEGCO,ATCO,N,OMEGAC,A,ITYPE)
      DOUBLE PRECISION OMEGA1,OMEGCO,OMEGAC,AT1,ATCO,W1,W2,G12,G13,A(1)
     1,AA
      ALPHA=.618
      IF(N.EQ.0) N=2
   10 CALL BTCF(N,A)
      W1=0.
      W2=OMEGCO
    1 OMEGAC=W1+ALPHA*(W2-W1)
      CALL FRQS(N,A,OMEGAC,OMEGCO,G12,G13,ITYPE)
      IF(G12-ATCO) 15,5,2
    2 IF(G12-1.01*ATCO) 5,5,3
    3 W1=OMEGAC
      GO TO 1
   15 W2=OMEGAC
      GO TO 1
    5 WRITE(6,60)OMEGAC,G12
   60 FORMAT(5X,'OMEGAC=',D20.12,5X,'ATCO=',D20.12)
      RETURN
      END
```

```
C
C       SUBROUTINE MPLX
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF FREQUUENCY RESPONSE
C             OF A MULTIPLEXER COMPOSED OF A MULTI-PORT CIRCULATOR AND
C             THE BUTTERWORTH NETWORKS.
C
C       USAGE
C         CALL MPLX(N,M,OMEGA,OMEGAE,G21M)
C
C         N       - THE ORDER OF THE BUTTERWORTH RESPONSE.
C         M       - THE NUMBER OF CHANNELS.
C         OMEGA   - THE RADIAN FREQUENCY.
C         OMEGAE  - THE EDGE FREQUENCIES  OF THE BAND-PASS RESPONSE.
C         DG21M   - THE TRANSDUCER POWER GAIN.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT VALUES ARE N,M, OMEGA,OMEGAE.
C          THE OUTPUT IS  G21M.
C
      SUBROUTINE MPLX(N,M,OMEGA,OMEGAE,G21M)
      DOUBLE PRECISION OMEGA,OMEGAB,OMEGAE(10,2),OMEGAO(10),G21(10),
     1G11(10),DG21M(10),B(10),G21M(M)
      DO 10 I=1,M
      OMEGAO(I)=(OMEGAE(I,1)*OMEGAE(I,2))**0.5
      B(I)=OMEGAE(I,2)-OMEGAE(I,1)
      OMEGAB=(OMEGAO(I)/B(I))*(OMEGA/OMEGAO(I)-OMEGAO(I)/OMEGA)
      G21(I)=10.*DLOG10(1.+ OMEGAB**(2*N))
      G11(I)=10.*DLOG10(1.+(1./OMEGAB)**(2*N))
      IF(I.NE.1) GO TO 5
      G21M(I)=G21(I)
      DG21M(I)=G11(I)
      GO TO 10
    5 DG21M(I)=DG21M(I-1)+G11(I)
      G21M(I)=G21(I)+DG21M(I-1)
   10 CONTINUE
      RETURN
      END
```

```
C
C
C       SUBROUTINE BPT
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE ELEMENT VALUES OF
C             A BANDPASS FILTER FROM THE LOWPASS FILTER.
C
C       USAGE
C         CALL BPT(N,OMEGA1,OMEGA2,ALC,AL,AC)
C
C         N       - THE ORDER OF THE FILTER.
C         OMEGA1  - THE EDGE FREQUENCIES  OF THE BAND-PASS RESPONSE.
C         OMEGA2  - THE EDGE FREQUENCIES  OF THE BAND-PASS RESPONSE.
C         ALC     - THE VALUES OF THE ELEMENTS OF LOW-PASS FILTER.
C         AL      - THE VALUES OF THE INDUCTANCES OF BANDPASS FILTER.
C         AC      - THE VALUES OF THE CAPACITANCES OF BANDPASS FILTER.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKS
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATIONS.
C          THE INPUT VALUES ARE N,OMEGA1,OMEGA2 AND ALC.
C          THE OUTPUT IS  AL AND AC.
C
      SUBROUTINE BPT(N,OMEGA1,OMEGA2,ALC,AL,AC)
      DOUBLE PRECISION OMEGA1,OMEGA2,ALC(N),AL(N),AC(N),OMEGAO,B
      OMEGAO=(OMEGA1*OMEGA2)**0.5
      B=OMEGA2-OMEGA1
      DO 10 J=1,N,2
      AL(J)=ALC(J)/B
   10 AC(J)=B/(ALC(J)*OMEGAO**2)
      DO 20 J=2,N,2
      AC(J)=ALC(J)/B
   20 AL(J)=B/(ALC(J)*OMEGAO**2)
      DO 40 J=1,N
   40 WRITE(6,50) J,AL(J),J,AC(J)
   50 FORMAT(10X,'AL(',I2,')=',D20.12,5X,'AC(',I2,')=',D20.12)
      RETURN
      END
```

```
C
C       SUBROUTINE BTCF
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE COEFFICIENTS OF
C             A BUTTERWORTH POLYNOMIAL WITH THE ORDER N.
C             RC LOAD.
C
C       USAGE
C         CALL BTCF(N,A)
C
C         N       - THE ORDER OF BUTTERWORTH RESPONSE.
C         A       - THE VALUES OF BUTTERWORTH COEFFICIENTS.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATION.
C          THE INPUT DATUM IS N.
C          THE OUTPUT DATUM ARE A.
C
      SUBROUTINE BTCF(N,A)
      DOUBLE PRECISION A(1),AA,PI
      PI=3.1415926535898
      NN=N-1
      AA=1.
      A(1)=AA
      DO 10 IU=1,NN
      AA=AA*DCOS((IU-1)*PI/(2*N))/DSIN(IU*PI/(2*N))
      A(IU+1)=AA
   10 CONTINUE
      NN=N+1
      A(NN)=1.
      RETURN
      END
```

```
C       SUBROUTINE BTNK
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE ELEMENT VALUES OF
C             AN OPTIMUM BUTTERWORTH NETWORK TERMINATED IN A RESISTIVE
C             GENERATOR WITH INTERNAL RESISTANCE R1 AND A PARALLEL
C             RC LOAD.
C
C       USAGE
C         CALL BTNK(N,R1,R,C,OMEGAC,ALC)
C
C         N       - THE ORDER OF BUTTERWORTH RESPONSE.
C         R1      - THE INTERNAL RESISTANCE OF THE SOURCE.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE BUTTERWORTH NETWORK IN
C                      RADIANS PER SECOND.
C         ALC     - THE VALUES OF THE ELEMENTS OF BUTTERWORTH NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATION.
C          THE INPUT DATUM ARE N,R1,R,C AND OMEGAC.
C          THE OUTPUT DATUM ARE ALC.
      SUBROUTINE BTNK(N,R1,R,C,OMEGAC,ALC)
      REAL*8 GAMMA,NT
      DOUBLE PRECISION R,R1,C,C1,OMEGAC,PI,XP,XQ,AK,B,DELTA,RA
     1,RB,RC,RD,RE,ALC(1)
      PI=3.1415926535898
      XP=R*C*OMEGAC
      XQ=2.*DSIN(PI/(2.*N))
      IF(XP.LT.XQ) GO TO 20
   10 DELTA=1.0-XQ/XP
      C1=C
      AK=1.-DELTA**(2*N)
      GO TO 30
   20 DELTA=0.
      AK=1.
      C1=XQ/(R*OMEGAC)
   30 B=DELTA**N
      NT=DSQRT(R1*(1.+B)/(R*(1.-B)))
      WRITE(6,50)DELTA,AK,NT
   50 FORMAT('1',2X,'DELTA= ',D20.12/3X,'AK=     ',D20.12/3X,'NT =',D23.1
     12/)
      ALC(1)=C1-C
      WRITE(6,60)ALC(1)
   60 FORMAT(3X,'C( 1)=',1X,D20.12)
      J=N/2
      DO 90 M=1,J
      RA=PI*(4*M-3)/(2.*N)
      RB=PI*(4*M-2)/(2.*N)
```

```
      RC=PI*(4*M-1)/(2.*N)
      MM=2*M
      ALC(MM)=4.*DSIN(RA)*DSIN(RC)/(C1*OMEGAC**2*(1.-2.*DCOS(RB)*
     4DELTA+DELTA**2))
      WRITE(6,65)MM,ALC(MM)
   65 FORMAT(3X,'L(',I2,')= ',D20.12)
      IF(MM-N) 70,90,90
   70 RD=PI*4*M/(2.*N)
      RE=PI*(4*M+1)/(2.*N)
      C1=4.*DSIN(RC)*DSIN(RE)/(ALC(MM)*OMEGAC**2*(1.-2.*DELTA*DCOS(RD)
     5+DELTA**2))
      MM1=MM+1
      ALC(MM1)=C1
      WRITE(6,80)MM1,ALC(MM1)
   80 FORMAT(3X,'C(',I2,')= ',D20.12)
   90 CONTINUE
  100 RETURN
      END
```

```
C
C       SUBROUTINE CBNK
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE ELEMENT VALUES OF
C             AN OPTIMUM CHEBYSHEV NETWORK TERMINATED IN A RESISTIVE
C             GENERATOR WITH INTERNAL RESISTANCE R1 AND A PARALLEL
C             RC LOAD.
C
C       USAGE
C         CALL CBNK(N,R1,R,C,OMEGAC,EP,ALC)
C
C         N       - THE ORDER OF CHEBYSHEV RESPONSE.
C         R1      - THE INTERNAL RESISSTANCE OF THE SOURCE.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE CHEBYSHEV NETWORK IN
C                      RADIANS PER SECOND.
C         EP      - THE RIPPLE COEFFICIENT OF  THE CHEBYSHEV RESPONSE.
C         ALC     - THE VALUES OF THE ELEMENTS OF CHEBYSHEV NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATION.
C          THE INPUT DATUM ARE N,R1,R,C,OMEGAC AND EP.
C          THE OUTPUT DATUM ARE ALC.
C
      SUBROUTINE CBNK(N,R1,R,C,OMEGAC,EP,ALC)
      REAL*8 GAMMA,NT
      DOUBLE PRECISION R,R1,C,C1,OMEGAC,PI,XP,XQ,AK,B,DELTA,RA
     1,RB,RC,RD,RE,ALC(1),ASINH,EP,A,A1
      PI=3.1415926535898
      XP=R*C*OMEGAC
      A=ASINH(1./EP)/N
      XQ=2.*DSIN(PI/(2.*N))/DSINH(A)
      IF(XP.LT.XQ) GO TO 20
      DELTA=EP*DSINH(N*ASINH((1.-XQ/XP)*DSINH(A)))
      C1=C
      AK=1.-DELTA**2
      GO TO 30
   20 DELTA=0.
      AK=1.
      C1=2.*DSIN(PI/(2*N))/(R*OMEGAC*DSINH(A))
   30 WRITE(6,35)C1
   35 FORMAT('1',2X,'C1=',4X,D20.12)
      A1=ASINH(DELTA/EP)
      J=N/2
      IF(2*J.EQ.N) GO TO 40
      BO=2.**(1-N)*DSINH(N*A)
      BO1=2.**(1-N)*DSINH(N*A1)
      GO TO 50
   40 BO=2.**(1-N)*DCOSH(N*A)
      BO1=2.**(1-N)*DCOSH(N*A1)
```

```
 50 NT=((R1*(BO+BO1))/(R*(BO-BO1)))**0.5
    WRITE(6,55)DELTA,AK,NT
 55 FORMAT(3X,'DELTA= ',D20.12/3X,'AK=     ',D20.12/3X,'NT =',D23.12/)
    ALC(1)=C1-C
    WRITE(6,60)ALC(1)
 60 FORMAT(3X,'C( 1)=',1X,D20.12)
    J=N/2
    DO 90 M=1,J
    RA=PI*(4*M-2)/(2.*N)
    RB=PI*(4*M-3)/(2.*N)
    RC=PI*(4*M-1)/(2.*N)
    F=4.*(DSINH(A)**2+DSINH(A1)**2+DSIN(RA)**2-2.*DSINH(A)*DSINH(A1)
   1*DCOS(RA))
    MM=2*M
    ALC (MM)=16.*DSIN(RB)*DSIN(RC)/(OMEGAC**2*F*C1)
    WRITE(6,65)MM,ALC (MM)
 65 FORMAT(3X,'L(',I2,')= ',D20.12)
    IF(MM-N) 70,90,90
 70 RD=PI*4*M/(2.*N)
    RE=PI*(4*M+1)/(2.*N)
    F=4.*(DSINH(A)**2+ DSINH(A1)**2+DSIN(RD)**2-2.*DSINH(A)*DSINH(A1)
   2*DCOS(RD))
    MM1=MM+1
    C1=16.*DSIN(RC)*DSIN(RE)/(OMEGAC**2*F*ALC(MM))
    ALC(MM1)=C1
    WRITE(6,80)MM1,ALC(MM1)
 80 FORMAT(3X,'C(',I2,')= ',D20.12)
 90 CONTINUE
100 RETURN
    END
```

```
C       SUBROUTINE FRQS1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE MAGANITUDE RESPONSE
C             OF A RATIONAL FUNCTION.
C
C       USAGE
C         CALL FRQS1(NA,NB,A,B,OMEGA,G,ITYPE)
C
C         NA      - THE DEGREE OF THE NUMERATOR POLYNOMIAL.
C         NB      - THE DEGREE OF THE DENOMINATOR POLYNOMIAL
C         A       - THE COEFFICIENTS OF THE NUMERATOR.
C         B       - THE CIEFFICIENTS OF THE DENOMINATOR.
C         OMEGA   - THE RADIAN FREQUENCY.
C         G       - THE MAGANITUDE EITHER IN RATIO AR IN DB.
C         ITYPE   - 0 G IS IN DB.
C                 - 1 G IS IN DB.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          POLY(N,A,Y,Q)
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE COMPUTATION.
C          THE INPUT DATUM ARE NA,NB,A,B,OMEGA AND ITYPE.
C          THE OUTPUT DATA IS G.
C
      SUBROUTINE FRQS1(NA,NB,A,B,OMEGA,G,ITYPE)
      DOUBLE PRECISION A(NA),B(NB),OMEGA,G,REAL,IMAG
      COMPLEX*16 Y,QA,QB
      Y=DCMPLX(0.D0,OMEGA)
      NA=NA-1
      NB=NB-1
      CALL POLY (NA,A,Y,QA)
      CALL POLY (NB,B,Y,QB)
      C=(REAL(QA)**2+IMAG(QA)**2)**(0.5)
      G=(REAL(QB)**2+IMAG(QB)**2)**(0.5)/G
      IF(ITYPE.EQ.0) G= 20.*DLOG10(G)
      NA=NA+1
      NB=NB+1
      RETURN
      END
```

```
C
C       SUBROUTINE FRQS
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE MAGNITUDE RESPONSE
C             OF A SYMMETRICAL DIPLEXER COMPOSED OF TWO CANONICAL
C             BUTTERWORTH NETWORKS AT A FIXED RADIAN FREQUNCY OMEGA.
C
C       USAGE
C         CALL FRQS(N,A OMEGAC,OMEGA,G12,G13,ITYPE)
C
C         N       - THE ORDER OF BUTTERWORTH RESPONSE.
C         A       - THE BUTTERWORTH COEFFICIENTS.
C         OMEGAC  - THE CUT-OFF FREQUENCY OF THE LOWPASS BUTTERWORTH
C                      NETWORK IN RADIAN.
C         OMEGA   - THE RADIAN FREQUENCY.
C         G12     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 2 AT
C                      THE FIXED FREQUENCY OMEGA.
C         G13     - THE TRANSDUCER POWER GAIN FROM PORT 1 TO PORT 3 AT
C                      THE FIXED FREQUENCY.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          CALL POLY(N,A,Y,Q)
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE POMPUTATION.
C          THE INPUT DATUM ARE N,A,OMEGAC,OMEGA.
C          THE OUTPUT DATUM ARE G12,G13.
C
      SUBROUTINE FRQS(N,A,OMEGAC,OMEGA,G12,G13,ITYPE)
      COMPLEX*16 Y,Y1,Y2,PP1,QQ1,PP2,QQ2,Q1,Q2,SD,S12,S13,Z1,Z2
      DOUBLE PRECISION OMEGA,OMEGAC,A(1),IMAG,G12,G13,REAL
      Y=DCMPLX(0.0D0 ,OMEGA)
      Y1=Y/OMEGAC
      CALL POLY(N,A,Y1,Q1)
      Y2=Y*OMEGAC
      CALL POLY (N,A,Y2,Q2)
      PP1=Q1-Y1**N
      QQ1=Q1+Y1**N
      PP2=Q2-1.
      QQ2=Q2+1.
      SD=PP1*QQ2+PP2*QQ1+QQ1*QQ2
      S12=2.*QQ2/SD
      S13=2.*(Y2**N)*QQ1/SD
      G12=(REAL(S12)**2+IMAG(S12)**2)**0.5
      G13=(REAL(S13)**2+IMAG(S13)**2)**0.5
      Z1=(Q1-Y1**N)/(Q1+Y1**N)
      Z2=(Q2-1.)/(Q2+1.)
      IF(ITYPE.NE.0) RETURN
      G12=-20.*DLOG10(G12)
      G13=-20.*DLOG10(G13)
      RETURN
      END
```

```
C
C       SUBROUTINE POLY
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF THE  VALUES OF
C             A COMPLEX   POLYNOMIAL AT A FIXED RADIAN COMPLEX
C             FREQUENCY Y.
C
C       USAGE
C         CALL POLY(N,A,Y,Q)
C
C         N       - THE ORDER OF THE POLYNOMIAL.
C         A       - THE POLYNOMIAL  COEFFICIENTS.
C         Y       - THE RADIAN COMPLEX FREQUENCY.
C         Q       - THE VALUE OF THE POLYNOMIAL AT Y.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE POMPUTATION.
C          THE INPUT DATUM ARE N,A AND Y.
C          THE OUTPUT DATUM IS Q.
C
      SUBROUTINE POLY (N,A,Y,Q)
      COMPLEX*16 Y,Q
      DOUBLE PRECISION A(1)
      NN=N+1
      Q=0.
      DO 10 IU=1,NN
      Q=Q+A(IU)*Y**(IU-1)
   10 CONTINUE
      RETURN
      END
```

```
C
C       SUBROUTINE DPLXAJ
C
C       PURPOSE
C          THE PROGRAM IS TO DESGIN A DIPLEXER COMPOSED OF A THREE-PORT
C             CIRCULATOR AND TWO BUTTERWORTH NETWORKS AND TO DETERMINE
C             THE ADJUSTMENT OF THE 3 DB CUT-OFF FREQUENCY IN EITHER
C             LOSS-PASS OR HIGH-PASS NETWORK.
C
C       USAGE
C         CALL DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
C
C         N       - THE ORDER OF BUTTERWORTH RESPONSE.
C         OMEGAC  - THE CROSSOVER FREQUENCY IN RADIANS PER SECOND.
C         OMGCL   - THE ADJUSTED FREQUENCY OF THE LOW-PASS NETWORK.
C         OMGCH   - THE ADJUSTED FREQUENCY OF THE HIGH-PASS NETWORK.
C         ITYPE   - 1, THE ADJUSTED NETWORK IS A HIGH-PASS NETWORK.
C                   2, THE ADJUSTED NETWORK IS A LOWPASS NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE POMPUTATION.
C          THE INPUT DATUM ARE N,OMEGAC,ITYPE.TYPE.
C          THE OUTPUT DATUM IS EITHER OMGCL OR OMGCH.
C
      SUBROUTINE DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
      DOUBLE PRECISION OMEGAC,OMGCL,OMGCH,OMGAJ
      OMGAJ=(2./(1.+ OMEGAC**(4*N)) -1.)**(1./(2*N))/OMEGAC
      GO TO (1,2),ITYPE
    1 OMGCH=OMGAJ
      OMGCL=OMEGAC
      WRITE(6,10)OMEGAC,OMGCL,OMGCH
   10 FORMAT('0',5X,'OMEGAC=',D20.12,5X,'OMGCL=',D20.12,5X,'OMGCH=',
     1D20.12//)
      GO TO 20
    2 OMGCL=1./OMGAJ
      OMGCH=1./OMEGAC
      WRITE(6,15)OMEGAC,OMGCL,OMGCH
   15 FORMAT('0',5X,'OMEGAC=',D20.12,5X,'OMGCL=',D20.12,5X,'OMGCH=',
     1D20.12//)
   20 RETURN
      END
```

```
C
C       SUBROUTINE DPLXCH
C
C       PURPOSE
C          THE PROGRAM IS FOR THE EVALUATION OF FREQUUENCY RESPONSE
C             OF A DIPLEXER COMPOSED OF A THREE-PORT CIRCULATOR AND
C             TWO BUTTERWORTH NETWORKS.
C
C       USAGE
C         CALL DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
C
C         N       - THE ORDER OF BUTTERWORTH RESPONSE.
C         OMGCL   - THE  CUT-OFF FREQUENCY OF THE LOW-PASS BUTTERWORTH
C                      NETWORK IN RADIANS PER SECOND.
C         OMGCH   - THE CUT-OFF FREQUENCY OF THE HIGH-PASS BUTTERWORTH
C                      NETWORK IN RADIAN PER SECOND.
C         G1      - THE TRANSDUCER POWER GAIN  OF THE LOW-PASS CHANNEL.
C         G2      - THE TRANSDUCER POWER GAIN OF THE HIGH-PASS CHANNEL.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED IN ALL THE POMPUTATION.
C          THE INPUT DATUM ARE N,OMGCL,OMGCH,OMEGA.
C          THE OUTPUT DATUM ARE G1,G2.
C
      SUBROUTINE DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
      DOUBLE PRECISION OMGCL,OMGCH,OMEGA,G1,G2,GA
      G1=10.*DLOG10(1.+(OMEGA/OMGCL)**(2*N))
      G2=10.*DLOG10(1.+(OMGCH/OMEGA)**(2*N))
      GO TO (1,2),ITYPE
    1 GA=10.*DLOG10(1.+(OMGCL/OMEGA )**(2*N))
      G2=G2+GA
      GO TO 20
    2 GA=10.*DLOG10(1.+(OMEGA /OMGCH)**(2*N))
      G1=G1+GA
   20 RETURN
      END
```

```
C
C       SUBROUTINE CRT
C
C       PURPOSE
C          THE PROGRAM IS FOR PRINTING THE CONFIGURATION AND THE
C             THE ELEMENT VALUES OF A CASCADE LOW-PASS LOSSLESS
C             NETWORK TERMINATED IN A RC PALLEL LOAD.
C
C       USAGE
C         CALL CRT(R,C,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         C       - THE CAPACITANCE OF THE LOAD.
C         ALC     - THE VALUES OF THE ELEMENTS OF CHEBYSHEV NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUT DATUM ARE R,C,ALC AND N.
C          THE OUTPUT DATUM ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT(R,C,ALC,N)
      DOUBLE PRECISION R,C,ALC(1)
      WRITE(6,10)R,C
   10 FORMAT('1',7X,7('-'),'R',7('-'),16X,'R=',D24.12,2X,'OHM'/
     1     2(7X,'|',15X,'|'/)3X,2('----',17X)/2(2(3X,'|'),15X,2('|',3X
     2     )/)3X,'|',4X,7('-'),'C',7('-'),16X,'C=     ',D20.12,2X,' F')
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(                  4(3X,'|',23X,'|'/)
     3       3X,'|',11('-'),'C',11('-'),'|',11X,'C(',I2,')=',D20.12,3X,
     4       'F'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT('+',2X,'|',23X,'L',11X,'L(',I2,')=',D20.12,2X,
     5' H')
   20 CONTINUE
      RETURN
      END
```

```
C
C       SUBROUTINE CRT1
C
C       PURPOSE
C          THE PROGRAM IS FOR PRINTING THE CONFIGURATION AND THE
C             THE ELEMENT VALUES OF A CASCADE LOW-PASS LOSSLESS
C             NETWORK TERMINATED IN A RESISTANCE LOAD.
C
C       USAGE
C         CALL CRT1(R,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE VALUES OF THE ELEMENTS OF CHEBYSHEV NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUT DATUM ARE R,ALC AND N.
C          THE OUTPUT DATUM ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT1(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(              4(3X,'|',23X,'|'/)
     3       3X,'|',11('-'),'C',11('-'),'|',11X,'C(',I2,')=',D20.12,3X,
     4       'F'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'L',11X,'L(',I2,')=',D20.12,2X,
     5' H')
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     1    4X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

```
C
C       SUBROUTINE CRT2
C
C       PURPOSE
C          THE PROGRAM IS FOR PRINTING THE CONFIGURATION AND THE
C             THE ELEMENT VALUES OF A CASCADE HIGH-PASS LOSSLESS
C             NETWORK TERMINATED IN A RESISTANCE LOAD.
C
C       USAGE
C         CALL CRT2(R,ALC,N)
C
C         N       - THE ORDER OF THE NETWORK.
C         R       - THE RESISTANCE OF THE LOAD.
C         ALC     - THE VALUES OF THE ELEMENTS OF CHEBYSHEV NETWORK.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
C
C       REMARKES
C          DOUBLE PRECISION IS USED.
C          THE INPUT DATUM ARE R,ALC AND N.
C          THE OUTPUT DATUM ARE THE VALUES OF THE ELEMENTS.
C
      SUBROUTINE CRT2(R,ALC,N)
      DOUBLE PRECISION R,ALC(1)
      WRITE(6,5)
    5 FORMAT('1',3X,'CIRCUIT CONFIGURATION'//)
      DO 20 M=1,N,2
      WRITE(6,30) M,ALC(M)
   30 FORMAT(4(3X,'|',23X,'|'/)
     33X,'|',11('-'),'L',11('-'),'|',11X,'L(',I2,')=',D20.12,3X,
     4'H'/3(3X,'|',23X,'|'/))
      MM=M+1
      IF(MM.GT.N) GO TO 20
      WRITE(6,40) MM,ALC(M+1)
   40 FORMAT(3X,'|',23X,'C',11X,'C(',I2,')=',D20.12,2X,
     5' F'/)
   20 CONTINUE
      WRITE(6,10)R
   10 FORMAT(3(3X,'|',23X,'|'/),
     1    4X,11('-'),'R',11('-'),11X,'R=',D24.12,2X,'OHM')
      RETURN
      END
```

```
C       SUBROUTINE PLOT1
C
C       PURPOSE
C          THE PROGRAM IS FOR THE PLOTTING ONE OR MORE CURVES
C             (UP TO SIX) ON ONE PLOT BY A LINE PRINTER.
C
C       USAGE
C         CALL PLOT1(X,Y,N,DY,M)
C
C         X       - A ONE-DIMENSIONAL ARRAY.
C         Y       - A TWO-DIMENSIONAL ARRAY IN WHICH EACH
C                     COLUM REPRESENTS THE  VALUES OF A VARIABLE.
C         N       - THE NUMBER OF POINTS FOR EACH CURVE TO BE PLOTTED.
C         DY      - THE SCALE FACTOR OF Y.
C                    DY WILL BE DETERMINED AUTOMATICLY IF SET DY=0.
C         M       - THE NUMEBER OF CURVES TO BE PLOTTED ON
C                    A SINGAL PLOT.
C
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
      SUBROUTINE PLOT1(X,Y,N,DY,M)
      INTEGER ROW(81),STAR,BLANK,POINT,S3,S4,S5,PLUS
      DATA STAR,BLANK,POINT,PLUS,S3,S4,S5,S6,N1,N2,N3,N4/1H*,1H ,1H.,1H+
     1,1H-,1H$,1H:,1H&,1H1,1H2,1H3,1H4/
      DIMENSION X(N),Y(N,M)
      YMIN=1.E10
      YMAX=-1.E-10
      DO 5 I=1,81
    5 ROW(I)=BLANK
      DO 10 J=1,M
      DO 10 I=1,N
      IF(Y(I,J).GT.YMAX) YMAX=Y(I,J)
   10 IF(Y(I,J).LT.YMIN) YMIN=Y(I,J)
      D=DY
      IF(DY.EQ.0.)      D=(YMAX-YMIN)/80.
      IF(YMAX.LT.0.OR .YMIN.GT.0.) GO TO 15
      NZ=-YMIN/D+1
      ROW(NZ)=POINT
   15 CONTINUE
      NN=N-4
      WRITE(6,20)
   20 FORMAT('1',5X,'X(I)',8X,'Y(I,1)')
      WRITE(6,25)
   25 FORMAT(37X,'+',8('---------+'))
      DO 90 L=1,NN,5
      DO 90 LL=1,5
      I=L+LL-1
      IF(I.NE.N)   GO TO 32
      DO 30 K=1,72,10
      ROW(K)=PLUS
      DO 30 KK=1,9
      K1=K+KK
   30 ROW(K1)=S3
      GO TO 34
   32 ROW(1)=S5
      ROW(81)=S5
```

```
      IF(LL.EQ.5) ROW(1)=PLUS
      IF(LL.EQ.5) ROW(81)=PLUS
   34 CONTINUE
      DO 50 J=1,M
      NY=(Y(I,J)-YMIN)/D+1.5
      IF(NY.GT.81) GO TO 50
      GO TO (41,42,43,44,45,46), J
   41 ROW(NY)=N1
      GO TO 50
   42 ROW(NY)=N2
      GO TO 50
   43 ROW(NY)=N3
      GO TO 50
   44 ROW(NY)=N4
      GO TO 50
   45 ROW(NY)=S4
      GO TO 50
   46 ROW(NY)=S5
   50 CONTINUE
      WRITE(6,60) X(I),Y(I,1),ROW
   60 FORMAT(1X,2E13.6,10X,81A1)
      DO 80 K=1,81
   80 ROW(K)=BLANK
   90 CONTINUE
      RETURN
      END
```

```
      FUNCTION IMAG(X)
      COMPLEX*16 X
      REAL*8 IMAG
      IMAG=X*(0.,-1.)
      RETURN
      END
C
      FUNCTION REAL(X)
      COMPLEX*16 X
      DOUBLE PRECISION REAL
      REAL=X
      RETURN
      END
C
      FUNCTION ASINH(X)
      DOUBLE PRECISION X,ASINH
      Y=X
      ASINH= ALOG(Y+(Y**2+1.)**0.5)
      RETURN
      END
```

```
C
C
C
      SUBROUTINE IMCS(R1,RLC,IE,IC,NE,OMEGA,Z,G,ITYPE)
      DIMENSION RLC(1),IE(1),IC(1)
      COMPLEX Z
      Z=(0.,0.)
      DO 50 I=1,NE
      IETYPE=IE(I)
      IF(IC(I).EQ.2) GO TO 20
      GO TO (11,13,15),IETYPE
   11 Z=Z+CMPLX(RLC(I),0.)
      GO TO 50
   13 Z=Z+CMPLX(0.,OMEGA*RLC(I))
      GO TO 50
   15 Z=Z+CMPLX(0.,1./(OMEGA*RLC(I)))
      GO TO 50
   20 GO TO (21,23,25),IETYPE
   21 Z=1./Z+CMPLX(1./RLC(I),0.)
      GO TO 30
   23 Z=1./Z+CMPLX(0.,1./(OMEGA*RLC(I)))
      GO TO 30
   25 Z=1./Z+CMPLX(0.,(OMEGA*RLC(I)))
   30 Z=1./Z
   50 CONTINUE
      R=Z
      X=(0.,-1.)*Z
      D=(R+R1)**2+X**2
      G=4.*R1*R/D
      IF(ITYPE.EQ.0) G=-10.*ALOG10(G)
      RETURN
      END
```

```
C
C
C
      DIMENSION  X(50),YG(50,6)
      DOUBLE PRECISION OMEG(3),ALC(10),OMEGA,OMEGAC,OMGCL,OMGCH,G1,
     1GA,OMGCH1
      READ, N,OMEGAC,ITYPE
      WRITE(6,15)N,OMEGAC,ITYPE
   15 FORMAT('1',5X,'N=',I3,10X,'OMEGAC=',D20.12,10X,'ITYPE=',I1//)
      CALL DPLXAJ(N,OMEGAC,OMGCL,OMGCH,ITYPE)
      DO 80 I=1,50
      OMEGA=I*0.04
      CALL DPLXCH(N,OMGCL,OMGCH,OMEGA,G1,G2,GA,ITYPE)
      X(I)=OMEGA
      YG(I,1)=G1
      YG(I,2)=G2
      YG(I,3)=GA
   80 CONTINUE
      OMEG(1)=OMEGAC
      OMEG(2)=1.
      OMEG(3)=1./OMEGAC
      DO 82 K=1,3
      CALL DPLXCH(N,OMGCL,OMGCH,OMEG(K),G1,G2,GA,ITYPE)
   82 WRITE(6,85)OMEG(K),G1,G2,GA
   85 FORMAT(6X,'OMEGA=',D20.12,5X,'G1=',D20.12,5X,'G2=',D20.12,5X,
     1,D20.12)
      CALL PLOT1(X,YG,50,0.25,2)
      CALL BTNK (N,1.D0,1.D0,0.D0,OMGCL,ALC)
      CALL CRT1 (1.D0,ALC,N)
      OMGCH1=1./OMGCH
      CALL BTNK (N,1D0,1.D0,0.D0,OMGCH1,ALC)
      DO 86 I=1,N
   86 ALC(I)=1./ALC(I)
      CALL CRT2 (1.D0,ALC,N)
      STOP
      END
```

```
C
C
C
      DOUBLE PRECISION OMEGAE(10,2),OMEGA,OMEGA1,OMEGA2,OMEGAO(10),
     1G21(10),G11(10),G21M(10),DG21M(10),B(10),
     1R1,R,C,ALC(10),AL(10),AC(10)
      DIMENSION  X(50),YG(50,6),ZG(50,6),YG1(50,6),ZG1(50,6)
      READ, N,OMEGA1,OMEGA2
      WRITE(6,10)N,OMEGA1,OMEGA2
   10 FORMAT('1',5X,'N=',I3,10X,'OMEGA1=',D20.12,10X,'OMEGA2=',D20.
      READ,M
      WRITE(6,15)M
   15 FORMAT(6X,'M=',I3//)
      DO 20 I=1,M
      READ, OMEGAE(I,1),OMEGAE(I,2)
   20 WRITE(6,25)I,OMEGAE(I,1),I,OMEGAE(I,2)
   25 FORMAT('0',5X,'OMEGAE(',I2,',1)=',D20.12,10X,'OMEGAE(',I2,',2
     120.12)
      WRITE(6,30)
   30 FORMAT('1',4X,'OMEGA',8X,'G21M'/)
      DO 80 I=1,50
      OMEGA=OMEGA1+(OMEGA2-OMEGA1)*I/50.
      X(I)=OMEGA
      CALL MPLX(N,M,OMEGA,OMEGAE,G21M)
      WRITE(6,35)OMEGA,(G21M(J),J=1,M)
   35 FORMAT(1X,5(D12.5))
      DO 40 K=1,M
   40 YG(I,K)=G21M(K)
   80 CONTINUE
      CALL BTNK (N,1.D0,1.D0,0.D0,1.D0,ALC)
      CALL CRT1 (1.D0,ALC,N)
      DO 100 I=1,M
      WRITE(6,99) I
   99 FORMAT('0',8X,'M=',I2/)
      CALL BPT (N,OMEGAE(I,1),OMEGAE(I,2),ALC,AL,AC)
  100 CONTINUE
      CALL PLOT1(X,YG,50,0.25,M)
      STOP
      END
```

```
C
C
C
      DOUBLE PRECISION OMEGA,G,A(10),B(10)
      COMPLEX*16 Y,QA,QB
      DIMENSION X(100),YG(100)
      READ(5,5)NA,NB
      READ(5,6) (A(I),I=1,NA)
      READ(5,6) (B(I),I=1,NB)
    5 FORMAT(2I2)
    6 FORMAT(8F10.0)
      WRITE(6,9)NA,NB
    9 FORMAT('1',3X,'NA=',I2,5X,'NB=',I2)
      WRITE(6,15)(I,A(I),I=1,NA)
   15 FORMAT(1X, 5(3X,'A(',I2,')=',D15.7))
      WRITE(6,16)(I,B(I),I=1,NB)
   16 FORMAT(1X, 5(3X,'B(',I2,')=',D15.7))
      WRITE(6,25)
   25 FORMAT('1',5X,'OMEGA',12X,'G'/)
      DO 10 I=1,100
      OMEGA=I*0.02
      X(I)=OMEGA
      CALL FRQS1(NA,NB,A,B,OMEGA,G,0)
      YG(I)=G
   10 WRITE(6,20) OMEGA,G
   20 FORMAT(1X, 2D15.7)
      CALL PLOTXY(X,YG,100)
      STOP
      END
```

```
C
C
C
      DOUBLE PRECISION R,R1,C,OMEGAC,ALC(20)
      READ,R,C,R1,OMEGAC,N
      WRITE(6,100)R,C,R1,OMEGAC,N
  100 FORMAT(3X,'R=',5X,D20.12/3X,'C=',5X,D20.12/3X,'R1=',4X,D20.12
     1/3X,'OMEGAC=',D20.12/3X,'N=  ',3X,I3/)
      CALL  BTNK(N,R1,R,C,OMEGAC,ALC)
      CALL CRT(R,C,ALC,N)
      STOP
      END
```

```
C
C
C
      DIMENSION RLC(20),IE(20),IC(20),X(50),YG(50,1)
      COMPLEX Z
      READ, R1
      WRITE(6,5) R1
    5 FORMAT('1',29X,'R1=',7X,E15.7//)
      READ,NE
      DO 10 I=1,NE
      READ, RLC(I),IE(I),IC(I)
   10 WRITE(6,15)I,RLC(I),I,IE(I),I,IC(I)
   15 FORMAT(30X,'RLC(',I2,')=',E17.7,10X,'IE(',I2,')=',I2,10X,'IC(',
     1       I2,')=',I2)
      J=1
      M=1
      N=50
      DO 80 I=1,N
      OMEGA=I*0.02
      CALL IMCS(R1,RLC,IE,IC,NE,OMEGA,Z,G,1)
      X(I)=OMEGA
      YG(I,J)=G
   80 WRITE(6,40)I,OMEGA,I,G
   40 FORMAT(1X,'OMEGA(',I2,')=',E15.7,10X,'G(',I2,')=',E15.7)
      YMIN=1.E10
      YMAX=-1.E-10
      DO 16 J=1,M
      DO 16 I=1,N
      IF(YG(I,J).GT.YMAX) YMAX=YG(I,J)
   16 IF(YG(I,J).LT.YMIN) YMIN=YG(I,J)
      DY= YMAX/YMIN
      RIPPLE=10.*ALOG10(DY)
      WRITE(6,20) YMAX,YMIN,RIPPLE
   20 FORMAT( 1X,'YMAX=',E15.7,5X,'YMIN=',E15.7,5X,'RIPPLE=',E15.7,'DB')
      CALL PLOT1(X,YG,50,0.,1)
      STOP
      END
```

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END
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