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EFFECT OF PHASE ERRORS IN STEPPED-FREQUENCY RADAR
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H E VANBRUNDT APR 88 NOSC/TR-1211

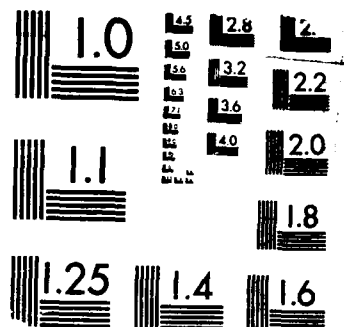
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Effect of Phase Errors in Stepped-Frequency Radar Systems

H. E. VanBrundt



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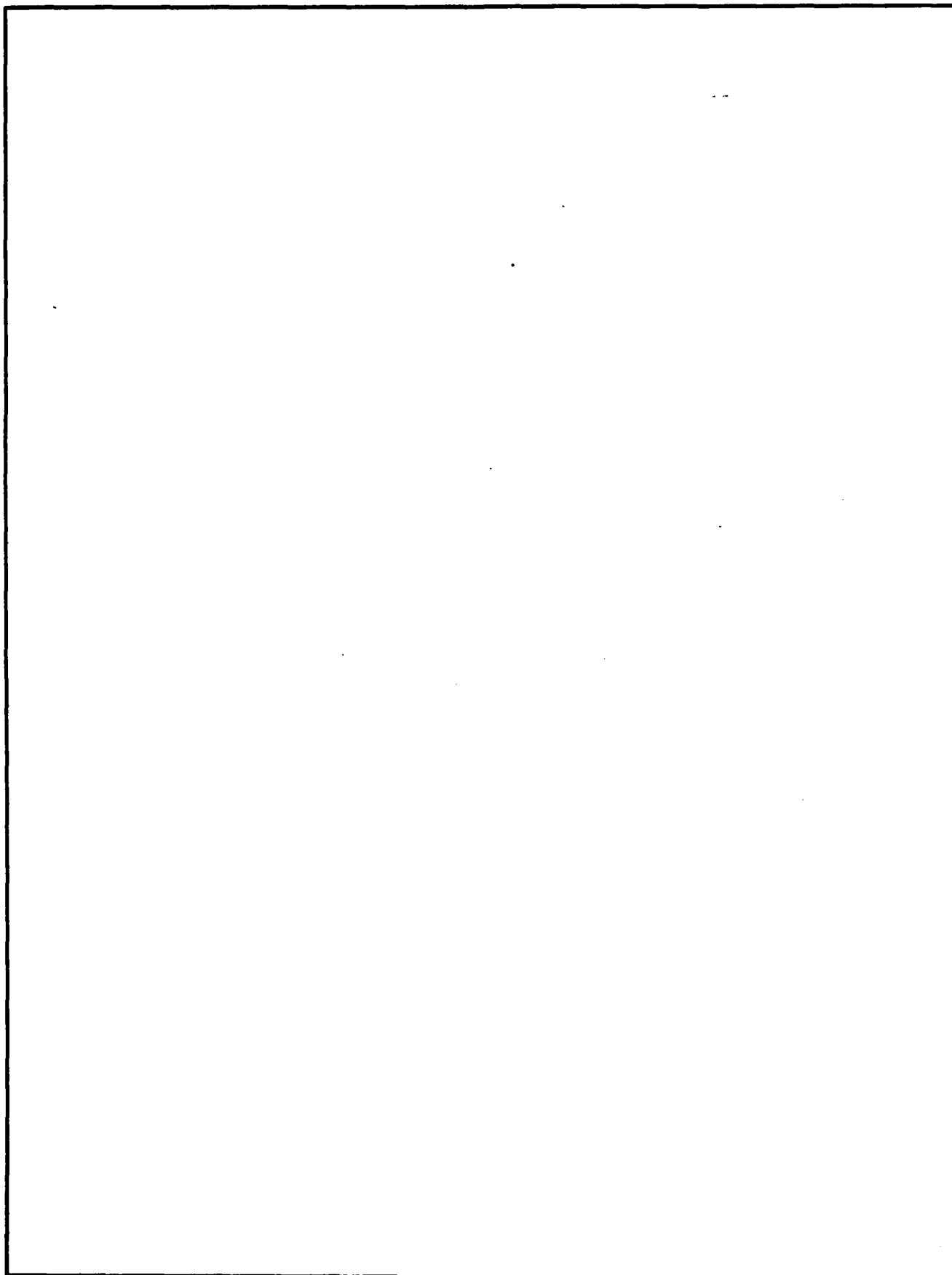
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INTRODUCTION

Stepped-frequency waveforms are being considered for inverse synthetic aperture radar (ISAR) imaging from ship and airborne platforms and for detailed radar cross section (RCS) measurements of ships and aircraft. These waveforms make it possible to achieve resolutions of 1.0 foot by using existing radar designs and processing technology.

One problem not yet fully resolved in using stepped-frequency waveform for ISAR imaging is the deterioration in signal level caused by random frequency error. Random frequency error of the stepped-frequency source results in reduced peak responses and increased null responses. The resulting reduced signal-to-noise ratio is range dependent.

Two of the major concerns addressed in this report are radar range limitations for ISAR and the error in calibration for RCS measurements caused by differences in range between a passive reflector used for an RCS reference and the target to be measured. In addressing these concerns, NOSC developed an analysis to assess the tolerable frequency error in terms of resulting power loss in signal power and signal-to-phase noise.

STEPPED-FREQUENCY RANGE PROFILE

Stepped-frequency waveforms make it possible to obtain highly resolved radar signatures and ISAR images of ship and air targets without requiring the wide instantaneous bandwidth on high-speed analog-to-digital conversion commonly associated with high range resolution. The approach described in detail by Chen *et al.** is to transmit a continuous train of radar pulses, stepped in frequency, pulse to pulse, in uniform periodic bursts. Typically, the burst period, n , is 256 pulses in extent.

Frequency steps are of uniform size, typically 1.0 MHz. Echoes from each pulse are mixed to baseband where the in-phase and quadrature outputs are sampled, one complex pair from each echo pulse. A discrete Fourier transform of each burst of n complex values produces a "synthetic" range-delay profile, when corrected to zero radial velocity, which is the equivalent of a range profile obtained from a short pulse or narrow chirp pulse of the same bandwidth.

The synthetic range profile H_l of length l of a single scatterer at range R obtained from a series of n complex echo signals G_k is obtained from the inverse Fourier transform according to Wehner.**

$$H_l = \frac{1}{n} \sum_{k=0}^{n-1} G_k e^{i\lambda l k} ; 0 \leq l \leq n-1 \quad (1)$$

where

$$\lambda = 2\pi/n \quad (2)$$

* C. Chen, M.J. Prickett, R.G. Rock, and D.R. Wehner, *Stepped Frequency Radar Target Imaging*, NOSC Technical Report TR 490, November 1979.

** D.R. Wehner, *Frequency Agile Magnetron Imaging Radar Concept*, NOSC Technical Document 417, February 1981.

is a parameter defined in terms of n numbers of transmitted pulse per burst and

$$G_k = e^{i2\pi\tau f_k} \quad (3)$$

is the echo function in the absence of frequency variations. The k^{th} frequency given in terms of the frequency step size Δk is

$$f_k = f_0 + k\Delta f \quad (4)$$

and

$$\tau = 2\pi R/c \quad (5)$$

is the range delay for a target range R at a propagation velocity c .

With these parameters, the finite series for H_l sums to

$$H_l = \frac{1}{n} e^{-i2\pi f_0 \lambda} \frac{\sin \frac{1}{2} n \lambda y}{\sin \frac{1}{2} \lambda y} e^{i \frac{1}{2} (n-1) \lambda y} \quad (6)$$

where

$$y = l - n\tau\Delta f \quad (7)$$

The magnitude of the normalized synthetic range profile then becomes

$$|H_l| = \left| \frac{1}{n} \frac{\sin \frac{1}{2} n \lambda y}{\sin \frac{1}{2} \lambda y} \right| \quad (8)$$

It is shown in Wehner, *op. cit.*, that the envelope of H_l in Eq. 6 for $l = n\tau\Delta f$ is identical to that for the matched-filter response of the periodic discrete-frequency coded waveform.

Responses to a point target will be a maximum for values of y corresponding to $0, \pm n, \pm 2n, \dots, \pm mn$.

RANGE-PROFILE DISTORTION PRODUCED BY FREQUENCY ERROR

In Eq. 2, the derivation of the synthetic range profile of a point target, it was assumed that the frequency-step size Δf was precise. In the presence of frequency variations, however, any deviation from Eq. 4 results in distortions. Random frequency error in f_k produces random phase error that reduces the peak value of the range profile of a point target and introduces noise along the profile. In other words, the overall effect is a reduction in the peak of the point target range profile and an increase in the nulls.

Let us now proceed to examine the effects of phase error produced by frequency variation on the synthetic range profile for single-scatterer targets.

In the presence of frequency variations, Eq. 3 can be rewritten as

$$G_k = e^{-i2\pi f_k \tau - i\nu x_k} \quad (9)$$

where νx_k is the random phase error produced by frequency error x_k in the k^{th} frequency step.

A random error x_k in frequency step k produces a random phase error in G_k of $(-2\pi\tau x_k)$. Therefore, ν in Eq. 9 will be defined as

$$\nu_s = 2\pi \tau x_k \quad (10)$$

Distortion produced by phase errors is obtained by substituting Eq. 9 into Eq. 3 and solving for the peak response of the expected value of $H_l(x_k)$, which occurs at

$$y = 0, \pm n, \pm 2n, \dots, \pm kn \quad (11)$$

And the variance σ_H^2 of the synthetic range profile at the nulls occurs at

$$y = \pm 1 \pm 2, \dots, \pm (n-1) \quad (12)$$

Assuming normal probability distribution of frequency error (see the Appendix), the results are magnitude of peak (see Wehner, *op. cit.*):

$$\langle H_l(x) \rangle^2 = C_f^2 \quad (13)$$

at positions of peak $|H_l|$:

$$\sigma_H^2 = \frac{C_f^2}{n^2} (1 - C_f^2) e^{-i4\pi\tau F} e^{i(n-1)2\pi m/n}; m < n \quad (14)$$

at positions of minimum $|H_l|$, where σ_H is the standard deviation of the random frequency variable x . The quantity

$$C_f = e^{-\nu_s^2 \sigma_s^2 / 2} \quad (15)$$

is the so-called characteristic function of the random frequency variable x , and σ_s refers to the standard deviation of the frequency error of the frequency synthesizer. The power is now defined through

$$Np = \left| \sigma_H^2 \right| = \frac{C_f^2}{n^2} (1 - C_f^2) \quad (16)$$

and the signal-to-phase noise is defined as the ratio of the square of the synthetic range profile to the power, where $\langle H_l(x) \rangle$ is evaluated at positions of peak H_l and σ_H^2 is evaluated at positions of minimum H_l .

Therefore,

$$\frac{S}{N_p} = \frac{\langle H_I(x) \rangle^2}{|\sigma_H^2|} = \frac{n^2}{1 - C_f^2} \quad (17)$$

Distortion in terms of signal power loss C_f^2 and signal-to-peak-noise ratio S/N_p is given in Table 1 for all values of n .

Table 1. Distortion versus $\nu\sigma$.

$\nu\sigma$	Signal Power Loss		$\frac{n^2}{1 - C_f^2}$	Ratio of Signal to Sideband Noise		
	C_f^2	$\text{dB}(C_f^2)$		$\text{dB}\left(\frac{n^2}{1 - C_f^2}\right)$		
				$n^2 = 64$	$n^2 = 256$	$n^2 = 1024$
0.01	0.9999	-0.0	$10000 \times n^2$	58	64	70
0.02	0.9996	-0.0	$2500 \times n^2$	52	58	64
0.05	0.9975	-0.0	$400 \times n^2$	44	50	56
0.1	0.9900	-0.0	$100 \times n^2$	38	44	50
0.2	0.9608	-0.2	$25.5 \times n^2$	32	38	44
0.5	0.7788	-1.1	$4.52 \times n^2$	25	31	36
1.0	0.3679	-4.3	$1.58 \times n^2$	20	26	32
2.0	0.0183	-17.4	$1.02 \times n^2$	18	24	30

CONCLUSIONS

Basic expressions were derived to assess the effects of phase noise produced by frequency error in radars that employ stepped-frequency waveforms for target imaging. Effects of phase noise were expressed in terms of signal attenuation and phase noise sidebands. A single expression for expected signal-to-phase-noise ratio was included. These basic expressions can be used to estimate a tolerable frequency error in ISAR measurements.

In a practical application, these expressions relate frequency stability of the frequency synthesizer in a stepped-frequency imaging radar to range performance. In addition, transmitter phase ripple produced by drive voltage ripple pulse to pulse (phase pushing) can be related to range performance for stepped-frequency imaging radar.

Another application of these expressions is in the analysis of noncoherent radars (magnetron radars) for target imaging.

Appendix

PHASE ERROR DISTORTION

The synthetic range profile generated by the inverse discrete Fourier transform of the complex echo samples from an n -element burst in the presence of frequency variation is

$$H_l(x) = \frac{1}{n} \sum_{m=0}^{n-1} e^{-i2\pi\tau F} e^{im\lambda y - ivx_m} \quad (A1)$$

where

$$y = l - m\tau \Delta f$$

The variance σ_H^2 of the synthetic range profile $H_l(x)$ is given by

$$\sigma_H^2 = \langle H_l^2 \rangle - \langle H_l \rangle^2 \quad (A2)$$

where σ_H is the standard deviation of the synthetic range profile $H_l(x)$. $\langle H_l^2 \rangle$ is the expected value of $H_l^2(x)$, and $\langle H_l \rangle^2$ is the square of the expected value of $H_l(x)$. C. Chen *et al.*, *op. cit.*, gives

$$|\langle H_l(x) \rangle|^2 = C_f^2 \quad (A3)$$

If $x_0, \dots, x_{n-1}$ are independent, then terms of the function

$$e^{imy - ivx_m} \text{ for } m = 0, \dots, n-1$$

are also all independent. Then

$$\text{Var}(H_l(x)) = e^{-4\pi i \tau F} \frac{1}{n^2} \sum_{m=0}^{n-1} e^{2im\lambda y} \text{Var} e^{-ivx_m} \quad (A4)$$

However

$$\begin{aligned} \text{Var}(e^{-ivx_m}) &= \langle (e^{-ivx_m})^2 \rangle - \langle e^{-ivx_m} \rangle^2 \\ &= E\{e^{-2ivx_m}\} - (E\{e^{-ivx_m}\})^2 \\ &= \text{Char}_{x_m}(-2\nu) - (\text{Char}_{x_m}(-\nu))^2 \end{aligned}$$

where

$$\text{Char}_{x_m}(-\nu) = E_{x_m}(e^{-ivx_m}) = \int_{-\infty}^{\infty} e^{-ix_m} dF(x_m)$$

We now choose $dF(x_m)$ to be the Gaussian measure. Hence

$$\begin{aligned}
 \text{Char}_{x_m}(-v) &= \int_{-\infty}^{\infty} e^{-ivx_m} P(x_m) dx_m \\
 &= \int_{-\infty}^{\infty} e^{-ivx_m} \frac{1}{\sqrt{2\pi\sigma}} e^{-x_m^2/2\sigma^2} dx_m \\
 &= \frac{1}{\sqrt{2\pi\sigma}} e^{-v^2\sigma^2/2} \int_{-\infty}^{\infty} e^{(x_m+iv\sigma^2)^2/2\sigma^2} dx_m \\
 &= e^{-v^2\sigma^2/2} \stackrel{\text{def}}{=} C_f
 \end{aligned}$$

and so

$$\langle e^{-ivx_m} \rangle^2 = \text{Char}_{x_m}^2(-v) = e^{-v^2\sigma^2} = C_f^2$$

Similarly, we find that

$$\text{Char}_{x_m}(-2v) = e^{-2v^2\sigma^2} = C_f^4$$

Therefore, Eq. A4 becomes

$$\begin{aligned}
 \text{Var}(H_l(x)) &= e^{-i4\pi\tau F} C_f^2 (C_f^2 - 1) \frac{1}{\pi^2} \sum_{m=0}^{n-1} e^{2i4m\lambda y} \\
 &= e^{-i4\pi\tau F} C_f^2 (C_f^2 - 1) \frac{1}{\pi^2} \frac{\sin^{1/2}(2n-1)2y}{\sin^{1/2}\lambda y} e^{i(n-1)\lambda y}
 \end{aligned}$$

We now define

$$\sigma_H^2 = \text{Var}(H_l(x))$$

to be the variance of the synthetic range profile H_l expressed as a function of y on l (the synthetic range) and the variance σ^2 of the frequency error.

Our present interest is in the random noise sideband (sidelobes) in the range profile when the expected value of H_l is zero. This occurs for values of y .

$$\langle H_l(x) \rangle_{y=m} = 0 ; m = 1, 2, \dots, n-1$$

The fact that $\langle H_l(x) \rangle$ is zero for interger values of y is a restriction placed on the argument of H_l .

$$\frac{1}{2} n \lambda y = m \pi ; m = 1, 2, \dots, n-1 ; m < n$$

Hence

$$\begin{aligned} \sigma_H^2 &= (\text{Var } H_1(x))_{y=m} = e^{-i4\pi\tau F} C_f^2 (C_f^2 - 1) \frac{1}{n^2} \frac{\sin(2n-1)m\pi/n}{\sin m\pi/n} e^{i(n-1)2\pi m/n} \\ &= - e^{-i4\pi\tau F} C_f^2 (C_f^2 - 1) \frac{1}{n^2} e^{i(n-1)2\pi m/n} \\ &= \frac{1}{n^2} e^{-i4\pi\tau F} e^{i(n-1)2\pi m/n} C_f^2 (1 - C_f^2) \end{aligned} \quad (\text{A5})$$

We now define the power N_p to be the magnitude of the variance of the synthetic range profile H_p as follows:

$$N_p = |\sigma_H^2| = \frac{1}{n^2} C_f^2 (1 - C_f^2) \quad (\text{A6})$$

and the signal-to-phase noise is defined by the ratio

$$\frac{S}{N_p} = \frac{\langle H_1(x) \rangle^2}{|\sigma_H|} = \frac{n^2}{1 - C_f^2} \quad (\text{A7})$$

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