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ON

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This is the Proceedings of the Fifth AFOSR Forum on Space Structures. The topics covered include modeling of spacecraft, wave propagation in large space structures, multiflexible body dynamic simulation, adaptive structures, electromechanical actuators for controlling flexible structures, system identification of suboptimal control parameters, integrated structural analysis and control, active control of elastic wave motion in structural networks, adaptive control of large space structures, analysis of performance degradation, optimal projection equations for fixed-order dynamic compensation, decentralized/relegated control for large space structures, Frobenius-Hankel norm framework for disturbance rejection and low order decentralized controller design, a method for truss structure vibration control, and robust eigenstructure assignment by a projection method. Also included are notes on the presentations and discussions and the list of attendees.			
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PROCEEDINGS
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Fifth AFOSR Forum
on
Space Structures

held on

August 20-21, 1987

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A Technical Report
from
The University of Virginia

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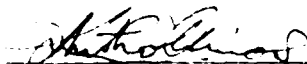
Preface

The fifth Forum on Space Structures, like its four predecessors, sought to promote intense dialog on contemporary issues in the interaction of controls and dynamics for large flexible spacecraft. To this end the meeting followed the forum tradition of open discussions interspersed by short (10 minute) presentations.

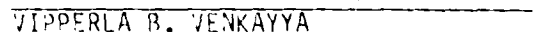
The Forum on Space Structures originated from a need, perceived by a group of Air Force supported research scientists and engineers, for improved communication, interaction, and collaboration between the structural dynamics and controls disciplines. They considered this essential if the well recognized potential benefits of active control of dynamic response were to be realized. Consequently, a group of these individuals gathered in an informal meeting at MIT in 1983. The free exchange of ideas that materialized from the open discussion format adopted, was hailed by all participants as worthy of repetition at least annually for the next several years. The subsequent meetings have continued to foster increased communication and interaction between the controls and dynamics communities. The fifth Forum was no less successful of its predecessors.

This document is an attempt to record the essence of the discussions for the benefit of other interested readers in the hope of extending whatever accomplishments were attained beyond the specific space and time confines of the meeting. Abstracts of the brief presentations have been reproduced with only minor editing. In addition, summary accounts of the discussions accompanying and following the presentations, as recalled by several participants, have been compiled and included herein. An executive summary has also been provided that summarizes the major highlights and achievements of the fifth Forum.

On behalf of the Air Force, we wish to express our appreciation to all who participated in the discussions, and in particular Dr Robert Kosut who took on the very difficult task of organizing and chairing the meeting, and did them both so well. We are also grateful to Professor Walter Pilkey who also accepted the burden of overseeing the production of this document. Lastly, we commend those participants who volunteered to and did take notes of the proceedings, and subsequently composed the accounts contained herein.



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Synopsis

The issues of discussion at the Fifth Forum provided clear indications of progress made in the Forum objectives since the inception of the meetings. Technical progress was evidenced by the transition of the focus from the topics that used to dominate the discussions to those that were then considered too esoteric and too far out to be meaningfully addressed by the audience.

In the modeling area, the primary concerns used to be on issues related to the synthesis of controllers for complex flexible structures. They included basic modeling approaches - distributed versus discrete parameterization, accommodation of parameter uncertainties and modeling errors, model reduction techniques and criteria, and system identification for model updating in both synthesis and adaptation. In contrast, the modeling issues of the Fifth Forum, with but one exception, dealt principally with performance representation and prediction, and with performance implications of actuator dynamics. The one exception was the Integrated Structural Analysis and Control (ISAAC) presentation which highlighted progress to date.

The significance of orbital effects such as gravity gradient and dynamic coupling between rigid and vibratory motions in the dynamics and stability of a class of spacecraft systems was the issue in Peter Bainum's presentation and accompanying discussions. These effects were also the motivating considerations in the modeling approach outlined by Len Meirovitch. Ed Haug's presentation also dealt with this general theme but from a simulation and computational algorithm standpoint. Jim Williams led a very lively discussion on travelling waves and their relevance to orbital performance. This is perhaps one of the perennial topics of interest at these meetings. But in spite of its popularity only a few attempts have been made to incorporate wave mechanics approaches in the modeling of large spacecraft dynamics. Dan Inman and Walter Pilkey both addressed the issue of actuator dynamics and their impact on system performance (damping). Finally the issue of in-flight adaptation, well recognized for control systems, was broadened by Michail Zak to include structural parameters, e.g. stiffness.

In the controls area, the shift in focus was more subtle, with controller quality issues such as robustness, stability, and implementability dominating over the purely algorithmic issues. The wave motion control issue raised by David Miller represented a significant change from previous discussions in which the emphasis was clearly on modeling considerations. Robert Kosut's discussion of Adaptive Controls provided a brief review of the state of development in modeling, parameter identification, robustness, stability and convergence. Stability analysis is currently feasible for slow adaptation, and extensions to faster adaptation rates represents the current thrust of research. In a slight departure from his prepared abstracts, Dave Hyland discussed issues from theory/practice considerations, including difficulty of modeling joint nonlinearities and local resonance conditions, both of which played a major role on the COFS-I project. He also described some experimental development programs at Harris including an analytical/experimental program being initiated under SDIO/IST funding. The object is

to develop and validate a hybrid design qualification methodology. His two abstracts dealt with the determination of performance bounds caused by modeling uncertainties. Decentralized control algorithm developments were discussed by Umit Ozguner, David Young and Juri Medanic. The first also included a discussion of experimental programs conducted or planned to validate the analytical developments. David Young's presentation was particularly interesting because it approached the decentralized control problem from the context of physical components of the system with local control requirements and arrived naturally at a hierarchical (interlocking) configuration for the global system controller. It provided reinforcement to the effort at MIT by Ed Crawley in which the hierarchical concept was assumed by physical intuition. John Junkins concluded the formal presentations with a discussion of his multi-criteria optimization scheme for robust, minimum control effort designs based on eigenvalue assignment.

The concluding general discussion touched on many of the issues raised during the meeting but focussed principally on the problems of validation of theory, and on transition to application. On the latter it was felt that transition is lagging the advanced technology developments by about 30 years. This was considered unsatisfactory for the structure/controls interaction disciplines. Some counter examples were however alluded to in the helicopter and fighter aircraft areas. On the theory validation issue there was a strong consensus on the need for flight experimentation to provide the feedback necessary for fine-tuning of the theories prior to transition. It was observed without further discussion that current funding trend seems to favor experimental over theoretical research projects.

Progress in the nontechnical objective of the Forum - promotion of improved communication between the Controls and Structures communities - was also evident in the degree of interaction that occurred. No longer were the two communities directing questions at each other from opposite camps or pointing accusative fingers at each other on specific issues, but all participated actively in addressing the various topics each contributing to the discussion from the vantage point of their own expertise.

A. K. Amos

MODELING

Modeling of Flexible Spacecraft Accounting for Orbital Effects

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INTRODUCTION

Many current investigations of the shape and orientation control of proposed flexible orbiting large space structures (LSS) do not incorporate the effects of the gravity-gradient and orbital dynamic coupling into the plant models. This means that for the corresponding linearized unforced, open-loop systems the poles of the rigid rotational modes are at the origin. The manner in which the orbitally induced coupling effects, due to gravity-gradient and gyroscopic effects, are introduced is clearly indicated in the continuum formulation of Santini [1] for predicting the motion of a general orbiting flexible body in orbit. These coupling terms reflect both coupling between the rigid and flexible motions and also intra-modal coupling effects. Elastic deformations are considered small as compared with characteristic body dimensions. Equations are developed for both the rigid and elastic (generic) motions, based on an a priori knowledge of the frequencies and shape functions of all modes included within the truncated system model. The orbitally induced coupling terms are seen to depend on volume integrals whose integrands are functions of the various components of the different modal shape functions together with the coordinates of the differential mass elements [1].

NUMERICAL STUDIES

This paper summarizes two recent numerical studies aimed at attempting to evaluate the effect of including/omitting the orbital coupling terms in the modelling of LSS systems. The first is based on a model of an orbiting shallow spherical shell system which contains a hybrid system for controlling both the shape and the orientation (Fig. 1). The passive spring-loaded dumbbell type damper attached at the shell's apex is also used to provide the favorable moment of inertia ratio required for gravity-gradient stabilization. Control laws are developed for the six shell mounted point actuators based on pole clustering techniques. A study was made to determine the effects of omitting the coupling terms in the development of control laws. Control laws based on pole placement were first developed based on the shell model which did not include the gravity-gradient and orbital coupling terms in the plant model. The control laws thus developed were then inserted into the previously devel-

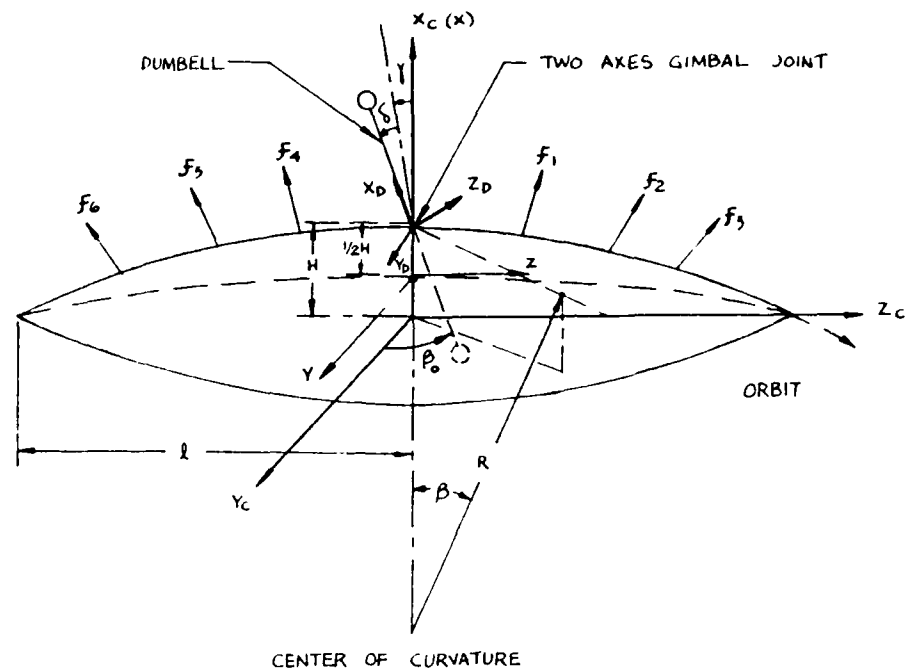


Fig. 1 Shallow spherical shell with dumbbell and actuators
OVERALL DESIGNED RESPONSE TIME CONSTANT OF THE SYSTEM = 615 SEC.

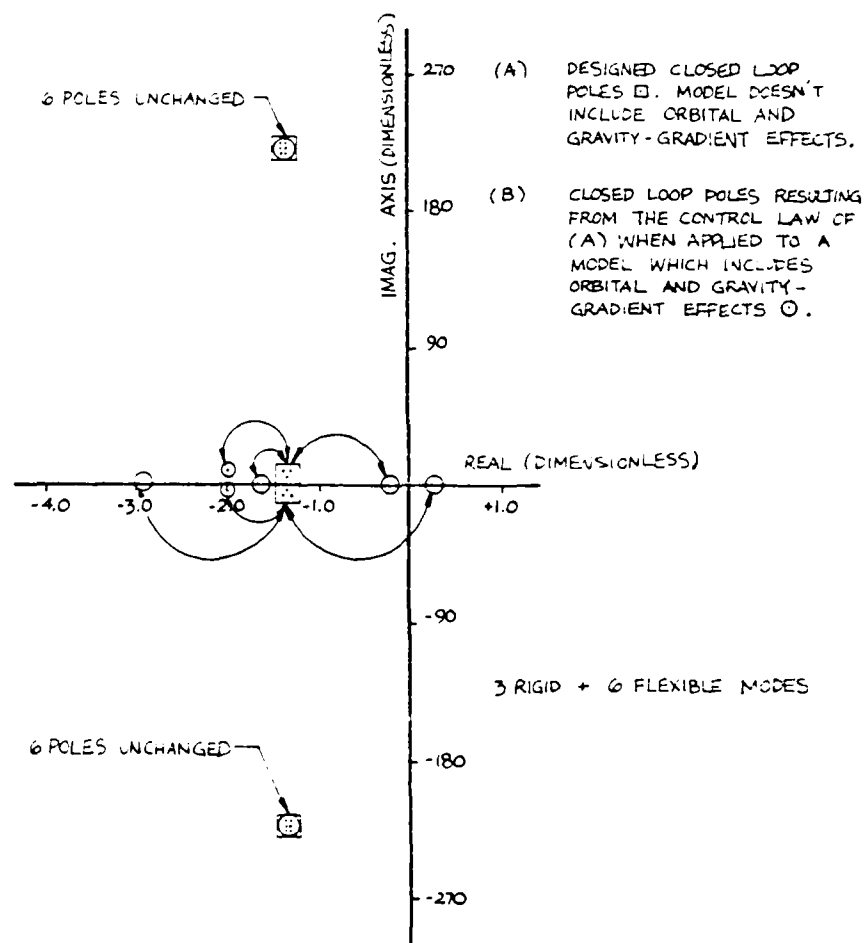


Fig. 2 Shift in the closed loop poles due to orbital and gravity-gradient effects - shallow spherical shell in 250 nmi. orbit

oped models which contain both first order gravity-gradient and orbital dynamic coupling terms. Typical results are illustrated in Fig. 2 and Ref. [2]. In all cases studied, there is a general tendency for some of the poles of the rigid modes to shift towards the imaginary axis when the gravity-gradient and orbital effects are superimposed into the plant. For the less robust systems (Fig. 2) instability may result. In general, there is no noticeable shift in the poles corresponding to the flexible modes. As expected, for the more robust systems, the relative effect of this shift is less apparent and stability may still be maintained, but at the expense of greater control force effort [2].

In the second study a computational algorithm was developed to evaluate the coefficients of the various coupling terms in the equations of motion as applied to the finite element model (FEM) of the 122m diameter Hoop/Column system [3]. For the discretized FEM the expressions for the volume integrals in the coupling coefficients were replaced by summations of similar terms taken over the total number of discrete masses for which eigenvector (mode shape) information was available from the FEM output. The details concerning these calculations are provided in Ref. [3]. Table 1 shows the comparison between the magnitudes of the components of the main coefficients in Euler's rigid rotational equations, R , and the components of the largest coupling coefficients, Q_n , for the seven flexible modes contained in the truncated FEM model. Two different values of modal amplitude factors, A_n , of 1m, and 1mm, respectively, were assumed for all the flexible modes. It appears that when the system is operating within the mission specifications (deflections of the order of mm), the finite element assumptions neglecting the orbital coupling are valid; when the deflections are of the order of meters the coupling between the flexible and rigid modes should be incorporated into the equations of motion.

In Table 2, a comparison of pertinent terms in the generic modal equations is given. $R_n \ddot{A}_n + \omega_n^2 A_n$ is compared with coupling terms represented by P_n . The comparison, after normalization of the different terms in the equations, shows that the time dependent amplitude of the modes can be approximated as an harmonic oscillator at least up to the point where $A_n = 1m$ for all n .

Table 1. Comparison of pertinent terms in the rigid modal equations

$A_n = 1m$ Direction	R	$\sum Q_n$
X	1.48922E-01	2.64238E-03
Y	1.49854E-01	8.81619E-03
Z	9.26317E-02	1.48015E-03
$A_n = 1mm$		
X	1.48922E-01	2.64238E-03
Y	1.49854E-01	8.81619E-03
Z	9.26317E-02	1.48015E-03

Table 2. Comparison of pertinent terms in the generic modal equations

$L = 1 \text{ m}$ Mode number	Frequency rad/s	Modal mass	R_i , kg	P_i , kg
7	0.7489859	153.157	0.629344×10^{-3}	1.9843×10^{-5}
8	1.3692409	8.232954	0.210346×10^{-4}	2.40287×10^{-7}
9	1.7471481	3.232954	0.33248×10^{-4}	8.7299×10^{-8}
10	3.214894	0.3046446	0.115957×10^{-3}	4.901252×10^{-5}
11	4.538031	1.992988	0.230747×10^{-3}	3.46589×10^{-5}
12	8.8926689	723.8216	0.350924×10^{-3}	3.226×10^{-5}
13	8.7942228	0.6861203	0.176624×10^{-3}	1.678578×10^{-5}
$L = 1 \text{ m}$ Mode number	Frequency rad/s	Modal mass	R_i , kg	P_i , kg
10	3.214894	0.3046446	0.115957	0.4901252×10^{-4}
11	4.538031	1.992988	0.230747	0.337854×10^{-4}
12	8.8926689	723.8216	0.350924	3.2362×10^{-5}

CONCLUDING COMMENTS

Even though the numerical analysis has been applied to two specific models of proposed orbiting LSS systems, the results indicate that one should not, a priori, neglect orbitally induced coupling terms in the analysis and design of open and closed-loop LSS systems in general.

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Additional Comments

Modeling of Flexible Spacecraft Accounting for Orbital Effects

by

Peter M. Bainum

In response to comments raised concerning the two examples presented and the importance need for these studies the following additional points should be emphasized:

- 1) For both sets of numerical examples presented it appears that the influence of orbitally induced coupling on the rigid body motions can be more important than the similarly induced effects on the flexible (vibrational) motions (compare the shift in the rigid poles of Fig. 2 with the results in Table 1 for the vibrational modal amplitudes of 1m. assumed for the first seven flexible modes, and also the results of Table 2).
- 2) The results shown in Fig. 2 are for an LSS shell type structure in LEO of 250n.mi. altitude. Similar results (not shown) for geosynchronous (GEO) orbit indicate a far less pronounced shift in the rigid poles. The gravity-gradient torques vary with the square of the orbital angular velocity - and are at least two orders of magnitude less at GEO as compared with LEO. Many proposed applications for LSS will be at LEO, in addition to the obvious widescale communications applications at GEO.
- 3) The main objective of the first paper cited (Ref. 2) was to compare the relative merits of a hybrid (passive gimballed damper with 6 point actuators) control system with that of a completely active control system, where the pole clustering technique was used for both cases to develop the active control laws. No attempt was made to optimize the control or synthesis technique nor placement location of the actuators. The author agrees with one of the questioners in the audience that, in retrospect, a different control technique would have been better, such as a carefully weighted LQR design. He also agrees that the results should be extended to include realistic sensor modeling and sensor dynamics and careful modeling of the solar pressure induced

disturbance torques on the vibrating and thermally deflected system. Only then can final conclusions be made concerning RMS pointing and shape requirements. The studies cited in Refs. 2 and 3, directed and supported by NASA Langley, were intended to gain insight into a complex problem and not to present a final design of the control system for an actual LSS flight system, which will probably depend on future developments in LSS sensor technology.

- 4) Regarding slew maneuvers, as long as the elastic deformations can be maintained small as compared with characteristic body undeformed dimensions (such as length or diameter) the techniques of Ref. 3 could also be used to evaluate the coupling terms during or immediately following a slew maneuver. For violent slews where large elastic deformations may result the nature of the coupling terms would change from those of Refs. 1 and 3 and a different formulation to represent the dynamics would be required. In connection with slewing, it should be noted that several proposed LSS systems, such as the Wrap Rib Antenna and the SCOLE, involve asymmetrical, offset systems. Desired orientations after line-of-sight maneuvers may not always correspond to gravitational equilibrium conditions, so that control requirements for slewing may vary greatly depending on the sense or direction of the slew.
- 5) One member of the audience raised questions about the need for the types of studies described in Ref. 2 and 3 since he thought that the effects of gravity induced effects on the closed loop dynamics would be small. The author agrees that with a robust enough, properly designed, and functioning (no fault) control and sensor system these effects can be properly compensated for (Fig. 6 of Ref. 2). In the event of failure of a part (or all) of the control sensor system the open-loop results of Ref. 3 might also provide useful. In conclusion the author would like to thank the many participants for their interest and helpful, constructive comments.

Wave Propagation in Large Space Structures

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INTRODUCTION

The first step in the mathematical analysis of any physical system is the selection of a mathematical model to represent the system. This selection is extremely important, since it not only determines in advance the scope of possible results of the analysis, but also heavily influences the design of auxiliary devices and systems, such as control systems. In the dynamic analysis of large space structures, mathematical models have consisted primarily of a set of vibration modes [1, 2]. The popularity of the modal vibration model of large space structures is due partly to the well-developed analytical techniques which can be applied to such a model, and partly to the success with which modal vibration models have been used to describe structures on earth. However, in view of the unprecedented size of large space structures and their potential technological importance, it is worthwhile to examine the limitations of modal vibration models and to consider the usefulness of other models. In particular, this paper is concerned with models which view large space structures as media in which wave propagation provides an accurate dynamic description.

WAVES AND MODES

The relationship between modal analysis and wave propagation analysis may be illustrated by the Venn diagram in Fig. 1. The relationship is analogous to the relationship between linear elasticity and non-linear elastic-plastic continuum theory. For linear problems, wave propagation analysis and modal analysis are mathematically equivalent, but the two approaches often give enlightening complementary perspectives. Wave analysis is applicable to many problems in which the concept of modal superposition cannot be applied, such as the problem of dynamic failure and other nonlinear problems. In many linear problems, such as the

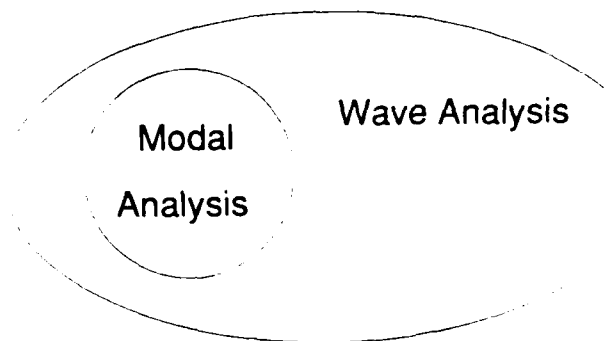


Fig. 1 Relationship between modal analysis and wave analysis.

linear analysis of joint dynamics or the development of linear control systems, wave propagation analysis may have computational or conceptual advantages.

Linear Analysis

In linear analysis, when wave propagation analysis and modal analysis are theoretically equivalent, the preference between a wave analysis and a modal analysis is assessed in accordance with the ratio between the characteristic structural length L and a characteristic length associated with a typical disturbance or loading function. The characteristic length associated with the disturbance function may be the wavelength λ introduced into the structure by a point disturbance source, the spatial extent over which a distributed disturbance source is applied, or the spatial extent over which a disturbance from a point source is effectively attenuated as it propagates through the structure. For the case of a point source disturbance in a nondissipative structure, three characteristic regimes of dynamic behavior may be defined. When $\lambda \gg L$, a quasistatic or rigid body dynamic analysis is likely to be adequate. For $\lambda \approx L$, the structure begins to deform dynamically, and a modal analysis consisting of globally defined modes is likely to be preferable. For $\lambda \ll L$, the globally defined modes cannot easily represent the short wavelength disturbances, and a wave analysis is likely to become preferable.

Structural Details

For disturbances with short wavelengths ($\lambda \ll L$), structural details such as individual members and joints become important in predicting dynamic response. An advantage of wave propagation analysis is that it can deal with local models of structural subsystems. By considering transmission of structural disturbances through a subsystem, a wave analysis can be based only on a local model of the subsystem and the adjacent portions of the structure to which it is attached. A modal analysis, on the other hand, must be based on a model of the entire structure, and requires a large number of accurately modeled modes in order to include the significance of structural details.

Time Delays

Recent results in the analysis of large tetrahedral lattice structures show that the time delay between an input at one location in the structure and the response at another location may be on the order of tens of seconds [3]. Such time delays, which may have important implications for control system design, are an essential part of a wave propagation analysis.

Dynamic Failure

Consideration of structural failure is an important aspect of the reliable implementation of large space structures. In a dynamic failure problem, the geometry and/or the material properties of a structure vary with time. Thus, a normal modal analysis cannot be applied. A wave propagation analysis can be used to predict the conditions under which dynamic failure will occur and to suggest some means of achieving failure arrest [4, 5]. The dynamic failure problem is an example of a problem which has no modal description, but which can be analyzed using wave propagation concepts.

CONCLUSION

The final judgement of any mathematical model must be based to a large extent on the ability of the model to predict those phenomena of interest. A good model is a model which may be used to design and operate a useful physical system. Since large space structures are envisioned to be complex structures which are to be used for a wide variety of purposes, it is unlikely that any single model (a modal model or a wave model) will be sufficient for all aspects of large space structure applications. Thus, it seems unwise at this point

to restrict attention to any single viewpoint. When large space structures come closer to actual deployment, operational reality alone will determine the relative merit of any model which is considered.

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A Recursive Method for Parallel Processor Multiflexible Body Dynamic Simulation

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INTRODUCTION

The purpose of this note is to outline a recently developed method for formulating and solving equations of motion of multibody dynamic systems that is well suited for high speed dynamic simulation using parallel processors. An outline of the approach is given here, with references cited to papers that develop the mathematical foundations. Examples involving a rigid body vehicle system real-time simulation, a geometrically nonlinear rotating blade, and a space manipulator are used to illustrate application of the method and to indicate computational efficiency that can be gained.

OUTLINE OF METHOD

References 1 and 2 present a recursive algorithm for dynamics of rigid body systems, either open loop or closed loop, based on the variational form of multibody equations of motion. In most recent formulations of equations of motion in terms of joint relative coordinates [3], equations of motion of bodies in a mechanical system are first written in terms of absolute coordinates. Kinematic relationships between absolute velocity, acceleration, and virtual displacement and corresponding variables in terms of relative coordinates are applied to the graph theoretic model of the system, to reduce absolute coordinate equations of motion to coupled equations of motion in all relative coordinates and base body absolute coordinates.

In Ref. 1, Bae observed that as this reduction proceeds from tree end bodies toward the base body, coefficients of relative coordinate variations must be zero, leading to equations that can be used to solve for associated relative coordinate accelerations as functions of inboard body absolute accelerations and velocities. This process is repeated, recursively, from tree end bodies to the base body, to obtain a reduced set of equations of motion associated with the base body. Upon solving these equations of motion for the base body accelerations, relative coordinate accelerations are recovered, moving from the base body outward toward tree end bodies. Following integration of these relative coordinate accelerations, position and velocity analysis is carried out

by moving from the base body outward toward tree end bodies, completing one step of the system equation formulation and integration process. Calculations associated with this recursive process are implemented with the aid of symbolic processing software to yield an order of magnitude speed up over absolute coordinate methods [1,2]. Equally important, the tree structure associated with this recursive algorithm leads to definition of parallel computation tasks that can be carried out effectively on a parallel processor [4]. This algorithm is currently yielding to real-time vehicle dynamic simulation.

A RIGID BODY VEHICLE EXAMPLE

As an illustration of the foregoing approach, consider the schematic representation of a four wheel vehicle with double-A arm suspension subsystems at all four corners and rack and pinion steering mechanisms, as shown in Fig. 1. This model represents the Army's new HMMWV multipurpose vehicle. It consists of a total of 15 bodies, 22 kinematic joints, and 8 closed kinematic loops. Following a process of cutting selected joints to open kinematic loops [2], a tree structure model of the vehicle is obtained, as shown in Fig. 2. This model leads to 10 parallel paths for concurrent processor implementation.

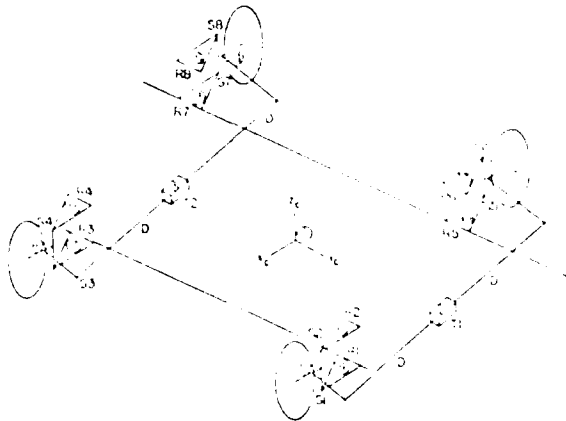


Figure 1 Schematic Representation of a Vehicle

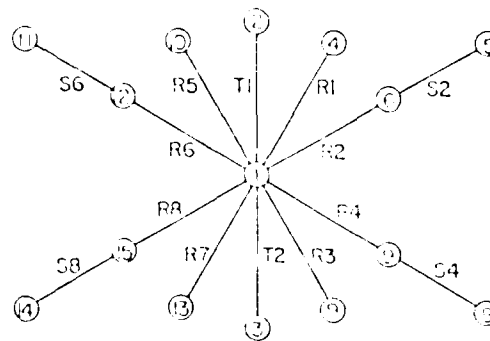


Figure 2 Tree Structure Corresponding to Vehicle

To illustrate parallel processor allocation, the parallel task graph of Fig. 3 is provided [4]. This model and algorithm is currently running on an Alliant FX8 mini supercomputer, including all nonlinear effects, with approximately 12 millisecond timesteps. This temporal resolution is more than adequate for real-time simulation of vehicle systems, accounting for approximately 50 Hertz frequency response.

A GEOMETRIC NONLINEAR FLEXIBLE BODY EXAMPLE

The algorithm outlined above for rigid body applications has recently been extended to multiflexible body systems [5]. Deformation modal coordinates are employed in this formulation to account for linear elastic deformation effects, relative to a moving reference frame.

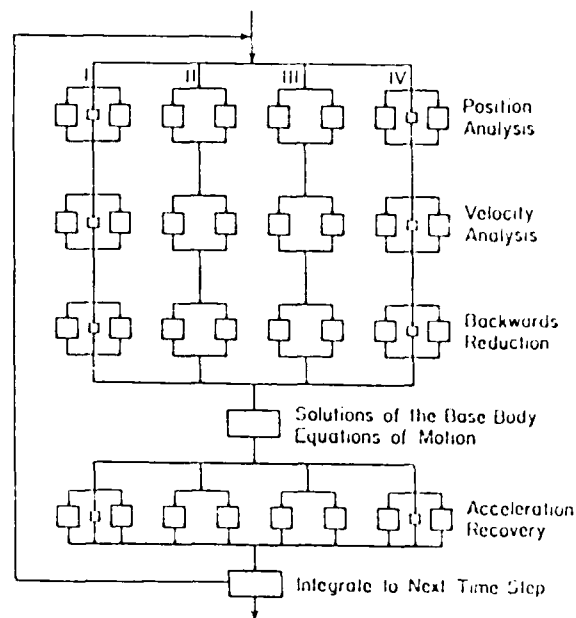


Figure 3 Parallel Task Graph of Recursive Algorithm for Vehicle

Nonlinear coupling due to elastic deformation and gross motion of a body reference frame are accounted for in the formulation. The second basic approach outlined above for rigid body systems is applicable, with the exception that deformation mode coordinates are eliminated in the same manner as relative coordinates in the case of rigid body dynamics. To illustrate use of the method, a long slender bar shown in Fig. 4 is constrained to rotate about the global z axis. Its motion is begun at rest in the configuration shown in Fig. 4. The shaft that rotates about the z axis is accelerated to a steady state angular velocity of four radians per second in a period of 15 seconds. Numerical results for tip deflection, relative to the undeformed state of the bar, are shown in Fig. 5 for several flexible body models.

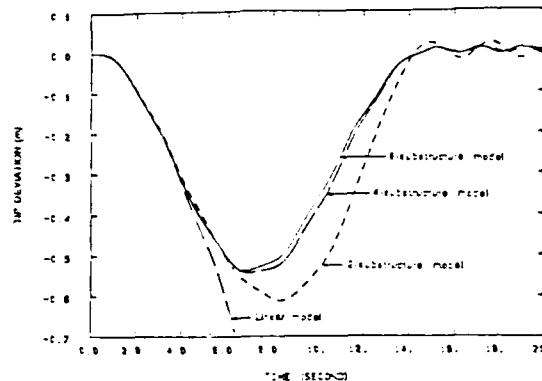
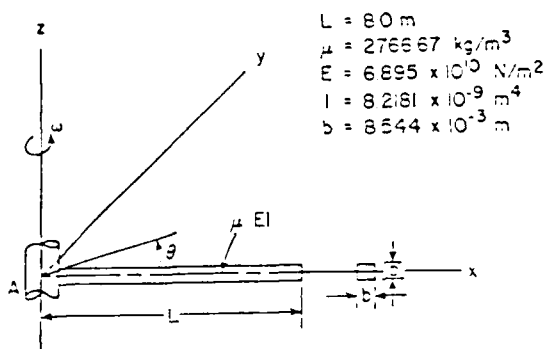


Figure 4 A Rotating Blade Figure 5 Tip Deflection of Rotating Blade

If the bar is modeled as a single body, results labeled "linear model" shown in Fig. 5 indicate divergence. As two, four, and six substructures are used in modeling the spinning bar, results that account for geometric stiffening are shown in Fig. 5. In these later models, each substructure is assigned a body reference frame that moves

with the substructure, approximating an Eulerian reference frame that moves with the system, hence permitting capture of geometric nonlinear effects. These results agree well with classical theory and numerical results obtained with the absolute coordinate DADS computer code. It is interesting to note that even though there are no parallel tasks to be exploited in this application, CPU time required for the recursive algorithm in the six substructure model is one-thirteenth that required for an absolute coordinate approach. This suggests that not only can high speed simulation of flexible body systems be achieved, but geometric nonlinear effects can be captured in the computational approach.

A FLEXIBLE MULTIBODY MANIPULATOR EXAMPLE

As a final example of interest in space structure applications, consider the multiflexible body model of a shuttle manipulator shown in Fig. 6 [4]. In this application, the two long intermediate bodies are flexible. The rigid base rotates about an axis fixed on the shuttle and the end effector is modeled as a rigid body. There are thus four moving bodies in the system, each connected to its predecessor body by a revolute joint. As a simple computational illustration [5] relative position actuators at the joints where specified and deviations of the flexible body simulation compared with those of a rigid body simulation in Fig. 7. As noted, substantial elastic deformation effects occur, due to the gross motion and nonlinear coupling effects in the model.

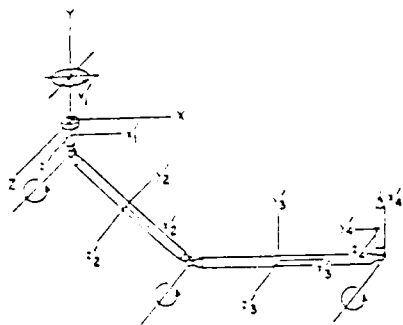


Figure 6 A Flexible Manipulator
with Angle Drivers

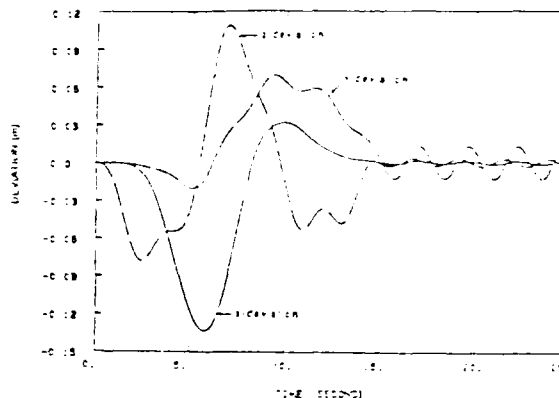


Figure 7 Tip Deviation of
Flexible Manipulator System

The real-time simulation capability currently functioning for realistic vehicle models, coupled with successful initial demonstration of capability to account for multiflexible body effects, suggest that high speed and perhaps real-time multiflexible body simulation will be possible in the near future, through exploitation of emerging parallel processor architectures.

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Maneuvering Equations in Terms of Quasi-Coordinate

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INTRODUCTION

The equations for the motion of a flexible spacecraft in space can be described conveniently in terms of quasi-coordinates. The nonlinear equations can be treated by a perturbation approach, resulting in a low-order nonlinear problem for the "rigid-body" maneuvering and a linear time-varying problem for angular perturbations from the rigid-body maneuver and elastic vibration.

THE EQUATIONS OF MOTION AND THE PERTURBATION APPROACH

The motion of a flexible spacecraft can be described in terms of the rigid-body translation and rotation relative to the inertial space of a set of body axes xyz embedded in the underformed body and the elastic motion relative to the body axes. Assuming that the rigid-body translation can be ignored, the equations of motion can be written in the form of a set of hybrid (ordinary and partial differential) Lagrange's equations in terms of quasi-coordinates [1]

$$\dot{\tilde{J}}\dot{\tilde{\omega}} + \int_D \rho(\tilde{r} + \tilde{u})\dot{\tilde{v}} dD = -\tilde{J}_v\dot{\tilde{\omega}} - \int_D \rho\tilde{v}\dot{\tilde{v}} dD - \tilde{\omega}\tilde{J}\dot{\tilde{\omega}} - \tilde{\omega}\int_D \rho(\tilde{r} + \tilde{u})\dot{\tilde{v}} dD + \tilde{M} \quad (1a)$$

$$\rho(\tilde{r} + \tilde{u})\dot{\tilde{\omega}} + \rho\dot{\tilde{v}} = -\rho\tilde{v}\dot{\tilde{\omega}} - \mathcal{L}\tilde{u} + \tilde{U} \quad (1b)$$

where $\tilde{\omega}$ = angular velocity vector, $\tilde{v}$ = elastic velocity vector, $\tilde{r}$ = position vector of a point in the body, $\tilde{u}$ = elastic displacement vector, ρ = mass density, $\tilde{M}$ = torque vector, $\tilde{U}$ = force density vector, $\mathcal{L}$ = differential operator matrix and $\tilde{J} = \int_D \rho(\tilde{r} + \tilde{u})(\tilde{r} + \tilde{u})^T dD$, $\tilde{J}_v = \int_D \rho(\tilde{r}\tilde{v}^T + \tilde{v}\tilde{r}^T) dD$. Note that symbols with an overtilde represent skew symmetric matrix counterparts of the corresponding vectors. Moreover, all quantities in Eqs. (1) are in terms of components about the body axes.

Equations (1) can be conveniently used to describe the maneuvering of flexible spacecraft. The equations are not only hybrid but also nonlinear. It should be noted, however, that an ideal maneuver is one in which the spacecraft is reoriented as if it were rigid. Hence, a reasonable assumption is that the rigid-body rotations are generally large and the elastic displacements are generally small. This

assumption permits a perturbation approach [2] to the solution, in which terms involving rigid-body motions are regarded as being of zero-order in magnitude and terms involving elastic motions are regarded as being of first-order in magnitude. Hence, we assume that

$$\underline{\omega} = \underline{\omega}_0 + \underline{\omega}_1, \underline{J} = \underline{J}_0 + \underline{J}_1, \underline{M} = \underline{M}_0 + \underline{M}_1 \quad (2a,b,c)$$

where $\underline{J}_0 = \int_D \rho \tilde{r} \tilde{r}^T dD$, $\underline{J}_1 = \int_D \rho (\tilde{r} \tilde{u}^T + \tilde{u} \tilde{r}^T) dD = \underline{J}_u$; the subscripts indicate the order of magnitude. Moreover, $\underline{u}$, $\underline{v}$, $\hat{\underline{U}}$ and $\underline{J}_v$ are small. Introducing Eqs. (2) into Eqs. (1), we obtain the zero-order equations

$$\underline{J}_0 \dot{\underline{\omega}}_0 = -\tilde{\underline{\omega}}_0 \underline{J}_0 \underline{\omega}_0 + \underline{M}_0 \quad (3)$$

which are recognized as Euler's equations, and the first-order equations

$$\begin{aligned} \underline{J}_0 \dot{\underline{\omega}}_1 + \int_D \rho \tilde{r} \dot{\underline{v}} dD = & -\underline{J}_1 \dot{\underline{\omega}}_0 - \underline{J}_v \underline{\omega}_0 - \tilde{\underline{\omega}}_1 \underline{J}_0 \underline{\omega}_0 - \tilde{\underline{\omega}}_0 \underline{J}_1 \underline{\omega}_0 - \tilde{\underline{\omega}}_0 \underline{J}_0 \underline{\omega}_1 \\ & - \tilde{\underline{\omega}}_0 \int_D \rho \tilde{r} \underline{v} dD + \underline{M}_1 \end{aligned} \quad (4a)$$

$$\rho \tilde{r}^T \dot{\underline{u}}_1 + \rho \dot{\underline{v}} = -\rho \tilde{u}^T \dot{\underline{\omega}}_0 - \rho \tilde{v}^T \underline{\omega}_0 - \mathcal{L} \underline{u} - \rho \tilde{r}^T \underline{\omega}_0 + \hat{\underline{U}} \quad (4b)$$

Equations (3) and (4) permit design of a maneuver strategy described by a low-order set of nonlinear ordinary differential equations and a linear set of hybrid equations. According to this, Eq. (3) represents the rigid-body maneuvering equations. The rigid-body maneuvering of spacecraft has been discussed extensively in the literature (see, for example [3]). On the other hand, Eqs. (4) govern angular perturbations from the rigid-body maneuver and elastic displacements. They represent a regulator problem.

As an illustration, we consider the single-axis maneuver of the spacecraft shown in Fig. 1. The pertinent quantities are as follows:

$$\begin{aligned} \underline{\omega}_0 &= \Omega(t) \underline{k}, \dot{\underline{\omega}}_0 = \dot{\Omega}(t) \underline{k}, \underline{r} = x \underline{i}, \underline{u} = u_y \underline{j} + u_z \underline{k}, \underline{v} = v_y \underline{i} + v_z \underline{k}, \\ \underline{\omega}_1 &= \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}, \underline{J}_0 = \text{diag} [J_{xx} \ J_{yy} \ J_{zz}], \underline{M}_0 = M_0 \underline{k}, \\ \underline{M}_1 &= M_x \underline{i} + M_y \underline{j} + M_z \underline{k}, \hat{\underline{U}} = \hat{U}_x \underline{i} + \hat{U}_y \underline{j} + \hat{U}_z \underline{k} \end{aligned} \quad (5)$$

Inserting Eqs. (5) into Eq. (3) and introducing an obvious kinematical relation, we obtain the maneuver equations

$$\underline{J}_{xx} \dot{\hat{\theta}}(t) = M_0(t), \dot{\hat{\theta}}_0(t) = \Omega(t) \quad (6a,b)$$

where $\hat{\theta}_0(t)$ is the rigid-body maneuver angle. Moreover, inserting Eqs. (5) into Eqs. (4) and adding appropriate kinematical relations, we obtain the regulator equations

$$\underline{J}_{xx} \dot{\underline{\omega}}_x = -(\underline{J}_{zz} - \underline{J}_{yy}) \Omega \underline{\omega}_y - 2\Omega \int_D \rho x v_z dD - \hat{\Omega} \int_D \rho x u_z dD + \underline{M}_x \quad (7a)$$

$$\underline{J}_{yy} \dot{\underline{\omega}}_y - \int_D \rho x \dot{v}_z dD = (\underline{J}_{zz} - \underline{J}_{xx}) \Omega \underline{\omega}_x - 2\Omega \int_D \rho x v_z dD - \hat{\Omega} \int_D \rho x u_z dD + \underline{M}_y \quad (7b)$$

$$\underline{J}_{zz} \dot{\underline{\omega}}_z + \int_D \rho x \dot{v}_y dD = \underline{M}_z, 0 = \rho \hat{\Omega} u_y + \rho \Omega v_y + \hat{U}_x, \quad (7c,d)$$

$$\rho x \dot{\omega}_z + \rho \dot{v}_y = \mathcal{L}_y u_y - \rho x \dot{\Omega} + \hat{U}_y, -\rho x \dot{\omega}_y + \rho \dot{v}_z = \mathcal{L}_z u_z + \hat{U}_z \quad (7e,f)$$

$$\dot{\theta}_1(t) = \dot{\omega}_1(t), \dot{u}(x,t) = \dot{v}(x,t) \quad (7g,h)$$

where $\theta_1(t) = [\theta_x(t) \theta_y(t) \theta_z(t)]^T$ is a vector of perturbation angles.

We observe that in this particular case Eqs. (7a), (7b) and (7f) are independent of Eqs. (7c), (7d) and (7e). This implies that the variables ω_z and u_y can be controlled independently of the variables ω_x , ω_y and u_z , which simplifies the control problem significantly.

Equations (6) permit a maneuver strategy as if the spacecraft were rigid. For a minimum-time maneuver, the control is bang-bang [4]. As a result of selecting a rigid-body maneuver, Ω and $\dot{\Omega}$ can be regarded as known functions of time. As can be observed from Eqs. (7), the effect of Ω and $\dot{\Omega}$ is to introduce time-dependent coefficients into the regulator equations. Hence, the net effect of the perturbation approach is to replace a hybrid nonlinear problem by an ordinary second-order nonlinear problem and a hybrid time-varying linear problem. It is clear that the latter alternative is much more advantageous.

It should be pointed out that in both cases described above spatial discretization is necessary to transform a hybrid system into a system of ordinary differential equations. Then, the question arises as to the nature of approximation and in particular whether controls designed for a discretized system are capable of controlling the actual structure.

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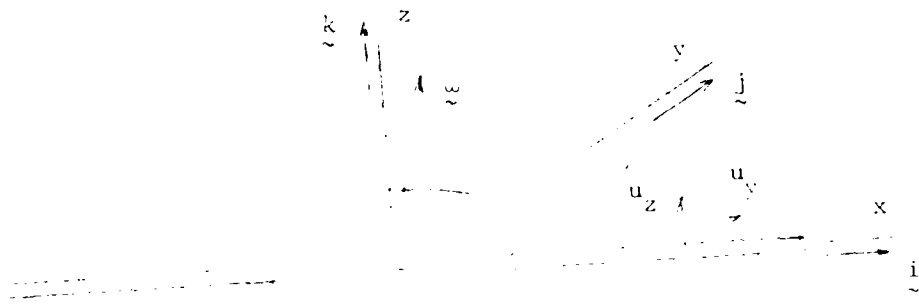


Figure 1

Concept of Adaptive Structures

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The concept of adaptive structures is brought up in connection with the need in ultra lightweight structural systems which maintain desired properties and configurations without human intervention when subjected to dynamic, thermal, and other environmental forces. Examples are large antenna structures and flexible robotic structures. In the both cases such adaptivity would allow less massive structural members to be employed under normal loading conditions. During special circumstances when unusually large loads are encountered, temporary stiffening would allow the use of less sturdy structures, resulting in large savings in their cost, and in increasing their mobility and efficiency.

Within the framework of a finite-dimensional representation of structural dynamics the adaptivity can be implemented by the dependence of the stiffness matrix $\{k\}$ upon the expected load $\{Q\}$, or expected (programmed) changes in configurations, i.e.,

$$\{k\} = \{k(t)\} \quad (1)$$

where the dependence upon time is programmed in advance. In order to sustain unexpected loads the adaptive structure can be provided by feedback force control, or by a parametrical stiffness control. In the last case instead of (1)

$$\{k\} = \{k(t, X_i)\} \quad (2)$$

in which X_i is a generalized coordinate.

A variable stiffness of a structural member can be achieved by appropriate changes in pre-stress. Indeed, as follows from finite elasticity:

$$E = \frac{\delta T_{xx}}{\delta \epsilon_{xx}} + \frac{1}{2} T_{xx}, \quad G = \frac{1}{2} \frac{\delta T_{xy}}{\delta \epsilon_{xy}} + \frac{1}{2} T_{xx}, \quad \text{etc} \quad (3)$$

in which E and G are the Young's and the shear moduli, respectively and T_{xx} is the normal pre-stress, etc.

Hence, the stiffness of an elastic member is increased by increasing the pre-stress T_{xx} . This can be implemented by cables, hydraulic actuators, piezoelectric actuators, or induced thermal gradients. A variable shape of a structural member (for instance, a snake-like robotic arm) is achieved by an appropriate combination of a pre-tension and bending moments which can be implemented by piezoelectrical actuators (for fast changes) and by thermal gradients (for slow changes).

Since adaptive structures are characterized by time-dependent stiffness, new physical phenomena (such as a parametrical resonance) should be expected. In addition, because of time-dependent coefficients in the governing equations for transient dynamics all the methodology associated with the modal approach as well as the stability analysis should be revised.

In application to robotic arms the advantages of structural adaptivity become obvious. Indeed, a rigid segment arm can operate with loads which are several times lighter than the arm itself, while a human arm (which is an example of an adaptive "structure") operates with loads several times heavier than its own weight. One of the specific problems which arises in connection with a flexible arm is the interaction between the rigid motion and elastic micromotions. Such an interaction leads, for instance, to a false impulse of the arm tip due to finite time of the elastic wave propagation of the torque applied at the arm root. (This undesirable effect can be minimized by introducing a pre-stiffening of the arm before the maneuver.) From the mathematical viewpoint rigid motions are governed by the Lagrange equations, while the elastic motion described by appropriate continuous models (like the Timoshenko beam, or the Kirchhoff's thin rod) are governed by partial differential equations. (Finite dimensional approximations for the elastic motions as well as the Bernoulli-Euler model may lead to a loss of the wave propagation effect.) Due to interactions between rigid and elastic motions the Lagrange ordinary differential equations and partial differential equations are coupled: the Lagrange equations include an additional (reactive) force depending upon the integral effect of the elastic motions, while the partial differential equations parametrically depend on the angular velocities of the rigid motions. Since the displacements due to the rigid motions are not small, the total system is non-linear.

Traditionally the robotic research is being performed by the community of electrical engineers. However, even a brief review of flexible robotic structures demonstrates the existence of complex problems which require deep understanding of structural dynamics and careful selection of appropriate mathematical models. That is why now is probably a good time for the community of structural engineers to offer their assistance in the development of a new generation of robots with adaptive flexible arms.

**Comments on
Electromechanical Actuators for Controlling Flexible
Structures**

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Two types of specific electromechanical actuators are described and discussed. A proof mass actuator and an electric motor are examined in terms of the amount of damping each produces in a specific structural control experiment. Theoretical and experimental values of actuator produced damping are examined. The effects of actuator dynamics on control law implementation are noted. In addition, a theoretical parameter study of the dynamic response of a DC motor controlling a flexible model of a beam are summarized.

The proof mass actuator hardware consists of a microprocessor controlled, solenoid like mass with displacement and acceleration sensors. Local digital control is used to cause the actuator to provide velocity feedback at a point. This form of control is capable of providing roughly a 10% increase in damping for various nodes. Transfer function analysis of the actuator reveals dynamics of the form

$$\frac{F(s)}{V(s)} = \frac{G_1 G_2 m_r s^2}{m_r s^2 + G_1 G_2 c_1 s + G_1 G_2 k}$$

where F is the force output, V is the input voltage and G_1 , G_2 , M_r , c_1 and k are the various physical parameters of the actuator and amplifier [1]. The actuator dynamics become important if the first natural frequency of the structure falls below the break frequency of the actuator (about 2.5 hertz).

A similar analysis is also presented for an electric motor used as a hinge and controlled for vibration suppression during slewing maneuvers. The actuator consists of a DC permanent magnet motor and tachometer of approximately 16%

of the mass of the structure [2]. The electric hinge is located at the 8 foot mark of a 12 foot long flexible aluminum beam. The motor and gear combination is first compensated for frictional losses, then active structural control was accomplished by viscous damping introduced through simple tachometer feedback. Damping increases as much as a factor of 3 (from 8.3% to 23%) were measured [3].

A very simple model of a motor controlling a load is used to illustrate how damping is introduced by the motor into the load dynamics. The damping introduced in this fashion constitutes a structure, control law and actuator interaction, as the equivalent viscous damping parameter depends on coefficients characterizing the load as well as the motor. A more detailed model of the motor illustrates that when the load inertia is comparable to, or greater than, the motor inertia, substantial actuator structure interaction occurs greatly effecting control law performance. The details of this interaction depend on all of the systems parameters and hence are not simple. State feedback optimal control is used to determine structure and control parameter that yields minimum power usage for constant speed of response. The procedure has been repeated for a flexible load [4].

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Comments in response to a questions raised during the presentation (Crawley)

A question was raised regarding the nature of the high gain instability of a proof mass actuator.

As was noted in the lecture, the proof mass actuator illustrates a high gain instability. That is, as the gain of the actuator is increased, the stability of the structure actuator system is lost. This phenomena is explained in terms of the interaction between the control and the structure. The actuator can be viewed simplistically as an added degree of freedom which "absorbs" vibration of the structure modes by adding damping to these modes. As the control gain is increased, more of the damping force of the actuator is transferred to the structural modes. This happens at the reduction of the damping force of the actuator mode. Eventually the actuator mode has no damping force and the system (structure plus actuator) beomes unstable (actually, marginally stable). If the gain is increased further the actuator modal damping becomes negative, causing an instability in the system [1]. This is predicted analytically and observed experimentally.

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System Identification of Suboptimal Feedback Control Parameters Based on Limiting-Performance/Minimum-Time Characteristics

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INTRODUCTION

Most active controllers developed to control large structures are subject to constraints. For instance, control characteristics of proof-mass actuators are dominated by the nature of the constraints. To find the optimal or suboptimal control laws for such controllers can be a formidable task. Intuition of a designer plays an important role [1] and the design method may vary dramatically in accordance with the constraints of the system. Therefore, one may wish to have a systematic methodology to solve the control problems subject to control force and state variable constraints. Limiting-performance / minimum-time (LP/MT) control calculates the optimal control force as a function of time for a known system with initial conditions, subject to certain constraints and external excitations while minimizing a given performance index. Although the LP/MT control gives optimal open loop control for the system, it is desirable to develop a closed loop control which has more practical value. Perhaps a system identification technique can be used to establish a suboptimal feedback control law based on the LP/MT control characteristics.

LIMITING-PERFORMANCE/MINIMUM-TIME CONTROL

A brief description of the problem which the method can solve is given here. Further information on the method can be found in [2] and [3]. A linear vibrating system with n degrees of freedom subject to arbitrary external excitations $\underline{f}(t)$ and control forces $\underline{u}(t)$ is expressed in the first order system of differential equations

$$\dot{\underline{s}}(t) = A\underline{s}(t) + B\underline{u}(t) + C\underline{f}(t) \quad (1)$$

where $\underline{s}(t)$ is an n -dimensional state vector, A , B , and C are $n \times n$, $n \times n_u$ and $n \times n_f$ constant coefficient matrices. The quantities n_u and n_f are the number of control forces and excitations, respectively.

Constraints are imposed on the dynamic system under study. The format of the constraints is

$$\underline{y}_L \leq Q_1 \underline{s} + Q_2 \underline{u} + Q_3 \underline{f} \leq \underline{y}_U \quad \text{for } t_0 \leq t \leq t_f \quad (2)$$

where $\underline{y}_L$ and $\underline{y}_U$ are nc -dimensional lower and upper constraint vectors.

Q_1 , Q_2 and Q_3 are $n \times n$, $n \times n_u$ and $n \times n_f$ constant coefficient matrices, and t_0 and t_f are given initial and final time.

The LP/MT control finds an optimal control $\underline{u}(t)$ which will transfer an initial state $\underline{s}(t_0) = \underline{s}_0$ to a desired final state $\underline{s}(t_f) = \underline{s}_f$ in the minimum time while extremizing a given performance index of the form

$$\text{Minimize } J = \{t_0 \leq t \leq t_f \mid p_1^T \underline{s} + p_2^T \underline{u} + p_3^T \underline{f} \mid\} \quad (3)$$

where p_1 , p_2 and p_3 are given n , n_u and n_f constant coefficient vectors.

SYSTEM IDENTIFICATION OF SUBOPTIMAL FEEDBACK CONTROL PARAMETERS

Since the LP/MT control gives the best possible or "limiting" response of a system, it would appear to be reasonable to base a control system on the LP/MT control characteristics. However, due to uncertainties in control problems, open loop control such as the LP/MT control may not be applicable in practice, unless real-time computing power for the LP/MT control is available. To overcome this difficulty, parameter identification to find suboptimal feedback control laws based on the LP/MT control characteristics is possible. Note that the prescribed constraints have been taken into account when the optimal LP/MT control is computed.

Consider a linear, time invariant dynamic system represented by a set of difference equations

$$\underline{s}(k+1) = G\underline{s}(k) + H\underline{u}(k) \quad (4)$$

where $\underline{s}(k)$ and $\underline{u}(k)$ are vectors of dimensions n and m , respectively; G and H are constant matrices of appropriate dimensions.

From the LP/MT control characteristics, the optimal time responses $\underline{s}^*(k)$ and $\underline{u}^*(k)$ are obtained. Consider a linear controller,

$$\underline{u}(k) = K\underline{s}(k) \quad (5)$$

where K is an $m \times n$ feedback gain matrix.

Since m controllers are considered and optimal control forces are available for each controller, it is possible to proceed controller by controller. For controller i ($i = 1, 2, \dots, m$), a suboptimal linear control law to be determined is described by

$$u_{s_i}(k) = \begin{cases} u_{\max_i} & \text{for } u_{s_i}^*(k) > u_{\max_i} \\ u_{s_i}^*(k) & \text{for } |u_{s_i}^*(k)| < u_{\max_i} \\ u_{\min_i} & \text{for } u_{s_i}^*(k) < u_{\min_i} \end{cases} \quad (6)$$

where

$$\begin{aligned} u_{s_i}(k) &= \text{suboptimal control force for controller } i \\ u_{s_i}^*(k) &= \underline{k}_i s^*(k) \\ \underline{k}_i &= i^{\text{th}} \text{ row of } K \text{ matrix} \end{aligned} \quad (7)$$

Here it is supposed that the controller i is constrained as (See Eq. (2))

$$u_{\min_i} \leq u_i(k) \leq u_{\max_i} \quad (8)$$

Define

$$re_i(k) = u_i^*(k) - u_{s_i}(k) \quad (9)$$

where $u_i^*(k)$ is an optimal control force for controller i .

Then a constant feedback gain matrix K in Eq.(5) is selected to minimize

$$RE_i = \sum_{k=1}^{N-1} [re_i(k)]^2, \quad i = 1, 2, \dots, m \quad (10)$$

This identification method handles problems efficiently, since once the optimal LP/MT control characteristics are known, it is not necessary to perform a complete structural analysis. It is expected that this parameter identification technique will give a stable feedback control law for a large structural system. To ensure stability the gain matrix K will be constrained when a least squares fit is performed. Also, to increase the fitness nonlinear gain matrices may be superimposed on a linear gain matrix.

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Notes on Presentations and Discussions in the Modelling Session

Contributed by
Edward J. Haug

P.M. BAINUM

Peter Bainum summarized the results from two preceding papers, as indicated in the abstract contained in the proceedings.

During discussion, Meirovitch inquired as to what rigid body modes are accounted for. Bainum implied only rotational rigid body modes were included, since only they have significant effect.

Bryson questioned the speaker regarding interpretation of poles and the significance of high frequency modes in the analysis. Bainum responded that the scope of the study was set by NASA-Langley, the sponsor.

Eubanks inquired about nonselfadjointness of the model. Bainum responded that he was not looking at nonselfadjointness. Junkins and Meirovitch clarified the issue regarding selfadjointness in open loop control.

Bryson extended his earlier comments regarding structural frequency ranges of interest and their coupling with linear effects. He argued that the degree of coupling at higher frequency ranges is insignificant. Bainum agreed that high frequency effects are essentially linear in structural response.

Amos inquired about the significance of coupling between deformation and rigid body motion. Bainum responded that the application being considered did not concern slewing effects; i.e., only small gross motion was of interest.

J.H. WILLIAMS, JR.

Jim Williams summarized considerations regarding wave propagation modeling and feedback control, as compared to modal techniques. He specifically related modeling to wave length and structural dimension parameters.

Meirovitch challenged Williams on how to excite modes in space. Williams responded that actuators provide a significant excitation of waves and modes.

Eubanks challenged Williams regarding classification of problems with only wave speed (wave length) and characteristic structural length. He argued that parameters associated with the nature of load application were equally important. Crowley reinforced his opinion, observing that the manner of load application dictates structural response and the approach taken to modeling.

Highland argued that the wave approach is important and has substantial potential application in control and should be given serious consideration. He voiced the opinion that the wave approach is not being given fair consideration, in comparison with wave approaches.

Meirovitch argued that dispersion of waves is important and does not appear to be fully addressed. Williams responded that dispersion is important, but nondispersive effects are also of considerable importance.

Haug and Craig argued that deformation shapes other than global vibration modes need to be given consideration for modeling structural response. Static correction modes used to characterize local deformation fields due to low transmission across structural component boundaries are significant and can lead to much improved modal models, as compared to truly normal vibration modes.

E.J. HAUG

Ed Haug summarized recent developments in recursive modeling for parallel processor multiflexible body application, as per the abstract contained in the proceedings.

Von Flotow observed that Cain and co-workers at Stanford have observed geometric stiffening effects outlined in the presentation. Haug responded

that the purpose of presenting these results was two-fold. First to counter an earlier assertion that multiflexible body codes could not capture geometric nonlinear effects and second to demonstrate that use of piecewise Eulerian reference frames with elastic structural models relative to these frames can account for geometric nonlinear effects, in a unified and computationally effective manner.

Siljak observed that a similar automotive simulation application has been used with individual suspension subsystems for high speed computational algorithms. Haug responded that the algorithm presented has led to an order of magnitude speed up in computation and an ability to account for much higher system frequency response. He agreed that alternate physical views of mechanical systems can lead to concurrent computational paths.

The question was raised as to how two orders of magnitude of computational speedup in the vehicle simulation reported was achieved. Haug responded that nearly an order of magnitude was gained through the recursive algorithm, apart from parallelism. An additional factor of approximately three was gained through parallel computation. The remaining speedup is associated with higher speed processors, which are expected to provide still further gains in the near future.

L. MEIROVITCH

Meirovitch expanded upon the quasi-coordinate formulation for maneuvering equations for spacecraft that is presented in the abstract of these proceedings.

Dwyer asked if he could start with a finite element modal characterization of a large scale finite element model of a structural component and

recover coupling effects. Meirovitch indicated that he has not had experience with such applications, but felt that it was possible.

Venkayya asked if the proportion of mass in the hub of the spacecraft influences the equations of motion. Meirovitch indicated that a more important effect is the regulator that couples gross motion with deformation effects.

W. PILKEY

Pilkey made a presentation on a linear programming approach to open loop controller design, as in the abstract in these proceedings. He emphasized that the open control design technique being presented is independent of the number of degrees of freedom of the structural model. He pointed out that well established linear programming methods for solution can be applied effectively.

Junkins inquired as to whether the approach treats bang-bang control. Pilkey indicated that his approach smooths bang-bang control effects.

Eubanks asked if the formulation is linear. Pilkey indicated that it is, unless nonlinear kinematic effects are accounted for.

Junkins pointed out that pole placement optimization yields nonlinear constraints. Pilkey agreed.

Notes on Presentations and Discussions in the Modelling Session

Contributed by
John L. Junkins

P. Bainum - Orbital Effects

Tune Q, R weight matrices to place closed loop poles. Used fixed stability margin for all eigenvalues. Look at effects of neglecting orbit-attitude/ vibration coupling effects. Which effects dominate depends upon how large deflections are.

Q. What are the bandwidths of actuators, sensors, and controllers?

A. Don't know, off hand.

Q. How can you design feedback controller without worrying about bandwidth and roll off characteristics?

A. We didn't pretend this was an optimal controller. We simply assumed a "for example" control law.

J. Williams - Wave Approach

Suggests wave approach is appropriate when wavelength (λ) of excitation is much less than characteristic length (L) of the structure.

E. Haug - Multi-body Dynamics

Presented a new formulation for chain type topologies which facilitates parallel processing. Reported two order of magnitudes speedup in computing motion of a system with 10 parallel paths, some of the speedup due to faster processor.

L. Meirovitch - Maneuvers Quasi-coordinates

Presented a version of Lagrange's equations using quasi-coordinates and a perturbation method. The zeroth order solution is the rigid body part, the flex body effects enter as a first order correction.

Q. Can the zeroth order equations be modified to include certain dominant flex body coupling effects?

A. Yes.

W. D. Pilkey - System Identification of Suboptimal Feedback Gains

Find discretized open loop control subject to inequality constraints on control forces, position of proof mass, response of the system. Curve fit typical response history to find linear feedback gains "consistent" with the optimal open loop solution.

Q. Your open loop trajectory is a function of the initial conditions; how do you account for this in trying to curve fit to determine gains?

A. A family of typical initial conditions are considered.

M. Zak - Adaptive Structures

Presented an idea, not results. Structures must bear peak loads temporarily. Consider temporary increase in stiffness to thereby save weight. He developed equations of motion very similar to Meirovitch's. Flexible motions are perturbations of rigid motions. Leads to nonlinear, time varying equations.

Q. The basic idea is not new, is it?

A. No, but it's time we did something with the idea.

Dan Inman - Actuators

Discussed Haviland/UVa's proof mass actuators. Actuator won't work below 2.5 Hz and has some high frequency instability which has not been carefully identified (by Inman). He invites suggestions for experiments.

Notes on Presentations and Discussions in the Modelling Session

Contributed by
Gary L. Slater

(I) M. JAK - Concept of Adaptive Structures

Q: Why are adaptive structures lighter and better?

A: To not waste the size - not think they should be.

Q: How do you design this to handle a given set of loads?

Q: How do you handle this in a finite element analysis - not really answered?

Author's Comments: In experiments this concept has been shown more efficient; lower weight.

Other Comments: But does this include power supply, etc.?

(II) D. INMAN - Comments on Actuators

Author's Comments: (I) Experimental data available from author on HITNET; also access to experiment.

(II) On proof mass actuator, if control law has bandwidth below actuator break frequency then the actuator dynamics are crucial to the analysis.

Q: From data shown, response does not seem too linear-amplitude dependent.

A: Yes, there is a heavy nonlinearity in hinge.

Q: What is maximum force for a given mass?

A: $F = m \cdot a$ Newton's.

CONTROLS

Integrated Structural Analysis and Control (ISAAC): Issues and Progress

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INTRODUCTION

The AFOSR-sponsored ISAAC program at the Aerospace Corporation has been pursuing issues which arise in the simultaneous design of structures and control systems for large space structures using a mathematical programming code (see, for example [6]). There are many practical advantages to such integrated design, such as tuning the structure to directly improve the closed loop performance measure. This talk will focus on key elements in our work, and in particular those elements which distinguish our emphasis from that of other workers in the field.

The idea is to simultaneously optimize some cost criterion and satisfy various design constraints. Other researchers have pursued similar approaches (see, for example, [4] and references therein). It is relatively easy to list the desired qualitative characteristics of good designs. Development of an implementation involves translation of these characteristics into quantitative measures, and then the choice of which characteristic(s) to implement as the cost criterion and which to implement as constraints. The next logical step is to become concerned with numerical issues: ideas involving the solution of mathematical programming problems can easily be very attractive in concept but unworkable computationally.

In our ISAAC work we have attempted to address both conceptual and computational issues. The following are issues we have considered which may be classified as conceptual: (1) definition of system robustness and other performance criteria, (2) structure, complexity and parametrization of controller, and (3) structure, complexity and parametrization of physical spacecraft structural model. Numerical issues we have considered include: (4) implementation of the constraint of closed-loop stability, (5) algorithms for updating structural models, (6) simplified continuum models for complex structures, and (7) evaluation of constraint and criterion functions and their derivatives. In our presentation we shall elaborate on these issues and indicate what we think could be accomplished in the future.

SUMMARY OF ISSUES

Definition of Criteria: The translation of design goals into criterion functions is central to the success of optimization-based design.

Disturbance attenuation, insensitivity to parameter variation, bandwidth, saturation bounds, tracking performance and stability are typical examples of desirable characteristics or limitations from the control design point of view. All of these can be implemented in a reasonably

direct manner using the frequency-dependent maximum singular values of transfer function matrices [5,7]. There is, for example, no need to use indirect measures of disturbance attenuation or robustness such as damping ratios as in [4].

From the structural designers' point of view it is necessary to meet constraints involving permissible ranges of parameter variations due to requirements on physical dimensions, loading, mass, etc. These constraints are generally easy to implement in an obvious way.

Controller Structure: Quadratic regulator theory has been used in other work (e.g. [4,8]), and this has dictated the compensator structure. Our approach using the Q-parametrization [9] of stabilizing compensators allows a greater variety of compensator structures. It does still leave open the question of how to best limit compensator dynamic order. [3] argues that in a digital implementation of such an approach there is no practical need to limit order.

Spacecraft Structural Model: We have worked both with continuum and finite element models. The mathematical programming approach involves imposing constraints on structural parameters based upon physical significance. The presence of a multitude of parameters in a preliminary design poses an obstacle both to understanding and computation. The use of continuum models [1] can provide a more ready conceptual understanding of the structure, an easier handle on the important design parameters, and means of more readily obtaining a feasible design point. There are also cases in which a reasonable continuum model cannot be found.

Closed Loop Stability Constraint: This is necessary. The quadratic regulator theory ensures closed loop stability, but it gives a fixed (and high dynamic order) compensator structure. Other approaches have involved checking stability conditions at each iteration step: for example, testing for positivity of certain matrices [8]. The Q-parameterization provides stability automatically, so that the constraint is implicit in the compensator structure.

Structural Model Updates: In the process of varying structural parameters when using frequency domain criteria, it becomes necessary to recompute the frequency response of the whole system. There are various possibilities for both approximate and exact frequency response updates, with or without modal analysis. We have investigated an approach using exact frequency response updates, which is computationally efficient for localized structural parameter variations. Conceptually similar techniques appear in the literature for exact eigenvalue updates with localized modifications [2].

Continuum Models: Complex structures may require the interconnection of several such models, perhaps with finite elements models as well. Efficient generation of frequency responses for such models would be very useful.

Evaluation of Criteria: Numerical evaluation of constraint and criterion functions can involve large computations at each step of the iteration.

tion, such as eigenvalue problems, singular value decompositions or multiple matrix inversions. One generally also needs to differentiate the results of these computations with respect to parameters.

CONCLUSIONS

There is obviously more to gain by simultaneously tuning structural and controller designs. Closed loop performance criteria which directly measure desired characteristics are available. The big issues are parametrization of controller and structure, and computational techniques for evaluation of criteria, constraints, and their derivatives.

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Integrated Structural Analysis and Control (ISAAC): Issues and Progress

Post-Forum Summary

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During our presentation at the forum, a number of concerns pertaining to our methodology and design example were expressed by several forum participants. In this summary we briefly address those concerns.

Compensator Complexity

We noted that Q-parametrization design generally results in a compensator which is full (each input affects each output) and has order higher than the design model of the structure. Based upon the work of Boyd, et. al.[1], we made the comment that if the resulting compensator could be constructed as a digital finite impulse response (FIR) filter, then it would not be necessary to reduce high compensator order. This is because VLSI FIR chips are already being marketed which contain large numbers of delay taps [1]. The concern was raised, however, that such implementations have not been qualified for spaceflight, and hence complex compensators would not be acceptable.

We agree that, when acceptable, compensator simplification is desirable. However, for extremely high performance mission requirements compensator complexity will be unavoidable due to increasing complexity of the design model of the structure. By the time that most missions requiring precision structural control are ready to fly, VLSI FIR hardware will most likely be space qualified due to requirements of advanced communications payloads.

TOYSAT Design Example

We showed a diagram of the TOYSAT model which is the basis of our development. This consisted of two 2-dimensional four-bay truss appendages attached to a central body. At each truss tip a vertical position sensor is located, and a rotation sensor is located at the upper center body. Here, vertical and horizontal are defined in the TOYSAT reference frame with the horizontal direction in parallel with the undeflected truss center line. Two force and one torque actuator are respectively collocated with the sensors. As presented, the design goal was to minimize truss tip vertical deflections and center body rotation responding to a vertically-oriented station-keeping thruster located at the bottom of the center body.

It was pointed out during the forum that no center body rotation would be induced by the thruster disturbance due to the position and orientation of the thruster.

We appreciate that this point was brought out. Earlier in our work the structural dynamics members of our research team chose the disturbance source to be a bang-bang slew torque. In the process of coding the design this was inadvertently taken to be a station-keeping thruster. We have since corrected the disturbance source in the design example so that all three outputs will respond to a center body slew torque.

We might add that even in the example presented, the design procedure would not be invalidated because the truss tips will respond to the thruster. The disturbance to output transfer matrix will be 3 by 1 with one element equal to zero. We have verified this by looking at our current computational results. The singular value of this matrix will be non-zero, and hence the objective function will reflect the truss tip responses.

Multiple Extrema

We emphasized the use of constrained optimization to select compensator and structural design parameters. The question of the possible existence of multiple extrema was raised.

At this point we have no analytical results governing the number of extrema and which are maxima or minima. The existence of multiple extrema is a common problem in mathematical programming. In most practical situations one must run the optimization for a variety of initialization points and pick the optimum from these, with no assurance of having found a global optimum. Engineering insight will improve the likelihood that a chosen initial point will converge to the global optimum. The Q- parametrization design method [2], by virtue of providing some ability to fix input-output structure, may facilitate such insight. If the form of the compensator is chosen to be Digital FIR [1], then most objectives and constraints will be convex in at least the controller parameters, although still not in the structural parameters. Convex programming problems have unique minima [3].

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Active Control of Elastic Wave Motion in Structural Networks

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Modal descriptions of dynamic behavior are the basis of the majority of proposed structural control designs. While modal analysis is a useful tool, limitations to its applicability exist. Modal analysis of dense spectrum systems, typical of proposed large space structures, is known to be extremely sensitive to small parameter perturbations. Some analyses indicate that hundreds of modes of such space structures will contribute to performance degradation. Many of these modes are considerably beyond the range where they may be confidently modelled using this technique. Ignorance of the eigen properties of these modes can lead to unstable control-structure interaction.

Recent work performed at the M.I.T. Space Systems Laboratory¹ has dealt with control design based on wave propagation models of flexible structures. Reflection and transmission properties of performance critical locations are actively altered in order to meet mission requirements regarding dynamic isolation, energy shunting, and vibration suppression. Using an input/output relation, the wave scattering characteristics of the location, or structural junction, can be actively altered in order to vary the path of power transmission or reduce power emanating from the junction.

Such a technique has several advantages. Wave control approaches the problem as feedforward disturbance rejection of incoming waves. In some applications, this can eliminate resonant behavior by creating matched terminations. Local models are insensitive to all but local modelling errors. Since a wave model is a local description, a guarantee of control stability is not based upon knowledge of global structural behavior. Instead, stability is judged based on the frequency dependent power generation/dissipation properties of the active junction. Global control performance is dependent upon the significance of the disturbance path containing the active junction. The control energy expended is of the order of the disturbance energy and only a few actuators and sensors are required to carry out most tasks.

Two wave control formulations have been developed. The first

involves specifying the desired scattering behavior and deriving the appropriate control. The second minimizes a cost based on the power emanating from a junction and the level of control effort used. These two formulations can be used separately or in combination with the specified behavior being used as a constraint in the cost minimization problem. An example would call for minimizing power emanating from an intersection of several structural members while prohibiting waves from departing the junction along one of the members. This application is a combination of vibration suppression and dynamic isolation. Both formulations are frequency domain techniques.

When employing frequency domain techniques, issues of causality must be dealt with. The first step in the solution of the cost minimization problem is to construct an integral over the entire frequency range whose integrand is composed of two quadratic terms. The first term is junction power flow expressed in wave mode amplitudes and the second term is control effort. Minimizing the integral yields an expression relating control effort to wave mode amplitudes. Junction deflections can be substituted for wave mode amplitudes to provide a causal set of measurements. The resulting relation is then placed in Wiener-Hopf form and solved accordingly to yield the optimal, causal solution. Alternatively, some researchers have proposed methods for estimating wave modes.

For some applications, such as the matched termination of a rod, wave control resembles direct velocity feedback. In general the solutions will be quite different. For example, solutions for beams in transverse bending yield feedback of half derivatives and half integrals of junction deflections which are easily implementable compensators yet not obvious solutions when modal models are employed. In theory, these wave controllers yield performance far exceeding direct velocity feedback even to the point of eliminating resonant behavior.

Many possibilities stem from the use of these local wave models. The compensators may be adapted based upon comparison of actual outgoing wave modes with their predicted levels. Measurements can be performed "upstream" to compensate for actuator and sensor lags. Vibrational energy can be shunted around performance critical locations or trapped in noncritical regions to be dissipated. Many of these possibilities are not readily observed using modal models yet are obvious using equally valid wave models. This promises to be an interesting area for research which addresses many of the performance and implementation requirements necessary for large space structure applications.

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Adaptive Control of Large Space Structures

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INTRODUCTION

In this talk we briefly describe some of the research issues involved in the design and analysis of adaptive control systems for large space structures (LSS). The need for adaptive control arises from many envisioned future LSS missions which impose stringent performance demands on tracking accuracy and structural vibration attenuation. Both active feedback control and passive damping will be a practical necessity, and moreover, their design will require a model of the LSS system whose accuracy is compatible with the performance demands. Structural variations from many sources, such as deployment, material fatigue, and even random variations in materials and manufacturing tolerances, will significantly degrade closed-loop performance, e.g., ACOSS(1981). Thus, the on-orbit dynamics of LSS will not be sufficiently like those obtained from either ground-testing or even from sophisticated computer generated modeling techniques, such as finite element modeling. Current structural modeling techniques are just not sufficiently accurate or able to account for all the possible sources of parameter variation. Therefore, under these conditions, it will be necessary to identify the LSS dynamics directly from on-orbit measurements, and simultaneously, tune or re-design the control. Hence, the control design cycle will be an *adaptive* process, typically starting with a nominal low-performance design based on a coarse model, and then re-designed from on-orbit data. The adaptation may take place either off-line or simultaneously while the system is operating, but in either case, the process will require on-orbit data.

STRUCTURE OF ADAPTIVE CONTROL SYSTEM

Adaptive control, as depicted in Figure 1, consists of a controller and two additional processes, namely: (1) a model estimator, and (2) a control design rule. The model estimator operates on the input-output data obtained from measurements of the system to be controlled, producing a model estimate from a pre-determined model set. The model estimate is transformed by the control design rule into a controller to be used in feedback with the actual system. The fundamental question is: when will it work? That is, under what conditions will the model estimate converge to a good model in the allowed model set. By a good model is meant one that produces a controller, via the control design rule, which when applied to the actual system yields acceptable closed-loop performance. Even if such a good model exists, the estimated model may not converge, even if the estimated model is initialized close to a good model. Moreover, if convergence is too slow, then unacceptable behavior can occur during the learning process. Thus, convergence alone is not a sufficient condition for establishing good performance. We also need information about the convergence rate, particularly how it can be affected by user choices of inputs, data filters, etc.

In summary, to evaluate an adaptive system, it is necessary to determine the relations among the following choices:

(1) a representative "true" system, (2) a parametric model set, (3) a model estimator or adaptive algorithm, (4) a set of closed-loop performance criteria, and (5) a control design approach, rule, or algorithm.

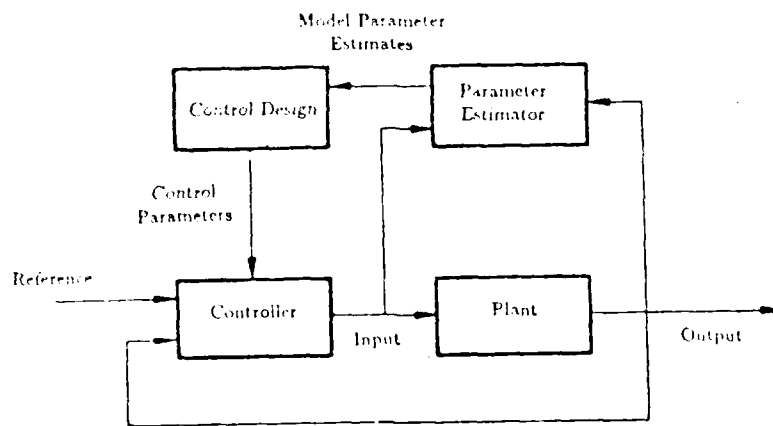


Figure 1: Adaptive Control

Model Parametrization

We will restrict our discussion here to parametric transfer function models. There are two basic choices for parameters: (1) physical parameters, and (2) canonical parameters. Physical parameters include masses, spring constants, electrical circuit quantities, etc. Canonical parameters are best exemplified by transfer function coefficients, which usually depend in a complicated manner on the physical parameters. Moreover, if the transfer function represents a sampled-data system, then the relation between the physical parameters and the transfer function coefficients becomes even more complicated, particularly when the sampling rate is low.

In parameter estimation, the above parametrizations offer different advantages and disadvantages. The estimation of transfer function coefficients can be very direct, because it is possible to express the prediction error as a linear function of these parameters, thus leading to simple estimation algorithms, such as recursive least squares. Physical parameters are directly meaningful, but because they enter in a complicated manner into the transfer function coefficients, the prediction error is also a complicated function of the physical parameters, making direct estimation schemes difficult to design. Often a system model can be characterized by a few physical parameters, whereas the transfer function may consist of very many coefficients.

One can now ask the question: which of the above model parametrizations is "better"? To answer this question, we first remark that *neither* is correct, because the true system is different than either model. Even the so called physical transfer function model is undoubtedly derived under certain assumptions, such as small deflections, linear elasticity, uniform mass density, etc. Hence, the choice should be made by considering the intended use of the model.

Model Evaluation

In control design, which is the case we consider here, what is needed is an estimate of the transfer function, whose accuracy is determined by the *closed-loop* performance, e.g., Doyle and Stein(1981), Safonov et al.(1981), Kosut et al.(1983), Vidyasagar(1985), Bhaya and Descor(1985). Thus, for control design, the parametrization can be viewed merely as an intermediate step. However, different parametrizations invoke parameter estimation algorithms which may have significantly different convergence rates and numerical stability properties, e.g., Ljung and Soderstrom(1983).

Hence, although closed-loop performance is the ultimate aim, other criteria must be considered, such as those pertaining to computation of the estimate.

To evaluate the model parametrization requires the following results from Ljung(1987) regarding parametric transfer function estimation:

- Least squares parameter estimation is equivalent, asymptotically in the length of the data record, to minimization of a weighted frequency domain criterion which penalizes the quadratic error between the true system transfer function and the parametric model transfer function. The weighting function depends on various user choices, such as the input spectrum, data filters, and the noise or disturbance model.
- The mean-square-error (MSE) between the true system transfer function and the ideal¹ estimated transfer function can be explicitly evaluated and depends on the user choices listed above.

One of the most significant aspects of the above results is that the MSE between a particular choice of model parametrization and the true system can be evaluated *without specifying how the estimate is computed or whether it converges*. The convergence question involves the analysis techniques reported in Anderson et al.(1986). Hence, conditions for convergence add further constraints on the user choices. It is important to emphasize that since the above results depend on the choice of a "true" representative system, they can be used for analysis and design of the estimator and estimation experiment.

Many research efforts have been undertaken to evaluate methods of system identification for distributed parameter and LSS type systems, e.g., to name a few, Goodson and Polis(1974), Banks, Crowley, and Kunisch(1983), Schaechter(1986), and Denman et al.(1986). The difference between these reported results and what we are emphasizing here, is the connection between the criteria for system identification and the intended use of the estimated model for control design.

As one example of the kind of analysis that might be accomplished, consider the use of simple continuum models, such as beams, shells, plates, etc., with parametrized spatially varying parameters. For example, a true system can be represented as a non-uniform elastic rod, whereas the model set is a uniform rod with the parameters to be estimated being the constant torsional mass and stiffness. In this case it is possible to determine what amount of distributed torsional mass and stiffness can be sufficiently well approximated by a uniform mass and stiffness for the purposes of control design.

The above simple example illustrates the difficulties involved in choosing a model parametrization. Obviously the extension to the LSS environment is not so straightforward because of the additional complexity.

Experiment Design

The MSE can be used to aid in designing the estimation experiment when the intended use of the estimated model is control design. The basic idea is to make the criterion for estimation similar to the criterion for control design. This is equivalent to minimizing a weighted norm of the MSE with respect to free parameters or choices in the experiment, such as, the input spectrum, model order, data filters, and so on, as described in Ljung(1987).

At the present time the use of the MSE as a measure of experiment design for the case when the intended use of the transfer function estimate is control synthesis is still in the beginning stages.

¹The ideal estimate minimizes the least squares criterion for a specified length of the data record. This is not the estimate which is recursively computed from on-line data. These two estimates are, at best, asymptotically equivalent, with the recursive estimate having a larger variance, the size of which is dependent on the specific algorithm for on-line estimation.

One of the major problems is that measures of closed loop performance are complicated functions of the MSE and the criterion for estimation.

Computing Model Uncertainty from Data

The requisite information for robust feedback stabilization is typically a nominal model of the plant together with an uncertainty profile. From the above discussion, computation of the MSE for model error allows for an *analysis* of the adaptive system. If the MSE can be *estimated* from on-line data, then a robust controller can be designed. Our current approach to estimating model uncertainty involves not only parametric estimation, but also non-parametric (spectral) methods of transfer function estimation, e.g., Jenkins and Watts(1968). The non-parametric methods provide estimates of model accuracy as a function of frequency, particularly over those frequency ranges where either the model structure is poorly known, or else the model accuracy in that range is unimportant for control design. The feasibility of using spectral techniques for the estimation of model error can be found in Kosut(1987).

Stability and Convergence Analysis

Conditions for stability and convergence and causes for divergence and instability in adaptive control systems are contained in Anderson et al.(1986). The stability analysis focuses on *slow adaptation* and the use of some of the classical methods for analyzing differential equations, e.g., linearization, the method of averaging, and Lyapunov's second method, see, e.g., Hale(1969). The material in the text represents modifications and refinements of earlier work by the authors.

To remove the restrictiveness of slow adaptation requires an understanding of the transient behavior of adaptive systems. Preliminary investigations are reported in Kosut et al.(1987).

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Majorant Analysis of Performance Degradation due to Uncertainty

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INTRODUCTION AND BACKGROUND

The problem addressed here is the determination of bounds on the degradation of system performance due to uncertainties and/or unmodeled and imperfectly modelled subsystem interactions. Such bounding techniques represent a fundamental systems analysis tool that is indispensable for further elucidation of decentralized controller architectures and robust design.

Extensive work has been carried out within the controls community in the area of frequency-domain analysis of robust stability giving rise to the H-infinity theory of robustness characterization and robust design [1-5]. However, on several occasions we have remarked that although the H-infinity world-view is a beautiful and compelling theory within its proper province, its fundamental assumptions render it inapplicable to structural vibration control which involves parametric and often nondestabilizing open-loop uncertainties. A principal difficulty is the conservatism of H-infinity robustness characterizations. A stability robustness analysis technique is called conservative if the predicted set of nondestabilizing perturbations is a proper subset of the actual set of nondestabilizing perturbations. Note that conservatism jointly depends upon both the definition of admissible perturbation classes and the robustness analysis technique.

The well-known conservatism of H-infinity theory does not arise because it operates in the frequency domain, per se, or because the infinity norm is employed, but rather because of the crudeness of H-infinity bounds. What is the fundamental source of this crudeness? Possibly this arises because the fundamental intent of H-infinity development was the extension of classical control design concepts to the multivariable case whether or not classical concepts are truly suited to the problem at hand.

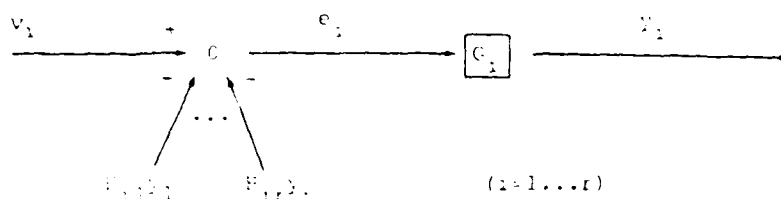
For example, in keeping with classical ideas, there has been widespread insistence upon couching all questions of performance and uncertainty in terms of simplistic (albeit traditional) unity gain feedback diagrams. Thus, singular value developments have lumped

uncertainty in a single block thereby obscuring the often complex structure of modelling error. Moreover, this feedback paradigm is maintained even for structured uncertainty approaches [6].

LARGE SCALE SYSTEM FORMULATION AND L_p BOUNDS

Here, we contend that to achieve a less confining point of view, the first step is to represent uncertain systems by means of a large-scale system input-output formulation as depicted in Figure 1.

LARGE-SCALE SYSTEM-INPUT-OUTPUT FORMULATION



$$(I - F_{11})v_1 = y_1$$

$$y_1 = F_{11}v_1$$

$$F_{11} = \text{block-diag}(G_{11}, \dots, G_{1r}) \quad G_{1k} \text{ known} \\ k=1, \dots, r$$

$$F = \begin{bmatrix} 0 & F_{12} & F_{13} & \dots \\ F_{21} & 0 & F_{23} & \\ F_{31} & F_{32} & 0 & \\ \vdots & & & \end{bmatrix} \in \mathcal{H}, \text{ some compact,} \\ \text{arcwise connected set with} \\ \text{off-diagonal block structure}$$

uncertain subsystem interactions or parametric uncertainties

$\mathcal{H}$ specifies uncertainty about F

Problem: bound output y or deviation from nominal, $y - y_0$,
for all $F \in \mathcal{H}$

Figure 1. Large-Scale System Input-Output Formulation

Referring to Figure 1, the overall system is represented by interconnected subsystems undergoing interactions. The subsystems, characterized by the operators G_k ($k=1, \dots, r$), represent the known dynamics of the system while the subsystem interactions, given by the operators H_{kj} , correspond to uncertainties. Note that the partitioned off-block-diagonal operator H is stipulated to belong to some compact arcwise connected set $\underline{H}$. The set $\underline{H}$ specifies both the character and extent of dynamical uncertainties.

The motivation for the above input-output formulation within the context of large-scale systems is obvious. But in addition, thanks to the Dynamic Inclusion Principle and related ideas elaborated by Siljak and his co-workers [7,8] the representation of Figure 1 is also suitable for parametric perturbations in monolithic systems, i.e., systems without explicit interconnections.

The problem now addressed is how to bound the degradation of the system output vector y or the prediction accuracy $y-y_0$ due to the uncertainties.

To give this problem mathematical form, we must use the block-matrix results of Ostrowski [9] and define the block-Lp norm matrix of a partitioned operator, as the nonnegative matrix whose elements are the Lp norms of the corresponding subblocks. With this definition, the principal problem is to bound the block-norm vector of the system output y over all variations of the uncertain perturbations. Note also that the existence of such a finite bound implies input-output stability (see [10]).

Now, the above articulation of uncertainties into numerous interactions permits more finely articulated methods of computing bounds beyond singular value analysis, namely, methods associated with the majorant analysis of Dahlquist [3,11].

The uncertain subsystem interaction format (introduced in Figure 1) in conjunction with majorant analysis gives an almost unlimited potential for formulating sharper bounds. Using a process of operator iteration, one can obtain a hierarchy of output bounds, where each successive member of the hierarchy requires more and more information but is less and less conservative (with respect to the set $\underline{H}$). Also, because we work in an operator setting, distinctions between the time and frequency domains are blurred. It is parochial to assert that only frequency-domain or time-domain methods must be used. What's needed is easy and fluent translation between the frequency and time-domain. Furthermore, the computational advantage of this kind of hierarchy is that each bound requires only the inversion of an M-matrix. This is quite straightforward and nicely tractable, even for many subsystems, since it involves computing a monotonically increasing sequence where each iteration involves an addition and a multiplication of low-order matrices.

BOUNDS FOR RESPONSE TO STOCHASTIC INPUTS

The above discussion has sketched the general development of majorant robustness analysis within an operator setting which employs L_p norms to describe the "size" of subsystem outputs. For systems with stochastic inputs and time independent parameter uncertainties, the main lines of development are analogous. However, in this case one needs to work with the Lyapunov equation for the steady-state second-moment matrix of response and then derive majorant bounds for the block-norm matrix of the second moment. The general setup for undertaking majorant analysis for parametrically uncertain stochastic systems is shown in Figure 2. Here, the block-diagonal matrix A represents the known subsystem or nominal system dynamics while the off-block-diagonal matrix G represents uncertain subsystem interactions or parametric uncertainties.

$$\begin{aligned}\dot{\mathbf{x}} &= (\mathbf{A} + \mathbf{G})\mathbf{x} + \mathbf{w} & \dot{\mathbf{Q}} &= (\mathbf{A} + \mathbf{G})\mathbf{Q} + \mathbf{Q}(\mathbf{A} + \mathbf{G})^T + \mathbf{V}\end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & \cdots \\ 0 & \mathbf{A}_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Known Subsystem Dynamics

$$\mathbf{G} = \begin{bmatrix} 0 & \mathbf{G}_{12} & \cdots \\ \mathbf{G}_{21} & 0 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Uncertain Subsystem Interactions

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_{12} & \cdots \\ \mathbf{V}_{21} & \mathbf{V}_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Noise Intensity

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_{12} & \cdots \\ \mathbf{Q}_{21} & \mathbf{Q}_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

State Covariance

Figure 2. Subsystem Interaction Model

Note that the disturbance intensity $\mathbf{V}$ and the second-moment matrix $\mathbf{Q}$ are partitioned conformably with $\mathbf{A}$ and $\mathbf{G}$. We bound performance degradation due to uncertain interactions $\mathbf{G}$ ranging over the admissible set $\mathcal{G}$ by bounding the block-norm matrix of $\mathbf{Q}$. To do this, however, requires additional algebraic tools, such as the Block-Kronecker Algebra which is a generalization of the matrix calculus reviewed by Brewer [12]. Because of the algebraic complexity of deriving majorants for the second-moment matrix, the Kronecker Algebra is far more than a mere notational convenience.

In particular, using the Block-Kronecker Algebra one first reduces the problem to a more tractable form and then applies majorant analysis to obtain a hierarchy of majorant bounds. As in the L_p bound analysis, each successive member of the hierarchy offers less and less conservative bounds.

We now consider in more detail the first two members of the majorant hierarchy in order to illustrate the specific forms of the modified Lyapunov equations that are obtained.

For example, Figure 3 shows the first member of the second-moment majorant hierarchy [13]. This gives the majorant $\hat{Q}$ as the solution of a simple nonnegative matrix equation, where $\star$ denotes the Hadamard (element by element) product and the row and column dimension of the equation is the number of subsystems. For the norm-bounded uncertainty set shown in Figure 3, the existence of a nonnegative solution implies a bound for the block-norm matrix of the second moment and robust stability, i.e., $(A+G)$ is stable for all perturbations G in the norm-bounded set.

$$H \star \hat{Q} = G \hat{Q} + \hat{Q} G^T + V$$

$$\bar{\sigma}(G_{ij}) \leq G_{ij}$$



$$Q \leq \hat{Q}$$



- Robust Stability
- Robust Performance

Figure 3. Majorant Lyapunov Equation

One particular advantage of the first member of the hierarchy is that it correctly shows the effect of wide frequency separation of subsystems on performance degradation and robust stability. In particular, the majorant equation will correctly predict that as frequency separation becomes sufficiently large, subsystems become effectively decoupled. Such predictions cannot be made by the small gain theorem for large-scale systems or by vector Lyapunov theory [7,8]. Thus, even the first member of the majorant hierarchy offers greatly reduced conservatism compared to previous results.

Furthermore, the second member of the second-moment majorant hierarchy, shown in Figure 4 gives even tighter bounds and can even predict the stabilizing effect of certain kinds of perturbations. The form of the majorant equation (top of Figure 4) is similar to the first member of the hierarchy except that the operation $H[Q]$ appears. This operator is precisely what would arise in the equation for the second-moment matrix for a system with Stratonovich noise parameters! So far, we have discussed a design analysis tool for predicting performance

degradation due to uncertainty. This crucial observation brings us to consideration of the link between majorant robustness analysis and MEOP design synthesis theory.

Second member of the hierarchy:

$$H \cdot \hat{Q} + H[\hat{Q}] = \langle \hat{Q} \rangle + \langle \hat{Q} \rangle G^T + V$$

$$J = \text{tr}[\hat{Q}R] + 2 \sum_{K=1}^r (\text{tr } \hat{P}_K) (\hat{Q} - \hat{Q}_K) K K$$

$$0 = A\hat{Q} + \hat{Q}A^T + H[\hat{Q}] + V$$

$$0 = A^T \hat{P} + \hat{P}A + H[\hat{P}] + R$$

where:

$$\hat{Q} = \hat{Q} - \frac{1}{n} \text{tr}(\hat{Q}) I \quad \text{off-diagonal part of } \hat{Q}$$

$$H[\cdot] = \text{Stratonovich model operator}$$

- Tighter bound—incorporates more information on A and G
- Predicts stability when $(A + A^T)$ stable, $G = -G^T$
- "Nominal" performance, $\text{tr}[\hat{Q}R]$, given by Stratonovich model

Figure 4. Second Member of the Majorant Hierarchy

THE LINK BETWEEN ANALYSIS AND SYNTHESIS

Figure 5 illustrates this link and the accompanying sequence of logical developments. Overall, one may regard the MEOP design synthesis theory as arising from a particular robustness analysis tool. Although any member of the second-moment majorant hierarchy might be chosen as the basis of a design synthesis theory, we choose the second member of the hierarchy (see lower right block in Figure 5) to serve as the point of departure because it is the simplest bound that also handles nondestabilizing uncertainties. Referring to the lower left block of Figure 5, it is seen that the second-moment equation of a multiplicative Stratonovich noise model essentially gives an approximation to the majorant equation and a smooth optimization problem. The Stratonovich second moment equation then leads to an auxiliary optimization problem (upper left block in Figure 5), namely, choose dynamic compensator gains to minimize the quadratic performance of a system having multiplicative stochastic parameters. Because of the Stratonovich modifications to the standard form of the Lyapunov equation that appear in the equation for $\bar{Q}$, the robust stability condition implied by the majorant equation is still enforced since the optimization problem imposes a robust performance constraint.

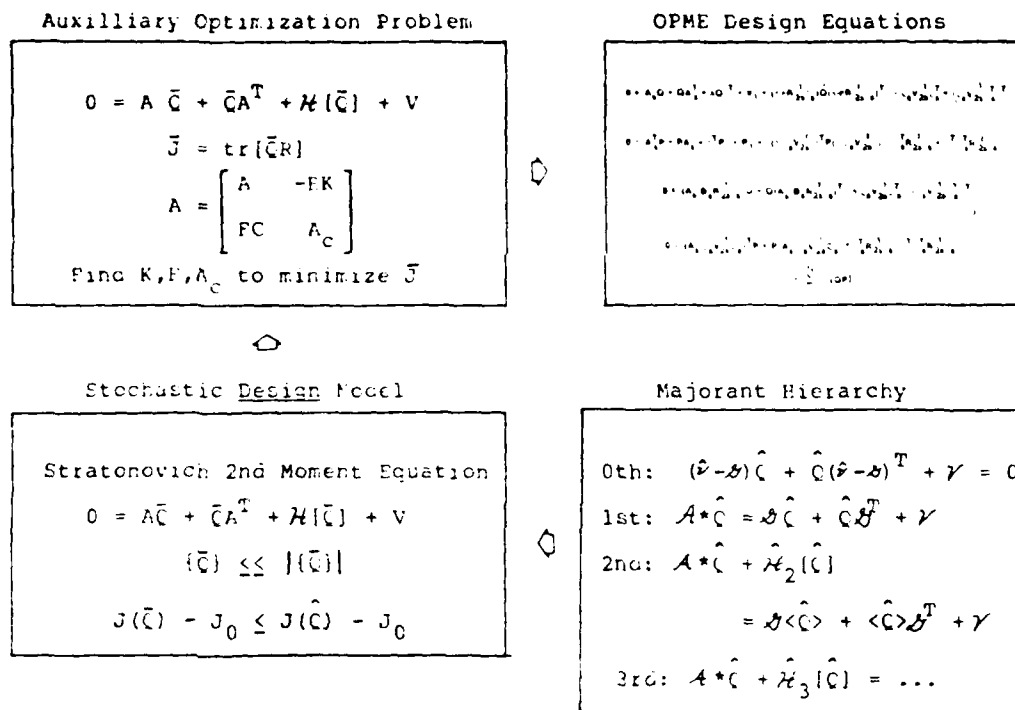


Figure 5. Majorant Hierarchy and Stratonovich Models
-- Link Between Analysis and Synthesis

This optimization of an apparently stochastic system actually approximates the majorant bound which was derived purely deterministically and leads to the rather elegant MEOP optimality conditions given in the upper right block in Figure 5.

Of course, the use of Stratonovich stochastic models was earlier indicated by maximum entropy principles and stochastic approximation theory, and this line of development still stands. But the import of the more recent majorant analysis developments is that there is a direct link between maximum entropy stochastic modelling and deterministic performance bounds. This link tends to blur the distinctions between stochastic and deterministic points of view. This is just as well: The task confronting the controls and systems theory community is not to resolve the stochastic versus deterministic debate one way or the other, but rather to rise above it. As the work described here suggests, there is a plane upon which these points of view are numerically indistinguishable.

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The Optimal Projection Equations for Fixed-Order Dynamic Compensation:
Existence, Convergence and Global Optimality

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INTRODUCTION

Despite significant advances in the cost and performance of digital computers over the last decade, there remains a need in several technological areas for low-order, high-performance controllers. In particular, this paper is motivated by the problem of vibration suppression in large flexible space structures. Such systems are infinite-dimensional (distributed parameter) in nature and hence any finite-dimensional controller is necessarily of reduced order. The need for low-order controllers is further driven by severe constraints on cost, weight and power in space systems, not to mention the restriction to space-qualified computational hardware.

A wide variety of approaches have been proposed to obtaining reduced-order controllers. A comparison of several approaches to controller reduction is given in [1]. These methods operate by first designing a high-order LQG controller and then obtaining a suitable low-order controller by means of controller reduction.

A more direct approach to designing reduced-order controllers involves optimizing the quadratic performance functional over the class of controllers of fixed order. The controller order may be determined by implementation constraints or can be varied for performance/throughput tradeoff studies.

A reformulation of the parameter optimization approach was given recently in [2]. The authors showed that the first order necessary conditions can be transformed to yield explicit gain expressions for extremal fixed-order controllers.

Regardless of how appealing the optimal projection formulation in [2-12] may appear to be, its contribution is vacuous unless certain serious questions can be resolved. These include:

1. Under what conditions on the problem data can the optimal projection equations be guaranteed a priori to possess a solution?

2. Given problem data, exactly how many solutions do the equations possess?
3. Of the possible solutions, what are their stability properties, what is their performance, and which is the global optimum?
4. How can numerical algorithms be constructed which can be guaranteed to converge to any desired solution especially the global minimum?

It seems clear that any attempt to address the above issues must utilize mathematical methods which are global in nature. To this end we have applied degree theory and associated homotopic continuation methods ([13-24]) to analyze the solutions to the optimal projection equations and to construct convergent, implementable algorithms for their computation. The purpose of this presentation is to report significant recent results in this regard.

HOMOTOPIC CONTINUATION AND DEGREE THEORY FOR THE OPTIMAL PROJECTION EQUATIONS

A homotopic continuation method for solving a problem is to first solve an easy "similar" problem, and then to continuously deform the easy problem into the original problem and to follow the path of solutions as the easy problem is deformed into the original problem. This is shown conceptually in Figure 1.

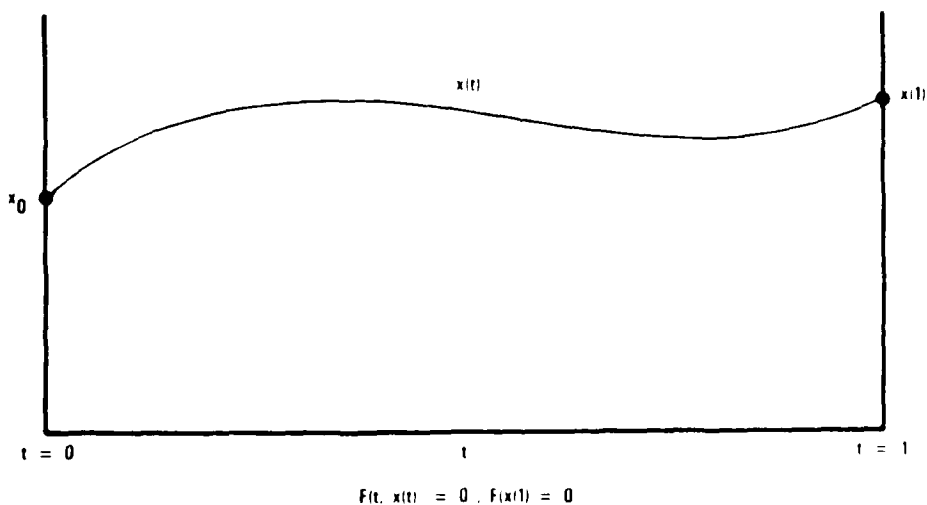


Figure 1

Topological degree theory is the basic tool for analyzing the existence and topology of continuation solutions.

Here we consider how these concepts have been recently applied by Richter [25] to solution of the Optimal Projection Equations (OPE). The basic computational problem is to find $P, Q, \hat{P}, \hat{Q}$ which satisfy:

$$0 = AQ + QA^T + V_1 - Q\bar{\Sigma}Q + \tau_1 Q\bar{\Sigma}Q\tau_1^T,$$

$$0 = A^T P + PA + R_1 - P\Sigma P + \tau_1^T P\Sigma P\tau_1,$$

$$0 = (A - \Sigma P)\hat{Q} + \hat{Q}(A - \Sigma P)^T + Q\bar{\Sigma}Q - \tau_1 Q\bar{\Sigma}Q\tau_1^T,$$

$$0 = (A - Q\bar{\Sigma})^T \hat{P} + \hat{P}(A - Q\bar{\Sigma}) + P\Sigma P - \tau_1^T P\Sigma P\tau_1$$

given $\Sigma, \bar{\Sigma}, \tau_1, \tau_1^T, R_1, V_1, A$, where n is the plant dimension and n_c is the desired controller dimension. To do this let

$$A(t) = \begin{bmatrix} L_1 & & \\ & B_{n_c} & \\ & & D_n \end{bmatrix} (1-t) + tA$$

$$R_1(t) = I(1-t) + tR_1, \quad V_1(t) = I(1-t) + tV_1$$

$$\Sigma(t) = \begin{bmatrix} \Sigma_0 & 0 \\ 0 & 0 \end{bmatrix} (1-t) + t\Sigma$$

$$\bar{\Sigma}(t) = \begin{bmatrix} \bar{\Sigma}_0 & 0 \\ 0 & 0 \end{bmatrix} (1-t) + t\bar{\Sigma}$$

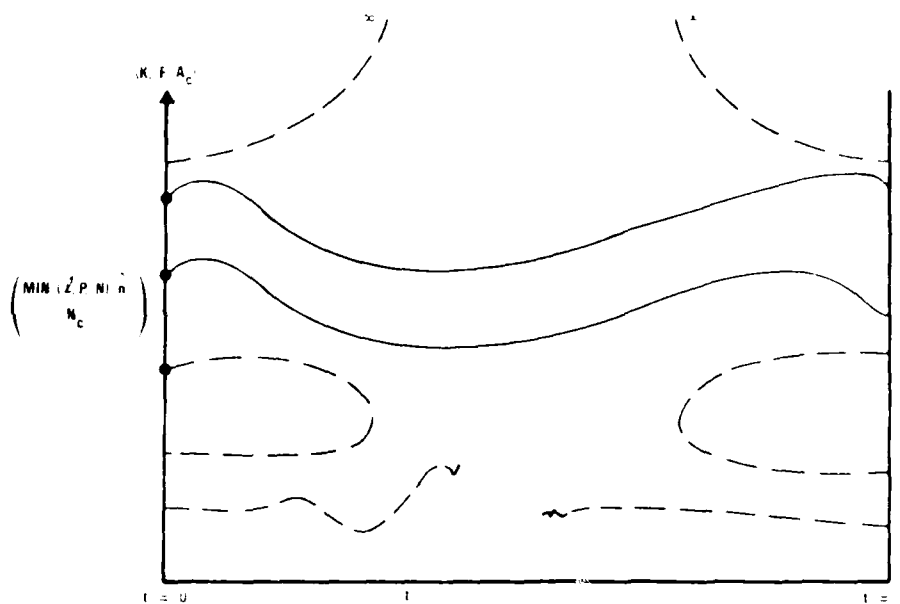
For $t=0$ the solution is easy to find. The object is to follow the path or paths of solutions $P(t), Q(t), \hat{P}(t), \hat{Q}(t)$ from $t=0$ to $t=1$.

In following these initial solutions from $t=0$ to $t=1$ there are several situations which could occur (see figure 2).

Using degree theory, Richter [25], has shown the following results:

Theorem 1

With respect to the above continuation method of OPE solution, the situations shown in dashed lines in Figure 2 cannot occur. That is, the only solutions to the OPE at $t=1$ (or for $0 < t < 1$) are those which are continuously derived from the solutions at $t=0$.



Topological degree theory $\Rightarrow$ The dashed paths do not exist

If $n_c \geq \min (L, P, N) - n$, there is only one solution ($\Rightarrow$ global minimum)

Figure 2

Theorem 2

Assume that the plant is stabilizable and detectable, $V_1 > 0$, $R_1 > 0$ and $n_u \leq n_c$ when n_u is the dimension of the unstable subspace of A . Then, in the class of nonnegative-definite solutions $Q, P, \hat{Q}, \hat{P}$ with

$$\text{rank } \hat{Q} = \text{rank } \hat{P} = \text{rank } \hat{Q}\hat{P} = n_c,$$

the optimal projection equations possess at most

$$\begin{pmatrix} \min(n, m, l) - n_u \\ n_c - n_u \end{pmatrix}, \quad n_c \leq \min(n, m, l),$$

1, otherwise,

stabilizing solutions. Each such solution is reachable via a homotopic path with starting point corresponding to diagonal initial data. Furthermore, if the plant is stabilizable by means of an n_c th-order dynamic compensator, then there exists at least one solution.

Note that when n_c is larger than the number of inputs or outputs, there is only one solution to the OPE and this solution corresponds to the global minimum of the quadratic performance index.

ALGORITHM DESCRIPTION AND NUMERICAL RESULTS

In a homotopy path following algorithm one follows the path of solutions of $F(x(t), t) = 0$ by integrating the initial value problem

$$\frac{dx}{dt} = F_x(x(t)) \frac{dF}{dt}(x(t), t); \quad x(0) = x_0.$$

For the optimal projection equations the solution $P, \hat{P}, Q, \hat{Q}$ can be easily determined once the projection τ is known so that

$P(t) = P(\tau(t)), Q(t) = Q(\tau(t))$ etc. Thus the derivatives of $P, Q, \hat{P}, \hat{Q}$ can be written in terms of derivatives of G^T and Γ where G^T and Γ are the factors of $\tau (\tau = G^T \Gamma)$. Thus we obtain

$$\text{vec} \begin{bmatrix} \Gamma' \\ G^{T'} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \text{vec} i(F, G^T),$$

which gives $2n_c n$ equation for Γ' and $G^{T'}$. $P', Q', \hat{P}'$ and $\hat{Q}'$ are then calculated from Γ' and $G^{T'}$ and finally $\Gamma(t+\Delta t)$ is updated by

$$\Gamma(t+\Delta t) = \Gamma(t) + \Gamma' \times \Delta t$$

and likewise for $G, P, Q, \hat{P}$ and $\hat{Q}$.

Note that each continuation step involves the solution of a linear system of dimension $2n_c n$. This means that for small controller dimension ($n_c \ll n$), the computational burden of the OPE homotopy algorithm can actually be less than the LQG reduction methods (which typically require two $n \times n$ Riccati and two $n \times n$ Lyapunov equation solutions).

Figure 3 summarizes the closed-loop stability results reported in [1] for LQG reduction methods along with results obtained using the homotopy method for solving the optimal projection equations. Here q_2 is a scale factor for the plant disturbance noise affecting controller authority. Clearly, LQG reduction methods experience increasing difficulty as authority increases, i.e., as the τ_1 terms become increasingly more important in coupling the control and reduction operations. In contrast, the OPE consistently yield closed-loop stable reduced-order controllers for all values of q_2 . Figure 4 shows the quadratic performance J , versus the observer bandwidth parameter q_2 for OPE designs of various orders. Even for very high control authority ($q_2 = 10^5$), when LQG reduction methods produce no stabilizing designs, the 2nd order OPE compensator shows a performance only 20% higher than a full-order LQG design.

Method	N_c	1	2	3	4	5	6	7	8	9	10
Enns	1	S	S	S	S	S	S	S	S	S	S
	2	S	S	S	S	S	S	S	S	S	S
	3	S	S	S	S	S	S	S	S	S	S
	4	S	S	S	S	S	S	S	S	S	S
	5	S	S	S	S	S	S	S	S	S	S
	6	S	S	S	S	S	S	S	S	S	S
	7	S	S	S	S	S	S	S	S	S	S
	8	S	S	S	S	S	S	S	S	S	S
	9	S	S	S	S	S	S	S	S	S	S
	10	S	S	S	S	S	S	S	S	S	S
	11	S	S	S	S	S	S	S	S	S	S
	12	S	S	S	S	S	S	S	S	S	S
	13	S	S	S	S	S	S	S	S	S	S
	14	S	S	S	S	S	S	S	S	S	S
	15	S	S	S	S	S	S	S	S	S	S
	16	S	S	S	S	S	S	S	S	S	S
	17	S	S	S	S	S	S	S	S	S	S
	18	S	S	S	S	S	S	S	S	S	S
	19	S	S	S	S	S	S	S	S	S	S
	20	S	S	S	S	S	S	S	S	S	S
	21	S	S	S	S	S	S	S	S	S	S
	22	S	S	S	S	S	S	S	S	S	S
	23	S	S	S	S	S	S	S	S	S	S
	24	S	S	S	S	S	S	S	S	S	S
	25	S	S	S	S	S	S	S	S	S	S
	26	S	S	S	S	S	S	S	S	S	S
	27	S	S	S	S	S	S	S	S	S	S
	28	S	S	S	S	S	S	S	S	S	S
	29	S	S	S	S	S	S	S	S	S	S
	30	S	S	S	S	S	S	S	S	S	S
	31	S	S	S	S	S	S	S	S	S	S
	32	S	S	S	S	S	S	S	S	S	S
	33	S	S	S	S	S	S	S	S	S	S
	34	S	S	S	S	S	S	S	S	S	S
	35	S	S	S	S	S	S	S	S	S	S
	36	S	S	S	S	S	S	S	S	S	S
	37	S	S	S	S	S	S	S	S	S	S
	38	S	S	S	S	S	S	S	S	S	S
	39	S	S	S	S	S	S	S	S	S	S
	40	S	S	S	S	S	S	S	S	S	S
	41	S	S	S	S	S	S	S	S	S	S
	42	S	S	S	S	S	S	S	S	S	S
	43	S	S	S	S	S	S	S	S	S	S
	44	S	S	S	S	S	S	S	S	S	S
	45	S	S	S	S	S	S	S	S	S	S
	46	S	S	S	S	S	S	S	S	S	S
	47	S	S	S	S	S	S	S	S	S	S
	48	S	S	S	S	S	S	S	S	S	S
	49	S	S	S	S	S	S	S	S	S	S
	50	S	S	S	S	S	S	S	S	S	S
	51	S	S	S	S	S	S	S	S	S	S
	52	S	S	S	S	S	S	S	S	S	S
	53	S	S	S	S	S	S	S	S	S	S
	54	S	S	S	S	S	S	S	S	S	S
	55	S	S	S	S	S	S	S	S	S	S
	56	S	S	S	S	S	S	S	S	S	S
	57	S	S	S	S	S	S	S	S	S	S
	58	S	S	S	S	S	S	S	S	S	S
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	70	S	S	S	S	S	S	S	S	S	S
	71	S	S	S	S	S	S	S	S	S	S
	72	S	S	S	S	S	S	S	S	S	S
	73	S	S	S	S	S	S	S	S	S	S
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	75	S	S	S	S	S	S	S	S	S	S
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	77	S	S	S	S	S	S	S	S	S	S
	78	S	S	S	S	S	S	S	S	S	S
	79	S	S	S	S	S	S	S	S	S	S
	80	S	S	S	S	S	S	S	S	S	S
	81	S	S	S	S	S	S	S	S	S	S
	82	S	S	S	S	S	S	S	S	S	S
	83	S	S	S	S	S	S	S	S	S	S
	84	S	S	S	S	S	S	S	S	S	S
	85	S	S	S	S	S	S	S	S	S	S
	86	S	S	S	S	S	S	S	S	S	S
	87	S	S	S	S	S	S	S	S	S	S
	88	S	S	S	S	S	S	S	S	S	S
	89	S	S	S	S	S	S	S	S	S	S
	90	S	S	S	S	S	S	S	S	S	S
	91	S	S	S	S	S	S	S	S	S	S
	92	S	S	S	S	S	S	S	S	S	S
	93	S	S	S	S	S	S	S	S	S	S
	94	S	S	S	S	S	S	S	S	S	S
	95	S	S	S	S	S	S	S	S	S	S
	96	S	S	S	S	S	S	S	S	S	S
	97	S	S	S	S	S	S	S	S	S	S
	98	S	S	S	S	S	S	S	S	S	S
	99	S	S	S	S	S	S	S	S	S	S
	100	S	S	S	S	S	S	S	S	S	S

S = The closed-loop system is stable
 U = Unstable

Figure 3. The optimal Projection Approach Was Compared to Several LQG Reduction Techniques over a Range of Controller Authorities for an Example of Enns

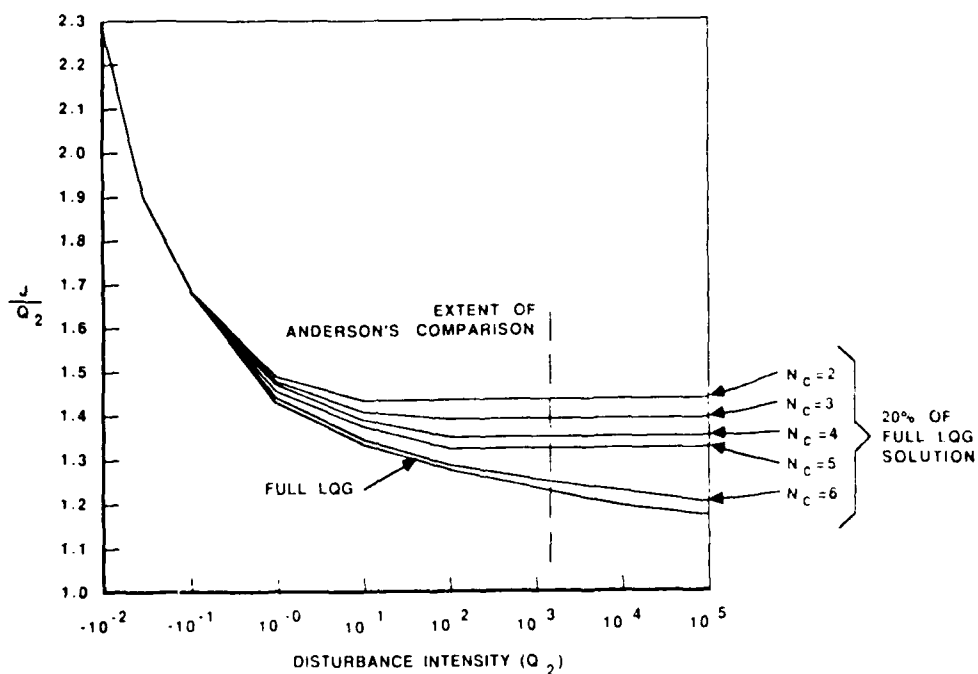


Figure 4

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Decentralized/Relegated Control for Large Space Structures

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Introduction

The complexity of present and envisaged space structures dictates the inevitable need for emphasis on control and structure interaction. Moreover, in view of present methodologies, stringent control requirements such as precise pointing and slewing, vibration suppression, and shape control indicate that much work remains to be done in this important area. In this presentation we summarize recent results in the decentralized, relegated control of large space structures, with concentration on topics of decentralization, relegation, servomechanism design, and multiple mirror system examples.

Decentralization and Relegation

General goals of this research have been focused in the area of large space structure (LSS) modeling and control. In particular, the work has been oriented to evaluating the use of decentralized control strategies as a fundamental context within which to develop controller design methodologies for a variety of control objectives for large flexible structures. It is within this context that we develop decentralized servocompensator design techniques which incorporate various implementation constraints, such as subsystem input bandwidth, sensitivity and robustness requirements, integrity constraints, and so on. The concept of relegation as coordinated with decentralized schemes is also being developed. Relegation is the assignment of control tasks and information channels in view of control effectiveness, on-line computational complexity, controller capabilities, and physical and structural constraints. Based on the system model formulation and proposed control objectives, we believe relegation to be extremely important for the control of large-scale systems in general and of large space structures in particular.

Advanced space-based detection, surveillance, and defense systems necessitate the construction of LSS with highly flexible components. Such systems must operate effectively under disturbances (for example, constant disturbances due to solar winds or sinusoidal disturbances due to the operation of equipment on board) and must be able to track moving targets or reference trajectories. Difficulties arise from the lack of significant inherent damping characteristics and from the fact that it is usually not possible to obtain precise linear-time-invariant models for such systems. Therefore, it is necessary to design on-line controllers which are robust to uncertainties in modeling, to achieve desired damping, disturbance rejection and tracking.

In the case of LSS, the solution of the servomechanism problem may require decentralized controllers for various reasons such as:

- a) The sensor-actuator locations on the structure may have a decentralized nature which makes it hard or impossible to implement feedback through a centralized location.
- b) Information transfer may be costly so that the designer may wish to impose decentralized feedback.
- c) On-line centralized feedback calculations may be too time consuming so that simple local feedback calculations may be preferred.

A LSS can in general be described by a generalized wave equation which leads to an infinite dimensional state-space model. Alternatively, finite-element methods can be employed to obtain a finite dimensional model directly. However, an accurate model may necessitate a large number of states, which makes it very hard or impossible to design a controller based on such a model. In this case, it is necessary to use model reduction techniques to obtain a model of manageable dimension.

Decentralized LQG/LTR

The above controller design problem, which is usually referred to as the robust servomechanism problem has been studied by various researchers in a general context. The linear-quadratic-Gaussian with loop-transfer-recovery (LQG/LTR) design methodology has been introduced recently and has attracted much interest. This methodology enables frequency domain controller design by using the well known LQG techniques.

Recently, a linear-quadratic optimal approach has been developed to solve this problem, where it has been shown that the resulting controller enjoys many desirable properties. In this presentation we consider this development with the LQG/LTR methodology for decentralized control systems. Specifically, bounds on the norm

of the *multiplicative error matrix* are calculated and the LQG-LTR methodology is used to design local controllers.

Vibration Compensation in Optical Tracking Systems

The focus of this work is on compensation for flexibility effects due to, for example, slew maneuvers, for flexible structure systems composed of multiple actuated mirrors. Primary applications for such systems include line of sight pointing systems on large flexible structures, space telerobotic systems, and space telescope systems. Two stages characterize the modeling problem for this study: 1) The description of the *rigid* slewing motion and associated mirror and ray optics; and 2) The description of the *flexible* dynamics. The rigid-motion ray-trace equations are developed by using the compact notation used to describe the motion of robotic manipulators, while flexible dynamics are obtained via standard finite-element techniques. The resulting hybrid model is suitable for analysis in a four stage process: 1) Relegation of control tasks (intimately related to the kinematics); 2) The standard slewing control problem; 3) Flexibility compensation using mirror actuators; and 4) Active vibration damping with additional (proof mass) actuation. In this presentation we describe results addressing the first and third stages of the above process.

Frobenius-Hankel Norm Framework for Disturbance Rejection and Low Order Decentralized Controller Design

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INTRODUCTION

There are many obvious reasons why simple controllers of low order are to be preferred to high order controllers. However, established controller synthesis techniques, such as LQG, frequency weighted LQG, and H_2 minimization result in controllers of high order, even of order higher than the plant. While this may be tolerated for systems of small dimension, it is obviously unacceptable where the plant itself is of high order, such as it is with even the simplest space structures.

There is, therefore, the desire to design low-order controllers for high order plants. Procedures to solve the low-order controller design problem can broadly be divided into two classes [1]: Direct methods, in which the parameters defining a low order controller are computed by some optimization or other procedure, and indirect methods, in which a high order controller is found first, and then a procedure is used to simplify it. Examples of direct methods are the parametric LQ designs [5] and the projective controls procedure developed by the authors [2,3,4]. In the case of indirect methods the LQG, the frequency weighted LQG, and the H_∞ -norm minimization approaches provide the high order controller and, each in its own right, captures many relevant performance, robustness design goals. However, as stressed in [1], procedures for reducing high order controllers to low order controllers are less well developed. In [1] the controller reduction problem is reduced in principle to the constrained L_∞ model matching problem:

$$K_r^*(s) = \arg \min \| T_1(s) - T_2(s) K(s) T_3(s) \|_\infty \quad (1)$$

$K_r(s)$ of order r or less

and $K_r(s)$ is the desired reduced order controller. Unfortunately, there is no convenient algorithm for the solution of the constrained model matching problem in the H_∞ or L_∞ spaces.

REDUCTIONS AND L_∞ -BOUNDS

Important relations have, however, been established between certain state space model reduction techniques and optimal H_∞ and L_∞ approximations that are useful in the solution of the controller reduction problem. They will be exploited here in a novel approach which can be used in direct methods as well as in indirect methods of solving the low-order controller design problem, and in many other control problems. For the controller reduction problem these results will essentially allow one to minimize a bound γ , and to ascertain that $\| T_1(s) - T_2(s) K(s) T_3(s) \|_\infty \leq \gamma$.

To highlight the ideas let A, B, C be a balanced realization [6] of a linear time-invariant strictly proper system with transfer function $G(s)$, in the sense that

$$G(s) = C(sI - A)^{-1} B \quad (2)$$

$$A\Sigma + \Sigma A^T + BB^T = 0 \quad (3)$$

$$A^T \Sigma + \Sigma A + C^T C = 0 \quad (4)$$

where Σ is the (balanced) observability and controllability grammian, with

$$\Sigma = \text{diag}\{\sigma_1, \sigma_1, \dots, \sigma_n\}, \quad \sigma_i \geq \sigma_{i+1}, \quad i = 1, \dots, n-1. \quad (5)$$

It is known that the k -dimensional reduced order model

$$G_r(s) = C_1(sI - A_{11})^{-1}B_1 \quad (6)$$

where A_{11}, B_1, C_1 come from the partition

$$A = \begin{bmatrix} \bar{A}_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C = [C_1 \quad C_2] \quad (7)$$

enjoys the following properties:

- (i) Systems (A_{ii}, B_i, C_i) , $i=1,2$ are asymptotically stable $\sigma_i > \sigma_{i+1}$ [7]
- (ii) $\|G(j\omega) - G_r(j\omega)\|_\infty \leq 2 \text{Trace}(\sigma_{k+1} + \dots + \sigma_n) = 2 \text{Trace} \Sigma_2$, [8,9] where

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}, \quad \Sigma_1 \in \mathbb{R}^{k \times k}. \quad (8)$$

For Hankel norm optimal approximations [8], similar results hold and it is known that if $G_r(s)$ is a k -dimensional Hankel norm optimal optimization, then

$$\|G(j\omega) - G_r(j\omega)\| \leq \text{Trace} \Sigma_2 \quad (9)$$

with the σ_i the Hankel singular values as in the balanced realization.

From the above considerations it is clear that, while using the balanced realization in reduced order modeling, one cannot expect L_∞ optimal solutions, the smaller the $\sigma_{k+1}, \dots, \sigma_n$ the better the quality of the approximation and the closed one is to the theoretical lower bound $\|G(j\omega) - G_r(j\omega)\| \geq \sigma_{k+1}$ [8].

FROBENIUS-HANKEL NORM DESIGN

Results relating balanced realizations to Hankel singular values, and Hankel singular values to a bound on the H_∞ -norm can be used to develop a new framework for the solution of many control problems including the low-order controller design problem and the controller reduction problem. The framework is best described on the simplest disturbance rejection problem described by

$$\dot{x} = Ax + Bu + Gv \quad (10)$$

$$y = Cx$$

$$z = Hx$$

where u is the control, v is the disturbance, y is the measured and z is the controlled input. Optimal disturbance rejection is formulated as a minimization problem involving the product of the controllability grammian associated with the disturbance and the observability grammian associated with the controller output subject to admissible feedback controls from y to u . This is in effect equivalent to the Frobenius norm defined on the Hankel singular values of the system, and is therefore called the Frobenius-Hankel norm. The ensuing approach can then be shown to exhibit the following characteristics:

A Frobenius norm associated with Hankel singular values, called the FH norm is used to define the optimal controller.

Controller order can be fixed in advance, resulting in the design of controllers of bounded dimension.

The optimal controllers are guaranteed to be stable if there exists at least one stabilizing controller of the considered order.

The optimal controller have guaranteed frequency domain performance as measured by the H_∞ -norm.

Computational aspects are significantly less burdensome than with all presently available controller reduction or low-order controller design techniques.

The approach can be used in many control problems such as model reduction, model reference design, disturbance rejection and others. The work was motivated by research on design of low-order controllers via projective controls approach. It immediately solves a disturbance rejection problem emanating from the projective controls approach and can be used to select the free parameters in the resulting controllers [3]. Design of decentralized low order controllers is a straightforward and important extension of the described framework. It enables, in particular, the allocation of the free parameters in the decentralized controllers obtained using the projective controls approach [4]. Finally, while it is of interest to determine the optimal FH norm solution, most important in our new approach is the availability of simple computational algorithm to find optimal solutions for constrained problems emanating from controller parameterization, or constraints on controller order.

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A Controlled Component Synthesis Method
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Decentralized control is intuitively appealing for vibration control of large flexible structures: It offers simplified control system implementations which basically require the feedback of local measurements to close the control loop for systems that may have a few hundred to thousands of control loops. Many decentralized control methods have been developed in the last decade, typically using one of these two approaches. The first is an *Information Constraint Approach* in which the control design uses the model of the large structure and seeks a feedback control solution to the problem, subject to constraints on the flow of the measurement information [1,2]. The control action decentralization is inherent in any feasible solution of the problem satisfying the imposed constraints. In this approach, the design utilizes a model of the large structure in an optimization problem of the order of the model dimension. Because of the need to exercise a model of the large structure in the design, the designer is forced to reduce the model order to cope with computational resource limitations, at the expense of causing spillovers in the controlled structure.

The second is a *Subsystem Decomposition Approach* in which the large structure model is first decomposed or partitioned into an interconnection of subsystem models [3-7]. Local control designs are produced by designing controllers for the local decoupled subsystem models which are derived from the large structure model by discarding the interconnections. These local designs are solved independently, thereby reducing the computational resource requirements for performing control designs for very large structures. The behavior of the locally controlled structure is analyzed using aggregation analysis methods: An aggregate variable is defined to represent the subsystem's dynamic interactions with the other subsystems. The order of the aggregate analysis problem is equal to the number of subsystems which is determined by the decomposition scheme.

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This paper introduces a new framework for the design of controllers for truss structure vibration control which is closely related to that of the Subsystem Decomposition Approach. The method developed herein deviates from conventional control system design practice in which a dynamic model of the open loop plant is often the initial data given to the control system designer. Instead, the controlled plant is assembled from the *controlled components* in the control design process. The development of this *controlled component synthesis method* is motivated on one hand by the well developed component mode synthesis methods [8-10] - a collection of structural analysis methods which has been demonstrated to be effective for solving large complex structural analysis problems for almost three decades, and on the other, stimulated by the subsystem decomposition viewpoint in large scale system theory. Connections between controlled component synthesis and existing large scale system decomposition techniques are established herein to build a control theoretic foundation for the developed method. A simple truss vibration control problem has been employed to illustrate the design procedures, as well as demonstrating the potentials of the developed method for controlling very large dimensional repetitive truss structures.

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**Robust Eigenstructure Assignment by a Projection Method:
Application Using Multiple Optimization Criteria**

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Introduction

We present new ideas which lead to feedback control laws for large flexible structures which are insensitive to model uncertainty. A pole placement method is presented which leads to near-unitary closed loop eigenvectors, and a new method is introduced to design the control while simultaneously considering three competing measures of optimality: Robustness *versus* Integral Square State Error Energy *versus* Integral Square Control Energy. We have established that the algorithms are applicable to at least moderately high-dimensioned systems. In the present discussion, controls for two coupled flexible bodies are considered. A 6x24 gain matrix is designed to control a 12 modes system using 6 actuators. We have also developed control laws for the R2P2 simulator at Martin Marietta; in this case 3 actuators are used to control a 12th order system. Simulation studies indicate that we have indeed achieved robust designs without significant difficulties associated with spillover into the uncontrolled modes.

In this present discussion, we overview several key ideas and numerical results. In the references, we provide details of the formulation, discussions of salient features, and connection to the available literature.

A Few Details

Consider a linear autonomous dynamical system (having n states and m actuators) with full state feedback:

$$\dot{x} = Ax + Bu, \quad u = -Gx \quad (1)$$

The closed loop system is then

$$\dot{x} = (A - BG)x \quad (2)$$

and the closed loop eigenvalues and eigenvectors satisfy

$$(A - BG - \lambda_i I)\phi_i = 0, \quad i = 1, 2, \dots, n \quad (3)$$

If we are solving the usual *forward* eigenvalue problem (given A , B , and G , find λ 's & ϕ 's), we would determine the eigenvalues from the characteristic equation: $\det(A - BG - \lambda I) = 0$. However, we seek to solve the *inverse* eigenvalue problem (given A , B , λ 's & ϕ 's, find G), we will find this problem easier, in one sense, since we will find it involves only linear operations. Notice that the condition of Eq. (3) can be re-written in *Sylvester form* as

$$(A - \lambda_i I)\phi_i = B h_i, i = 1, 2, \dots, n \quad (4)$$

$$\text{where } h_i = G \phi_i, i = 1, 2, \dots, n \quad (5)$$

Introducing the following matrix definitions:

$$H = [h_1 \ h_2 \ \dots \ h_n], \Phi = [\phi_1 \ \phi_2 \ \dots \ \phi_n], \Lambda = \text{diag}(\lambda_1 \ \dots \ \lambda_n) \quad (6)$$

then Eqs. (4) and (5) become

$$A\Phi - \Phi\Lambda = BH \quad (7)$$

$$\text{and } H = G\Phi \quad (8)$$

Referring to Eqs. (7), (8), consider the following algorithm:

- (1) Choose the closed loop eigenvalue matrix Λ , and the parameter matrix H .
- (2) Solve Eq. (7) for Φ .
- (3) Solve Eq. (8) for G .

As a practical matter this algorithm is oversimplified, because there are an infinity of possible choices for the matrix H , and most of them lead to poor control laws! Since it is difficult to guess the matrix H , perhaps the problem would be easier to solve if the parameters which must be guessed had more obvious physical meaning. This is precisely the path pursued in references [1, 2] where H is found to make the resulting closed loop modal matrix as near as possible (in the least square sense) to a specified *target* set of eigenvectors. Actually, three target eigenvector sets are considered:

- (i) The modal matrix constructed [1, 2] from open loop eigenvectors.
- (ii) The unitary matrix nearest [2] the modal matrix of open loop eigenvectors.
- (iii) The unitary matrix (left singular vectors) resulting [1] from singular value decomposition of the matrix: $[U_1 U_2 \dots U_n]$, where U_i is a basis matrix spanning $(A - \lambda_i I)^{-1} B$.

The explicit algorithms and justifications for determining H corresponding to the above three choices of target eigenvectors are given in the references. It is also evident that an initial choice can be subjected to further optimization to improve some other performance measure and/or satisfy prescribed constraints. It is of significance to record here the following observations:

- (a) For cases in which the closed loop eigenvalues are to be only slightly perturbed from their open loop values, then the choice (i) has been found to lead to the smallest gains, but not necessarily the most robust design.
- (b) For cases in which some of the closed loop eigenvalues are moved deep into the left half plane, then choices (ii) and (iii) lead to consistently smaller condition numbers than (i) for the closed loop modal matrix, and usually smaller gain norms as well. To date a consistent pattern has not emerged as to which is best, both lead to robust designs with small gain norms.

Multiple Objective Optimization

State Error Energy:
$$J_s(H) = \int_0^{\infty} \mathbf{x}^T \mathbf{Q}_s \mathbf{x} dt = \text{trace}(\mathbf{P}_s \mathbf{X}_o)$$

Control Effort Energy:
$$J_u(H) = \int_0^{\infty} \mathbf{u}^T \mathbf{Q}_u \mathbf{u} dt = \text{trace}(\mathbf{P}_u \mathbf{X}_o)$$

Stability Robustness Measure: $J_e(H) = k(\Phi)$

where
$$\mathbf{P}_s^{ij} = -\mathbf{Q}_s^{ij} / (\lambda_i^H + \lambda_j), \quad \mathbf{X}_o = E\{\mathbf{x}_o \mathbf{x}_o^T\}$$

$$\mathbf{P}_u^{ij} = -\mathbf{h}_i^H \mathbf{Q}_u \mathbf{h}_j / (\lambda_i^H + \lambda_j)$$

$$k(\Phi) = |\Phi| |\Phi^{-1}| = \bar{\sigma}(\Phi) / \underline{\sigma}(\Phi)$$

= condition number: the ratio of the maximum and minimum singular values of the closed loop modal matrix Φ .

Multiple Index Optimization: Generation of Tradeoff Surfaces

Step 1. Optimization of Primary Objective Function ...

Find the $\mathbf{H}_e^*$ matrix which minimizes the robustness index $J_e(H)$.

Step 2. Evaluation of Secondary Objective Function Gradients

Define judicious directions in the control gain space to evaluate the design which minimizes the primary objective function. Establish a two parameter family of gain variations (along the gradients of the two secondary indices):

$$\mathbf{q} = \text{col}(h_{11}, h_{21}, \dots, h_{mn}) = \text{col}(\mathbf{H}), \quad \mathbf{q}_e^* = \text{col}(\mathbf{H}_e^*)$$

$$\nabla_s = \text{col}\left(\frac{\partial J_s}{\partial q_1}, \frac{\partial J_s}{\partial q_2}, \dots, \frac{\partial J_s}{\partial q_{mn}}\right) \Big|_{\mathbf{q}=\mathbf{q}_e^*}$$

$$\nabla_u = \text{col}\left(\frac{\partial J_u}{\partial q_1}, \frac{\partial J_u}{\partial q_2}, \dots, \frac{\partial J_u}{\partial q_{mn}}\right) \Big|_{\mathbf{q}=\mathbf{q}_e^*}$$

$$q(\alpha_s, \alpha_u) = |\mathbf{q}_e^*| \left(\frac{\mathbf{q}_e^*}{|\mathbf{q}_e^*|} + \alpha_s \frac{\nabla_s}{|\nabla_s|} + \alpha_u \frac{\nabla_u}{|\nabla_u|} \right)$$

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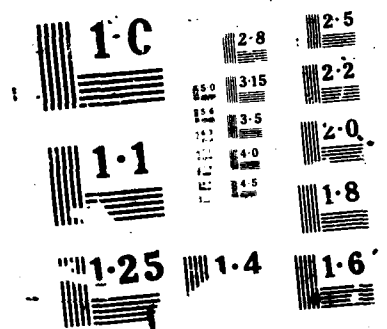
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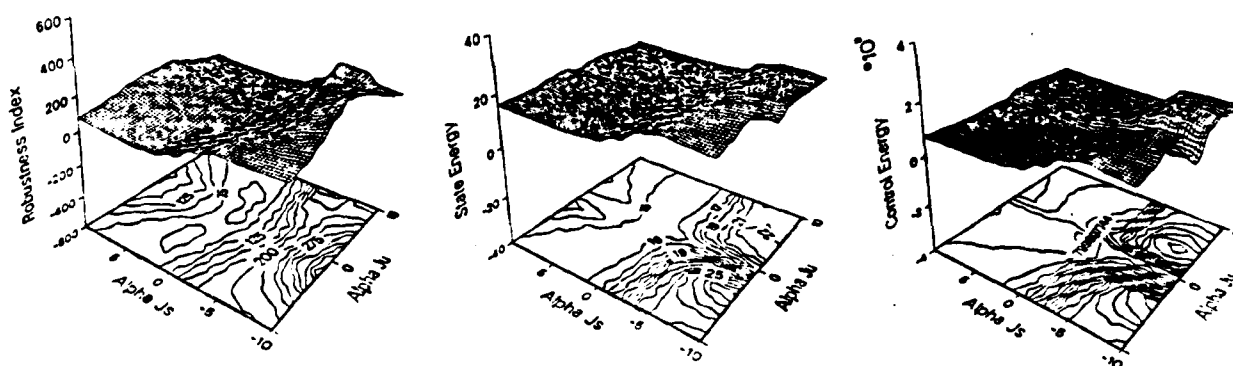
Step 3. Generation of the Performance Evaluation Surfaces

A family of judicious gain variations is generated by sweeping (α_s, α_u) , since (∇_s, ∇_u) are the directions of steepest variation of the two secondary indices: The surfaces $J_e(\alpha_s, \alpha_u)$, $J_s(\alpha_s, \alpha_u)$, $J_u(\alpha_s, \alpha_u)$, and implicitly, $J_e(J_s, J_u)$ provide perspective on the optimal design and its neighbors.

Three typical performance variations are shown in the figures below. In this case, the closed loop eigenvalue positions (for a 24 th order system having 6 actuators) were held fixed; so these surfaces indicate design freedom *beyond pole placement*. It is evident that even with fixed pole locations, significant variations in all three indices result from this family of judicious gain variations; it is also interesting to note the gain region (lower left quadrant) which is relatively flat and near-optimal for all three indices.

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Robustness Index Expected State Error Energy Expected Control Energy
 Robustness, State Energy, & Control Energy Variations: Beyond Pole Placement

**Notes on Presentations and Discussions in the
Control Session**

**Contributed by
John L. Junkins**

Alan Jenkins - Integrated Control

Formulations and algorithms for simultaneous optimization of structures and controllers were developed. Continuum and equivalent continuous modeling "LQG" and "Q" control parameterization. Believes "Q" is superior.

Q. Wouldn't linking finite element parameters yield as much reduction in the number of model parameters as the equivalent continuum approach?

A. Not sure (several members of the audience were confident that the answer is "yes".)

Q. Shouldn't the order of the compensator be established early, or, rather than carrying through very high order designs before considering order reduction at the end?

A. There is disagreement on this answer.

Q. Do you really believe it is feasible to optimize the structure and control system simultaneously? How do you deal with different objectives in the two designs?

A. Using multiple objective optimization techniques.

Bob Kosut - Adaptive Control

Q. How many parameters can you really adapt?

A. At most, a half a dozen.

Ed Crawley - Why use Control Experiments to learn "what we don't know"

Recommends performers of experiments carefully document three things:

1. What we expected
2. What we got
3. What went wrong

**Notes on Presentations and Discussions in the
Control Session**

Contributed by
Gary L. Slater

A.B. JENKIN (ISAAC)

- Q: In control performance index, shouldn't one include the mass of the power supply, etc. in the overall optimization?
- A: Yes, it must be included, although it is not shown here and is difficult to do.
- Q: Analysis uses controller of fairly high order. There is a problem here as there are no flight qualified hardware of this type.
- A: Mil Spec FIR filters are 'almost' available.
- Q: What are you doing for structural design? I do not understand.
- A: Put areas, masses into the performance index, e.g. in a truss type structure or use areas.
- Q: How do you do the Q-factorization? Previous results show some difficulty.
- Q: Have you thought about local minima? How would you handle this?
- A: Typically start at different locations and see if convergence is to the same solution. Do not know how high a problem this is.

Summary of a fairly long discussion:

- Q: What is the correct perf-index for the control-structure interaction?
- A: That's a difficult question.

D. MILLER: Waves

- Q: You're talking about sinusoidal waves - there is no advantage over modal analysis. Wave approach is good for 'discontinuous' waves where you need an infinite number of modes.
- A: The advantage is that you only need local properties here. Do not need modes at all.
- Q: What kind of sensors are required?
- Q: Implication that modal analysis does not work is incorrect!

G. HYLAND: Theory and Practice

Q: What about nonlinearities in joints?

A: Joint nonlinearities still there. Stiffness $\approx$ 10% lower at low amplitudes. Testing shows 'modes' do still exist however.

Q: How many sensors/actuators on plate?

A: 4 Actuators, 16 Sensors. Data not yet published - will be soon.

Q: What kind of actuators?

A: Coil suspended. Rare earth magnet mounted on ____ (?)

Q: There are similar experiments done on circular plate? Is square plate easier?

A: No number of modes is roughly similar. Key is the cuts (Note: Plate has diagonal cuts at corners). Help prevent buckling effects due to slight curvature in plate.

Q: Shown that by changing sensors, you change the zeros and therefore the effect of the actuators - Did you look at this?

A: We looked at sensor location but cannot comment on how we got final location.

D. YOUNG - Controlled Component Synthesis Method

Comment (#1) Good to see control engineer adapt algorithm to structure - rather than other way around. Need to look at time response more carefully. Time response shown may have "beats".

(#2) This is addressing one problem - looking at behavior of local structure - Short length scale. Need something done to look at longer length scales.

(#3) This is a 'local' design technique. There is a trade off between centralized design and losing information about global structure.

J. JUNKINS

Q: Can you draw conclusions when # actuators = # sensors.

A: In that case you get IMSC.

Q: Back to trade-off between robustness and performance - What about performance?

A: Fixed 'A' gives approximately same performance (factor of 3 or 4) but do give great difference in robustness.

ÖZGÜNER - Decentralized/Relegated Control

Q: Please comment on mirrors on beam; "servo-compensated"?

A: Small motor on each mirror; Just use disturbance model in feedback compensation.

Q: What sensors pick up vibration?

A: Use a tip accelerometer.

Summary of Presentations in the Control Session

Contributed by
K. David Young

1. Professor Umit Ozguner, Ohio State University, outlined the Advanced Beam Control Experiments to be carried out at AFWAL. The first of this series of experiments will involve a cylindrical beam suspended vertically with a disk attached to the free end. Four proof mass actuators are mounted on the disk to control excite the beam/disk structure. Three well-known control design methodologies will be selected for the validation of control system design performance against experimental data on a controlled flexible structure. Umit presented a framework for control authority relegation - research supported by SDIO LLNL. A Relegator is a key element in his relegation scheme - it serves to optimize the set points for the subsystems in the large scale system. The developed control authority relegation scheme has been applied to the slewing and vibration control of a flexible structure with multiple mirrors attached, in particular, to a slewing mirror structure experiment at OSU. Modeling aspects, such as the optical ray tracing, the integration of actuator dynamics with structural dynamics, as well as multivariable servomechanism design results were presented.
2. Professor Juri Medanic, University of Illinois, presented an overview of the current status of decentralized control of large scale systems. In particular, he summarized a projective control approach for reduced order controller design, and some results on a Linear Quadratic frequency-shaped optimal regulator approach, both of which were supported by SDIO LLNL. These two approaches, when integrated avoid exciting unmodelled dynamics in control system design for flexible structures. Juri highlighted a new control design framework for disturbance rejection that the Illinois group is currently developing. This framework utilizes a Frobenius-Hankel norm to produce a low order controller which allows the disturbance rejection problem to be solved using a projective control approach.

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