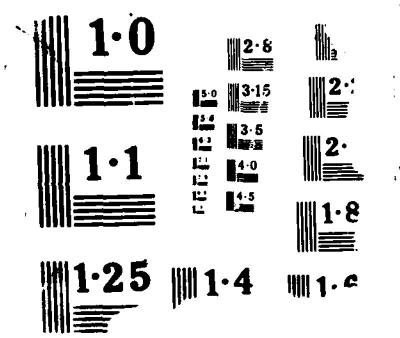
AD-A194 912   SENSOR AND ACTUATOR SELECTION FOR LARGE SPACE STRUCTURE CONTROL(U) AIR COMMAND AND STAFF COLL MAXWELL AFB AL M L DELORENZO APR 88 ACSC-88-0725   1/1

UNCLASSIFIED   F/G 22/2   NL


1·0
2·8
3·15
3·5
4·0
4·5
2·
1·1
1·8
1·25
1·4

AIR COMMAND AND STAFF COLLEGE

STUDENT REPORT

SENSOR AND ACTUATOR SELECTION FOR LARGE SPACE STRUCTURE CONTROL

MAJOR MICHAEL L. DELORENZO 88-0725

"insights into tomorrow"

DISCLAIMER

The views and conclusions expressed in this document are those of the author. They are not intended and should not be thought to represent official ideas, attitudes, or policies of any agency of the United States Government. The author has not had special access to official information or ideas and has employed only open-source material available to any writer on this subject.

This document is the property of the United States Government. It is available for distribution to the general public. A loan copy of the document may be obtained from the Air University Interlibrary Loan Service (AUL/LDEX, Maxwell AFB, Alabama, 36112-5564) or the Defense Technical Information Center. Request must include the author's name and complete title of the study.

This document may be reproduced for use in other research reports or educational pursuits contingent upon the following stipulations:

- Reproduction rights do not extend to any copyrighted material that may be contained in the research report.

- All reproduced copies must contain the following credit line: "Reprinted by permission of the Air Command and Staff College."

- All reproduced copies must contain the name(s) of the report's author(s).

- If format modification is necessary to better serve the user's needs, adjustments may be made to this report--this authorization does not extend to copyrighted information or material. The following statement must accompany the modified document: "Adapted from Air Command and Staff College Research Report ____(number)____ entitled ____(title)____ by ____(author)."

- This notice must be included with any reproduced or adapted portions of this document.

REPORT NUMBER 88-0725

TITLE SENSOR AND ACTUATOR SELECTION FOR LARGE SPACE STRUCTURE CONTROL

AUTHOR(S) MAJOR MICHAEL L. DELORENZO

FACULTY ADVISOR LT COL LARRY ROSELAND, ACSC/EDW

SPONSOR MAJOR DAVID WAGIE, HQ USAF ACADEMY/DFAS

Submitted to the faculty in partial fulfillment of
requirements for graduation.

AIR COMMAND AND STAFF COLLEGE
AIR UNIVERSITY
MAXWELL AFB, AL 36112-5542

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT STATEMENT "A" Approved for public release; Distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) 88-0725	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION ACSC/EDC 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Maxwell AFB AL 36112-5542	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
SENSOR AND ACTUATOR SELECTION FOR LARGE SPACE STRUCTURE CONTROL (U)

12. PERSONAL AUTHOR(S)
DeLorenzo, Michael L., Major, USAF

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
	FROM ____ TO ____	1988 April	39

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This paper presents an algorithm which aids the controls engineer in specifying a sensor and actuator configuration for regulation of large scale, linear, stochastic systems such as a large space structure (LSS). The algorithm uses a linear quadratic gaussian (LQG) controller, an efficient weight selection technique based upon successive approximation, and a measure of sensor and actuator effectiveness to provide a final sensor and actuator configuration. This configuration enables the closed-loop system to meet output specifications with minimal input power. The algorithm involves no complex gradient calculations and has proven numerically tractable for large linear models. Additionally, the algorithm provides the controls engineer information on the important design issues of actuator sizing, reliability, redundancy, and optimal number.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION	
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
ACSC/EDC Maxwell AFB AL 36112-5542	(205) 293-2867	

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

PREFACE

The potential benefits of a large space structure (LSS) for communications, power generation, surveillance, astronomy, and space defense are unlimited. Consequently, the problem of controlling an LSS while it orbits the earth is receiving increased attention in both the military and civilian communities. The problem is greatly complicated by the extreme size and flexibility of the LSS and the difficulties in testing an LSS control system in the 1-g environment of the earth's surface. As a result, most of the LSS control work to date is theoretical and based upon computer simulation. This paper is no exception. It proposes an algorithm which aids the controls engineer in specifying a configuration for sensors (motion or rate detectors) and actuators (force or torque generators) to control an LSS modeled as a linear stochastic system (i.e., a computer model).

Subject to clearance, this manuscript will be submitted to the American Institute for Aeronautics and Astronautics' (AIAA) Journal of Guidance, Control, and Dynamics for consideration. As a result, the paper is written at a technical level assuming the reader has graduate exposure to structures, dynamics, and optimal stochastic control. Additionally, the paper is tailored to the AIAA's formatting requirements which differ from the ACSC requirements. These differences are most noticeable in line spacing, margins, table and figure format, heading format, reference citation, page number location, and paragraph indentation. Also, EDC has authorized substituting the AIAA required abstract for the Executive Summary.

The author gratefully acknowledges Major David Wagie and the United States Air Force Academy Department of Astronautics for their willingness to sponsor this work. The author also expresses hearty thanks to Lt Col Larry Roseland for his willingness to serve as the faculty advisor for this project and his helpful comments. Additionally, the typing and proofing assistance provided by Mrs. Cathy DeLorenzo were invaluable to the timely completion of the project, and the proofing time contributed by Majors David Cooley and Tom Angle and Mrs. Anne Schuckert went well beyond the call of duty.

ABOUT THE AUTHOR

Major Michael L. Delorenzo is a 1974 graduate of the United States Air Force Academy. From 1974-1978 he served as a gyroscope test engineer for the 6585th Test Group, Holloman AFB, New Mexico. While serving at Holloman, he received an MS in Electrical Engineering from New Mexico State University. From 1978-1980 Major DeLorenzo was an instructor at the United States Air Force Academy in the Department of Astronautics and Computer Science. He entered Purdue University in 1980 to pursue a PhD in Dynamics and Control. Upon receiving his degree in 1983, Major DeLorenzo returned to the Air Force Academy as Associate Professor and Research Director for the Department of Astronautics. Presently, Major DeLorenzo is attending Air Command and Staff College at Maxwell AFB, Montgomery, Alabama, with a follow-on assignment to Arnold Engineering Development Center, Arnold AFS, Tullahoma, Tennessee. Major DeLorenzo's primary research interest is sensor and actuator selection for the control of large scale systems. His published works in the area are documented in References 10-14 of this paper.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLES

FIGURES

ABSTRACT

This paper presents an algorithm which aids the controls engineer in specifying a sensor and actuator configuration for regulation of large scale, linear, stochastic systems such as a large space structure (LSS) model. The algorithm uses a linear quadratic gaussian (LQG) controller, an efficient weight selection technique based upon successive approximation, and a measure of sensor and actuator effectiveness to specify a final sensor and actuator configuration. This configuration enables the closed-loop system to meet output specifications with minimal input power. The algorithm involves no complex gradient calculations and is numerically tractable for large linear models as demonstrated by the solar optical telescope example in this paper. Additionally, the algorithm provides the controls engineer information on the important design issues of actuator sizing, reliability, redundancy, and optimal number.

1.0 INTRODUCTION

The advent of the Space Shuttle makes the large space structure (LSS) an imminent reality. These future space structures will be measured in kilometers and, of necessity, will be lightweight and highly flexible (light damping). Standard LSS missions will include power generation, surveillance, astronomy, and communications. These missions will require stringent pointing accuracy, shape control, and vibration suppression. To satisfy these demanding mission requirements, the LSS will almost certainly require an active, regulator-type controller with multiple sensors and actuators located throughout the structure.[1-4] Furthermore, given the size of an LSS, there will be a large set of admissible sensor and actuator locations. The controls engineer then faces the problem of selecting a limited number of sensor and actuator locations to "best achieve" the LSS mission. The term "best achieve" in this paper means achieving the LSS output specifications with minimal actuator power.

The solution to this sensor and actuator selection (SAS) problem needs at least the following ingredients:

(1) A specific closed-loop control law structure
(2) A technique for systemically adjusting (tuning) control law parameters to achieve output specifications with minimal power
(3) A technique to evaluate the effectiveness of possible sensor and actuator configurations in achieving output specifications with minimal power

This paper incorporates the above ingredients into an algorithm which solves the SAS problem for an LSS modeled as a linear stochastic system. Some necessary background information on linear stochastic systems and linear quadratic gaussian (LQG) control theory are presented in Section 2.0 along with a formal statement of the SAS problem. Section 3.0 contains two important selection theorems, four pertinent facts, and the general flow of the SAS algorithm. Results from the SAS algorithm applied to the controller design for a large space telescope are presented in Section 4.0, and Section 5.0 contains concluding remarks.

2.0 BACKGROUND

The SAS algorithm presented in this paper addresses controlling an LSS described by the following linear stochastic model:

$$
\begin{aligned}
&\dot{x}(t) = Ax(t) + Bu(t) + Dw(t), \quad x \in \underline{R}^{n}, \quad u \in \underline{R}^{m}, \quad w \in \underline{R}^{p} \\
&x(t_0) = x_0 \\
(2.1) \qquad &D \equiv [\ B \quad D_0\ ], \quad w(t) \equiv [\ w_m^T(t) \quad w_0^T(t)\ ]^T \\
&y(t) = Cx(t), \quad y \in \underline{R}^{k} \quad \text{(system outputs)} \\
&z(t) = Mx(t) + v(t), \quad z \in \underline{R}^{l} \quad \text{(system measurements)}
\end{aligned}
$$

with noise characteristics

$$
(2.2) \qquad E \begin{bmatrix} x_0 \\ w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} x_0^T, & w^T(\tau), & v^T(\tau) \end{bmatrix} = \begin{bmatrix} X_0 & 0 & 0 \\ 0 & W\delta(t-\tau) & 0 \\ 0 & 0 & V\delta(t-\tau) \end{bmatrix}
$$

$$
E\, x_0 \equiv 0, \quad E\, w(t) \equiv 0, \quad E\, v(t) \equiv 0, \quad W>0, \quad V>0
$$

The notation $\underline{R}^i$ implies a real vector space of dimension i. The expectation operator is denoted by E. The superscript T represents matrix transposition. The Dirac delta function is denoted by δ, and W>0 implies W is a positive definite matrix. The n-dimensional vector x(t) represents the state of the system, while the m-dimensional vector u(t) contains actuator signals. The regulated system outputs are defined by the k-dimensional vector y(t), and the l-dimensional vector z(t) represents the system measurements (sensor information). The white noise vector process w(t) represents unmodeled system behavior $D_o w_o(t)$ and unmodeled actuator behavior $B w_a(t)$. Unmodeled sensor behavior is accounted for by the white noise vector process v(t). The matrices A, B, C, D, M, W, and V are time-invariant and appropriately dimensioned. The matrix B has no zero columns, and the matrices C and M have no zero rows. Furthermore, the matrices A, B, C, D, and M satisfy the following stabilizability and detectability conditions:

(2.3) (A,B), (A,D) stabilizable; (A,C), (A,M) detectable.

For simplicity, the model described by (2.1)-(2.3) will be identified by S(n,k,m,l). The arguments n, k, m, and l represent the number of system states, outputs, actuators, and sensors respectively.

S(n,k,m,l) specifications usually take the form

(2.4) $$\lim_{t\to\infty} Ey_i^2(t) = E y_{\infty i}^2 \le \sigma_i^2, \quad i = 1,2,\ldots k$$

(2.5) $$\lim_{t\to\infty} Eu_i^2(t) = E u_{\infty i}^2 \le \mu_i^2, \quad i = 1,2,\ldots m$$

where y_i represents the ith output for the system, and u_i represents the ith input. The quantities σ_i^2, μ_i^2 are mean square or variance constraints on the ith output and input. These specifications are ideally suited for the LSS case where y_i could represent a line of sight pointing error, a deflection from a desired shape, etc., and u_i could be the output from a thruster, torque actuator, etc.

The ingredients of the SAS solution stated in the introduction require the choice of a specific closed-loop control law which is readily adjusted to meet the output specifications with minimal power. LQG control theory provides such a law. This theory originated in the 1960s with the foundational work of Kalman, Bucy, and others. Since that time, many papers and texts have been written further clarifying and expanding the theory. Refs. 7-9 are a few examples. The following paragraphs present the fundamental results of the theory.

For a system S(n,k,m,l), LQG theory guarantees a stable, steady-state, closed-loop control system minimizing

(2.6a) $\underline{V} = E_{\infty}(y^T Q\, y + u^T R\, u)$ or

(2.6b) $\underline{V} = \sum_{i=1}^{k} E_{\infty} y_i^2 q_i + \sum_{i=1}^{m} E_{\infty} u_i^2 r_i$

where q_i and r_i are the ith diagonal entries of the arbitrary, diagonal, and positive definite weighting matrices Q and R. The defining equations for this steady-state controller are

(2.7a) $u(t) = -R^{-1} B^T K\, \hat{x}(t) = G\, \hat{x}(t)$

$$\text{(2.7b)}\quad \begin{aligned} &\dot{\hat{x}}(t) = A_c\,\hat{x}(t) + F\,z(t), \quad F = P\,M^T V^{-1} \\ &A_c = A + BG - FM, \quad \hat{x}(t_0) = x_0 \end{aligned}$$

$$\text{(2.7c)}\quad KA + A^T K - KBR^{-1}B^T K + C^T QC = 0$$

(Steady state Control Riccati Equation)

$$\text{(2.7d)}\quad PA^T + AP - PM^T V^{-1} MP + DWD^T = 0$$

(Steady state Filter Riccati Equation)

Substituting this controller into S(n,k,m,l), the following closed-loop system S(2n, k+m, m+l, 0) results:

$$\text{(2.8a)}\quad \begin{aligned} &\dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{B}\,\bar{w}, \quad \bar{x} = (x^T\ \hat{x}^T)^T, \quad \bar{w} = (w^T\ v^T)^T \\ &\bar{y} = \bar{C}\,\bar{x}, \quad \bar{V} = \bar{y}^T \bar{Q}\,\bar{y}, \quad \bar{y} = (y^T\ u^T)^T \end{aligned}$$

$$\text{(2.8b)}\quad \bar{A} = \begin{bmatrix} A & BG \\ FM & A_c \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} D & 0 \\ 0 & F \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \\ 0 & G \end{bmatrix},$$

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}$$

Given the noise characteristics of S(n,k,m,l), the steady-state variance matrix for S(2n, k+m, m+l, 0) is known to be

$$\text{(2.9)}\quad E_\infty\{(\bar{x} - 0)(\bar{x} - 0)^T\} = E_\infty\{\bar{x}\,\bar{x}^T\} = \begin{bmatrix} \hat{X}+P & \hat{X} \\ \hat{X} & \hat{X} \end{bmatrix}$$

$$\text{(2.10)}\quad \hat{X} = \lim_{t \to \infty} \hat{X}(t) = E_\infty\{(\hat{x} - 0)(\hat{x} - 0)^T\} = E_\infty\{\hat{x}\,\hat{x}^T\}$$

where $\hat{X}$ is the solution of the steady-state Lyapunov equation

$$\text{(2.11)}\quad \hat{X}(A + BG)^T + (A + BG)\hat{X} + PM^T V^{-1} MP = 0$$

Also, coupling the definition of $\bar{x}$ in (2.8a) with (2.9) gives

(2.12) $X = \lim_{t \to \infty} X(t) = E_\infty\{ x x^T \} = \hat{X} + P$

The following facts, documented in Refs. 10-14, show the LQG control structure is extremely powerful in attacking the SAS problem:

F1. Analytical expressions for $E_\infty y_i^2$ and $E_\infty u_i^2$ exist and require no additional major calculations beyond those of the controller:

(2.13) $E_\infty y_i^2 = \left[C(P+\hat{X})C^T \right]_{ii}$, i=1,...k, and $_{ii}$ implies the ith diagonal element of the matrix

(2.14) $E_\infty u_i^2 = \left[G\hat{X}G^T \right]_{ii}$, i=1,...m

F2. A successive approximation algorithm (LQGWTS) developed in Refs. 10, 13, and 14 can be used to adjust the elements of Q and R so the LQG controller achieves the output specifications $E_\infty y_i^2 = \sigma_i^2$, i = 1,...k with minimal power (i.e., $\sum_{i=1}^{m} E_\infty u_i^2 = \min$). LQGWTS involves no gradient calculations or complex calculation beyond the LQG controller and is numerically tractable for large systems as shown in Refs. 10, 13, and 14.

F3. A measure developed in Refs. 10 and 11 identifies the effectiveness of each sensor and actuator in the present LQG controller configuration. The mathematical expressions for these effectiveness values are

(2.15) $\underline{V}_i^{sen} = m_i^T PLPm_i V_{ii}^{-1}$ (where m_i is the ith column of M^T)

(2.16) $\underline{V}_i^{act} = \left[G\hat{X}G^T R \right]_{ii} - \left[B^T (K+L) B W^{\infty} \right]_{ii}$

where W_∞ is the partition of W corresponding to actuator noise sources ($w_\infty(t)$), and L satisfies the following:

$$L(A-PM^T V^{-1} M) + (A-PM^T V^{-1} M)^T L + G^T RG = 0$$

F2 and F3 represent recent contributions to LQG theory. Their derivations are in Ref. 10 but are omitted here for the sake of brevity. However, Appendix A presents a brief summary of the weight selection work, and Appendix B provides a short discussion of the effectiveness values.

With the above background, the SAS problem addressed in this paper can be mathematically stated:

SAS Problem Statement

<u>Given</u>: A system (S,n,$\underline{k}$,m,l) which has only $\bar{m}$ out of m actuators and l out of l sensors available to design a steady state LQG regulator which must achieve a set of output variances (σ^2).

<u>Required</u>: Specify the closed-loop system which satisfies the following:

(2.17) $$\text{Min} \sum_{i=1}^{\bar{m}} E\, u_i^2 \text{ subject to } E\, y_i^2 = \sigma_i^2 \;\forall\; i=1,\ldots k$$

3.0 SAS ALGORITHM

Before presenting the algorithm, two selection theorems and some pertinent facts should be stated.

Theorem 1 Deletion of Noisy Actuators

For a system S(n,k,m,l) regulated by the LQG controller defined in (2.7), the deletion of an actuator is not sufficient for $\underline{V}(m-1) > \underline{V}(m)$ where

$\underline{V}(m)$ = the value of (2.6) for a system S(n,k,m,l) under LQG regulation

$\underline{V}(m-1)$ = the value of (2.6) for a system S(n,k,m-1,l) under LQG regulation

The proof of Theorem 1 is given in Ref. 10. More specifically, Theorem 1 says it is possible for an LQG controller to do better (i.e., maintain output specifications at a lower input power) when a noisy actuator is deleted. This means the desired number of actuators in a control system might be less than the allowable (i.e., $< \bar{m}$).

Theorem 2 Deletion of Noisy Sensors

> For a system S(n,k,m,l) under the regulation of the LQG controller defined in (2.7), deletion of a sensor cannot reduce the $\underline{V}$ of (2.6).

Theorem 2 clearly says that noisy sensors never degrade the performance of an LQG controller. The proof of this theorem is contained in Refs. 10, 12, and 15. However, intuitively, it makes sense because the LQG controller uses a Kalman filter, and if a sensor is too noisy the Kalman filter essentially ignores the information from that sensor. Therefore, Theorem 2 indicates the number of sensors in the controller design should be $\bar{l}$ (i.e., the allowable number).

The following facts are important to the algorithm:

F4. Data in Refs. 10 and 16 show relative $\underline{V}_i^{act}$ and $\underline{V}_i^{sen}$ rankings can change as functions of Q and R.

F5. Deleting sensors and actuators changes the Q and R needed to achieve output specifications at minimal power.

F6. The results from Refs. 10, 11, 15, and 17 show the SAS problem is highly coupled and simultaneous SAS works better than sequential selection.

F7. $\underline{V}_i^{sen}$ and $\underline{V}_i^{act}$ do not predict the loss of measurability or controllability.

With these theorems and facts stated, the general flow of the SAS algorithm can be outlined.

SAS Algorithm (General Flow)

1. Given the system S(n,k,m,l) specify the number of sensors l (l<l) and actuators m (m<m) that can be used in the controller design.

2. Run the subroutine LQGWTS for the full system S(n,k,m,l). (LQGWTS selects the weighting matrices Q and R so the LQG controller enables the closed-loop system to satisfy (2.17).)

3. With the resulting closed-loop system, calculate $\underline{V}^{sen}$ and $\underline{V}^{act}$ for each sensor and actuator using (2.15) and (2.16).

4. Rank the actuators and sensors according to the algebraic value of $\underline{V}^{sen}$ and $\underline{V}^{act}$.

5. Modify the current system to eliminate the lowest ranking sensor and actuator and all others of "nearly" the same ranking, if measurability and/or controllability of the system are not disturbed.

6. If the appropriate number of actuators and sensors are deleted (i.e., m-m and l-l) stop. However, in light of Theorem 1, fewer actuators than m may be better, and actuator deletion should continue until

$$(1/m\cdot) \sum_{i=1}^{m\cdot} E\, u_i^2 \qquad (3.1)$$

is no longer decreasing. (Note, m· is the number of actuators currently in the design (m· < m), and Equation 3.1 is the average power per actuator.)

7. Run LQGWTS on the reduced system and return to 3.

F4 and F5 dictate the order of Steps 2, 3, and 4 in the above general flow. More specifically, since the ranking of effectiveness values can vary as a function of Q and R, the effectiveness values should be based on the Q and R that produce a closed-loop system satisfying (2.17).

Step 5 of the general flow results from facts F6 and F7. Caution should be exercised in deleting sensors and actuators with "nearly" the same effectiveness values since F4 and F5 indicate any perturbation in the sensor and actuator configuration can alter relative effectiveness rankings of the sensors and actuators. Also, the requirement for checking measurability and controllability is severe. However, as a result of the work by Skelton and Hughes in Ref. 18, the controllability and measurability of systems in modal form (i.e., the form of the telescope model in Appendix C) can be done by inspection for systems of any dimension.

Theorems 1 and 2 provide the rationale for Step 6. If $\bar{l}$ sensors are allowed, then according to Theorem 2, $\bar{l}$ sensors should be used. Theorem 1 invalidates this philosophy for actuators.

4.0 SPACE TELESCOPE EXAMPLE

This section presents the results of the SAS algorithm when applied to the model of a solar optical telescope. Before discussing the results, a brief model description is presented.

The solar optical telescope model was developed in 1980 by the Charles Stark Draper Laboratory (CSDL) primarily for providing a minimum complexity structure to evaluate LSS control design techniques. The outputs for the model are the telescope line of sight angles about the x and y axes (see Figure 4.0) and the focal length (defocus) of the lenses located at the top and bottom of the telescope.

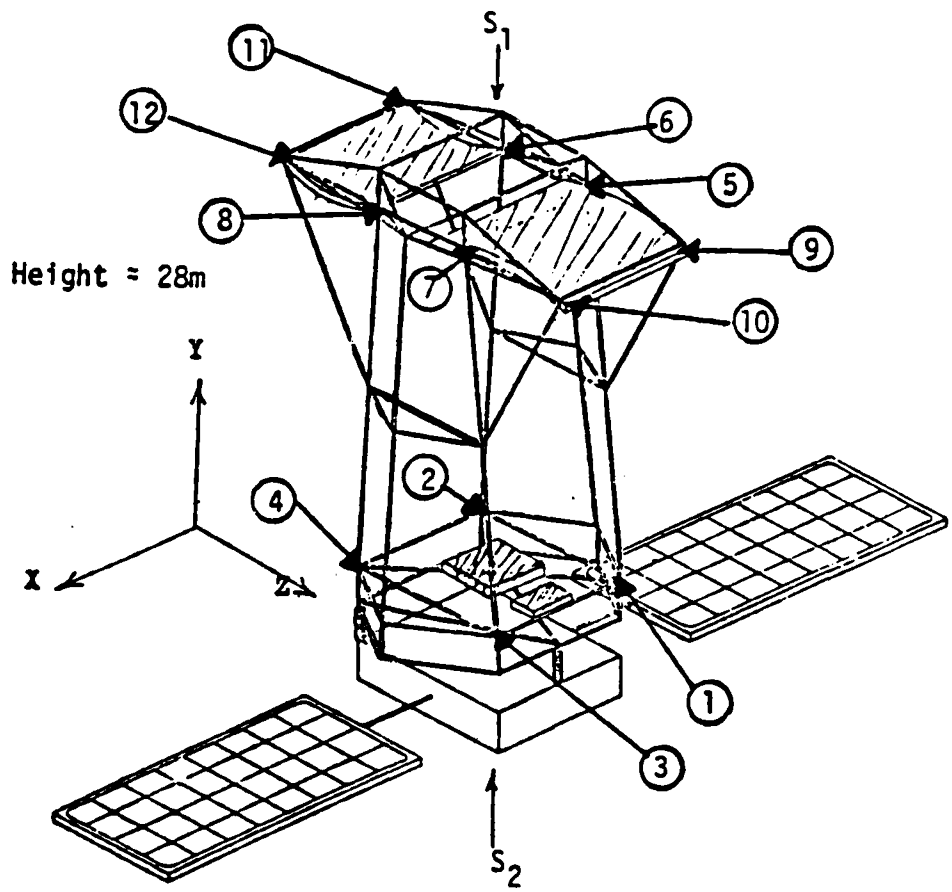


Figure 4.0 Solar optical telescope

The 12 nodes shown in Figure 4.0 represent the spatial locations for 45 admissible sensors and 21 admissible actuators. Tables 4.0 - 4.2 provide specific type, location, direction, and label information for the admissible sensors and actuators and also a listing of the output specifications (σ).

Table 4.0 Telescope sensor description

Sensor #	Type	Nodal Location	Direction	Label
1	Line of Sight Angle	---	X	LOS_x
2	"	---	Y	LOS_y
3	Defocus	---	---	Defocus
4	Linear Displacement	1	Y	Y1
5	"	1	Z	Z1
6	"	2	Z	Z2
7	"	3	X	X3
8	"	3	Y	Y3
9	"	3	Z	Z3
10	"	4	Z	Z4
11	"	5	X	X5
12	"	5	Y	Y5
13	"	5	Z	Z5
14	"	6	Z	Z6
15	"	7	Y	Y7
16	"	7	Z	Z7
17	"	8	Z	Z8
18	"	9	Z	Z9
19	"	10	Z	Z10
20	"	11	X	X11
21	"	11	Y	Y11
22	"	11	Z	Z11
23	"	12	Y	Y12
24	"	12	Z	Z12
25	Linear Rate	1	Y	LRY1
26	"	1	Z	LRZ1
27	"	2	Z	LRZ2
28	"	3	X	LRX3
29	"	3	Y	LRY3
30	"	3	Z	LRZ3
31	"	4	Z	LRZ4
32	"	5	X	LRX5
33	"	5	Y	LRY5
34	"	5	Z	LRZ5
35	"	6	Z	LRZ6
36	"	7	Y	LRY7
37	"	7	Z	LRZ7
38	"	8	Z	LRZ8
39	"	9	Z	LRZ9
40	"	10	Z	LRZ10
41	"	11	X	LRX11
42	"	11	Y	LRY11
43	"	11	Z	LRZ11
44	"	12	Y	LRY12
45	"	12	Z	LRZ12

Table 4.1 Telescope actuator description

Actuator (Force)#	Nodal Location	Direction	Label
1	1	Y	FY1
2	1	Z	FZ1
3	2	Z	FZ2
4	3	X	FX3
5	3	Y	FY3
6	3	Z	FZ3
7	4	Z	FZ4
8	5	X	FX5
9	5	Y	FY5
10	5	Z	FZ5
11	6	Z	FZ6
12	7	Y	FY7
13	7	Z	FZ7
14	8	Z	FZ8
15	9	Z	FZ9
16	10	Z	FZ10
17	11	X	FX11
18	11	Y	FY11
19	11	Z	FZ11
20	12	Y	FY12
21	12	Z	FZ12

Table 4.2 Telescope specifications

Output	Title	Label	Value
σ_1	Optical line of sight	LOS_x	65.2 sec
σ_2	"	LOS_y	"
σ_3	Defocus	Defocus	0.001 mm

CSDL used the program NASTRAN to develop data for the first 44 mode shapes of the telescope and provide location information for the two sinusoidal disturbances S_1 and S_2 shown in Figure 4.0. The final design model chosen for this example was a 10 mode, 20 state, linear stochastic model coupled with a 2 mode, 4 state linear stochastic model of the disturbances (i.e., a model $S_{t-1-}(24,3,21,45)$). The development of this model from the NASTRAN data is presented in Ref. 10. The model data is in Appendix C.

The SAS problem for the telescope is

Telescope SAS Problem

> <u>Given</u>: $S_{t=1} = (24,3,21,45)$ with only 12 actuators and 12 sensors available for designing an LQG regulator to achieve the (σ) of Table 4.2.
>
> <u>Required</u>: Specify the closed-loop system which satisfies (2.17).

Table 4.3 shows the iterations of the SAS algorithm when applied to the space telescope. Three observations are worth noting. First, in Iteration #1 of the algorithm, the measurements of the outputs were deleted. This might appear disturbing, but two things should be remembered: the algorithm guarantees the system remains measureable and the goal of the Kalman-Bucy filter is to estimate the state of the system and not necessarily the output. In the telescope example, the noise present on the output measurements made them unattractive to the filter. Second, despite sensor deletion, the average input value decreased from Iterations 1-3. This is dramatic empirical proof of Theorem 1 and indicates the controller is more efficient at holding the system outputs to specification with 19 sensors than it is with 20 or 21. Furthermore, the rise of average actuator power at Iteration 4 suggests the optimal number of actuators for this design lies between 19 and 15. Third, the algorithm converged to a configuration in only 10 iterations. A direct search algorithm could have taken thousands of iterations even with the aid of gradient calculations.

Table 4.3 Telescope SAS algorithm results

Iteration Number	Identified Sensor (V_i^{sen})	Identified Actuator (V_i^{act})	Ave. Input Value	Num sen/act
1	Y3(.008197) Y1(.008195) LOSY(.000015) LOSX(.000002) DEFOCUS(0)	FX11(-7.980)	7.483	45/21
2	Z5(.02955) Z7(.02955)	FX5(-8.796)	7.393	40/20
3	Y12(.03848) Y7(.03848) Y11(.03848) Y5(.03484) Z2(.03835) Z4(.03835) Z1(.03570) Z3(.03570) Z6(.03460) Z8(.08460)	FY7(-4.003) FY12(-4.003) FY11(-4.003) FY5(-4.003)	7.221	38/19
4	Z10(.08602) Z9(.08602) X3(.08277)	FZ11(-4.8877) FZ12(-4.8877)	7.789	28/15
5	LRY3(.10815) LRY1(.10813)	FZ9(-5.414)	8.155	25/13
6	LRZ7(.12273) LRZ5(.11553)	---	8.339	23/12
7	LRZ1(.16805) Z11(.16129) LRZ6(.13838) LRZ8(.13615)	---	8.359	21/12
8	LRX3(.2237) LRZ2(.2191) LRZ4(.2077) LRZ3(.1960)	---	8.416	17/12
9	LRZ9(.47256)	---	8.492	13/12
10	---	---	8.535	12/12

Figure 4.1 shows the sensor and actuator configuration chosen by the SAS algorithm for the telescope example.

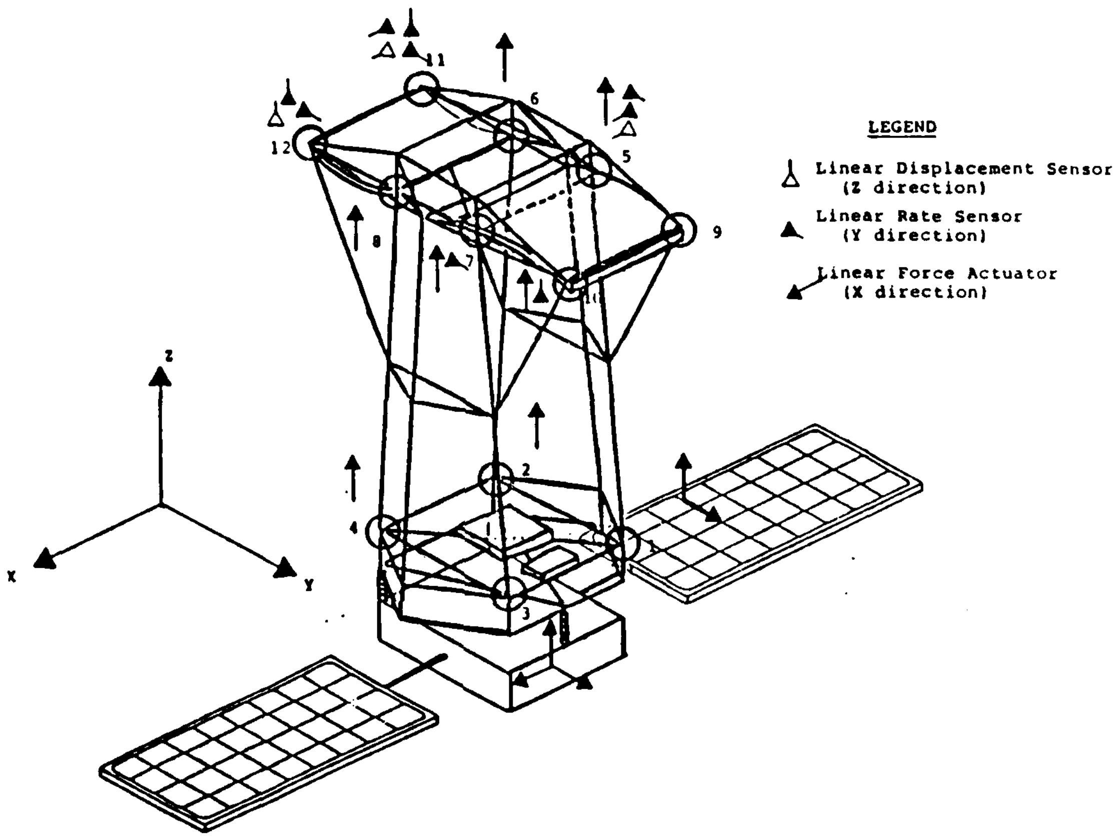


Figure 4.1 Telescope sensor and actuator configuration

Some striking features of this data are no force actuators in the x and y direction at the top of the telescope (i.e., nodes 5-12) and no sensors at the base of the telescope (i.e., nodes 1-4). It appears the algorithm is making use of the telescope's rigidity in the z direction (i.e., LOS direction) and the flexibility (i.e., motion sensitivity) of the telescope's upper panel in designing an efficient regulator. Several other 12 sensor and 12 actuator configurations have been tested and none performed as well as this configuration.[10]

Table 4.4 shows the output specifications achieved by this regulator and the root-mean-square (rms) power of each actuator.

Table 4.4 Telescope performance

Output #	$\sqrt{\overline{E_\infty y_i^2}}$	Actuator #	$\sqrt{\overline{E_\infty u_i^2}}$ (minimum achievable)
1(LOSx)	65.2 sec	1(FY1)	.0244N
2(LOSy)	"	2(FZ1)	.0303N
3(DEFOCUS)	.0002mm	3(FZ2)	.0297N
		4(FX3)	.0324N
		5(FY3)	.0251N
		6(FZ3)	.0265N
		7(FZ4)	.0338N
		8(FZ5)	.0245N
		9(FZ6)	.0242N
		10(FZ7)	.0221N
		11(FZ8)	.0273N
		12(FZ10)	.0437N

The rms power, expressed in Newtons (N), gives the controls engineer actuator sizing information for the controller design. It also provides a measure of the importance of each actuator in the controller. This information is useful in reliability and redundancy considerations for the actuator configuration.

A final note is the value of the defocus output. The value (.0002mm) is well below the σ of .001mm. This seems to indicate the controller is working too hard and less actuator power would be required if the defocus operated at .001mm. However, for this model, the defocus output is linearly dependent upon LOS_x and LOS_y (i.e., the defocus output could have been deleted from the model). The algorithm LQGWTS looks for output dependency and assigns dependant outputs a zero weighting to delete them from the model.

5.0 CONCLUSION

In summary, the SAS algorithm aids the controls engineer in specifying a sensor and actuator configuration for regulation of large scale, linear, stochastic systems. The algorithm uses an LQG controller, an efficient weight selection technique based upon successive approximation, and a measure of sensor and actuator effectiveness to specify a final sensor and actuator configuration. This configuration enables the closed-loop system to meet output specifications with minimal input power. Also, the algorithm involves no complex gradient calculations and has proven numerically tractable for large linear models. This tractability is demonstrated by the space telescope example presented in Section 4.0 and the hoop-column antenna example of Refs. 10 and 12. Finally, the algorithm provides the controls engineer information on the important design issues of actuator sizing, reliability, redundancy, and optimal number.

REFERENCES

1. Likins, P.W., Dynamics and Control of Flexible Space Vehicles, Jet Propulsion Laboratory, TR32-1329, Rev. 1, January 1970.

2. Balas, M.J., "Trend in Large Space Structure Control Theory: Fondest Hopes, Wildest Dreams," IEEE Transactions on Automatic Control, Vol. AC-27, Number 3, June 1982.

3. Kissel, G.J., and Lin, J.G., "Spillover Prevention via Proper Synthesis/Placement of Actuators and Sensors," American Control Conference, Arlington, VA, June 1982.

4. Wu, Y.W., Rice, R.B., and Juang, J.N., "Sensor and Actuator Placement for Large Flexible Space Structures," Proceedings of the Joint Automatic Controls Conference, Denver, CO, June 1979.

5. Johnson, T.L., "Principles of Sensor and Actuator Location in Distributed Systems," Proceedings of NCKU/AAS International Symposium on Engineering Sciences and Mechanics, Tainan, Taiwan, December 1981.

6. Gupta, N.A., and Hall, W.E., Jr., "Design and Evaluation of Sensor Systems for State and Parameter Estimation," Journal of Guidance and Control, Vol. 1, Number 6, November-December 1978.

7. Kwakernaak, N., and Sivan, R., Linear Optimal Control Systems, Wiley-Interscience, John Wiley and Sons, New York, 1972.

8. Bryson, A.E., Jr. and Ho, Y.C., Applied Optimal Control, Waltham, MA, Ginn and Company, 1969.

9. Athans, M., and Falb, P.L., Optimal Control, New York, Mcgraw-Hill, 1966.

10. DeLorenzo, M.L., "Selection of Noisy Sensors and Actuators for Regulation of Linear Systems," PhD thesis, School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN, May 1983.

11. Skelton, R.E. and DeLorenzo, M.L., "Selection of Noisy Actuators and Sensors in Linear Stochastic Systems," Journal on Large Scale Systems, Theory and Applications, Vol. 4, April 1983, pp. 109-136.

12. Skelton, R.E. and DeLorenzo, M.L., "Space Structure Control Design by Variance Assignment," Journal of Guidance, Control, and Dynamics, Vol. 8, Number 4, July-August 1985, pp. 454-462.

13. DeLorenzo, M.L. and Skelton, R.E., "Weight Selection for Variance Constrained LQG Regulators with Application to the Hoop-Column Antenna," Fourth VPI & SU/AIAA Symposium on Dynamics and Control of Large Structures, Blacksburg, VA, June 1983.

14. Skelton, R.E., and DeLorenzo, M.L., "On Selection of Weighting Matrices in the LQG Problem," 20th Annual Allerton Conference on Circuit and System Theory, Allerton, IL, October 1982.

15. Chiu, D., Optimal Sensor/Actuator Selection, Number, and Placement for Linear Stochastic Systems, Purdue University Technical Report, Contract No. 955369, West Lafayette, IN, May 1981.

16. Juang, J.N., and Rodriguez, G., "Formulations and Applications of Large Structure Actuator and Sensor Placements," Second VPI & SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, June 21-23 1979.

17. Ichikawa, A., and Ryan, E.P., "Sensor and Controller Location Problems for Distributed Parameter Systems," Automatica, Vol. 15, 1979, pp. 347-352.

18. Hughes, P.C., and Skelton, R.E., "Stability, Controllabilty and Observability of Linear Matrix-Second-Order Systems," Proceedings of the Joint Automatic Controls Conference, Denver, CO, June 1979, pp. 56-62.

19. Norris, G.A., "Selection of Non-Ideal Noisy Sensors and Actuators in the Control of Linear Systems," master's thesis, School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN, December 1987.

APPENDIX A LQG WEIGHT SELECTION

The successive approximation technique (LQGWTS) adjusts the diagonal Q and R matrices in the LQG controller so the closed-loop system satisfies variance constraints on inputs, outputs, or both. The technique evolved through Refs. 14, 13, and 10. This appendix presents a summary of the LQGWTS' theory applying to the minimum power SAS problem.

For the minimum power problem, each iteration of the SAS algorithm requires LQGWTS to design a closed-loop system meeting output variance specifications, while minimizing actuator power ($\sum_{i=1}^{m} E u_i^2$). LQGWTS does this by first setting R equal to the identity matrix. This allows the LQG controller to consider all actuators equally and permits the term $\sum_{i=1}^{m} E u_i^2$ to appear directly in the cost function $\underline{V}$ of (2.6), thus assuring its minimization. LQGWTS then adjusts Q using the following iterative equations:

$$q_i(1) = 1/\sigma_i^2, \quad i = 1,\ldots k$$

$$\text{(A.1)} \quad q_i(j+1) = [\, E y_i^2(j)/\sigma_i^2 \,]^{pwr(j)} \cdot q_i(j), \quad j = 1,2,\ldots$$

$$pwr(1) = 1, \quad pwr(j) = j-1 \;\forall\; j>1$$

where q_i is the ith diagonal entry of Q, and j is the jth iteration of LQGWTS.

The update expression in (A.1) always adjusts q_i in the right direction. The exponent pwr(j) is used to speed up convergence (i.e., increase step size). To prevent overshoot LQGWTS incorporates a descent function defined as

(A.2) $\text{descent}(j+1) = \max[(\underline{E}\, y_i^2(j+1)/\sigma_i^2 - 1),\ 0]$

If descent(j+1) > descent(j), LQGWTS returns to the jth iteration, resets the pwr function to 1 (i.e., the smallest step size), and continues.

As mentioned in Section 4.0, LQGWTS checks for dependent outputs and removes them from the model. The check LQGWTS applies is

(A.3) If $q_i < \epsilon/\sigma_i$ then $q_i = 0$ $(\epsilon << 1)$

where setting $q_i = 0$ removes the ith output from the model.

When used for the telescope SAS example, LQGWTS converged in 9 or less iterations for each iteration of the SAS algorithm. The major advantage of LQGWTS is its lack of cumbersome gradient calculations. Additionally, it has demonstrated "good" convergence properties on substantial problems in Refs. 10, 12, 13, and 14.

APPENDIX B ACTUATOR AND SENSOR EFFECTIVENESS VALUES

The effectiveness values in (2.15) and (2.16) are derived from closed-loop input cost analysis (CICA) and closed-loop output cost analysis (COCA). Both CICA and COCA are extensions of component cost analysis (CCA) developed by Skelton and co-workers in the late seventies and early eighties. CICA and COCA, along with the sensor and actuator effectiveness values, are discussed in detail in Refs. 10-12. This appendix briefly presents the important details applying to the telescope SAS problem.

The fundamental concept behind CICA and COCA is to identify the contribution each input and output of (2.8) is making to the cost functional $\underline{V}$ of (2.6). With these contributions identified, they are combined to determine the effectiveness of each input and output to the control effort.

Using the closed-loop model (2.8), the following input and output costs are defined:

(B.1) $\underline{V}_i^{u} = 1/2E_{\infty}\{(\partial\bar{y}^T\bar{Q}\,\bar{y}/\partial u_i)u_i\}$ (i.e., contribution ith control is making to $\underline{V}$)

(B.2) $\underline{V}_i^{w_a} = 1/2E_{\infty}\{(\partial\bar{y}^T\bar{Q}\,\bar{y}/\partial w_{a_i})w_{a_i}\}$ (i.e., contribution ith actuator noise is making to $\underline{V}$)

(B.3) $\underline{V}_i^{v} = 1/2E_{\infty}\{(\partial\bar{y}^T\bar{Q}\,\bar{y}/\partial v_i)v_i\}$ (i.e., contribution ith sensor noise is making to $\underline{V}$)

Applying these definitions to the closed-loop model (2.8) yields the following formulas:

(B.4) $\underline{V}_i^{u} = [G\hat{X}G^T R]_{ii}$, i = 1,....m (ii subscript means ith diagonal element of matrix)

(B.5) $\underline{V}_i^{\hat{w}} = [B^T (K+L) B\hat{W}]_{ii}$, i = 1,.....m

(B.6) $\underline{V}_i^{v} = [F^T LFV]_{ii}$, i = 1,....l

where the matrix L satisfies the following steady state Lyapunov equation:

(B.7) $L(A-FM) + (A-FM)^T L + G^T RG = 0$

Equations (B.4)-(B.6) and some useful properties are derived in Refs. 10 and 11. These equations are the basis for $\underline{V}_i^{act}$ and $\underline{V}_i^{sen}$ defined in (2.15) and (2.16).

As noted above, $\underline{V}_i^{u}$ represents the contribution that u_i is making to $\underline{V}$. Since the function of LQG theory is to use u_i to minimize $\underline{V}$ (for a given Q and R), a large $\underline{V}_i^{u}$ means u_i is important to the minimization effort. Alternatively, the contribution the ith actuator noise source makes to $\underline{V}$ ($\underline{V}_i^{\hat{w}}$) is clearly an undesirable contribution. These facts coupled with additional rationale discussed in Refs. 10-13 produce

(B.8) $\underline{V}_i^{act} = \underline{V}_i^{u} - \underline{V}_i^{\hat{w}}$

Substituting (B.4) and (B.5) into (B.8) gives (2.16). A modification to (B.8) accounting for constrained actuator power is presented in Ref. 10. A modification is also developed in Ref. 19 to account for noise correlations.

As previously noted, $\underline{V}_i^{v}$ is the contribution the ith sensor noise source makes to $\underline{V}$. Therefore, those sensors with larger values for $\check{\underline{V}}_i$ are contributing more noise to $\underline{V}$ than those with a smaller $\check{\underline{V}}_i$ and, at first glance, appear to be candidates for deletion. However, the sensor measurements for systems S(n,k,m,l) under LQG regulation are passed through a Kalman-Bucy filter. One of the purposes of this filter is to de-emphasize or throw out measurements which have more noise than estimation information. Therefore, any noise source making a "large" contribution to $\underline{V}$ comes from a sensor which is making an even larger contribution to the estimation information necessary to minimize $\underline{V}$. Consequently, the following effectiveness value is defined:

(B.9) $$\underline{V}_i^{een} = \check{\underline{V}}_i$$

Substituting (B.6) and the definition for F in (2.8) into (B.9) yields (2.15). Additionally, Ref. 19 presents a modification to (B.9) accounting for noise correlations.

APPENDIX C TELESCOPE MODEL DATA

This appendix presents the contents of the A, B, C, D, M, W, and V matrices for the telescope model S(24,3,21,45).

$$\text{(C.1)} \quad A = \begin{bmatrix} \Omega & S & O_{10} \\ O_4 & \Omega_d & O_{10} \\ O_4 & 0 & \Phi \end{bmatrix}, \quad B = \begin{bmatrix} O_8 \\ BE \\ O_6 \\ BR \end{bmatrix}, \quad D = \begin{bmatrix} O_8 & O_{18} \\ BE & I_2 \\ O_6 & O_2 \\ BR & O_2 \end{bmatrix},$$

$$\text{(C.2)} \quad C = [\; CE \quad 0^{12} \; CR \quad 0^2], \quad M = \begin{bmatrix} CE & 0^{12} & CR & 0^2 \\ ME & 0^{12} & MR & 0^2 \\ 0^8 & ME & 0^6 & MR \end{bmatrix},$$

$$\text{(C.3)} \quad W = \begin{bmatrix} W^e & O_{21} \\ O_2 & W^o \end{bmatrix}, \quad V = \begin{bmatrix} V_1 & O_2 & O_2 \\ O_{22} & V_2 & O_{22} \\ O_{21} & O_{21} & V_3 \end{bmatrix}$$

The terms in the above matrices are defined below:

$$\Omega = \begin{bmatrix} O_8 & I_8 \\ -w^2 & -2\sigma \end{bmatrix},$$

O_8 = null matrix with 8 rows
I_8 = 8x8 identity matrix
w^2 = 8x8 diagonal matrix of squared modal frequencies
2σ = 8x8 diagonal modal damping matrix

w^2 = diag[.835, 2.736, 3.971, 4.378, 7.746, 13.175, 13.339, 59.112] (rad^2/sec^2)

2σ = diag[.00183, .00331, .00399, .00418, .00557, .00726, .00730, .0154] (rad/sec)

S = 8x2 coupling matrix (defined in table C.0)

$$\Omega_d = \begin{bmatrix} O_2 & I_2 \\ -w_d^2 & -2\sigma_d \end{bmatrix},$$

w_d^2 = 2x2 diagonal matrix of squared disturbance frequencies
$2\sigma_d$ = 2x2 diagonal disturbance damping matrix

w_d^2 = diag[3947.8, 986.96] (rad^2/sec^2)

$2\sigma_d$ = diag[.1257, .0628] (rad/sec)

$$\Phi = \begin{bmatrix} O_2 & I_2 \\ O_2 & O_2 \end{bmatrix},$$

Φ = telescope rigid body rotational modes (θ_x, θ_y)

BE = 8x21 matrix transpose of modal eigenvectors (defined in table C.1)

BR = 2x21 matrix transpose of rigid body eigenvectors (defined in table C.2)

CE = 3x8 matrix of linear combinations of modal eigenvectors required by system output (defined in table C.3)

0^{12} = null matrix with 12 columns

CR = 3x2 matrix of rigid body contribution to system outputs (defined in table C.4)

ME = 21x8 matrix of linear combinations of modal eigenvectors required by system measurements (defined in table C.5)

MR = 21x2 matrix of rigid body contribution to system measurements (defined in table C.6)

W^a = $(0.1) I_{21}$ (N^2), W^b = $(3.95) I_2$ (N^2),

V_1 = $(.0001) I_2$ (rad^2), V_2 = $(.000001) I_{22}$ (m^2),

V_3 = $(1 \times 10^{-7}) I_{21}$ (m^2/s^2)

Table C.0 The S matrix (8x2)

row\col:	1	2
1	-1.5105E-03	6.1789E-03
2	8.0569E-04	-9.4151E-05
3	-5.1719E-03	-7.4586E-04
4	1.0263E-03	1.4033E-03
5	7.8788E-03	-5.0280E-03
6	-3.0956E-03	-4.2834E-03
7	-3.5394E-03	-5.3071E-04
8	1.3823E-04	2.2141E-02

Table C.1 The BE matrix (8x21)

row\col:	1	2	3	4	5
1	6.740E-05	-1.520E-03	-1.507E-03	1.946E-03	-6.628E-05
2	-4.615E-03	9.353E-04	7.917E-04	-7.844E-03	4.612E-03
3	-2.582E-04	-5.140E-03	-5.175E-03	2.359E-06	-2.581E-04
4	-4.927E-03	-7.887E-04	1.479E-03	1.084E-06	-4.930E-03
5	2.786E-04	7.798E-03	7.885E-03	-7.174E-06	2.784E-04
6	1.557E-02	2.626E-03	-4.516E-03	-6.149E-05	1.558E-02
7	-3.792E-04	-3.451E-03	-3.544E-03	1.705E-02	4.814E-04
8	-6.065E-05	8.113E-05	1.475E-04	4.850E-04	6.065E-05

row\col:	6	7	8	9	10
1	1.521E-03	1.507E-03	-6.035E-03	8.661E-05	-1.517E-03
2	-9.354E-04	-7.911E-04	-1.575E-03	-4.944E-03	9.175E-04
3	-5.140E-03	-5.175E-03	-3.486E-08	-3.288E-04	-5.152E-03
4	-7.889E-04	1.480E-03	-4.465E-09	-3.893E-04	-3.367E-04
5	7.798E-03	7.885E-03	1.040E-07	4.498E-04	7.831E-03
6	2.607E-03	-4.546E-03	2.331E-06	1.257E-03	1.212E-03
7	3.468E-03	3.514E-03	-4.789E-04	-5.916E-04	-3.473E-03
8	-8.125E-05	-1.471E-04	-3.086E-04	6.402E-05	9.512E-05

row\col:	11	12	13	14	15
1	-1.510E-03	-8.696E-05	1.517E-03	1.510E-03	-1.522E-03
2	8.049E-04	4.942E-03	-9.180E-04	-8.050E-04	1.042E-03
3	-5.172E-03	-3.290E-04	-5.152E-03	-5.172E-03	-5.132E-03
4	1.026E-03	-3.896E-04	-3.366E-04	1.027E-03	-1.928E-03
5	7.879E-03	4.502E-04	7.831E-03	7.879E-03	7.786E-03
6	-3.095E-03	1.250E-03	1.190E-03	-3.124E-03	6.250E-03
7	-3.540E-03	5.986E-04	3.481E-03	3.519E-03	-3.405E-03
8	1.417E-04	-6.462E-05	-9.552E-05	-1.408E-04	6.075E-05

row\col:	16	17	18	19	20
1	1.522E-03	-6.313E-03	8.542E-05	-1.504E-03	-8.536E-05
2	-1.043E-03	1.452E-02	-4.945E-03	6.931E-04	4.945E-03
3	-5.132E-03	2.594E-07	-3.290E-04	-5.195E-03	-3.290E-04
4	-1.929E-03	-3.986E-07	-3.892E-04	2.617E-03	-3.894E-04
5	7.786E-03	-5.702E-07	4.501E-04	7.933E-03	4.504E-04
6	6.236E-03	-1.038E-05	1.256E-03	-8.125E-03	1.248E-03
7	3.446E-03	1.473E-03	-5.928E-04	-3.611E-03	6.008E-04
8	-6.123E-05	-5.088E-04	5.993E-05	1.824E-04	-5.937E-05

row\col:	21
1	1.503E-03
2	-6.904E-04
3	-5.195E-03
4	2.618E-03
5	7.933E-03
6	-8.162E-03
7	3.558E-03
8	-1.824E-04

Table C.2 The BR matrix (2x21)

row\col:	1	2	3	4	5
1	4.931E-06	3.908E-06	-4.849E-06	0	4.931E-06
2	-4.576E-07	3.273E-06	3.273E-06	-5.119E-06	4.576E-07

row\col:	6	7	8	9	10
1	3.908E-06	-4.849E-06	0	-1.258E-05	2.156E-06
2	-3.273E-06	-3.273E-06	1.148E-07	-4.576E-07	3.273E-06

row\col:	11	12	13	14	15
1	-3.097E-06	-1.258E-05	2.156E-06	-3.097E-06	8.286E-06
2	3.273E-06	4.576E-07	-3.273E-06	-3.273E-06	3.273E-06

row\col:	16	17	18	19	20
1	8.286E-06	0	-1.258E-05	-9.227E-06	-1.258E-05
2	-3.273E-06	1.296E-05	-4.576E-07	3.273E-06	4.576E-07

row\col:	21
1	-9.227E-06
2	-3.273E-06

Table C.3 The CE matrix (3x8)

row\col:	1	2	3	4	5
1	-2.532E-07	-3.478E-07	1.295E-06	2.283E-04	6.032E-06
2	3.253E-04	2.145E-04	3.471E-07	-4.422E-07	1.066E-06
3	-5.025E-08	1.100E-06	5.215E-06	2.761E-06	1.552E-05

row\col:	6	7	8
1	-7.338E-04	-2.296E-06	1.021E-06
2	6.735E-06	-8.727E-04	2.360E-04
3	-1.016E-05	4.061E-07	4.992E-08

Table C.4 The CR matrix (3x2)

row\col:	1	2
1	1	0
2	0	1
3	0	0

Table C.5 The ME matrix (21x8)

row\col:	1	2	3	4	5
1	6.740E-05	-4.615E-03	-2.582E-04	-4.927E-03	2.78[illegible]E-04
2	-1.520E-03	9.353E-04	-5.140E-03	-7.887E-04	7.798E-03
3	-1.507E-03	7.917E-04	-5.175E-03	1.479E-03	7.885E-03
4	1.946E-03	-7.844E-03	2.359E-06	1.083E-06	-7.174E-06
5	-6.628E-05	4.612E-03	-2.581E-04	-4.930E-03	2.784E-04
6	1.521E-03	-9.354E-04	-5.140E-03	-7.889E-04	7.798E-03
7	1.507E-03	-7.911E-04	-5.175E-03	1.480E-03	7.885E-03
8	-6.035E-03	-1.575E-03	-3.486E-08	-4.465E-09	1.040E-07
9	8.661E-05	-4.944E-03	-3.288E-04	-3.893E-04	4.498E-04
10	-1.517E-03	9.175E-04	-5.152E-03	-3.367E-04	7.831E-03
11	-1.510E-03	8.049E-04	-5.172E-03	1.026E-03	7.879E-03
12	-8.696E-05	4.942E-03	-3.290E-04	-3.896E-04	4.502E-04
13	1.517E-03	-9.180E-04	-5.152E-03	-3.366E-04	7.831E-03
14	1.510E-03	-8.050E-04	-5.172E-03	1.027E-03	7.879E-03
15	-1.522E-03	1.042E-03	-5.132E-03	-1.928E-03	7.786E-03
16	1.522E-03	-1.043E-03	-5.132E-03	-1.929E-03	7.786E-03
17	-6.313E-03	1.452E-02	2.594E-07	-3.986E-07	-5.702E-07
18	8.542E-05	-4.945E-03	-3.290E-04	-3.892E-04	4.501E-04
19	-1.504E-03	6.931E-04	-5.195E-03	2.617E-03	7.933E-03
20	-8.536E-05	4.945E-03	-3.290E-04	-3.894E-04	4.504E-04
21	1.503E-03	-6.904E-04	-5.195E-03	2.618E-03	7.933E-03

row\col:	6	7	8
1	1.557E-02	-3.792E-04	-6.065E-05
2	2.626E-03	-3.451E-03	8.113E-05
3	-4.516E-03	-3.544E-03	1.475E-04
4	-6.149E-05	1.705E-02	4.850E-04
5	1.558E-02	4.814E-04	6.065E-05
6	2.607E-03	3.468E-03	-8.125E-05
7	-4.546E-03	3.515E-03	-1.471E-04
8	2.331E-06	-4.789E-04	-3.086E-04
9	1.257E-03	-5.916E-04	6.402E-05
10	1.212E-03	-3.473E-03	9.512E-05
11	-3.095E-03	-3.540E-03	1.417E-04
12	1.250E-03	5.986E-04	-6.462E-05
13	1.190E-03	3.481E-03	-9.552E-05
14	-3.124E-03	3.519E-03	-1.408E-04
15	6.250E-03	-3.405E-03	6.075E-05
16	6.236E-03	3.446E-03	-6.123E-05
17	-1.037E-05	1.473E-03	-5.088E-04
18	1.256E-03	-5.928E-04	5.993E-05
19	-8.125E-03	-3.611E-03	1.824E-04
20	1.248E-03	6.008E-04	-5.937E-05
21	-8.162E-03	3.558E-03	-1.824E-04

Table C.6 The MR matrix (21x2)

row\col:	1	2
1	5.632E+00	-4.545E-07
2	4.463E+00	4.000E+00
3	-5.537E+00	4.000E+00
4	0	-5.632E+00
5	5.632E+00	4.545E-07
6	4.463E+00	-4.000E+00
7	-5.537E+00	-4.000E+00
8	0	1.437E+01
9	-1.437E+01	-4.545E-07
10	2.463E+00	4.000E+00
11	-3.537E+00	4.000E+00
12	-1.437E+01	4.545E-07
13	2.463E+00	-4.000E+00
14	-3.537E+00	-4.000E+00
15	9.463E+00	4.000E+00
16	9.463E+00	-4.000E+00
17	0	1.437E+01
18	-1.437E+01	-4.545E-07
19	-1.054E+01	4.000E+00
20	-1.437E+01	4.545E-07
21	-1.054E+01	-4.000E+00

END
DATED
FILM
8-88
DTIC