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**Some Dynamic Properties of a Rigid Two-Bladed
Fully Gimballed Rotor with Teetering Feedback**

by
Harvey R. Chaplin

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AVIATION AND SURFACE EFFECTS DEPARTMENT

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CONTENTS

	Page
NOTATION	v
ABSTRACT	1
INTRODUCTION	1
EQUATIONS.	3
STABILITY.	4
STEADY STATE RESPONSE TO FIRST-HARMONIC EXCITATION	7
BEHAVIOR NEAR STABILITY BOUNDARIES WITH NO EXCITATION.	9
STEADY MOTION OF CONTROL AXIS.	10
DECAY OF CONTROL RESPONSE ERROR.	12
DISCUSSION OF NUMERICAL EXAMPLES	13
CONCLUSIONS.	31

FIGURES

1. Stability boundaries when $D = 1$	6
2. Decaying control response error with behavior near the $A_0 = 0$ stability boundary.	14
3. Decaying control response error with behavior near the $\sigma = 0$ stability boundary	15
4. Decaying control response error with near-optimum gain settings ($K_1 K = 0.25$, $K \leq 0.2$).	17
5. Decaying control response error with near-optimum $K_1 K$ (0.25) but excessive K (0.5)	20
6. Decaying control response error with near-optimum gain settings but reduced feather stability.	21

FIGURES (Continued)

	Page
7. Decaying control response error with near-optimum gain settings but reduced damping	22
8. Decaying control response error with excessive (but still stable) gain settings.	23
9. Response to a steady motion of the control axis	24
10. Response to a steadily increasing pitching moment	27

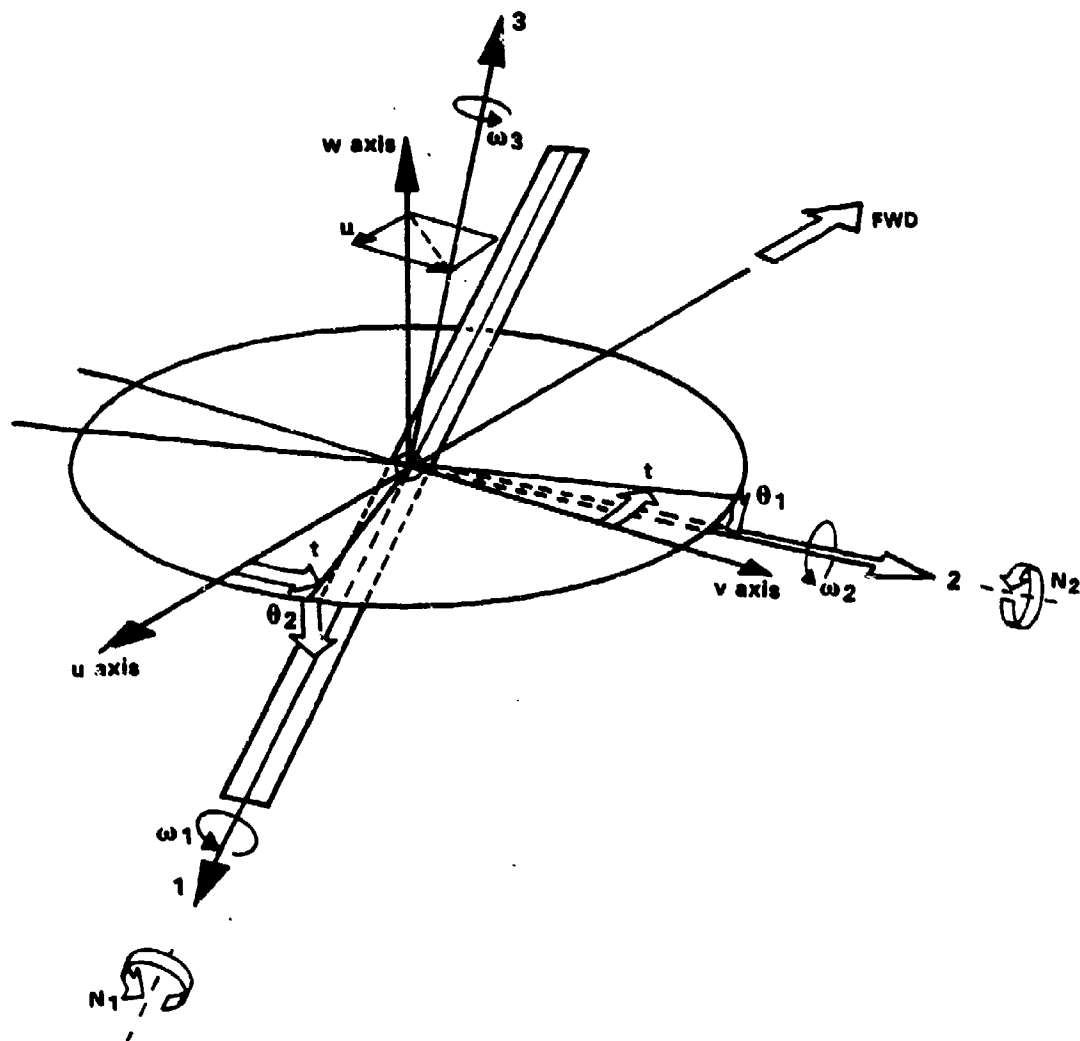


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NOTATION

$1, 2, 3$	Body axis system coincident with principal inertial axes
u, v, w	Nonrotating reference axes
u', v', w'	Control axis system pitched by angle u_0 and rolled by angle v_0 relative to u, v, w
u_0, v_0	(See u', v', w')
u	Pitch angle displacement of axis 3
v	Roll angle displacement of axis 3
u_t, v_t	Pitch, roll angle displacement of the thrust vector
$\bar{\theta}_1$	Feather angle measured positive down from $u'-v'$ plane to axis 2
$\bar{\theta}_2$	Teeter angle measured positive down from $u'-v'$ plane to axis 1
θ_1, θ_2	Feather and teeter angles measured from the u, v plane
$\Omega_1, \Omega_2, \Omega_3$	Dimensional angular velocities around axes 1, 2, 3
$\omega_1, \omega_2, \omega_3$	Nondimensional angular velocities around axes 1, 2, 3 = $\Omega_1/\Omega_3, \Omega_2/\Omega_3, 1$
τ	Nondimensional time equals azimuth angle measured from u axis to axis 1
$(\dot{})$	Derivative with respect to t
I_1, I_2, I_3	Moments of inertia about axes 1, 2, 3
ϵ_1	Angle by which damping moment vector (i.e., moment proportional to $-\dot{\omega}_2$) lags axis 2
ϵ_2	Angle by which feedback moment vector (i.e., moment proportional to $-K\theta_2$) lags axis 2
ϵ_3	Angle by which forcing function moment vector lags axis 2
N_1, N_2	Nondimensional moments about axes 1, 2 (dimensional moments divided by $I_1\Omega_3^2, I_2\Omega_3^2$)

C	Rotor blade chord
R	Rotor radius
A_0, A_1, A_2, A_3	Coefficients of the stability quartic
A	Feather stability coefficient = $-\frac{\partial N_1}{\partial \omega_2} = \frac{I_3 - I_2}{I_1} + \epsilon_1 \frac{I_2}{I_1} B$
B	Teeter damping coefficient = $-\frac{\partial N_2}{\partial \omega_2} \left(\pm \frac{0.35 \pi \rho R^5}{I_2} \frac{C}{R} \right)$
D	Teeter stability coefficient = $\frac{\partial N_2}{\partial \omega_1} = \frac{I_3 - I_1}{I_2}$
K	Teeter feedback coefficient = $-\frac{\partial N_2}{\partial \theta_2}$
$K_1 K$	Feather feedback coefficient = $-\frac{\partial N_1}{\partial \theta_2} \left(K_1 = \epsilon_2 \frac{I_2}{I_1} \right)$
M	Nondimensional teeter moment forcing function
EM	Nondimensional feather moment forcing function $\left(E = \epsilon_3 \frac{I_2}{I_1} \right)$
a	Pitch offset angle from w' axis to mean position of axis 3
b	Roll offset angle from w' axis to mean position of axis 3
ϕ_u	Pitch lag angle from u'-v' plane to plane containing instantaneous motion of axis 1
ϕ_v	Roll offset angle from u'-v' plane to plane containing instantaneous motion of axis 1
p	Nondimensional pitch rate of the control axis



REFERENCE AXIS SYSTEMS

ABSTRACT

A rigid two-bladed helicopter rotor mounted with no mechanical constraint of its teetering and feathering motions is analyzed with regard to the stability and controllability achievable with a simple feedback moment linearly proportional to teeter angle. Subject to confirmation by a detailed design study, it is tentatively concluded that the level of stability and control practically achievable with such a system would be adequate for tip-jet-driven very-heavy-lift helicopters. It is further concluded that the outlook for relatively low vibration levels is favorable.

INTRODUCTION

There is generally conceded to be an upper limit on helicopter disk loading (for reasons of downwash effects on personnel, objects, and ground surface in the landing/takeoff area and damage to the helicopter from re-circulated debris) and on rotor tip speed (for reasons of compressibility effects on rotor aerodynamics). If disk loading and tip speed are held fixed, the following proportional relationships with rotor radius R apply:

Maximum takeoff weight	$\sim R^2$
Rotor horsepower	$\sim R^2$
Rotor rpm	$\sim 1/R$
Rotor torque	$\sim R^3$

With torque increasing so much faster than total helicopter weight, the weights of the main rotor transmission and the anti-torque system tend to become excessive when one attempts to design conventional simple helicopters in the very-heavy-lift range.

The tip-jet-driven helicopter is a possible solution that has been known for many years. Though less efficient in energy terms when considering the main rotor alone, the elimination of the transmission and drastic reduction of anti-torque system requirements may lead to a significant overall advantage.

Tip-jet-driven rotors are constrained, however, to a small number of rotor blades (preferably two, at most three) to maintain an adequate ratio of blade cross-section area to blade planform area for efficient ducting of the jet gases through the blades to the tip jet nozzles. The control of a very large two-bladed rotor through conventional mechanical cyclic pitch becomes a challenging problem. Control actuation loads can become very large, and reacting these loads on the fuselage can entail severe vibrations, especially when the helicopter flies lightly loaded and fuselage mass and inertias are relatively low. These considerations have stimulated interest in the possibility of a fully gimballed rotor; that is, one which is mechanically unconstrained in both teetering and feathering motion. Control moments might be applied to such a rotor by circulation control on the blades using a portion of the jet-drive gases, or by trailing edge flaps with actuation forces reacted within the blade itself.

Previous analyses* of a rigid two-bladed fully gimballed helicopter rotor showed that with appropriate mass balancing such a rotor can be stable at zero and low advance ratios in the sense that transient disturbances will damp to zero, and responses to harmonic excitation will damp to a steady state. (If the harmonic excitation includes a one-per-rev component of feathering moment not coincident with the rotor damping axis, the steady state response will include a progressive precession of the axis of rotation.)

A more desirable level of stability is one in which the rotor would return to a preselected orientation with the fuselage after a transient disturbance and in which small feathering moment excitations, even if not precisely coincident with the damping axis, would not produce progressive precession. The purpose of

* As reported by H.R. Chaplin (DTNSRDC/TM-16-80/16, "Some Dynamic Properties of a Rigid Two-Bladed Fully Gimballed Tip Jet Helicopter Rotor with Circulation Control," Aug 1980).

this effort is to examine the effectiveness of a feedback moment proportional to teeter angle (as measured from a plane fixed to the fuselage) as a means of achieving this type of stability.

EQUATIONS

The Euler equations* are written in nondimensional form as:

$$\dot{\omega}_1 = -A\omega_2 - K_1 K\theta_2 + EM(t) \quad (1)$$

$$\dot{\omega}_2 = -B\omega_2 + D\omega_1 - K\theta_2 + M(t) \quad (2)$$

$$\omega_3 = \text{constant}$$

These equations are equivalent to the equations in the previous work, but the nomenclature differs. As written here, M is the nondimensional disturbing aerodynamic moment due to gusts or non-zero advance ratio, and $-K\theta_2$ is the nondimensional feedback moment. (Coefficients are defined in the Notation.)

A nonrotating axis system u, v, w coinciding with the principal axis system 1, 2, 3 at time zero is defined. Pitch, roll velocity of axis 3 is given for small-amplitude motions by

$$\dot{u} = \omega_2 \cos t + \omega_1 \sin t \quad (3)$$

$$\dot{v} = -\omega_1 \cos t + \omega_2 \sin t \quad (4)$$

Feather and teeter angles relative to the $u-v$ plane are given by

$$\theta_1 = v \cos t - u \sin t \quad (5)$$

$$\theta_2 = u \cos t + v \sin t \quad (6)$$

* Symon, K.R., "Mechanics," Second Edition, Addison-Wesley Publishing Co., Inc., Reading, MASS (May 1961), p. 451.

Differentiating these equations and combining with Eqs. (3) and (4) gives

$$\dot{\theta}_1 = -\omega_1 - \theta_2$$

$$\dot{\theta}_2 = \omega_2 + \theta_1$$

$$\ddot{\theta}_2 = \dot{\omega}_2 - \omega_1 - \theta_2 \quad (7)$$

STABILITY

Equations (1), (2), and (7) now form a set of three linear differential equations relating the three variables ω_1 , ω_2 , θ_2 . Rearranging, these equations become

$$(\ddot{\theta}_2 + \theta_2) + \omega_1 - \dot{\omega}_2 = 0 \quad (7)$$

$$K_1 K \theta_2 + \dot{\omega}_1 + A \omega_2 = E M \quad (1)$$

$$K \theta_2 - D \omega_1 + (\dot{\omega}_2 + B \omega_2) = M \quad (2)$$

This is a fourth-order linear system that responds to the excitation $M(t)$ and has additional solutions of the form $e^{\lambda_1 t}$, where the λ_1 's are roots of the determinant

$$\begin{vmatrix} \lambda^2 + 1 & 1 & -\lambda \\ K_1 K & \lambda & A \\ K & -D & \lambda + B \end{vmatrix} = 0$$

or

$$\lambda^4 + A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 = 0 \quad (8)$$

where

$$\left. \begin{aligned} A_0 &= A(D+K) - K_1KB \\ A_1 &= B - K_1K(1-D) \\ A_2 &= 1 + K + AD \\ A_3 &= B \end{aligned} \right\} \quad (9)$$

The well-known criteria for stability are:

$$1. \text{ All of the coefficients } A_0, A_1, A_2, A_3 \text{ must be positive; and} \quad (10)$$

$$2. \text{ The quantity } \sigma = A_1A_2A_3 - A_1^2 - A_0A_3^2 \text{ must also be positive.} \quad (11)$$

For practical rotor systems, the teeter stability coefficient $D = (I_3 - I_1)/I_2$ will always be very slightly less than unity. Insight into the stability criteria is facilitated by examining the case of $D = 1$:

$$\left. \begin{aligned} A_0 &= A(1+K) - K_1KB \\ A_1 &= B \\ A_2 &= 1 + K + A \\ A_3 &= B \\ \sigma &= KB^2(1-A + K_1B) \end{aligned} \right\} \quad \text{when } D = 1 \quad (12)$$

The teeter damping coefficient B is necessarily positive on physical grounds as long as the predominant lift curve slope of the rotor blades is positive. (Only for a very severely stalled rotor might this not be so.) It happens that $\sigma/B^2 + A_0 + 1 = A_2$; therefore, if σ and A_0 are positive, A_2 is also positive.

The stability criteria thus reduce (for A positive, $D = 1$) to

1. $K_1KB/A \leq 1 + K$ to assure A_0 is positive, and
2. $K_1KB/A \geq (1 - 1/A)$ to assure σ is positive, as necessary and sufficient conditions for stability when $D = 1$.

Stability boundaries are illustrated in Figure 1. The region of primary interest for a helicopter rotor is the first quadrant with K_1 , K , and A all positive.

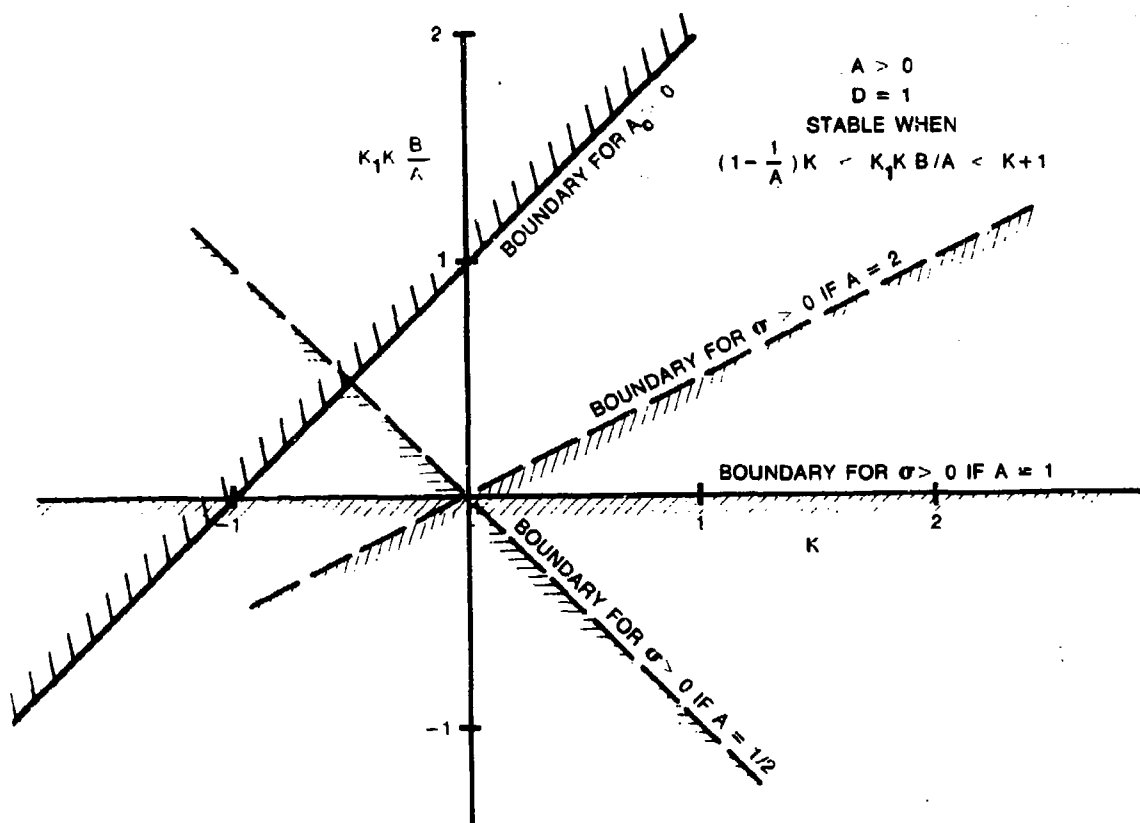


Fig. 1. Stability boundaries when $D = 1$.

It is interesting to note, however, that stability is possible with either K_1 or K or with both negative. (There is also a limited region of stability with A negative; boundaries with A negative are not illustrated in Figure 1, but are easy to draw. Since with no feedback system (i.e., with $K=0$) A must be positive for stability, this region is analogous to the "fly-by-wire" airplane designed to be unstable without its feedback system in operation.)

STEADY STATE RESPONSE TO FIRST-HARMONIC EXCITATION

In the previous study* it was found that the steady-state response of a rotor without feedback to first-harmonic excitation was a two-per-rev circular wobble of axis 3 superimposed on a steady progressive precession of axis 3. Since a two-per-rev circular wobble of axis 3 corresponds to a pure harmonic motion of teeter angle θ_2 and, hence, pure harmonic moments from the feedback terms (and since we expect progressive precession to be arrested by the feedback system), we would expect a stationary two-per-rev wobble to be the primary steady-state response of the rotor with feedback. The simplest approach is to assume such a motion and solve for the excitation required to produce it. Suppose

$$\begin{aligned}
 u &= \frac{1}{2} \sin 2t + a \\
 v &= -\frac{1}{2} \cos 2t + b \\
 \omega_1 &= -\sin t \\
 \omega_2 &= \cos t \\
 \theta_2 &= a \cos t + (b + \frac{1}{2}) \sin t \\
 \theta_1 &= -a \sin t + (b - \frac{1}{2}) \cos t
 \end{aligned} \tag{13}$$

* H.R. Chaplin, DTNSRDC/TM-16-80/16, Aug 1980.

Substituting these quantities into Eqs. (1) and (2) yields

$$a = \frac{EB + (1-A)}{K(K_1 - E)}$$

$$b + \frac{1}{2} = \frac{E(D-1)}{K(K_1 - E)}$$

$$M = \frac{K_1 B + (1-A)}{K_1 - E} \cos t + \frac{K_1 (D-1)}{K_1 - E} \sin t$$

Again, noting that D is nearly unity in practical cases, the case D = 1 provides simple insight.

a. The wobble circle is tangent to the u axis.

b. The Center of the circle is located at

$$u = a = \frac{EB + (1-A)}{K(K_1 B + (1-A))} \bar{M}$$

$$v = b = -\frac{1}{2} \frac{K_1 - E}{K_1 B + (1-A)} \bar{M}$$

with

$$D = 1$$

(14)

$$M = \bar{M} \cos t$$

c. $\theta_2 = a \cos t$

d. $\theta_1 = 2b \cos t - a \sin t$

e. The thrust vector, which is normal to the plane described by axis 1, lies at $u_T = u = a$, $v_T = 0$, stationary. (Contrary to the interpretation given in the previous work,* the wobble of axis 3 does not imply a wobbling thrust vector. The thrust vector is stationary under first-harmonic steady-state excitation.)

*H.R. Chaplin, DTNSRDC/TM-16-80/16, Aug 1980.

BEHAVIOR NEAR STABILITY BOUNDARIES WITH NO EXCITATION

These conclusions are of course valid only for combinations of coefficients satisfying the stability criteria derived earlier and, in fact, only if A_0 and σ are both positive and finite. It can be noticed immediately that when $\sigma \rightarrow 0$ the quantities $a/\bar{M}$ and $b/\bar{M}$ approach infinity; or tracking back through the derivation of $a/\bar{M}$ and $b/\bar{M}$, it can be seen that a stationary two-per-rev circular wobble (with any center location and any radius) is a solution of Eqs. (1) and (2) if $M = 0$ and $\sigma = 0$. (This observation remains valid when $D \neq 1$.) The thrust vector is stationary. The nature of the decay when σ is very small will be discussed later.

For the case $A_0 = 0$, it is apparent from Eq. (8) that $\lambda_1 = 0$ (i.e., $\omega_1, \omega_2, \theta_2$ constant) is a solution. From Eqs. (7) and (1), setting derivatives to zero, it is seen that this solution has the properties

$$\theta_2 = \omega_1 = \text{constant}$$

$$\omega_2 = -\frac{K_1 K}{A} \omega_1 = -\frac{D + K}{B} \omega_1$$

$$\theta_1 = -\omega_2$$

This is a pure rotation about the w-axis. In the u-v plane it is a one-per-rev circular wobble with the center at $u=v=0$ and can have any arbitrary radius.

If $A_0 \rightarrow 0$, $\lambda_1 = -\frac{A_0}{A_1}$ is a solution; that is,

$$\theta_2 \doteq \text{constant} \times e^{\lambda_1 t}$$

$$\theta_1 \doteq \frac{K_1 K}{A} \theta_2$$

In the $u-v$ plane this is a logarithmic spiral which approaches the origin as $t \rightarrow \infty$, if A_0 and A_1 are positive. The thrust vector also describes a logarithmic spiral.

STEADY MOTION OF CONTROL AXIS

Let us define a relative teeter angle by rewriting Eq. (6).

$$\bar{\theta}_2 = (u-u_0) \cos t + (v-v_0) \sin t \quad (6')$$

This is the angle between axis 1 and a new reference plane that is pitched and rolled by angles u_0 , v_0 relative to the $u-v$ plane. Other things being equal, any of the steady-state motions in the $u-v$ plane considered previously are simply displaced by the amounts u_0 and v_0 . We will call axis w' normal to this plane and coinciding with w when $u_0 = v_0 = 0$ the "control axis."

Let us consider a steady pitch-rate displacement of the control axis

$$u_0 = pt$$

Ideally, we might hope for a steady-state response $u = pt + a$, $v = 0$ with axis 3 tracking the control axis with constant lag angle $-a$; however, the preceding studies lead us to expect that there will be a two-per-rev circular wobble and a roll offset angle b superimposed on this ideal response. It is easily confirmed from Eqs. (3) and (4) that the motion

$$\omega_1 = p \sin t$$

$$\omega_2 = p \cos t$$

is a pure translation $u = pt$ in the $u-v$ plane; whereas the motions

$$\left[\begin{array}{l} \omega_1 = q \sin t \\ \omega_2 = -q \cos t \end{array} \right] \quad \text{and} \quad \left[\begin{array}{l} \omega_1 = r \cos t \\ \omega_2 = r \sin t \end{array} \right]$$

are stationary two-per-rev circular wobbles.

Taking

$$\omega_1 = p \sin t + q \sin t + r \cos t$$

$$\omega_2 = p \cos t - q \cos t + r \sin t$$

and from Eqs. (3), (4), (5), and (6)

$$\bar{\theta}_2 = (b-q/2) \sin t + (a-r/2) \cos t$$

$$\bar{\theta}_1 = (b+q/2) \cos t - (a+r/2) \sin t$$

and Eqs. (1) and (2), with $M = 0$ and $D = 1$,

$$q = \frac{K_1 B - (1+A)}{K_1 B + (1-A)} p$$

$$r = \frac{2K_1}{K_1 B + (1-A)} p$$

$$a = \frac{B}{K} q + \frac{1}{2} r - \frac{B}{K} p$$

$$b = \frac{1}{2} q + \frac{(1-A)}{K_1 K} r$$

The diameter of the wobble circle is $\sqrt{q^2 + r^2}$.

Now, as previously stated, the pitch lag and roll offset are $-a$ and b , respectively, in terms of the mean position of axis 3; however, the rotor thrust vector, in general, does not lie along axis 3. It lies (with steady state motion) along a line perpendicular to the plane described by axis 1. From the above expression for θ_2 it is seen that

$$\left. \begin{array}{l} \text{Thrust Vector} \\ \text{Pitch Lag} \end{array} \right\} \phi_u = -a + r/2 = \frac{B}{K} \frac{2}{K_1 B + (1-A)} p + \frac{p}{2} \sin 2t \quad (15)$$

$$\left. \begin{array}{l} \text{Thrust Vector} \\ \text{Roll Offset} \end{array} \right\} \phi_v = b - q/2 = \frac{1-A}{K} \frac{2}{K_1 B + (1-A)} p + \frac{p}{2}(1 + \cos 2t) \quad (16)$$

$$\bar{\theta}_1 = p \frac{2(1-A)/K}{K_1 B + (1-A)} \sin t - p \frac{2 B/K}{K_1 B + (1-A)} \cos t$$

$$\bar{\theta}_2 = p \frac{K_1 B - (1+A) + 2(1-A)/K}{K_1 B + (1-A)} \cos t - p \frac{2(K_1 B - B/K)}{K_1 B + (1-A)} \sin t$$

This represents a mean lag and offset proportional to p plus a two-per-rev wobble of the thrust vector of amplitude $p/2$. The thrust vector precesses in surges at angular rates oscillating between zero and $2p$.

DECAY OF CONTROL RESPONSE ERROR

In practical cases, A , B , and D are all of order unity, allowing Eqs. (15) and (16) to be rewritten approximately as

$$\phi_u \doteq \frac{2}{K_1 K} p + \frac{p}{2} \sin 2t$$

$$\phi_v \doteq 0 + \frac{p}{2}(1 + \cos 2t)$$

for steady-state movement of the control axis. If the control axis movement is suddenly arrested, the quantity ϕ_u can be thought of as a control response error, which will decay approximately as follows (considering only the mean position of

the thrust vector and ignoring the two-per-rev wobble):

$$\dot{\phi}_u \approx -\frac{2}{K_1 K} \dot{\phi}_u$$

$$\phi_u \approx \text{constant} \times e^{-\frac{K_1 K}{2} t}$$

This provides a useful approximation within the range $K < 0.5$, $K_1 K < 0.25$, wherein the wobble is sufficiently small relative to the error that the thrust vector can approach the control axis in a more or less straight line fashion. With $K_1 K = 0.25$, the error is reduced by about half per rotor revolution.

DISCUSSION OF NUMERICAL EXAMPLES

A digital simulation of Eqs. (1) and (2) has been coded on an HP-9836 micro-computer, and a number of illustrative example cases have been executed. These cases are presented in Figures 2 through 10 as graphs showing traces of the motion of the thrust vector on the u-v plane.

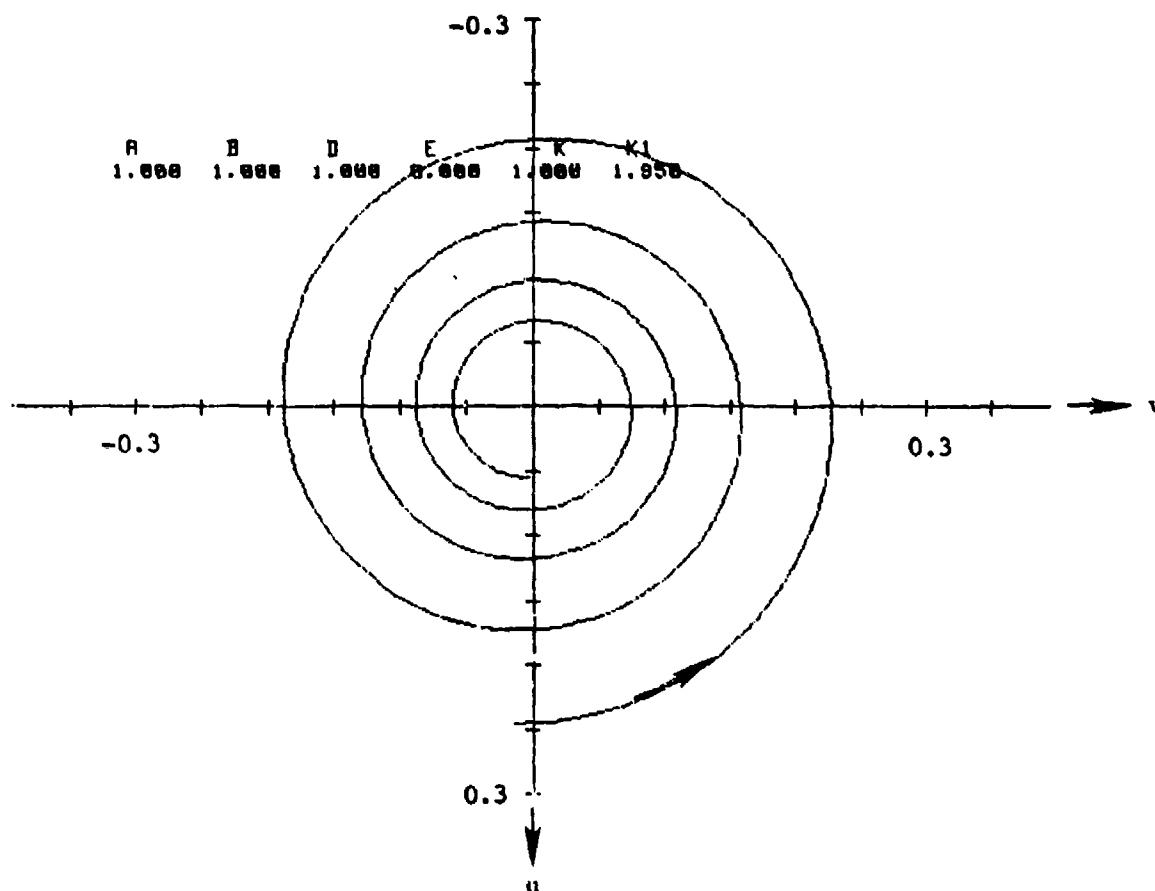


Fig. 2. Decaying control response error with behavior near the $A_0 = 0$ stability boundary.

In this example the feedback constants K_1 and K are set to values where the rotor is barely stable. The control axis was moved away from the origin and returned to the origin to introduce a disturbance, and several revolutions were allowed to elapse before the trace was started. Four revolutions of the one-per-rev spiral decay of the thrust axis trajectory around the control axis are shown.

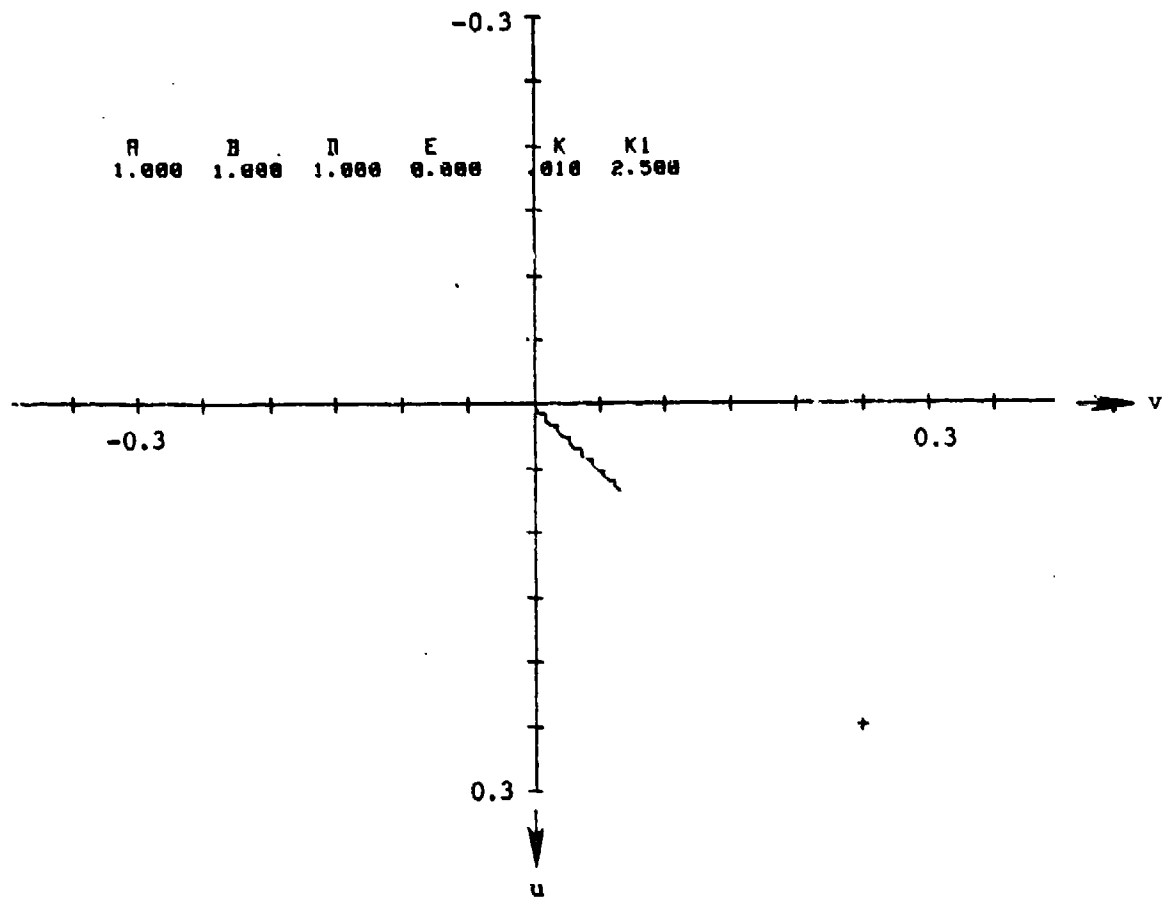


Fig. 3. Decaying control response error with behavior near the $\sigma = 0$ stability boundary.

(a) $K = 0.01$, $K_1 K = 0.025$.

With the rotor at steady state and the thrust vector at the origin, the control axis ("+") was suddenly moved to $u = v = 0.25$. Four rotor revolutions of response are shown. With both K and $K_1 K$ very small, the response is nearly pure (i.e., straight-line motion toward the control axis) but very slow.

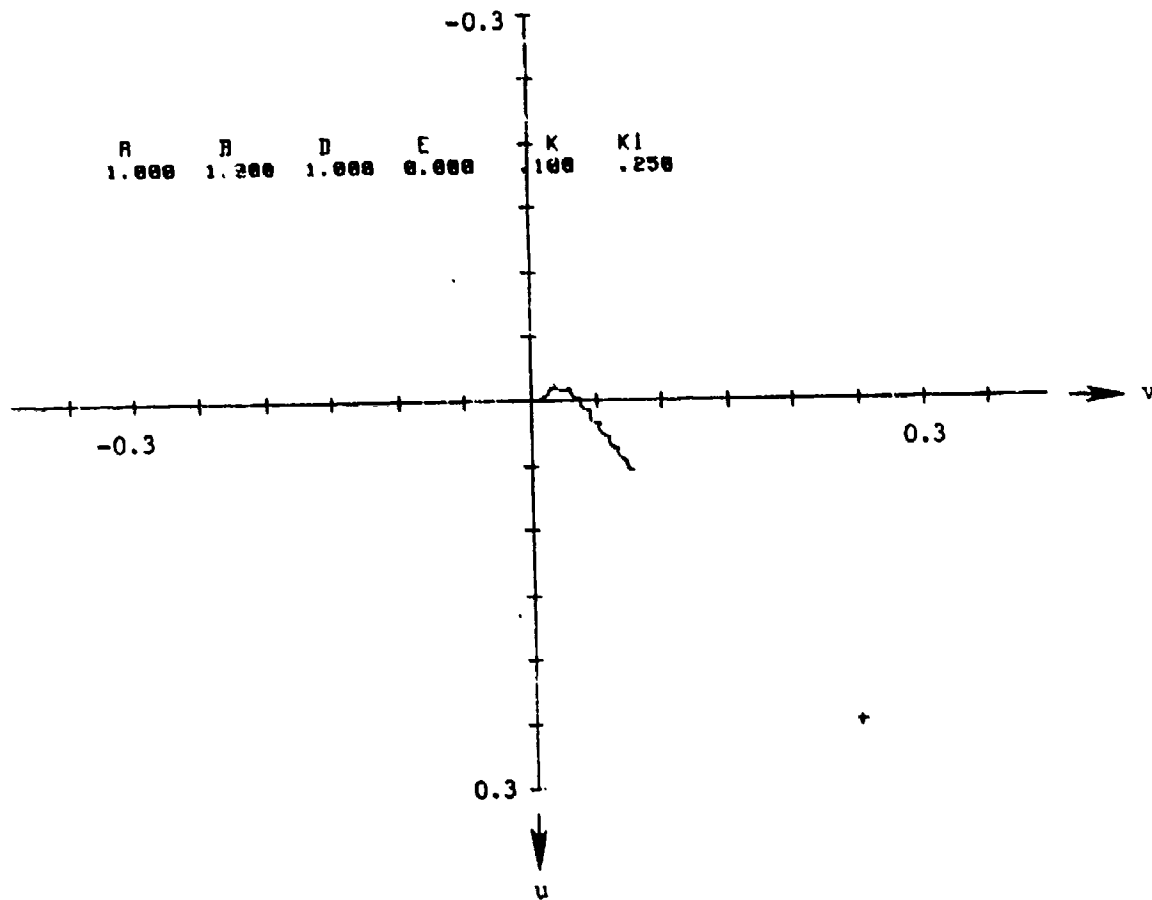


Fig. 3. Decaying control response error with behavior near the $\sigma = 0$ stability boundary. (continued)

(b) $K = 0.1$, $K_1 K = 0.025$.

The conditions for this example were the same as for Figure 3a, except that K was not so small, even though $K_1 K$ was the same. The initial response (first revolution) was essentially at right angles to the desired direction. This unwanted excursion damped quickly and the subsequent response was relatively pure but slow.

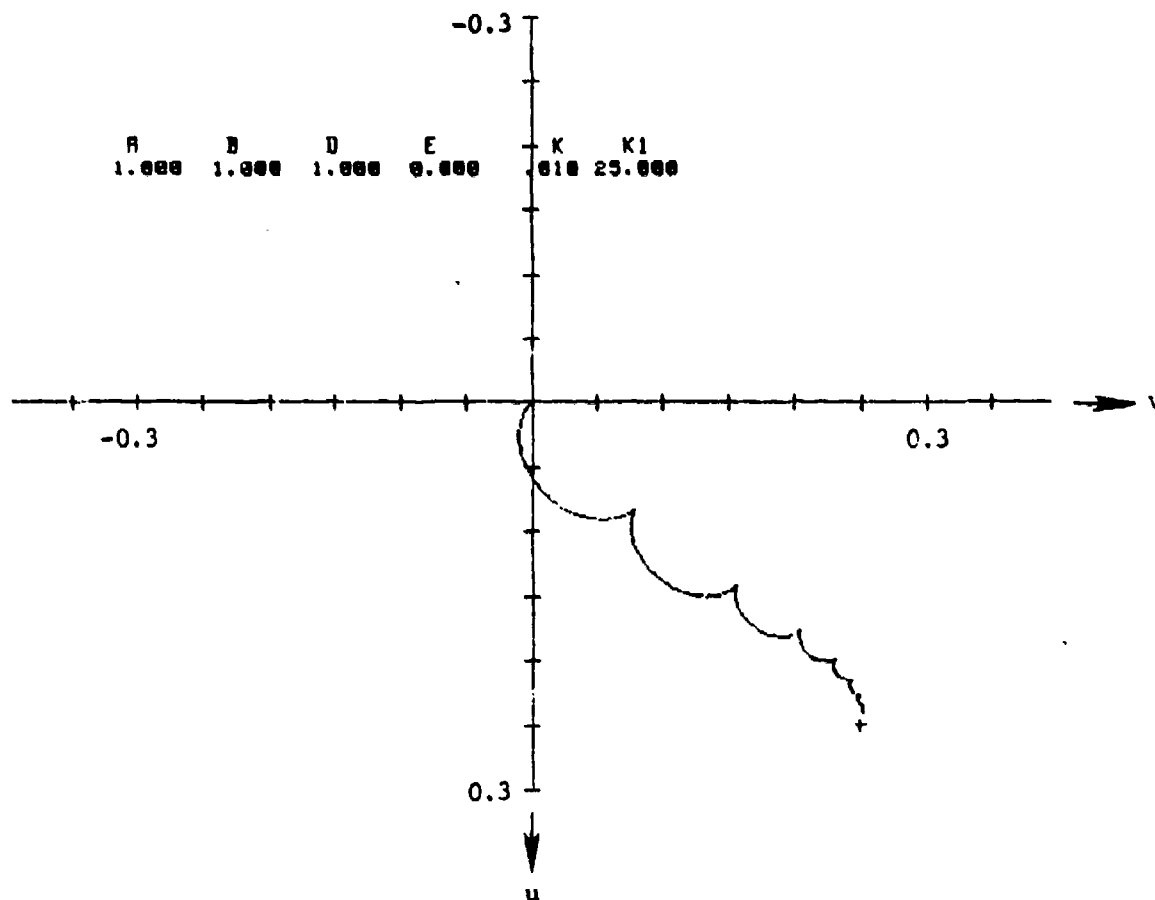


Fig. 4. Decaying control response error with near-optimum gain settings ($K_1 K = 0.25$, $K \leq 0.2$).
(a) $K = 0.01$.

The next several examples are similar to the preceding ones, but with gain settings well into the stable range. With $K_1 K = 0.25$, the error decays by about half in each rotor revolution (two "swoops" of the thrust vector occur per revolution; four revolutions are shown). With $K < 0.2$, the response is relatively pure, i.e., excursions at right angles to the desired direction are relatively small. The largest excursion occurs in the first revolution. With $K = 0.01$, it lies to the right.

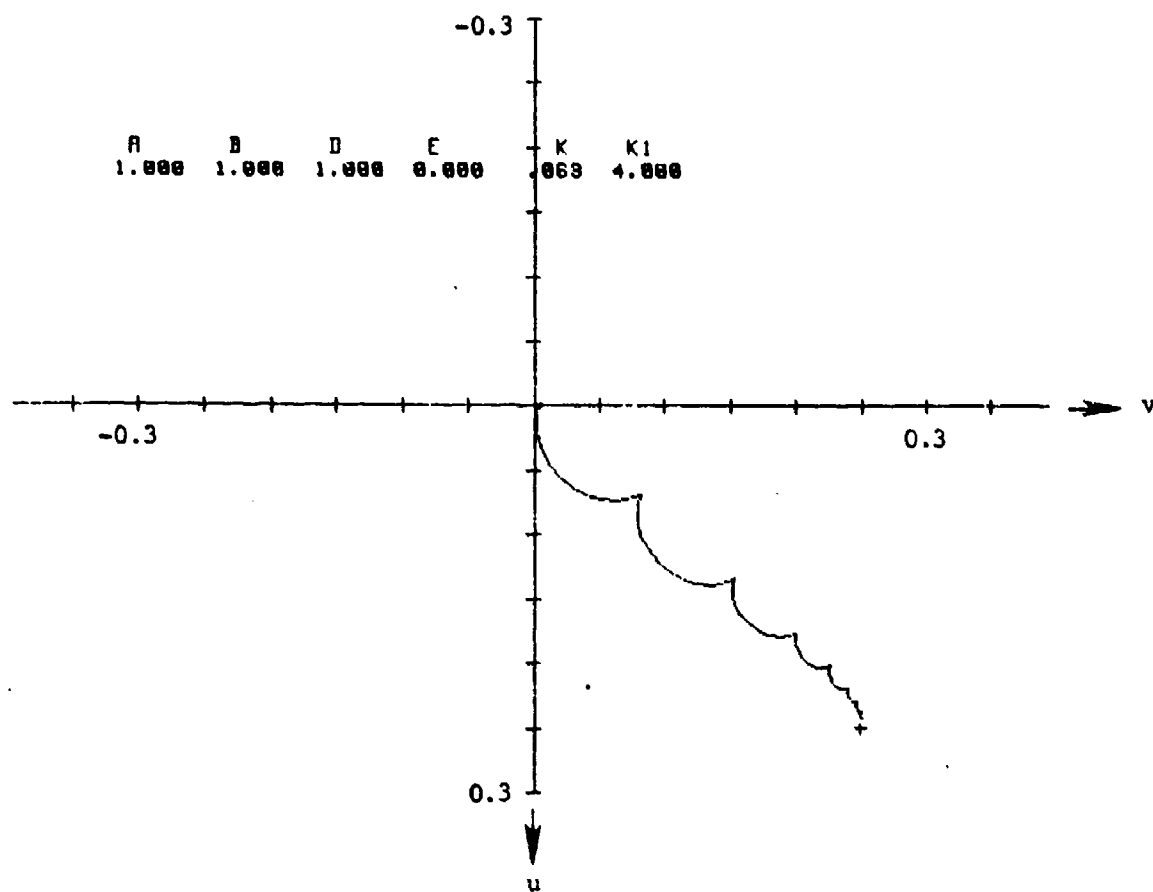


Fig. 4. Decaying control response error with near-optimum gain settings ($K_1 K = 0.25$, $K \leq 0.2$). (continued)
(b) $K = 0.0625$.

With $K = 0.0625$, the initial excursion is still to the right but smaller.
(Actually, the very first excursion is a nearly imperceptible one to the left.)

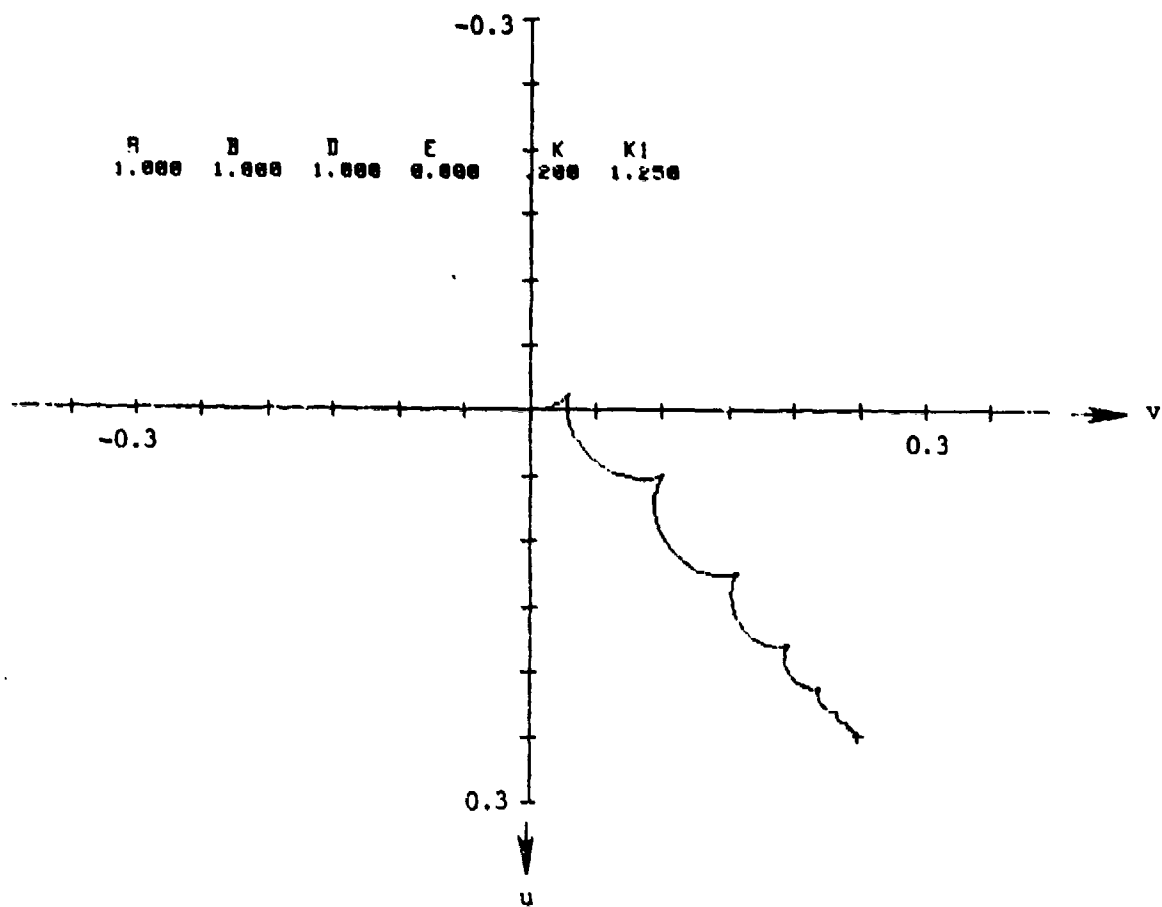


Fig. 4. Decaying control response error with near-optimum gain settings ($K_1 K = 0.25$, $K \leq 0.2$). (continued).

(c) $K = 0.2$

With $K = 0.2$, the initial excursion extends far enough to the left to begin to become slightly objectionable.

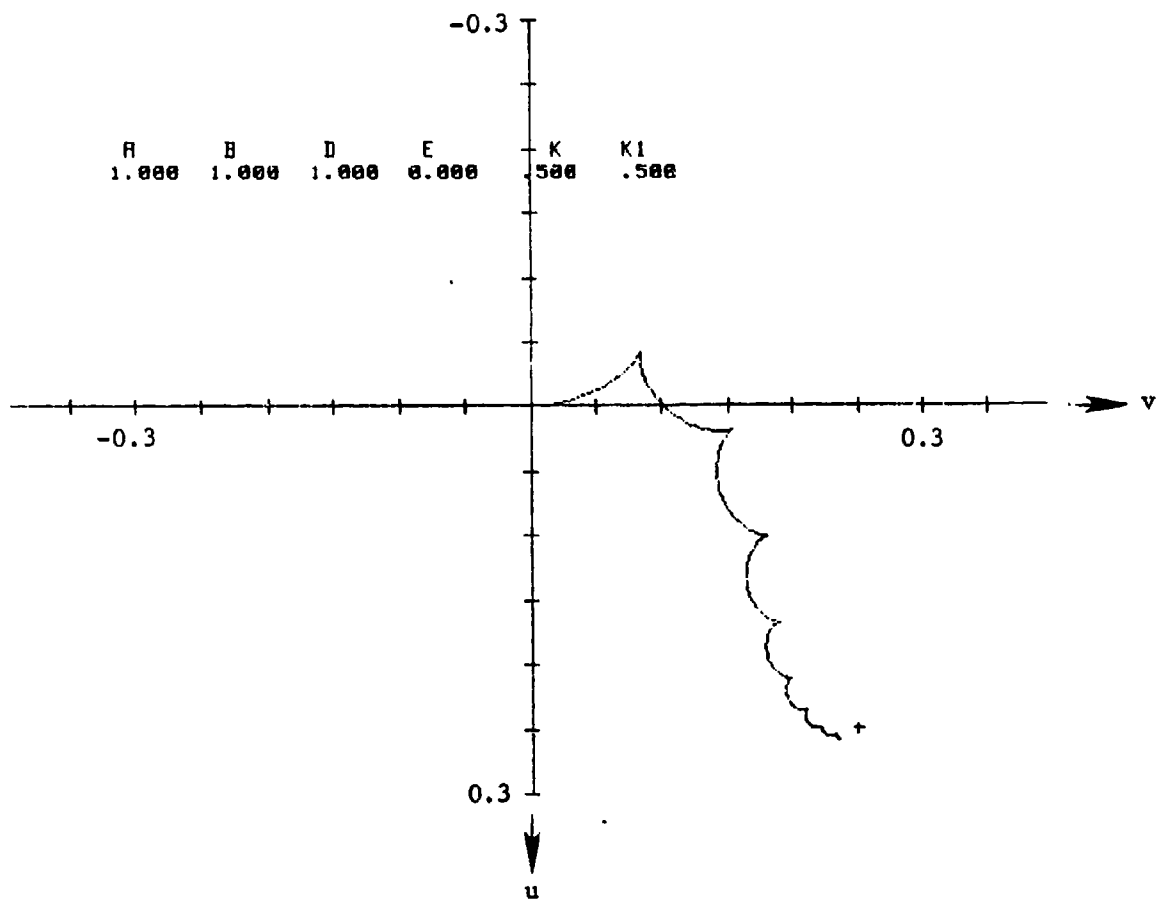


Fig. 5. Decaying control response error with near-optimum K_1K (0.25) but excessive K (0.5).

With excessive teetering moment feedback, there are excessive excursions to the right and left of the desired trajectory.

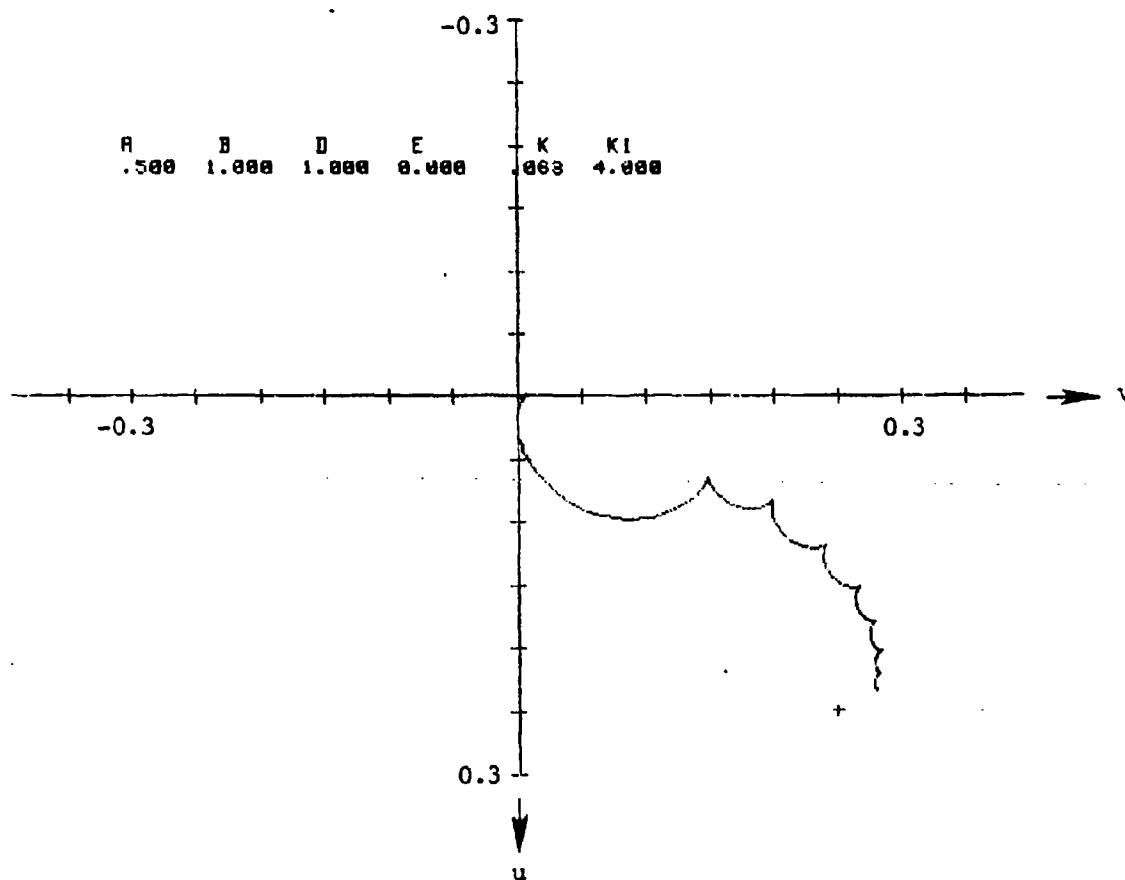


Fig. 6. Decaying control response error with near-optimum gain settings but reduced feather stability.

A too-low value of the feather stability coefficient A also leads to excessive excursions to the left and right of the desired trajectory.

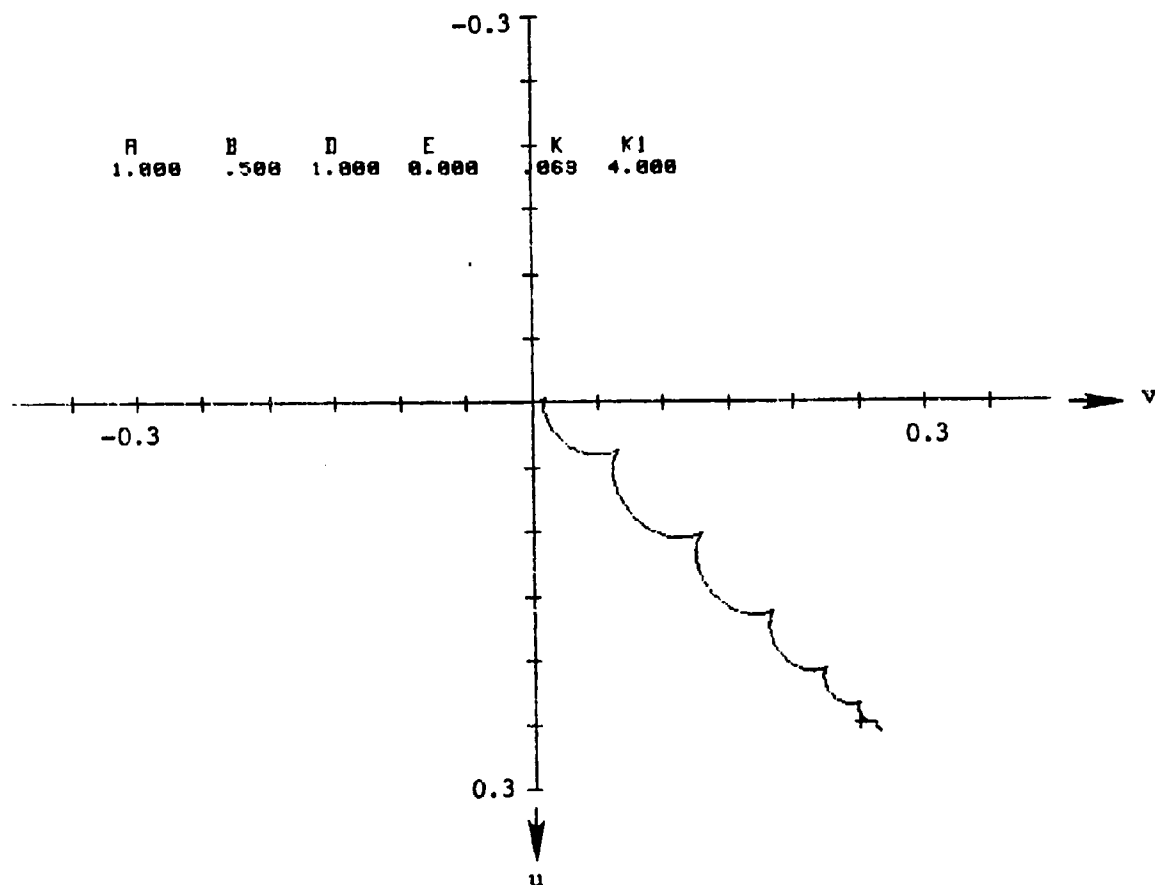


Fig. 7. Decaying control response error with near-optimum gain settings but reduced damping.

This example is the same as the one presented in Figure 4b, except that the damping coefficient B has been halved. The response is relatively quite pure, but the trajectory overshoots its objective slightly. This could be avoided by reducing K_1K .

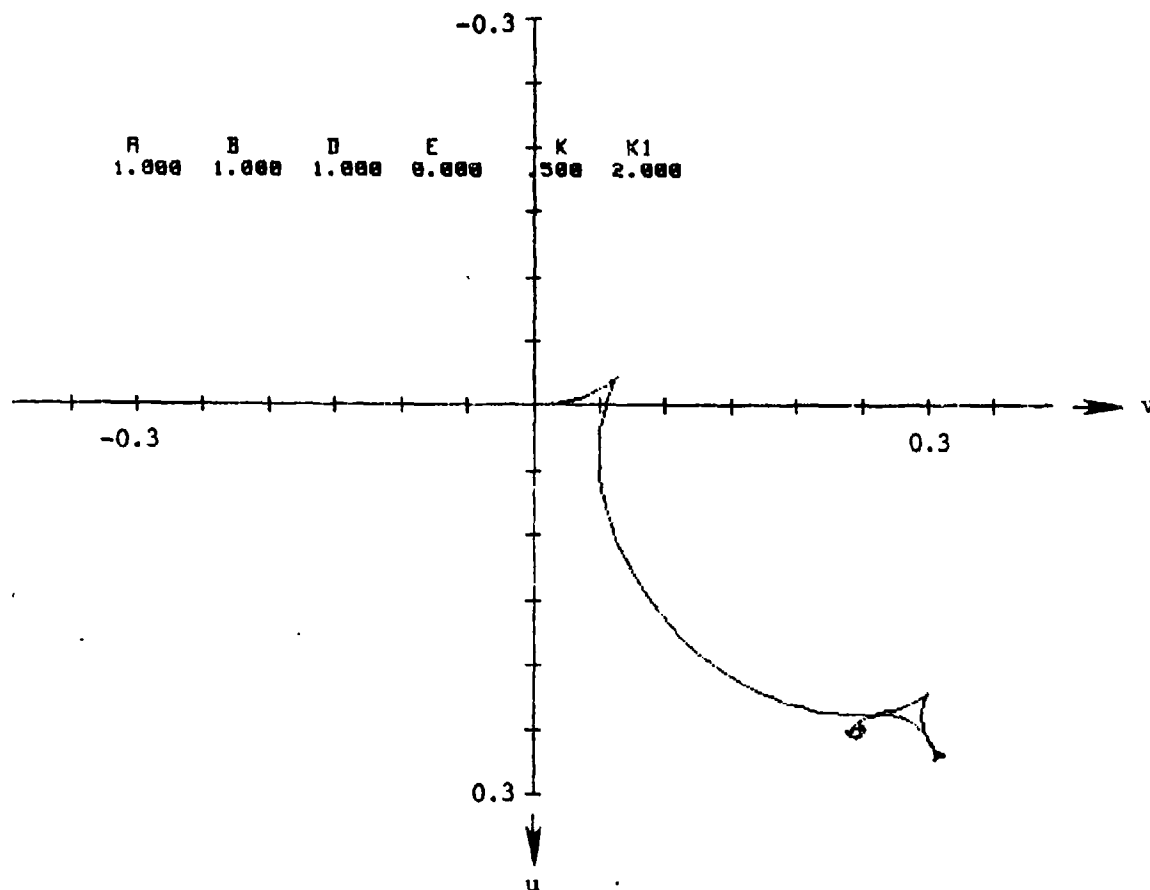


Fig. 8. Decaying control response error with excessive (but still stable) gain settings.

With excessive gains, the response becomes erratic and jerky. In the example shown, the error is reduced by about 80 percent in one revolution, but the flying quality would be very unpleasant. If K were further increased to 0.9 or K_1 were further increased to 2.7, thus approaching the $A_0 = 0$ stability boundary, the response would degenerate to a spiral of initially large amplitude around the control axis as shown in Figure 2.

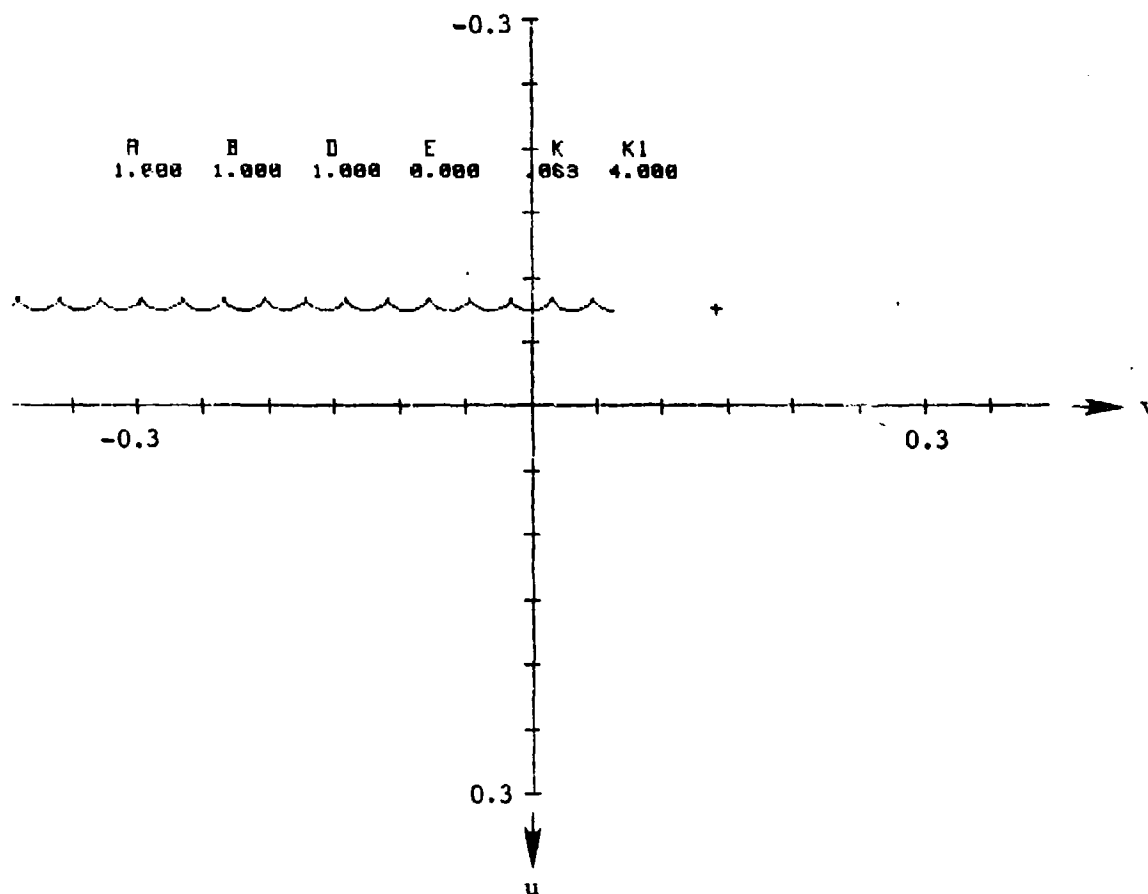


Fig. 9. Response to a steady motion of the control axis.

(a) Near-optimum gain settings.

In this example, the control axis ("+") is moving steadily from left to right at a rate of 0.01 times the rotor rotation rate. The thrust vector lags behind, moving at the same average rate superimposed on a small two-per-rev wobble as dictated by Eqs. (15) and (16).

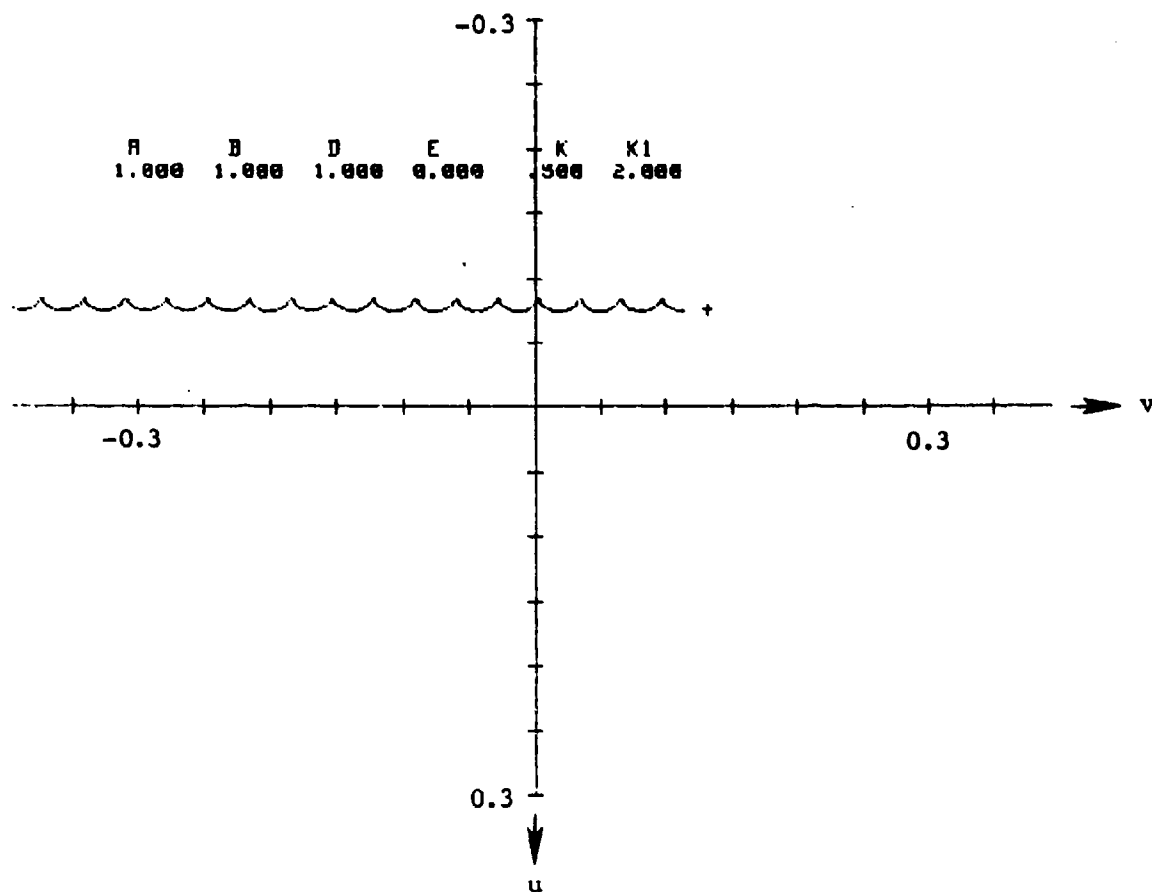


Fig. 9. Response to a steady motion of the control axis. (continued)
(b) Increased gain settings.

With increased gains, the steady state response is the same except that the lag is reduced.

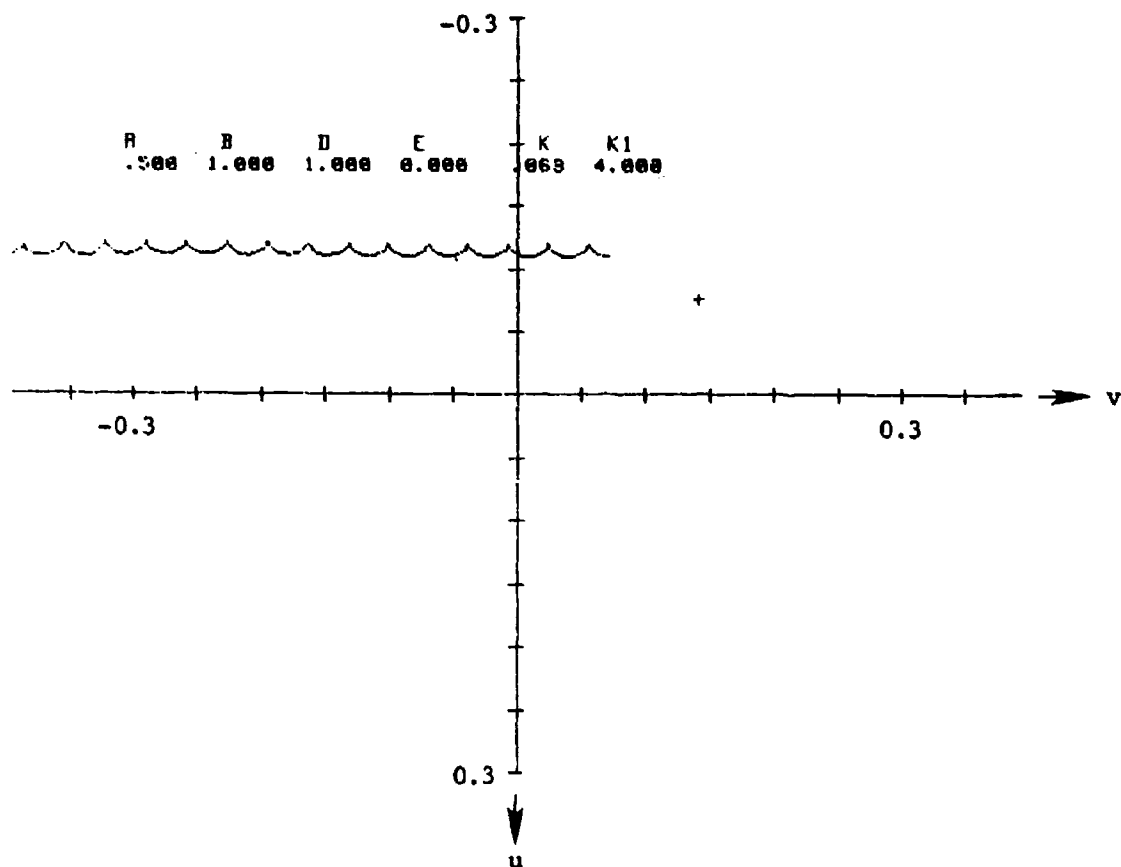


Fig. 9. Response to a steady motion of the control axis. (continued)
(c) Reduced feather stability.

With feather stability coefficient A less than 1.0, the lag is very slightly reduced, and the thrust vector trajectory is noticeably offset to the left of the control axis trajectory. (With A greater than 1.0, the lag would be increased and the offset would be to the right.)

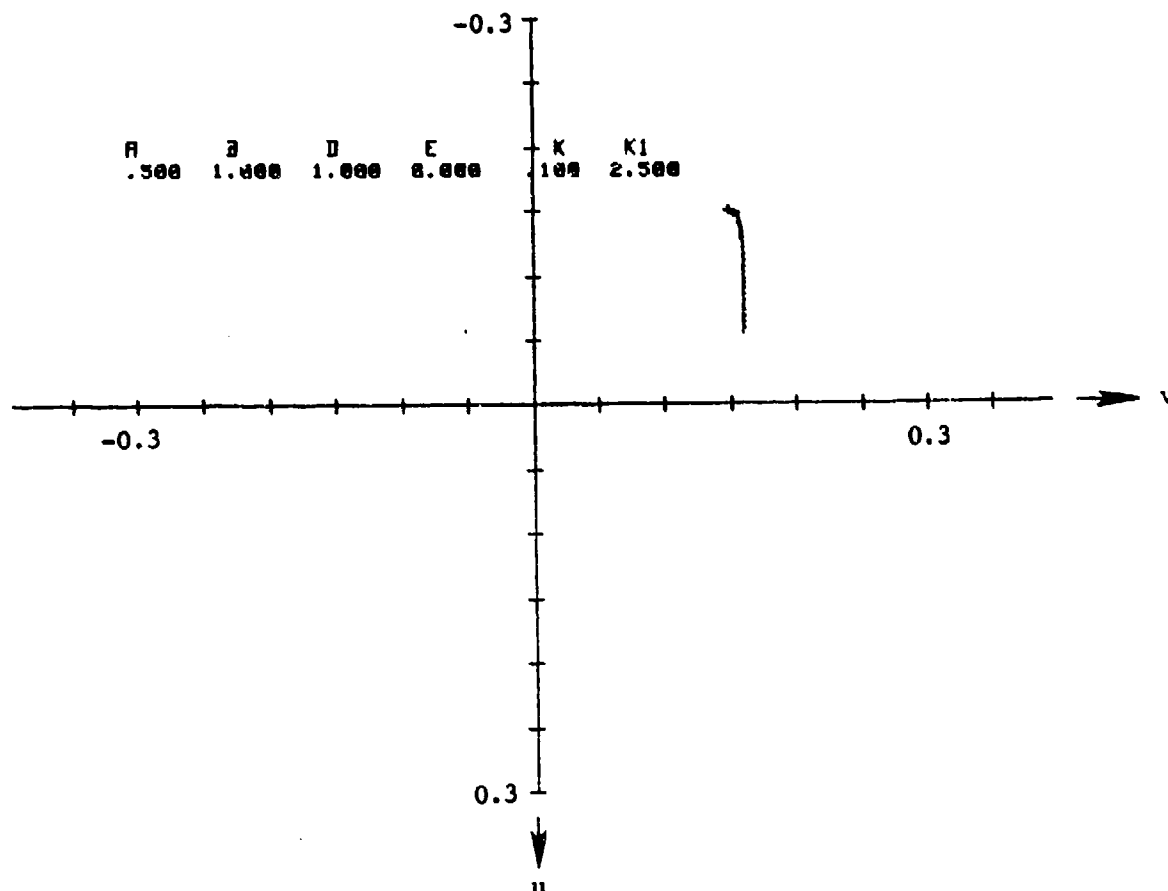


Fig. 10. Response to a steadily increasing pitching moment.

(a) $A = 0.5$, $E = 0$.

In this example and Figures 10b and 10c, the rotor is initially at steady state with the control axis and thrust vector at $u = u_T = -0.15$, $v = v_T = 0.15$. In the course of 12 revolutions shown, the nondimensional pitching moment is steadily increased from zero to 0.03125. (i.e., $M = 0.0625 \tau/24/\pi \cos \tau$). With A less than 1.0, $E = 0$, the thrust vector precesses in the positive pitch angle direction. There is also a small roll offset associated with the rate of pitching moment increase. (A constant pitching moment produces a pitch angle response but no roll offset; see Eq. (14).) With A greater than 1.0, $E = 0$, the thrust vector would precess in the negative pitch angle direction. However, as discussed in the next example, this is not physically possible.

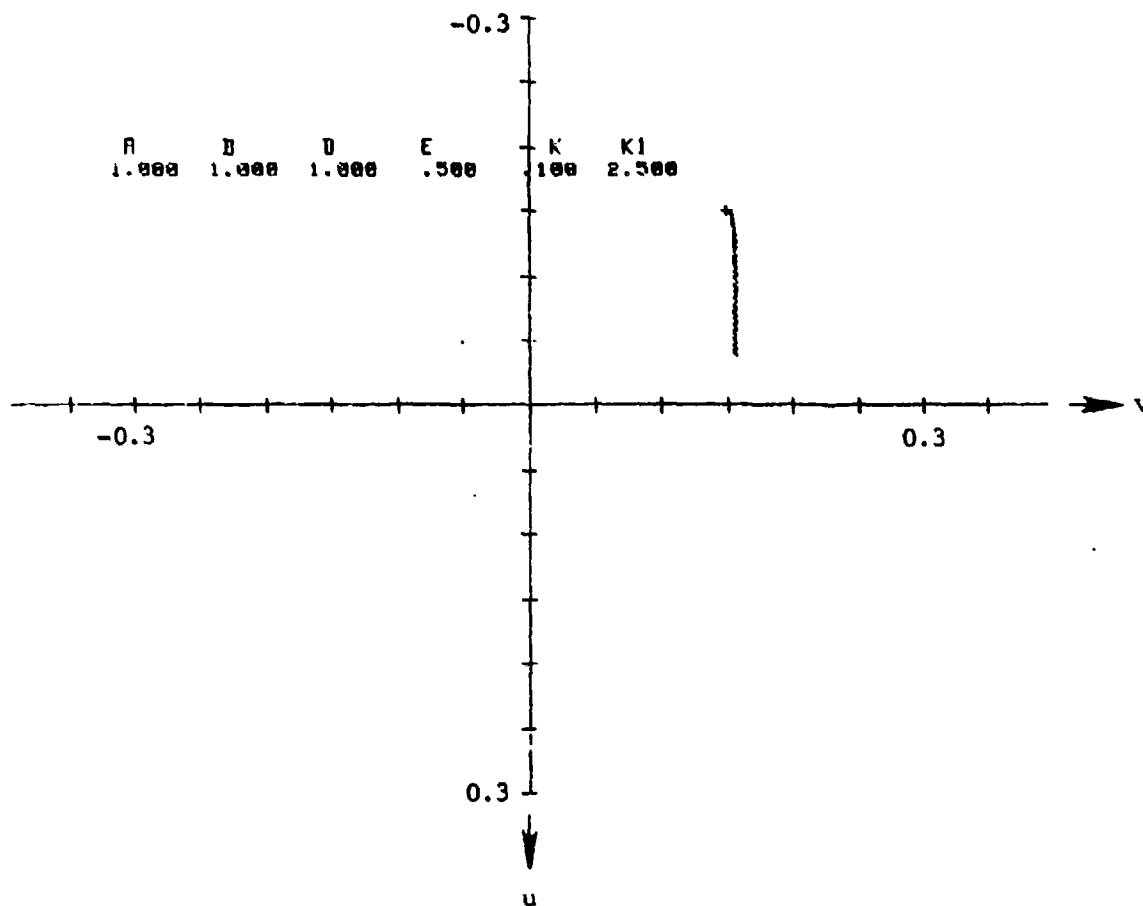


Fig. 10. Response to a steadily increasing pitching moment. (continued)
(b) $A = 1.0$, $E = 0.5$.

The effect of $E > 0$ is very similar to the effect of $A < 1.0$. The design parameters A and E are not independent as will be seen by reviewing their definitions in the Notation list and noting that the angles ϵ_1 and ϵ_3 are, for a rotor with conventional aerodynamics, the same angle. Since $(I_3 - I_2)/I_1$ is always less than unity, $A = 1.0$ is not attainable without $E > 0$; the condition $EB + (1 - A) = 0$ necessary to prevent a positive pitch angle response to a positive pitching moment is not attainable at all. In fact, $EB + (1 - A) = 1 - (I_3 - I_2)/I_1$. This is not an undesirable characteristic so long as $(I_3 - I_2)/I_1$ is not too much less than unity.

It is the necessary characteristic for airspeed stability in hover. With the control axis held vertical, a small airspeed in any direction will cause the thrust vector to tilt in the opposite direction, reducing the airspeed. A value too much below unity, however, would necessitate a large horizontal tail for speed stability at post-transition speeds (where pitching moment on the rotor decreases with increasing speed), large control inputs for roll trim at high forward speed, and pitch trim at transition speed.

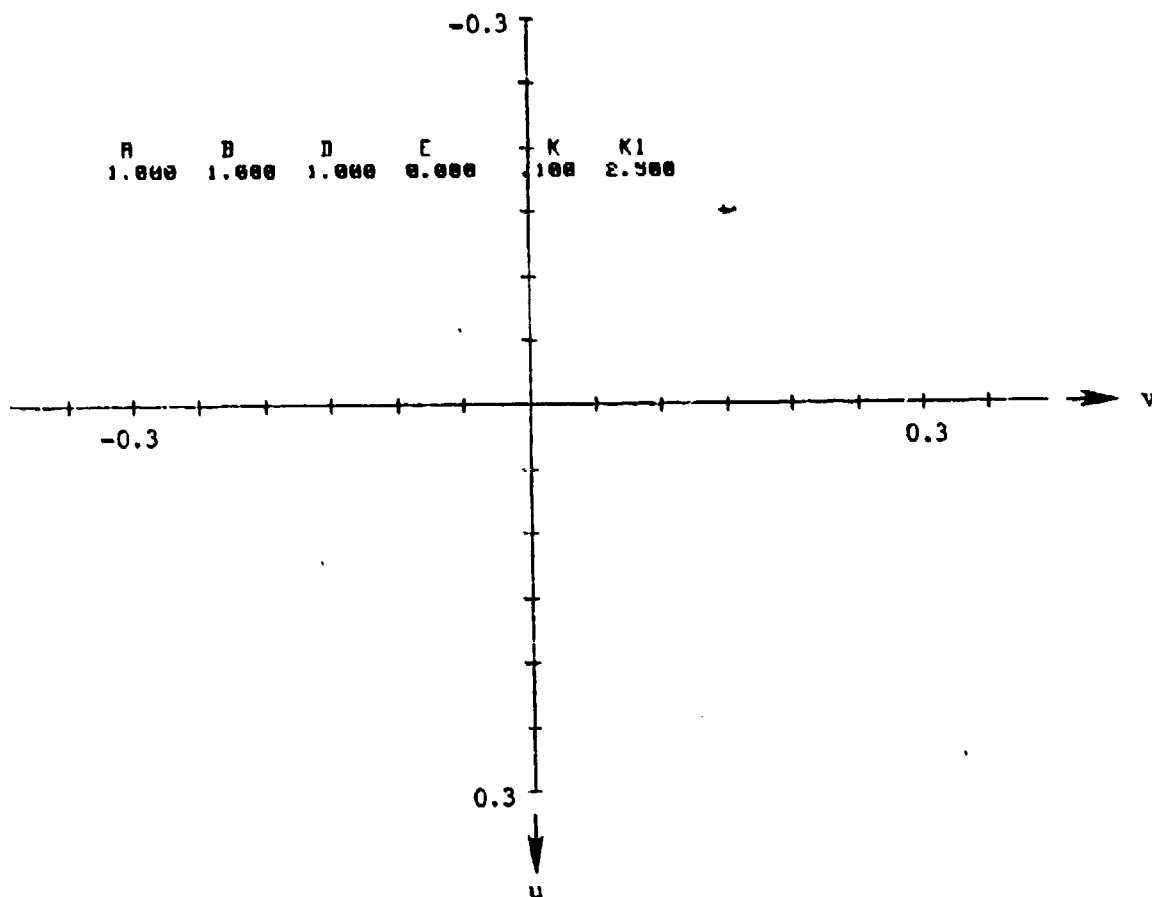


Fig. 10. Response to a steadily increasing pitching moment. (continued)
(c) $A = 1.0$, $E = 0$.

As expected, the only response with $A = 1$, $E = 0$ is a very small positive roll angle response to the rate of pitching moment increase.

CONCLUSIONS

1. A very simple feedback system providing a negative teetering moment and negative feathering moment proportional to teeter angle is sufficient to provide the desired type of stability and control. The required levels of feedback gains for good flying qualities appear to be easily attainable with either a circulation control rotor or a rotor with moveable trailing edge flaps.

2. Further requirements for good flying qualities are:

- a. The quantity $(I_3 - I_2)/I_1$ should be only slightly less than unity, and
- b. The damping axis should be nearly coincident with principal axis 2.

Previous studies have suggested that these requirements can be met with a mass-balancing of the rotor, which adds appreciably but not unacceptably to its weight. A detailed design study is needed to confirm this tentative conclusion.

3. If the requirements of item 2 are met, then a rotor of this type would have the advantage of very low coupling between longitudinal and lateral stability and control, compared to a conventional teetering or articulated rotor.

4. A rotor of this type (fully gimballed) is inherently free of vibrations due to hub moment oscillations (except possibly very small oscillations associated with the feedback system if a mechanical feedback actuation is employed). In all of the steady-state/fixed-control-axis situations considered herein, it is further totally free of any oscillations of the direction of the thrust vector. Small oscillations of the thrust vector direction do occur in transient situations and during steady-state precession of the thrust vector to a new position. Two-per-rev oscillations of the thrust vector amplitude and the in-plane force amplitude at high forward speed would be similar to those of any other two-bladed rotor. All in all, the outlook for relatively low vibration levels appears to be favorable.