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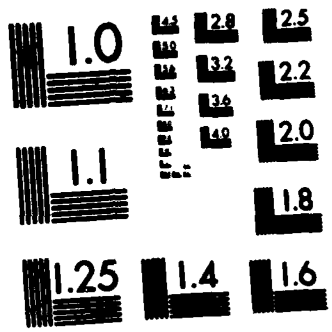

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Monterey, California

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NUMBER OF SAMPLES NEEDED TO OBTAIN
DESIRED BAYESION CONFIDENCE INTERVALS
FOR A PROPORTION

by

Robert B. Manion

March 1988

Thesis Advisor: G. F. Lindsay

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NUMBER OF SAMPLES NEEDED TO OBTAIN DESIRED BAYESIAN CONFIDENCE INTERVALS FOR A PROPORTION

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Submitted in partial fulfillment of the requirements for the degree of

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This thesis analyzes a Bayesian method for determining the number of samples that are needed to produce a desired confidence interval size for a proportion or probability. It compares the necessary sample size from Bayesian methods with that from classical methods and develops computer programs relating sample size and confidence interval size when a Beta prior distribution is employed. Tables and graphs are developed to assist an experimenter in determining the number of samples needed to produce desired confidence in this estimate of a proportion or probability.

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I. INTRODUCTION

The Naval Air Systems Command has established the Age Exploration Program for F/A-18 aircraft using Reliability-Centered Maintenance procedures in an effort to reduce maintenance costs by specifying only maintenance insuring flight integrity. Among other features of this program, fleet leader aircraft are sampled on a regular basis, with emphasis on inspection for cracks in selected structural components. Because of the potential dangers presented by cracks in aircraft, the Engineering Support Office at North Island Naval Air Station is concerned with determining the actual probability of detection of these cracks for each of its aircraft inspectors. Their proposal is to prepare test specimens (with cracks) which may be used to sample an inspector's detection performance, leading to estimates of detection probability. This thesis responds to their question of how many trials are necessary to estimate detection probability, and to the more general question of the sample size needed to estimate a proportion or probability using a set of Bernoulli trials.

There are many ways to produce estimators for unknown parameters such as our parameter; the probability of detection. Some of these methods have excellent properties. After examining North Island's problem, we came to the conclusion that the best way to estimate the unknown probability would be to use a confidence interval. "A confidence interval for an unknown parameter gives both an indication of the numerical value of an unknown parameter as well as a measure of how confident we are of that numerical value." [Ref. 1:p. 383] It is important to note that the size of the confidence interval depends upon the number of samples used to determine the confidence interval.

The primary focus of our study will be to determine the number of samples needed to obtain a specific confidence interval size for a proportion or probability. It will be seen that the approaches used throughout this work can be applied to more situations than just North Island's problem. By using the extensive tables and graphs included in the appendices of this document, the decision maker can relate the necessary number of samples to the appropriate confidence interval size that is warranted by his situation.

There are various methods that can be used to find a sample size to estimate a proportion or a probability. In Chapter II we will describe how we can use Classical methods to determine sample size. We will explain how we can determine a point estimate and how this point estimate can be used to obtain a confidence interval. Then we will use the confidence interval to determine the number of samples necessary to achieve a desired confidence interval size. In the next chapter we will describe the prior, sampling, and posterior distributions as they are related to Bayes Theorem. Also, in Chapter III we will introduce the Beta density function as our prior distribution. Then using the Binomial as our sampling distribution, we will show that the posterior density function is also Beta. In Chapter IV we will include an explanation of how a decision maker can determine his parameters for the Beta prior distribution. We will develop a set of graphs that can be used by this decision maker to determine the necessary sample size to obtain a desired 95% confidence interval size for the proportion. The next chapter will explain how we can use different Beta prior distributions with the same mean to determine the required sample size for estimating a proportion.

Finally, we will present a summary of what we accomplished and some suggestions for further research in using Bayesian methods to reduce the necessary number of samples to estimate a proportion or probability.

II. FINDING A SAMPLE SIZE TO ESTIMATE A PROPORTION USING CLASSICAL METHODS

This chapter will explain how we can use classical methods to find a sample size to estimate a proportion or probability. First, we will describe how we can attain a point estimate for a probability. Then we will use this point estimate to establish a confidence interval for the proportion. The confidence interval for the proportion that is derived from the point estimate will provide a measure of how accurate this point estimate is. Next, we will use the confidence interval to determine how many samples we will need for a particular interval size.

A. THE POINT ESTIMATE FOR A PROPORTION

"Typically, in a problem of parameter estimation we assume we have available a random sample of a random variable X, whose probability law is assumed known, except for the values for the parameters of the probability law. The problem then is to use the observed numbers to guess (estimate) these unknown parameter values." [Ref. 1:p. 359] From this one can say in general, that the estimator of an unknown parameter will be a function of the random variable X. One method that can be used to estimate our unknown proportion is to obtain a point estimate. "Basically, point estimation concerns the choosing of a statistic, that is, a single number calculated from sample data (and perhaps other information) for which we have some expectation, or assurance, that it is 'reasonably close' to the parameter it is supposed to estimate". [Ref. 2:p. 186] If we were to consider North Island's problem, we could calculate a point estimate for our detection probability P_d by assuming that each inspection conducted by a specific aircraft inspector was a Bernoulli trial with the same parameter P_d. We will assume that each trial is independent. If we conduct n inspections on n cracked aircraft components, and let

X_i = 1 if crack in component is discovered

X_i = 0 otherwise

then $X_1, X_2, \ldots, X_n$ is a random sample of a Bernoulli random variable X. Once our trials are completed we have observed sample values $x_1, x_2, \ldots, x_3$ and we can estimate our proportion by the following,

$$\hat{p}_d = \frac{1}{n}\sum_{i=1}^{n} x_i, \tag{2.1}$$

or

$$\hat{p}_d = \frac{k}{n}$$

where

$$k = \sum_{i=1}^{n} x_i$$

This is the point estimate for our proportion.

B. DETERMINING THE CONFIDENCE INTERVAL FOR A PROPORTION

A common classical method for obtaining a 95% confidence interval for detection probability P_d is to use the normal approximation to the binomial distribution, which is

$$\hat{p}_d - 1.96\sqrt{\frac{\hat{p}_d(1-\hat{p}_d)}{n}} \le p_d \le \hat{p}_d + 1.96\sqrt{\frac{\hat{p}_d(1-\hat{p}_d)}{n}} .$$

[Ref. 3:p. 112] (2.2)

"A good rule of thumb is to use the normal approximation to the binomial only when np and n(1-p) are both greater than 5." [Ref. 2:p. 112] With Equation 2.2 we can compute a 95% confidence interval for our proportion. For example, suppose that 20 items with cracks are inspected and 15 are identified as having cracks. Then from Equation 2.1 our point estimate of detection P_d is 0.75 and the 95% confidence interval is, from Equation 2.2,

$$0.75 - 0.19 \le P_d \le 0.75 + 0.19 ,$$

or

$$0.56 \le P_d \le 0.94 .$$

This says that as a result of our sample of 20 items, we are 95% certain that this interval (0.56 to 0.94) contains the true value of our proportion P_d. One can observe that the interval size for this example is 0.38. One can also observe from Equation 2.2 that the greater the sample size n, the smaller will be the size of the confidence interval.

C. FINDING SAMPLE SIZE FROM CONFIDENCE INTERVAL

The size of our sample can be determined by specifying how accurate we wish our estimate to be. This is reflected by the size of our confidence interval. Note that the interval is

$$P_d \pm A,$$

where

$$A = 1.96\sqrt{\frac{\hat{p}_d\left(1-\hat{p}_d\right)}{n}}. \tag{2.3}$$

Hence, if we desire a confidence interval of $\pm$ A, we simply solve Equation 2.3 for n and get

$$n = \left(\frac{1.96}{A}\right)^2 \hat{p}_d\left(1-\hat{p}_d\right). \tag{2.4}$$

If we are without any prior data we must guess at our sample result P_d, in order to determine n. However, sample size from Equation 2.4 is maximized at $P_d = 0.5$, so if we do not want to guess, worst case planning suggests that we use

$$n = \left(\frac{1.96}{A}\right)^2 (0.5)\,(0.5)$$

or

$$n = 0.9604/A^2.$$

This yields the results seen in Table 1 which show the required number of samples to obtain a 95% confidence interval of various sizes when $P_d = 0.5$.

This is conservative. If we agreed that the probable detection probability was closer to 0.7, we would use

$$n = \left(\frac{1.96}{A}\right)^2 (0.7)\,(0.3)$$

or

$$n = 0.8067/A^2$$

This yields the results seen in Table 2 which shows the required number of samples to obtain a 95% confidence interval sizes when $P_d = 0.7$.

TABLE 1

NUMBER OF SAMPLES NECESSARY TO OBTAIN
A DESIRED 95% CONFIDENCE INTERVAL SIZE
USING A POINT ESTIMATE OF $P_d = 0.5$

Desired 95% Confidence Interval Size = 2A	Required Sample Size n
0.05	1537
0.10	384
0.15	171
0.20	96
0.25	62
0.30	43
0.35	32

TABLE 2

NUMBER OF SAMPLES NECESSARY TO OBTAIN
A DESIRED 95% CONFIDENCE INTERVAL SIZE
USING A POINT ESTIMATE OF $P_d = 0.7$

Desired 95% Confidence Interval Size = 2A	Required Sample Size n
0.05	1291
0.10	323
0.15	143
0.20	81
0.25	52
0.30	36
0.35	26

In Chapter III we will discuss how we can use Bayesian Methods to estimate a proportion, and how by the use of prior information, the needed number of observations may be less than that shown in Tables 1 and 2 above.

III. BAYESIAN APPROACH TO ESTIMATE A PROPORTION

Another way we can determine a sample size to estimate a proportion is to use a Bayesian approach. The general idea behind a Bayesian approach to estimation is that we have some knowledge about possible values of the parameter prior to taking the observations, and this information may be aggregated with the experimental results to provide a better estimate (smaller confidence interval) than that from the experimental results alone.

In this chapter we will describe the Bayesian approach that will be used throughout this writing. We will accomplish this by describing the three parts of the Bayesian method that are related by Bayes Theorem: the prior distribution, the sampling distribution and the posterior distribution. Next we will explain our rationale for selecting the Beta distribution as our prior density function and the Binomial as our sampling distribution. Finally, we will use the Beta prior distribution and the Binomial sampling distribution to derive our posterior density function, yielding the known result that the posterior density function is a Beta distribution with different parameters than the prior distribution.

A. BAYES THEOREM AND ITS PARTS

An alternative method to estimate a proportion is to use a Bayesian approach. This Bayesian approach makes use of the expertise of engineers, scientists and others who generally have sound intuition concerning the problem area that is being analyzed. These experts can place subjective bounds on the range of the possible values of the parameters to be estimated. By using this expert intuition we can achieve the same confidence interval size with fewer samples.

Bayesian methods are derived from Bayes Theorem. If we let Y be a continuous random variable with density function f(Y) so that

$$\int_{-\infty}^{\infty} f(Y)\,dY = 1,$$

and we are given effect k, Bayes theorem states that

$$Pr(Y \mid k) = \frac{Pr(k \mid Y) f(Y)}{\int_0^n Pr(k \mid Y)(Y)\,dY}.$$

[Ref. 4:p. 558] (3.1)

Equation 3.1 can be broken into three parts. The sampling distribution is $P_r(k|Y)$. The sampling distribution is the probability function from which the observations of k are to be taken. The prior distribution is f(Y). "The prior distribution of a parameter θ is a probability function or probability expressing our degree of belief about the value of θ, prior to observing a sample of a random variable k whose distribution function depends on θ." [Ref. 1:p. 553] The posterior distribution is $f(Y|k)$ and its mean value is our Bayesian Estimate.

B. THE SELECTION OF THE PRIOR

We will use the posterior distribution of our Bayesian approach to estimate θ. The value θ is our proportion and can take on any value between 0 and 1. Therefore, our prior probability distribution must be continuous.

The expertise of those familiar with the problem area may provide prior bounds (on the proportion) that are closer than 0 and 1.0, and the prior probability distribution should reflect this information. Two ways to set bounds on θ (the unknown probability) and consequently establish a prior probability distribution would be to use the Uniform distribution or the Beta distribution. The uniform density function is

$$f(\theta) = \frac{1}{\theta_{hi} - \theta_{lo}}$$

where,

$$0 \le \theta_{lo} \le \theta \le \theta_{hi} \le 1, \qquad (3.2)$$

and the Beta density function is

$$f(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} , \qquad (3.3)$$

where $0 \le \theta \le 1$,

$$\alpha,\ \beta > 0.$$

Figure 1 shows a Uniform distribution for a random variable that is bounded between 0.142 and 0.858, and a Beta distribution that has 98% of its density between 0.142 and

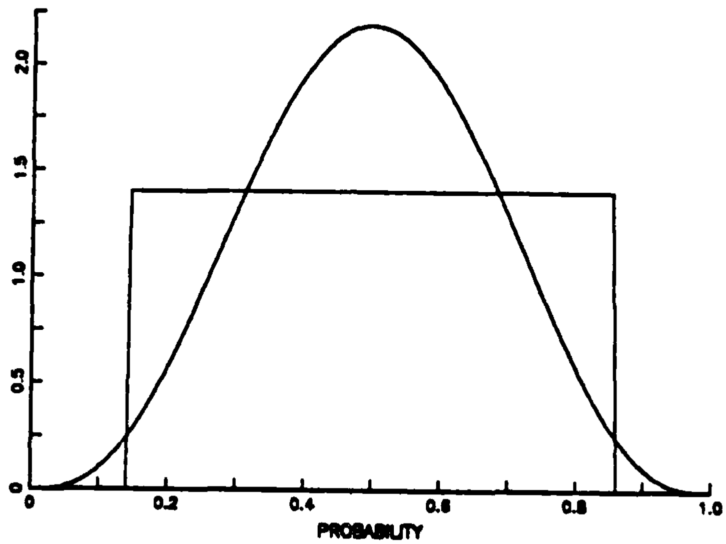

Figure 1 Beta Distribution ($\alpha = 4$, $\beta = 4$) and Uniform Distribution a = 0.142, b = 0.858

0.858 (viz., 1% in each tail), with parameters $\alpha = 4$, $\beta = 4$. The points (0.142 and 0.858) that bound 98% of the density of the Beta distribution were determined by calculating the inverse cumulative distribution function of a Beta distribution with parameters $\alpha = 4$, $\beta = 4$, at 0.01 and 0.99. This method can be duplicated using other values of α and β to insure that 98% of the density will lie between the two resulting points.

By using the Beta distribution, our experts will be able to better control their prior beliefs. If a group of experts feel that the likelihood of θ occurring in a particular section is greater than that of it occurring in another section, then by selecting the appropriate parameters α and β of the Beta distribution, they can institute a prior distribution to accommodate their desires. For these and other reasons we will use the Beta distribution as our prior distribution. If the experimentor simply gives bounds on θ as he would for a uniform distribution, a Beta prior (as a two parameter distribution) may be "fit" to those bounds. Also, one should remember that the Beta distribution can be skewed to one side or the other, based on the values of α and β. This should be taken into account when selecting the prior.

The probability function from which we will take our observations of k will be the binomial distribution. This is because the binomial distribution counts the number of success for n Bernoulli trials, viz.,

$$Pr(k \mid p) = \binom{n}{k} p^k (1-p)^{n-k} \quad , \ k = 0, 1, \ldots, n \ . \tag{3.4}$$

C. DERIVATION OF THE POSTERIOR DISTRIBUTION

Using a prior distribution that is Beta (α, β) and a sample distribution that is binomial (n, p), we have from Equations 3.1, 3.3, and 3.4, posterior distribution:

$$Pr(p \mid k) = \frac{\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}\binom{n}{k}p^{k}(1-p)^{n-k}}{\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}\binom{n}{k}p^{k}(1-p)^{n-k}dp}$$

where k is the number of successes. If we combine terms, our posterior becomes

$$Pr(p \mid k) = \frac{\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\binom{n}{k}p^{\alpha+k-1}(1-p)^{\beta+n-k-1}}{\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\&\beta)}\binom{n}{k}\int_0^1 p^{\alpha+k-1}(1-p)^{\beta+n-k-1}dp}$$

Now we can cancel out terms. Notice that the combinatorial $\binom{n}{k}$ cancels out. We now have

$$Pr(p \mid k) = \frac{p^{\alpha+k-1}(1-p)^{\beta+n-k-1}}{\int_0^1 p^{\alpha+k-1}(1-p)^{\beta+n-k-1}dp}.$$

This can be rewritten as

$$Pr(p \mid k) = \frac{p^{\alpha+k-1}(1-p)^{\beta+n-k-1}}{\frac{\Gamma(\alpha+k)\Gamma(\beta+n-k)}{\Gamma(\alpha+k+\beta+n-k)}}$$

or

$$Pr(p \mid k) = \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+k)\Gamma(\beta+n-k)} p^{\alpha+k-1}(1-p)^{\beta+n-k-1},$$

which is our posterior distribution. The posterior derived above is a Beta distribution with parameters α + k and

$\beta + n - k$ and is a well-known result from Bayesian statistics. [Ref. 1:p. 565]

In the Bayesian approach the point estimate of p is the mean of the posterior, or $E[p|X]$, and a 95% confidence interval on that parameter θ is provided by the 2.5 and 97.5 percentiles of the posterior distribution. Thus, with a Beta prior and Bernoulli trials, the size of the resulting confidence interval depends upon the parameters of the prior (α and β), the sample size n, and k, the number of successes in the sample.

As in the classical method, we need to know the number of successes to determine sample size n. Therefore, we are going to make the assumption that k, the number of successes, will equal the mean of our prior distribution multiplied by the number of samples or

$$\left(\frac{\alpha}{\alpha+\beta}\right)n.$$

This will result in the most conservative value of k if $\alpha = \beta$ because it maximizes the variance of the Beta posterior distribution, and it should result in a "fairly" conservative value otherwise. Making this assumption, we now have

$$Pr(p \mid k) = \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+(\frac{\alpha}{\alpha+\beta})n)\Gamma(\beta+n-(\frac{\alpha}{\alpha+\beta})n)} p^{\alpha+(\frac{\alpha}{\alpha+\beta})n-1}(1-p)^{\beta+n-(\frac{\alpha}{\alpha+\beta})n-1}$$

which becomes

$$Pr(p \mid k) = \frac{\Gamma(\alpha + \beta + n)}{\Gamma(\alpha + (\frac{\alpha}{\alpha + \beta})n)\Gamma(\beta + (\frac{\beta}{\alpha + \beta})n)} p^{\alpha + (\frac{\alpha}{\alpha+\beta})n-1}(1-p)^{\beta(\frac{\beta}{\alpha+\beta})n-1}.$$

If, at this point, we let $\alpha^* = \alpha + (\frac{\alpha}{\alpha + \beta})\, n$ and let $\beta^* = B + (\frac{\beta}{\alpha + \beta})\, n$, we get

$$Pr(p \mid k) = \frac{\Gamma(\alpha^* + \beta^*)}{\Gamma(\alpha^*)\Gamma(\beta^*)} p^{\alpha^*-1}(1-p)^{\beta^*-1}. \tag{3.5}$$

Equation 3.5 is the posterior distribution that we will use throughout the remainder of this thesis. One should note that Equation 3.5 is Beta (α^*, β^*).

In the next chapter we will discuss how we developed tables, graphs and computer programs that can be used by an experimenter to determine the necessary sample size to estimate a proportion.

IV. PROVIDING THE DECISION MAKER THE MEANS TO DETERMINE THE APPROPRIATE SAMPLE SIZE

In this chapter we will direct our attention to using the Bayesian approach as a way to find the sample size to estimate a proportion. The decision maker will be asked for information about subjective bounds for the unknown proportion p. This information can be related to a prior Beta distribution. From this Beta distribution and a specification of the decision maker's desired 95% confidence interval size (which he wishes after the sampling), the necessary sample size may be determined. Our goal is to provide tables and curves to facilitate the decision maker in his determination of n.

First, we will describe the tables with which the decision maker can select the parameters for his prior Beta distribution that are the most appropriate for his subjective bounds. Next, we will describe how we constructed these tables. When this is completed we will discuss our methodology for developing the curves which can be used by our decision maker to determine the appropriate sample size for a proportion. Finally, we will use an example to describe how the decision maker can use the curves and computer programs to determine sample size.

Throughout this chapter we will explain how our computer programs assisted us in our analysis and describe how these programs can be used to assist another analyst in determining the sample size necessary to estimate a proportion.

At this point it is necessary to note that all programs presented in this writing were written in APL and can be duplicated on any computer capable of running an APL workspace. It should also be noted that these programs use extensive looping and may require a significant amount of time to run on some computers.

A. SELECTION OF PARAMETERS FOR THE BETA PRIOR DISTRIBUTION USING THE DECISION MAKER'S SUBJECTIVE BOUNDS ON THE UNKNOWN PROPORTION

Before we can employ our Bayesian approach to determine a sample size, we need to find the values for α and β, the parameters of the Beta prior distribution, that best fit our decision maker's subjective bounds. To determine these values, the decision maker could use a set of tables such as those found in Appendix A. He could simply scan these tables until he found the values in the columns labelled $P._{lo}$ and $P._{hi}$ that best reflect his subjective bounds for the unknown proportion.

As an example we have reproduced one of these tables as Table 3. If the decision maker believes that the true value of the proportion is somewhere between 0.14 and 0.86 he would go down the table until he found values in the third and fourth columns that are near 0.14 and 0.86 respectively. In this example the decision maker would select the fourth row with $P._{lo} = 0.142270$ and $P._{hi} = 0.857730$. We can see

TABLE 3

MEANS, VARIANCES, AND 98% BOUNDS FOR
BETA DISTRIBUTION WITH α = 4

α	β	P.lo	P.hi	Mean	Var
4	1	.421318	.997491	.800000	.026667
4	2	.222072	.967318	.666667	.031746
4	3	.173070	.915270	.571429	.030612
4	4	.142270	.857730	.500000	.027778
4	5	.120950	.801798	.444444	.024691
4	6	.105262	.749974	.400000	.021818
4	7	.093214	.702884	.363636	.019284
4	8	.083660	.660417	.333333	.017094
4	9	.075895	.622193	.307692	.015216
4	10	.069455	.587759	.285714	.013605
4	11	.064028	.556669	.266667	.012222
4	12	.059390	.528514	.250000	.011029
4	13	.055381	.502936	.235294	.009996
4	14	.051880	.479621	.222222	.009097
4	15	.048797	.458298	.210526	.008310
4	16	.046061	.438734	.200000	.007619
4	17	.043615	.420729	.190476	.007009
4	18	.041417	.404110	.181818	.006468
4	19	.039430	.388727	.173913	.005986
4	20	.037625	.374451	.166667	.005556
4	21	.035978	.361170	.160000	.005169
4	22	.034470	.348784	.153846	.004821
4	23	.033083	.337208	.148148	.004507
4	24	.031804	.326367	.142857	.004222
4	25	.030620	.316193	.137931	.003964

the corresponding values for the parameters that best fit that our decision maker's subjective bounds are α = 4 and β = 4.

In Chapter III we described a method that would insure 98% of our Beta prior density was between two possible values for our proportion. This method involved taking the inverse cumulative distribution function for a specific Beta distribution at 0.01 and 0.99. This is the method we used to design a computer program that could construct a set of tables to determine the values of α and β.

To create Table 3 and Appendix A we used the APL program entitled SENSE located in Appendix C. SENSE makes use of the APL subroutines BQUAN, NQUAN and BETA which are located in Appendix D. These subroutines calculate the inverse cumulative distribution function of a Beta distribution at 0.01 and 0.99, yielding the bound values ($P._{lo}$ and $P._{hi}$) in our tables. SENSE uses nested loops to vary the values of α from 1 to 5 and vary β from 1 to 50.

More extensive tables could be created by increasing the loops controlling the maximum values of our parameters. This can be accomplished in accordance with the comments provided at the beginning of SENSE.

Next, we will show how these parameters and the decision maker's desired 95% confidence interval size may be used to find the necessary sample size.

B. DETERMINING SAMPLE SIZE WITH GRAPHS

In this section we will develop a set of graphs. These graphs can be used by a decision maker to facilitate his determination of sample size n using the parameters of the Beta prior distribution and a desired 95% confidence interval size. To accomplish this we will first explain our methodology in developing the graphs. Then by use of an example, we will explain how the decision maker can use these graphs to determine the required sample size necessary to estimate a proportion.

In Chapter III we derived our posterior density function which was a Beta distribution with parameters $\alpha*$, $\beta*$. It was also stated in Chapter III that we assumed k, the number of successes, to be that which would result from the mean value of the Beta prior distribution, or $\left(\frac{\alpha}{\alpha+\beta}\right)n$. This assumption results in the parameters of our posterior Beta distribution being

$$\alpha^* = \alpha + \left(\frac{\alpha}{\alpha+\beta}\right)n \tag{4.1}$$

and

$$\beta^* = \beta + \left(\frac{\beta}{\alpha+\beta}\right)n \tag{4.2}$$

where α and β are the parameters of our prior distribution and n is the sample size.

Once we have obtained the parameters of our Beta posterior distribution we can compute the inverse cumulative distribution function at 0.025 and 0.975 for a Beta distribution with parameters $\alpha*$ and $\beta*$. This will yield the lower and upper bounds of a 95% confidence interval. We can then subtract the lower bound from the upper bound to determine the size of the confidence interval. Table 4 demonstrates (for a Beta prior with $\alpha = \beta = 4$) what happens as sample size n is increased from 1 to 1000. It can be seen, as in the classical method, that when sample size increases, the confidence interval size decreases. Hence, with enough values of n we could create a table that would tell us the value of n when we reached our desired confidence interval size.

At this point it is important to realize that our computer program uses subroutine BQUAN to calculate the inverse cumulative density function at 0.025 and 0.975. BQUAN has a shortcoming, in that, it cannot compute the inverse cumulative distribution function for large values of $\alpha*$ and $\beta*$. Hence, it is necessary that we use another method to determine the bounds of our confidence interval for large parameters.

The Beta distribution has the following relationship with the F-distribution. That is

$$X_r = \frac{\alpha^* F_r(2\alpha^*, 2\beta^*)}{\beta^* + \alpha^* F_r(2\alpha^*, 2\beta^*)}$$

[Ref. 5:p. 151 and p. 380] (4.3)

where X_r is the cumulative distribution function for the Beta posterior distribution at the r^{th} quantile, and $F_r(a,b)$ is the distribution function for an F-distribution with a,b degrees of freedom. Equation 4.3 was previously mentioned in Chapter III. We also used a software package developed by Dr. Peter W. Zehna of the Naval Postgraduate School to evaluate the F-distribution at r. This was done in the following manner:

1. We selected integer values of sample size n that were at, or near, 500 and 1000. These values were selected to insure that α^* and β^* were also integers.

2. We computed α^* and β^* using Equations 4.1 and 4.2 respectively.

3. The values of $F_{0.025}$ $(2\alpha^*, 2\beta^*)$ and $F_{0.975}$ $(2\alpha^*, 2\beta^*)$, where α^* and β^* were calculated using n near 500, were placed in a vector, along with the value of n and saved in the APL workspace. This vector is referred to as the X vector.

4. The values of $F_{0.025}$ $(2\alpha^*, 2\beta^*)$ and $F_{0.975}$ $(2\alpha^*, 2\beta^*)$, where α^* and β^* were calculated using n near 1000, were placed in a vector along with the value of n and saved in the APL workspace. This vector is referred to as the Y vector.

To employ our method we developed an APL program named CHARTPLUS located in Appendix E. CHARTPLUS is the main program used in our analysis and it accomplishes several functions. First, CHARTPLUS provides the subjective bounds associated with the parameters of the Beta prior distribution. It creates a table similar to Table 4. Finally, CHARTPLUS makes a vector of the lower bounds, the upper bounds and the confidence interval size, for each sample size.

To use CHARTPLUS, the user is required to enter the parameters of the prior Beta distribution, a vector of various sample sizes, and the X and Y vectors described

TABLE 4

THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVAL, WITH BETA PRIOR (α = 4, β = 4) AND k = ($\alpha/\alpha + \beta$) n SUCCESS IN THE SAMPLE

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	4.5000	4.5000	.1990	.8010	.6021
2	5.0000	5.0000	.2120	.7880	.5760
3	5.5000	5.5000	.2235	.7765	.5529
4	6.0000	6.0000	.2338	.7662	.5324
5	6.5000	6.5000	.2430	.7570	.5140
6	7.0000	7.0000	.2513	.7487	.4973
7	7.5000	7.5000	.2589	.7411	.4821
8	8.0000	8.0000	.2659	.7341	.4683
9	8.5000	8.5000	.2722	.7278	.4555
10	9.0000	9.0000	.2781	.7219	.4438
15	11.5000	11.5000	.3020	.6980	.3961
20	14.0000	14.0000	.3195	.6805	.3610
25	16.5000	16.5000	.3331	.6669	.3338
30	19.0000	19.0000	.3440	.6560	.3120
35	21.5000	21.5000	.3530	.6470	.2940
40	24.0000	24.0000	.3606	.6394	.2787
45	26.5000	26.5000	.3672	.6328	.2656
50	29.0000	29.0000	.3729	.6271	.2542
55	31.5000	31.5000	.3779	.6221	.2441
60	34.0000	34.0000	.3824	.6176	.2352
65	36.5000	36.5000	.3864	.6136	.2272
70	39.0000	39.0000	.3900	.6100	.2199
75	41.5000	41.5000	.3934	.6066	.2133
80	44.0000	44.0000	.3964	.6036	.2072
85	46.5000	46.5000	.3992	.6008	.2017
90	49.0000	49.0000	.4017	.5983	.1966
100	54.0000	54.0000	.4063	.5937	.1874
110	59.0000	59.0000	.4103	.5897	.1793
120	64.0000	64.0000	.4139	.5861	.1723
130	69.0000	69.0000	.4170	.5830	.1660
140	74.0000	74.0000	.4198	.5802	.1603
150	79.0000	79.0000	.4224	.5776	.1552
160	84.0000	84.0000	.4247	.5753	.1506
170	89.0000	89.0000	.4268	.5732	.1463
180	94.0000	94.0000	.4288	.5712	.1424
190	99.0000	99.0000	.4306	.5694	.1388
200	104.0000	104.0000	.4323	.5677	.1354
504	256.0000	256.0000	.4568	.5433	.0865
1000	504.0000	504.0000	.4692	.5308	.0617

earlier in this section. CHARTPLUS starts a loop that evaluates each value of our vector of sample sizes. Typically, our vectors contained values of sample sizes from 1 to 200. CHARTPLUS can evaluate vectors with other values of sample size. However, a 'good rule of thumb' is to limit the maximum sample size to 200. This is due to BQUAN's inability to compute large values of $\alpha*$ and $\beta*$.

After CHARTPLUS initiates looping it calls subroutine INTER2 (see Appendix E). INTER2 calculates $\alpha*$ and $\beta*$ using Equations 4.1 and 4.2. Then INTER2 calls subroutines BQUAN, NQUAN and BETA to calculate the inverse cumulative distribution functions for the Beta posterior distribution with parameters $\alpha*$, $\beta*$ at 0.025 and 0.975. This gives us the upper and lower values of our confidence interval. INTER2 then subtracts the lower bound from the upper bound to obtain the confidence interval size and returns to CHARTPLUS.

CHARTPLUS then formats the output and creates vectors in the APL workspace of the lower bounds, the upper bounds and the confidence interval sizes. CHARTPLUS continues to loop until our sample size vector is exhausted. Then CHARTPLUS calls subroutine CHARTER.

CHARTER is located in Appendix E and uses the X and Y vectors to calculate $\alpha*$, $\beta*$, the lower and upper bounds of the confidence interval and the interval size for the values of sample size n at, or near, 500 and 1000. CHARTER then formats the output in the exact same way as CHARTPLUS and concatenates each of the three vectors created in CHARTPLUS with the lower bound, upper bound, and interval size for n at or near 500 and 1000.

If we plot our vector of lower bounds and our vector of upper bounds as a function of our sample size vector we can obtain graphs shown in Figure 2 and in Appendix B. It is important that the decision maker realize these figures are not the confidence intervals. The actual confidence intervals must be determined after the samples are taken.

If we plot our vector of confidence interval sizes as a function of our sample size vector we obtain graphs as shown in Figure 3 and in Appendix B. These graphs can prove to be useful to the decision maker as seen in the following example,

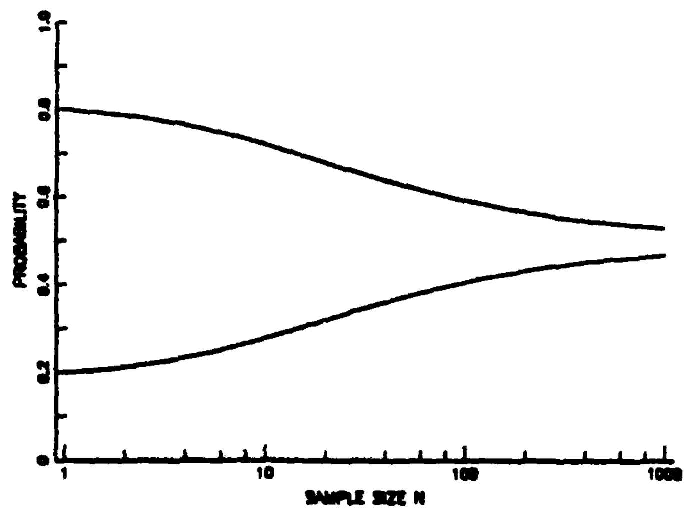

Figure 2 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 4$

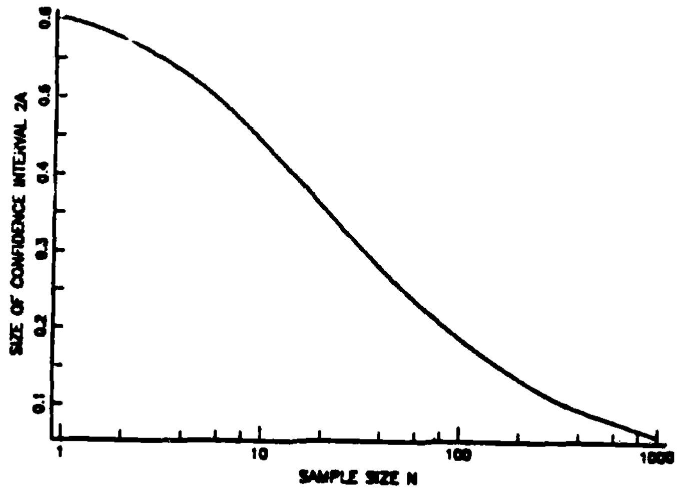

Figure 3 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 4$

Suppose the decision maker's prior Beta distribution parameters are $\alpha = 4$, $\beta = 4$, the same as determined in Section A of this chapter. In addition, suppose the decision maker desires the size of the 95% confidence interval for estimating the proportion to be 0.20. Then the decision maker can use Figure 3 to determine the most appropriate sample size to meet these criteria. The decision maker can find 0.20 on the ordinate, move across the graph to where 0.20 intercepts the curve, and read approximately 87 off the abscissa. This is the most appropriate sample size for n that reflects both the decision maker's subjective bounds and his desired 95% confidence interval size.

In the next section we will introduce a pair of computer programs that can be used to obtain the same results as graphing the confidence interval size for specific parameters of the Beta prior distribution.

C. DETERMINING SAMPLE SIZE USING COMPUTER PROGRAMS

Because it is possible that the decision maker may be without graphic capability and has input values not provided in the tables here, we have developed a pair of computer programs that can be used to facilitate his determination of n using the parameters of the Beta prior distribution and a desired 95% confidence interval size. We will do this by explaining the computer programs in detail. Then we will provide an example that will demonstrate how a decision maker can use these programs to determine the needed sample size to estimate a proportion.

The APL program SCHARTS was developed to assist the user in finding an interval of sample sizes. This interval contains the exact number of samples necessary to achieve the decision maker's desired 95% confidence interval size and is determined using the parameters that best fit his subjective bounds.

SCHARTS, located in Appendix F, requires the user to input the parameters of Beta prior distribution and a vector of various sample sizes. SCHARTS analyzes the sample size vector and identifies the two elements in the vector between which the exact number of samples required lies. If the vector of sample sizes fails to contain this exact number necessary to satisfy the decision maker's criteria, SCHARTS will inform the user.

SCHARTS is a modification of CHARTPLUS, uses the same subroutines (with the exception of CHARTER), and in general cannot evaluate sample sizes greater than 200.

Once we have found the interval containing the required number of samples, we use the APL program entitled CHARTS located in Appendix E. CHARTS allows the user to enter new sample size vectors of any length and produces parameters α*, β*, together with the lower and upper bounds of the confidence interval, and the 95% confidence interval size for each element in the vector.

CHARTS asks the user to input the parameters of the Beta prior distribution and a vector (of any length) of various sample sizes. The user should select a vector that contains all the integer values of the interval identified by SCHARTS. If the user inputs these elements in numerical order his output will be in the order of decreasing confidence interval size. This will allow the user to select the smallest value of sample sizes that meets or surpasses his desired 95% confidence interval size.

Suppose, continuing our example, that our decision maker's Beta prior distribution has the parameters α =4,

β = 4 and his desired 95% confidence interval size is 0.20. Then he can use SCHARTS and CHARTS in the following manner to determine the sample size that will meet his goals.

We will identify our vector of sample sizes as C. We assign C the values in the following APL session shown in Figure 4.

```
      C←30 40 50 60 70 80 90
      SCHARTS
ENTER ALPHA AND BETA PARAMETERS
⎕:
      4 4
ENTER VECTOR OF SAMPLE SIZES
⎕:
      C
LIMITS FOR 0.20
ALPHA BETA  N    CI SIZE
  4     4   80    .20725
  4     4   90    .19655


CHARTS
ENTER VALUES OF ALPHA AND BETA PARAMETERS
⎕:
      4 4
ENTER NEW SAMPLE SIZE VECTOR, MUST ENTER AT LEAST 2 NUMBERS
⎕:
      81 82 83 84 85 86 87 88 89
     N          A*           B*          P.LO   P.HI   CI SIZE
  81.0000     44.5000     44.5000     .3970  .6030  .20610
  82.0000     45.0000     45.0000     .3975  .6025  .20497
  83.0000     45.5000     45.5000     .3981  .6019  .20386
  84.0000     46.0000     46.0000     .3986  .6014  .20276
  85.0000     46.5000     46.5000     .3992  .6008  .20169
  86.0000     47.0000     47.0000     .3997  .6003  .20063
  87.0000     47.5000     47.5000     .4002  .5998  .19958
  88.0000     48.0000     48.0000     .4007  .5993  .19856
  89.0000     48.5000     48.5000     .4012  .5988  .19755
```

Figure 4 An APL Session using the Programs SCHARTS and CHARTS

We see that when n is 87 we have surpassed our decision maker's desired confidence interval size. Hence, 87 is the number of samples he should take.

In the next chapter we will discuss other uses of the Bayesian approach to find the number of samples needed to obtain a desired 95% confidence interval size.

V. SOME ADDITIONAL WAYS THAT THE BAYESIAN METHOD CAN BE USED TO DETERMINE SAMPLE SIZE

In this chapter we will discuss other ways that our Bayesian approach can be used to assist in finding the number of samples required to obtain a desired 95% confidence interval size for a proportion.

First, we will discuss the relationship between different Beta prior distributions with the same mean and their sample sizes. We will accomplish this by deriving an equation that illustrates this relationship. Then we will describe a computer program that can be used to graph this relationship. We will also explain how analysts can use this graph to determine sample size. Finally, we will illustrate what happens to the sample size necessary to achieve a desired 95% confidence interval size for a proportion when the α parameter of the prior distribution is held constant and the β parameter is varied. We will do this by use of a graph with which the analyst can determine the number of samples required for a particular 95% confidence interval size as β is varied.

A. FINDING THE NECESSARY SAMPLE SIZE FOR BETA PRIOR DISTRIBUTIONS WITH THE SAME MEAN

Suppose our prior density is Beta (α, β), so that the mean for our prior density is

$$\frac{\alpha}{\alpha + \beta} = Q$$

If at some sample size n we obtain the desired confidence interval size on our Beta posterior distribution, then for any other prior density, Beta (α', β'), whose mean is equal to Q, we can determine the new sample size n' by

$$n' = n + \frac{\alpha - \alpha'}{Q}. \tag{5.1}$$

We can show this by assuming that our desired confidence interval size is achieved when our posterior is Beta ($\alpha*$, $\beta*$). Let us also assume that

$$\alpha^* = \alpha + \left(\frac{\alpha}{\alpha+\beta}\right)n \tag{5.2}$$

and

$$\beta^* = \beta + \left(\frac{\beta}{\alpha+\beta}\right)n \tag{5.3}$$

Now, if we have a different Beta prior with parameters α' and β', one can reason that there exists some sample size value n' that, when used in our Bayesian approach, will result in our posterior distribution being Beta ($\alpha*$, $\beta*$). Hence, we want

$$\alpha^* = \alpha' + \left(\frac{\alpha'}{\alpha'+\beta'}\right)n' \tag{5.4}$$

and

$$\beta^* = \beta' + \left(\frac{\beta'}{\alpha'+\beta'}\right)n' \tag{5.5}$$

We must allow n' to be a continuous number. When we compare Equation 5.2 with 5.4 we can state

$$\alpha' + \left(\frac{\alpha'}{\alpha'+\beta'}\right)n' = \alpha + \left(\frac{\alpha}{\alpha+\beta}\right)n$$

If the means of our two Beta prior distributions equal Q we have

$$\alpha' + Q \times n' = \alpha + Q \times n.$$

When we solve for n' we get

$$n' = n + \frac{\alpha - \alpha'}{Q}.$$

It can be shown that this value of the new sample size n' can be substituted into Equation 5.5 to obtain β*.

We used Equation 5.1 in developing the APL program named SMEAN located in Appendix G. SMEAN provides the user with the necessary sample size to obtain a desired 95% confidence interval size when $\alpha = 1$. It also provides the α parameter when the necessary sample size is zero. Therefore, the user has two points he can plot on a graph. In addition, SMEAN provides the user with the slope of the line connecting these two points.

SMEAN asks the user to provide the number of samples necessary to obtain a desired 95% confidence interval size for a proportion. In addition, it asks for the parameters of the Beta prior distribution.

Figure 5 was constructed using SMEAN, from a Beta prior distribution with parameters $\alpha = 4$, $\beta = 4$. The necessary sample sizes for each confidence interval size were determined using Figure 3 in Chapter IV.

The analyst can use Figure 5 to determine the sample size required for any α parameter. He can do this by locating the α parameter that he wants on the abscissa, then

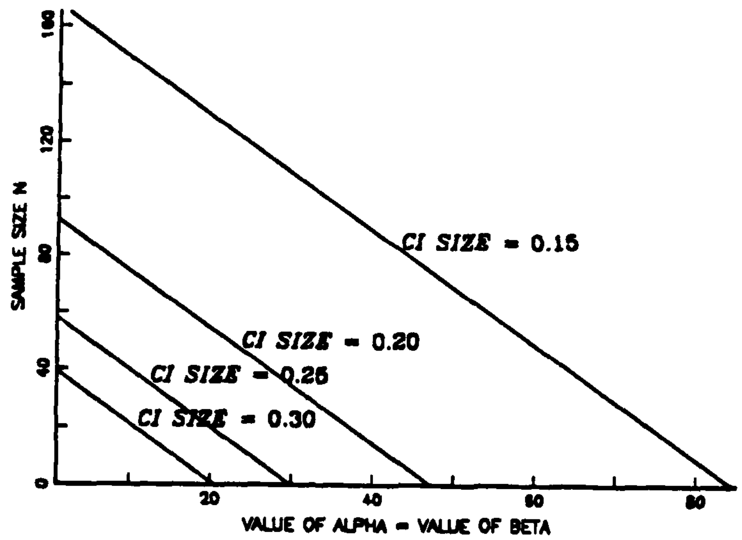

Figure 5 Number of samples needed for estimation of a proportion for a Beta Prior with Mean = 0.5

selects the desired 95% confidence interval size line, and reads the corresponding ordinate.

If the analyst has no graphic capabilities, he can use the APL program entitled GENERAL located in Appendix G. GENERAL provides the user with the required sample size for a desired 95% confidence interval size for different Beta distributions which have the same mean. The user must know the necessary sample size to obtain a desired 95% confidence interval size for at least one of these Beta prior distributions.

In the last section of this chapter we will show what happens to the required sample size when the α parameter of the prior distribution is held constant and the β parameter is varied.

B. DETERMINING THE REQUIRED SAMPLE SIZE AS THE β PARAMETER IS VARIED AND THE α PARAMETER IS HELD CONSTANT

In this section we demonstrate what happens to the sample size required to obtain a desired 95% confidence interval size as the β parameter of the prior distribution is varied and the α parameter is held constant.

We accomplish this by constructing Figures 6 and 7. These figures were constructed using the APL program CHARTS. Through trial and error we entered vectors with different sample sizes for n until we reached the exact 95% confidence interval size for a particular α parameter of the prior density. Then we changed our β parameter and repeated this process. We used approximately 20 different values of β for each α we evaluated. Then we plotted our results. It should be mentioned that to do this we treated n as if it were a continuous variable.

The analyst can use these charts by varying the β parameter of the prior distribution on the abscissa. Then, he can find the curve for his α parameter and determine how his sample size changes on the ordinate.

Similar graphs can be developed to see the effect of varying α and holding β constant using the methods discussed in this section.

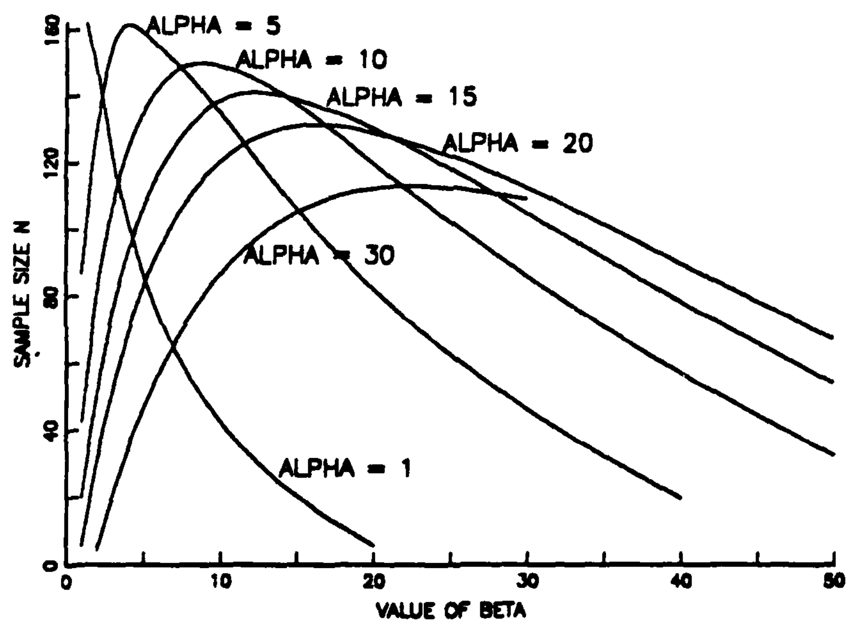

Figure 6 Number of Samples needed for Estimation of a Proportion when the α Parameter of the Beta Prior is Constant, the β Parameter Varied, and Desired Confidence Interval Size is 0.20.

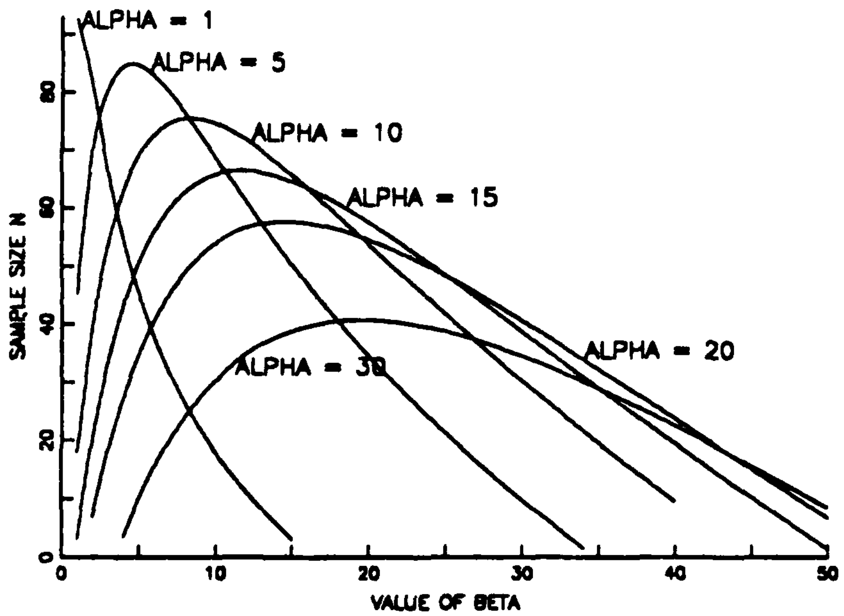

Figure 7 Number of Sample Needed for Estimation of a Proportion When the α Parameter of the Beta Prior is Constant, the β Parameter is Varied, and the Desired Confidence Interval Size is 0.15.

In the next chapter we will summarize what we have accomplished, and suggest some additional research in using Bayesian methods to reduce the number of observations needed to estimate a proportion.

VI. SUMMARY AND SUGGESTIONS FOR FURTHER STUDY

In this chapter we will summarize how we developed tables and graphs, through Bayesian methods, which can be used by a decision maker to relate confidence interval size and the corresponding number of samples needed to produce that or a smaller confidence interval for a proportion. Included in this summary will be a comparison of the results obtained using the tables and graphs with the Classical Methods mentioned in Chapter II. Finally, we will make recommendations for some additional research in using Bayesian methods to reduce the number of observations needed to estimate a proportion.

A. COMPENDIUM

In this paper we described a method that uses the Beta distribution to place bounds on the possible outcomes for an unknown proportion. Equipped with this method we were able to create tables that could be used to find the appropriate parameters α and β to give the Beta distribution that fits a decision maker's subjective bounds.

Our next step was to evaluate the posterior Beta distribution using various sample sizes. We did this by calculating the lower bound, the upper bound, and the 95% confidence interval size for each of the various sample sizes. We then plotted the 95% confidence interval as a function of sample size and obtained the graphs in Appendix B. The decision maker can use these graphs to determine the number of samples needed to obtain a desired confidence interval size.

As an example, if the decision maker wanted the size of the 95% confidence interval to be 0.20 and his subjective bounds on the proportion were 0.14 to 0.86, the parameter on the Beta prior would be $\alpha = 4$, $\beta = 4$ and the number of observations needed would be 87. If the decision maker's subjective bounds were "tighter", then using our tables and graphs would result in even fewer samples to obtain a final confidence interval of the sample size. For example, if subjective bounds reflected $\alpha = 15$, $\beta = 15$, the number of samples needed would be reduced to 65. These results compare quite favorably to those obtained using non-Bayesian methods where the number of samples needed is 96.

In the next section we suggest some additional studies to enhance our understanding of the ways Bayesian methods may be used to reduce the number of samples required to estimate a proportion.

B. RECOMMENDATIONS FOR FURTHER RESEARCH

This paper dealt solely with 95% confidence intervals. It would extend the usefulness of this approach if tables and graphs could be developed for other confidence interval sizes, such as 90% and 99%.

The techniques that we discussed used the Beta distribution for our prior density function. Other density functions, such as the Uniform distribution, could be considered for the prior density function. Here, the subjective bounds could define the prior uniform (rectangular) distribution for the proportion.

An addition to our research would be the development of an APL program that could determine the inverse cumulative density function of the Beta distribution for large values of α and β. This could result in a more extensive set of tables and graphs which could be used to determine sample size.

It is sincerely hoped that the tables, graphs and computer programs embodied in this thesis will be beneficial to the Engineering Support Office at North Island Naval Air Station and others faced with the problem of determining the number of samples necessary to estimate proportion.

APPENDIX A. TABLES THAT CAN BE USED TO DETERMINE THE PARAMETERS TO FIT A DECISION MAKER'S SUBJECTIVE BOUNDS

Table 5. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 1 AND BETA LESS THAN OR EQUAL TO 25

α	β	P.lo	P.hi	Mean	Var
1	1	.010000	.990000	.500000	.083333
1	2	.005013	.899997	.333333	.055556
1	3	.003345	.784555	.250000	.037500
1	4	.002509	.683645	.200000	.026667
1	5	.002008	.601237	.166667	.019841
1	6	.001674	.534319	.142857	.015306
1	7	.001435	.479547	.125000	.012153
1	8	.001256	.434215	.111111	.009877
1	9	.001116	.396255	.100000	.008182
1	10	.001005	.364105	.090909	.006887
1	11	.000913	.336590	.083333	.005876
1	12	.000837	.312811	.076923	.005072
1	13	.000773	.292081	.071429	.004422
1	14	.000718	.273864	.066667	.003889
1	15	.000670	.257741	.062500	.003447
1	16	.000628	.243376	.058824	.003076
1	17	.000591	.230503	.055556	.002762
1	18	.000558	.218904	.052632	.002493
1	19	.000529	.208402	.050000	.002262
1	20	.000502	.198850	.047619	.002061
1	21	.000478	.190126	.045455	.001886
1	22	.000457	.182128	.043478	.001733
1	23	.000437	.174770	.041667	.001597
1	24	.000419	.167979	.040000	.001477
1	25	.000402	.161692	.038462	.001370

Table 6. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 1 AND BETA BETWEEN 25 AND 50

α	β	P.lo	P.hi	Mean	Var
1	26	.000386	.155855	.037037	.001274
1	27	.000372	.150423	.035714	.001188
1	28	.000359	.145354	.034483	.001110
1	29	.000347	.140614	.033333	.001039
1	30	.000335	.136171	.032258	.000976
1	31	.000324	.131999	.031250	.000917
1	32	.000314	.128075	.030303	.000864
1	33	.000305	.124376	.029412	.000816
1	34	.000296	.120883	.028571	.000771
1	35	.000287	.117581	.027778	.000730
1	36	.000279	.114454	.027027	.000692
1	37	.000272	.111488	.026316	.000657
1	38	.000264	.108672	.025641	.000625
1	39	.000258	.105993	.025000	.000595
1	40	.000251	.103444	.024390	.000567
1	41	.000245	.101014	.023810	.000541
1	42	.000239	.098695	.023256	.000516
1	43	.000234	.096480	.022727	.000494
1	44	.000228	.094361	.022222	.000472
1	45	.000223	.092334	.021739	.000452
1	46	.000218	.090392	.021277	.000434
1	47	.000214	.088530	.020833	.000416
1	48	.000209	.086742	.020408	.000400
1	49	.000205	.085025	.020000	.000384
1	50	.000201	.083375	.019608	.000370

Table 7. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 2 AND BETA LESS THAN OR EQUAL TO 25

α	β	P.lo	P.hi	Mean	Var
2	1	.100635	.994987	.666667	.055556
2	2	.058903	.941097	.500000	.050000
2	3	.041999	.859132	.400000	.040000
2	4	.032682	.777928	.333333	.031746
2	5	.026763	.705686	.285714	.025510
2	6	.022665	.643365	.250000	.020833
2	7	.019658	.589942	.222222	.017284
2	8	.017357	.544034	.200000	.014545
2	9	.015538	.504353	.181818	.012397
2	10	.014065	.469816	.166667	.010684
2	11	.012847	.439543	.153846	.009298
2	12	.011824	.412826	.142857	.008163
2	13	.010952	.389095	.133333	.007222
2	14	.010199	.367890	.125000	.006434
2	15	.009544	.348838	.117647	.005767
2	16	.008967	.331633	.111111	.005198
2	17	.008457	.316023	.105263	.004709
2	18	.008001	.301800	.100000	.004286
2	19	.007592	.288790	.095238	.003917
2	20	.007223	.276844	.090909	.003593
2	21	.006888	.265840	.086957	.003308
2	22	.006582	.255670	.083333	.003056
2	23	.006303	.246245	.080000	.002831
2	24	.006046	.237485	.076923	.002630
2	25	.005810	.229324	.074074	.002450

Table 8. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 2 AND BETA BETWEEN 25 AND 50

α	β	P.lo	P.hi	Mean	Var
2	26	.005591	.221702	.071429	.002287
2	27	.005388	.214568	.068966	.002140
2	28	.005200	.207877	.066667	.002007
2	29	.005024	.201589	.064516	.001886
2	30	.004859	.195668	.062500	.001776
2	31	.004706	.190084	.060606	.001674
2	32	.004561	.184809	.058824	.001582
2	33	.004425	.179818	.057143	.001497
2	34	.004297	.175089	.055556	.001418
2	35	.004176	.170601	.054054	.001346
2	36	.004062	.166337	.052632	.001278
2	37	.003954	.162280	.051282	.001216
2	38	.003851	.158416	.050000	.001159
2	39	.003754	.154732	.048780	.001105
2	40	.003662	.151214	.047619	.001055
2	41	.003574	.147853	.046512	.001008
2	42	.003490	.144637	.045455	.000964
2	43	.003409	.141558	.044444	.000923
2	44	.003333	.138608	.043478	.000885
2	45	.003260	.135777	.042553	.000849
2	46	.003190	.133060	.041667	.000815
2	47	.003123	.130449	.040816	.000783
2	48	.003058	.127938	.040000	.000753
2	49	.002997	.125522	.039216	.000725
2	50	.002938	.123196	.038462	.000698

Table 9. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 3 AND BETA LESS THAN OR EQUAL TO 25

α	β	P.lo	P.hi	Mean	Var
3	1	.233813	.996655	.750000	.037500
3	2	.140868	.958001	.600000	.040000
3	3	.105640	.894360	.500000	.035714
3	4	.084730	.826930	.428571	.030612
3	5	.070804	.763676	.375000	.026042
3	6	.060840	.706770	.333333	.022222
3	7	.053348	.656315	.300000	.019091
3	8	.047507	.611743	.272727	.016529
3	9	.042823	.572323	.250000	.014423
3	10	.038982	.537343	.230769	.012680
3	11	.035775	.506171	.214286	.011224
3	12	.033057	.478264	.200000	.010000
3	13	.030723	.453166	.187500	.008961
3	14	.028698	.430493	.176471	.008074
3	15	.026923	.409923	.166667	.007310
3	16	.025356	.391187	.157895	.006648
3	17	.023961	.374055	.150000	.006071
3	18	.022711	.358335	.142857	.005566
3	19	.021586	.343864	.136364	.005120
3	20	.020567	.330500	.130435	.004726
3	21	.019640	.318123	.125000	.004375
3	22	.018793	.306630	.120000	.004062
3	23	.018016	.295930	.115385	.003780
3	24	.017301	.285945	.111111	.003527
3	25	.016640	.276606	.107143	.003299

Table 10. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 3 AND BETA BETWEEN 25 AND 50

α	β	P.lo	P.hi	Mean	Var
3	26	.016028	.267853	.103448	.003092
3	27	.015460	.259633	.100000	.002903
3	28	.014930	.251899	.096774	.002732
3	29	.014436	.244610	.093750	.002575
3	30	.013973	.237728	.090909	.002431
3	31	.013539	.231221	.088235	.002299
3	32	.013131	.225058	.085714	.002177
3	33	.012747	.219215	.083333	.002065
3	34	.012385	.213665	.081081	.001961
3	35	.012043	.208389	.078947	.001864
3	36	.011719	.203366	.076923	.001775
3	37	.011412	.198578	.075000	.001692
3	38	.011121	.194010	.073171	.001615
3	39	.010844	.189646	.071429	.001542
3	40	.010581	.185474	.069767	.001475
3	41	.010330	.181481	.068182	.001412
3	42	.010091	.177656	.066667	.001353
3	43	.009863	.173988	.065217	.001297
3	44	.009645	.170469	.063830	.001245
3	45	.009436	.167088	.062500	.001196
3	46	.009236	.163839	.061224	.001150
3	47	.009045	.160713	.060000	.001106
3	48	.008861	.157704	.058824	.001065
3	49	.008684	.154805	.057692	.001026
3	50	.008515	.152011	.056604	.000989

Table 11. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 4 AND BETA LESS THAN OR EQUAL TO 25

α	β	P.lo	P.hi	Mean	Var
4	1	.421318	.997491	.800000	.026667
4	2	.222072	.967318	.666667	.031746
4	3	.173070	.915270	.571429	.030612
4	4	.142270	.857730	.500000	.027778
4	5	.120950	.801798	.444444	.024691
4	6	.105262	.749974	.400000	.021818
4	7	.093214	.702884	.363636	.019284
4	8	.083660	.660417	.333333	.017094
4	9	.075895	.622193	.307692	.015216
4	10	.069455	.587759	.285714	.013605
4	11	.064028	.556669	.266667	.012222
4	12	.059390	.528514	.250000	.011029
4	13	.055381	.502936	.235294	.009996
4	14	.051880	.479621	.222222	.009097
4	15	.048797	.458298	.210526	.008310
4	16	.046061	.438734	.200000	.007619
4	17	.043615	.420729	.190476	.007009
4	18	.041417	.404110	.181818	.006468
4	19	.039430	.388727	.173913	.005986
4	20	.037625	.374451	.166667	.005556
4	21	.035978	.361170	.160000	.005169
4	22	.034470	.348784	.153846	.004821
4	23	.033083	.337208	.148148	.004507
4	24	.031804	.326367	.142857	.004222
4	25	.030620	.316193	.137931	.003964

Table 12. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 4 AND BETA BETWEEN 25 AND 50

α	β	P.lo	P.hi	Mean	Var
4	26	.029521	.306628	.133333	.003728
4	27	.028498	.297619	.129032	.003512
4	28	.027544	.289119	.125000	.003314
4	29	.026651	.281088	.121212	.003133
4	30	.025815	.273487	.117647	.002966
4	31	.025030	.266283	.114286	.002812
4	32	.024291	.259447	.111111	.002669
4	33	.023594	.252951	.108108	.002537
4	34	.022936	.246770	.105263	.002415
4	35	.022314	.240882	.102564	.002301
4	36	.021725	.235267	.100000	.002195
4	37	.021166	.229907	.097561	.002096
4	38	.020636	.224784	.095238	.002004
4	39	.020131	.219884	.093023	.001917
4	40	.019650	.215192	.090909	.001837
4	41	.019192	.210695	.088889	.001761
4	42	.018754	.206381	.086957	.001689
4	43	.018337	.202240	.085106	.001622
4	44	.017937	.198261	.083333	.001559
4	45	.017554	.194436	.081633	.001499
4	46	.017188	.190754	.080000	.001443
4	47	.016836	.187209	.078431	.001390
4	48	.016499	.183793	.076923	.001340
4	49	.016174	.180499	.075472	.001292
4	50	.015863	.177321	.074074	.001247

Table 13. **MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 5 AND BETA LESS THAN OR EQUAL TO 25**

α	β	P.lo	P.hi	Mean	Var
5	1	.690656	.997992	.833333	.019841
5	2	.294314	.973237	.714286	.025510
5	3	.236324	.929196	.625000	.026042
5	4	.198202	.879050	.555556	.024691
5	5	.170965	.829035	.500000	.022727
5	6	.150443	.781662	.454545	.020661
5	7	.134388	.737798	.416667	.018697
5	8	.121467	.697596	.384615	.016906
5	9	.110835	.660900	.357143	.015306
5	10	.101929	.627435	.333333	.013889
5	11	.094356	.596893	.312500	.012638
5	12	.087838	.568971	.294118	.011534
5	13	.082166	.543388	.277778	.010559
5	14	.077185	.519890	.263158	.009695
5	15	.072776	.498252	.250000	.008929
5	16	.068845	.478276	.238095	.008246
5	17	.065318	.459787	.227273	.007636
5	18	.062136	.442633	.217391	.007089
5	19	.059250	.426681	.208333	.006597
5	20	.056621	.411812	.200000	.006154
5	21	.054216	.397923	.192308	.005753
5	22	.052008	.384923	.185185	.005389
5	23	.049972	.372730	.178571	.005058
5	24	.048090	.361275	.172414	.004756
5	25	.046345	.350492	.166667	.004480

Table 14. MEANS, VARIANCES AND 98% BOUNDS FOR A BETA DISTRIBUTION WITH ALPHA = 5 AND BETA BETWEEN 25 AND 50

α	β	P.lo	P.hi	Mean	Var
5	26	.044722	.340326	.161290	.004227
5	27	.043210	.330727	.156250	.003995
5	28	.041796	.321647	.151515	.003781
5	29	.040472	.313048	.147059	.003584
5	30	.039229	.304892	.142857	.003401
5	31	.038061	.297146	.138889	.003232
5	32	.036960	.289781	.135135	.003076
5	33	.035921	.282769	.131579	.002930
5	34	.034939	.276086	.128205	.002794
5	35	.034009	.269709	.125000	.002668
5	36	.033128	.263619	.121951	.002550
5	37	.032291	.257795	.119048	.002439
5	38	.031495	.252222	.116279	.002335
5	39	.030738	.246883	.113636	.002238
5	40	.030016	.241765	.111111	.002147
5	41	.029328	.236853	.108696	.002061
5	42	.028670	.232136	.106383	.001981
5	43	.028041	.227602	.104167	.001904
5	44	.027439	.223241	.102041	.001833
5	45	.026863	.219043	.100000	.001765
5	46	.026310	.215000	.098039	.001701
5	47	.025779	.211103	.096154	.001640
5	48	.025270	.207344	.094340	.001582
5	49	.024780	.203716	.092593	.001528
5	50	.024309	.200212	.090909	.001476

APPENDIX B

GRAPHS OF BOUNDS OF CONFIDENCE INTERVALS AND SIZE OF CONFIDENCE INTERVALS FOR VARIOUS BETA PRIOR DISTRIBUTION

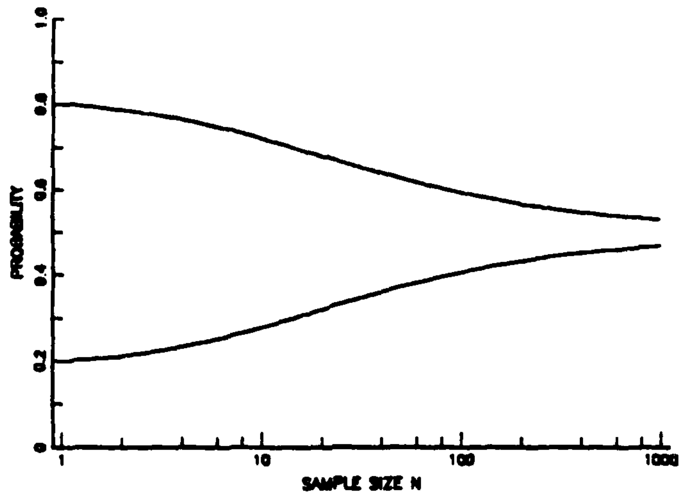

Figure 8 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 4$

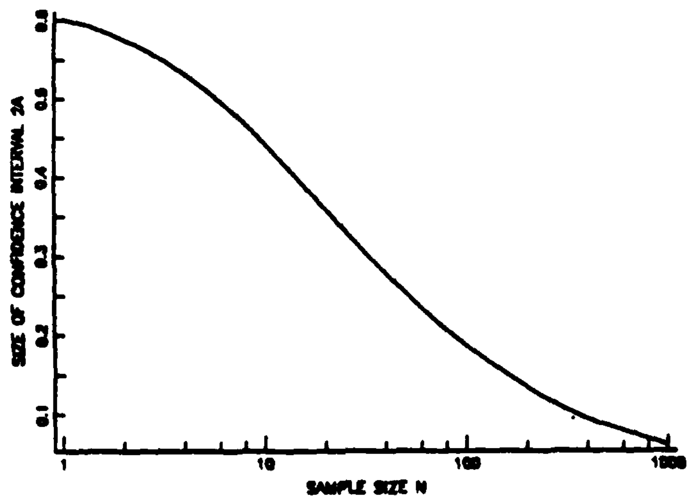

Figure 9 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 4$

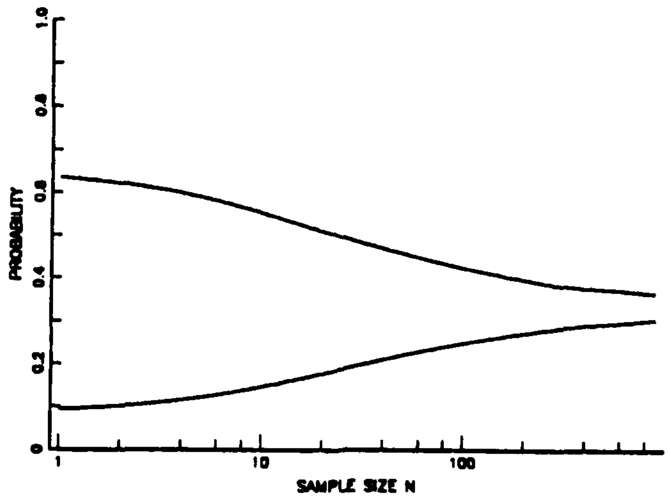

Figure 10 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 3$, $\beta = 6$

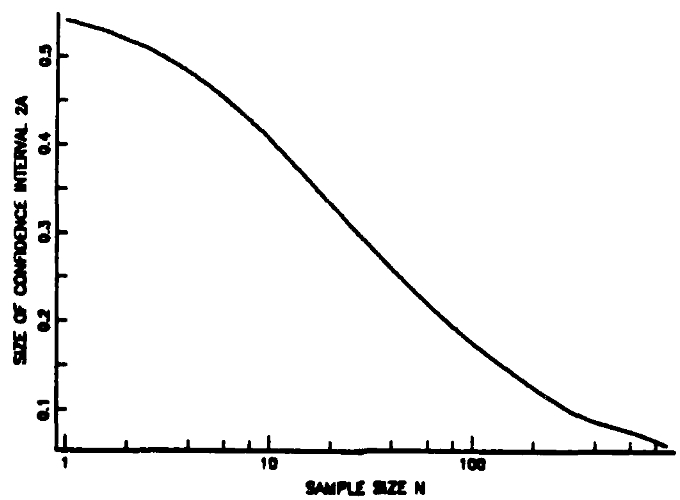

Figure 11 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 3$, $\beta = 6$

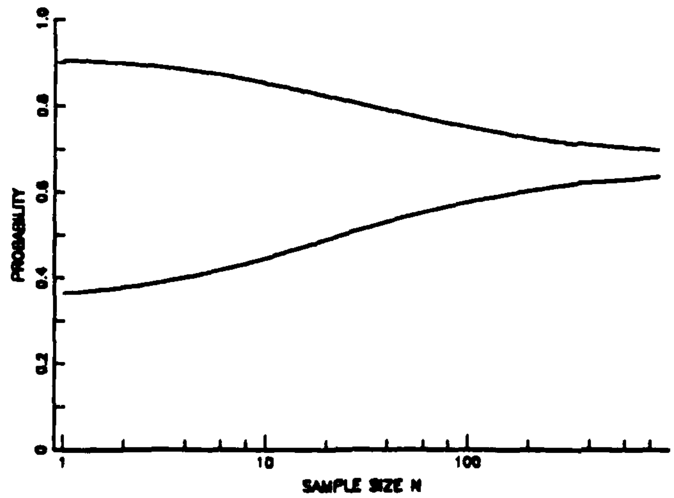

Figure 12 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 3$

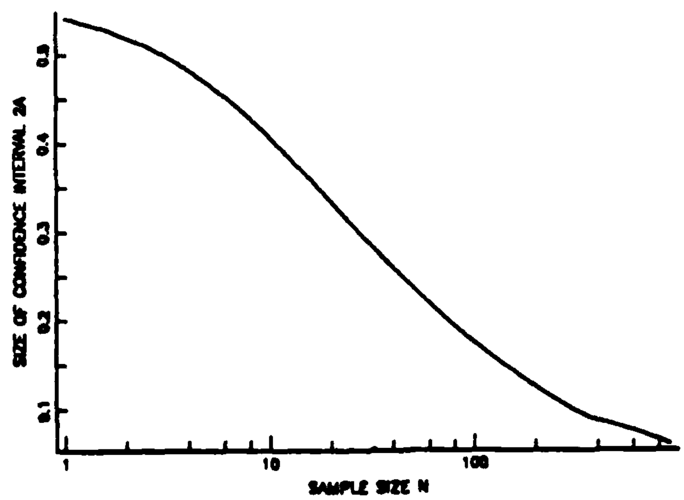

Figure 13 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 3$

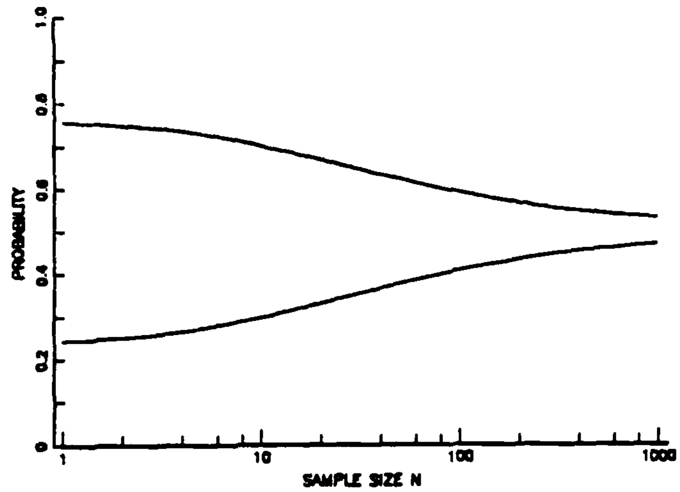

Figure 14 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 6$

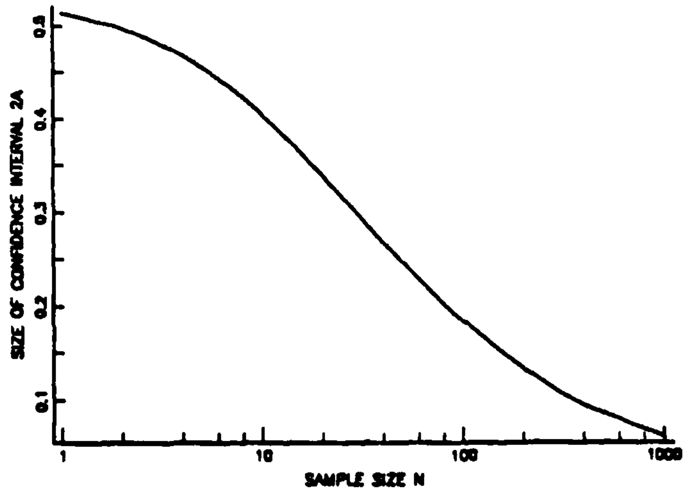

Figure 15 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 6$

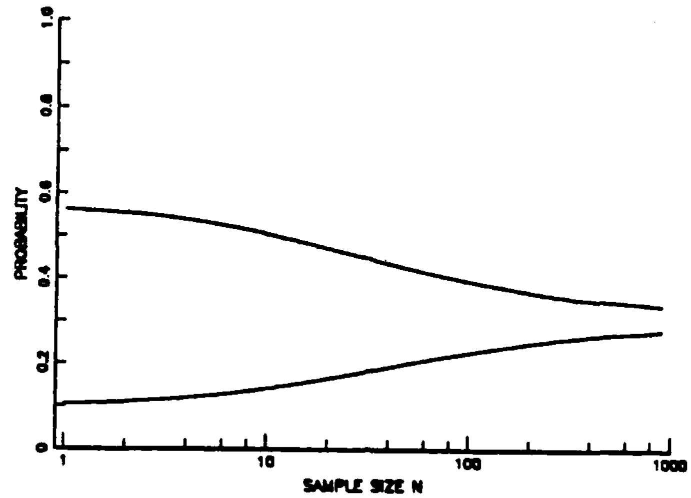

Figure 16 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 9$

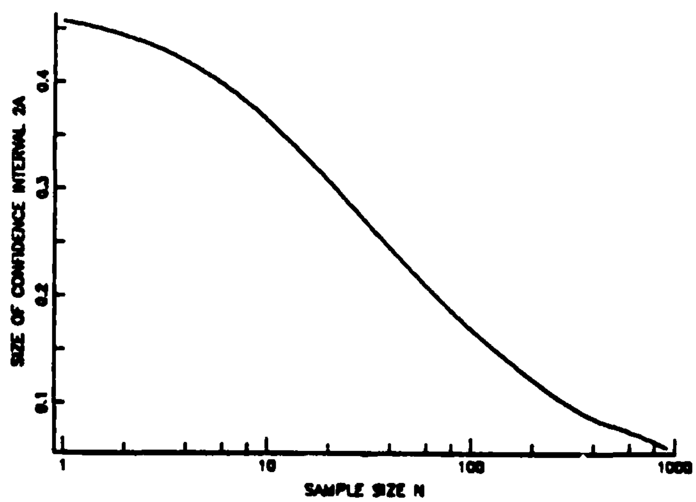

Figure 17 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 4$, $\beta = 9$

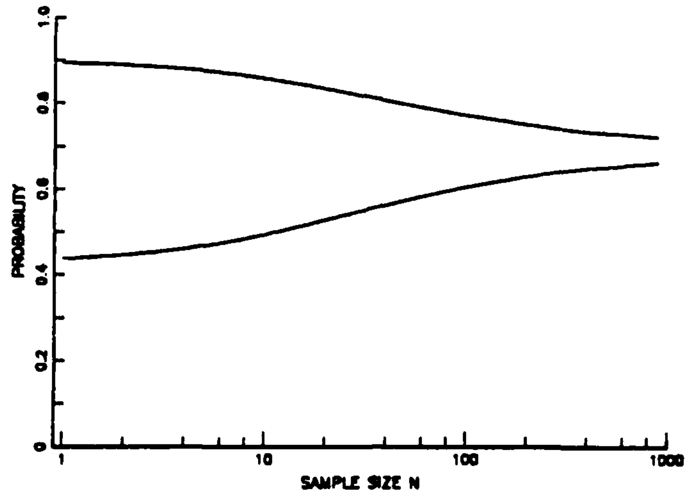

Figure 18 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 9$, $\beta = 4$

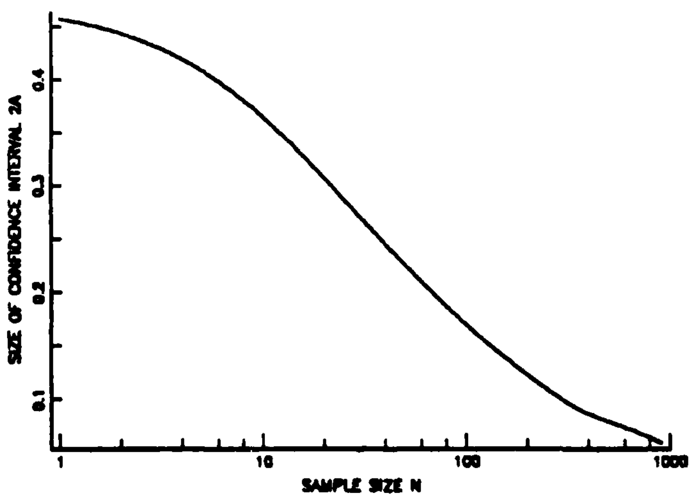

Figure 19 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 9$, $\beta = 4$

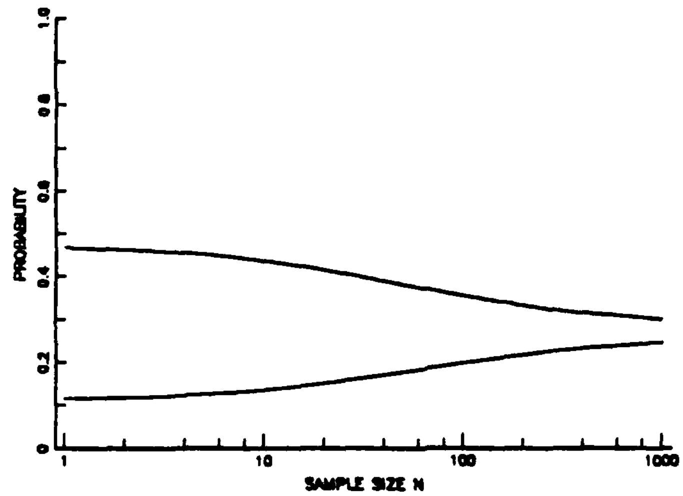

Figure 20 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 16$

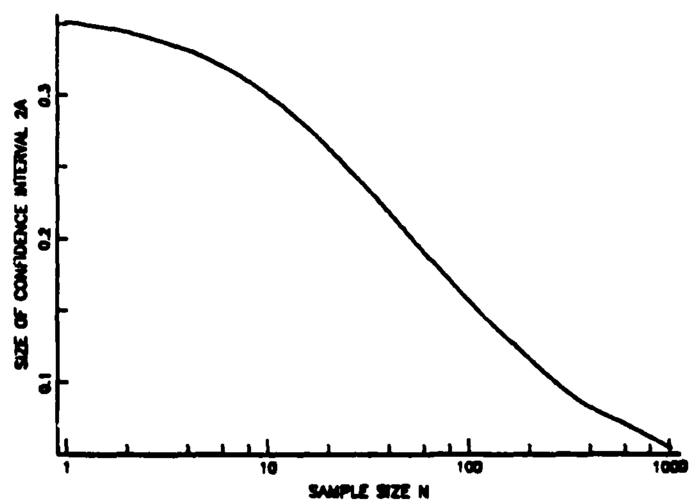

Figure 21 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 16$

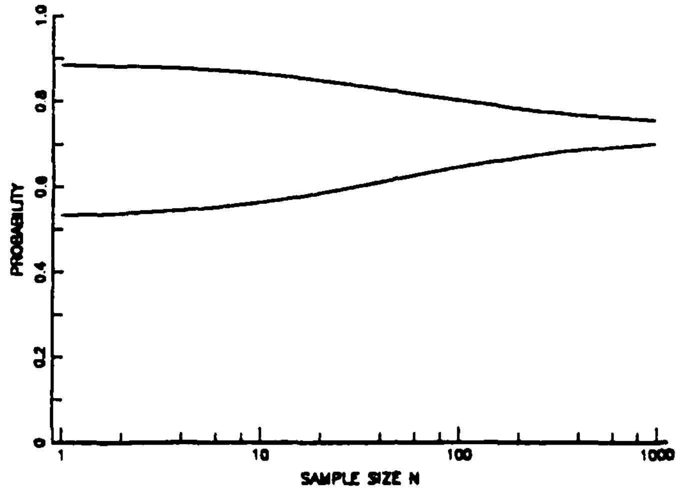

Figure 22 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 16$, $\beta = 6$

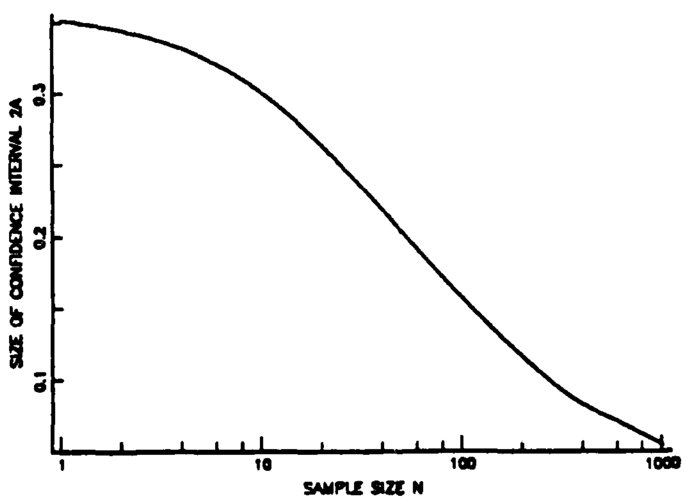

Figure 23 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 16$, $\beta = 6$

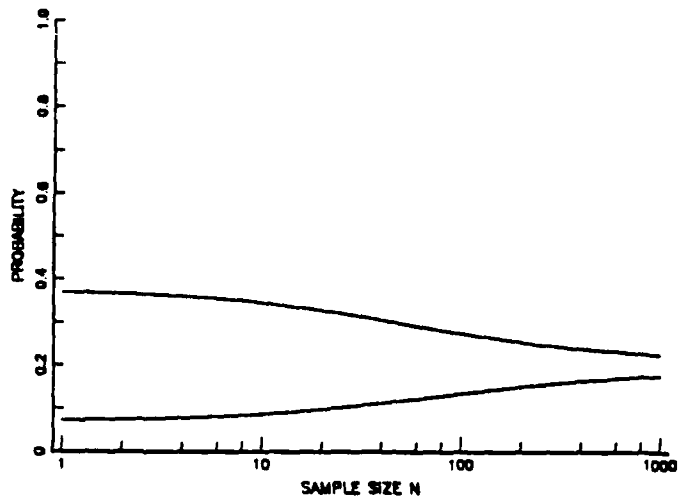

Figure 24 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 20$

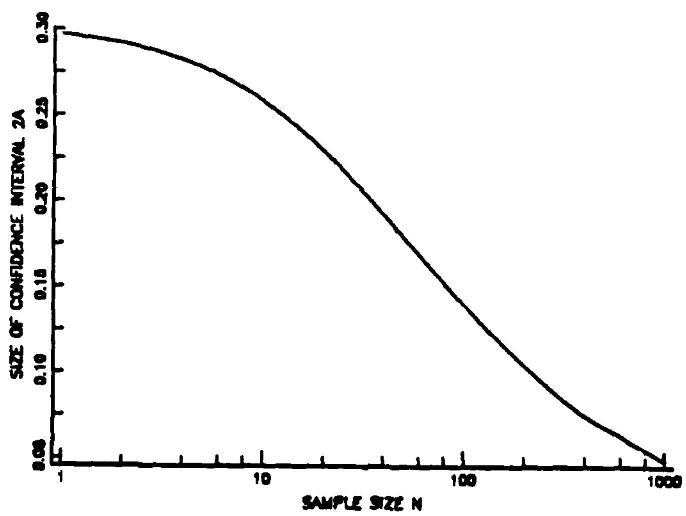

Figure 25 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 20$

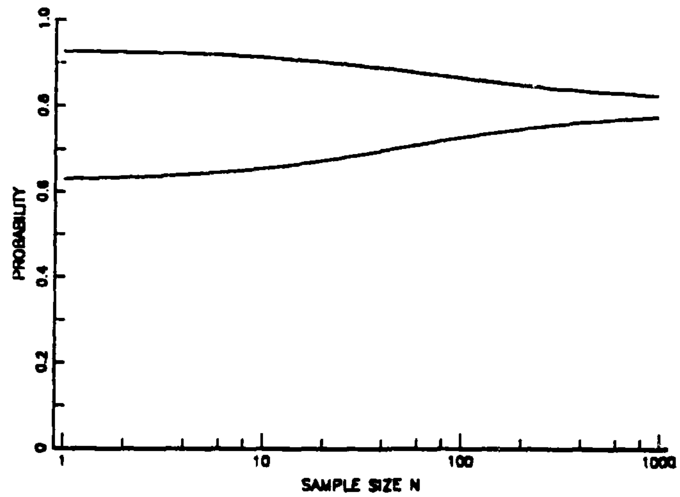

Figure 26 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 20$, $\beta = 5$

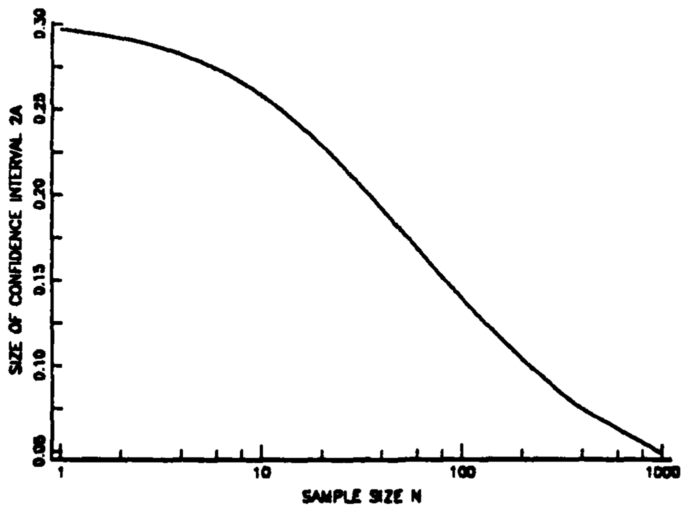

Figure 27 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 20$, $\beta = 5$

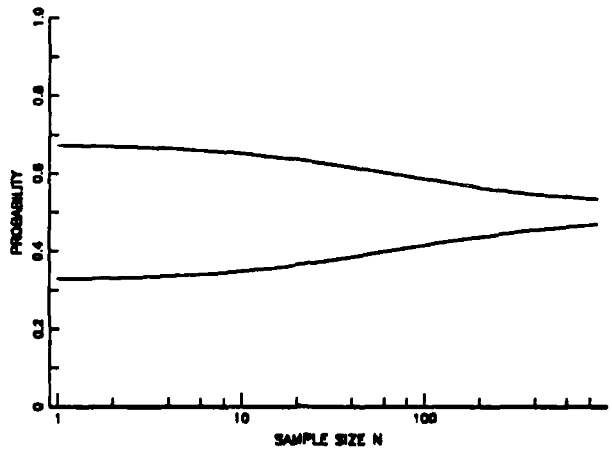

Figure 28 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 15$, $\beta = 15$

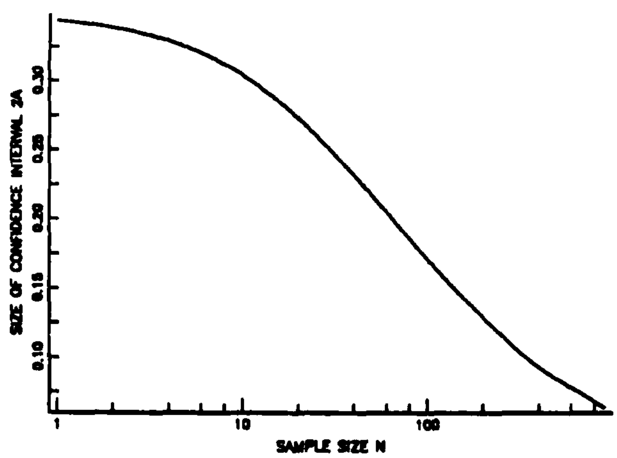

Figure 29 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 15$, $\beta = 15$

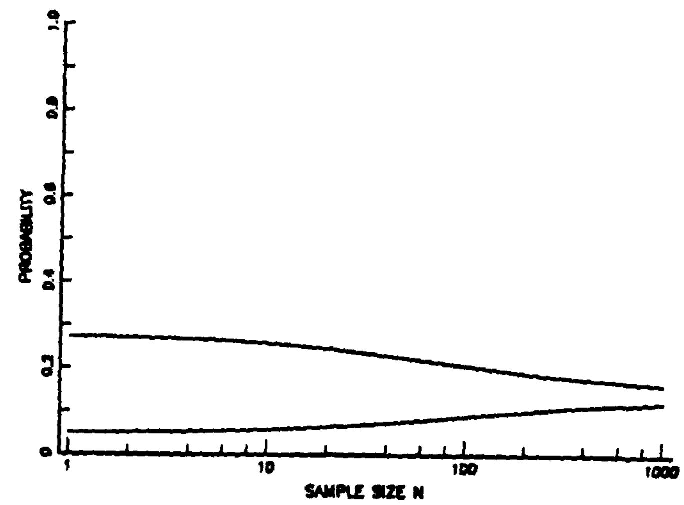

Figure 30 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 30$

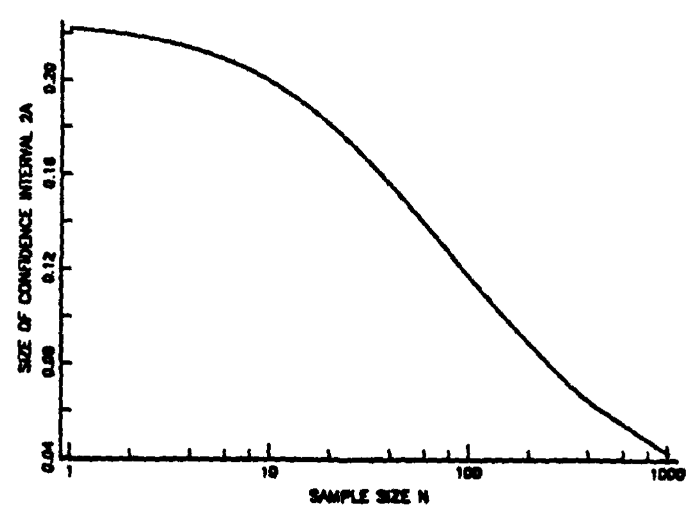

Figure 31 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 30$

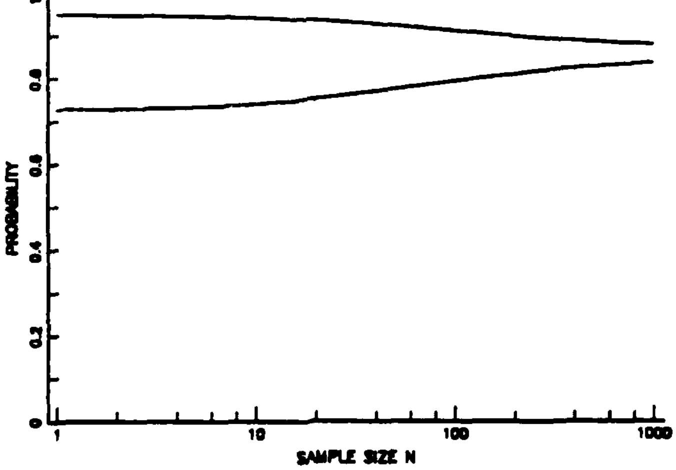

Figure 32 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 30$, $\beta = 5$

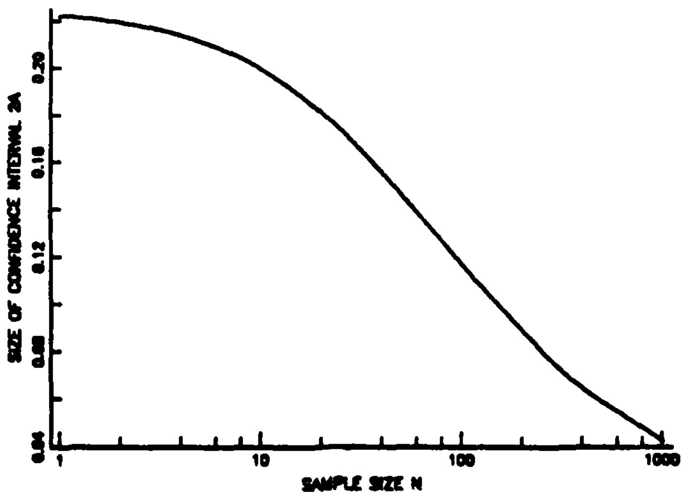

Figure 33 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 30$, $\beta = 5$

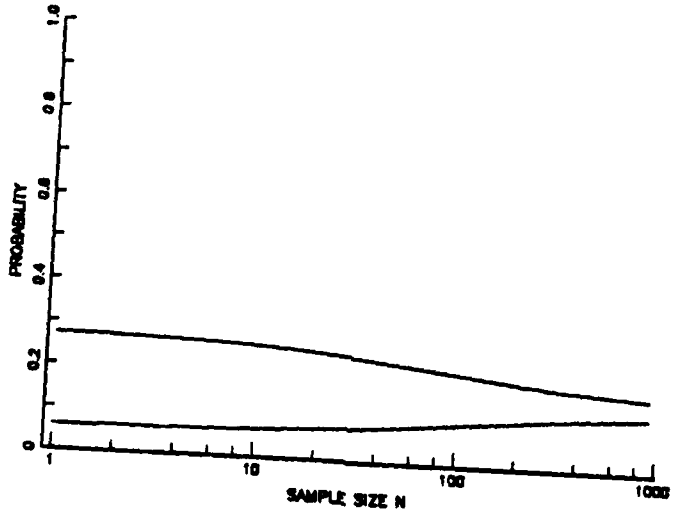

Figure 34 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 34$

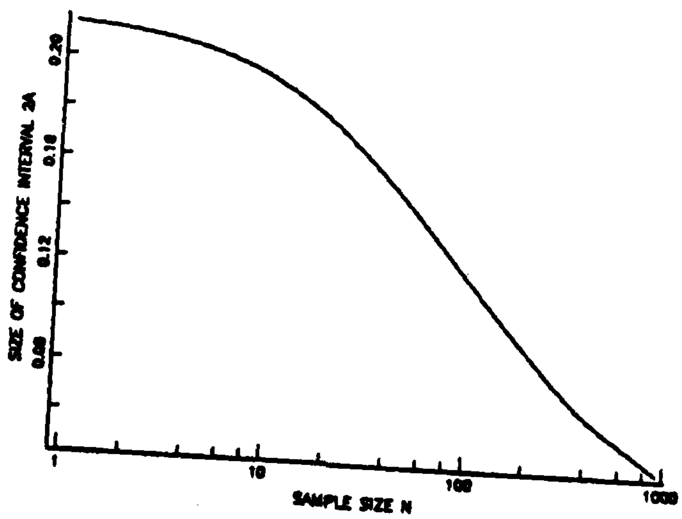

Figure 35 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 6$, $\beta = 34$

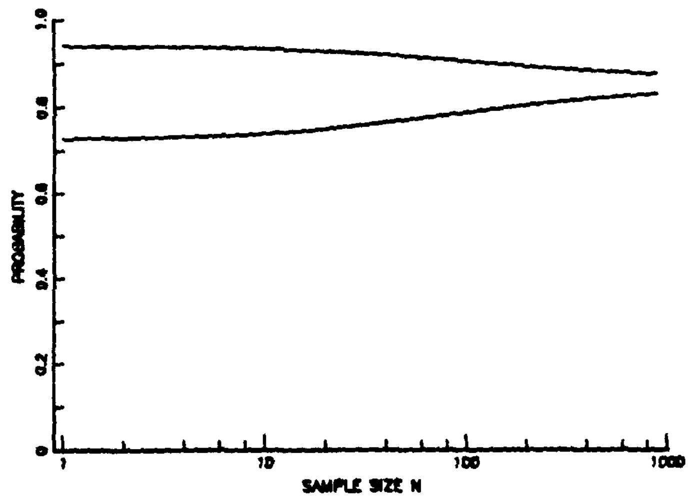

Figure 36 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 34$, $\beta = 6$

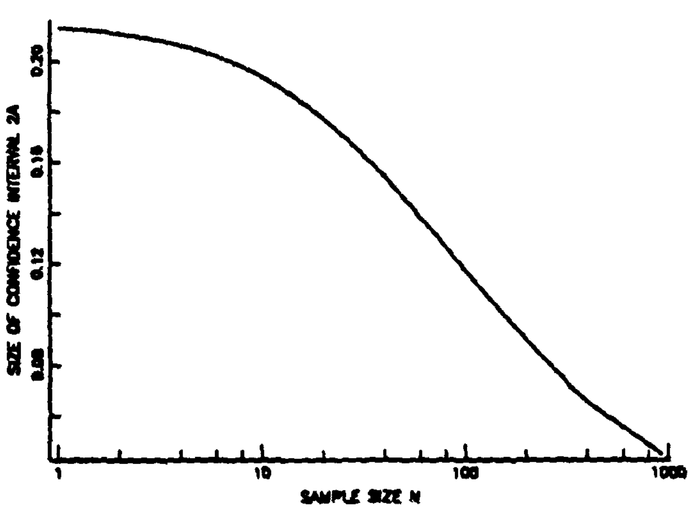

Figure 37 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 34$, $\beta = 6$

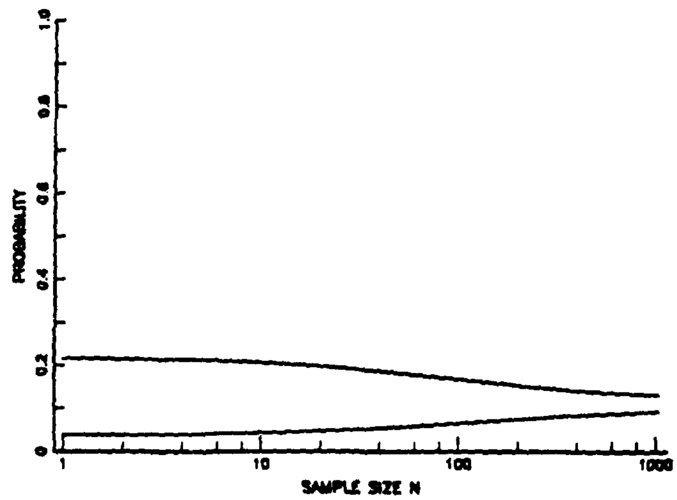

Figure 38 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 40$

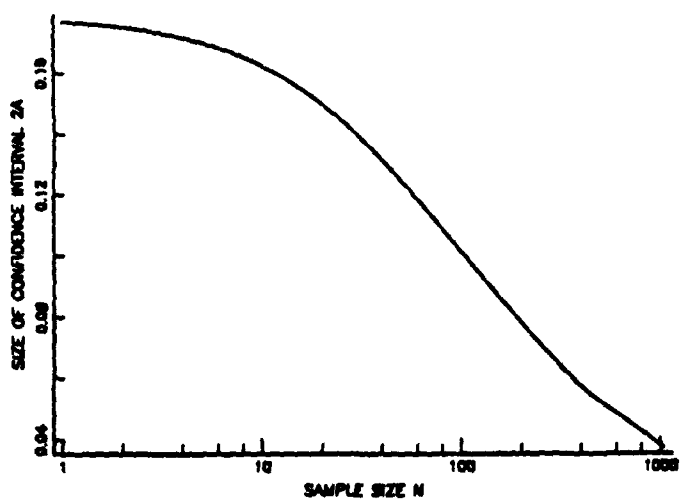

Figure 39 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 40$

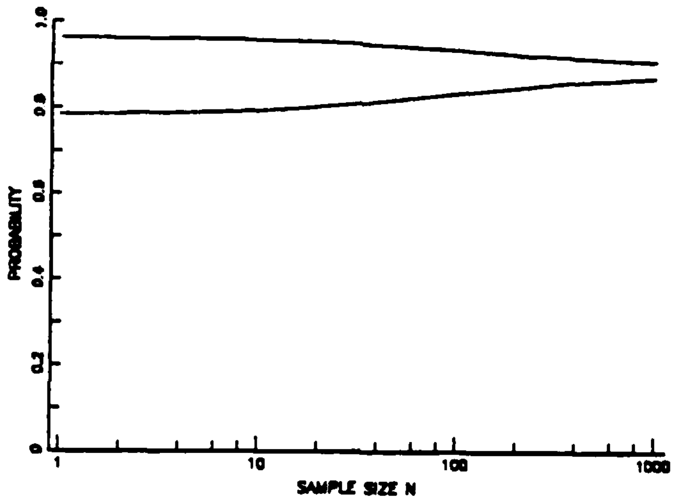

Figure 40 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 40$, $\beta = 5$

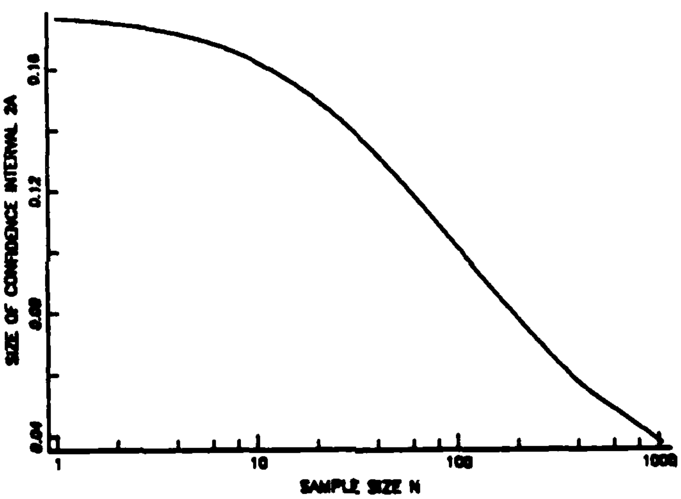

Figure 41 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 40$, $\beta = 5$

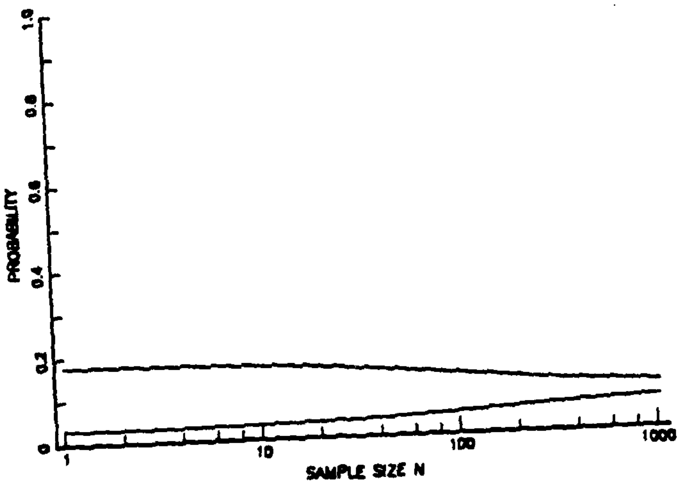

Figure 42 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 50$

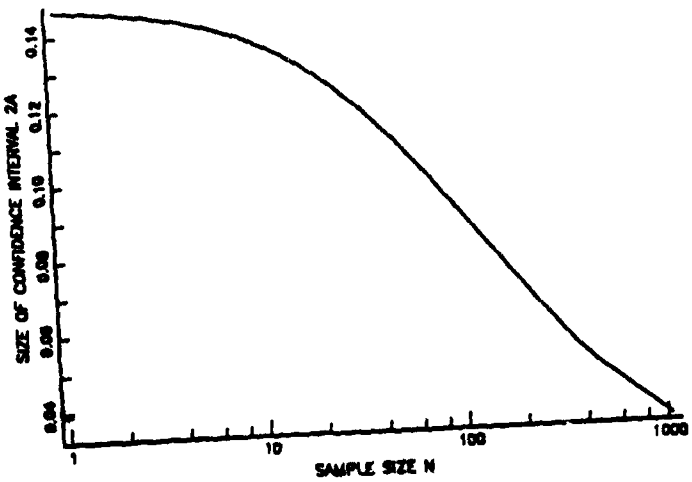

Figure 43 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 5$, $\beta = 50$

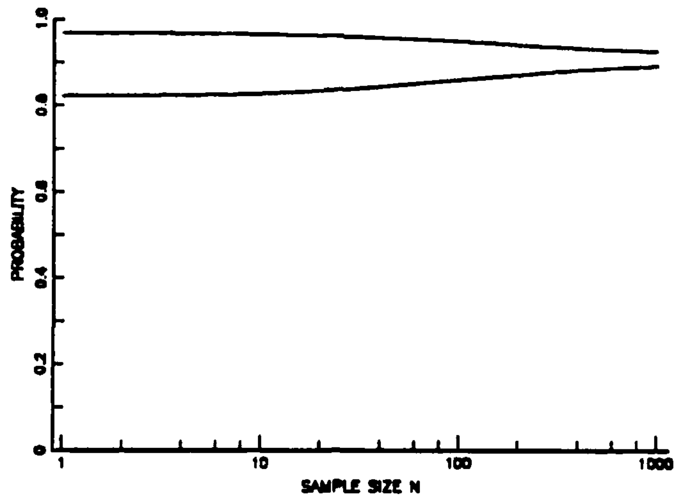

Figure 44 The Relation of Sample Size and the Bounds of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 50$, $\beta = 5$

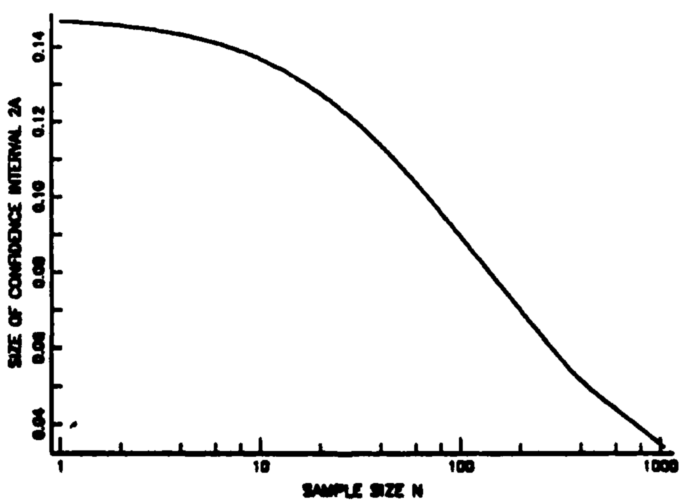

Figure 45 The Relation of Sample Size and the Size of the Bayesian 95% Confidence Interval with a Beta Prior Distribution Having Parameters $\alpha = 50$, $\beta = 5$

APPENDIX C. THE APL PROGRAM USED TO CONSTRUCT TABLES THAT DETERMINE PARAMETERS OF THE BETA PRIOR DISTRIBUTION

```
      ∇ SENSE
[1]   ⍝ THIS PROGRAM CAN BE USED TO ESTABLISH BOUNDS FOR THE BETA
[2]   ⍝ DISTRIBUTION. THE USER CAN BE ASSURED THAT 98 PERCENT OF THE
[3]   ⍝ DENSITY OF THE BETA DISTRIBUTION IS BETWEEN P.LO AND P.HI FOR
[4]   ⍝ EACH SET OF CORRESPONDING PARAMETERS. A MORE EXTENSIVE SET OF
[5]   ⍝ TABLES CAN BE CREATED BY INCREASING THE NUMBERS FOR THE MAX VALUE
[6]   ⍝ OF N IN LINE 22 AND THE MAX VALUE OF K IN LINE 24.
[7]    K←0
[8]   LOOP1:K←K+1
[9]    ⎕←'   ALPHA          BETA          P.LO        P.HI       MEAN     VARIANCE'
[10]   →(K>5)/NEXT
[11]   N←0
[12]   DDUMM←0
[13]   →(DDUMM=0)/NEXT2
[14]  NEXT:N←1
[15]  NEXT2:AL←K
[16]  LOOP2:N←N+1
[17]   VNUM←N×K
[18]   VDEN1←(N+K)*2
[19]   VDEN2←N+K+1
[20]   VDEN←VDEN1×VDEN2
[21]   VAR←VNUM÷VDEN
[22]   BE←N
[23]   AVE←K÷(K+N)
[24]   PARA←AL,BE
[25]   PCT←PARA BQUAN 0.01 0.99
[26]   OUTP←K,N,PCT,AVE,VAR
[27]   6 0 14 0 15 6 11 6 10 6 10 6 ⍕OUTP
[28]   →(N<25)/LOOP2
[29]   ⎕←' '
[30]   →(K<5)/LOOP1
      ∇
```

APPENDIX D. THE APL PROGRAMS DESIGNED AT NAVAL POSTGRADUATE SCHOOL TO COMPUTE THE INVERSE CDF OF BETA DISTRIBUTION

```
      ∇ V←A BQUAN P;E;U;S;D;L;Z;DENS;I;PP;M;X;F;C2;C3;C4
[1]   ⍝ IMPLEMENTATION OF CARTER, 1947, BIOMETRIKA FOR APPROXIMATE INVERSE BETA
[2]   ⍝ 11/5/86 BEST FOR A[1]≤2×A[2], AND SEEMS TO WORK FINE
[3]   ⍝ 12/27/86 ADDED 2 NEWTON-RAPHSON ITERATIONS; ADD MORE FOR GREATER ACC.
[4]    →((⌊/A)<1)/SMALL
[5]    E←NQUAN 1-P
[6]    U←⌽¯1+2×A
[7]    S←+/÷U
[8]    D←-/÷U
[9]    L←(¯3+E*2)÷6
[10]   Z←((S÷2)×E×(L+2÷S)*0.5)-D×(L+(5÷6)-S÷3)-(D*2)×((2÷S)*0.5)×E×(11+E*2)÷144
[11]   V←÷1+(÷/⌽A)×*2×Z×I←1
[12]  LOOP:DENS←A[1]×(A[1]!¯1++/A)×(V*A[1]-1)×(1-V)*A[2]-1
[13]   V←V-((A BETA V)-P)÷DENS
[14]   →((I←I+1)≤2)/LOOP
[15]   →0
[16]  ⍝ MY VERSION FOR THE BETA QUANTILES WHEN A⌊B<1. 12/31/86
[17]  ⍝ MODIFIED 1/1/87 WITH A CORNISH-FISHER TYPE EXPANSION.
[18]  ⍝ MODIFIED 1/3/87 TO USE MEAN AND STANDARD DEVIATION, AND NORMAL QUANTILE
[19]  ⍝ WHEN ONE PARAMETER IS GREATER THAN ONE (FOR ONE SIDE). OTHER SIDE (OR
[20]  ⍝ BOTH) USES THE DENSITY WHICH IS UNBOUNDED, FOLLOWED BY CORNISH-FISHER.
[21]  SMALL:V←X←(⍴,P)⍴0
[22]   PP←P≤M←A[2]÷+/A
[23]   X[PP/⍳⍴X]←((PP/P)÷(((A[1]<1),A[1]≥1)/1,A[1])×A[1]!¯1++/A)*÷A[1]
[24]   X[(~PP)/⍳⍴X]←1-((1-(~PP)/P)÷(((A[2]<1),A[2]≥1)/1,A[2])×A[2]!¯1++/A)*÷A[2]
[25]   X[(X=1)/⍳⍴X]←1-1E¯15
[26]   →((⌈/A)≥1)/ONE
[27]  START:F←(A[1]!¯1++/A)×A[1]×(X*A[1]-1)×(1-X)*A[2]-1
[28]   C2←((1-A[1])÷X)+(A[2]-1)÷1-X
[29]   C3←(2×C2*2)+((A[1]-1)÷X*2)+(A[2]-1)÷(1-X)*2
[30]   C4←(6×C2*3)+(7×C2×(C3-2×C2*2))+((1-A[1])÷X*3)+(A[2]-1)÷(1-X)*3
[31]   F←(P-(A BETA X))÷F
[32]   V←X+F+((C2×F*2)÷2)+(C3×(F*3)÷6)+C4×(F*4)÷24
[33]   V[(V>1)/⍳⍴V]←1
[34]   →0
[35]  ONE:M←1-M
[36]   S←(M×(1-M)÷1++/A)*0.5
[37]   →((A⍳⌈/A)=2)/4+⎕LC
[38]   X[PP/⍳⍴X]←M+S×NQUAN PP/P
[39]   X[(X≤0)/⍳⍴X]←1E¯15
[40]   →START
[41]   X[(~PP)/⍳⍴X]←M+S×NQUAN(~PP)/P
[42]   X[(X≥1)/⍳⍴X]←1-1E¯15
[43]   →START
      ∇
```

```
      ∇ Z←NQUAN P;A;B;C;D;Q;T;S;R;F
[1]   ⍝ IMPLEMENTS ALGORITHM AS 111 BY BEASLEY   SPRINGER, APPLIED STAT, 1977
[2]   ⍝ FOR A VECTOR INPUT OF FRACTIONS, RETURNS CORRESPONDING NORMAL QUANTILES
[3]   ⍝ WITH CLAIMED ACCURACY BETTER THAN 1.5×10*¯8. FOR GREATER ACCURACY,
[4]   ⍝ ESPECIALLY FOR EXTREME P VALUES, ADD ONE OR MORE NEWTON-RAPHSON LOOPS.
[5]    →(∨/(P≤0),(P≥1))/ERR
[6]    →(∨/((|Q←,P-0.5)≤0.42))/3+⎕LC
[7]    S←Z←,Q
[8]    →EXT
[9]    T←(0.42≥|Q)/Z←,Q
[10]   →(F←((⍴,T)=⍴,P))/2+⎕LC
[11]   S←(0.42<|Q)/Q
[12]   A← 2.50662823884 ¯18.6150006252 41.39119773534 ¯25.44106049637
[13]   B← ¯8.4735109309 23.08336743743 ¯21.06224101826 3.13082909833
[14]   T←T×(((T*2)∘.*0,⍳3)+.×A)÷1+((T*2)∘.*⍳4)+.×B
[15]   Z[(0.42≥|Q)/⍳⍴,Q]←T
[16]   →(F=1)/0
[17] EXT:C← ¯2.78718931138 ¯2.29796479134 4.85014127135 2.32121276858
[18]   D← 3.54388924762 1.63706781897
[19]   S←(×S)×((R∘.*0,⍳3)+.×C)÷1+(((R←(|⍟0.5-|S)*0.5)∘.* 1 2)+.×D)
[20]   Z[(0.42<|Q)/⍳⍴Q]←S
[21]   →0
[22] ERR:'ONE OR MORE P VALUES ARE OUT OF RANGE.'
      ∇


      ∇BETA[⎕]
      ∇ U←A BETA X;Y;W;N;OD;EV;Z;I
[1]   ⍝ 12/27/86 EVALUATES THE BETA CDF, PARAMETERS A, AT VECTOR X USING THE
[2]   ⍝ BOUVER-BARGMAN CONTINUED FRACTION AT DEPTH VARYING FROM 7 TO 21.
[3]   ⍝ 11TH ANNUAL SYMPOSIUM ON THE INTERFACE OF COMPUTER SCIENCE AND
[4]   ⍝ STATISTICS, 1978, P 325. BECAUSE OF THE RANGE OF !, +/A≤255. SEEMS TO
[5]   ⍝ GIVE A GOOD 8 OR MORE DECIMALS.
[6]    Y←X≤(A[1]÷+/A)
[7]    U←(⍴,X)⍴0
[8]    N←7++/(⌈/A)>(2×⍳4),10×⍳10
[9]    →((+/Y)=0)/FLIP
[10]   W←Y/X←,X
[11]   OD←W∘.×((⍳N)×A[2]-⍳N)÷×/(N,2)⍴A[1]+⍳2×I←N
[12]   EV←-W∘.×(×⌿((2,N)⍴(A[1]+0,⍳N-1),(+/A)+0,⍳N-1))÷×/(N,2)⍴A[1]+0,⍳(2×N-Z←1)
[13] L:Z←1+EV[;I]÷1+OD[;I]÷Z
[14]   →((I←I-1)>0)/L
[15]   U[Y/⍳⍴U]←(÷Z)×(A[1]!¯1++/A)×(W*A[1])×(1-W)*A[2]
[16]   →((+/Y)=⍴X)/0
[17] FLIP:A←⌽A
[18]   W←1-(~Y)/X
[19]   OD←W∘.×((⍳N)×A[2]-⍳N)÷×/(N,2)⍴A[1]+⍳2×I←N
[20]   EV←-W∘.×(×⌿((2,N)⍴(A[1]+0,⍳N-1),(+/A)+0,⍳N-1))÷×/(N,2)⍴A[1]+0,⍳(2×N-Z←1)
[21] L1:Z←1+EV[;I]÷1+OD[;I]÷Z
[22]   →((I←I-1)>0)/L1
[23]   U[(~Y)/⍳⍴U]←1-(÷Z)×(A[1]!¯1++/A)×(W*A[1])×(1-W)*A[2]
      ∇
```

APPENDIX E. THE APL PROGRAMS USED TO CONSTRUCT GRAPHS THAT DETERMINE SAMPLE SIZE NEEDED FOR A DESIRED CONFIDENCE INTERVAL SIZE

```
      ∇ CHARTPLUS
[1]    ⍝ THIS PROGRAM COMPUTES THE PARAMETERS OF A BETA POSTERIOR DISTRIBUTION
[2]    ⍝ FOR A PARTICULAR BETA PRIOR DISTRIBUTION USING VARIOUS SAMPLE SIZES.
[3]    ⍝ IT PROVIDES A TABLE THAT FURNISHES THE SAMPLE SIZE USED TO CALCULATE
[4]    ⍝ THE PARAMETERS OF THE BETA POSTERIOR DISTRIBUTION (DENOTED BY N), THE
[5]    ⍝ PARAMETERS OF THE BETA POSTERIOR DISTRIBUTION (DENOTED BY A* AND B*),
[6]    ⍝ THE LOWER AND UPPER BOUNDS FOR A 95 PERCENT CONFIDENCE INTERVAL(DE-
[7]    ⍝ NOTED P.LO AND P.HI) AND THE SIZE OF THE CONFIDENCE INTERVAL.CHARTPLUS
[8]    ⍝ USES SUBROUTINE INTER2 TO CALCULATE THE LOWER AND UPPER BOUNDS OF THE
[9]    ⍝ THE CONFIDENCE INTERVAL. CHARTPLUS USES SUBROUTINE CHARTER TO PERFORM
[10]   ⍝ THESE CALCULATIONS FOR SAMPLE SIZE NUMBERS AT OR NEAR 500 AND 1000.
[11]   ⍝ THIS PROGRAM ALSO PROVIDES A VECTOR OF SAMPLE SIZES (DENOTED SAMSZ), A
[12]   ⍝ VECTOR OF THE LOWER BOUNDS FOR EACH SAMPLE SIZE (DENOTED LBND),A VEC-
[13]   ⍝ TOR OF UPPER BOUNDS (DENOTED UBND), AND A VECTOR OF CONFIDENCE INTER-
[14]   ⍝ VAL SIZES (DENOTED INTV). THESE VECTORS CAN BE USED TO PRODUCE GRAPHS
[15]   ⍝ IN GRAFSTAT.
[16]    ⎕←'ENTER PARAMETERS OF THE BETA PRIOR DISTRIBUTION'
[17]    A←⎕
[18]    ⎕←'ENTER VECTOR OF VARIOUS SAMPLE SIZES'
[19]    C←⎕
[20]    ⎕←'ENTER X MATRIX'
[21]    X←⎕
[22]    ⎕←'ENTER Y MATRIX'
[23]    Y←⎕
[24]    SD←0.01,0.99
[25]    ⎕←'THE VALUES OF ALPHA AND BETA PRIOR ARE'
[26]    ⎕←A
[27]    ⎕←'THE PRIOR BELIEF OF P.LOWER AND P.UPPER ARE'
[28]    V←A BQUANT SD
[29]    ⎕←V
[30]    ⎕←' '
[31]    ⎕←' '
[32]    ⎕←' '
[33]    ⎕←' '
[34]    ⎕←'   N       A*          B*       P.LO    P.HI   CI SIZE'
[35]    K←0
[36]    R←⍴C
[37]    SAMSZ←0
[38]    INTV←0
[39]    LBND←0
[40]    UBND←0
[41]  LOOP:K←K+1
[42]    DF←C[K]
[43]    A INTER2 DF
[44]    SAMSZ←SAMSZ,DF
[45]    INTV←INTV,CI
```

```
[46]   LBND←LBND,PR2[1]
[47]   UBND←UBND,PR2[2]
[48]   STUFF←DF,FY,PR2,CI
[49]   4 0 11 4 11 4 7 4 7 4 7 4 ⍕STUFF
[50]   →(K<R)/LOOP
[51]   X CHARTER Y
[52]   SAMSZ←1↓SAMSZ
[53]   INTV←1↓INTV
[54]   LBND←1↓LBND
[55]   UBND←1↓UBND
[56]   ⎕←' '
       ∇


       ∇ B INTER2 N
[1]   ⍝ THIS SUBROUTINE IS USED TO COMPUTE UPPER AND LOWER BOUNDS FOR A 95
[2]   ⍝ PERCENT CONFIDENCE INTERVAL. THE LEVEL OF CONFIDENCE CAN BE CHANGED
[3]   ⍝ BY USING DIFFERENT VALUES IN LINES 11 AND 14 OF THIS SUBROUTINE.
[4]   ⍝ FOR EXAMPLE IF WE DESIRED A 90 PERCENT CONFIDENCE INTERVAL WE COULD
[5]   ⍝ CHANGE 0.025 IN LINE 11 TO 0.05 AND CHANGE 0.975 IN LINE 14 TO 0.95.
[6]   ⍝ THIS WOULD RESULT IN THE LOWER AND UPPER BOUNDS FOR A 90 PERCENT
[7]   ⍝ CONFIDENCE INTERVAL. IN ADDITION, THIS SUBROUTINE CALCULATES THE
[8]   ⍝ VALUES OF ALPHA* AND BETA*, THE PARAMETERS FOR THE BETA POSTERIOR
[9]   ⍝ DISTRIBUTION.
[10]   RY←B[1]+((B[1]÷(B[1]+B[2]))×N)
[11]   QY←B[2]+((B[2]÷(B[1]+B[2]))×N)
[12]   FY←RY,QY
-13-   FY BQUAN 0.025
[14]   P1←V
[15]   →(P1≥1)/HOPE
[16]   V←FY BQUAN 0.975
[17]   P2←V
[18]   DUM←0
[19]   →(DUM=0)/HOP
[20] HOPE:P1←0
[21] HOP:PR2←P1,P2
[22]   CI←PR2[2]-PR2[1]
       ∇


       ∇ X CHARTER Y
[1]   ⍝ THIS SUBROUTINE WAS DESIGNED TO WORK WITH A SOFTWARE PACKAGE
[2]   ⍝ DEVELOPED BY DR. PETER W. ZEHNA OF THE NPS. IN ORDER TO USE THIS
[3]   ⍝ SUBROUTINE THE USER MUST SELECT INTEGER VALUES OF SAMPLE SIZE N AT,
[4]   ⍝ OR NEAR 500 AND 1000, SO THAT THE BETA POSTERIOR PARAMETERS ARE
[5]   ⍝ ALSO INTEGERS. THESE PARAMETERS ARE CALCULATED IN THE FOLLOWING
[6]   ⍝ MANNER:
[7]   ⍝        ALPHA* = ALPHA + ((ALPHA)÷(ALPHA + BETA)) × N
[8]   ⍝        BETA*  = BETA  + ((BETA)÷(ALPHA + BETA)) × N.
[9]   ⍝ THEN THE USER MUST FIND THE CDF OF THE F-DISTRIBUTION AT 0.025 AND
[10] ⍝ 0.975 USING 2 × ALPHA* AND 2 × BETA* DEGREES OF FREEDOM IN BOTH .
[11] ⍝ CASES AND FOR EACH VALUE OF N.
[12] ⍝
[13] ⍝ THE X VECTOR IS COMPRISED OF THE FOLLOWING ELEMENTS:
[14] ⍝      X = (CDF OF F AT 0.025,CDF OF F AT 0.975,N AT, OR NEAR 500)
[15] ⍝ (REMEMBER DEGREES OF FREEDOM ARE COMPUTED AT N NEAR 500)
```

```
[16] ⍝
[17] ⍝ THE Y VECTOR IS COMPRISED OF THE FOLLOWING ELEMENTS:
[18] ⍝      Y = (CDF OF F AT 0.025,CDF OF F AT 0.975,N AT, OR NEAR 1000)
[19] ⍝ (REMEMBER DEGREES OF FREEDOM ARE COMPUTED AT N NEAR 1000)
[20]  RM←X[3]
[21]  NM←A[1]+((A[1]÷(A[1]+A[2]))×RM)
[22]  KM←A[2]+((A[2]÷(A[1]+A[2]))×RM)
[23]  TM←NM,KM
[24]  MC←X[1]
[25]  JC←NM×MC
[26]  SC←KM+JC
[27]  AB←JC÷SC
[28]  MCM←X[2]
[29]  LM←NM×MCM
[30]  SM←KM+LM
[31]  ASB←LM÷SM
[32]  AM←AB,ASB
[33]  AEM←ASB-AB
[34]  SAMSZ←SAMSZ,RM
[35]  INTV←INTV,AEM
[36]  LBND←LBND,AB
[37]  UBND←UBND,ASB
[38]  POE←RM,TM,AM,AEM
[39]  4 0 11 4 11 4 7 4 7 4 7 4 ⍕POE
[40]  RBM←Y[3]
[41]  NCM←A[1]+((A[1]÷(A[1]+A[2]))×RBM)
[42]  KEM←A[2]+((A[2]÷(A[1]+A[2]))×RBM)
[43]  MTM←NCM,KEM
[44]  MJC←Y[1]
[45]  JCC←NCM×MJC
[46]  SCC←KEM+JCC
[47]  ABM←JCC÷SCC
[48]  MCM2←Y[2]
[49]  LAM←NCM×MCM2
[50]  SMM←KEM+LAM
[51]  ASBM←LAM÷SMM
[52]  MAM←ABM,ASBM
[53]  AEMS←ASBM-ABM
[54]  SAMSZ←SAMSZ,RBM
[55]  INTV←INTV,AEMS
[56]  LBND←LBND,ABM
[57]  UBND←UBND,ASBM
[58]  APOE←RBM,MTM,MAM,AEMS
[59]  4 0 11 4 11 4 7 4 7 4 7 4 ⍕APOE
      ∇
```

APPENDIX F. THE APL PROGRAMS USED TO DETERMINE SAMPLE SIZE FOR USERS WITHOUT GRAPHIC CAPABILITIES

```
      ∇ SCHARTS
[1]   ⍝ THIS PROGRAM IS USED TO DETERMINE AN INTERVAL WITHIN WHICH THE
[2]   ⍝ EXACT SAMPLE SIZE FOR A DESIRED CONFIDENCE INTERVAL LIES.
[3]    ⎕←'ENTER ALPHA AND BETA PARAMETERS'
[4]    A←⎕
[5]    ⎕←'ENTER VECTOR OF SAMPLE SIZES'
[6]    C7←⎕
[7]    SD←0.01,0.99
[8]    A BQUANT SD
[9]    K←0
[10]   R←⍴C7
[11]   RK←0
[12]  LOOP:K←K+1
[13]   KN←K-1
[14]   M←K+1
[15]   DF←C7[K]
[16]   A INTER2 DF
[17]   ALF←A[1]
[18]   BAIT←A[2]
[19]   RK←RK,CI
[20]   CIS←RK[M]
[21]   SIS←RK[2]
[22]   →(SIS<0.2)/LINK
[23]  ⍝STUFF←ALF,C7[K],FY,PR2,CI
[24]   →(CIS<0.15)/TTEND
[25]   →(CIS≤0.2)/OUTS
[26]   →(K<R)/LOOP
[27]   →(CIS>0.2)/TTEND
[28]  OUTS:BOZO←0
[29]   ⎕←'LIMITS FOR 0.20'
[30]   ⎕←'ALPHA BETA  N     CI SIZE'
[31]   STUFF1←ALF,BAIT,C7[KN],RK[K]
[32]   STUFF←ALF,BAIT,C7[K],RK[M]
[33]  ⍝OUTS: 9 4 11 4 11 4 7 4 7 4 8 5 ⍕STUFF
[34]   3 0 5 0 6 0 10 5 ⍕STUFF1
[35]   3 0 5 0 6 0 10 5 ⍕STUFF
[36]   FUN←0
[37]   →(FUN<1)/POOP
[38]  LINK:⎕←'SORRY NO GO FOR 0.20'
[39]   →(SIS<0.15)/FOOT
[40]  POOP:K←K+1
[41]   DF←C7[K]
[42]   KN←K-1
[43]   M←K+1
[44]   A INTER2 DF
[45]   RK←RK,CI
[46]   CIS←RK[M]
[47]   →(CIS≤0.15)/OUTP
```

```
 →(K<R)/POOP
FOOT:⎕←'SORRY NO GO FOR 0.15'
 GUM←0
 →(GUM=0)/TEND
TTEND:⎕←'SORRY NO GO ON THIS ONE FOR 0.20 OR 0.15'
 DUM←0
 →(DUM=0)/TEND
OUTP:BOZO←0
 ⎕←'LIMITS FOR 0.15'
 ⎕←'ALPHA BETA   N     CI SIZE'
 STUFF←ALF,BAIT,C7[KN],RK[K]
 3 0 5 0 6 0 10 5 ⍕STUFF
 STUFF1←ALF,BAIT,C7[K],RK[M]
 3 0 5 0 6 0 10 5 ⍕STUFF1
TEND:⎕←'PROGRAM COMPLETE'
 ∇


 ∇ CHARTS
⍝ THIS PROGRAM PROVIDES AN ABRIDGED VERSION OF CHARTPLUS AND IS
⍝ CAPABLE OF COMPUTING CONFIDENCE INTERVAL SIZES FOR CONTINUOUS
⍝ VALUES OF N (NUMBER OF SAMPLES REQUIRED)
 ⎕←'ENTER VALUES OF ALPHA AND BETA PARAMETERS'
 A←⎕
 ⎕←'ENTER NEW SAMPLE SIZE VECTOR, MUST ENTER AT LEAST 2 NUMBERS'
 C7←⎕
 ⎕←'      N       A*          B*          P.LO    P.HI   CI SIZE'
 K←0
 R←⍴C7
LOOP:K←K+1
 DF←C7[K]
 A INTER2 DF
 STUFF←DF,FY,PR2,CI
 9 4 11 4 11 4 7 4 7 4 8 5 ⍕STUFF
 →(K<R)/LOOP
 ∇
```

APPENDIX G. THE APL PROGRAMS USED TO FIND SAMPLE SIZE FOR DIFFERENT BETA PRIOR DISTRIBUTIONS WITH THE SAME MEANS

```
      ∇ SMEAN
[1]   ⍝ THIS PROGRAM ALLOWS ONE TO CALCULATE THE POINTS NECESSARY TO PLOT
[2]   ⍝ A LINE THAT CAN BE USED TO DETERMINE THE REQUIRED NUMBER OF SAMPLES
[3]   ⍝ THAT ATTAIN A DESIRED CONFIDENCE INTERVAL SIZE FOR BETA PRIOR DIS-
[4]   ⍝ TRIBUTIONS WITH THE SAME MEANS. TO USE THIS PROGRAM THE USER MUST
[5]   ⍝ KNOW THE REQUIRED SAMPLE SIZE FOR AT LEAST ONE SET OF PARAMETERS.
[6]   ⎕←'INPUT ALPHA AND BETA'
[7]   A1A2←⎕
[8]   ⎕←'INPUT THE NUMBER OF SAMPLES NEEDED TO ATTAIN THE DESIRED INTERVAL SIZE
      '
[9]   SNUM←⎕
[10]  EV←A1A2[1]÷(A1A2[1]+A1A2[2])
[11]  XINT←A1A2[1]÷(SNUM×EV)
[12]  NUMPR←SNUM÷((A1A2[1]-1)÷EV)
[13]  NSLOP←NUMPR
[14]  DSLOP←1-XINT
[15]  SLOPE←NSLOP÷DSLOP
[16]  NSLOPE←¯1×SLOPE
[17]  ⎕←'THE NECESSARY SAMPLE SIZE NEEDED AT ALPHA = 1 FOR A MEAN OF ',(⍕EV),'
      IS ',(⍕NUMPR)
[18]  ⎕←'THE VALUE OF ALPHA FOR WHICH NO SAMPLES ARE NEEDED (X INTERCEPT) FOR A
                BETA DISTRIBUTION WITH A MEAN OF ',(⍕EV),' IS ',(⍕XINT)
[19]  ⎕←'AS ALPHA IS INCREASED BY ONE THE NECESSARY SAMPLE SIZE OF THIS BETA
               DISTRIBUTION IS DECREASED BY ',(⍕NSLOPE)
      ∇


      ∇ GENERAL
[1]   ⍝ THIS PROGRAM CAN BE USED TO DETERMINE A REQUIRED SAMPLE SIZE FOR A
[2]   ⍝ BETA PRIOR DISTRIBUTION THAT HAS THE SAME MEAN AS A SECOND
[3]   ⍝ BETA PRIOR DISTRIBUTION BUT HAS DIFFERENT PARAMETERS. TO USE THIS
[4]   ⍝ PROGRAM THE USER MUST KNOW THE REQUIRED SAMPLE SIZE FOR THE SECOND
[5]   ⍝ BETA DISTRIBUTION.
[6]   ⎕←'INPUT ORIGINAL AND SECOND ALPHAS'
[7]   A1A2←⎕
[8]   ⎕←'INPUT NUMBER OF SAMPLES REQUIRED FOR ORIGINAL ALPHA'
[9]   N←⎕
[10]  ⎕←'INPUT THE MEAN (SHOULD BE THE SAME FOR BOTH ALPHAS)'
[11]  EV←⎕
[12]  NSAM←N÷((A1A2[1]-A1A2[2])÷EV)
[13]  11 5 ⍕NSAM
      ∇
```

APPENDIX H. TABLES SHOWING EFFECT OF SAMPLE SIZE WHEN THE NUMBER OF SUCCESSES K = (THE MEAN OF THE BETA PRIOR) TIMES (THE NUMBER OF TRIALS)

Table 15. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 4, BETA = 4)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	4.5000	4.5000	.1990	.8010	.6021
5	6.5000	6.5000	.2430	.7570	.5140
10	9.0000	9.0000	.2781	.7219	.4438
15	11.5000	11.5000	.3020	.6980	.3961
20	14.0000	14.0000	.3195	.6805	.3610
30	19.0000	19.0000	.3440	.6560	.3120
40	24.0000	24.0000	.3606	.6394	.2787
50	29.0000	29.0000	.3729	.6271	.2542
60	34.0000	34.0000	.3824	.6176	.2352
70	39.0000	39.0000	.3900	.6100	.2199
80	44.0000	44.0000	.3964	.6036	.2072
90	49.0000	49.0000	.4017	.5983	.1966
100	54.0000	54.0000	.4063	.5937	.1874
110	59.0000	59.0000	.4103	.5897	.1793
120	64.0000	64.0000	.4139	.5861	.1723
130	69.0000	69.0000	.4170	.5830	.1660
140	74.0000	74.0000	.4198	.5802	.1603
150	79.0000	79.0000	.4224	.5776	.1552
160	84.0000	84.0000	.4247	.5753	.1506
170	89.0000	89.0000	.4268	.5732	.1463
180	94.0000	94.0000	.4288	.5712	.1424
190	99.0000	99.0000	.4306	.5694	.1388
200	104.0000	104.0000	.4323	.5677	.1354
504	256.0000	256.0000	.4568	.5433	.0865
1000	504.0000	504.0000	.4692	.5308	.0617

Table 16. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 3, BETA = 6)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	3.3333	6.6667	.0940	.6353	.5412
2	3.6667	7.3333	.1020	.6217	.5197
3	4.0000	8.0000	.1093	.6097	.5005
4	4.3333	8.6667	.1159	.5991	.4832
5	4.6667	9.3333	.1220	.5896	.4676
6	5.0000	10.0000	.1276	.5810	.4534
7	5.3333	10.6667	.1328	.5732	.4404
8	5.6667	11.3333	.1376	.5661	.4285
9	6.0000	12.0000	.1421	.5596	.4175
10	6.3333	12.6667	.1463	.5536	.4073
15	8.0000	16.0000	.1638	.5292	.3654
20	9.6667	19.3333	.1771	.5114	.3343
25	11.3333	22.6667	.1877	.4976	.3099
30	13.0000	26.0000	.1963	.4865	.2902
35	14.6667	29.3333	.2036	.4774	.2738
40	16.3333	32.6667	.2098	.4697	.2599
45	18.0000	36.0000	.2152	.4632	.2480
50	19.6667	39.3333	.2199	.4574	.2375
55	21.3333	42.6667	.2241	.4524	.2283
60	23.0000	46.0000	.2279	.4479	.2200
65	24.6667	49.3333	.2313	.4439	.2126
70	26.3333	52.6667	.2344	.4403	.2059
75	28.0000	56.0000	.2372	.4370	.1998
80	29.6667	59.3333	.2398	.4340	.1942
90	33.0000	66.0000	.2444	.4287	.1843
100	36.3333	72.6667	.2483	.4241	.1758
110	39.6667	79.3333	.2518	.4201	.1683
120	43.0000	86.0000	.2549	.4167	.1617
130	46.3333	92.6667	.2577	.4135	.1559
140	49.6667	99.3333	.2601	.4108	.1506
150	53.0000	106.0000	.2624	.4082	.1459
160	56.3333	112.6667	.2644	.4060	.1415
504	171.0000	342.0000	.2932	.3747	.0815
900	303.0000	606.0000	.3031	.3643	.0612

Table 17. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 6, BETA = 3)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	6.6667	3.3333	.3647	.9060	.5412
2	7.3333	3.6667	.3783	.8980	.5197
3	8.0000	4.0000	.3903	.8907	.5005
4	8.6667	4.3333	.4009	.8841	.4832
5	9.3333	4.6667	.4104	.8780	.4676
6	10.0000	5.0000	.4190	.8724	.4534
7	10.6667	5.3333	.4268	.8672	.4404
8	11.3333	5.6667	.4339	.8624	.4285
9	12.0000	6.0000	.4404	.8579	.4175
10	12.6667	6.3333	.4464	.8537	.4073
15	16.0000	8.0000	.4708	.8362	.3654
20	19.3333	9.6667	.4886	.8229	.3343
25	22.6667	11.3333	.5024	.8123	.3099
30	26.0000	13.0000	.5135	.8037	.2902
35	29.3333	14.6667	.5226	.7964	.2738
40	32.6667	16.3333	.5303	.7902	.2599
45	36.0000	18.0000	.5368	.7848	.2480
50	39.3333	19.6667	.5426	.7801	.2375
55	42.6667	21.3333	.5476	.7759	.2283
60	46.0000	23.0000	.5521	.7721	.2200
65	49.3333	24.6667	.5561	.7687	.2126
70	52.6667	26.3333	.5597	.7656	.2059
75	56.0000	28.0000	.5630	.7628	.1998
80	59.3333	29.6667	.5660	.7602	.1942
90	66.0000	33.0000	.5713	.7556	.1843
100	72.6667	36.3333	.5759	.7517	.1758
110	79.3333	39.6667	.5799	.7482	.1683
120	86.0000	43.0000	.5833	.7451	.1617
130	92.6667	46.3333	.5865	.7423	.1559
140	99.3333	49.6667	.5892	.7399	.1506
150	106.0000	53.0000	.5918	.7376	.1459
160	112.6667	56.3333	.5940	.7356	.1415
504	342.0000	171.0000	.6253	.7068	.0815
900	606.0000	303.0000	.6357	.6969	.0612

Table 18. **THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 6, BETA = 6)**

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	6.5000	6.5000	.2430	.7570	.5140
2	7.0000	7.0000	.2513	.7487	.4973
3	7.5000	7.5000	.2589	.7411	.4821
4	8.0000	8.0000	.2659	.7341	.4683
5	8.5000	8.5000	.2722	.7278	.4555
6	9.0000	9.0000	.2781	.7219	.4438
7	9.5000	9.5000	.2836	.7164	.4329
8	10.0000	10.0000	.2886	.7114	.4227
9	10.5000	10.5000	.2934	.7066	.4132
10	11.0000	11.0000	.2978	.7022	.4044
15	13.5000	13.5000	.3164	.6836	.3673
20	16.0000	16.0000	.3306	.6694	.3388
25	18.5000	18.5000	.3420	.6580	.3160
30	21.0000	21.0000	.3513	.6487	.2973
35	23.5000	23.5000	.3592	.6408	.2816
40	26.0000	26.0000	.3660	.6340	.2681
45	28.5000	28.5000	.3718	.6282	.2564
50	31.0000	31.0000	.3770	.6230	.2461
55	33.5000	33.5000	.3815	.6185	.2369
60	36.0000	36.0000	.3856	.6144	.2287
65	38.5000	38.5000	.3894	.6106	.2213
70	41.0000	41.0000	.3927	.6073	.2146
75	43.5000	43.5000	.3958	.6042	.2084
80	46.0000	46.0000	.3986	.6014	.2028
85	48.5000	48.5000	.4012	.5988	.1975
90	51.0000	51.0000	.4036	.5964	.1927
100	56.0000	56.0000	.4080	.5920	.1840
110	61.0000	61.0000	.4118	.5882	.1764
120	66.0000	66.0000	.4152	.5848	.1697
130	71.0000	71.0000	.4182	.5818	.1637
140	76.0000	76.0000	.4209	.5791	.1582
150	81.0000	81.0000	.4233	.5767	.1533
160	86.0000	86.0000	.4256	.5744	.1488
170	91.0000	91.0000	.4276	.5724	.1447
180	96.0000	96.0000	.4295	.5705	.1409
190	101.0000	101.0000	.4313	.5687	.1374
200	106.0000	106.0000	.4329	.5671	.1342
504	258.0000	258.0000	.4569	.5431	.0862
1008	510.0000	510.0000	.4693	.5307	.0613

Table 19. **THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 4, BETA = 9)**

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	4.3077	9.6923	.1049	.5623	.4574
2	4.6154	10.3846	.1101	.5536	.4435
3	4.9231	11.0769	.1150	.5458	.4308
4	5.2308	11.7692	.1195	.5386	.4192
5	5.5385	12.4615	.1237	.5321	.4084
6	5.8462	13.1538	.1276	.5260	.3984
7	6.1538	13.8462	.1313	.5204	.3891
8	6.4615	14.5385	.1348	.5152	.3805
9	6.7692	15.2308	.1380	.5104	.3723
10	7.0769	15.9231	.1411	.5059	.3647
15	8.6154	19.3846	.1544	.4870	.3326
20	10.1538	22.8462	.1649	.4725	.3076
25	11.6923	26.3077	.1735	.4611	.2876
30	13.2308	29.7692	.1807	.4516	.2710
35	14.7692	33.2308	.1868	.4438	.2570
40	16.3077	36.6923	.1921	.4370	.2449
45	17.8462	40.1538	.1968	.4312	.2344
50	19.3846	43.6154	.2009	.4261	.2252
55	20.9231	47.0769	.2046	.4215	.2169
60	22.4615	50.5385	.2079	.4174	.2095
65	24.0000	54.0000	.2109	.4138	.2028
70	25.5385	57.4615	.2137	.4105	.1967
75	27.0769	60.9231	.2162	.4074	.1912
80	28.6154	64.3846	.2186	.4046	.1861
90	31.6923	71.3077	.2228	.3997	.1769
100	34.7692	78.2308	.2264	.3954	.1690
110	37.8462	85.1538	.2296	.3917	.1621
120	40.9231	92.0769	.2324	.3884	.1560
130	44.0000	99.0000	.2350	.3855	.1505
140	47.0769	105.9231	.2373	.3828	.1455
150	50.1538	112.8462	.2394	.3804	.1410
160	53.2308	119.7692	.2413	.3783	.1369
170	56.3077	126.6923	.2431	.3763	.1332
507	160.0000	360.0000	.2688	.3480	.0792
910	284.0000	639.0000	.2783	.3378	.0595

Table 20. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 9, BETA = 4)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	9.6923	4.3077	.4377	.8951	.4574
2	10.3846	4.6154	.4464	.8899	.4435
3	11.0769	4.9231	.4542	.8850	.4308
4	11.7692	5.2308	.4614	.8805	.4192
5	12.4615	5.5385	.4679	.8763	.4084
6	13.1538	5.8462	.4740	.8724	.3984
7	13.8462	6.1538	.4796	.8687	.3891
8	14.5385	6.4615	.4848	.8652	.3805
9	15.2308	6.7692	.4896	.8620	.3723
10	15.9231	7.0769	.4941	.8589	.3647
15	19.3846	8.6154	.5130	.8456	.3326
20	22.8462	10.1538	.5275	.8351	.3076
25	26.3077	11.6923	.5389	.8265	.2876
30	29.7692	13.2308	.5484	.8193	.2710
35	33.2308	14.7692	.5562	.8132	.2570
40	36.6923	16.3077	.5630	.8079	.2449
45	40.1538	17.8462	.5688	.8032	.2344
50	43.6154	19.3846	.5739	.7991	.2252
55	47.0769	20.9231	.5785	.7954	.2169
60	50.5385	22.4615	.5826	.7921	.2095
65	54.0000	24.0000	.5862	.7891	.2028
70	57.4615	25.5385	.5895	.7863	.1967
75	60.9231	27.0769	.5926	.7838	.1912
80	64.3846	28.6154	.5954	.7814	.1861
90	71.3077	31.6923	.6003	.7772	.1769
100	78.2308	34.7692	.6046	.7736	.1690
110	85.1538	37.8462	.6083	.7704	.1621
120	92.0769	40.9231	.6116	.7676	.1560
130	99.0000	44.0000	.6145	.7650	.1505
140	105.9231	47.0769	.6172	.7627	.1455
150	112.8462	50.1538	.6196	.7606	.1410
160	119.7692	53.2308	.6217	.7587	.1369
170	126.6923	56.3077	.6237	.7569	.1332
507	360.0000	160.0000	.6520	.7312	.0792
910	639.0000	284.0000	.6622	.7217	.0595

Table 21. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 6, BETA = 16)

SAMPLE SIZE n	α^*	β^*	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	6.2727	16.7273	.1156	.4672	.3515
2	6.5455	17.4545	.1183	.4629	.3446
3	6.8182	18.1818	.1208	.4590	.3381
4	7.0909	18.9091	.1233	.4552	.3320
5	7.3636	19.6364	.1256	.4517	.3261
6	7.6364	20.3636	.1278	.4484	.3206
7	7.9091	21.0909	.1299	.4452	.3153
8	8.1818	21.8182	.1319	.4422	.3103
9	8.4545	22.5455	.1338	.4394	.3055
10	8.7273	23.2727	.1357	.4366	.3010
15	10.0909	26.9091	.1439	.4248	.2808
20	11.4545	30.5455	.1508	.4151	.2643
25	12.8182	34.1818	.1567	.4070	.2503
30	14.1818	37.8182	.1618	.4002	.2384
35	15.5455	41.4545	.1663	.3943	.2280
40	16.9091	45.0909	.1702	.3891	.2189
45	18.2727	48.7273	.1738	.3845	.2107
50	19.6364	52.3636	.1770	.3804	.2035
55	21.0000	56.0000	.1799	.3768	.1969
60	22.3636	59.6364	.1825	.3734	.1909
65	23.7273	63.2727	.1849	.3704	.1854
70	25.0909	66.9091	.1872	.3676	.1804
75	26.4545	70.5455	.1893	.3651	.1758
80	27.8182	74.1818	.1912	.3627	.1715
90	30.5455	81.4545	.1947	.3585	.1638
100	33.2727	88.7273	.1977	.3548	.1570
110	36.0000	96.0000	.2005	.3515	.1510
120	38.7273	103.2727	.2029	.3486	.1457
130	41.4545	110.5455	.2051	.3460	.1409
140	44.1818	117.8182	.2071	.3436	.1365
150	46.9091	125.0909	.2090	.3415	.1325
160	49.6364	132.3636	.2107	.3395	.1288
170	52.3636	139.6364	.2122	.3377	.1255
180	55.0909	146.9091	.2137	.3360	.1223
506	144.0000	384.0000	.2356	.3115	.0759
1012	282.0000	752.0000	.2460	.3003	.0542

Table 22. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 16, BETA = 6)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	16.7273	6.2727	.5328	.8844	.3515
2	17.4545	6.5455	.5371	.8817	.3446
3	18.1818	6.8182	.5410	.8792	.3381
4	18.9091	7.0909	.5448	.8767	.3320
5	19.6364	7.3636	.5483	.8744	.3261
6	20.3636	7.6364	.5516	.8722	.3206
7	21.0909	7.9091	.5548	.8701	.3153
8	21.8182	8.1818	.5578	.8681	.3103
9	22.5455	8.4545	.5606	.8662	.3055
10	23.2727	8.7273	.5634	.8643	.3010
15	26.9091	10.0909	.5752	.8561	.2808
20	30.5455	11.4545	.5849	.8492	.2643
25	34.1818	12.8182	.5930	.8433	.2503
30	37.8182	14.1818	.5998	.8382	.2384
35	41.4545	15.5455	.6057	.8337	.2280
40	45.0909	16.9091	.6109	.8298	.2189
45	48.7273	18.2727	.6155	.8262	.2107
50	52.3636	19.6364	.6196	.8230	.2035
55	56.0000	21.0000	.6232	.8201	.1969
60	59.6364	22.3636	.6266	.8175	.1909
65	63.2727	23.7273	.6296	.8151	.1854
70	66.9091	25.0909	.6324	.8128	.1804
75	70.5455	26.4545	.6349	.8107	.1758
80	74.1818	27.8182	.6373	.8088	.1715
90	81.4545	30.5455	.6415	.8053	.1638
100	88.7273	33.2727	.6452	.8023	.1570
110	96.0000	36.0000	.6485	.7995	.1510
120	103.2727	38.7273	.6514	.7971	.1457
130	110.5455	41.4545	.6540	.7949	.1409
140	117.8182	44.1818	.6564	.7929	.1365
150	125.0909	46.9091	.6585	.7910	.1325
160	132.3636	49.6364	.6605	.7893	.1288
170	139.6364	52.3636	.6623	.7878	.1255
180	146.9091	55.0909	.6640	.7863	.1223
506	384.0000	144.0000	.6885	.7644	.0759
1012	752.0000	282.0000	.6997	.7540	.0542

Table 23. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 5, BETA = 20)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	5.2000	20.8000	.0732	.3702	.2971
2	5.4000	21.6000	.0750	.3669	.2919
3	5.6000	22.4000	.0767	.3637	.2869
4	5.8000	23.2000	.0784	.3606	.2823
5	6.0000	24.0000	.0799	.3577	.2778
6	6.2000	24.8000	.0815	.3550	.2736
7	6.4000	25.6000	.0829	.3524	.2695
8	6.6000	26.4000	.0843	.3499	.2656
9	6.8000	27.2000	.0857	.3476	.2619
10	7.0000	28.0000	.0870	.3453	.2583
15	8.0000	32.0000	.0930	.3353	.2424
20	9.0000	36.0000	.0980	.3271	.2291
25	10.0000	40.0000	.1024	.3202	.2178
30	11.0000	44.0000	.1063	.3143	.2080
35	12.0000	48.0000	.1098	.3091	.1994
40	13.0000	52.0000	.1128	.3046	.1918
45	14.0000	56.0000	.1156	.3006	.1850
50	15.0000	60.0000	.1181	.2970	.1789
55	16.0000	64.0000	.1204	.2938	.1733
60	17.0000	68.0000	.1225	.2908	.1683
65	18.0000	72.0000	.1245	.2881	.1636
70	19.0000	76.0000	.1263	.2856	.1593
75	20.0000	80.0000	.1280	.2834	.1554
80	21.0000	84.0000	.1296	.2813	.1517
85	22.0000	88.0000	.1310	.2793	.1483
90	23.0000	92.0000	.1324	.2775	.1451
100	25.0000	100.0000	.1349	.2742	.1392
110	27.0000	108.0000	.1372	.2712	.1340
120	29.0000	116.0000	.1392	.2686	.1294
130	31.0000	124.0000	.1411	.2663	.1252
140	33.0000	132.0000	.1427	.2641	.1214
150	35.0000	140.0000	.1443	.2622	.1179
160	37.0000	148.0000	.1457	.2604	.1147
170	39.0000	156.0000	.1470	.2588	.1118
180	41.0000	164.0000	.1483	.2573	.1090
190	43.0000	172.0000	.1494	.2559	.1065
200	45.0000	180.0000	.1505	.2546	.1041
500	105.0000	420.0000	.1669	.2352	.0683
1000	205.0000	820.0000	.1761	.2250	.0489

AD-A195 087 NUMBER OF SAMPLES NEEDED TO OBTAIN DESIRED BAYESION CONFIDENCE INTERVALS FOR A PROPORTION(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA R B MANION MAR 88 2/2

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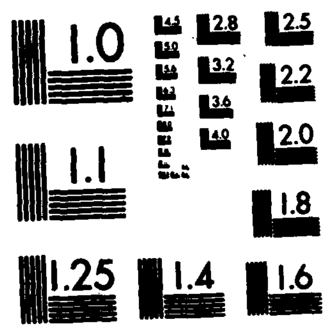
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Table 24. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 20, BETA = 5)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	20.8000	5.2000	.6298	.9268	.2971
2	21.6000	5.4000	.6331	.9250	.2919
3	22.4000	5.6000	.6363	.9233	.2869
4	23.2000	5.8000	.6394	.9216	.2823
5	24.0000	6.0000	.6423	.9201	.2778
6	24.8000	6.2000	.6450	.9185	.2736
7	25.6000	6.4000	.6476	.9171	.2695
8	26.4000	6.6000	.6501	.9157	.2656
9	27.2000	6.8000	.6524	.9143	.2619
10	28.0000	7.0000	.6547	.9130	.2583
15	32.0000	8.0000	.6647	.9070	.2424
20	36.0000	9.0000	.6729	.9020	.2291
25	40.0000	10.0000	.6798	.8976	.2178
30	44.0000	11.0000	.6857	.8937	.2080
35	48.0000	12.0000	.6909	.8902	.1994
40	52.0000	13.0000	.6954	.8872	.1918
45	56.0000	14.0000	.6994	.8844	.1850
50	60.0000	15.0000	.7030	.8819	.1789
55	64.0000	16.0000	.7062	.8796	.1733
60	68.0000	17.0000	.7092	.8775	.1683
65	72.0000	18.0000	.7119	.8755	.1636
70	76.0000	19.0000	.7144	.8737	.1593
75	80.0000	20.0000	.7166	.8720	.1554
80	84.0000	21.0000	.7187	.8704	.1517
85	88.0000	22.0000	.7207	.8690	.1483
90	92.0000	23.0000	.7225	.8676	.1451
100	100.0000	25.0000	.7258	.8651	.1392
110	108.0000	27.0000	.7288	.8628	.1340
120	116.0000	29.0000	.7314	.8608	.1294
130	124.0000	31.0000	.7337	.8589	.1252
140	132.0000	33.0000	.7359	.8573	.1214
150	140.0000	35.0000	.7378	.8557	.1179
160	148.0000	37.0000	.7396	.8543	.1147
170	156.0000	39.0000	.7412	.8530	.1118
180	164.0000	41.0000	.7427	.8517	.1090
190	172.0000	43.0000	.7441	.8506	.1065
200	180.0000	45.0000	.7454	.8495	.1041
500	420.0000	105.0000	.7648	.8331	.0683
1000	820.0000	205.0000	.7750	.8239	.0489

Table 25. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 15, BETA = 15)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	15.5000	15.5000	.3280	.6720	.3440
2	16.0000	16.0000	.3306	.6694	.3388
3	16.5000	16.5000	.3331	.6669	.3338
4	17.0000	17.0000	.3354	.6646	.3291
5	17.5000	17.5000	.3377	.6623	.3246
6	18.0000	18.0000	.3399	.6601	.3202
7	18.5000	18.5000	.3420	.6580	.3160
8	19.0000	19.0000	.3440	.6560	.3120
9	19.5000	19.5000	.3459	.6541	.3081
10	20.0000	20.0000	.3478	.6522	.3044
15	22.5000	22.5000	.3562	.6438	.2876
20	25.0000	25.0000	.3634	.6366	.2732
25	27.5000	27.5000	.3696	.6304	.2609
30	30.0000	30.0000	.3750	.6250	.2500
35	32.5000	32.5000	.3798	.6202	.2404
40	35.0000	35.0000	.3841	.6159	.2319
45	37.5000	37.5000	.3879	.6121	.2242
50	40.0000	40.0000	.3914	.6086	.2172
55	42.5000	42.5000	.3946	.6054	.2108
60	45.0000	45.0000	.3975	.6025	.2050
65	47.5000	47.5000	.4002	.5998	.1996
70	50.0000	50.0000	.4027	.5973	.1946
75	52.5000	52.5000	.4050	.5950	.1900
80	55.0000	55.0000	.4072	.5928	.1857
85	57.5000	57.5000	.4092	.5908	.1816
90	60.0000	60.0000	.4111	.5889	.1779
100	65.0000	65.0000	.4145	.5855	.1710
110	70.0000	70.0000	.4176	.5824	.1648
120	75.0000	75.0000	.4204	.5796	.1593
130	80.0000	80.0000	.4229	.5771	.1543
140	85.0000	85.0000	.4252	.5748	.1497
150	90.0000	90.0000	.4272	.5728	.1455
160	95.0000	95.0000	.4292	.5708	.1417
170	100.0000	100.0000	.4310	.5690	.1381
180	105.0000	105.0000	.4326	.5674	.1348
190	110.0000	110.0000	.4341	.5659	.1317
200	115.0000	115.0000	.4356	.5644	.1288
510	270.0000	270.0000	.4579	.5421	.0842
900	465.0000	465.0000	.4679	.5321	.0642

Table 26. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 5, BETA = 30)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	5.1429	30.8571	.0505	.2725	.2220
2	5.2857	31.7143	.0514	.2706	.2192
3	5.4286	32.5714	.0523	.2687	.2164
4	5.5714	33.4286	.0532	.2670	.2138
5	5.7143	34.2857	.0540	.2653	.2112
6	5.8571	35.1429	.0549	.2636	.2088
7	6.0000	36.0000	.0557	.2620	.2064
8	6.1429	36.8571	.0564	.2605	.2041
9	6.2857	37.7143	.0572	.2591	.2019
10	6.4286	38.5714	.0579	.2577	.1997
15	7.1429	42.8571	.0614	.2513	.1899
20	7.8571	47.1429	.0644	.2458	.1814
25	8.5714	51.4286	.0671	.2411	.1740
30	9.2857	55.7143	.0695	.2369	.1674
35	10.0000	60.0000	.0717	.2332	.1615
40	10.7143	64.2857	.0738	.2299	.1562
45	11.4286	68.5714	.0756	.2269	.1513
50	12.1429	72.8571	.0773	.2243	.1469
55	12.8571	77.1429	.0789	.2218	.1429
60	13.5714	81.4286	.0804	.2195	.1392
65	14.2857	85.7143	.0817	.2175	.1357
70	15.0000	90.0000	.0830	.2155	.1325
75	15.7143	94.2857	.0842	.2138	.1295
80	16.4286	98.5714	.0854	.2121	.1267
85	17.1429	102.8571	.0864	.2105	.1241
90	17.8571	107.1429	.0874	.2091	.1216
100	19.2857	115.7143	.0893	.2064	.1171
110	20.7143	124.2857	.0910	.2041	.1131
120	22.1429	132.8571	.0925	.2019	.1094
130	23.5714	141.4286	.0939	.2000	.1061
140	25.0000	150.0000	.0952	.1982	.1031
150	26.4286	158.5714	.0964	.1966	.1003
160	27.8571	167.1429	.0975	.1952	.0977
170	29.2857	175.7143	.0985	.1938	.0953
180	30.7143	184.2857	.0995	.1925	.0931
190	32.1429	192.8571	.1004	.1914	.0910
200	33.5714	201.4286	.1012	.1903	.0891
525	80.0000	480.0000	.1151	.1730	.0579
1015	150.0000	900.0000	.1224	.1646	.0422

Table 27. **THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 30, BETA = 5)**

SAMPLE SIZE n	α^*	β^*	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	30.8571	5.1429	.7275	.9495	.2220
2	31.7143	5.2857	.7294	.9486	.2192
3	32.5714	5.4286	.7313	.9477	.2164
4	33.4286	5.5714	.7330	.9468	.2138
5	34.2857	5.7143	.7347	.9460	.2112
6	35.1429	5.8571	.7364	.9451	.2088
7	36.0000	6.0000	.7380	.9443	.2064
8	36.8571	6.1429	.7395	.9436	.2041
9	37.7143	6.2857	.7409	.9428	.2019
10	38.5714	6.4286	.7423	.9421	.1997
15	42.8571	7.1429	.7487	.9386	.1899
20	47.1429	7.8571	.7542	.9356	.1814
25	51.4286	8.5714	.7589	.9329	.1740
30	55.7143	9.2857	.7631	.9305	.1674
35	60.0000	10.0000	.7668	.9283	.1615
40	64.2857	10.7143	.7701	.9262	.1562
45	68.5714	11.4286	.7731	.9244	.1513
50	72.8571	12.1429	.7757	.9227	.1469
55	77.1429	12.8571	.7782	.9211	.1429
60	81.4286	13.5714	.7805	.9196	.1392
65	85.7143	14.2857	.7825	.9183	.1357
70	90.0000	15.0000	.7845	.9170	.1325
75	94.2857	15.7143	.7862	.9158	.1295
80	98.5714	16.4286	.7879	.9146	.1267
85	102.8571	17.1429	.7895	.9136	.1241
90	107.1429	17.8571	.7909	.9126	.1216
100	115.7143	19.2857	.7936	.9107	.1171
110	124.2857	20.7143	.7959	.9090	.1131
120	132.8571	22.1429	.7981	.9075	.1094
130	141.4286	23.5714	.8000	.9061	.1061
140	150.0000	25.0000	.8018	.9048	.1031
150	158.5714	26.4286	.8034	.9036	.1003
160	167.1429	27.8571	.8048	.9025	.0977
170	175.7143	29.2857	.8062	.9015	.0953
180	184.2857	30.7143	.8075	.9005	.0931
190	192.8571	32.1429	.8086	.8996	.0910
200	201.4286	33.5714	.8097	.8988	.0891
525	480.0000	80.0000	.8270	.8849	.0579
1015	900.0000	150.0000	.8354	.8776	.0423

Table 28. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 6, BETA = 34)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	6.1500	34.8500	.0595	.2726	.2132
2	6.3000	35.7000	.0603	.2710	.2107
3	6.4500	36.5500	.0611	.2695	.2084
4	6.6000	37.4000	.0619	.2680	.2061
5	6.7500	38.2500	.0627	.2666	.2039
6	6.9000	39.1000	.0634	.2652	.2018
7	7.0500	39.9500	.0642	.2639	.1997
8	7.2000	40.8000	.0649	.2626	.1977
9	7.3500	41.6500	.0656	.2613	.1958
10	7.5000	42.5000	.0662	.2601	.1939
15	8.2500	46.7500	.0694	.2546	.1852
20	9.0000	51.0000	.0722	.2498	.1776
25	9.7500	55.2500	.0747	.2456	.1709
30	10.5000	59.5000	.0770	.2419	.1648
35	11.2500	63.7500	.0791	.2385	.1594
40	12.0000	68.0000	.0810	.2355	.1545
45	12.7500	72.2500	.0828	.2328	.1500
50	13.5000	76.5000	.0844	.2303	.1458
55	14.2500	80.7500	.0859	.2280	.1420
60	15.0000	85.0000	.0874	.2259	.1385
65	15.7500	89.2500	.0887	.2239	.1352
70	16.5000	93.5000	.0899	.2221	.1322
75	17.2500	97.7500	.0911	.2204	.1293
80	18.0000	102.0000	.0922	.2188	.1267
85	18.7500	106.2500	.0932	.2174	.1242
90	19.5000	110.5000	.0942	.2160	.1218
100	21.0000	119.0000	.0960	.2134	.1174
110	22.5000	127.5000	.0977	.2112	.1135
120	24.0000	136.0000	.0992	.2091	.1099
130	25.5000	144.5000	.1005	.2072	.1067
140	27.0000	153.0000	.1018	.2055	.1037
150	28.5000	161.5000	.1030	.2040	.1010
160	30.0000	170.0000	.1041	.2025	.0985
170	31.5000	178.5000	.1051	.2012	.0961
180	33.0000	187.0000	.1061	.2000	.0939
190	34.5000	195.5000	.1069	.1988	.0919
200	36.0000	204.0000	.1078	.1977	.0900
520	84.0000	476.0000	.1217	.1807	.0590
920	144.0000	816.0000	.1281	.1733	.0452

Table 29. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 34, BETA = 6)

SAMPLE SIZE n	α^*	β^*	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	34.8500	6.1500	.7274	.9405	.2132
2	35.7000	6.3000	.7290	.9397	.2107
3	36.5500	6.4500	.7305	.9389	.2084
4	37.4000	6.6000	.7320	.9381	.2061
5	38.2500	6.7500	.7334	.9373	.2039
6	39.1000	6.9000	.7348	.9366	.2018
7	39.9500	7.0500	.7361	.9358	.1997
8	40.8000	7.2000	.7374	.9351	.1977
9	41.6500	7.3500	.7387	.9344	.1958
10	42.5000	7.5000	.7399	.9338	.1939
15	46.7500	8.2500	.7454	.9306	.1852
20	51.0000	9.0000	.7502	.9278	.1776
25	55.2500	9.7500	.7544	.9253	.1709
30	59.5000	10.5000	.7581	.9230	.1648
35	63.7500	11.2500	.7615	.9209	.1594
40	68.0000	12.0000	.7645	.9190	.1545
45	72.2500	12.7500	.7672	.9172	.1500
50	76.5000	13.5000	.7697	.9156	.1458
55	80.7500	14.2500	.7720	.9141	.1420
60	85.0000	15.0000	.7741	.9126	.1385
65	89.2500	15.7500	.7761	.9113	.1352
70	93.5000	16.5000	.7779	.9101	.1322
75	97.7500	17.2500	.7796	.9089	.1293
80	102.0000	18.0000	.7812	.9078	.1267
85	106.2500	18.7500	.7826	.9068	.1242
90	110.5000	19.5000	.7840	.9058	.1218
100	119.0000	21.0000	.7866	.9040	.1174
110	127.5000	22.5000	.7888	.9023	.1135
120	136.0000	24.0000	.7909	.9008	.1099
130	144.5000	25.5000	.7928	.8995	.1067
140	153.0000	27.0000	.7945	.8982	.1037
150	161.5000	28.5000	.7960	.8970	.1010
160	170.0000	30.0000	.7975	.8959	.0985
170	178.5000	31.5000	.7988	.8949	.0961
180	187.0000	33.0000	.8000	.8939	.0939
190	195.5000	34.5000	.8012	.8931	.0919
200	204.0000	36.0000	.8023	.8922	.0900
520	476.0000	84.0000	.8193	.8783	.0590
920	816.0000	144.0000	.8267	.8719	.0451

Table 30. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 5, BETA = 40)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	5.1111	40.8889	.0385	.2154	.1769
2	5.2222	41.7778	.0391	.2142	.1751
3	5.3333	42.6667	.0396	.2130	.1734
4	5.4444	43.5556	.0402	.2118	.1717
5	5.5556	44.4444	.0407	.2107	.1700
6	5.6667	45.3333	.0412	.2096	.1684
7	5.7778	46.2222	.0417	.2086	.1669
8	5.8889	47.1111	.0422	.2076	.1654
9	6.0000	48.0000	.0427	.2066	.1639
10	6.1111	48.8889	.0432	.2056	.1625
15	6.6667	53.3333	.0454	.2012	.1558
20	7.2222	57.7778	.0474	.1973	.1499
25	7.7778	62.2222	.0492	.1939	.1447
30	8.3333	66.6667	.0509	.1908	.1399
35	8.8889	71.1111	.0525	.1881	.1356
40	9.4444	75.5556	.0539	.1856	.1317
45	10.0000	80.0000	.0552	.1833	.1281
50	10.5556	84.4444	.0565	.1812	.1248
55	11.1111	88.8889	.0576	.1793	.1217
60	11.6667	93.3333	.0587	.1775	.1188
65	12.2222	97.7778	.0597	.1759	.1161
70	12.7778	102.2222	.0607	.1743	.1136
75	13.3333	106.6667	.0616	.1729	.1113
80	13.8889	111.1111	.0624	.1715	.1091
85	14.4444	115.5556	.0632	.1703	.1070
90	15.0000	120.0000	.0640	.1691	.1051
100	16.1111	128.8889	.0655	.1669	.1014
110	17.2222	137.7778	.0668	.1649	.0982
120	18.3333	146.6667	.0680	.1632	.0952
130	19.4444	155.5556	.0691	.1615	.0925
140	20.5556	164.4444	.0701	.1601	.0900
150	21.6667	173.3333	.0711	.1587	.0877
160	22.7778	182.2222	.0719	.1575	.0855
170	23.8889	191.1111	.0728	.1563	.0835
180	25.0000	200.0000	.0735	.1552	.0817
190	26.1111	208.8889	.0743	.1542	.0799
200	27.2222	217.7778	.0750	.1533	.0783
540	65.0000	520.0000	.0870	.1378	.0508
1035	120.0000	960.0000	.0931	.1305	.0374

Table 31. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 40, BETA = 5)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	40.8889	5.1111	.7846	.9615	.1769
2	41.7778	5.2222	.7858	.9609	.1751
3	42.6667	5.3333	.7870	.9604	.1734
4	43.5556	5.4444	.7882	.9598	.1717
5	44.4444	5.5556	.7893	.9593	.1700
6	45.3333	5.6667	.7904	.9588	.1684
7	46.2222	5.7778	.7914	.9583	.1669
8	47.1111	5.8889	.7924	.9578	.1654
9	48.0000	6.0000	.7934	.9573	.1639
10	48.8889	6.1111	.7944	.9568	.1625
15	53.3333	6.6667	.7988	.9546	.1558
20	57.7778	7.2222	.8027	.9526	.1499
25	62.2222	7.7778	.8061	.9508	.1447
30	66.6667	8.3333	.8092	.9491	.1399
35	71.1111	8.8889	.8119	.9475	.1356
40	75.5556	9.4444	.8144	.9461	.1317
45	80.0000	10.0000	.8167	.9448	.1281
50	84.4444	10.5556	.8188	.9435	.1248
55	88.8889	11.1111	.8207	.9424	.1217
60	93.3333	11.6667	.8225	.9413	.1188
65	97.7778	12.2222	.8241	.9403	.1161
70	102.2222	12.7778	.8257	.9393	.1136
75	106.6667	13.3333	.8271	.9384	.1113
80	111.1111	13.8889	.8285	.9376	.1091
85	115.5556	14.4444	.8297	.9368	.1070
90	120.0000	15.0000	.8309	.9360	.1051
100	128.8889	16.1111	.8331	.9345	.1014
110	137.7778	17.2222	.8351	.9332	.0982
120	146.6667	18.3333	.8368	.9320	.0952
130	155.5556	19.4444	.8385	.9309	.0925
140	164.4444	20.5556	.8399	.9299	.0900
150	173.3333	21.6667	.8413	.9289	.0877
160	182.2222	22.7778	.8425	.9281	.0855
170	191.1111	23.8889	.8437	.9272	.0835
180	200.0000	25.0000	.8448	.9265	.0817
190	208.8889	26.1111	.8458	.9257	.0799
200	217.7778	27.2222	.8467	.9250	.0783
540	520.0000	65.0000	.8622	.9130	.0508
1035	960.0000	120.0000	.8695	.9069	.0375

Table 32. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 5, BETA = 50)

SAMPLE SIZE n	α^*	β^*	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	5.0909	50.9091	.0311	.1781	.1469
2	5.1818	51.8182	.0315	.1772	.1457
3	5.2727	52.7273	.0319	.1764	.1445
4	5.3636	53.6364	.0323	.1756	.1433
5	5.4545	54.5455	.0326	.1748	.1422
6	5.5455	55.4545	.0330	.1740	.1410
7	5.6364	56.3636	.0333	.1733	.1400
8	5.7273	57.2727	.0337	.1725	.1389
9	5.8182	58.1818	.0340	.1718	.1378
10	5.9091	59.0909	.0343	.1711	.1368
15	6.3636	63.6364	.0359	.1679	.1320
20	6.8182	68.1818	.0373	.1650	.1277
25	7.2727	72.7273	.0386	.1624	.1238
30	7.7273	77.2727	.0398	.1601	.1202
35	8.1818	81.8182	.0410	.1579	.1170
40	8.6364	86.3636	.0421	.1560	.1139
45	9.0909	90.9091	.0431	.1542	.1111
50	9.5455	95.4545	.0440	.1525	.1085
55	10.0000	100.0000	.0449	.1510	.1061
60	10.4545	104.5455	.0457	.1495	.1038
65	10.9091	109.0909	.0465	.1482	.1017
70	11.3636	113.6364	.0472	.1469	.0997
75	11.8182	118.1818	.0480	.1457	.0978
80	12.2727	122.7273	.0486	.1446	.0960
85	12.7273	127.2727	.0493	.1436	.0943
90	13.1818	131.8182	.0499	.1426	.0927
100	14.0909	140.9091	.0510	.1407	.0897
110	15.0000	150.0000	.0521	.1391	.0870
120	15.9091	159.0909	.0531	.1376	.0845
130	16.8182	168.1818	.0540	.1362	.0822
140	17.7273	177.2727	.0548	.1349	.0801
150	18.6364	186.3636	.0556	.1338	.0782
160	19.5455	195.4545	.0563	.1327	.0764
170	20.4545	204.5455	.0570	.1317	.0747
180	21.3636	213.6364	.0577	.1307	.0731
190	22.2727	222.7273	.0583	.1299	.0716
200	23.1818	231.8182	.0589	.1290	.0702
550	55.0000	550.0000	.0693	.1150	.0457
1045	100.0000	1000.0000	.0746	.1086	.0339

Table 33. THE EFFECT OF SAMPLE SIZE ON 95% BAYESIAN CONFIDENCE INTERVALS, WITH BETA PRIOR (ALPHA = 50, BETA = 5)

SAMPLE SIZE n	α *	β *	LOWER BOUND	UPPER BOUND	DESIRED SIZE 2A
1	50.9091	5.0909	.8219	.9689	.1469
2	51.8182	5.1818	.8228	.9685	.1457
3	52.7273	5.2727	.8236	.9681	.1445
4	53.6364	5.3636	.8244	.9677	.1433
5	54.5455	5.4545	.8252	.9674	.1422
6	55.4545	5.5455	.8260	.9670	.1410
7	56.3636	5.6364	.8267	.9667	.1400
8	57.2727	5.7273	.8275	.9663	.1389
9	58.1818	5.8182	.8282	.9660	.1378
10	59.0909	5.9091	.8289	.9657	.1368
15	63.6364	6.3636	.8321	.9641	.1320
20	68.1818	6.8182	.8350	.9627	.1277
25	72.7273	7.2727	.8376	.9614	.1238
30	77.2727	7.7273	.8399	.9602	.1202
35	81.8182	8.1818	.8421	.9590	.1170
40	86.3636	8.6364	.8440	.9579	.1139
45	90.9091	9.0909	.8458	.9569	.1111
50	95.4545	9.5455	.8475	.9560	.1085
55	100.0000	10.0000	.8490	.9551	.1061
60	104.5455	10.4545	.8505	.9543	.1038
65	109.0909	10.9091	.8518	.9535	.1017
70	113.6364	11.3636	.8531	.9528	.0997
75	118.1818	11.8182	.8543	.9520	.0978
80	122.7273	12.2727	.8554	.9514	.0960
85	127.2727	12.7273	.8564	.9507	.0943
90	131.8182	13.1818	.8574	.9501	.0927
100	140.9091	14.0909	.8593	.9490	.0897
110	150.0000	15.0000	.8609	.9479	.0870
120	159.0909	15.9091	.8624	.9469	.0845
130	168.1818	16.8182	.8638	.9460	.0822
140	177.2727	17.7273	.8651	.9452	.0801
150	186.3636	18.6364	.8662	.9444	.0782
160	195.4545	19.5455	.8673	.9437	.0764
170	204.5455	20.4545	.8683	.9430	.0747
180	213.6364	21.3636	.8693	.9423	.0731
190	222.7273	22.2727	.8701	.9417	.0716
200	231.8182	23.1818	.8710	.9411	.0702
550	550.0000	55.0000	.8850	.9307	.0457
1045	1000.0000	100.0000	.8914	.9254	.0339

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