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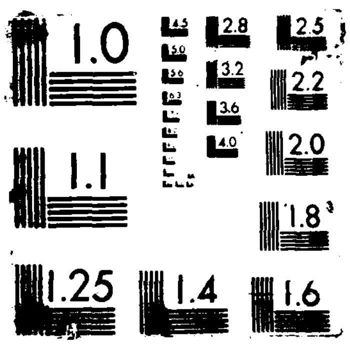
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AN AVERAGING ALGORITHM FOR MODES

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1 Introduction

The MODES model, discussed in PDRC Report 84-06, consists of the LIFTCAP and MRMATE models. LIFTCAP is a network flow with side constraints model while MRMATE is a network flow model. MODES uses Benders' decomposition as a solution technique that separates MRMATE and LIFTCAP and adds constraints at each iteration to the LIFTCAP model. Benders' thus represents a finitely convergent algorithm.

This report presents an alternative procedure that maintains problem structure of both LIFTCAP and MRMATE. Problem sizes are maintained at each iteration of the solution procedure. It ensures asymptotic convergence to a global optimal solution. Due to its nature it is termed the "averaging method". The steps involved in implementing this procedure are discussed and it is compared to Benders' decomposition technique.

Computational results are also presented. These reflect the performance of averaging versus Benders' decomposition technique.

2 An averaging algorithm for MODES

The averaging algorithm for MODES works as follows

1. Solve LIFTCAP with zero costs on arcs (or some reasonable estimates) and generate a LIFTCAP set of channels c_0.

2. Solve MRMATE with capabilities c_0 to generate a solution x_1 and a set of duals u^1, v^1 on the channels and MRs respectively. Set t = 1.

3. Set the costs = u^t on the channels in LIFTCAP (there is one dual value associated with each channel). Solve LIFTCAP to generate solution c_t. A lower bound to the problem is given by LB_t = LIFTCAP objective value + $\sum$ MR * v^t.

4. Set
$$c^t = \sum_{j=1}^{t} \frac{c_j}{t} = (1 - \frac{1}{t})c^{t-1} + \frac{c_t}{t}.$$
Solve MRMATE with these capabilities c^t. The objective function value provides an upper UB_t bound to the problem. The solution to MRMATE yields duals u_t, v_t.

5. Set

$$u^t = \sum_{j=1}^{t} \frac{u_j}{t} = (1 - \frac{1}{t})u^{t-1} + \frac{u_t}{t}$$

and

$$v^t = \sum_{j=1}^{t} \frac{v_j}{t} = (1 - \frac{1}{t})v^{t-1} + \frac{v_t}{t}.$$

6. If $UB_t - LB_t <$ epsilon, then stop with channel capabilities c^t and allocations x_t. Else, go to step 3.

While implementing the algorithm, store the maximum among lower bounds generated and the minimum among upper bounds generated across iterations.

2.1 Justification for the averaging procedure

The averaging procedure presented in Section 2.1 is essentially that of fictitious play of a two person game. Its asymptotic convergence relies on the proof of [Robinson] for the two person game. This section discusses how the LIFTCAP and MRMATE interaction can be considered as a two person game. LIFTCAP attempts to generate a set of channel capabilities that, when passed to MRMATE, yields the minimum penalty MR allocation to channels. However, since the LIFTCAP feasible region is a polytope, the particular channel capabilities required can be expressed as a convex combination of basic solutions to LIFTCAP. Now consider the dual feasible region of MRMATE for a given LIFTCAP solution. Since it is unaffected by channel capabilities generated by LIFTCAP, any dual solution can be expressed as a convex combination of dual extreme points (assuming a bounded dual region achieved by the use of dummy nodes in MRMATE).

Let

U = dual feasible region of MRMATE,

C = primal feasible region of LIFTCAP,

u^k = dual extreme points of MRMATE,

c^l = primal extreme points of LIFTCAP,

f_k and g_l = the multipliers associated with extreme points u^k and c^l.

The objective function value of MRMATE is $\max_{u \epsilon U} uc$

$$= \max_{u \epsilon U} \min_{c \epsilon C} uc$$
$$= \max_f \min_g \sum \sum_g f_k u^k * g_l c^l$$
$$= \max \ \min \sum \sum \ f_k g_l u^k c^l$$

The decomposition of LIFTCAP and MRMATE can be considered as a two person game. Given all its extreme points u^k, LIFTCAP generates a mixed strategy g that minimizes its objective function value. Similarly, given extreme points c^l, MRMATE generates a mixed strategy f that maximizes the dual objective function value. This produces a two person mixed strategy game with extreme points of the dual of MRMATE being the options of player 1 and extreme points of the primal of LIFTCAP reflecting the options of player 2.

[Robinson] has shown that fictitious play of a two person game converges asymptotically to the overall optimal solution to the problem. The algorithm in Section 2 is precisely fictitious play of a game applied to the LIFTCAP/MRMATE game. Therefore, the averaging algorithm converges to the overall optimal solution of MODES.

3 Comparison with Bender's decomposition

3.1 Problem Size and Structure

The averaging procedure maintains problem sizes for both MRMATE and LIFTCAP. Thus, MODES could be run for as many iterations as required without problems of memory availability. This is in contrast with Benders' decomposition which adds constraints to LIFTCAP thereby increasing problem size at each iteration. Benders', in current implementations, can be run for only about 10 to 20 iterations before numerical stability problems prevent furthur iterations with the network flow with side constraints LIFTCAP solution code.

Furthermore, the LIFTCAP model structure is maintained across iterations. Channel capabilities generated at each iteration have an intuitive interpretation as the best possible asset assignment for the given set of costs (MRMATE dual) allocated.

3.2 Flexibility

The averaging algorithm works for any set of weights ($0 < f < 1$) used to create a new set of channel capabilities or MRMATE duals at each iteration. Thus, the sequence of channels selected can be controlled by adjusting the weighting parameter.

If MODES is initialed with a good set of asset allocations, then the weights on the new sets of channels could be set very small. As the iterations progress, however, the weighting parameter could be increased to introduce better channel assignments.

3.3 Computational Results

An example problem was used to produce test results from the use of Benders' and averaging. The problem consists of 100 nodes in LIFTCAP with 1600 variables and an MRMATE problem that has 150 MRs and 300 channels. Figure 1 shows upper bounds generated at each iteration by the two procedures.

Figure 2 shows lower bounds generated by averaging and Figure 3 shows upper bounds, while Figure 4 shows upper and lower bounds for averaging across a hundred iterations.

Figure 5 gives the fraction of MRs allocated to channels for the two procedures. The averaging procedure appears to perform well, probably because it results in more non zero channels very quickly, thereby improving the MR allocations as compared to Benders'. This is another desirable feature of this algorithm. Continued testing of this procedure is necessary before strong conclusions can be made regarding averaging versus Benders'.

4 Conclusions and extensions

This report outlines a procedure to solve MODES without adding Benders' constraints to the LIFTCAP model. The procedure is asymptotically optimal and maintains problem size and structure at each iteration. Some

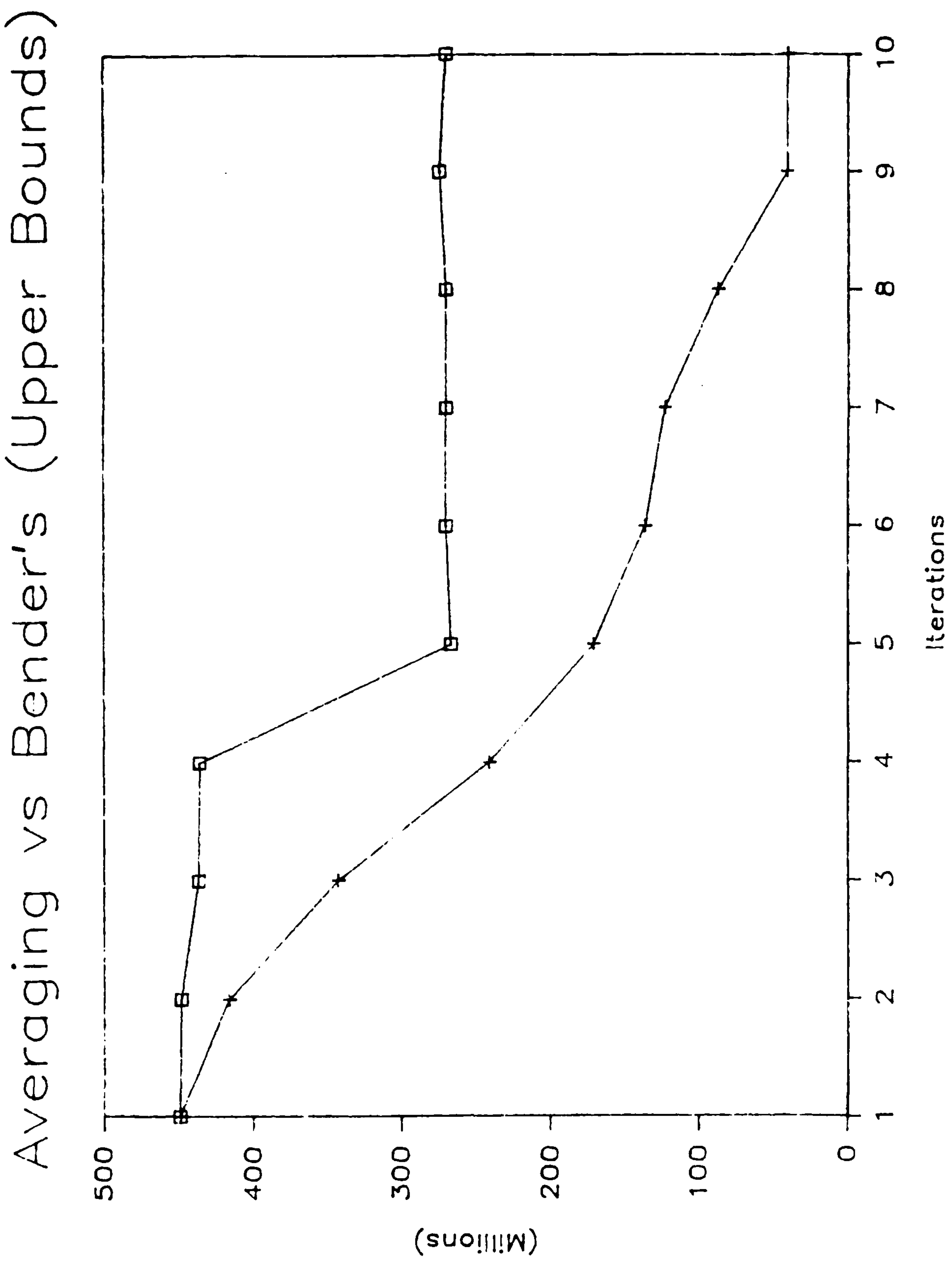

Figure 1: Upper Bounds for Benders' vs Averaging

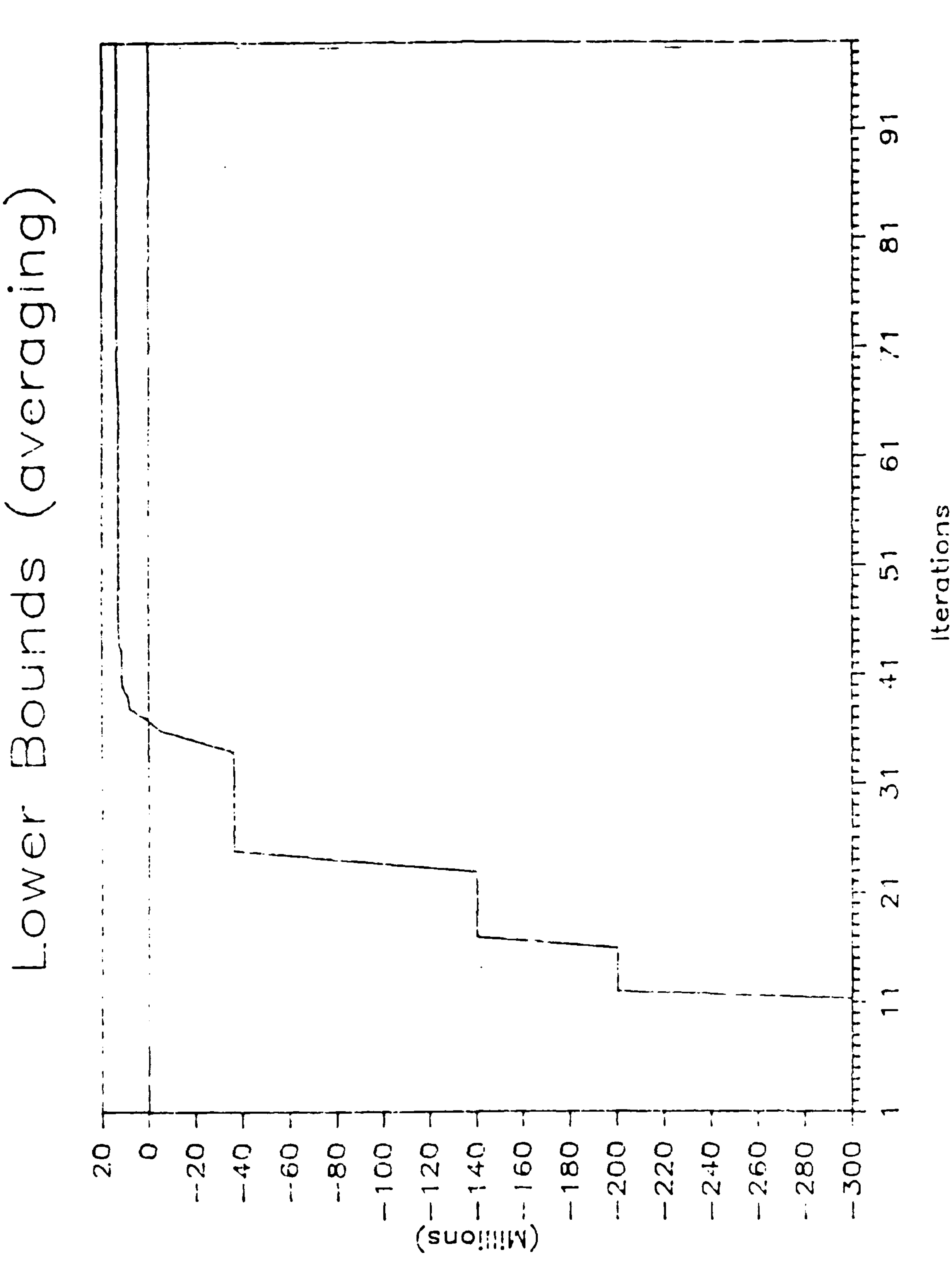

Figure 2: Lower Bounds for Averaging

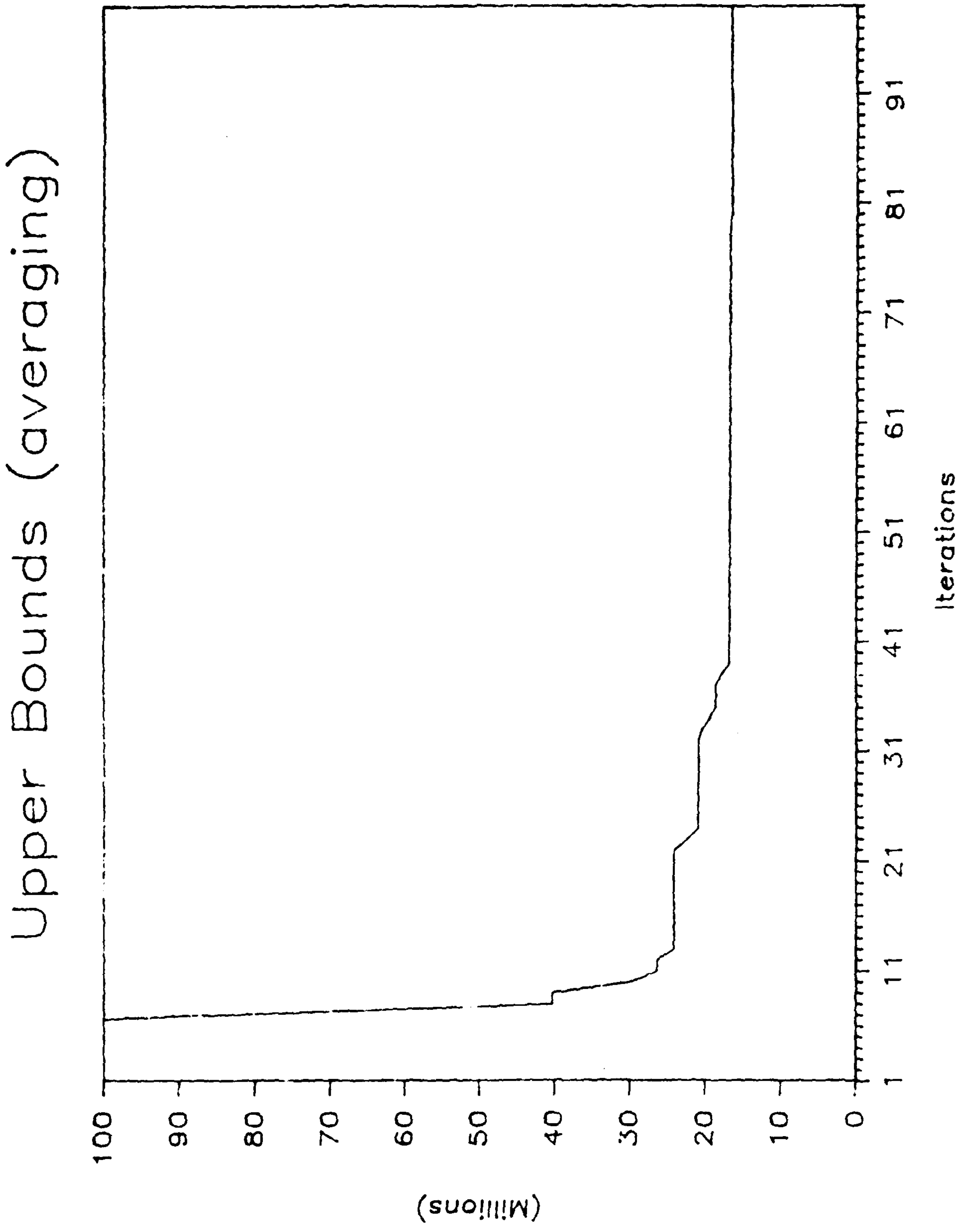

Figure 3: Upper Bounds for Averaging

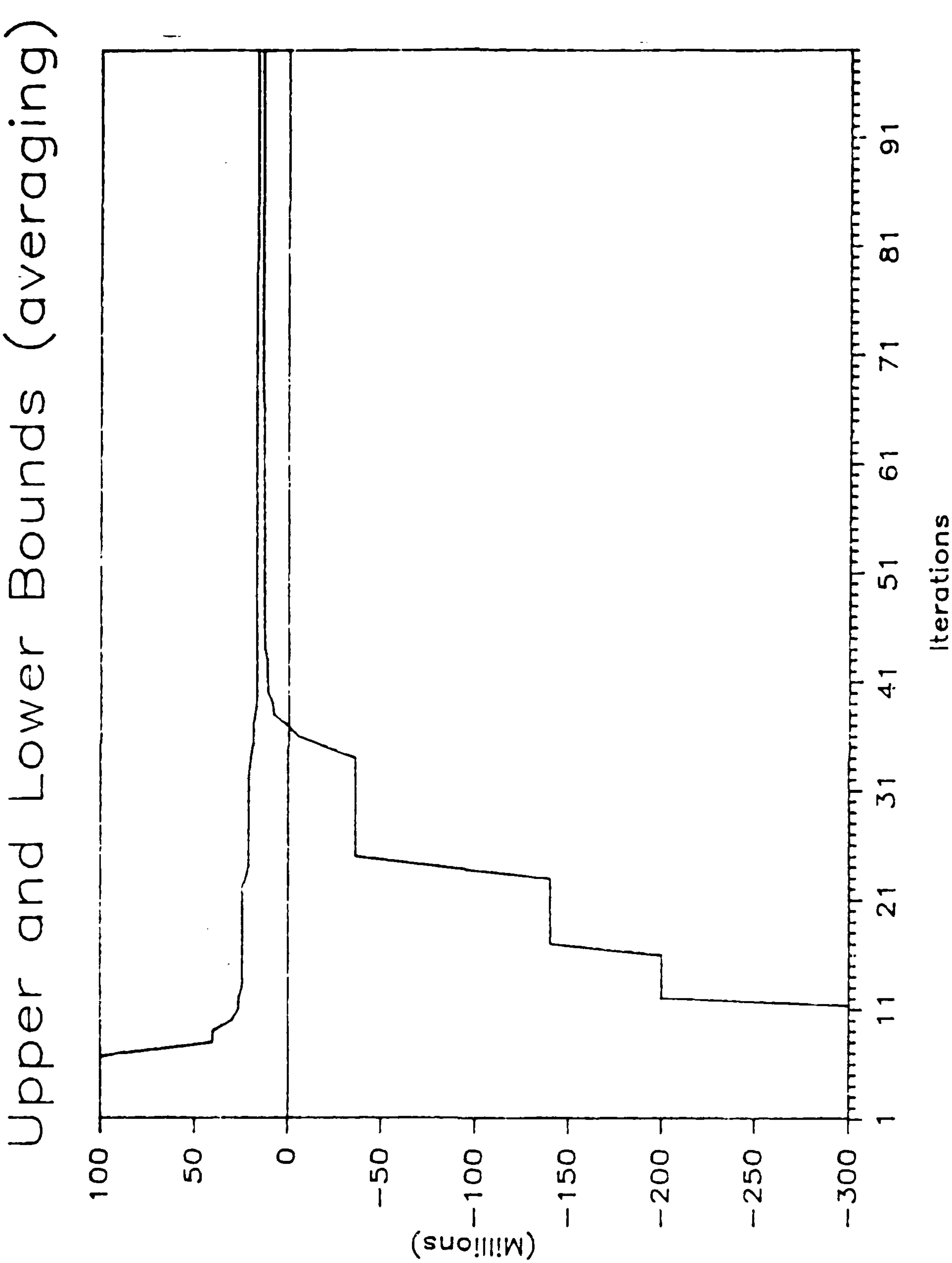

Figure 4: Upper and Lower Bounds for Averaging Across Iterations

Iteration	Bender's	Averaging
1	42.55	93.76
2	37.18	87.48
3	26.58	94.98
4	40.22	94.96
5	41.46	98.68
6	36.18	98.77

Figure 5: Fraction of MRs assigned to channels

preliminary results were presented which demonstrate the effect of this procedure over Benders' decomposition technique.

Further research is in progress to determine suitable choices of weighting parameters at each iteration. Furthermore, the most suitable procedure for MODES might be a hybrid approach that combines a few iterations of Benders' with the averaging procedure.

References

[Robinson] Robinson, J., "An Iterative Method of Solving a Game," Annals of Mathematics, 54, No. 2, pp. 296-301, 1951.

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