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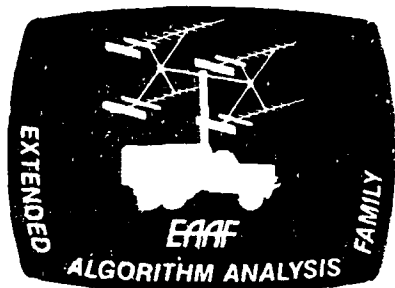
BEARING ERROR AND THE
CENTRAL LIMIT THEOREM

TECHNICAL MEMORANDUM No. 32

MARC

Mathematical Analysis Research Corporation

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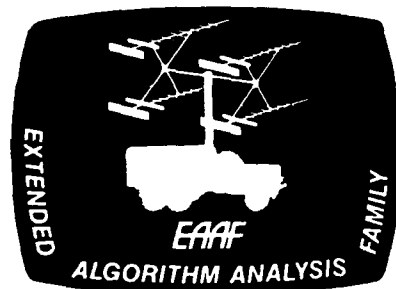
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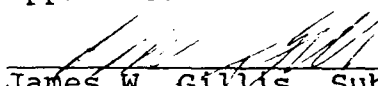
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Mathematical Analysis Research Corporation

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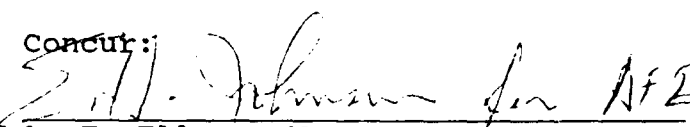


James W. Gillis, Subgroup Leader
Algorithm Analysis Subgroup




Edward J. Records, Supervisor
USAMS Task

Concur:



A. F. Ellman, Manager
Ground Data Systems Section



Fred Vote, Manager
Advanced Tactical Systems

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

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PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

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Bearing Error and the Central Limit Theorem

INTRODUCTION

Bearings are frequently computed as the average of a number of readings. Most of fixing theory assumes that the bearing error is normally distributed. Averages are usually much closer to being normally distributed than individual readings. The amount closer to normality depends on the amount of independence between readings and on the number of readings:

- i) If the readings are 100% dependent, then they are the same and hence the distribution of the average is no closer to normality than the original readings.
- ii) If the readings are 100% independent, then the convergence to normality is very fast as is shown in the accompanying examples. The exact speed of convergence depends on the shape of the original curve but not very much (This convergence is predicted by the Central Limit Theorem but the Central Limit Theorem does not address speed of convergence, hence the graphs provided here).

As a first approximation one may assume that some of the sources of error would be dependent and some independent. The independent error of the readings would get smaller and closer to normality as the number of readings being averaged increases. The dependent error of the readings would remain. The amount of the error that is independent could be judged using the standard deviation of the readings in comparison with the angular standard deviation. (Note: In order to compare the two, the standard deviation of the readings would need to be averaged).

THE GRAPHS

The method used to analyze the speed of convergence is demonstrated in Figure 1. The probability density functions for a Uniform distribution on the interval $[a,b]$ are presented. Each curve represents the density function for the average of a different number of readings (n). Figure 1 has the curves for $n=1$, $n=2$ and $n=3$. Notice that as the number of readings being used increases, the apparent Normality of the curve is increasing.

Figure 2 presents two curves. The lower curve is the density function for an average of three independent Uniform distributions. Superimposed on it is the associated Normal curve calculated using the Central Limit Theorem.

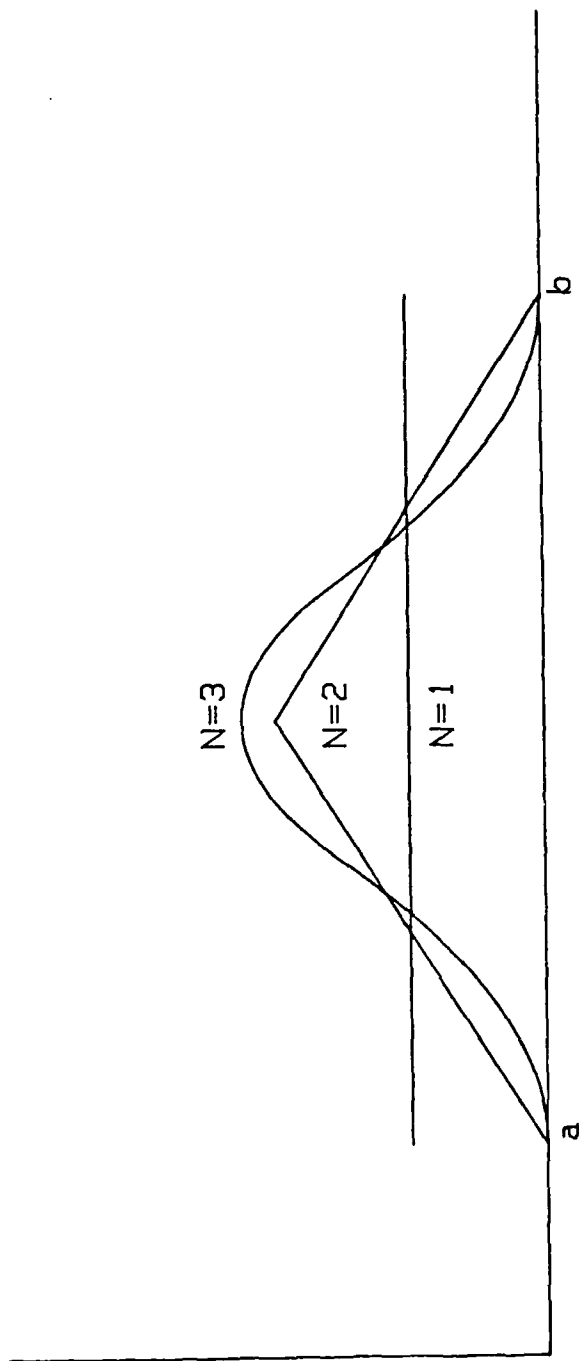
The speed of convergence for non-symmetric curves is portrayed in Figure 3. Two of the curves are the density functions of a Quadratic distribution averaging different numbers of readings. The $n=1$ case is almost a straight line, while the $n=2$ case is a decent approximation to a Normal

curve. The curve superimposed on the $n=2$ case is the associated Normal, again determined by the Central Limit Theorem. (The Normal has mean and standard deviation determined by the quadratic equation used in the distribution.) The speed of convergence for non-symmetric density functions is not as rapid as that for symmetric ones. Comparing Figures 1 and 3 is a bit misleading since the non-symmetric example is concave down, a factor which benefits the speed of convergence.

Figure 4 presents a non-symmetric density function which is concave up. The curves pictured are for a Quadratic distribution with $n=1$ and $n=2$. Superimposed on the $n=2$ case is the associated Normal. Note how much slower the rate of convergence seems for non-symmetric, concave up density functions.

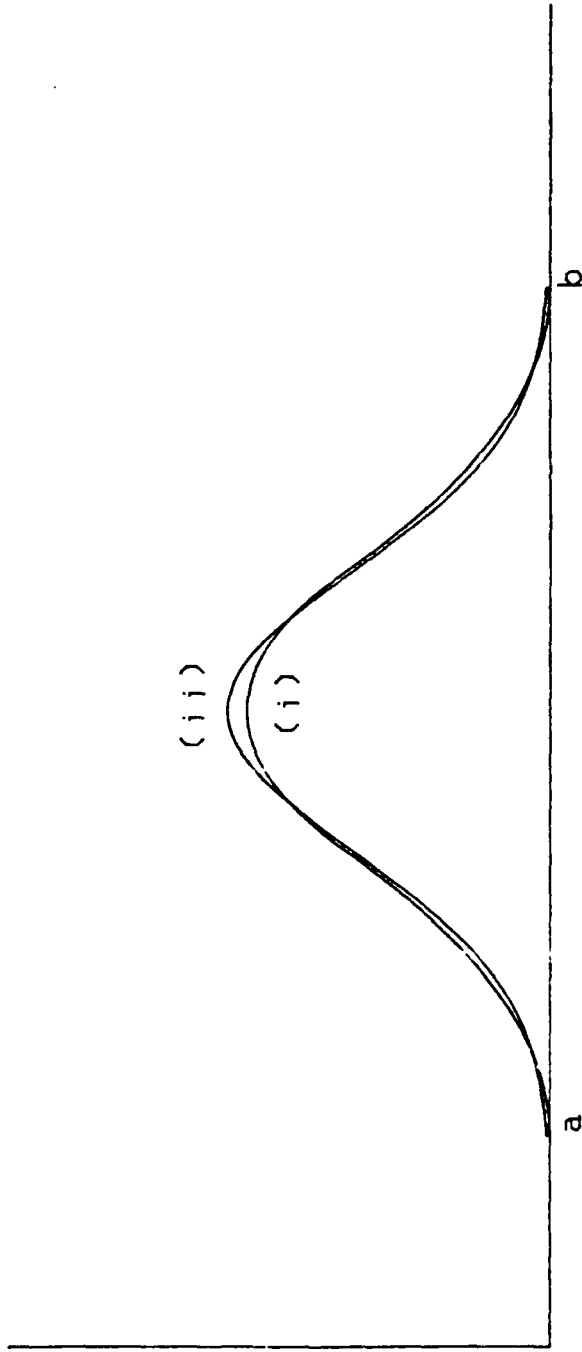
In general, symmetric density functions converge faster than do non-symmetric ones, but concavity plays a larger role. Figures 5 and 6 illustrate this problem. Figure 5 is a Quadratic distribution with a concave up density function and Figure 6 is a Quadratic distribution with a concave down density function. On both figures, the associated Normal is superimposed as a comparison with the $n=2$ case. The concave up density function barely resembles a Normal distribution, while the concave down density function is quite similar to its Normal.

FIGURE 1



(i) UNIFORM DISTRIBUTION; $N=1$, $N=2$, $N=3$ -- INDEPENDENT

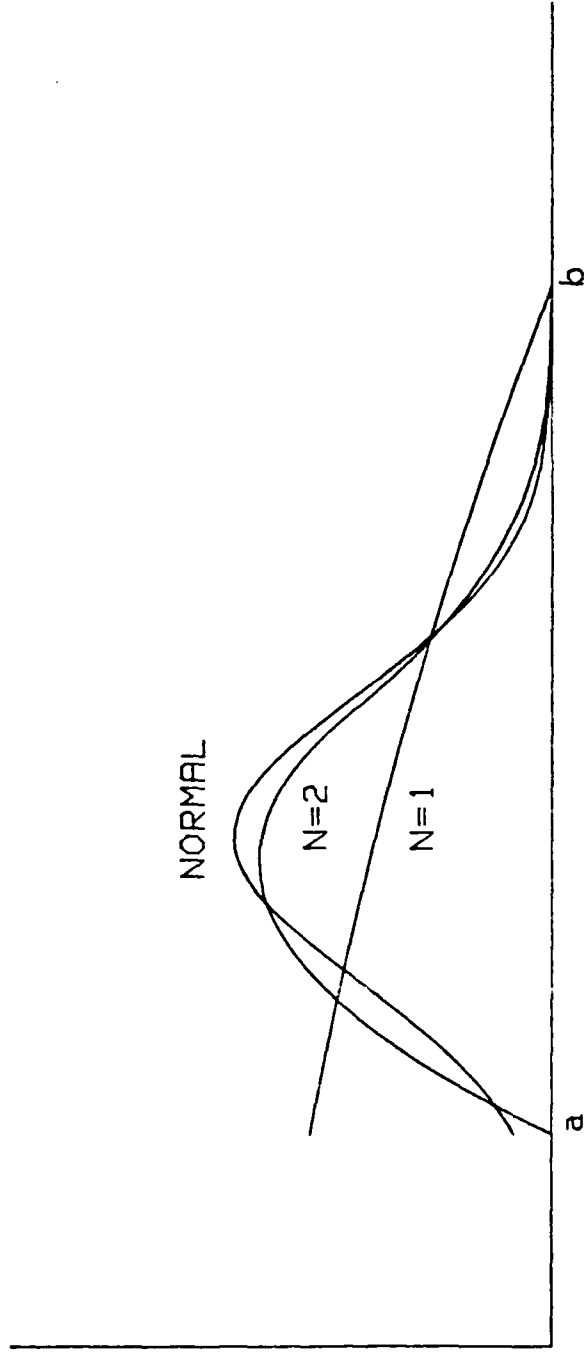
FIGURE 2



(i) UNIFORM DISTRIBUTION; $N=3$ -- INDEPENDENT

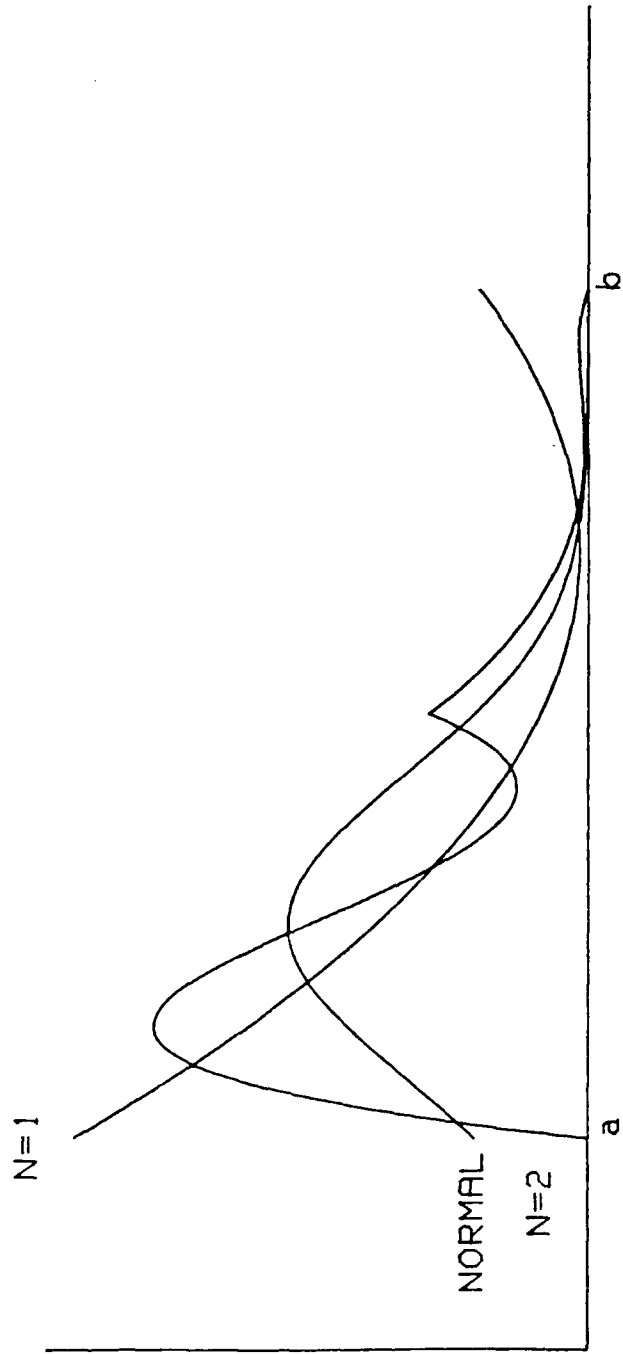
(ii) NORMAL DISTRIBUTION; $MEAN=(a+b)/2$

FIGURE 3 .



- (i) QUADRATIC DISTRIBUTION; $N=1$, $N=2$ -- INDEPENDENT
- (ii) NORMAL DISTRIBUTION; $\text{MEAN} = a + (0.35 * (b - a))$

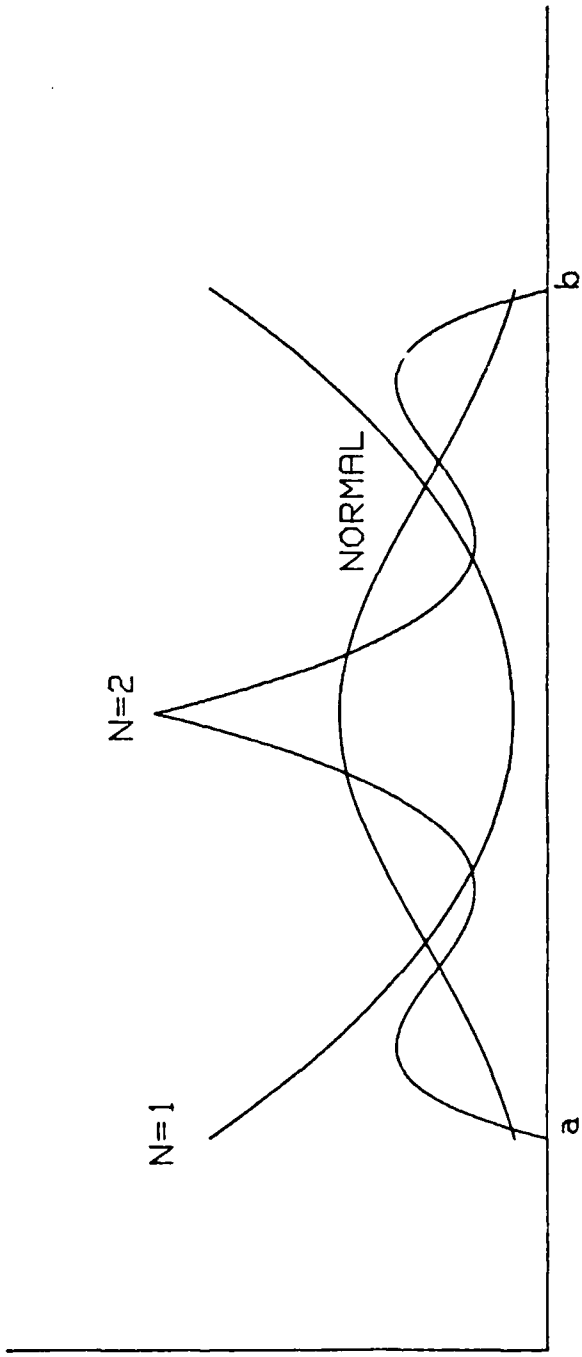
FIGURE 4



(i) QUADRATIC DISTRIBUTION; $N=1$, $N=2$ -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; $\text{MEAN} = a + (0.25 * (b - a))$

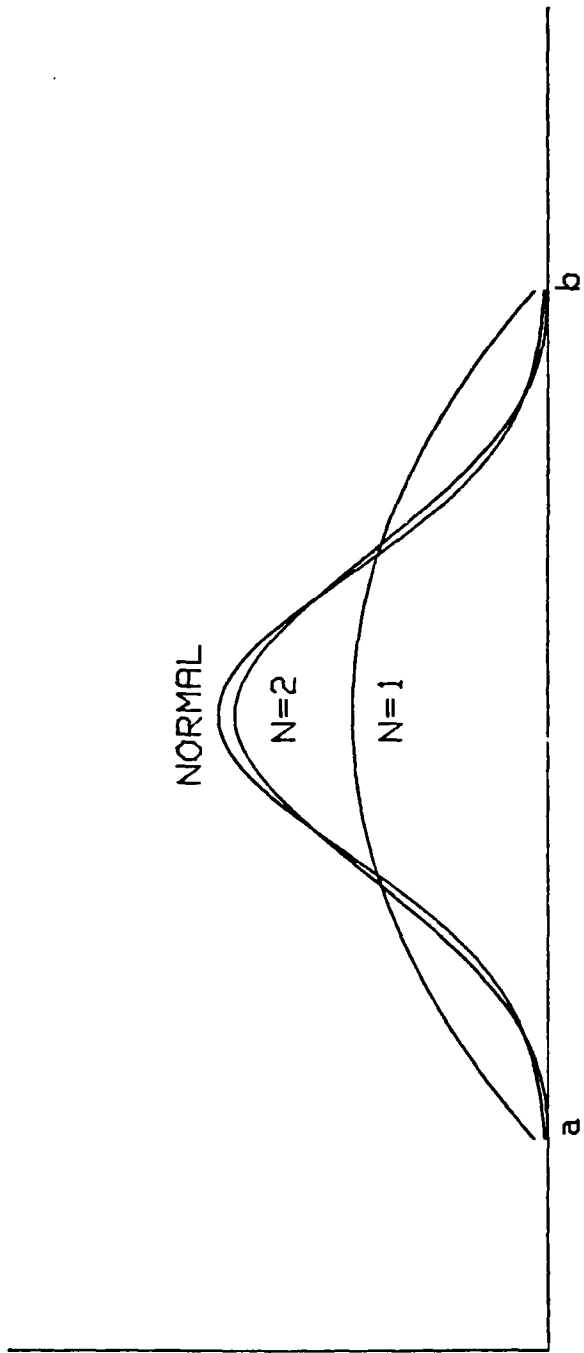
FIGURE 5



(i) QUADRATIC DISTRIBUTION; $N=1$, $N=2$ -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; $\text{MEAN}=(a+b)/2$

FIGURE 6



(i) QUADRATIC DISTRIBUTION; $N=1$, $N=2$ -- INDEPENDENT

(ii) NORMAL DISTRIBUTION; $\text{MEAN}=(a+b)/2$

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