



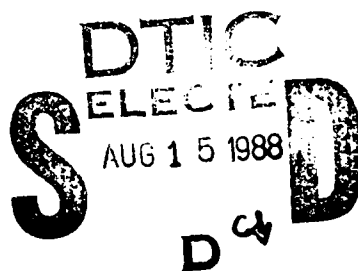
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Array Processing That Uses a Normal-Mode Model for Signal Representation

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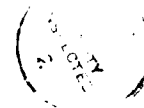
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INTRODUCTION

GENERAL

Most established theory for processing sonar arrays is based on fairly simple models of the signal arrival structure. For low frequencies and large aperture arrays these models become complex and may break down. It then becomes appropriate to reexamine the basic theory for how array processing should be done.

We shall assume that the frequency is low enough, and the aperture is large enough, so that ray theory is not appropriate. The approach is to postulate such an array and examine it from a signal processing viewpoint. The analysis is based on the normal-mode expansion of the received signal wave form. We shall assume that the sound speed profile and bottom effects are constant in the immediate neighborhood of the array. They may, however, vary substantially between the array and the source.

Under these assumptions, the wave front cannot be regarded as simply a plane crossing the array at a real angle. If sufficient *a priori* information is available, it may be practical to correct the wave front model. The application of such corrections is sometimes referred to as matched field processing. However, this approach has three disadvantages. First, adequate *a priori* information may not be available. Second, the process of computing and applying the corrections is quite formidable. Third, the usual matched field processing approach does not lend itself to instructive physical or geometric interpretation.

BACKGROUND

The normal-mode model of ocean propagation attempts to represent the signal field by an equation of the form

$$p(r, z_{\text{source}}, z_{\text{receiver}}) = \pi \sum_i U_i(z_{\text{source}}) U_i(z_{\text{receiver}}) H(\lambda_i r) \quad (1)$$

where

p is the pressure field
 r is the horizontal range from the source to the receiver
 z_{source} is the depth of the source
 z_{receiver} is the depth of the receiver
 U_i is an appropriate eigenfunction
 H is a Hankel function
 λ_i is a wave number

For our purposes

$$H(x) \approx \sqrt{\frac{2}{\pi x}} e^{(\sqrt{-1} (x - \pi/4))} \quad (2)$$

Equation 1 represents a solution of the wave equation under some circumstances. It is useful when the frequency is low enough so that the number of important modes is small. This is precisely where ray theory begins to fail.

The main thrust of this paper is to consider the implications of Eq. 1. The first point to consider is that U_i represents a mode function which moves through the neighborhood of the array as prescribed by the rest of the equation. More than that, any energy which travels a substantial horizontal distance through the neighborhood of the array must travel in one or more of these modes. Thus, the array will see two kinds of energy. The first will be energy which originates close to the array. It may not reach all or most of the sensors in the array. The second kind of energy will be energy which originates far from the array. It must be moving across the array in one or more of the mode functions. Even if the acoustic environment of the remote source is very different from the neighborhood of the array, the only way source energy can move to and across the array is in these modes. Thus, the remote environment is relevant only to the extent that it determines how the modes will be excited.

FORMAL IMPLICATIONS

Because of the form of $H(x)$, each mode moves across the array in a simple cylindrical pattern. The horizontal component of the processing can use the simple cylindrical model which naive plane-wave processing would assume. This divides the processing for a single mode into two operations. First, filters must extract the information from vertical staves of the array by using the model of that mode. Second, filters must combine the staves by using a cylindrical model with the proper speed of sound for that mode.

This contrasts in an interesting manner with the plane-wave approach. In the latter, the staves may be processed for a vertical angle of arrival and then for an azimuth. The processing for modes is in some ways analogous to the processing for vertical angles of arrival. However, there is a continuous set of vertical angles to consider. There is only a discrete set of modes, and no concept of interpolation seems needed. The vertical angle of arrival may change as the signal moves across the array. But once the energy is identified with a mode, it stays in that mode. All staves can be processed in an identical manner.

In most cases, the important modes can be represented as real-valued functions. Since the data must be in the frequency domain to begin with, this simplification is probably not great.

Some other simplifications appear profound. For example, $U_i(z_{\text{source}})$ contains all the information on how the modes are excited. If the sound-velocity profile changes significantly between the source and the array, then other excitation patterns may occur. However, the effects of this more complex excitation act only to change the source excitation effects. They enter into the equation only as changes to $U_i(z_{\text{source}})$. The mode shapes which we must process for are still defined simply by $U_i(z_{\text{receiver}})$.

We have already noted that $U_i(z_{\text{receiver}})$ represents the signal pressure pattern across a staff in the array. The fact that it does not change within the array aperture contrasts with the way a ray would bend.

Equation 2 implies similar simplifications. At the ranges of interest, the amplitude effect of $\sqrt{2/(\pi r)}$ can usually be ignored. The only other attenuation of the mode will result from an imaginary component of λ_i . Within the aperture of the array, this attenuation is nearly constant for the important modes. This leaves only the phase factor, which propagates horizontally in a simple manner. The only subtlety is that each mode travels at a different speed, as shown by the values of λ_i .

If we assume that the sound velocity profile does not change significantly between the source and the array, an even more interesting point emerges. The relative amplitudes of the modes contain all the information about the source depth, while the phases of the modes contain all the information about the range. The utility of this idea is, however, somewhat speculative.

The ideas of conventional beam-forming can be easily generalized to deal with the mode-forming problem. In that case, the pressure field must be divided into components which correspond to the modes. Let n denote the number of modes which we wish to attempt to recover. We shall assume that the number of elements in the array is larger than n .

Let x denote a column vector of complex pressures which occur at the sensor positions due to the signal. Let v_i denote the pressure which is computed from $U_i(z_{\text{receiver}}) H(\lambda_i r)$. (U and H are continuous functions, while v is a set of samples from that product.) Note that if the sensors are in a vertical stave, the H factor can be ignored. Finally, let α_i denote a set of complex weights which multiply the v_i . In the simplest case, $\alpha_i = U_i(z_{\text{source}})$. The required expansion is then

$$x \approx \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n \quad (3)$$

We can simplify the notation. Let

$$V = [v_1 \ v_2 \ \cdots \ v_n] \quad (4)$$

and

$$a = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \quad (5)$$

So the number of rows in V is the number of elements in the stave, and the number of columns in V is n . a is an n element complex vector. Equation (3.1) can now be written as

$$x \approx V a \quad (6)$$

Let x^H denote the complex conjugate transpose of a vector. The problem is now to find a vector, a , which will minimize ϵ , where

$$\epsilon = (x - V a)^H (x - V a) \quad (7)$$

$$\epsilon = x^H x - x^H V a - a^H V^H x + a^H V^H V a \quad (8)$$

$$\epsilon = (a - (V^H V)^{-1} V^H x)^H (V^H V) (a - (V^H V)^{-1} V^H x) + x^H x - x^H V (V^H V)^{-1} V^H x \quad (9)$$

Obviously, the minimum occurs when

$$a_o = (V^H V)^{-1} V^H x \quad (10)$$

and

$$\epsilon_o = x^H x - x^H V (V^H V)^{-1} V^H x \quad (11)$$

Many readers will recognize $V (V^H V)^{-1} V^H$ as the projection operator for the matrix V . This lends a convenient geometric interpretation to the process.

INTRASTAVE PROCESSING (CONVENTIONAL)

The first problem to consider is processing of a single vertical stave of sensors. In that case, the complex weights in V can be taken simply from the U_i functions. The estimation operator $(V^H V)^{-1} V^H$ can be used to produce a complex amplitude for each mode. That is, for each stave, a data vector, x , can be identified and a coefficient vector, $a = (V^H V)^{-1} V^H x$, can be estimated. These complex amplitudes can then be used as input to a horizontal beam former or interrogated directly.

The U_i functions are mutually orthogonal. This means that if the staves were densely sampled and ran from the surface to the bottom, then $V^H V$ would be diagonal. In most cases, it should be possible to design the array so that $V^H V$ will be well conditioned.

HORIZONTAL PROCESSING

If an array of staves is to be used, the values of a represent estimates of the complex pressures to be associated with the modes at each stave. For each mode, horizontal beams can be steered by multiplying the pressure by an appropriate phase rotator. The processing takes place as if the array and the signals existed only in a plane, and the beam forming contains no novel elements. (Remember that each mode travels at a different speed. The correct propagation speed must be used for each mode. These speeds can easily be inferred from the λ_i values.)

If the array consists of individual elements, as in a flat disc, then the signal separation into modal coefficients must be done on the basis of speeds of propagation. It is probably best to form a matrix V based on those speeds and process the data as indicated in Eq. 10.

INTERMODE PROCESSING

At this point, we encounter novel processing problems. The beam-formed values of α_i must be combined to form a single power estimate. It is not currently clear whether a coherent or an incoherent combination should be selected. We must examine both cases.

If the relative phases of the modes are perturbed in a random manner, incoherent summation is the best we can do. However, the summation should not give equal weight to each mode. The relative amplitudes are prescribed by the function $U_i(z_{\text{source}})$. Estimates of the relative noise amplitudes to be expected on each mode may be made from historical data or from theoretical models. Let v_i denote the noise power expected on the i th

mode. We shall assume that the noise pressures appearing in each mode are independent. (This assumption is only approximately true. It is not clear whether it will be worthwhile to correct for this assumption.) Reasoning similar to that used in derivation of the Eckart filter shows that each α_i should be multiplied by $U_i(z_{\text{source}})/v_i$. The results should be magnitude-squared and summed in the normal manner.

It may be that the modes can be treated in a more deterministic manner. This is the case discussed by T.C. Yang.* Here, matched filter theory comes into play. The result now becomes sensitive to the assumed range as well as the assumed depth. The simplest case is where the sound velocity profile is constant between the source and receiver, so that Eq. 1 holds. For each depth to be considered, a set of $U_i(z_{\text{source}})$ functions should be used, and the source depth which best matches the relative amplitudes of α_i should be selected. The expected relative phases of the modes can be found from $H_i(\lambda, r)$. The range can be selected which best matches the phases of the α_i 's. This set of amplitudes and phases can now serve as the template for a matched filter, and the detection processing can proceed as it would for any known signal problem. This is physically equivalent to matched field processing.

Since there will probably not be more than a dozen significant modes, this means that up to a dozen complex multiplications would have to be done at this stage for each candidate range and depth. This may provide a dramatic reduction in computation rate relative to the way matched field processing is usually done. It is possible that this requirement can be further reduced by clever logic to locate the probable depth and range.

ADAPTIVE PROCESSING

Adaptive processing presents new and interesting problems, especially if the modes are found to be less than perfectly coherent. The theoretical details are beyond the scope of this paper. They are being addressed in another paper which the author is preparing. The results will be briefly discussed here.

We must consider the complex noise covariance matrix, C . The simplest approach seems to be to look for the maximum likelihood estimator for a . We shall assume that the noise is complex, zero mean, and Gaussian. The best estimator for a seems to be

$$a_0 = (V^H C^{-1} V)^{-1} V^H C^{-1} x \quad (12)$$

Note that if $C = I$, then the result is the same as Eq. 10. a_0 is the maximum likelihood estimator for the pressure amplitudes associated with each mode.

CONCLUSIONS

Large, low-frequency hydrophone arrays present difficult signal processing challenges. If the received signal field is treated as a summation of modes rather than acoustic rays,

* In "A method of range and depth estimation by modal decomposition," J. Acoust. Soc. Am. Vol. 82 No. 5, Nov. 1987, 1736-1745.

considerable simplifications occur. These simplifications are both conceptual and computational.

From a conceptual point of view, the important point is that the modes spread in a simple cylindrical manner. It is also easy to separate vertical processing considerations from horizontal processing considerations.

From a computational point of view, the key idea involves projection of the data into a signal subspace. This greatly reduces the amount of data to be handled by the subsequent processing without loss of useful information.