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A PARAMETER ESTIMATION ALGORITHM AND EXTENSIVE NUMERICAL SIMULATIONS FOR THE CAP MODEL

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The inviscid two-invariant cap model is considered for geological materials such as concrete. A systematic constrained optimization procedure based on the Marquardt-Levenberg algorithm and the Armijo step-size rule is developed to determine values of the model parameters from available experimental data. The predictive capabilities of the cap model and the efficiency of the parameter estimation procedure are assessed through extensive numerical simulations based on well-documented experimental concrete data from the University of California.

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PREFACE

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SECTION 1

INTRODUCTION

The inviscid, two-invariant associative cap model was originally proposed by DiMaggio and Sandler [1,2] and an algorithm to implement the model in stress analysis programs was proposed by Sandler and Rubin [3]. To assess the predictive capabilities of the inviscid cap model, the extensive and well-documented data obtained in the experimental program at the University of Colorado [4] has been selected. A characteristic of this experimental work is the exercise of truly three dimensional non-conventional stress paths. Due to the non-conventional nature of the experimental data, standard fitting procedures based on the use of conventional tests to independently fit the cap surface, failure envelope and hardening law (see e.g. [5,6]) cannot be used. Hence, to obtain values for the cap parameters, an alternative constrained optimization procedure which employs a modified *Marquardt-Levenberg* algorithm and *Armijo* step-size rule is developed. This approach makes the fitting process completely systematic and renders the optimal values of the parameters in a least square sense.

In the simulations reported herein, six (6) tests are used to fit the seven parameters of the cap model, and the resulting model is exercised to predict the remaining sixty-one (61) tests. The resulting numerical predictions agree remarkably well, both *qualitatively* and *quantitatively*, with the experimental results.

SECTION 2

BASIC FORMULATION OF THE INVISCID CAP MODEL

The two-invariant, rate-independent elastoplastic associative cap model is characterized by the following constitutive equations:

$$\epsilon = \epsilon^e + \epsilon^p$$

$$\sigma = \hat{\sigma}(\epsilon^e) \quad (\textit{elastic response})$$

$$\dot{\epsilon}^p = \dot{\lambda} \frac{\partial \phi(\sigma, \kappa)}{\partial \sigma} \quad (\textit{associative flow rule}) \tag{1}$$

$$\phi(\sigma, \kappa) \leq 0 \quad (\textit{yield condition})$$

where ϵ, ϵ^e, and ϵ^p denote the total, elastic and plastic strain tensors; σ denotes the stress tensor and $\phi(\sigma,\kappa) = 0$ is the yield surface in stress space. In addition, κ is the hardening parameter which for the cap model is related to the plastic volume change by a *hardening law* as described below. Loading/unloading conditions may be expressed in a compact manner by requiring that

$$\phi(\sigma, \kappa) \leq 0, \quad \dot{\lambda} \geq 0, \quad \dot{\lambda}\phi(\sigma, \kappa) \equiv 0 \tag{2}$$

This is the so-called Kuhn-Tucker form of unilateral constraint conditions. Note that if $\phi < 0$ then $\dot{\lambda} = 0$ and the process is elastic. On the other hand, for loading, $\dot{\lambda} > 0$ and $\phi = 0$. In this latter case, $\dot{\lambda}$ is determined by requiring that $\dot{\phi} = 0$; the so-called *consistency condition* leads to the classical elastoplastic tangent modulus.

The basic characteristic of the cap model is the form of the yield function $\phi(\sigma,\kappa)$ which is specified in terms of two functions F_e and F_c. The function F_e denotes the so-called *failure envelope surface* whereas the function F_c is referred to as the *hardening cap*. Functional forms for F_e and F_c are (see Fig. 1)

$$\phi(\sigma, \kappa) \equiv \begin{cases} \sqrt{J_{2D}} - F_e(J_1) \leq 0 & (\textit{failure envelope}) \\ \sqrt{J_{2D}} - F_c(J_1, \kappa) \leq 0 & (\textit{cap surface}) \end{cases} \tag{3}$$

where $J_1 \equiv tr\,\sigma$, $J_{2D} \equiv \frac{1}{2}\mathbf{s}:\mathbf{s}$ ($\mathbf{s}$: stress deviator) and

$$F_e(J_1) \equiv \alpha - \gamma \exp(-\beta J_1) + \theta J_1$$

$$F_c(J_1, \kappa) \equiv \frac{1}{R}\sqrt{[X(\kappa) - L(\kappa)]^2 - [J_1 - L(\kappa)]^2} \tag{4}$$

$$L(\kappa) \equiv \langle\kappa\rangle = \begin{cases} \kappa & if \quad \kappa > 0 \\ 0 & if \quad \kappa \leq 0 \end{cases} \quad (\ \langle \textit{McAuley bracket} \rangle\ )$$

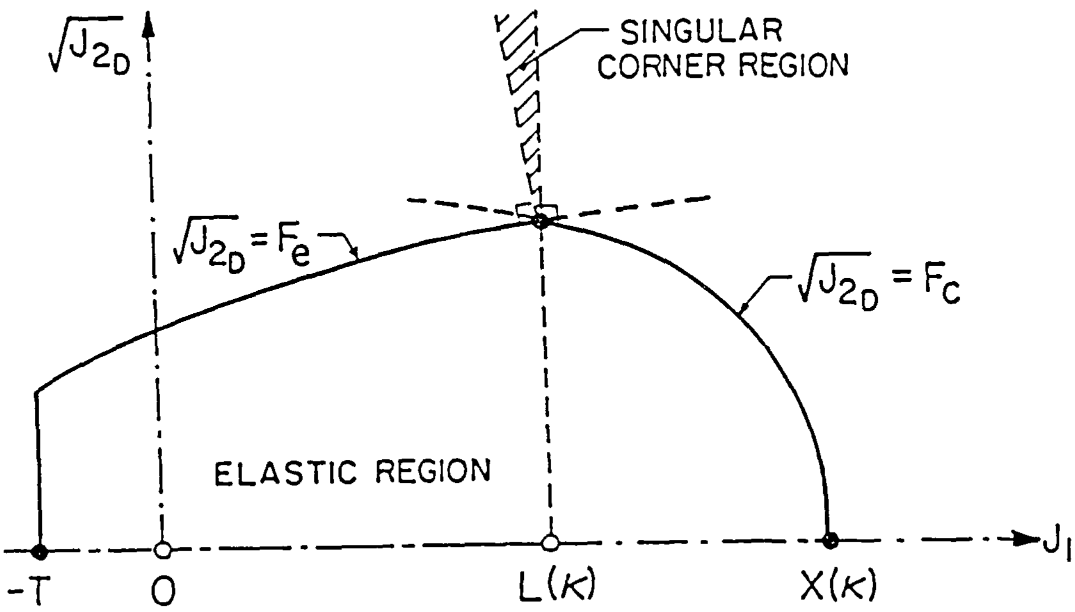

Figure 1. The yield surface for cap model. F_e and F_c denote the failure envelope and the hardening cap surface, respectively. The shaded area is the "singular corner region".

Finally, the *hardening parameter* κ is related to the *plastic volume change* $\epsilon_v^p \equiv tr\, \boldsymbol{\epsilon}^p$ by the hardening law

$$\epsilon_v^p(X) \equiv W \, \{ \, 1 - \exp\left[-D \, X(\kappa) \right] \, \} \tag{5}$$

where $X(\kappa)$ is defined by

$$X(\kappa) \equiv \kappa + R \, F_e(\kappa) \tag{6}$$

In the above expressions, $\alpha, \beta, \gamma, \Theta, W, D$, and R are material parameters which characterize the two-invariant cap model considered here.

SECTION 3

PARAMETER ESTIMATION AND NUMERICAL SIMULATIONS

In order to assess the capability of the two-invariant cap model in predicting response behavior for actual materials such as concrete and geomaterials, model parameters need to be estimated from available experimental data. In this section, a parameter estimation procedure and an assessment of the predictive capability of the cap model are presented. This is followed by extensive numerical simulations for the Colorado concrete data.

3.1. PARAMETER ESTIMATION. MARQUARDT-LEVENBERG ALGORITHM.

It is characteristic of currently employed parameter estimation procedures for the cap model (see e.g. [5,6]) to fit *separately* the failure envelope, cap surface, and hardening law parameters. Typically, asymptotic failure points from TE, TC, SS, CTC, CTE, RTE, RTC and PL† are used with a least-square fit procedure to estimate the failure parameters; whereas iso-plastic volumetric strain contours are employed to estimate the cap shape parameter R. The hardening law parameters D and W are determined from HC tests.‡ Although this procedure provides a parameter fitting directly associated with the physical construction of the cap model, it has the following two major drawbacks: (a) a large amount (more than 20 tests) of conventional experimental data are required (e.g. CTC, CTE etc.), and (b) it is not possible to utilize some existing nonconventional experimental work; e.g., the results from the "Colorado" experimental program [4]. Hence, a more flexible and systematic parameter estimation procedure is needed. This is the objective of the following section.

Optimization algorithm The basic idea of the procedure advocated here is to regard the optimal fitting process for given experimental data as a least-square *constrained optimization* problem. In this context, the objective function $\Pi: \mathbb{R}^N \rightarrow \mathbb{R}$ is simply the sum-of-squares error function defined as

$$\Pi(\boldsymbol{\Psi}) \equiv \sum_{I=1}^{N} ||\boldsymbol{\sigma}_I(\boldsymbol{\Psi}, \boldsymbol{\epsilon}_I) - \boldsymbol{\sigma}_I^{*}||^2 \tag{7}$$

where

N : *number of observations*

† TE stands for triaxial extension. TC triaxial compression. SS simple shear. CTC conventional triaxial compression. CTE conventional triaxial extension. RTE reduced triaxial extension. RTC reduced triaxial compression. and PL proportional loading.

‡ HC represents hydrostatic compression test.

$\boldsymbol{\sigma}_I$: *stress response from constitutive model considered*

$\boldsymbol{\sigma}^*_I$: *observed stress response*

$\boldsymbol{\Psi}$: *parameter vector (in* $\mathbb{R}^7$ *for cap model)*

I: I^{th} *data point*

In the following, this procedure will be illustrated using the cap model. It is, however, generally applicable to any constitutive model. The constraints imposed on the optimization problem emanate from physical restrictions placed on the cap parameters. For example, for a physically meaningful model one should have $\alpha > 0,\ \gamma > 0,\ \alpha > \gamma,\ \Theta > 0,\ \beta > 0,\ R > 0,\ D > 0,\ W > 0.$ These constraints define a *feasible domain* $\Xi \subset \mathbb{R}^7$, which is a *convex* polygon. The resulting constrained optimization problem is then expressed as

$$\text{Find}: \quad \min\ \Pi(\boldsymbol{\Psi}) \quad \text{subject to} \quad \boldsymbol{\Psi} \in \Xi \tag{8}$$

There exists a wide variety of algorithms for solving the standard convex optimization problem (8) (e.g., see [9] for a review). The algorithm employed here is the well-known *Marquardt-Levenberg* algorithm together with the *Armijo* step-size rule [7-10]. This algorithm is essentially a hybrid of Newton and steepest descent (gradient) methods. It combines the ability of the steepest descent method to converge from an initial guess, which may be outside the region of convergence of other methods, with the asymptotic quadratic convergence characteristics of Newton's method near the solution. The Marquardt-Levenberg algorithm can be summarized in the following form:

$$\boldsymbol{\Psi}_{i+1} = \boldsymbol{\Psi}_i + \lambda_i \mathbf{h}_i \tag{9}$$

$$\mathbf{h}_i = -[\mathbf{H}_i + \eta_i \mathbf{D}_i]^{-1}\, \nabla_i \Pi \tag{10}$$

$$\mathbf{H}_i = 2\, \mathbf{Q}_i^T \mathbf{Q}_i \quad \textit{(approx. Hessian)} \tag{11}$$

$$\mathbf{Q}_i = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\Psi}_i} \quad \textit{(sensitivity matrix)} \tag{12}$$

$$\eta_i = \textit{Marquardt parameter}$$

$$\mathbf{D}_i = \textit{diagonal matrix of } \mathbf{H}_i \textit{ or simply } \mathbf{I}$$

$$\lambda_i = \underset{\omega \in (0,1),\, k \in \mathbb{N}}{\textit{argmin}} \left\{ \omega^k \;\middle|\; \boldsymbol{\Psi}_{i+1} \in \Xi,\ \Pi(\boldsymbol{\Psi}_{i+1}) < \Pi(\boldsymbol{\Psi}_i) \right\} \tag{13}$$

$$i = i^{th} \textit{ iteration}$$

For problems where $\mathbf{Q}_i$ may not be easily constructed analytically the derivatives are typically computed by means of forward differences. However, central differences provide greater accuracy in the vicinity of the solution (minimum); thus, central rather than forward differences are employed in computing $\mathbf{Q}_i$ when the solution is closely approached.

In addition, to minimize the number of function evaluations (stress responses), a *rank one* update to the sensitivity matrix is used periodically (similar to the Quasi-Newton method)

$$\mathbf{Q}_{i+1} = \mathbf{Q}_i + \frac{1}{||\Delta\mathbf{\Psi}_{i+1}||^2}\left[\boldsymbol{\sigma}(\mathbf{\Psi}_{i+1}) - \boldsymbol{\sigma}(\mathbf{\Psi}_i) - \mathbf{Q}_i\,\Delta\mathbf{\Psi}_{i+1}\right]\Delta\mathbf{\Psi}_{i+1}^T \tag{14}$$

where $\Delta\mathbf{\Psi}_{i+1} \equiv \mathbf{\Psi}_{i+1} - \mathbf{\Psi}_i$. In Eq. (10), for a given value of η_i, *Cholesky factorization* of $\mathbf{H}_i + \eta_i \mathbf{D}_i$ is employed to check for positive definiteness. If the factorization breaks down, i.e. $\mathbf{H}_i + \eta_i \mathbf{D}_i$ is not positive definite, then η_i is increased. The algorithm summarized above can be systematically applied to any set of experimental data to obtain the optimal fit for the constitutive model under consideration in a least square sense.

Error measurement During the optimization process, a root-mean-square (RMS) type of error measurement is adopted. The optimization process is considered to reach its optimum when the RMS measure is minimized. The relevant measures are defined as follows:

$$\Delta_N \equiv \left[\frac{\Pi}{N}\right]^{\frac{1}{2}} \quad (\textit{RMS of error}) \tag{15}$$

$$\Gamma_N \equiv \left[\sum_{I=1}^{N}\frac{||\boldsymbol{\sigma}_I^*||^2}{N}\right]^{\frac{1}{2}} \quad (\textit{RMS of observed responses}) \tag{16}$$

$$\delta_N \equiv \frac{\Delta_N}{\Gamma_N} \quad (\textit{normalized relative RMS error}) \tag{17}$$

Remark 3.1. It is interesting to examine the sensitivity of the response under perturbations in cap model parameters. A finite difference sensitivity matrix Q is defined in dimensionless form:

$$Q_{ij} = \frac{\Delta\sigma_i / \sigma_i}{\Delta\Psi_j / \Psi_j} \tag{18}$$

where σ_i is a stress component (i = 1,...,6) and Ψ_j is a parameter component (j = 1,...,7), respectively. A standard sensitivity analysis reveals that the response of the cap model is relatively insensitive to changes in the model parameters. By ordering the model parameters according to relative sensitivity in the response, one obtains in decreasing order of sensitivity:

$$W \rightarrow D \rightarrow R \rightarrow \alpha \rightarrow \theta \rightarrow \gamma \rightarrow \beta \tag{19}$$

In summary, one obtains the following relative degree of sensitivity (from large to small):

hardening parameters → *cap parameters* → *failure parameters* □

3.2. PREDICTIVE CAPABILITIES. "COLORADO" CONCRETE DATA.

In this section, we first examine the consistency of the "Colorado concrete" data [4], next we estimate the model parameters by exercising the procedure described above, finally we assess the predictive capability of the inviscid cap model.

Colorado concrete data. This experimental program on concrete was performed at the University of Colorado (1983) and is well-documented [4]. The program consists of six major series of nonconventional multiaxial stress-strain curves. The total number of experiments is 67. The data are characterized by the following properties: (a) characteristic uniaxial compressive strength $f_c' \approx 4$ ksi, (b) mean pressure $\leq$ 8 ksi (c) *truly triaxial* states of stress for concrete, (d) nonconventional complicated stress paths, and (e) quasi-static loading.

The six major series of tests consist of the following:

(1) A series of 12 cyclic triaxial tests, consisting of cyclic hydrostatic preloading to various stress levels, followed by proportional deviatoric stress cycles without reversal along triaxial compression, simple shear, and triaxial extension paths.

(2) A series of 8 cyclic triaxial tests, consisting of cyclic hydrostatic preloading to various stress levels, followed by proportional deviatoric stress cycles with reversal along the same deviatoric paths as in Series 1.

(3) A series of 17 tests consisting of hydrostatic loading, followed by proportional stress deviation, followed by a circular stress path within the deviatoric plane.

(4) A series of 22 axisymmetric triaxial tests to explore load path effects. In addition to proportional and hydrostatic-deviatoric paths, this series contained staircase-type loadings to explore convergence to the proportional path, tests with hydrostatic stress increments with and without hydrostatic preloading, and tests under non-proportional loadings.

(5) A series of 6 tests within the deviatoric plane, as well as a number of other tests specifically designed to check the meaning of loading and unloading.

(6) A series of 2 tests of piecewise-uniaxial loadings.

Assessment of data consistency. Basically, the measures employed here are the same as those discussed in the previous section. For convenience, these measures are summarized as follows:

$$\Delta_N \equiv \left[\sum_{I=1}^{N} \frac{||\Delta\epsilon_I||^2}{N} \right]^{\frac{1}{2}} \qquad (see(15)) \tag{20}$$

$$\Gamma_N \equiv \left[\sum_{I=1}^{N} \frac{||\epsilon_I^e||^2}{N} \right]^{\frac{1}{2}} \qquad (see(16)) \tag{21}$$

$$\delta_N \equiv \frac{\Delta_N}{\Gamma_N} \qquad (see(17)) \tag{22}$$

Here ϵ_i^A refers to a strain measurement of test 'A'. An assessment of consistency for the "Colorado" concrete data may be obtained from the replicates of experiments available in the reported results [4]. The present analysis generally indicates reasonable consistency of the data. However, some serious discrepancies between replicates are also observed. See Table 1 below.

Table 1. *Consistency of the Colorado concrete data* [4].

Tests	δ %	Major Path
1-1 & 1-10	13.5	CTC
1-4 & 1-7	31.1	TC
1-6 & 1-9	51.3	TE
2-3 & 2-4	9.6	SS
2-7 & 2-8	13.5	SS
3-1 & 3-2	244.3	Circular
3-3 & 3-4	47.2	Circular
3-10 & 3-11	92.9	Circular
4-1 & 4-2	10.9	Axisymmetric
4-6 & 4-7	54.2	Axisymmetric

Model parameter estimation procedure. The actual data employed in the optimization process based on the Marquardt-Levenberg algorithm are obtained by arbitrarily selecting *one* test out of each of the six major series. Thus, a total number of 6 tests is used in the actual fit of the model. The quality of the fitting is satisfactory. Typical values of the RMS error found from back-prediction using the optimal material properties are: $\delta = 16\%$ for test 1-1 (CTC), $\delta = 8.5\%$ for test 2-3 (SS), $\delta = 26\%$ for test 3-11 (circular), $\delta = 11.5\%$ for test 4-11 (axisym.), etc.. From this optimization procedure, we obtain the following set of parameters which best fits the observed experiments: $\alpha = 3.86\,ksi$, $\Theta = .11$, $\gamma = 1.16\,ksi$, $\beta = .44\,ksi^{-1}$, $R = 4.43$, $D = .0032\,ksi^{-1}$, $W = .42$, $X^o = 16\,ksi$.

Predictive capability. After the optimal model parameters are obtained, the resulting cap model is used to predict the response of *every* other Colorado test which is not included in the optimization process (total number = 61). It is emphasized that the "prediction" here has nothing to do with optimal fitting, but is obtained by exercising the cap model using previously estimated parameters. In general, considering the experimental data scatter, the predicted response is in good agreement with the experimental results. It is noted that the *overall qualitative* behavior for the Colorado concrete data is captured. Values of the RMS error corresponding to a selected sample of simulations are summarized in Table 2 below.

The overall RMS and standard deviation of error for 61 tests are 26.6% and 14%, respectively. A comparison between experimental and predicted stress-strain curves is contained in Figures 2-11.

Table 2. *Results of prediction. Inviscid case*

Tests	δ %	Major Path
1-2	12.4	SS
1-3	14.1	TE
2-2	17.	TE
2-4	11.7	SS
3-5	15.	Circular
3-17	11.6	Circular
4-7	14.	Axisymmetric
4-12	11.4	Axisymmetric
5-1	14.	Unsymmetric
5-2	17.	Unsymmetric

Assessment and evaluation. From the above fitting and prediction exercises, it may be concluded that the inviscid cap model generally exhibits good fitting and predictive capabilities for the Colorado concrete data. The simulations reported herein capture the *overall* qualitative behavior of the experimental response.

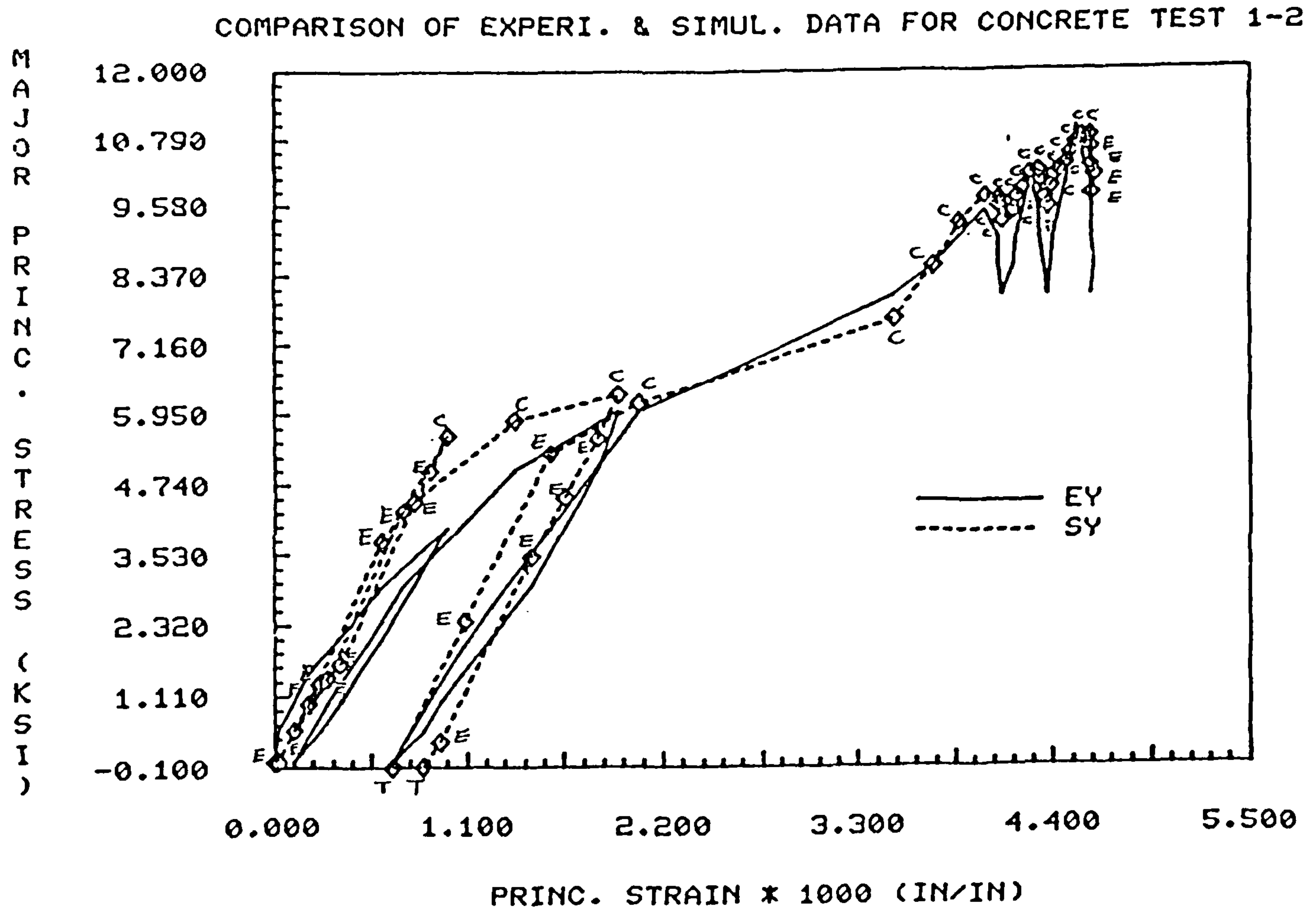

Figure 2. Comparison of the experimental and simulated data for concrete test 1-2. This is a cyclic simple shear test. The vertical axis is the major principal stress and the horizontal axis is one of the three principal strains. "EY" (solid-line) and "SY" (dash-line) represent the experimental and the simulated response in Y-direction, respectively. The diamond symbols signify the data points along "SY", in which "E" stands for the elastic mode, "C" for the cap mode and "T" for the tension cutoff mode. The r.m.s. error measure δ = 12.4%.

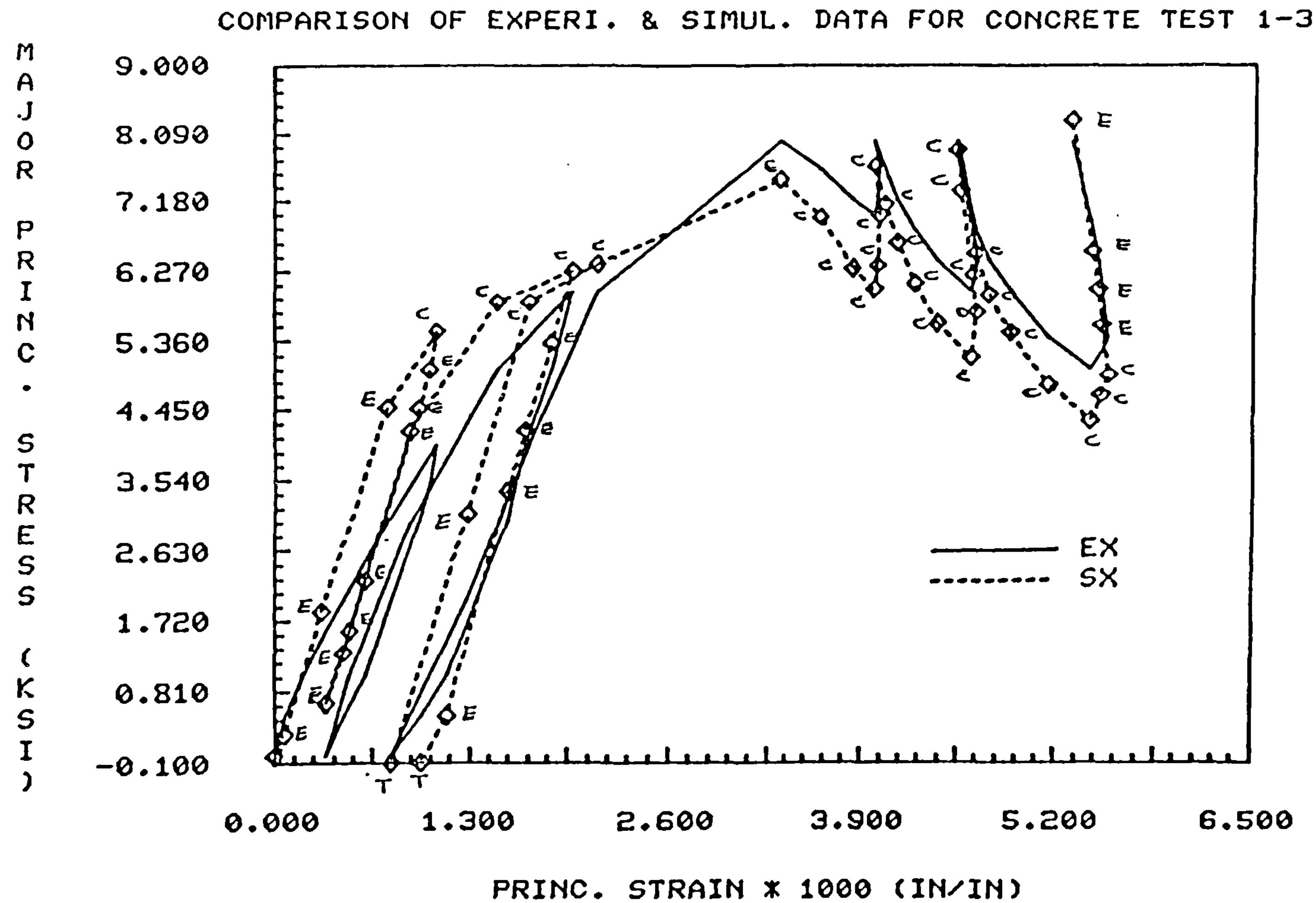

Figure 3. Comparison of the experimental and simulated data for concrete test 1-3. This is a cyclic triaxial extension test. The vertical axis is the major principal stress and the horizontal axis is one of the three principal strains. "EX" and "SX" represent the experimental and the simulated response in X-direction, respectively. The r.m.s. error measure δ = 14.1%.

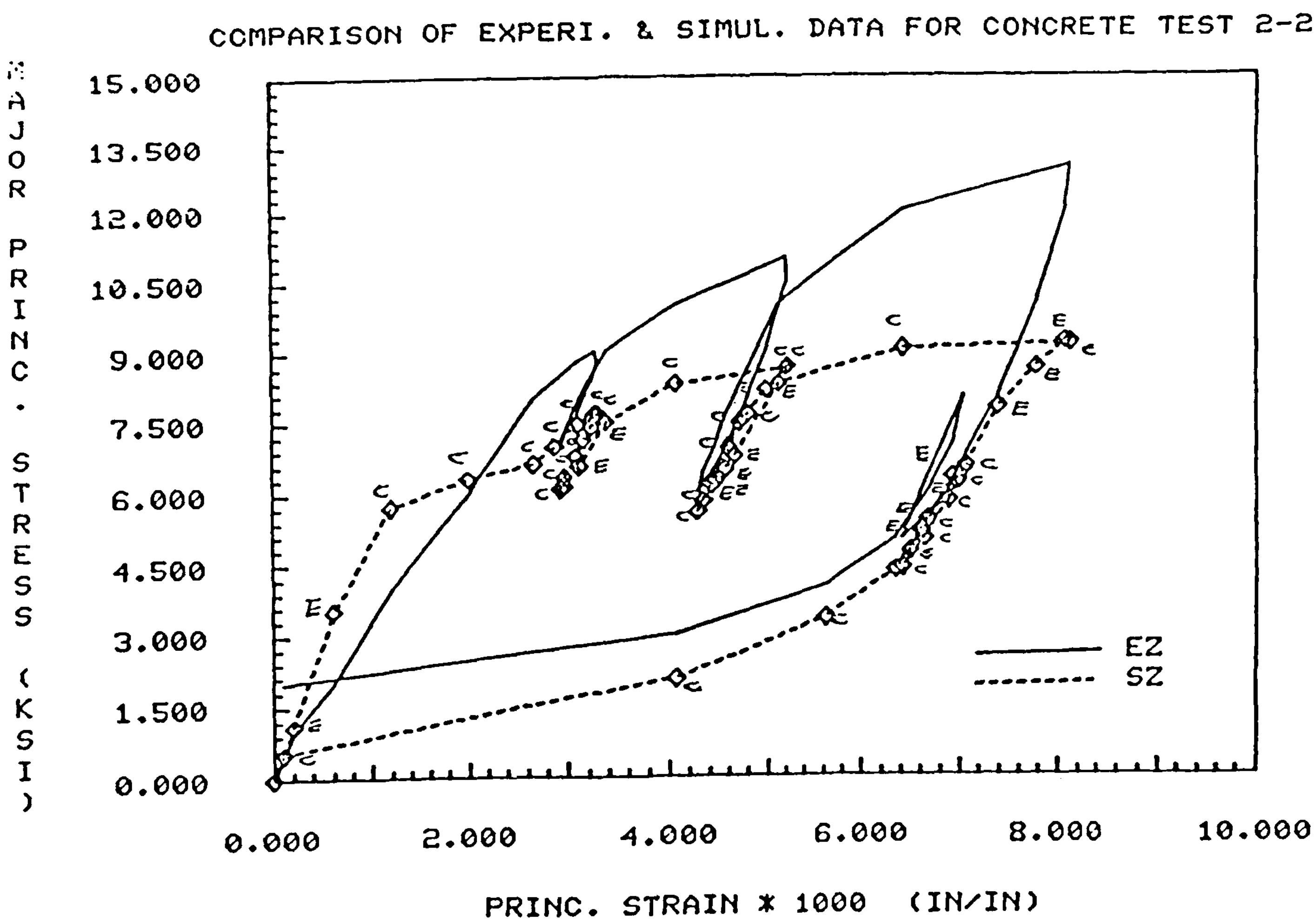

Figure 4. Comparison of the experimental and simulated data for concrete test 2-2. This is a cyclic triaxial extension test with stress reversal about the hydrostatic axis. "EZ" and "SZ" represent the experimental and the simulated response in Z-direction, respectively. The r.m.s. error measure δ = 17%.

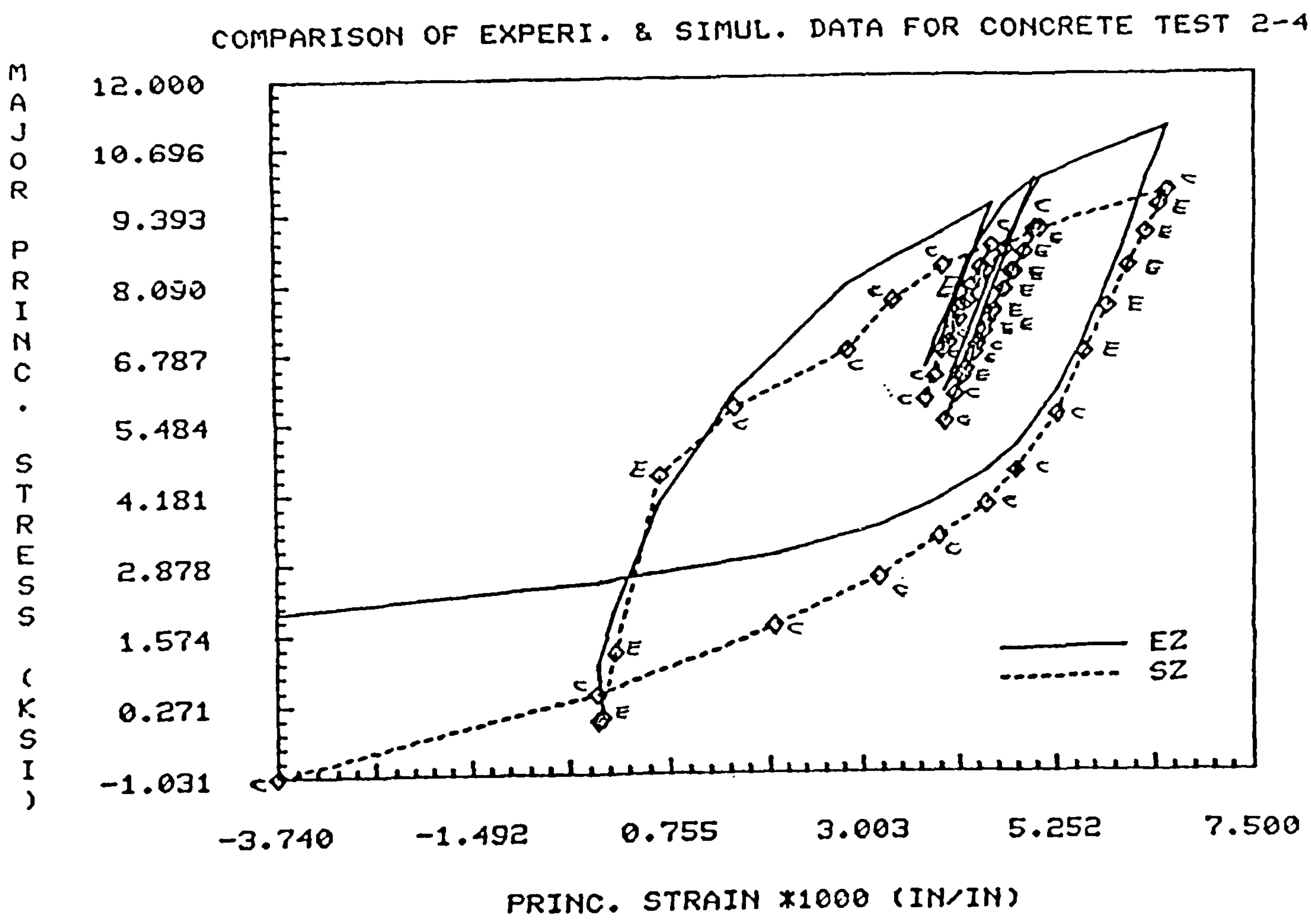

Figure 5. Comparison of the experimental and simulated data for concrete test 2-4. This is a cyclic simple shear test with stress reversal with respect to the hydrostatic axis. The r.m.s. error measure δ = 11.7%.

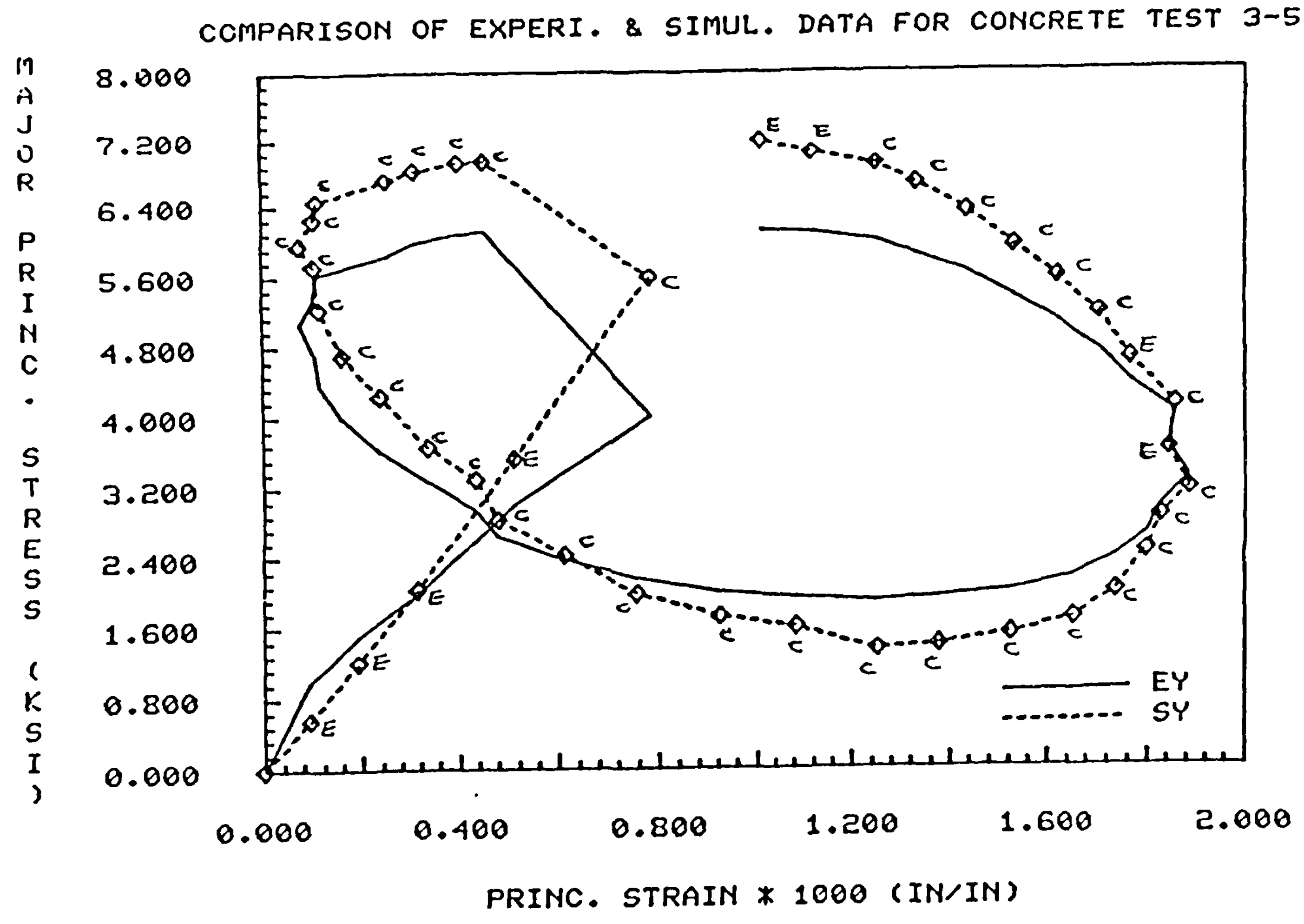

Figure 6. Comparison of the experimental and simulated data for concrete test 3-5. This is a circular stress path on the 12 KSI octahedral plane. The r.m.s. error measure $\delta = 15\%$.

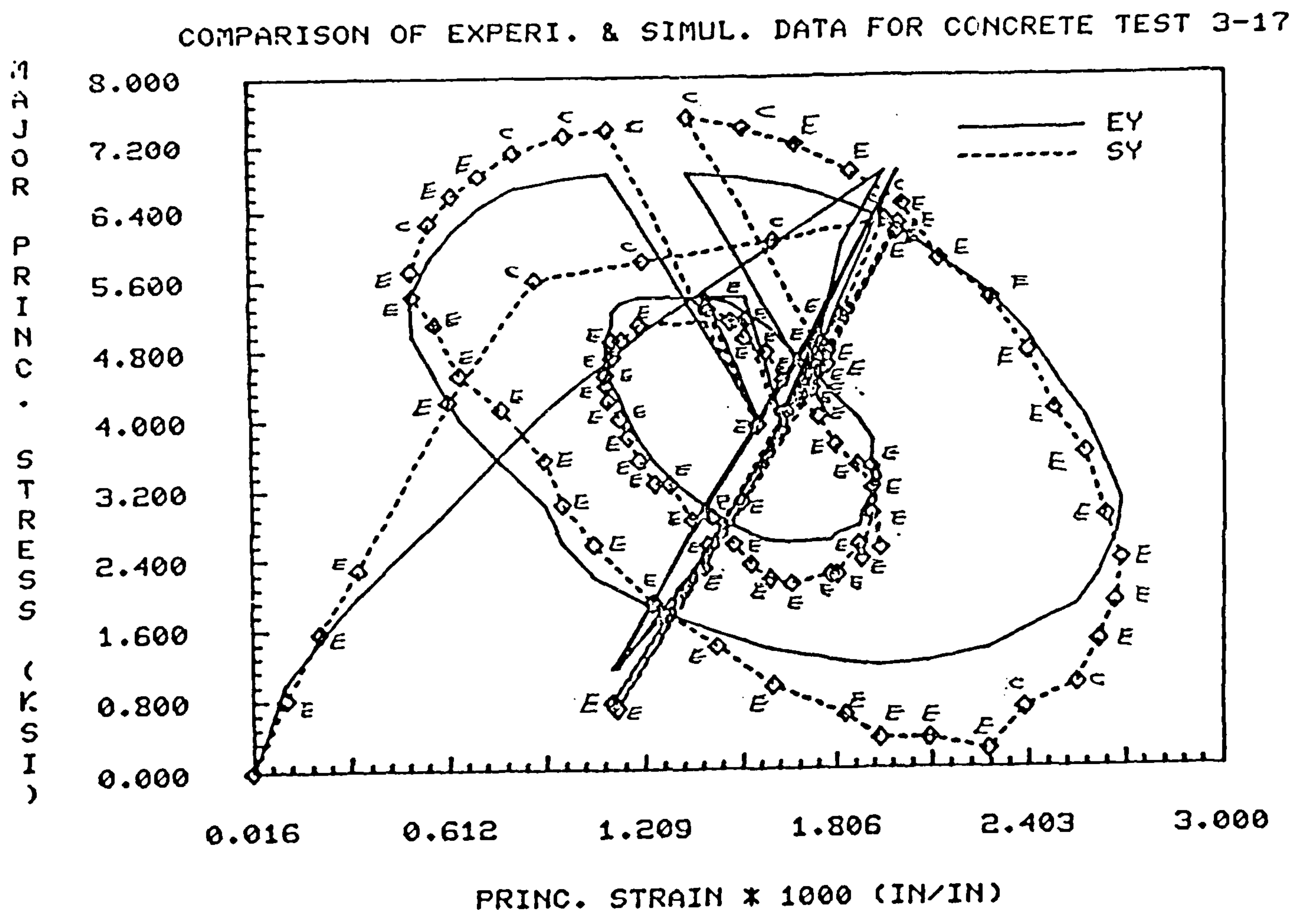

Figure 7. Comparison of the experimental and simulated data for concrete test 3-17. This is a proportional loading path followed by cyclic circular stress path on two octahedral planes, and finally followed by another proportional loading path. The r.m.s. error measure $\delta = 11.6\%$.

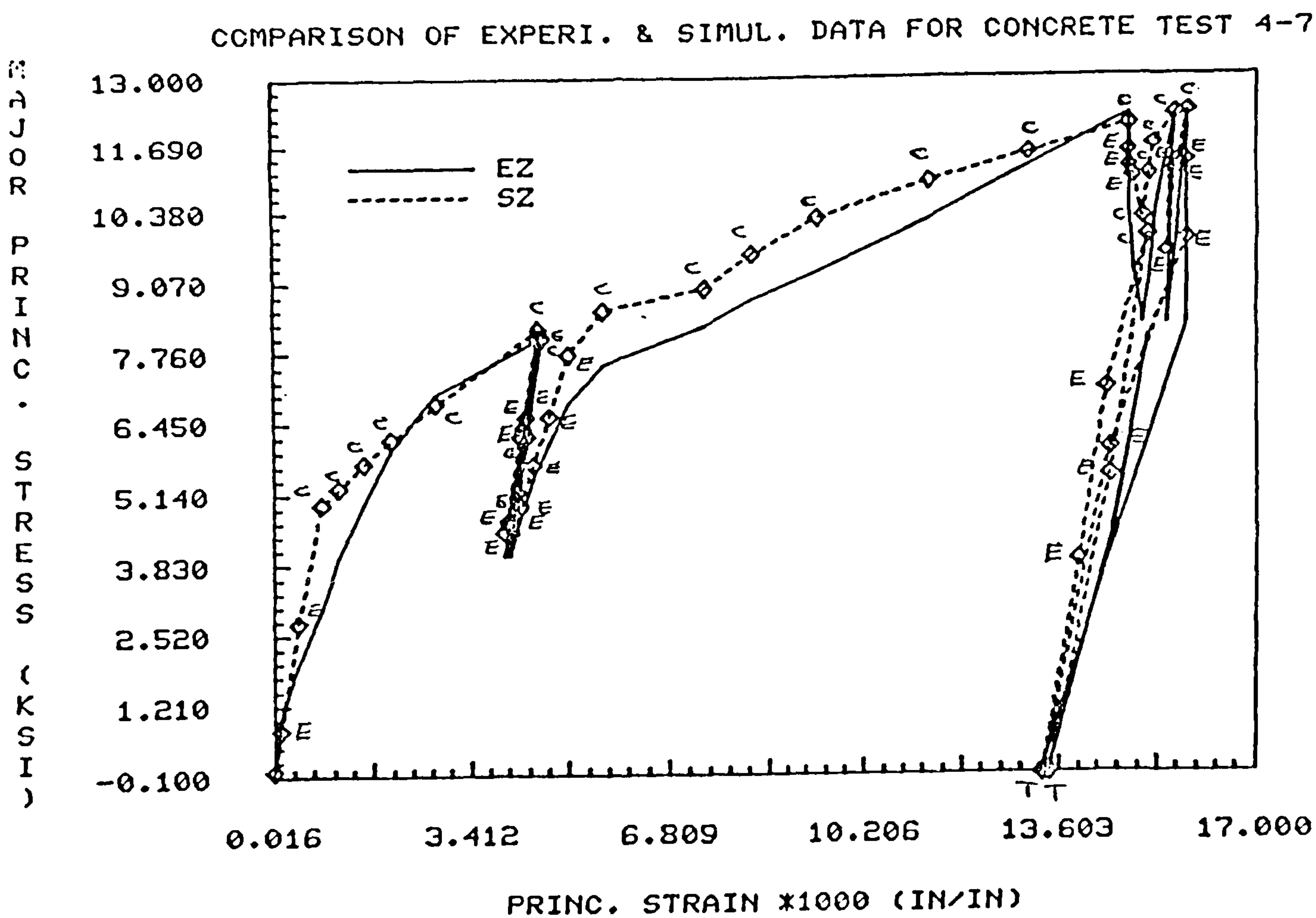

Figure 8. Comparison of the experimental and simulated data for concrete test 4-7. This is a cyclic axisymmetric triaxial compression test. The r.m.s. error measure $\delta = 14\%$.

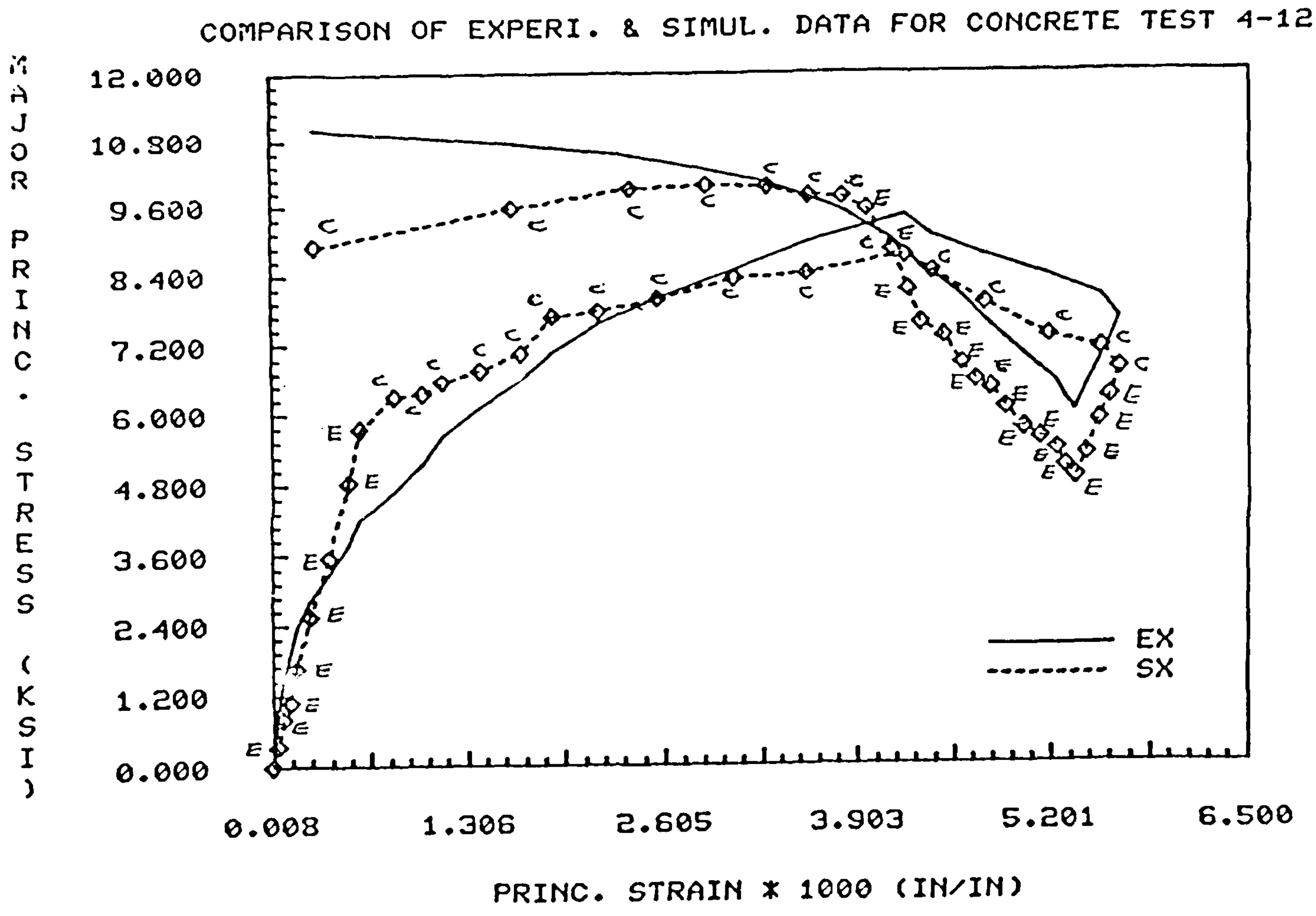

Figure 9. Comparison of the experimental and simulated data for concrete test 4-12. This is another cyclic axisymmetric triaxial test. The r.m.s. error measure $\delta = 11.4\%$.

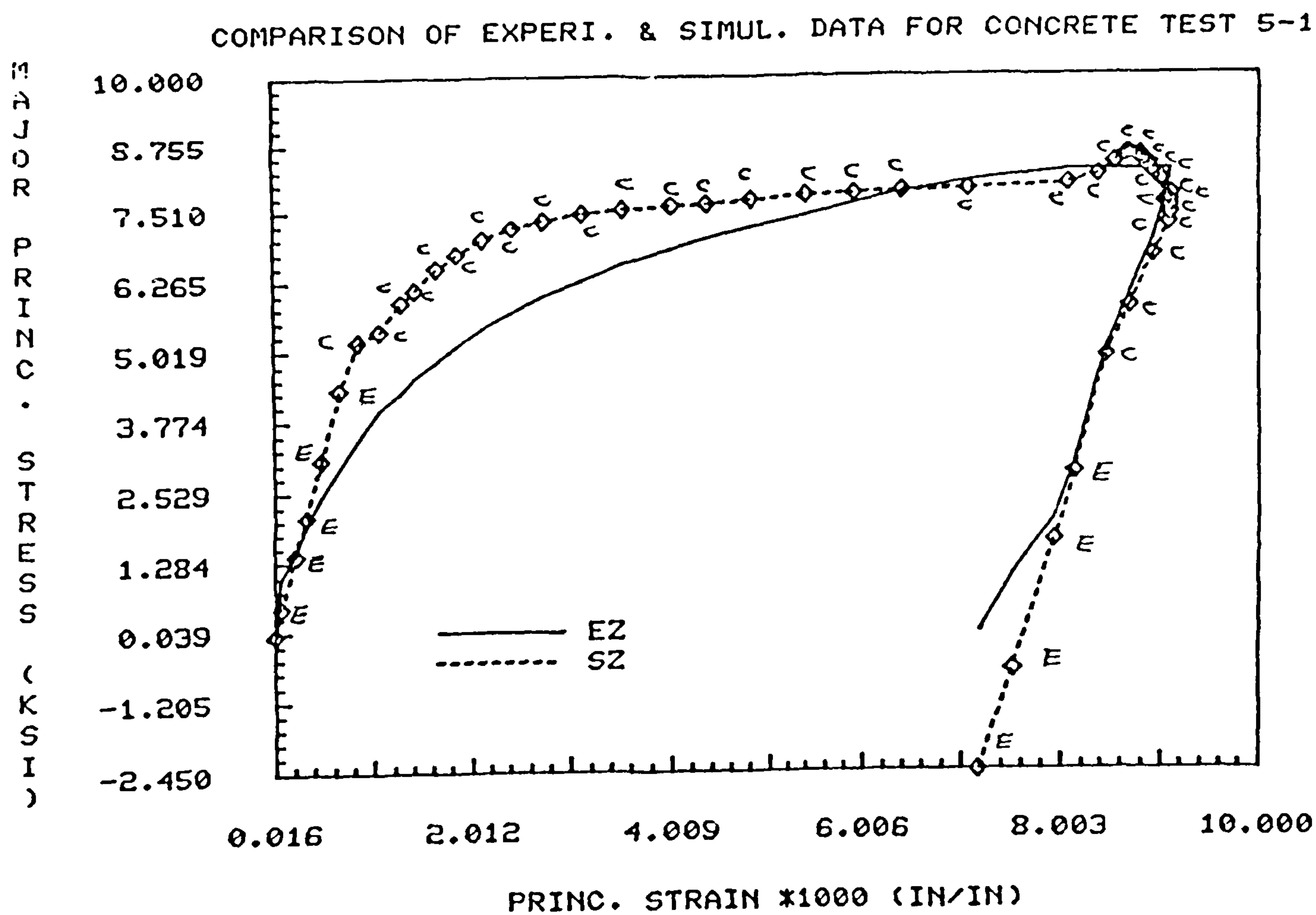

Figure 10. Comparison of the experimental and simulated data for concrete test 5-1. This is an unsymmetric triaxial test. The r.m.s. error measure δ = 14%.

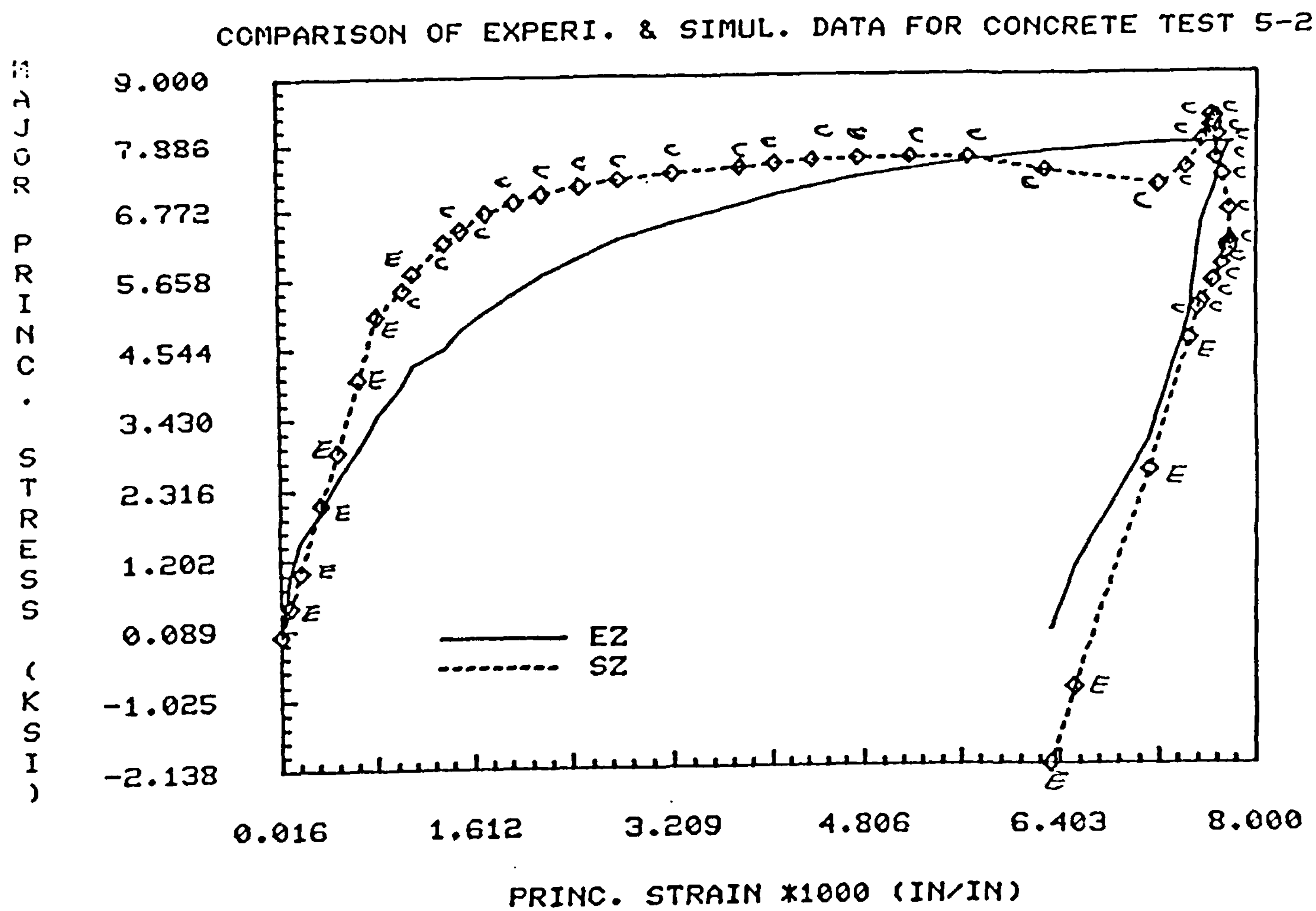

Figure 11. Comparison of the experimental and simulated data for concrete test 5-2. This is an unsymmetric triaxial test. The r.m.s. error measure δ = 17%.

SECTION 4

CLOSURE

A systematic estimation procedure for the parameters involved in the cap model to given experimental data has been developed, based on a modified Marquardt-Levenberg optimization algorithm. This procedure has been applied to the extensive experimental program carried out at the University of Colorado and reported in [4]. It is emphasized that due to the *nonconventional* character of this experimental data, standard fitting procedures (e.g., Desai [5,6]) based on conventional tests can not be employed. The numerical simulations performed on the basis of these data support the good predictive capabilities of the cap model for concrete materials.

SECTION 5

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APPENDIX A

LISTING OF PARAMETER ESTIMATION PROGRAM

```
c     *****************
      program fit
c     *****************
c.... Program for parameter fitting from experimental test data
c     for the cap model
      implicit double precision(a-h,o-z)
      common/fix/bulkm,shearm,zm
      common/aa/n(6),mm
      common/prop/ltype,tcut,fcut
      common ystar(500,6),w(500,6)
      dimension f(3000),q(21000),work(6063)
c.... Specify elastic material parameters and initial cap (z) parameter:
      read(5,*) bulkm,shearm,zm
      write(6,2004) bulkm,shearm,zm
c.... Input size of experimental steps
      read(5,*) (n(j),j=1,6)
      mm=n(1)+n(2)+n(3)+n(4)+n(5)+n(6)
      do 10 j=1,6
      if(n(j).gt.500) stop 5
   10 continue
c.... Input Experimental Data for y ( sig-33)
      do 20 j=1,6
      read(5,1000) (ystar(l,j),l=1,n(j))
   20 continue
c.... Call Optimization Algorithm --
c     Modified Levenberg-Marquardt Algorithm.
      call opt(f,q,work)
      stop
c     Format Statement
 1000 format(8f10.0)
 2004 format(//
     * t5,'BULK MODULUS      = ',d14.6/
     * t5,'SHEAR MODULUS     = ',d14.6/
     * t5,'INITIAL CAP ( Z ) = ',d14.6/)
      end
```

```
c     **************************************
      subroutine opt(f,q,work)
c     **************************************
c.... Program to calc. the optimal values for cap model
c     by the Modified Levenberg-Marquardt algorithm.
c.... The optimization criterion is with respect to the
c     least square norm.
      implicit double precision(a-h,o-z)
      external func
      common/fix/bulkm,shearm,zm
      common/prop/ltype,tcut,fcut
      common/aa/n(6),mm
      common/bb/indi
      common/ha/old(7)
      common ystar(500,6),w(500,6),y(500,6)
      dimension parm(4),para(7),f(1),q(mm,1),g(28),work(1)
      dimension soss(6),sosorr(6),rmsso(6),rmsyy(6),rell(6)
      dimension eigval(7),eigvec(7,7),wk(7)
c.... Parameters for IMSL — ZXSSQ
      ixjac=mm
      read(5,*) nsig,eps,delta,maxfn,iopt
      if(iopt.eq.2) read(5,*) (parm(l),l=1,4)
c.... Initial guess for parameters alpha ~ w
      read(5,*) (para(j),j=1,7)
      write(6,2000) (para(j),j=1,7)
c.... Input weighting matrix W
      read(5,*) iflag
      if(iflag.ne.1) then
        do 31 j=1,6
          do 30 k=1,n(j)
   30       w(k,j)=1.
   31     continue
      else
        do 32 j=1,6
          read(5,1000) (w(k,j),k=1,n(j))
   32     continue
      endif
c.... Call optimization package ZXSSQ
      indi=1
c     (Save the old parameters)
      do 40 k=1,7
   40 old(k)=para(k)
c
      call zxssq(func,mm,7,nsig,eps,delta,maxfn,iopt,
     *  parm,para,ssq,f,q,ixjac,g,work,infer,ier)
c.... Print out output data
      write(6,2001) (para(k),k=1,7)
c.... SOS   : sum of squares of residuals
c     SOSOR : sum of squares of observed responses (ystar)
      sos=0.
      sosor=0.
      do 50 i=1,6
      sosorr(i)=0.
   50 soss(i)=0.
```

```
      do 200 j=1,6
      do 100 k=1,n(j)
      diff=y(k,j)-ystar(k,j)
      soss(j)=soss(j)+diff**2
      sosorr(j)=sosorr(j)+ystar(k,j)**2
 100  continue
      sos=sos+soss(j)
      sosor=sosor+sosorr(j)
 200  continue
c.... Overall estimators
      rmssos=sqrt(sos/mm)
      rmsy=sqrt(sosor/mm)
      rel=rmssos/rmsy
c.... Test-j estimators
      do 210 j=1,6
      rmsso(j)=sqrt(soss(j)/n(j))
      rmsyy(j)=sqrt(sosorr(j)/n(j))
      rell(j)=rmsso(j)/rmsyy(j)
 210  continue
c
      write(6,2007) sos
      write(6,2004) rmssos
      write(6,2005) rel
c
      m1=n(1)+n(2)
      m2=m1+n(3)
      m3=m2+n(4)
      m4=m3+n(5)
c.... Testj
      do 220 j=1,6
      write(6,2007) soss(j)
      write(6,2004) rmsso(j)
      write(6,2005) rell(j)
 220  continue
c
c.... Compute the condition number for G == Q sup T * Q == 1/2 H
c     Cond(G) = (max lambda) / (min lambda)
      call eigrs(g,7,0,eigval,eigvec,7,wk,ierr)
      if (eigval(1).eq.0.0d0) stop 'zero eigval'
      cond=eigval(7)/eigval(1)
      write(6,3100) cond
      return
c
 1000 format(8f10.0)
 2000 format(//
     * 20x,'THE INITIAL GUESS FOR PARAMETERS'//
     * 10X,'ALPHA',7X,'THETA',8X,'GAMA',8X,'BETA',
     * 11X,'R',11X,'D',11X,'W'/
     * /5X,7F12.6)
 2001 format(//
     * 20x,'THE OPTIMAL VALUES  OF PARAMETERS ARE'//
     * 10X,'ALPHA',7X,'THETA',8X,'GAMA',8X,'BETA',
     * 11X,'R',11X,'D',11X,'W'/
     * /5X,7F12.6)
```

```
 2004  format(//20x,'THE TRUE ROOT-MEAN-SQUARE OF PHI = ',D15.7//)
 2005  FORMAT(//20x,'THE NORMALIZED RELATIVE ERROR     = ',d15.7)
 2007  format(///20x,'TRUE SUM OF SQUARES = ',d15.7)
 3100  format(//20x,'CONDITION NUMBER OF G = ',d15.7)
       end
c     ********************************
      subroutine func(para,in,ip,f)
c     ********************************
c.... Function evaluation (stress response) and residual
c     computation.
       implicit double precision(a-h,o-z)
       common/fix/bulkm,shearm,zm
       common/prop/ltype,tcut,fcut
       common/aa/n(6),mm
       common/bb/indi
c.... 500: max. no. of data pts in each test
c     6  : 6 strain components
c     6  : 6 tests
       common/ab/del(500,6,6)
       common/ha/old(7)
       common ystar(500,6),w(500,6),y(500,6)
       dimension para(1),f(1),delp(7),ytemp(500),deltem(500,6)
c.... Preserve total increments
       do 10 i=1,7
   10  delp(i)=para(i)-old(i)

c.... Check if constraints are violated:
c     para(1) > 0 required
   20  if (para(1).le.0.d0) then
        go to 30
c     para(3) > 0 required
       elseif (para(3).le.0.d0) then
        go to 30
c     para(3) < para(1) required
       elseif (para(3).gt.para(1)) then
        go to 30
c     para(3) > 0.1 * para(1) preferred
       elseif (para(3).lt.0.1*para(1)) then
        go to 30
c     para(2) > 0 required
       elseif (para(2).lt.0.d0) then
        go to 30
c     para(4) >= 0.21 preferred
       elseif (para(4).lt.0.21d0) then
        go to 30
c     para(4) <= 2 preferred
       elseif (para(4).gt.2.0d0) then
        go to 30
c     para(5) >= 1.6 preferred
       elseif (para(5).lt.1.6d0) then
        go to 30
c     para(6) and para(7) > 0 required
       elseif (para(6).le.0.d0.or.para(7).le.0.d0) then
        go to 30
```

```
      else
c     if O.K.
        go to 50
      endif
c.... Half the increments for parameters if constraints are violated.
   30  do 40 i=1,7
      delp(i)=delp(i)/2.
   40  para(i)=old(i)+delp(i)
      go to 20
c.... Update the old parameters
   50  do 60 i=1,7
   60  old(i)=para(i)
c
      do 70 j=1,6
        if (indi.eq.1) go to 63
         do 62 k=1,n(j)
          do 61 kk=1,6
          deltem(k,kk) = del(k,kk,j)
   61      continue
   62      continue
   63    call main(ytemp,n(j),para,deltem,j,indi)
        do 65 k=1,n(j)
        y(k,j) = ytemp(k)
        ytemp(k)=0.0
          do 64 kk=1,6
          del(k,kk,j) = deltem(k,kk)
          deltem(k,kk) = 0.0
   64      continue
   65    continue
   70  continue
      indi=indi+1
c
      knt=0
      do 200 j=1,6
      do 100 i=1,n(j)
      k=knt+i
      f(k)=(ystar(i,j)-y(i,j))*sqrt(w(i,j))
  100  continue
      knt=knt+n(j)
  200  continue
      return
      end
```

```
c    ***************************************************
      subroutine main(y,n,para,del,ino,ind)
c    ***************************************************
c.... DEL  : the specified strain increment vectors.
       implicit double precision(a-h,o-z)
       common/state/sig0(6)
       common/sta/geop,xint
c.... Y    : the response vector
c.... PARA : the parameter vector
       dimension del(500,6),sig(6)
       dimension y(1),para(1)
       common/fix/bulkm,shearm,zm
c.... Material parameters :
       common/prop/ltype,tcut,fcut
       common/elas/bulk,shear
       common/par1/alpha,theta,gama,beta,r
       common/par2/d,w,z
c.... Definition for parameters ( just for convenience ).
       bulk=bulkm
       shear=shearm
       alpha=para(1)
       theta=para(2)
       gama=para(3)
       beta=para(4)
       r=para(5)
       d=para(6)
       w=para(7)
       z=zm
c.... IND : flag, if ind=1 , read strain increment data
c.... INO : identifier for test # ino ( 1-6 ).
       if(ind.ne.1) go to 200
       if(ino.ne.1) go to 50
c.... Read common input data: ltype,tcut,sig0,geop,xint
c.... Read material type and tension cutoff criterion
c.... TCUT is in terms of live stresses.
       read(5,*) ltype,tcut
c.... Input the initial states of stress and strain
       read(5,*) (sig0(k),k=1,6)
c.... Input the geostatic pressure and XINT( the initial cap )
       read(5,*) geop,xint
c.... Strain controlled CAP model
c.... Read input data del and initial strain.
  50   do 100 i=1,n
       read(5,*) (del(i,k),k=1,6)
 100   continue
c.... Call preprocessor INITEL to calculate elint from
c     given xint and fcut(in total stress)
c.... XINT := the inital X value for the initail cap.
c.... Z    := the  X value for the characteristic initial cap.
c       i.e. the  X value when EVP = 0.
 200   if(ino.ne.1) go to 210
       call initel(xint,elint,nocon1,nocon2)
c.... Assign initial state of stress and strain accordingly
c
```

```
 210  continue
      do 220 k=1,6
      sig(k)=sig0(k)
 220  continue
      el=elint
c.... Call 3-D strain history driver
      call drv3D(n,y,del,el,sig)
      return
      end
```

```
c     ******************************************************************
      subroutine drv3D(n,y,del,el,sig)
c     ******************************************************************
c.... This routine is a 3-D strain history driver and the
c     variable increments are deps stored in array del.
      implicit double precision(a-h,o-z)
      common/sta/geop
      dimension del(500,6),sig(1),deps(6),y(1)
      common/prop/ltype,tcut,fcut
      common/elas/bulk,shear
      common/par1/alpha,theta,gama,beta,r
      common/par2/d,w,z
c
      do 200 i=1,n
        do 100 k=1,6
        deps(k)=del(i,k)
100     continue
      call cap(sig,deps,geop,el,mtype,it,nocon,sj1,sj2,xl,evpi
     * ,ej1,ej2d,f1ej1)
      y(i)=sig(3)
 200  continue
      return
      end
```

```
c     ******************************************************
      subroutine cap(sig,deps,geop,el,mtype,it,nocon,sj1,sj2,
      xl,evpi,ej1,ej2d,f1ej1)
c     ******************************************************
c.... For full three-dimensional stresses and strains
c      computations by using the CAP model
c.... Strain controlled algorithm.
c.... Stresses and strains are sig and eps, respectively.
c.... geop = geostatic pressure (overburden stress)
c.... el = hardening parameter
c.... mtype :  0 = tension cutoff, 1 = elastic, 2 = failure,
c....           3 = cap mode,      4= cone mode
c.... it = # of iterations for CAP mode calculation
c.... nocon = 1 indicates no convergence under max iterations
c....             (nit) restriction. Otherwise = 0
c.... eps = error tolerance parameter
c.... ltype : 1 = soil, 2 = rock
        implicit double precision(a-h,o-z)
        common/prop/ltype,tcut,fcut
        common/elas/bulk,shear
        common/par1/alpha,theta,gama,beta,r
        common/par2/d,w,z
        dimension sig(1),deps(1),s(6),de(6)
        data eps/1.d-6/
c.... Statement function for exponential with negatively large
c.... argument for large caps
        exps(z)=dexp(dmax1(-500.,z))
c.... Failure envelope function for sj2
        f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
        d1(sj1)=theta+gama*beta*exps(-beta*sj1)
c.... Cap statement functions for f2 functional forms
c.... capl=l(k) : intersection point of f1 & f2,
c.... x(k) : intersection of f2 & j1 axis
        capl(el)=dmax1(0.0,el)
        ra(capi)=r
        x(el)=dmax1(0.,el+ra(capl(el))*f1(el))
        evp(xl)=w*(1.0-exps(d*(z-xl)))
        f2(sj1,xl,capi)=dsqrt((xl-capi)**2-(sj1-capi)**2)
     *  /ra(capi)
c.... Elastic moduli functions
        bmod(sj1,ev)=bulk
        smod(sj2,ev)=shear
c
        it=0
        nocon=0
        dev=deps(1)+deps(2)+deps(3)
        devb3=dev/3.0
        do 1 k=1,3
    1   de(k)=deps(k)-devb3
        do 2 k=4,6
    2   de(k)=deps(k)
        press=(sig(1)+sig(2)+sig(3))/3.0
        do 3 k=1,3
    3   s(k)=sig(k)-press
```

```
      do 4 k=4,6
    4  s(k)=sig(k)
      sj1t=3.*(press+geop)
      temp=0.
      do 11 k=1,3
   11  temp=temp+0.5*s(k)*s(k)
      do 12 k=4,6
   12  temp=temp+s(k)*s(k)
      sj2l=dsqrt(temp)
      capi=capl(el)
      xl=x(el)
      evpi=evp(xl)
c.... Elastic material properties
      threek=3.*bmod(sj1t,evpi)
      g=smod(sj2l,evpi)
      twog=2.*g
c.... Elatic trial
      sj1=threek*dev+sj1t
      do 13 k=1,6
   13  s(k)=s(k)+twog*de(k)
      ratio=1.0
      mtype=1
c.... Tension limit test
      tencut=dmax1(fcut,tcut+3.*geop)
      if(sj1.gt.tencut) go to 10
      sj1=tencut
      ratio=0.0
      sj2=0.
      mtype=0
c     If no contraction
      if(ltype.eq.2.or.el.le.0.0d0) go to 200
c.... Tension dilatancy coding for soils with el.ge.0.0
c.... Dilatancy controlled by contracting cap up to el.ge.0.0
      ell=dmin1(0.0,el-eps*f1(el))
      xll=x(ell)
      denom=evp(xll)-evpi
      if(denom.lt.0.0d0) go to 5
      el=0.0
      go to 200
    5  devp=dev-(sj1-sj1t)/threek
      denom=dmin1(denom,devp)
      el=el+devp*(ell-el)/denom
      el=dmax1(0.0,el)
      go to 200
c.... Check if failure envelope mode is invoked
   10  continue
      temp=0.
      do 14 k=1,3
   14  temp=temp+0.5*s(k)*s(k)
      do 15 k=4,6
   15  temp=temp+s(k)*s(k)
c     Calc. J2'E
      sj2=dsqrt(temp)
      sj2e=sj2
```

```
c    If cap mode
       if(sj1.gt.capi) go to 40
       ej2d=sj2
c.... TMISES is the sj2 value at the corner point(tmises>=fj1)
       tmises=f2(capi,xl,capi)
       ej1=sj1
       fj1=f1(sj1)
       f1ej1=fj1
       fe=sj2-dmin1(fj1,tmises)
c    If elastic
       if(fe.le.0.0d0) go to 200
c.... If k0<0 (small cap) , no contraction allowed.
c    k=k0 and J1=J1E (von Mises transition)
       if (el.lt.0.0d0) then
         mtype=2
         go to 30
c.... For k0>=0:
c    If J1E=L(k0), J1=L(k0)=J1E, k=k0
       elseif (dabs(sj1-capi).le.1.d-6) then
         mtype=4
         go to 30
       endif
c.... Failure envelope surface calculation ( f1 )
       mtype=2
       elold=el
c.... Iterate to find new k & J1 .
       call proj(deps,el,sj1,2,sj2,nocon,it,threek,g)
c.... Consistency check for cap model:
       if(ltype.eq.2.or.elold.eq.0.d0) el=elold
       if(sj1.gt.el) el = sj1
       if (ltype.eq.1.and.elold.gt.0.0d0) then
         if(dabs(el-sj1).le.1.d-6) mtype=4
         el = max(el,0.0d0)
       endif
       el = max(el,0.0d0)
   30  fj1=f1(sj1)
       sj2=dmin1(fj1,tmises)
       ratio=sj2/sj2e
       go to 200
c.... CAP mode calculation
   40  if(sj1.gt.xl) go to 50
c    If elastic
       if(sj2.le.f2(sj1,xl,capi)) go to 200
   50  mtype=3
       call proj(deps,el,sj1,3,sj2,nocon,it,threek,g)
       ratio=0.0
       if(sj2e.ne.0.0d0) ratio=sj2/sj2e
  200  continue
c.... Update dev. stresses .
       do 300 k=1,6
  300  s(k)=s(k)*ratio
       press=sj1/3.-geop
c.... Calc. live stresses
       do 400 k=1,3
```

```
 400   sig(k)=s(k)+press
       do 410 k=4,6
 410   sig(k)=s(k)
c.... calc. X and vol. plastic strain .
       xl=x(el)
       evpi=evp(xl)
       return
       end
c      ********************************************************
       subroutine proj(deps,el,sj1,mtype,sj2,nocon,it,threek,g)
c      ********************************************************
c.... Subprogram to calc. the k and J1 iteratively by modified
c     Regula Falsi Secant Method.
       implicit double precision(a-h,o-z)
       common/prop/ltype,tcut,fcut
       common/elas/bulk,shear
       common/par1/alpha,theta,gama,beta,r
       common/par2/d,w,z
       dimension deps(1)
       data nit/600/
       data eps/1.d-6/
c.... Statement function for exponential with negatively large
c.... argument for large caps
       exps(z)=dexp(dmax1(-500.,z))
c.... Failure envelope function for sj2
       f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
       d1(sj1)=theta+gama*beta*exps(-beta*sj1)
c.... Cap statement functions for f2 functional forms
c.... capl=l(k) : intersection point of f1 & f2,
c.... x(k) : intersection of f2 & j1 axis
       capl(el)=dmax1(0.0,el)
       ra(capi)=r
       x(el)=dmax1(0.,el+ra(capl(el))*f1(el))
       evp(xl)=w*(1.0-exps(d*(z-xl)))
       f2(sj1,xl,capi)=dsqrt((xl-capi)*(xl-capi)-(sj1-capi)*(sj1-capi))
     *  /ra(capi)
       d2(sj1,xl,capi)=-(sj1-capi)/ra(capi)/dsqrt((xl-capi)**2-
     *  (sj1-capi)**2)
c.... Elastic moduli functions
       bmod(sj1,ev)=bulk
       smod(sj2,ev)=shear
c*********************************************************************
c*********************************************************************
       nocon=0
       it=0
       sj1e=sj1
       sj2e=sj2
       xl=x(el)
       evpi=evp(xl)
c.... Convergence criterion
       conv=eps*0.1
c.... Failure mode
c     Initial guess
       if (mtype.eq.2) then
```

```
          ell=sj1e
          elr=el
        else
          go to 45
        endif
c.... tcut > -1
        xll=x(ell)
        devpl=evp(xll)-evpi
        sj1l=sj1e-threek*devpl
        ql=-(sj1l+1.)/(sj1e+1.)
c
        xlr=xl
        devpr=evp(xlr)-evpi
        sj1r=sj1e-threek*devpr
        qr=(xlr-sj1r)/(xlr-j1e)
        go to 47
c.... Cap mode
   45   ell=el
        elr=sj1e
        if(sj1e.ge.xl) ql=(el-sj1e)/(el-xl)
        if(sj1e.lt.xl) ql=2.*sj2e/(sj2e+f2(sj1e,xl,capi))-1.0
        xr=x(elr)
        sj1r=sj1e-threek*(evp(xr)-evpi)
        qr=(xr-sj1r)/(elr-xr)
   47   qold=0.0
c.... Modified Regula Falsi Method
        do 80 it=1,nit
c       Secant method
        el=(qr*ell-ql*elr)/(qr-ql)
        xl=x(el)
        devp=evp(xl)-evpi
        sj1=sj1e-threek*devp
        capi=capl(el)
        if(mtype.eq.3) go to 48
c     If Failure mode
c     el >=-1
        if(sj1.gt.el) qc=-(sj1+1.)/(el+1.)
        if(sj1.le.sj1e) qc=(xl-sj1)/(xl-sj1e)
        if(sj1.gt.el.or.sj1.le.sj1e) go to 60
        sj2=f1(sj1)
        go to 49
c     If Cap mode
   48   continue
        if(sj1.ge.xl) qc=(el-sj1)/(el-xl)
        if(sj1.le.capi) qc=(xl-sj1)/(capi-xl)
        if(sj1.ge.xl.or.sj1.le.capi) go to 60
        sj2=f2(sj1,xl,capi)
   49   if (mtype.eq.2) then
c**** If cone mode (at corner pt.) inside failure mode*******
          if(dabs(el-sj1).le.1.d-6) then
c.... Correct treatment
            slope=(sj1-sj1e)/(sj2e-sj2)*g/(3.*threek)
            desp=devp/(3.*slope)
          else
```

```
          desp=devp/(3.*d1(sj1))
        endif
      else
        desp=devp/(3.*d2(sj1,xl,capi))
      endif
      a=sj2-g*desp
      error=sj2e-a
      qc=error/(sj2e+a)
c.... Convergence criteria
      if(dabs(error).le.conv) go to 90
   60 if(qc.gt.0.0d0) go to 70
      if (mtype.eq.3) then
c     k too large
        elr=el
        qr=qc
        if(qold.lt.0.0d0) ql=0.5*ql
      else
c     k too small
        ell=el
        ql=qc
        if(qold.lt.0.0d0) qr=0.5*qr
      endif
      go to 80
c
   70 if (mtype.eq.3) then
c     k too small
        ell=el
        ql=qc
        if(qold.gt.0.0d0) qr=0.5*qr
      else
c     k too large
        elr=el
        qr=qc
        if(qold.gt.0.0d0) ql=0.5*ql
      endif
   80 qold=qc
c
c.... If no convergence within NIT iterations:
      nocon=1
c     If cap mode:
      if (mtype.eq.3) then
        sj1=dmin1(sj1,xl)
        if(sj1.lt.capl(elr)) sj1=capi
        sj2=dmin1(sj2e,f2(sj1,xl,capi))
c     If failure envelope mode:
      else
        sj1=dmin1(sj1,el)
      endif
c
c  89 continue
   90 return
      end
c     ************************************************
      subroutine initel(xint,elint,nocon1,nocon2)
```

```
c     ********************************************
c.... This routine uses secant method to find initial
c     value of el(hardening parameter) for a given
c     initial x(el) value.
c.... Also, it solves FCUT,the intersection of F1 and
c     J1-axis.
      implicit double precision(a-h,o-z)
      common/prop/ltype,tcut,fcut
      common/elas/bulk,shear
      common/par1/alpha,theta,gama,beta,r
      common/par2/d,w,z
      data eps,nit/1.d-6,60/
c.... Statement function for exponential with negatively
c     large argument for large caps
      exps(z)=dexp(dmax1(-500.,z))
c.... Failure envelope function for sj2
      f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
c.... Cap statement functions
      capl(el)=dmax1(0.0,el)
      ra(capi)=r
      x(el)=el+ra(capl(el))*f1(el)
c.... Elastic modulus function
      bmod(sj1,ev)=bulk
c.... Find initial el
c     Solve f(k)=x(k)-xint=0 , not related to Z.
c.... xint is reset so that within the convergence criteria
c     xint is positive ( because we assume x>0, l(k)>=0. )
c.... nocon1=1 means no convergence for initial el iteration
c     nocon2=1 means no convergence for fcut iteration
c
      xint=dmax1(xint,eps*0.0001*bmod(0.,0.))
c.... Make initial guess k0
      el0=xint*0.1
      fl0=x(el0)-xint
c.... Make second initial guess k1
      el1=(xint-0.1*dmax1(dabs(xint),f1(xint)))*0.05
c.... Set up convergence criterion
      conv=dmin1(1.d-7,xint)
c.... Secant iteration
      do 100 it=1,nit
      fl1=x(el1)-xint
      if(dabs(fl1).lt.conv.or.dabs(el1-el0).lt.conv) go to 200
      el2=el1-fl1*(el1-el0)/(fl1-fl0)
      el0=el1
      fl0=fl1
      el1=el2
  100 continue
      nocon1=1
  200 elint=el1
c.... Find fcut
c     Solve f1(fcut)=0
c.... Make first initial guess for fcut
      fcut=dmin1(0.,elint)
      del=f1(fcut)
```

```
      if(del.eq.0.d0) go to 600
c.... Make two better initial guesses for fcut
      do 300 it=1,nit
      el0=fcut-del
      fl0=f1(el0)
      if(fl0.lt.0.d0) go to 400
      del=10.*del
      fcut=el0
  300 continue
c.... Secant iterations
  400 do 500 it=1,nit
      fl1=f1(fcut)
      if(dabs(fl1).lt.conv.or.dabs(fcut-el0).lt.conv) go to 600
      el2=fcut-fl1*(fcut-el0)/(fl1-fl0)
      el0=fcut
      fl0=fl1
      fcut=el2
  500 continue
      nocon2=1
  600 return
      end
```

```
c      ***********************************
       subroutine dprint(y,n1,n2,name)
c      ***********************************
c.... Program for printing response y (sig-33).
       implicit double precision(a-h,o-z)
       dimension y(1)
       character*6 name
c
       write(6,2000) name
c.... Print out 8 columns each time.
       do 100 j=n1,n2,8
c.... JH  : the right-most index.
       jh=j+7
       if(jh.gt.n2) jh=n2
       write(6,2001) (n,n=j,jh)
       write(6,2002) (y(k),k=j,jh)
  100  continue
       return
c.... Format
 2000  format(////20x,a6/
     *  20x,'======')
 2001  format(//8x,8i15)
 2002  format(/8x,8d15.7)
       end
```

APPENDIX B

EXAMPLE INPUT AND OUTPUT FOR APPENDIX A

```
INPUT BULK MODULUS, SHEAR MODULUS, AND INITIAL CAP (Z) PARAMETER:
2100. 1700. 0.
INPUT NO. OF OBSERVATIONS FOR 6 TESTS:
47 49 45 48 49 48
INPUT OBSERVED (EXPERIMENTAL) STRESS RESPONSES FOR 6 TESTS:
 NO. 1
1.     3.     5.    6.     5.     4.     3.     1.
8.8    9.     8.8   8.6    8.3    8.     9.     10.
11.    10.5   9.    8.     10.    12.    13.    12.
10.    8.     10.   12.    13.    14.    14.9
 NO. 2
1.     2.     4.    6.     8.     8.5    9.     9.5
9.     8.5    8.    7.5    7.     6.5    7.     7.5
8.     8.5    9.    9.5    10.    9.5    9.     8.5
8.     7.5    7.    6.5    6.     6.5    7.     7.5
8.     9.     10.   11.    10.5   10.    9.     8.
7.     6.     5.    4.5    4.     3.5    3.     2.5
2.
 NO. 3
1.     2.     4.    6.     8.     10.    10.12  10.09
9.99   9.84   9.625 9.36   9.06   8.73   8.37   8.
7.63   7.275  6.94  6.63   6.375  6.16   6.     5.91
5.89   5.91   6.    6.16   6.375  6.63   6.94   7.275
7.63   8.     8.37  8.73   9.06   9.36   9.625  9.84
9.99   10.09  10.12 10.    8.
 NO. 4
1.     2.     3.    4.     5.     6.     7.     8.
8.708  9.414  10.122 10.828 11.536 12.246 12.95 13.656
12.246 10.828 9.414 8.     7.823  7.646  7.293  6.939
7.293  7.646  8.    8.     8.     8.     8.     8.
8.     8.     8.    8.     8.     8.     8.     8.
8.     7.823  7.646 8.     7.292  6.584  5.172  4.466
NO. 5
1.     1.5    2.    3.     4.     4.566  5.132  5.698
6.262  6.828  5.414 4.     3.717  3.434  3.151  2.869
2.586  3.293  4.    3.717  3.434  3.151  2.869  2.586
3.293  4.     5.    6.     7.     7.5    8.     9.
10.    8.     6.    4.     4.708  5.414  6.122  6.828
6.828  6.828  6.828 6.828  6.828  6.828  6.828  6.828
6.828
 NO. 6
3.6    3.6    2.92  2.24   1.56   .88    0.     0.
0.     0.     0.    0.     0.     0.     0.     0.
0.     .88    1.56  2.24   2.92   3.6    3.6    3.6
3.6    3.6    3.6   2.92   2.24   1.56   .88    0.
1.     1.5    2.    3.     3.6    4.     5.     6.
INPUT CONVERGENCE CRITERION AND MAXIMUM NO. OF FUNCTION EVALUATIONS:
5 1.d-11 0.000001 500 2
INPUT PARAMETERS FOR MARQUARDT-LEVENBERG ALGORITHM:
```

INPUT INITIAL GUESS FOR MATERIAL PARAMETERS:
3.2 .09 1.0 .49 4. 0.004 0.2
INPUT OPTION FOR WEIGHTING MATRIX (0 : WEIGHT = IDENTITY):
0
INPUT OPTION FOR SOIL (1) OR ROCK (2); AS WELL AS TENSION CUTOFF VALUE
1 -0.3
INPUT INITIAL STRESS STATE:
0. 0. 0. 0. 0. 0.
INPUT GEOSTATIC OVERBURDEN PRESSURE AND INITIAL CAP POSITION:
0. 16.
INPUT STRAIN HISTORY OF 6 TESTS:
NO. 1

-0.0000020	-0.0000587	-0.0000635	0.	0.	0.
0.0002395	0.0002086	0.0001715	0.	0.	0.
0.0004127	0.0004370	0.0004799	0.	0.	0.
0.0003060	0.0003555	0.0003940	0.	0.	0.
-0.0001604	-0.0002206	-0.0001891	0.	0.	0.
-0.0003913	-0.0003993	-0.0003953	0.	0.	0.
-0.0001187	-0.0001047	-0.0001095	0.	0.	0.
-0.0000720	-0.0000330	-0.0000827	0.	0.	0.
0.0001420	0.0000992	0.0000880	0.	0.	0.
0.0004722	0.0004828	0.0004767	0.	0.	0.
0.0005390	0.0006185	0.0006685	0.	0.	0.
0.0004835	0.0005567	0.0005853	0.	0.	0.
-0.0001502	-0.0002710	-0.0001843	0.	0.	0.
-0.0001722	-0.0002279	-0.0002293	0.	0.	0.
-0.0001348	-0.0001654	-0.0001603	0.	0.	0.
-0.0004710	-0.0004949	-0.0005204	0.	0.	0.
-0.0001411	-0.0001104	-0.0001218	0.	0.	0.
-0.0001776	-0.0001480	-0.0002248	0.	0.	0.
0.0004733	0.0004259	0.0004466	0.	0.	0.
0.0004973	0.0005316	0.0005792	0.	0.	0.
0.0004338	0.0005852	0.0005394	0.	0.	0.
0.0011258	0.0011877	0.0012631	0.	0.	0.
0.0000354	-0.0000029	0.0003382	0.	0.	0.
0.0000676	0.0000314	0.0002335	0.	0.	0.
-0.0000763	-0.0000882	0.0001768	0.	0.	0.
0.0000006	-0.0000269	0.0001764	0.	0.	0.
0.0000336	0.0000646	-0.0000060	0.	0.	0.
0.0000244	0.0000372	-0.0000439	0.	0.	0.
0.0000506	0.0000774	-0.0000951	0.	0.	0.
0.0000724	0.0000719	-0.0001010	0.	0.	0.
-0.0001322	-0.0001861	0.0003957	0.	0.	0.
-0.0001670	-0.0002238	0.0009633	0.	0.	0.
-0.0002107	-0.0002359	0.0014727	0.	0.	0.
0.0000704	0.0001000	-0.0000485	0.	0.	0.
0.0002085	0.0002957	-0.0004022	0.	0.	0.
0.0001969	0.0002268	-0.0003118	0.	0.	0.
-0.0002632	-0.0003922	0.0006584	0.	0.	0.
-0.0003644	-0.0004242	0.0016212	0.	0.	0.
-0.0003102	-0.0003345	0.0019130	0.	0.	0.
0.0001826	0.0002295	-0.0000325	0.	0.	0.
0.0003423	0.0004182	-0.0004839	0.	0.	0.
0.0003900	0.0004675	-0.0005386	0.	0.	0.

-0.0002378	-0.0003693	0.0005416	0.	0.	0.
-0.0002967	-0.0003534	0.0006050	0.	0.	0.
-0.0002038	-0.0002265	0.0004095	0.	0.	0.
-0.0003311	-0.0003540	0.0012834	0.	0.	0.
-0.0006453	-0.0006992	0.0026053	0.	0.	0.

NO. 2

0.0000605	0.0001083	0.0000454	0.	0.	0.
0.0002414	0.0002434	0.0001962	0.	0.	0.
0.0006576	0.0005363	0.0005038	0.	0.	0.
0.0009995	0.0008484	0.0008523	0.	0.	0.
0.0012717	0.0011145	0.0011642	0.	0.	0.
-0.0000126	0.0000866	0.0004583	0.	0.	0.
-0.0002257	0.0000549	0.0006188	0.	0.	0.
-0.0002288	0.0000196	0.0007454	0.	0.	0.
0.0001748	0.0000072	-0.0000809	0.	0.	0.
0.0002013	0.0000030	-0.0001115	0.	0.	0.
0.0002151	0.0000220	-0.0001045	0.	0.	0.
0.0002809	0.0000040	-0.0001376	0.	0.	0.
0.0005792	0.0000319	-0.0001188	0.	0.	0.
0.0006429	0.0000219	-0.0001788	0.	0.	0.
-0.0001008	0.0000160	0.0001356	0.	0.	0.
-0.0001274	0.0000082	0.0001300	0.	0.	0.
-0.0001354	-0.0000062	0.0001382	0.	0.	0.
-0.0002059	-0.0000085	0.0001024	0.	0.	0.
-0.0002097	0.0000104	0.0001523	0.	0.	0.
-0.0001877	-0.0000040	0.0002033	0.	0.	0.
-0.0002341	0.0000050	0.0002506	0.	0.	0.
0.0001462	0.0000058	-0.0001113	0.	0.	0.
0.0001768	-0.0000107	-0.0001103	0.	0.	0.
0.0001798	0.0000064	-0.0001008	0.	0.	0.
0.0001866	-0.0000034	-0.0001223	0.	0.	0.
0.0001946	0.0000161	-0.0001244	0.	0.	0.
0.0001869	0.0000088	-0.0001155	0.	0.	0.
0.0002393	0.0000143	-0.0001289	0.	0.	0.
0.0004460	0.0000048	-0.0001845	0.	0.	0.
-0.0000910	0.0000106	0.0001064	0.	0.	0.
-0.0001403	0.0000132	0.0001417	0.	0.	0.
-0.0001539	0.0000090	0.0001467	0.	0.	0.
-0.0001847	0.0000051	0.0001365	0.	0.	0.
-0.0003387	0.0000249	0.0002432	0.	0.	0.
-0.0003679	0.0000020	0.0003542	0.	0.	0.
-0.0005787	0.0000484	0.0017548	0.	0.	0.
0.0001755	0.0000317	-0.0000571	0.	0.	0.
0.0001739	0.0000169	-0.0000843	0.	0.	0.
0.0003868	0.0000054	-0.0002295	0.	0.	0.
0.0003718	0.0000153	-0.0002529	0.	0.	0.
0.0004210	-0.0000016	-0.0002611	0.	0.	0.
0.0005352	0.0000149	-0.0003012	0.	0.	0.
0.0018660	0.0000722	-0.0004459	0.	0.	0.
0.0010116	0.0000614	-0.0003765	0.	0.	0.
0.0011949	0.0000972	-0.0004134	0.	0.	0.
0.0012990	0.0001705	-0.0006715	0.	0.	0.
0.0018603	0.0002503	-0.0009851	0.	0.	0.
0.0020935	0.0004362	-0.0016850	0.	0.	0.

0.0029288	0.0006565	-0.0027520	0.	0.	0.
NO. 3					
0.0000484	0.0000377	0.0000252	0.	0.	0.
0.0002150	0.0002914	0.0001842	0.	0.	0.
0.0005414	0.0004079	0.0004483	0.	0.	0.
0.0007421	0.0005248	0.0007087	0.	0.	0.
0.0009765	0.0012642	0.0008171	0.	0.	0.
-0.0000970	0.0016179	0.0008264	0.	0.	0.
0.0000007	-0.0000162	0.0001920	0.	0.	0.
0.0001209	-0.0000960	0.0000430	0.	0.	0.
0.0001261	-0.0000576	-0.0000534	0.	0.	0.
0.0000925	-0.0000849	0.0000311	0.	0.	0.
0.0000845	-0.0000320	-0.0000264	0.	0.	0.
0.0002109	-0.0000471	-0.0000221	0.	0.	0.
0.0001669	-0.0000023	-0.0001151	0.	0.	0.
0.0001409	-0.0000190	0.0000355	0.	0.	0.
0.0001269	0.0000137	0.0000463	0.	0.	0.
0.0002552	0.0001032	-0.0001915	0.	0.	0.
0.0000561	0.0000454	-0.0000306	0.	0.	0.
0.0001549	0.0000741	-0.0000281	0.	0.	0.
0.0000986	0.0000611	-0.0000475	0.	0.	0.
0.0000517	0.0000666	-0.0000549	0.	0.	0.
-0.0000806	0.0000684	-0.0000474	0.	0.	0.
0.0000517	0.0001330	-0.0000737	0.	0.	0.
-0.0000616	0.0000843	-0.0000197	0.	0.	0.
-0.0000121	0.0001012	-0.0000002	0.	0.	0.
-0.0000821	0.0000329	-0.0000058	0.	0.	0.
-0.0000560	0.0000838	-0.0000020	0.	0.	0.
-0.0000832	0.0000919	0.0000286	0.	0.	0.
-0.0000732	0.0000191	0.0000442	0.	0.	0.
-0.0000191	0.0001280	0.0000552	0.	0.	0.
-0.0000628	0.0000374	0.0000650	0.	0.	0.
-0.0000772	0.0000058	0.0000605	0.	0.	0.
-0.0000528	-0.0000068	0.0000588	0.	0.	0.
-0.0000730	0.0000128	0.0000582	0.	0.	0.
-0.0000642	-0.0000708	0.0000813	0.	0.	0.
-0.0000509	-0.0000516	0.0000428	0.	0.	0.
-0.0000317	-0.0000462	0.0004019	0.	0.	0.
-0.0000125	-0.0001712	-0.0003004	0.	0.	0.
-0.0000096	0.0000530	0.0000690	0.	0.	0.
0.0000333	-0.0000684	0.0000731	0.	0.	0.
0.0000159	-0.0000923	0.0000391	0.	0.	0.
0.0000992	-0.0000855	0.0000068	0.	0.	0.
0.0000079	-0.0000511	0.0000643	0.	0.	0.
0.0000813	-0.0000230	-0.0000395	0.	0.	0.
-0.0000199	-0.0000122	-0.0000426	0.	0.	0.
0.0001928	0.0001901	-0.0003665	0.	0.	0.
NO. 4					
0.0000791	0.0001324	0.0000957	0.	0.	0.
0.0001569	0.0001952	0.0001699	0.	0.	0.
0.0002074	0.0002416	0.0001982	0.	0.	0.
0.0003594	0.0003230	0.0003006	0.	0.	0.
0.0004221	0.0003794	0.0004216	0.	0.	0.
0.0006202	0.0005900	0.0006006	0.	0.	0.

0.0006292	0.0006308	0.0006382	0.	0.	0.
0.0012222	0.0011276	0.0011728	0.	0.	0.
0.0001410	0.0000790	0.0009865	0.	0.	0.
-0.0000600	0.0001391	0.0009415	0.	0.	0.
-0.0000592	0.0000187	0.0009028	0.	0.	0.
-0.0001154	-0.0000927	0.0008714	0.	0.	0.
-0.0001138	-0.0000733	0.0009108	0.	0.	0.
-0.0002066	-0.0001771	0.0009852	0.	0.	0.
-0.0001900	-0.0002405	0.0007927	0.	0.	0.
-0.0002473	-0.0001590	0.0011455	0.	0.	0.
0.0002762	0.0002745	-0.0001501	0.	0.	0.
0.0003008	0.0002316	-0.0001202	0.	0.	0.
0.0002369	0.0001790	-0.0001658	0.	0.	0.
0.0003884	0.0003313	-0.0002903	0.	0.	0.
0.0002549	0.0000901	0.0000210	0.	0.	0.
0.0005766	-0.0000051	-0.0000250	0.	0.	0.
0.0009615	-0.0000561	-0.0000813	0.	0.	0.
0.0011829	-0.0000638	-0.0001446	0.	0.	0.
-0.0000458	0.0001162	0.0000787	0.	0.	0.
-0.0001230	0.0000959	0.0000463	0.	0.	0.
-0.0001584	0.0000261	0.0000648	0.	0.	0.
0.0001326	-0.0000401	0.0000151	0.	0.	0.
0.0000471	-0.0000579	-0.0000228	0.	0.	0.
0.0000889	-0.0000764	-0.0000028	0.	0.	0.
0.0000512	0.0000139	0.0000052	0.	0.	0.
0.0000813	-0.0000568	-0.0000072	0.	0.	0.
0.0000810	-0.0000235	-0.0000170	0.	0.	0.
0.0002334	-0.0001039	-0.0000867	0.	0.	0.
0.0001176	-0.0000608	-0.0000148	0.	0.	0.
0.0001083	-0.0000388	-0.0000258	0.	0.	0.
0.0005572	-0.0001531	0.0000138	0.	0.	0.
0.0016937	-0.0006312	0.0000094	0.	0.	0.
0.0000237	0.0003188	-0.0000019	0.	0.	0.
-0.0002057	0.0002319	0.0000001	0.	0.	0.
-0.0005429	0.0005126	0.0000196	0.	0.	0.
0.0000422	0.0002010	-0.0000207	0.	0.	0.
-0.0000105	0.0003828	-0.0000166	0.	0.	0.
0.0000669	-0.0001148	0.0000507	0.	0.	0.
0.0000888	0.0000783	-0.0001115	0.	0.	0.
0.0000906	0.0001283	-0.0000956	0.	0.	0.
0.0002193	0.0009332	-0.0003970	0.	0.	0.
0.0001673	0.0003156	-0.0003535	0.	0.	0.

NO. 5

0.0000602	-0.0000033	-0.0000030	0.	0.	0.
0.0000662	0.0001508	0.0001436	0.	0.	0.
0.0000986	0.0001254	0.0001093	0.	0.	0.
0.0002091	0.0001187	0.0001403	0.	0.	0.
0.0004602	0.0005228	0.0005385	0.	0.	0.
-0.0000782	-0.0000767	0.0003576	0.	0.	0.
-0.0000840	-0.0000768	0.0004826	0.	0.	0.
-0.0000645	-0.0000581	0.0007508	0.	0.	0.
-0.0000853	-0.0000996	0.0008040	0.	0.	0.
-0.0001174	-0.0001384	0.0009033	0.	0.	0.
0.0001483	0.0001664	-0.0003725	0.	0.	0.

0.0003256	0.0003237	-0.0004044	0.	0.	0.
0.0002350	0.0000300	-0.0000478	0.	0.	0.
0.0005129	-0.0000342	-0.0000830	0.	0.	0.
0.0006768	-0.0000320	-0.0000767	0.	0.	0.
0.0008572	-0.0000982	-0.0001384	0.	0.	0.
0.0012622	-0.0001020	-0.0001551	0.	0.	0.
-0.0002812	0.0001777	0.0001875	0.	0.	0.
-0.0003534	0.0001597	0.0002252	0.	0.	0.
-0.0000682	0.0002975	-0.0000719	0.	0.	0.
-0.0000849	0.0004480	-0.0000842	0.	0.	0.
-0.0000731	0.0006175	-0.0000746	0.	0.	0.
-0.0001584	0.0009155	-0.0001398	0.	0.	0.
-0.0001798	0.0013772	-0.0001679	0.	0.	0.
0.0002120	-0.0003489	0.0002209	0.	0.	0.
0.0001899	-0.0004123	0.0002372	0.	0.	0.
0.0001160	0.0000869	0.0001063	0.	0.	0.
0.0003453	0.0002330	0.0003364	0.	0.	0.
0.0005674	0.0004565	0.0004510	0.	0.	0.
0.0007839	0.0008205	0.0005003	0.	0.	0.
0.0007683	0.0007891	0.0005170	0.	0.	0.
0.0015877	0.0017248	0.0011667	0.	0.	0.
0.0013860	0.0014737	0.0010068	0.	0.	0.
-0.0000516	-0.0000565	-0.0001214	0.	0.	0.
-0.0001216	-0.0001476	-0.0000936	0.	0.	0.
-0.0002875	-0.0003229	-0.0003298	0.	0.	0.
-0.0000104	0.0000052	0.0002081	0.	0.	0.
-0.0001008	-0.0000906	0.0001869	0.	0.	0.
-0.0000930	-0.0000756	0.0002327	0.	0.	0.
-0.0001250	-0.0001354	0.0002326	0.	0.	0.
-0.0001757	-0.0000526	0.0000376	0.	0.	0.
-0.0002167	0.0000949	0.0000693	0.	0.	0.
-0.0001410	0.0000656	-0.0000065	0.	0.	0.
-0.0001834	0.0000666	0.0000742	0.	0.	0.
-0.0001515	0.0000727	0.0000440	0.	0.	0.
-0.0002026	0.0000717	0.0000534	0.	0.	0.
-0.0002599	0.0000473	0.0000907	0.	0.	0.
-0.0003525	0.0001030	0.0000933	0.	0.	0.
-0.0003709	0.0000825	0.0001300	0.	0.	0.

NO. 6

-0.0000625	-0.0000546	0.0001171	0.	0.	0.
-0.0000476	-0.0000387	0.0003012	0.	0.	0.
-0.0000532	-0.0000341	0.0003818	0.	0.	0.
-0.0000675	-0.0000629	0.0004775	0.	0.	0.
-0.0001220	-0.0000960	0.0005752	0.	0.	0.
-0.0000531	0.0001651	0.0000414	0.	0.	0.
-0.0000679	0.0002344	0.0000331	0.	0.	0.
-0.0000715	0.0003692	-0.0000080	0.	0.	0.
-0.0001069	0.0003996	0.0000018	0.	0.	0.
-0.0001335	0.0004520	0.0000100	0.	0.	0.
0.0000151	0.0001397	-0.0001865	0.	0.	0.
0.0000176	0.0000723	-0.0002161	0.	0.	0.
0.0000234	0.0000978	-0.0002795	0.	0.	0.
0.0000688	0.0002153	-0.0003166	0.	0.	0.
-0.0000229	0.0003285	-0.0006315	0.	0.	0.

0.0005334	0.0000008	-0.0000703	0.	0.	0.
0.0004023	-0.0000019	-0.0001729	0.	0.	0.
0.0004588	0.0000356	-0.0001098	0.	0.	0.
0.0004159	-0.0000104	-0.0001238	0.	0.	0.
0.0005309	0.0000204	-0.0001665	0.	0.	0.
0.0001729	-0.0001561	0.0000129	0.	0.	0.
0.0000965	-0.0002153	0.0000149	0.	0.	0.
0.0001470	-0.0002790	0.0000144	0.	0.	0.
0.0001908	-0.0003715	0.0000025	0.	0.	0.
0.0003892	-0.0007686	-0.0001695	0.	0.	0.
0.0000529	-0.0000881	0.0003802	0.	0.	0.
-0.0000426	-0.0001118	0.0004453	0.	0.	0.
0.0000148	-0.0001371	0.0005146	0.	0.	0.
0.0000144	-0.0001193	0.0003962	0.	0.	0.
-0.0000554	-0.0001868	0.0005284	0.	0.	0.
-0.0001609	-0.0000049	0.0001260	0.	0.	0.
-0.0002267	0.0000196	0.0001397	0.	0.	0.
-0.0002755	0.0000271	0.0000800	0.	0.	0.
-0.0004377	0.0000032	0.0001573	0.	0.	0.
-0.0009679	-0.0000286	0.0003732	0.	0.	0.
0.0000160	0.0000371	-0.0001910	0.	0.	0.
0.0000396	0.0000453	-0.0002006	0.	0.	0.
0.0000427	0.0000660	-0.0002739	0.	0.	0.
0.0000475	0.0000792	-0.0003740	0.	0.	0.
0.0000935	0.0001432	-0.0006179	0.	0.	0.
0.0004870	0.0003082	0.0002324	0.	0.	0.
0.0001613	0.0001642	0.0001151	0.	0.	0.
0.0001853	0.0002125	0.0001662	0.	0.	0.
0.0003383	0.0004073	0.0003093	0.	0.	0.
0.0002077	0.0002818	0.0001864	0.	0.	0.
0.0001044	0.0001421	0.0001103	0.	0.	0.
0.0005185	0.0006779	0.0013489	0.	0.	0.
0.0007016	0.0011151	0.0017758	0.	0.	0.

BULK MODULUS = 0.210000d+04
SHEAR MODULUS = 0.170000d+04
INITIAL CAP (Z) = 0.000000d+00

THE INITIAL GUESS FOR PARAMETERS:

ALPHA	THETA	GAMA	BETA
3.200000	0.090000	1.000000	0.490000

R	D	W
4.000000	0.004000	0.200000

THE OPTIMAL VALUES OF PARAMETERS ARE:

ALPHA	THETA	GAMA	BETA
3.865751	0.100000	1.163779	0.443505

R	D	W
4.433298	0.003223	0.429271

TRUE SUM OF SQUARES = 0.6758328d+03

THE TRUE ROOT-MEAN-SQUARE OF PHI = 0.1537222d+01

THE NORMALIZED RELATIVE ERROR = 0.2221052d+00

CONDITION NUMBER OF G = 0.1611413d+05

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