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**THE ROLE OF COLLATERAL INFORMATION ABOUT
EXAMINEES IN ITEM PARAMETER ESTIMATION**

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and

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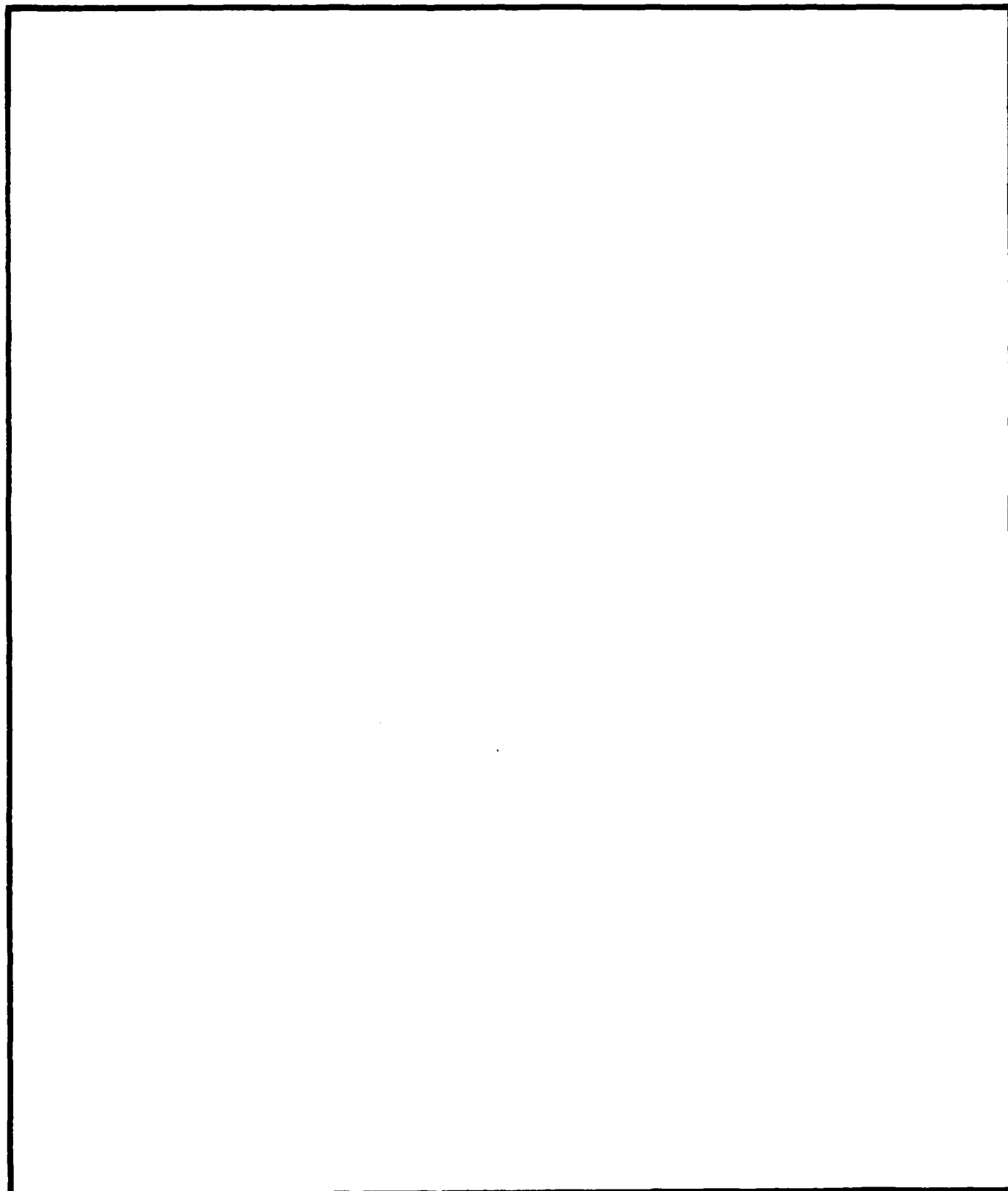
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in Item Parameter Estimation

Robert J. Mislevy and Kathleen M. Sheehan

Educational Testing Service

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The Role of Collateral Information about Examinees
in Item Parameter Estimation

Abstract

Standard procedures for estimating item parameters in item response theory (IRT) ignore collateral information that may be available about examinees, such as their standing on demographic and educational variables. This paper describes circumstances under which collateral information about examinees may be used to make inferences about item parameters more precise, and circumstances under which it must be used to obtain correct inferences.

Key words: Item response theory, Maximum likelihood, Missing data, Missing information principle.

Introduction

In typical applications of item response theory (IRT), information is available about examinees in addition to their item responses. Familiar examples include values of demographic variables such as age and gender, and educational variables such as courses taken and grades received. Although such collateral variables are sometimes employed to make testing more efficient, as when younger students are administered easier items than older students, it is standard practice to ignore them when estimating item parameters. Questions arise as to whether collateral information about examinees can be exploited during the course of item parameter estimation, and whether it should be. This paper brings together results from the psychometric and statistical literature to answer these and related questions.

The focus is on maximum likelihood item parameter estimation under the "mixed model" approach; that is, item parameters are treated as fixed effects and examinee parameters as random effects. Bock and Aitkin (1981) refer to the resulting item parameter estimates as "marginal maximum likelihood" (MML) estimates. After reviewing MML estimation when no collateral information is present (Case 0), attention turns to the situations listed below. In each case, we contrast the results of item parameter estimation expected when using the collateral information about examinees, with those expected when ignoring it.

Case 0: No collateral information.

Under an IRT model satisfying local independence, the probability of the pattern $\underline{x} = (x_1, \dots, x_n)$ of right/wrong responses to n test items from an examinee with proficiency parameter θ is given as

$$f(\underline{x}|\theta, \underline{\beta}) = \prod_j^n f_j(x_j|\theta, \beta_j) , \quad (1)$$

where each f_j is a function of known form in θ and the unknown, possibly vector-valued, item parameter β_j , and $\underline{\beta} = (\beta_1, \dots, \beta_n)$. The probability of $\underline{x}$ for a randomly selected examinee from a population in which θ has density $g(\theta|\alpha)$ with parameters α is the expectation of (1):

$$h(\underline{x}|\underline{\beta}, \alpha) = E_{\theta} f(\underline{x}|\theta, \underline{\beta}) = \int f(\underline{x}|\theta, \underline{\beta}) g(\theta|\alpha) d\theta .$$

The probability of the data matrix $\underline{X} = (\underline{x}_1, \dots, \underline{x}_N)$ from a random sample of N examinees from the population is then given as

$$P(\underline{X}|\underline{\beta}, \alpha) = \prod_i \int f(\underline{x}_i|\theta, \underline{\beta}) g(\theta|\alpha) d\theta . \quad (2)$$

After $\underline{X}$ has been observed, (2) can be interpreted as a likelihood function, say $L_0(\underline{\beta}, \alpha|\underline{X})$, and provides the basis for marginal

maximum likelihood (MML) estimation of $\underline{\beta}$ and α (e.g., Bock and Aitkin, 1981; Tsutakawa, 1984).

If g belongs to a known parametric family with a finite number of parameters, standard maximum likelihood results such as asymptotic normality and consistency in N hold for $\hat{\underline{\beta}}$ and $\hat{\alpha}$ under widely satisfied regularity conditions such as identifiability of parameters and continuous first and second derivatives (Kendall and Stuart, 1979).

The Information Matrix

Define the gradient $s(\theta, \underline{x})$, or s for short, associated with a single observation $(\theta, \underline{x})$ as the column vector

$$\begin{aligned} s(\theta, \underline{x}) &= [s'_{\underline{\beta}}(\theta, \underline{x}), s'_{\alpha}(\theta, \underline{x})]' \\ &= \left[\frac{\partial}{\partial \underline{\beta}'} \ln f(\underline{x}|\theta, \underline{\beta}), \frac{\partial}{\partial \alpha'} \ln g(\theta|\alpha) \right]'. \end{aligned}$$

Under regularity conditions, the MLE $(\hat{\underline{\beta}}, \hat{\alpha})$ satisfies the likelihood equation

$$\underline{0} = \sum_i E_{\theta}(s|\underline{x}_i) = \sum_i \int s(\theta, \underline{x}_i) p(\theta|\underline{x}_i, \underline{\beta}, \alpha) d\theta,$$

with $p(\theta|\underline{x}_i, \underline{\beta}, \alpha) = h^{-1}(\underline{x}_i|\underline{\beta}, \alpha) f(\underline{x}_i|\theta, \underline{\beta}) g(\theta|\alpha)$ by Bayes theorem.

This MLE is approximately normal for large N , with mean $(\underline{\beta}, \alpha)$ and

covariance given by the inverse of the expected information matrix $N I_X$, where

$$\begin{aligned} I_X &= E_X[E_\theta(s|\underline{x}) E_\theta(s'|\underline{x})] \\ &= \sum_{\ell} E_\theta(s|\underline{x}_\ell) E_\theta(s'|\underline{x}_\ell) h(\underline{x}_\ell|\underline{\beta}, \alpha) , \end{aligned} \quad (3)$$

the summation running over all possible response vectors $\underline{x}_\ell$.

Since $E_X[E_\theta(s|\underline{x})] = \underline{0}$, I_X can also be written as $\text{Var}_X[E_\theta(s|\underline{x})]$. The off-diagonal block of I_X for $\underline{\beta}$ with α is zero, so the asymptotic covariance matrix of $\underline{\beta}$ can be obtained by inverting $I_X(\underline{\beta}) = E_X[E_\theta(s_\beta|\underline{x}) E_\theta(s'_\beta|\underline{x})] = \text{Var}_X[E_\theta(s_\beta|\underline{x})]$.

Recent results on the structure of information matrices in the presence of missing data (Little and Rubin, 1987; Louis, 1982; Orchard and Woodbury, 1972) apply if $(\theta, \underline{x})$ is viewed as the hypothetical complete datum, $\underline{x}$ as the observed data, and the latent variable θ as missing data. Mislevy and Sheehan (1988) use Orchard and Woodbury's "missing information principle" (MIP) to show that an upper bound for I_X is the expected information $I_{\theta X} = E_X E_\theta(ss')$ that would obtain if values of θ were observed along with values of $\underline{x}$:

$$I_{\theta X} = E_X E_\theta(ss')$$

$$\begin{aligned}
 &= E_X[E_\theta(s|\underline{x}) E_\theta(s'|\underline{x})] + E_X[\text{Var}_\theta(s|\underline{x})] \\
 &= I_X + I_{\theta|X} . \qquad (4)
 \end{aligned}$$

$I_{\theta|X}$, the missing information, is the expected variance of the complete-data gradient vector over θ , given $\underline{x}$. It expresses variation in ss' over possible values of θ that could have given rise to the observed data $\underline{x}$, averaged over $\underline{x}$. If $\underline{x}$ determined θ with complete accuracy, $\text{Var}_\theta(s|\underline{x}) = 0$ for all $\underline{x}$, and no information would be lost. As values of θ are less well determined by $\underline{x}$, however, this variance increases and information about $\underline{\beta}$ and α decrease commensurately.

Alternative Population Structures

The preceding sections address estimation when the examinee population can be expressed in terms of a density of known parametric form, possibly with unknown parameters that are identified. Two alternative situations are also relevant to present concerns: θ distributions whose parameters are not identified, and nonparametric estimates of θ distributions.

A parameter is identified if different values imply different probabilities for $\underline{X}$. It is possible for some parameters to be identified while others are not. The concern in this presentation is that $\underline{\beta}$ be identified, for even if α is not, this, along with the regularity conditions for f , insures that $\hat{\underline{\beta}}$ enjoys the asymptotic properties mentioned above. In particular, the large-

sample variance of $\hat{\underline{\beta}}$ is still given by the inverse of $I_X(\underline{\beta})$. A consistent estimate for g is still obtained if one extends the notion of consistency to convergence to the set of distributions that (i) includes the correct distribution and (ii) imply the same probabilities for $\underline{x}$ (de Leeuw and Verhelst, 1986), but in this case asymptotic normality is not meaningful for the population distribution.

A consistent nonparametric estimate $\hat{g}$ of g can be obtained along with $\hat{\underline{\beta}}$ if Kiefer and Wolfowitz's (1956) regularity conditions are met. One useful characterization is a step function with jumps at a finite number of points, where both the values of the points and the heights of the jumps are estimated; they correspond to α in the parametric solution described above. The number of points depends upon the IRT model and the number of test items. Engelen (1987) and Lindsay (1987) have worked out the details for the 1-parameter logistic (Rasch) IRT model.

Case 1: Collateral information, independent of examinee and item sampling.

Suppose that collateral information y exists about examinees. Limiting ourselves to categorical y for convenience, we can express g as a finite mixture of K subpopulation densities:

$$g(\theta|\underline{\alpha}) = \sum_{k=1}^K \pi_k g_k(\theta|\alpha_k) , \quad (5)$$

where α_k is the possibly vector-valued parameter of the density in subpopulation k , and π_k is the proportion in which subpopulation k is represented in the mixture. If, for example, each g_k is normal with mean μ_k and common variance σ^2 , then $\alpha_k = (\mu_k, \sigma^2)$; and g is the density of the mixture of K normal components, with $\underline{\alpha} = (\pi_1, \dots, \pi_K, \mu_1, \dots, \mu_K, \sigma^2) = (\underline{\pi}, \underline{\mu}, \sigma^2)$.

Define $\underline{y}_i$ as $(y_{i1}, \dots, y_{iK})$, with $y_{ik} = 1$ if examinee i is associated with subpopulation k and 0 if not. Suppose a random sample $(\underline{X}, \underline{Y})$ comprised of N values of $(\underline{x}, \underline{y})$ is drawn from the population. Let N_k be the number of observations and $\underline{X}_k$ the item responses from Subpopulation k . In this situation there are two ways to estimate $\underline{\beta}$ by MML: using $\underline{Y}$ and ignoring $\underline{Y}$.

Analysis Using $\underline{Y}$

If the subpopulation structure is dealt with explicitly, the likelihood function induced by $(\underline{X}, \underline{Y})$ is

$$\begin{aligned}
 L_{1U}(\underline{\beta}, \underline{\pi}, \underline{\alpha} | \underline{X}, \underline{Y}) &= \prod_i P(\underline{y}_i | \underline{\pi}) \int p(\underline{x}_i | \underline{\theta}, \underline{\beta}) p(\underline{\theta} | \underline{y}_i, \underline{\alpha}) d\theta \\
 &= \prod_k \pi_k^{N_k} \times \prod_{k=1}^K \prod_{i=1}^{N_k} \int p(\underline{x}_i | \underline{\theta}, \underline{\beta}) g_k(\underline{\theta} | \alpha_k) d\theta
 \end{aligned}$$

$$= L(\underline{\pi}|\underline{Y}) \prod_k L(\underline{\beta}, \alpha_k | \underline{X}_k) . \quad (6)$$

Noting the isolation of information about $\underline{\pi}$ in the first factor, we have the consistency of $\hat{\underline{\pi}}$ in N from standard results on the multinomial distribution. The second factor is the product of K likelihoods with the same form as (2), related by the presence of $\underline{\beta}$ in each. Bradley and Gart's (1962) results on maximum likelihood in associated populations apply to this situation. Informally, if the usual ML identifiability and regularity conditions hold in each of a fixed number of subpopulations for the parameters involved in each, and sample sizes increase at the same rate in all subpopulations, then standard asymptotic properties hold for ML estimates of the joint parameter set. Bradley and Gart's theorem and the results from Case 0 thus imply consistency for $\hat{\underline{\beta}}$ and each $\hat{\alpha}_k$ in the problem at hand, with no additional regularity conditions beyond those required by Case 0.

The usual way of defining an information matrix in this setting is to condition on the observed values of N_k , giving $N I_{XY}$ where

$$I_{XY} = N^{-1} \sum_i \sum_{\ell} [E_{\theta}(s|\underline{x}_{\ell}, \underline{y}_i) E_{\theta}(s'|\underline{x}_{\ell}, \underline{y}_i)] h(\underline{x}_{\ell}|\underline{y}_i, \underline{\beta}) \quad (7)$$

with

$$E_{\theta}(s|\underline{x}_{\ell}, \underline{y}_i) = \int s(\theta, \underline{x}_{\ell}) p(\theta|\underline{x}_{\ell}, \underline{y}_i) d\theta ,$$

$$h(\underline{x}_{\ell}|\underline{y}_i) = \int f(\underline{x}_{\ell}|\theta, \underline{\beta}) \prod_k [g_k(\theta|\alpha_k)]^{y_{ik}} d\theta ,$$

and, again by Bayes theorem,

$$p(\theta|\underline{x}_{\ell}, \underline{y}_i) = h^{-1}(\underline{x}_{\ell}|\underline{y}_i) f(\underline{x}_{\ell}|\theta, \underline{\beta}) \prod_k [g_k(\theta|\alpha_k)]^{y_{ik}} .$$

By applying the MIP repeatedly, Mislevy and Sheehan (1988) show that $I_{\theta XY}(\underline{\beta}) = I_{\theta X}(\underline{\beta}) \geq I_{XY}(\underline{\beta}) \geq I_X(\underline{\beta})$, where $A \geq B$ means that the matrix difference $A-B$ is at least positive semidefinite. Additional detail about the relationships among expected information matrices will be given under Case 2.

Analysis Ignoring $\underline{y}$

Following Rubin (1976), we define "ignoring $\underline{y}$ " to mean acting as though the observed item responses arose from the marginal distribution of $\underline{X}$, or $\int p(\underline{X}, \underline{Y}) d\underline{Y}$. That is,

$$L_{1I}(\underline{\beta}, \underline{\alpha}|\underline{X}) = E_Y[L_{1U}(\underline{\beta}, \underline{\pi}, \underline{\alpha}|\underline{x}, \underline{Y})]$$

$$= E_Y[\prod_i P(\underline{y}_i|\underline{\pi}) \int p(\underline{x}_i|\theta, \underline{\beta}) p(\theta|\underline{y}_i, \underline{\alpha}) d\theta]$$

$$= \prod_i \int p(\underline{x}_i | \theta, \underline{\beta}) g(\theta | \underline{\alpha}) d\theta$$

$$= L_0(\underline{\beta}, \underline{\alpha} | \underline{X}) .$$

In other words, ignoring $\underline{Y}$ means estimating $\underline{\beta}$ without regard to y -values that have been observed, using x -values only and proceeding as if they had been collected under Case 0.

Rubin's (1976) theorems on inference and missing data can be used to verify whether L_{1I} yields the correct likelihood inferences about $\underline{\beta}$, now viewing $\underline{Y}$ as the missing data and $\underline{X}$ as the observed data. A sufficient condition is that the missing data be missing at random (MAR): the probability of the observed pattern of missingness must be the same for all possible values of the missing variables. MAR is satisfied in Case 1, since y -values are ignored (missing) regardless of their values. Therefore, Case 0 holds if $\underline{Y}$ is ignored; consistent estimates of $\underline{\beta}$ and g are obtained; and the information matrix is given by (3).

Case 2: Collateral information used in examinee
sampling but not item administration

Suppose that the examinee population has the same mixture structure given in (5), but that examinee sampling is carried at rates that differ from the π_k s. One possibility is stratified sampling in accordance with collateral variables rather than random from the population as a whole. In particular, examinees

are sampled from subpopulation k at predetermined rates π_k^* , which are not necessarily equal to π_k and which may take the value zero. One possibility is convenience sampling, in which the researcher may have little knowledge about the actual subpopulation sampling rates his procedures imply. A practical example is this: item parameters for a test intended for all sixth graders in the nation are estimated from the responses of students who happened to be present on a particular day in schools that elected to cooperate, in only the states of Ohio and Indiana. In this section and the next, increasing N indefinitely will mean increasing all subsample sizes in the proportions π^* . Again, MML estimation can be carried out either using or ignoring the values y of the collateral variables of examinees in the sample.

Analysis Using $\underline{Y}$

When subpopulation membership is employed under Case 2, the MML likelihood function has the same form as under Case 1:

$$L_{2U}(\underline{\beta}, \underline{\alpha} | \underline{x}, \underline{y}) = \prod_k \pi_k^* \times \prod_{k=1}^K \prod_{i=1}^{N_k} \int p(\underline{x}_i | \underline{\theta}, \underline{\beta}) g_k(\underline{\theta} | \underline{\alpha}_k) d\theta$$

$$= L(\underline{\pi}^* | \underline{Y}) \prod_k L(\underline{\beta}, \underline{\alpha}_k | \underline{X}_k), \quad (8)$$

where $L(\underline{\beta}, \alpha_k | X_k) = 1$ if $\pi_k^* = 0$. The expected information matrix for $\underline{\beta}$ has the same form as in Case 1 when $\underline{y}$ is used, namely $N I_{XY}(\underline{\beta})$ as given in (7).

Analysis Ignoring $\underline{Y}$

As in Case 1, MAR is satisfied when $\underline{Y}$ is ignored. The appropriate likelihood function to maximize is obtained by marginalizing over the distribution of θ in accordance with actual sampling rates:

$$g^*(\theta | \underline{\alpha}^*) = \sum_{k=1}^K \pi_k^* g_k(\theta | \alpha_k) , \quad (9)$$

where $\underline{\alpha}^* = (\alpha_1, \dots, \alpha_K, \pi_1^*, \dots, \pi_K^*)$. The likelihood is

$$L_{2I}(\underline{\beta}, \underline{\alpha}^* | \underline{x}) = \prod_{k=1}^K \prod_{i=1}^{N_k} \int f(\underline{x}_i | \theta, \underline{\beta}) g^*(\theta | \underline{\alpha}^*) d\theta$$

$$= \prod_i^N \int f(\underline{x}_i | \theta, \underline{\beta}) g^*(\theta | \underline{\alpha}^*) d\theta . \quad (10)$$

The problem reverts again to Case 0, now with respect to a density g^* obtained by reweighting the constituents of the original population. Under the aforementioned regularity

conditions, consistency and asymptotic normality hold for $\hat{\underline{\beta}}$ as an estimate of $\underline{\beta}$, and consistency holds for $\hat{g}^*$ as an estimate of g^* . A consistent estimate of g , however, may not be forthcoming. To reconstruct g , one would need consistent estimates of each α_k and each π_k . But it may be that some subpopulations have sampling rates of zero, so that their parameters cannot be recovered; or it may be that the effective sampling rates are not known.

Thus, MML estimation with samples that are not representative of the targeted population in the manner of Case 2 yields consistent estimates of item parameters but generally not of the original population. The estimate of g^* is a computational byproduct of consistent estimation of $\underline{\beta}$, of little interest in its own right.

The expected information matrix obtained under sampling rates $\underline{\pi}^*$ when ignoring subpopulation membership during estimation is $N I_{X(Y)}$, where

$$I_{X(Y)} = \sum_{\ell} E_{\theta}(s|\underline{x}_{\ell}) E_{\theta}(s'|\underline{x}_{\ell}) h^*(\underline{x}_{\ell}|\underline{\beta}, \underline{\alpha}^*) , \quad (11)$$

with $h^*(\underline{x}_{\ell}) = \int f(\underline{x}_{\ell}|\theta, \underline{\beta}) g^*(\theta|\underline{\alpha}^*) d\theta$. $I_{X(Y)}$ differs from I_X only in that expectations are taken with respect to the subpopulation weights in the sampling scheme rather than their naturally occurring weights.

Relationships Among Information Matrices

In analogy to $I_{\theta X}(\underline{\beta})$, define by $I_{\theta XY}(\underline{\beta})$ the block of the expected information matrix for item parameters that would obtain when sampling in accordance with prespecified rates for y if values of θ were observed as well as those of x . Mislevy and Sheehan (1988) show that $I_{\theta XY}(\underline{\beta}) = I_{\theta X}(\underline{\beta})$, and, following Orchard and Woodbury (1972), begin to explicate the relationship among $I_{\theta XY}(\underline{\beta})$, $I_{XY}(\underline{\beta})$, and $I_{X(Y)}(\underline{\beta})$ by partitioning the variation of s_{β} for a fixed value of x :

$$\begin{aligned} E_Y E_{\theta}(s_{\beta} s_{\beta}' | \underline{x}, y) &= E_Y E_{\theta}(s_{\beta} | \underline{x}, y) E_Y E_{\theta}(s_{\beta}' | \underline{x}, y) \\ &+ \text{Var}_Y[E_{\theta}(s_{\beta} | \underline{x}, y)] + E_Y[\text{Var}_{\theta}(s_{\beta} | \underline{x}, y)] . \end{aligned}$$

The first term on the right is the squared average value of s_{β} over all values of y and θ . The second is the variance of the average values of s_{β} with respect to θ as y varies. It represents variation in $E_{\theta}(s_{\beta})$ explained by y beyond that explained by x . The final term is the variation in $E_{\theta}(s_{\beta})$ remaining unexplained after both x and y have been accounted for. Taking expectations over x gives

$$I_{\theta XY}(\underline{\beta}) = I_{X(Y)}(\underline{\beta}) + I_{Y|X}(\underline{\beta}) + I_{\theta|XY}(\underline{\beta}) .$$

By appropriately combining these terms, we have two instances of the missing information principle, first when both θ and y are missing:

$$I_{\theta XY}(\underline{\beta}) = I_{X(Y)}(\underline{\beta}) + \left[I_{Y|X}(\underline{\beta}) + I_{\theta|XY}(\underline{\beta}) \right] \\ = I_{X(Y)}(\underline{\beta}) + I_{\theta|X(Y)}(\underline{\beta}) ;$$

then when only θ is missing:

$$I_{\theta XY}(\underline{\beta}) = \left[I_{X(Y)}(\underline{\beta}) + I_{Y|X}(\underline{\beta}) \right] + I_{\theta|XY}(\underline{\beta}) \\ = I_{XY}(\underline{\beta}) + I_{\theta|XY}(\underline{\beta}) .$$

The portion of missing information about $\underline{\beta}$ that is recovered by using $\underline{Y}$, then, is $I_{XY}(\underline{\beta}) - I_{X(Y)}(\underline{\beta}) = I_{Y|X}(\underline{\beta})$ --yet another instance of the MIP, with $(\underline{X}, \underline{Y})$ viewed as the complete data and $\underline{X}$ as the incomplete data. When $\underline{y}$ and θ are independent, this term is zero because for each $\underline{x}$, $E_{\theta}(\underline{s}_{\underline{\beta}})$ takes the same value at all values of $\underline{y}$. No information about $\underline{\beta}$ is lost by ignoring $\underline{Y}$ in this case. When $\underline{y}$ and θ are not independent, the degree to which information about $\underline{\beta}$ increases depends not simply upon the strength of their relationship, but on the strength of their relationship conditional on $\underline{x}$. There is less to be gained by using collateral information when θ is already well determined by $\underline{x}$ alone.

These results indicate that greater benefit accrues from using collateral information as it relates more strongly to the latent variable, and as less information is available from x . Informal analyses suggest that in typical educational and psychological settings, readily available collateral variables can account for a third of the population variance and increase the precision of β about as much as two to six additional test items (Mislevy, 1987). This gain is substantial in applications such as educational assessment surveys, where an examinee might receive only five items, and it may be beneficial in adaptive testing, where he may receive fifteen well-chosen items. The proportional gain is less impressive with individual achievement tests, where test lengths of 60 to 100 items are common.

Example 1: Recovering Missing Information

This example uses artificial data to illustrate the relationships among $I_{\theta XY}(\beta)$, $I_{XY}(\beta)$, and $I_{X(Y)}(\beta)$, and the source of information about β recoverable by using collateral variables. Example 3 will give some indications of this effect with actual data.

Suppose that examinee responses to two items follow the Rasch model; that is, $P(x_j | \theta, \beta_j) = [x_j(\theta - \beta_j)] / [1 + \exp(\theta - \beta_j)]$. Suppose further that $\beta_1 = \beta_2 = 0$, and that the examinee population consists of two equally-represented subpopulations with known distributions: in Subpopulation 1, $\theta = -1$ with probability .5 and $\theta = 0$ with

probability .5; in Subpopulation 2, $\theta=0$ or $+1$, each with probability .5. In the total population, then, θ takes the values $(-1,0,+1)$ with probabilities $(.25,.50,.25)$. Table 1 shows the likelihoods of each of the four possible response patterns at each of the three possible θ values. Table 2 gives, for both subpopulations and the population as a whole, expected counts of each response pattern for samples of 1000 examinees from each of the two subpopulations.

Tables 1 and 2 about here

Whereas likelihoods in Table 1 indicate the probability of a response pattern given θ , the posterior probabilities in Table 3 indicate the probability that an examinee producing a given response pattern has a particular value of θ . The posteriors in the first panel are conditional on knowing that an examinee belongs to Subpopulation 1, or $P(\theta|\underline{x},y=1)$. They are obtained through Bayes theorem, multiplying the pattern's likelihood at each θ (from Table 1) by that θ 's probability in Subpopulation 1, then normalizing. Note that the column for $\theta=+1$ contains only zeros, since $P(\theta=+1|y=1)=0$. The second panel in Table 3 gives posteriors conditional on membership in Subpopulation 2, or $P(\theta|\underline{x},y=2)$, and the third panel gives the posteriors that obtain when subpopulation membership is not known, or $P(\theta|\underline{x}) = P(\theta|\underline{x},y=1)P(y=1) + P(\theta|\underline{x},y=2)P(y=2)$.

Table 4 gives the value of $s(\theta, \underline{x})$ for Item 1 at each item/ θ combination. The averages of these values within response patterns also appear, for each subpopulation and for the population as a whole. These averages correspond to $\int s(\theta, \underline{x}) p(\theta | \underline{x}, y) d\theta$ for $y=1$ and $y=2$, and $\int s(\theta, \underline{x}) p(\theta | \underline{x}) d\theta$, with integrations realized as summations since θ can take only three values. For example,

$$\begin{aligned} E(s_{\beta_1} | \underline{x}=00, y=1) &= s_{\beta_1}(\theta=-1, \underline{x}=00) P(\theta=-1 | \underline{x}=00, y=1) \\ &+ s_{\beta_1}(\theta=0, \underline{x}=00) P(\theta=0 | \underline{x}=00, y=1) \\ &+ s_{\beta_1}(\theta=+1, \underline{x}=00) P(\theta=+1 | \underline{x}=00, y=1) \\ &= .158 \times .681 + .294 \times .319 + .430 \times 0 = .202 \end{aligned}$$

As seen in (4), $I_{\theta X}$ is defined as $E_X E_{\theta}(ss')$. The element for β_1 in $N I_{\theta X}$ is the sum of squared elements in the conditional-on- θ columns of Table 4, with each square weighted by (i) the expected population count of the response pattern, from Table 2, and (ii) the posterior population probability of that θ , from the third panel in Table 3. The contribution from $\underline{x}=(0,0)$, for example, is calculated as

$$\begin{aligned} N P(\underline{x}=00) E_{\theta} [s_{\beta_1}(\theta, \underline{x}=00)^2] \\ = N P(\underline{x}=00) [s_{\beta_1}(\theta=-1, \underline{x}=00)^2 P(\theta=-1 | \underline{x}=00) \\ + s_{\beta_1}(\theta=0, \underline{x}=00)^2 P(\theta=0 | \underline{x}=00)] \end{aligned}$$

$$\begin{aligned}
& + s_{\beta_1}(\theta=+1, \underline{x}=00)^2 P(\theta=+1 | \underline{x}=00)] \\
& = 553.4 \times (.158^2 \times .483 + .294^2 \times .452 + .430^2 \times .065) \\
& = 34.94.
\end{aligned}$$

Summing such values over all four response patterns gives 154.00-- the expected information about β_1 when observing both $\underline{x}$ and θ from a total sample of 2000 comprised of 1000 from each subpopulation.

Tables 3 and 4 about here

The element for β_1 in $I_{X(Y)}$ is calculated without knowledge of examinees' θ s, and one must accordingly average $s(\theta, \underline{x})$ over θ values within $\underline{x}$ before squaring. The average uses as weights $p(\theta | \underline{x})$, the posterior probabilities when y is not known. The contribution from $\underline{x}=(0,0)$ to $N I_{X(Y)}$ is calculated as

$$\begin{aligned}
& N P(\underline{x}=00) \{E_{\theta}[s_{\beta_1}(\theta, \underline{x}=00)]\}^2 \\
& = N P(\underline{x}=00) [s_{\beta_1}(\theta=-1, \underline{x}=00) P(\theta=-1 | \underline{x}=00) \\
& \quad + s_{\beta_1}(\theta=0, \underline{x}=00) P(\theta=0 | \underline{x}=00) \\
& \quad + s_{\beta_1}(\theta=+1, \underline{x}=00) P(\theta=+1 | \underline{x}=00)]^2 \\
& = 553.4 \times (.158 \times .483 + .294 \times .452 + .430 \times .065)^2 \\
& = 31.12.
\end{aligned}$$

Summing such values over $\underline{x}$ gives 139.58. The information about β_1 lost when θ is not observed is thus $154.00 - 139.58 = 14.42$ --about a 9-percent drop.

The element in I_{XY} is obtained in a manner similar to that for $I_{X(Y)}$, except that since y is now used, calculations are carried out separately within subpopulations, then summed. The squares being summed within response patterns are now averages over θ weighted by subpopulation posteriors $p(\theta|\underline{x}, y)$. For $\underline{x}=(0,0)$, the contribution to $N I_{XY}$ is

$$\begin{aligned} & N P(\underline{x}=00|y=1) P(y=1) \{E_{\theta}[s_{\beta_1}(\theta, \underline{x}=00)|y=1]\}^2 + \\ & \quad N P(\underline{x}=00|y=2) P(y=2) \{E_{\theta}[s_{\beta_1}(\theta, \underline{x}=00)|y=2]\}^2 \\ & = 392.2 \times (.158 \times .681 + .294 \times .319 + .430 \times 0)^2 + \\ & \quad 161.2 \times (.158 \times 0 + .294 \times .775 + .430 \times .225)^2 \\ & = 32.89 . \end{aligned}$$

Summing over $\underline{x}$ gives 146.26--about a 5-percent drop from the upper limit of 154.00 in $I_{\theta X}$. Thus about half of the information about β_1 lost by not observing θ is recovered by using y .

Case 3: Collateral information present, and used in both examinee sampling and item administration

Assume the same finite mixture structure as in Cases 1 and 2 for the examinee population as a whole, and suppose again that examinees are sampled at rates π_k^* that may differ from π_k . In

contrast to Cases 1 and 2, suppose that examinees are administered test items as a function of their y values. The use of such designs is promoted in IRT, since for a given number of observed responses, they can increase the amount of information available about items and examinees. In targeted testing, for example, pupils in a lower grade may be administered an easier set of items than the overlapping set administered to pupils in a higher grade. Let λ_{jk} be the rate at which Item j is presented to examinees in Subpopulation k , and define $\pi_{jk}^{**} = \pi_k^* \lambda_{jk}$. For convenience, assume that for each j there exists at least one k such that $\pi_{jk}^{**} > 0$. Let $\underline{\beta}_{(k)}$ be the parameters of the items that are presented to subpopulation k , and note that for two subpopulations k and k' , $\underline{\beta}_{(k)}$ and $\underline{\beta}_{(k')}$ need not be disjoint.

Analysis Using $\underline{Y}$

If MML estimation is carried out using $\underline{Y}$, the likelihood function for $\underline{\beta}$ and $\underline{\alpha}$ is a product over subpopulations:

$$\begin{aligned}
 L_{3U}(\underline{\beta}, \underline{\alpha} | \underline{X}, \underline{Y}) &= \prod_{k=1}^K \prod_{i=1}^{N_k} \int f(x_i | \theta, \underline{\beta}_{(k)}) g_k(\theta | \alpha_k) d\theta \\
 &= \prod_k L(\underline{\beta}_{(k)}, \alpha_k | \underline{X}_k) .
 \end{aligned} \tag{12}$$

The results of Case 0 apply for each subpopulation in which $\pi_k^* \neq 0$ separately, so by Bradley and Gart's (1962) theorem for associated

populations, consistency holds for those $\hat{\alpha}_k$ s and for $\hat{\beta}$. The information matrix is $N I_{XY}$, where I_{XY} is calculated as in (7).

Analysis Ignoring $\underline{Y}$

When $\underline{Y}$ is ignored under Case 3, it is not the only datum that is missing; also missing are the (hypothetical) responses of examinees to the items they were not presented. The observed pattern of missingness consists therefore of y-values for all examinees, and the identifications of the items they were not presented. But because each examinee's y-value was used to determine which items he or she would be presented, the probability of this missingness pattern would have been different had the y-values been different. MAR does not hold. Of course MAR is merely sufficient, not necessary, for ignorability. Rather than demonstrating that Rubin's (1976) more complicated necessary condition for ignorability fails, we discuss the locus of the problem and present a counterexample.

An attempt to estimate $\underline{\beta}$ in Case 3 without accounting for subpopulation membership would proceed as in Case 2, first defining the mixture $g^*(\theta|\underline{\alpha}^*)$ as in (9), then maximizing a facsimile of (10):

$$L_{3I}^*(\underline{\beta}, \underline{\alpha}^* | \underline{x}) = \prod_{k=1}^K \prod_{i=1}^{N_k} \int f(\underline{x}_i | \theta, \underline{\beta}_{(k)}) g^*(\theta | \underline{\alpha}^*) d\theta \quad (13)$$

Equation (13) differs crucially from (10) in that for a given item, the distribution g^* used in marginalization over θ need not be the correct mixture for the sample of examinees administered that item. The expectation of the first derivative of the likelihood using $\underline{Y}$ (Equation 12) with respect to Item j is

$$N \sum_k \pi_{jk}^{**} \sum_{\ell} \int [s_{\beta_j}(\theta, \underline{x}_{\ell}) \sum_k \pi_{jk}^{**} p(\theta | \underline{x}_{\ell}, y_k = 1)] d\theta h(\underline{x}_{\ell} | y_k = 1) ,$$

whereas the corresponding expectation based on a likelihood ignoring $\underline{Y}$ (Equation 13) is

$$N \sum_k \pi_{jk}^{**} \sum_{\ell} \int [s_{\beta_j}(\theta, \underline{x}_{\ell}) \sum_k \pi_k^* p(\theta | \underline{x}_{\ell}, y_k = 1)] d\theta h(\underline{x}_{\ell} | y_k = 1) .$$

In general these gradients are not proportional, and need not have the same vanishing points. In particular, the expected maximizing values of L_{3I}^* for large N need not coincide with the expected maximizing values of L_{3U} , which are the true values of $\underline{\beta}$. Discrepancies remain as N increases.

Example 2: Inconsistent MML Item Parameter Estimates

Let G be an equally-weighted mixture of two components: G_1 , whose mass is concentrated at $\theta = -1$, and G_2 , whose mass is concentrated at $\theta = +1$. Let y_{i1} be 1 if Examinee i belongs to

Component 1 (i.e., has $\theta = -1$) and 0 if not; define y_{i2} analogously. Consider two test items, numbered 1 and 2, to which examinee responses follow the Rasch model with item parameters $\beta_1 = -1$ and $\beta_2 = +1$. Suppose that Item 1 is administered to a large sample of examinees from Subpopulation 1 only, and Item 2 is presented to an equally large sample from Subpopulation 2 only. Consonant with their modeled expectations, half the responses to each item are observed to be correct.

MML estimation of $\underline{\beta} = (\beta_1, \beta_2)$, using $\underline{y}$ and taking G_1 and G_2 as known, simplifies to solving the following subpopulation marginal probabilities for β_1 and β_2 . For Item 1,

$$\begin{aligned} .5 &= \int P(x_1=1|\beta_1, \theta) p(\theta|y_1=1) d\theta \\ &= P(x_1=1|\beta_1, \theta=-1) , \end{aligned}$$

and, for Item 2,

$$\begin{aligned} .5 &= \int P(x_2=1|\beta_2, \theta) p(\theta|y_2=1) d\theta \\ &= P(x_2=1|\beta_2, \theta=+1) , \end{aligned}$$

whence $\hat{\beta}_1 = -1$ and $\hat{\beta}_2 = +1$, the true parameter values.

MML estimation of $\underline{\beta}$ ignoring $\underline{y}$ but treating the true mixed population distribution G as known requires, for $j=1,2$, solving for β_j in

$$.5 = \int P(x_j=1|\beta_j, \theta) \sum_k p(\theta|y_k=1) d\theta$$

$$= [P(x_j=1|\beta_j, \theta=-1) \times .5] + [P(x_j=1|\beta_j, \theta=+1) \times .5] ,$$

whence $\hat{\beta}_1 = \hat{\beta}_2 = 0$. Thus the incorrect assumption implicit in ignoring $\underline{Y}$ under Case 3, that the examinees taking an item are a random sample of the total population, leads to inconsistent item parameter estimates.

Example 3: 1983/84 NAEP Reading Items

This section uses an example from Mislevy (1987) to illustrates points from Cases 1-3 above. The data are responses to ten items from the 1983/84 National Assessment of Educational Progress survey of reading (Beaton, 1987). The survey addressed performance levels in three subpopulations of students: Age 9/Grade 4, Age 13/Grade 8, and Age 17/Grade 11. Under the NAEP item-sampling design, the first five items were administered to examinees from just the Age 13 and Age 17 samples, while the remaining five were administered to examinees from all three samples. Case 3, as described above, obtains. The subpopulation sample sizes are 900, 1087, and 1159.

Mislevy and Bock's (1983) BILOG computer program was employed to estimate the parameters of the 3-parameter logistic (3PL) model

from the item responses. The item parameters of Item j under the 3PL are $\beta_j = (a_j, b_j, c_j)$, and the form of the model is

$$P(x_j=1|\theta, a_j, b_j, c_j) = c_j + (1-c_j)/(1+\exp[-1.7a_j(\theta-b_j)]) ,$$

where $x_j=1$ represents a correct response and $x_j=0$ an incorrect one. The b parameter indicates item difficulty; all other things being equal, an item with a higher value of b is harder than an item with a lower value.

Aside from a mild prior distribution on item parameter estimates, MML was used to estimate item parameters in two ways, ignoring and using subpopulation membership. In the first run, $g(\theta)$ was characterized as a histogram over ten prespecified points, without differentiation by subpopulation membership. This solution approximates nonparametric estimation of g . The second run used a modification of BILOG (described in Mislevy, 1987) to obtain estimates that were similar in all ways except that examinees were distinguished as to subpopulations, the distributions of which were each approximated by 10-point histograms. The unit-size and origin of the θ scale were set in both runs by standardizing the sample as a whole.

The differentiated-population solution yielded subpopulation means of $-.932$, $.0708$, and $.654$, with corresponding standard deviations of $.712$, $.727$, and $.823$. Subpopulation membership thus accounts for roughly 40-percent of the variation among examinees.

Item parameter estimates from the two runs are shown in Table 5. As discussed in the analysis of Case 3, the estimates from the second (differentiated-population) run are consistent, while those from the first (undifferentiated population) run need not be. Estimates of a and c parameters vary little in the two solutions, usually exhibiting differences less than twice the standard error of the undifferentiated solution. The same is true for b estimates of the last five items, which were administered to all examinees. More noticeable differences appear in the b estimates of the first five items, which were administered to only the two older subpopulations.

Table 5 about here

The b estimates from the differentiated solution exceed those of the undifferentiated solution by about three standard errors on the average; the undifferentiated solution makes these items seem too easy. The differentiated population effectively discounts rates of correct response to these items relative to the other five, since it is only the higher ability subpopulations to whom the last five have been administered.

Although the undifferentiated-population estimates are inconsistent, they are close enough to the correct estimates to make it meaningful to compare the relative precisions of the two sets. Table 5 also shows the ratios of squared standard errors;

they show average gains of about 10-percent for a and c parameters, and about 40-percent for b's. A few of these values are less than one, suggesting lower precision in terms of estimated variances from the differentiated population run. While expected information always increases when collateral information is employed, estimated variances can decrease for two reasons. First, only estimated information can be used in practice; this is a consistent estimate of expected information, but it has a distribution that can yield apparent decreases--especially if the expected increase is small. Second, while expected information for a parameter increases, variances are inverses of information matrices. Changes in other elements in the information matrix can make a variance larger rather than smaller, even though the corresponding element in the information matrix was larger.

Results for the Rasch Model

The preceding sections assume the propriety of examinee distributions in item parameter estimation, and discuss merely whether one should incorporate collateral information as well. This assumption might appear at odds with a primary motivating goal in the development of IRT, namely characterizing test items in ways that do not depend on the parameters of individual examinees or their distributions. In one sense, all IRT models that posit distinct parameters for items and examinees exhibit this property: the response probability for a given examinee on a

given item depends solely on the characteristics of that item and that examinee.

Estimating item parameters in the presence of examinee parameters is a thorny statistical problem, however, because attempting to accumulate information about an item by administering it to another examinee introduces a new unknown parameter. In an instance of the "infinitely many incidental parameters" problem (Neyman and Scott, 1948), joint ML estimates of the parameters of a fixed number of items are not consistent in the number of examinees (Andersen, 1973). MML eliminates examinee parameters by marginalizing over their distribution, thus providing consistent item parameter estimates in the broad variety of situations discussed above. For most IRT models, standard statistical theory offers no serious alternative to marginalization to deal with the problem.

The exception is the family of Rasch models, in which item and examinee parameters are distinct algebraically as well as conceptually. The existence of nontrivial sufficient statistics for examinee parameters in Rasch models makes it possible to estimate item parameters by conditional maximum likelihood, or CML (Andersen, 1973, 1977). In the Rasch model for dichotomous items, for example, the distribution of response patterns among those with the same total score t depends on β but not θ . The marginal probability for Case 0 can be factored as follows:

$$\begin{aligned}
P(\underline{X}|\underline{\beta},\alpha) &= \prod_i \int f(\underline{x}_i|\theta,\underline{\beta}) g(\theta|\alpha) d\theta \\
&= \left[\prod_i p(\underline{x}_i|\underline{t}_i,\underline{\beta}) \right] \left[\prod_i \int p(\underline{t}_i|\theta,\underline{\beta}) g(\theta|\alpha) d\theta \right] \\
&= P(\underline{X}|\underline{T},\underline{\beta}) P(\underline{T}|\underline{\beta},\alpha) .
\end{aligned} \tag{14}$$

Now MML estimates are obtained by viewing the marginal probability as a likelihood after $\underline{X}$ has been observed, and maximizing both factors jointly for $\underline{\beta}$ and, if required, for α as well. Resulting estimates of $\underline{\beta}$ are consistent and asymptotically normal. CML estimates are obtained by maximizing only the first factor on the right in (14). Resulting estimates are also consistent and asymptotically normal, but involve neither assuming nor estimating anything about examinee abilities or their distributions.

The connection between MML and CML has been explored in recent papers by Cressie and Holland (1983), Engelen (1987), Kelderman (1984), de Leeuw and Verhelst (1986), Lindsay (1987), and Tjur (1982). The relationship can be expressed in terms of a log-linear model for counts of response patterns, the parameters of which are the usual parameters $\underline{\beta}$ for items and parameters $\underline{\alpha}$ that are functions of the moments of the population distribution.

In the so-called semiparametric MML solution, one obtains ML estimates for $\underline{\beta}$ along with a nonparametric ML estimate of g by maximizing with respect to both $\underline{\beta}$ and $\underline{g}$, subject to constraints on $\underline{g}$ that enforce inequalities that must hold among the moments of any distribution. One obtains the CML solution for $\underline{\beta}$ by maximizing with respect to $\underline{\beta}$ and $\underline{g}$ without enforcing these constraints. If, for a given dataset, the inequalities just happen to be satisfied, CML and semiparametric MML estimates of $\underline{\beta}$ are numerically identical. Empirical observations suggest that even when they are not satisfied, the differences between the two sets of item parameter estimates are virtually indistinguishable. Since the constraints are satisfied in the limit, CML and semiparametric estimates of $\underline{\beta}$ have the same limiting distribution. In the semiparametric MML solution, the second factor in (14) is asymptotically uninformative.

Assuming a parametric form for g makes the second factor of (14) asymptotically informative, but it generally provides little information compared to that conveyed by the first factor (Bartholomew, 1988). Even the strong assumptions of normal distributions for both θ s and β s leads to estimates that often prove quite serviceable in applied work (Cohen, 1979). The only way to get substantially different MML and CML estimates of $\underline{\beta}$ under Case 0 is to make assumptions about g that are (i) capriciously strong and (ii) outrageously inconsistent with the

data. MML and CML item parameter estimation for Rasch item parameters under Case 0 arrive at the same destination, even if traveling different philosophical paths.

Analogous results for Cases 1-3 follow readily. The following notation can be used to write the marginal likelihood using collateral information that applies in all cases. Let $\underline{\beta}_{(i)}$ denote the parameters of the items administered to Examinee i , and let $p_i(\theta)$ denote the subpopulation distribution that pertains to Examinee i . Proceeding as for (14), we obtain

$$\begin{aligned} P(\underline{\beta}, \underline{\alpha} | \underline{X}, \underline{Y}) &= \left[\prod_i p(\underline{x}_i | \underline{t}_i, \underline{\beta}_{(i)}) \right] \left[\prod_i \int p(\underline{t}_i | \theta, \underline{\beta}_{(i)}) p_i(\theta | \underline{\alpha}) d\theta \right] \\ &= P(\underline{X} | \underline{T}, \underline{\beta}) P(\underline{T} | \underline{\beta}, \underline{\alpha}, \underline{Y}) . \end{aligned}$$

A number of interesting implications follow. Note first that the probability again breaks into a conditional factor--exactly the same factor that appears under Case 0--and a marginal factor, thus supporting CML estimation.

Second, collateral information appears only in the marginal factor, so CML estimation proceeds in the same manner regardless of whether collateral information is available, and, if it is, regardless of which collateral-information case obtains.

Third, the number of sets of moment equations that must be satisfied if a nonparametric MML estimate of each g_k is required

is the number of subpopulations with nonzero sampling rates. There is more potential for slight differences between CML and MML estimates of β than under Case 0, but again this source of information vanishes in the limit.

Fourth, while one can use MML estimates of β as virtually interchangeable with CML estimates in all cases, a caveat is in order if item administration was based on collateral examinee information. The results of Case 3 imply that the correct MML estimates are ones that condition on the collateral information.

Summary

When no collateral information about examinees exists, the marginal maximum likelihood (MML) estimates of the IRT parameters of a fixed number of items and the parameters of the examinee population parameters are consistent in N , the examinee sample size. Their expected information matrix is bounded from above by the information matrix that would obtain if values of the latent examinee proficiency variables were observed along with item responses.

When collateral information about examinees is available but not used in either examinee sampling or item assignment, MML item parameter estimation may proceed either ignoring or using it. If it is ignored, consistent estimates of item parameters and of the composite examinee population are still obtained. If it is used, consistent estimates of item parameters, conditional examinee

distributions, and the composite examinee distribution are obtained. The expected information about item parameters when the collateral information is used equals or exceeds the expected information when it is ignored.

When collateral information is used to sample examinees but not to assign items, MML estimates that ignore this information are consistent for item parameters but not for the composite examinee population. MML estimates that do use the collateral information are consistent for item parameters and for the examinee subpopulations that are included in the sample, but not generally for the composite examinee population. Again, expected information about parameters when collateral information is used equals or exceeds that expected when it is ignored.

When collateral information is used to sample examinees and to assign items to them, MML estimation of item parameters is not generally consistent if the collateral information is ignored. To achieve consistent MML item parameter estimates, one must take collateral variables into account.

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Table 1
Response Pattern Probabilities

$\underline{x}$	$P(\underline{x}; \theta)$		
	$\theta = -1$	$\theta = 0$	$\theta = +1$
0 0	.534	.250	.072
0 1	.197	.250	.197
1 0	.197	.250	.197
1 1	.072	.250	.534

Table 2
Expected Counts of Response Patterns
for Subpopulation Samples of 1000

$\underline{x}$	Subpopulation 1	Subpopulation 2	Total
0 0	392.2	161.2	553.4
0 1	223.3	223.3	446.6
1 0	223.3	223.3	446.6
1 1	161.2	392.2	553.4
Total	1000.0	1000.0	2000.0

Table 3

Posterior Probabilities for θ Given $\underline{x}$ Subpopulation 1: $P(\theta|\underline{x}, y=1)$

$\underline{x}$	$\theta=-1$	$\theta=0$	$\theta=+1$
0 0	.681	.319	0
0 1	.440	.560	0
1 0	.440	.560	0
1 1	.225	.775	0

Subpopulation 2: $P(\theta|\underline{x}, y=2)$

$\underline{x}$	$\theta=-1$	$\theta=0$	$\theta=+1$
0 0	0	.775	.225
0 1	0	.560	.440
1 0	0	.560	.440
1 1	0	.319	.681

Composite population: $P(\theta|\underline{x})$

$\underline{x}$	$\theta=-1$	$\theta=0$	$\theta=+1$
0 0	.483	.452	.065
0 1	.220	.559	.220
1 0	.220	.559	.220
1 1	.065	.452	.483

Table 4
Contributions to Gradient for β_1

$\underline{x}$	$s_{\beta_1}(\theta, \underline{x})$			$E_{\theta}[s_{\beta_1}(\theta, \underline{x})]$		
	$\theta = -1$	$\theta = 0$	$\theta = +1$	Subpop 1	Subpop 2	Composite
0 0	.158	.294	.430	.202	.325	.237
0 1	.158	.294	.430	.234	.354	.294
1 0	-.430	-.294	-.158	-.354	-.234	-.294
1 1	-.430	-.294	-.158	-.325	-.202	-.237

Table 5
Item Parameter Estimates and Precision Gains

Item	Run 1: Ignoring $\underline{Y}$		Run 2: Using $\underline{Y}$		$(SE_U/SE_I)^2$
	$\hat{a}$	$SE_I(\hat{a})$	$\hat{a}$	$SE_U(\hat{a})$	
1	.511	.049	.588	.055	.79
2	2.328	.245	2.585	.281	.76
3	1.419	.146	1.704	.139	1.10
4	.820	.069	.842	.059	1.37
5	1.432	.131	1.676	.145	.82
6	1.056	.082	1.234	.078	1.11
7	.338	.033	.322	.131	1.13
8	2.186	.147	2.939	.134	1.20
9	1.529	.126	1.442	.094	1.80
10	1.402	.127	1.605	.118	<u>1.16</u>
Mean					1.11
Item	$\hat{b}$		$\hat{b}$		$(SE_U/SE_I)^2$
	$\hat{b}$	$SE_I(\hat{b})$	$\hat{b}$	$SE_U(\hat{b})$	
1	-1.149	.165	-.842	.132	1.56
2	-.614	.056	-.383	.049	1.31
3	-.139	.056	.051	.038	2.56
4	.109	.059	.210	.053	1.24
5	-.756	.082	-.482	.062	1.75
6	1.033	.061	1.074	.053	1.32
7	.047	.119	.257	.129	.85
8	1.716	.075	1.671	.062	1.46
9	-.746	.069	-.820	.064	1.16
10	-.361	.055	-.365	.046	<u>1.43</u>
Mean					1.46
Item	$\hat{c}$		$\hat{c}$		$(SE_U/SE_I)^2$
	$\hat{c}$	$SE_I(\hat{c})$	$\hat{c}$	$SE_U(\hat{c})$	
1	.220	.044	.213	.044	1.00
2	.232	.041	.223	.043	.91
3	.238	.029	.240	.027	1.15
4	.143	.027	.120	.026	1.08
5	.193	.046	.205	.044	1.09
6	.180	.013	.205	.012	1.17
7	.161	.026	.203	.025	1.08
8	.126	.008	.123	.007	1.31
9	.237	.037	.208	.038	.95
10	.181	.028	.180	.027	<u>1.08</u>
Mean					1.08

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