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The set of linearized magnetohydrodynamic equations has been reduced to the reflectivity equation for the compressional magnetic perturbations in the framework of the Radoski model. It has been shown that the reflection coeficient is a function of the curvature of the magnetic field lines, the inhomogeneities of the magnetic field, and the inhomogeneities of the Alfven velocity. An interesting property of the reflectivity equation is that, near Alfven resonant magnetic force lines, this equation reduces to the curvature free Budden equation. Near Alfven resonances the curvature does not play a significant role and Budden's asymptotics in time can be applied to the wave field near the magnetic force lines where the Alfven dispersion relation holds.

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ALFVEN WAVES IN A COLD PLASMA WITH CURVED MAGNETIC FIELDS

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ABSTRACT

The set of linearized magnetohydrodynamic equations is reduced to the reflectivity equation for the compressional magnetic perturbations in the framework of the Radoski model. It is shown that the reflection coeficient is a function of the curvature of the magnetic field lines, the inhomogeneities of the magnetic field, and the inhomogeneities of the Alfven velocity. An interesting property of the reflectivity equation is that, near Alfven resonant magnetic force lines, this equation is reduced to the curvature free Budden equation. We show that near Alfven resonances the curvature does not play a significant role and Budden's assymtotics in time can be applied to the wave field near the magnetic force lines where the Alfven dispersion relation holds.

1. PERSONNEL:

The following specialists are involved in space physics research in the Geophysics Program of the Henry Krumb School of Mines:

1. Philip Carrion: Wave propagation in the Earth's magnetosphere and Alfven resonances.
2. Akira Hasegawa: Waves and instabilities in space plasma.
3. Waldo Patton: Numerical approach to solving the MHD and kinetic equations. Satellite data processing.
4. Manju Prakash: Wave-particle interactions.

Professor P. Gross who is an expert in different aspects of plasma theory and fluid dynamics serves as a consultant to the project.

2. ACCOMPLISHMENTS:

During the past year the following topics were analyzed:

1. Alfven resonances in a cold plasma magnetized by a curved magnetic field.

Main results:

It was shown that a set of linearized MHD equations can be reduced to the reflectivity equation for compressional magnetic perturbation. The reflection coefficients show that the waves are reflected or scattered by the inhomogeneities of the ambient magnetic field, by the curvature of the magnetic field, and by the inhomogeneities of the Alfven velocity. It was shown that near the Alfven resonant field line where the Alfven dispersion relation holds, the reflectivity equation is reduced to the Budden curvature free equation (Appendix 1) and that Budden solutions are valid near the Alfven resonant lines.

2. Observation and analysis of Pc 3 - Pc 4 pulsations using the geostationary satellite GOES-6.

Main results:

It was shown that the polarization of Pc 3 pulsations during October 1984 (the month of GOES-6 observations) are polarized linearly in the azimuthal direction. Whereas Pc 4 are typically elliptically polarized, our study of wave characteristics observed at the spacecraft indicates that Pc 3 pulsations can be caused by a Kelvin-Helmholtz instability occurring at the magnetopause whereas most observed Pc 4 pulsations could not be produced by this type of instability and indicates that an internal mechanism generates Pc 4 pulsations (Appendix 2).

3. Wave-particle interaction in space plasma

Main results:

We have studied the response of charged particle flux in ultra-low- frequency (UFL) electromagnetic field superimposed on the reference dipole field. We have investigated both the linear and the nonlinear behavior of particle flux by tracing numerically the motion of a test particle in phase space. The linear response has been compared with that obtained analytically from the kinetic equations. The calculated particle fluxes were compared with the observed behavior in the presence of geomagnetic pulsations. Based on these comparisons we inferred wave characteristics such as the azimuthal wave number "m", the parallel wave vector "k", and wave localization in the radial direction.

Despite many experimental efforts to infer the properties of pulsations from particle flux oscillations, there have been very few efforts to deduce them theoreticaly. Therefore our calculations have added a new dimension to the theoretical understanding of the phenomenon.

In addition, our calculations were used to study particle diffusion. We have also studied particle diffusion arising from the resonance interaction between wave and particle motion in the dipole field. Diffusion characteristics are considerably different from those of quasilinear diffusion because of chaotic particle behavior and nonlinear effects.

The following publications have been sponsored by this grant:

1. P. Carrion, "Alfven waves in a cold plasma with curved magnetic fields", Phys. Fluids (in press)
2. P. Carrion, "Compressional and toroidal resonances in the magnetosphere", (in press)
3. P. Carrion, A. Hasegawa, L. Lanzerotti, "Study of Alfven standing waves at a geostationary orbit", (in press)

Talks and expanded abstracts:

1. W. Patton, and P. Carrion, "Analysis of Geomagnetic Pulsations using GOES 6 data", EOS v 66, p 339
2. B. Jalali and P. Carrion, Coupling of ULF modes in a magnetized plasma with complex geometry" EOS, v 66, p 339
3. M. Prakash, A. Hasegawa, P. Carrion, "Particle Dynamics in Ultra Low Frequency Field", (Appendix 3)

ALFVEN WAVES IN A COLD PLASMA WITH CURVED MAGNETIC FIELDS

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ABSTRACT

The set of linearized magnetohydrodynamic equations is reduced to the reflectivity equation for the compressional magnetic perturbations. It is shown that scattering is caused by the curvature of the magnetic field lines, by the inhomogeneities of the magnetic field, and by the inhomogeneities of the Alfven velocity. An interesting property is that near Alfven resonant lines the equation is reduced to the curvature free Budden equation.

INTRODUCTION

Let us consider the set of linearized MHD equations that govern the propagation of small perturbations in a perfectly conductive magnetized fluid:

$$\rho\xi''_{tt} = -\nabla\bar{p} + \frac{1}{\mu_0}[(b\cdot\nabla)B + (B\cdot\nabla)b] \tag{1}$$

$$b'_t = -\nabla \times E \tag{2}$$

$$E + [v \times B] = 0 \tag{3}$$

$$b = -B(\nabla\cdot\xi) + (B\cdot\nabla)\xi - (\xi\cdot\nabla)B \tag{4}$$

where ρ is the density, ξ is the 3-D displacement vector, B and b are the unperturbed and the perturbed magnetic fields, respectively, v is the particle velocity, E is the electric field and $\bar{p}$ is the total pressure:

$$\bar{p} = p + \frac{b\cdot B}{\mu_0} \tag{5}$$

We consider an anbounded medium and assume that the Sommerfeld radiation condition at infinity is satisfied. The initial conditions express the causality of the electromagnetic field and the particle displacement and velocity that the field, displacement and velocity are all zero for time less then zero.

If the plasma is incompressible ($\nabla \cdot \xi = 0$) and homogeneous, it is easy to obtain from (1)-(5) that

$$\nabla^2 \overline{p} = 0 \tag{6}$$

Applying the divergence theorem to (6) yields:

$$\begin{cases} \overline{p} = \text{const} \\ Av = 0 \end{cases} \tag{7}$$

where A is the local scalar Alfven operator:

$$A = [\mu_0 \rho \frac{\partial^2}{\partial t^2} - (B \cdot \nabla)^2] \tag{8}$$

If the plasma is not homegeneous, equation (7) is no longer correct. Hasegawa and Uberoi [1] considered an inhomogeneous cold plasma in which the magnetic field lines of the unperturbed magnetic field are straight but the magnetic field is not homogeneous. They showed that in this case the MHD plasma permits both singular modes and a continuous spectrum. No dispersion relation exists. Bartson [2] showed that in the case of inhomogeneous plasma, Alfven oscillations are damped as the inverse of time.

Uberoi [3] showed that the governing equation for Alfven waves in incompressible fluids is similar to that governing the electrostatic oscillations of a cold plasma. Hasegawa and Chen [4] and Hasegawa [5] considered different aspects of Alfven-wave propagation in nonuniform plasmas. They treated the problem of damped and undamped surface modes for continuous and discontinuous density profiles. Radoski and McClay [6] considered the propagation of Alfven waves in cold plasmas magnetized by a curved magnetic field. They discussed a possible method of avoiding spatially dependent eigenfrequencies which are not acceptable; they introduced an electric field parallel to the ambient magnetic field. Radoski [7] solved the initial value problem for noncollective Alfven resonances and showed that with time the energy in the radial mode is converted into azimuthal motion. Recently the transient initial-boundary value problem for plasmas with curved magnetic fields has been treated by Carrion [8] and Allan et al [9]. They used the Radoski model and studied numerically the conversion of Alfven modes.

The knowledge of the behavior of Alfven waves appears to be imprtant not only for the laboratory plasma but for space plasma as well helping to better understand low frequency wave propagation (Lanzerotti et al [10], Chen and Hasegawa [11]; Radoski [12]).

In this paper I use the Radoski model which incorporates a cold plasma magnetized by a magnetic field whose force lines are circles. First, I reduce a set of linearized MHD equations (1)-(5) to the reflectivity equation. The reflectivity equation contains a term which is proportional to the reflection coefficient of Alfven waves in the single scattering approximation. This reflection coefficient is a function of the curvature of the ambient magnetic field and of the inhomogeneities of the density and the Alfven velocity. The reflectivity equation is transformed into the Schroedinger equation whose eigenvalues determine the coupling between two motions (in axial and radial directions). The scattering potential of the Schroedinger equation is studied and the solution at the singular points of the Schroedinger operator is analyzed.

REFLECTIVITY EQUATION AND THE REFLECTION COEFFICIENT

We assume that the unperturbed magnetic field is circular. In the cylindrical coordinate system (R, φ, Z) we can write the magnetic field as:

$$B = (0, B_{\varphi}, 0)$$

Substituting (4) into (1) and assuming a dependency of the wave field on time and space of the form:

$$e^{i(n\varphi + \lambda z + \omega t)}$$

where n is an integer and λ and ω are real numbers we obtain:

$$(\omega^2\mu_0\rho - \frac{n^2B^2}{R^2})\xi = \mu_0\nabla(\bar{F}) + F \tag{9}$$

where F is:

$$F = (B\cdot\nabla)[(\xi\cdot\nabla)B + B(\nabla\cdot\xi)] - (b\cdot\nabla)B \tag{10}$$

Equation (9) shows the coupling between the surface mode and the Alfven shear wave (toroidal mode) whose dispersion relation is:

$$\omega^2\mu_0\rho = n^2B^2/R^2$$

In a uniform plasma the toroidal mode is decoupled from the other compressional modes: the magnetosonic and the ion acoustic waves. We assume that the plasma is cold and consider the coupling between the toroidal mode and the poloidal (compressional) mode. We assume that there are only radial inhomogeneities in the density and in the magnetic field. Since the plasma is cold we neglect the pressure P in (1). Projecting equation (1) onto the radial and the axial directions, I obtain:

$$\Theta\xi_R = 1/B[\frac{\partial}{\partial R} + \alpha]b_\varphi \tag{11}$$

$$\Theta\xi_z = 1/B(i\lambda\, b_\varphi) \tag{12}$$

where

$$\alpha = \frac{\partial}{\partial R}\ln(B) \qquad (13)$$

and θ is the Alfven algebraic operator:

$$\theta = \omega^2/v_A^2 - n^2/R^2 \qquad (14)$$

$(n = 0, 1, 2 \ldots)$

where v_A is the Alfven velocity. For $\alpha = 0$ and $R \to \infty$ (curvature-free magnetic field) equations (11)-(12) become similar to those discussed by Kivelson and Southwood[13]. We see that the shear Alfven mode is coupled with the compressional (poloidal) mode. b_φ represents the compressional magnetic perturbation. Elimination ξ_R and ξ_z one finds an equation for b_φ :

$$b_\varphi'' - [\ln(\theta / RB)]' b_\varphi' + b_\varphi(\alpha/R + \alpha' - \alpha(\ln\theta)' + \theta - \lambda^2) = 0 \qquad (15)$$

where all derivatives are with respect to R. Equation (15) is known in wave theory as the "reflectivity equation" since the second term on the left hand side can be considered to be the reflection coefficient in the single scattering approximation, provided that the radial derivatives of θ and B are smooth. Let us analyze the singularities of equation (15). I exclude the point R = 0 which can be thought of as a magnetic shell of infinite cuvature. Then the obvious singularity is $\theta(R\ ) = 0$. The Taylor expansion of θ yields:

$$\theta(R) = \theta(R\ ) + \left.\frac{\partial\theta}{\partial R}\right|_{R=R_0} (R - R_0) + O(|R - R_0|^2) \qquad (16)$$

This means that

$$[\ln(\theta)]' \cong 1/(R-R_0)$$

Setting

$$x = R - R_0$$

I obtain:

$$b_\varphi'' - (1/x)b_\varphi' + b_\varphi(\beta x - \lambda^2 - \alpha/x + \alpha/R_0 + \alpha') = 0 \qquad (17)$$

where $\beta = \theta'|_{R=R_0}$

I assume first that near the Alfven resonance line, α is very small ($\alpha \to 0$). Then equation (17) can be reduced to Budden's equation:

$$b_\varphi'' - (1/x)b_\varphi' + b_\varphi(\beta x - \lambda^2) = 0 \qquad (18)$$

which is the same as for the curvature-free magnetic field. Now I can draw and interesting conclusiton that if the wave is propagationg from $+\infty$

in the radial direction, it will be reflected or scattered by the radial inhomogeneities of the magnetic field and by the ingomogeneities, the Alfven velocity profile (Alfven impedance) as well as by the curvature of the magnetic field. As the curvature increases the reflected wave has larger amplitude. However, near the Alfven resonance line the governing equation is reduced to the curvature-free Budden equation. This indicates that near the Alfven resonant shell the curvature does not play an important role and the time limit solutions for b_φ and are:

$$\begin{cases} b_\varphi \to 0 & \text{faster than } \frac{1}{t} \\ \xi_R \to 0 & \text{as } \frac{1}{t} \end{cases}$$

The Budden solutions applied to magnetospheric resonances were discussed by Kivelson and Southwood[13]. They introduced the idea of cavity resonances in the magnetosphere. An interesting property of the Budden solutions is that the magnetic shell which corresponds to the Alfven resonant condition behaves like a reflecting interface with an energy transformation occuring in the magnetic shell (Alfven resonator). The problem is that the Alfven resonant condition:

$$\frac{\omega}{v_A} = \frac{n}{R} \qquad n = 1, 2, \ldots \qquad (19)$$

implies that a number of magnetic shells will resonate. In general (19) represents a trancendental algebraic equation

$$\begin{cases} y = v_A \ (R) \\ y = \dfrac{R\omega}{n} \end{cases} \qquad (20)$$

The first equation is an arbitrary curve, whereas the second is a straight line whose slope is $\frac{\omega}{n}$. This means that the intersections of the line and the curve provides the solutions $R = R_0$ where R_0 is the positions of one of the resonating shells. Suppose now that the source is not monohromatic but is bandlimited. Its spectrum then can be presented as:

$$\sigma(\omega) = \begin{cases} \neq 0 & \omega \in [\omega_L, \omega_H] \\ 0 & \text{elsewhere} \end{cases}$$

where ω_L and ω_H are the low-cut off frequency and the high-cut off frequency respectively. Then the resonating shells will not by infinitesimal but of a finite width. Carrion[14] discussed this property for different sources.

Finally equation (15) can be transformed into the Schroedinger equation. This can be important for different aspects of Alfven wave theory and especially for the estimation of the unknown parameters of the plasma using the observed scattering or transmitted Alfven waves.

In order to do so I introduce a new variable u:

$$\sqrt{\frac{\theta}{RB}}\quad u = b_{\varphi} \tag{21}$$

Substituting (21) into (15) yields:

$$- u'' + u\,V = - \lambda^2 u \tag{22}$$

where V is the Schroedinger scattering potential:

$$V = \frac{\frac{d^2}{dR^2}\sqrt{\frac{RB}{\theta}}}{\sqrt{\frac{RB}{\theta}}} - \alpha/\dot{R} - \alpha' + \alpha(\ln\theta)' - \theta \tag{23}$$

The scattering potential is singular at the Alfven resonating shells. From the theory of Schroedinger operator it follows that strong reflection and mode conversion usually occur only at the locations of the singularities of the scattering potential.

The eigenvalues of the Schroedinger operator are λ ; they determine the coupling of the radial and the axial particle motions.

For $|\lambda| >> |V|$ (weak scattering or large coupling) a solution for b_{φ} is

$$b_{\varphi} = \sqrt{\theta/(RB)}\quad \exp(-\lambda\, R)$$

which decays exponentially with R. For small λ the coupling is small. This indicates that there is a finite eigenvalue $\lambda : (0 < \lambda < \infty)$ which provides the "best" coupling.

CONCLUSION

We have shown that the set of linearized MHD equations can be reduced to a single reflectivity equation for the compressional magnetic perturbation. These reflection coefficient in the single scattering approximation is a function of the curvature of the ambient magnetic field, and of the inhomogeneities of the Alfven velocity as well as of the density of the medium. An interesting property of this equation is that, near the Alfven resonant line, it reduces the curvature-free equation. The reflectivity equation can be transformed to the Schroedinger equation whose eigenvalues determine the coupling between the toroidal and the poloidal modes. The Schroedinger scattering potential is singular at the location of the Alfven resonating magnetic shells which means that at these positions reflections and mode conversions occur.

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APPENDIX 2

STUDY OF ALFVEN STANDING WAVES AT A GEOSTATIONARY ORBIT

Philip M. Carrion

Akira Hasegawa

Louis J. Lanzerotti

ABSTRACT

Low frequency HM waves (in the range of Pc3 - Pc4) were measured on board the geostationary satellite GOES-6 during the month of October 1984. Fourier analysis of the three components of the measured magnetic field on the day side shows that the Pc3 pulsations are mostly polarized in the azimuthal direction whereas Pc4 pulsations show elliptic polarization. The cross-correlation analysis demonstrates that the correlation between the transverse component of the wave field and the component of the magnetic field directed along the ambient field is very weak. This means that in the equatorial region where the GOES-6 is located, no compressional modes were observed in the Pc3-4 ranges on the day side. A theoretical model which predicts the polarization of these pulsations is discussed.

INTRODUCTION

Alfven waves on the day side of the magnetosphere can be excited by a variety of sources. These include external sources such as waves excited at the magnetopause by the Kelvin-Helmholtz instability and internal sources such as different magnetospheric plasma instabilities.

The main characteristics of Alfven standing waves have been described by several authors (see for example Lanzerotti and Southwood 1979, Southwood and Hughes 1983, Orr 1984). The data we analyze were observed by the GOES-6 which carries on board the Space Environment Monitor (SEM) instrument package. The SEM includes a magnetometer and an energetic particle detector. GOES-6 travels in a geosynchronous orbit (6.67 Earth radii) at roughly 100 West Longitude.

We analyzed the three components of the perturbed magnetic field on the day side in the Pc3-4 micropulsation ranges (Pc3=10-45 sec., Pc4=45-150 sec.) No compressional waves in these ranges on the day side were found. However the azimuthal and radial components of the Pc3-4 waves found were strongly correlated; they represent Alfven shear standing modes. The observed Pc3 micropulsations with the periods near 30 sec. were all azimuthally polarized. However the observed Pc4 pulsations were elliptically polarized with their radial component predominant. The rotation of polarization as a function of local time is discussed.

THEORY

Let us consider a cold plasma magnetized by an ambient magnetic field directed along the z-axis. Assuming that the effective dielectric constant, ε, as well as the magnetic field vary only in the x direction, the momentum equation combined with the frozen-in condition yields (see Hasegawa and Chen, 1976):

$$\frac{d}{dx}\left[\varepsilon \frac{B_o^2}{\varepsilon - k_y^2 B_o^2} \frac{d}{dx} W_x\right] + \varepsilon W_x = -\mu_o \sigma(\omega) B_o \delta(x-x_o) \quad (1)$$

where B_o is the magnetic flux density, W_x is the x-component of the 3-D displacement vector, $\sigma(\omega)$ is a Fourier transform of the time function (source wavelet) and x_o is the location of the source. The effective dielectric constant can be presented by:

$$\varepsilon(x) = \mu_o \rho_o (\omega^2 - \omega_o^2) \quad (2)$$

where ω_o is the Alfven resonant frequency:

$$\omega_o = k_z V_A$$

and

$$V_A = \frac{B_o}{\sqrt{\mu_o \rho_o}}$$

is the Alfven velocity and ρ_o is the mass density. Near the Alfven resonant frequency ($\omega_o = k_z V_A$) the effective dielectric constant tends to zero.

Let us consider the following case. Suppose that the source is a surface current located at x = 0 which is the position of the magnetopause. Then inside the magnetosphere near the Alfven resonant frequency equation (1) becomes:

$$\frac{d^2}{dx^2} W_x + \frac{d}{dx} \ln \frac{\varepsilon B_o^2}{\varepsilon - K_y^2 B_o^2} \frac{d}{dx} W_x = 0 \qquad (3)$$

Assuming that $|\varepsilon| << k_y^2 B_o^2$ which is a very good approximation near the Alfven resonant frequency where the equation (3) can be reduced to:

$$\left\{ \frac{d^2}{dx^2} + \gamma \frac{d}{dx} \right\} W_x = 0 \qquad (4)$$

where $$\gamma = \frac{d}{dx} \ln \varepsilon(x)$$

is the gradient of the effective dielectric constant near the resonant frequency. A general solution for W_x is:

$$W_x = - K_y C \int_0^x \frac{dx}{\varepsilon(x)} \qquad (5)$$

where C is a complex function which does not depend on x

The next step is to calculate the y-component of the displacement. Since

$$\frac{d}{dx} W_x \simeq -i\kappa_y W_y \qquad (\varepsilon \to 0) \tag{6}$$

equation (4) can be reduced to

$$\left\{ \frac{d}{dx} + \gamma \right\} W_y = 0 \tag{7}$$

Equation (7) has the solution:

$$W_y = \frac{A}{\varepsilon} \tag{8}$$

where A is a complex function which does not depend on x. Formula (8) shows that W_y has a simple pole at the resonant frequency, which implies that near the resonant frequency

$$\frac{|W_x|}{|W_y|} = \frac{|\kappa_y C|}{|A|} \varepsilon \int_0^x \frac{dx}{\varepsilon(x)} \to 0 \qquad (\varepsilon \to 0) \tag{9}$$

and the polarization of HM waves is strictly azimuthal (the y-axis represents the azimuthal direction, whereas the x-axis corresponds to the radial inward direction).

Under the assumption of a linear dependance of the dielectric constant similar results were obtained by Hasegawa et al. (1983). The integration in (5) smooths the singularity and this means that even if the effective dielectric constant varies non-linearly in the radial direction the azimuthal component of the hydromagnetic pulsations will predominate and:

$$|W_y| >> |W_x| \quad \text{near } \varepsilon = 0$$

DISCUSSION

Figure (1) shows typical hydromagnetic pulsations observed as a function of time by the GOES-6 satellite. The upper curve displays the compressional mode (the magnetic field variations directed perpendicular to the orbit), the middle curve, the radial component;and the lower curve, the azimuthal component.

Figures (2) shows the power spectra of all components of the magnetic field variation. This figure illustrates the fact that in the range of Pc3 pulsations the polarization of the pulsations is azimuthal. (The upper curve is the power spectrum of the azimuthal component (letter N). Figure 3 is the hodogram of the Pc3 pulsation presented by Figure 1. It again demonstrates the azimuthal polarization of the Pc 3 pulsation. During the entire month of October 1984 all day side Pc 3 pulsations were polarized in the azimuthal direction. Our observations show that the observed Pc3 pulsations can be generated by a narrow band surface current located at the magne- topouse due to the occurence of the Kelvin-Helmholtz instability. Fgure 4 depicts a typical Pc4 micropulsation as a function of local time. Figure 5 is the power spectrum of all components of the observed geomagnetic pulsation. The upper curve is the power spectrum of the radial component of the magnetic perturbation. This indicates that the observed micropulsation is radially polarized. Our obs vations indicate that more likely the Pc4 micropulsations can be also generated by the Kelvin-Helmholtz instability and the resonant magnetic tube is relatively far away from the point of observation. If the point of observation is removed from the resonant tube the polarization of HM waves can be elliptical or even predominantly radial. Figure 6 is a hodogram of a micropulsation (Pc4) corresponding to the morning hours. Figure 7 is an observed Pc 4 pulsation correspnding to the afternoon local hours. Our observations indicate that statistically a switch of polarization at about local noon occurs. This switch of polarization is intrinsic for the Kelvin-Helmholtz type instabilities. Similar results were reported by Lanzerotti et al.(1981).

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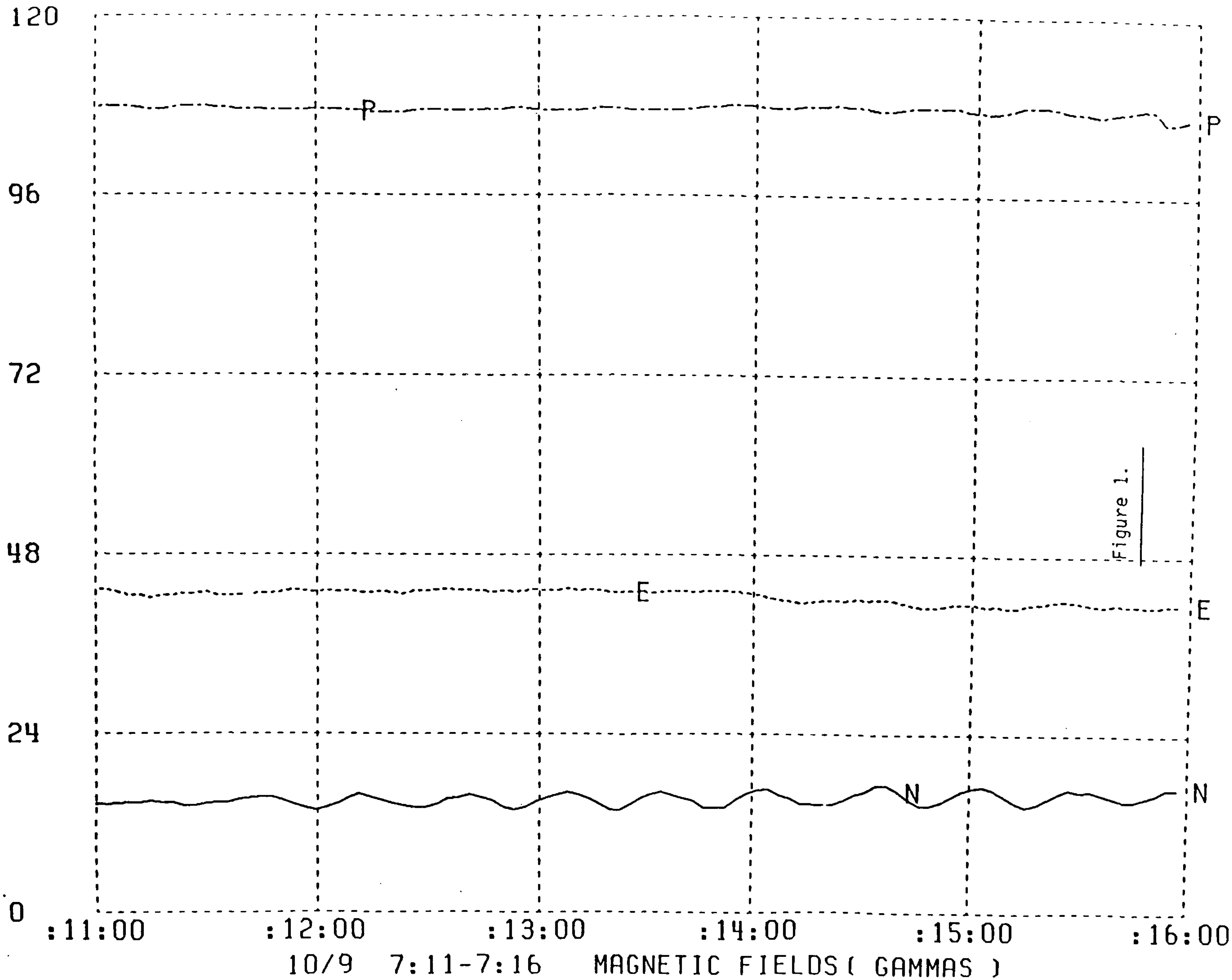
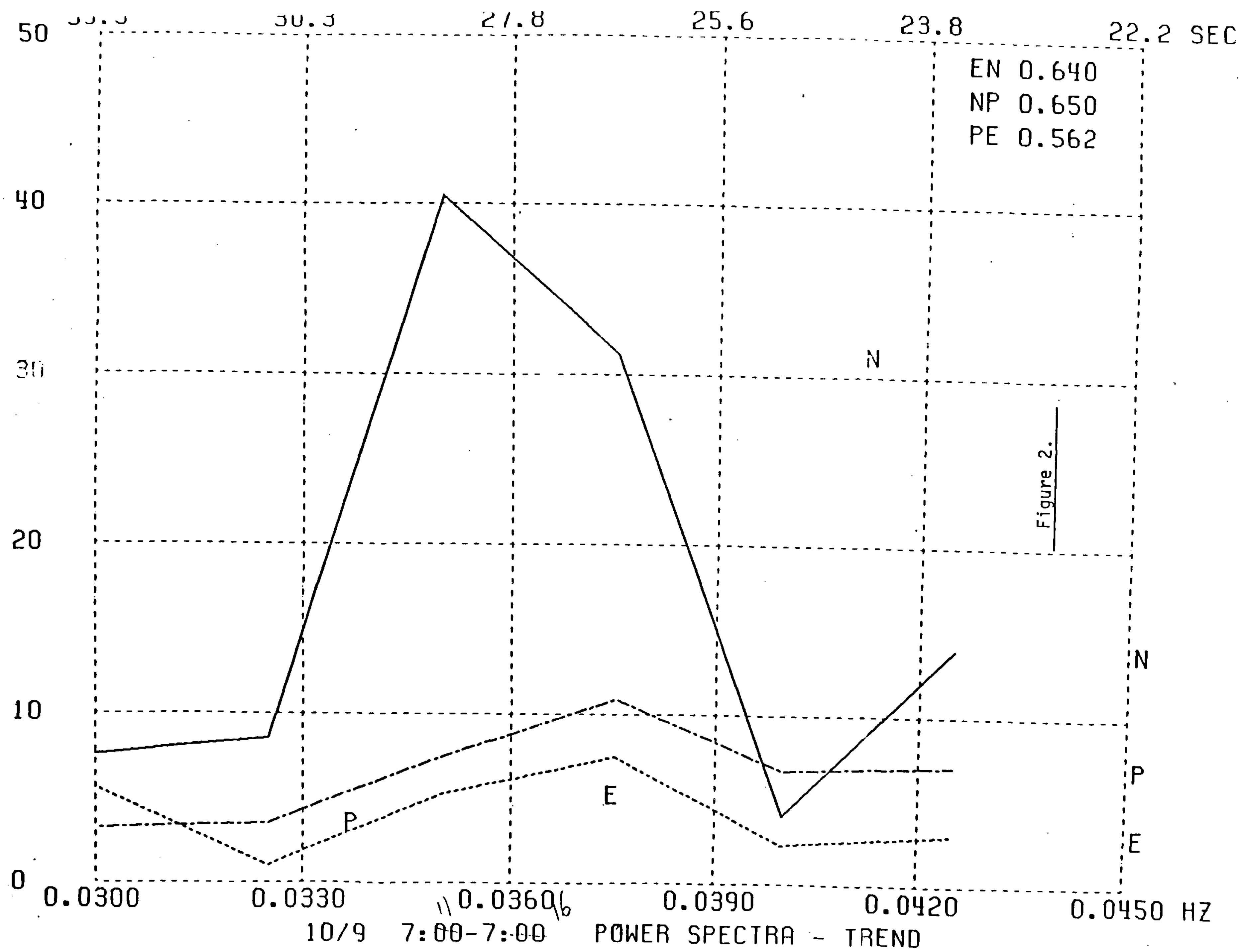
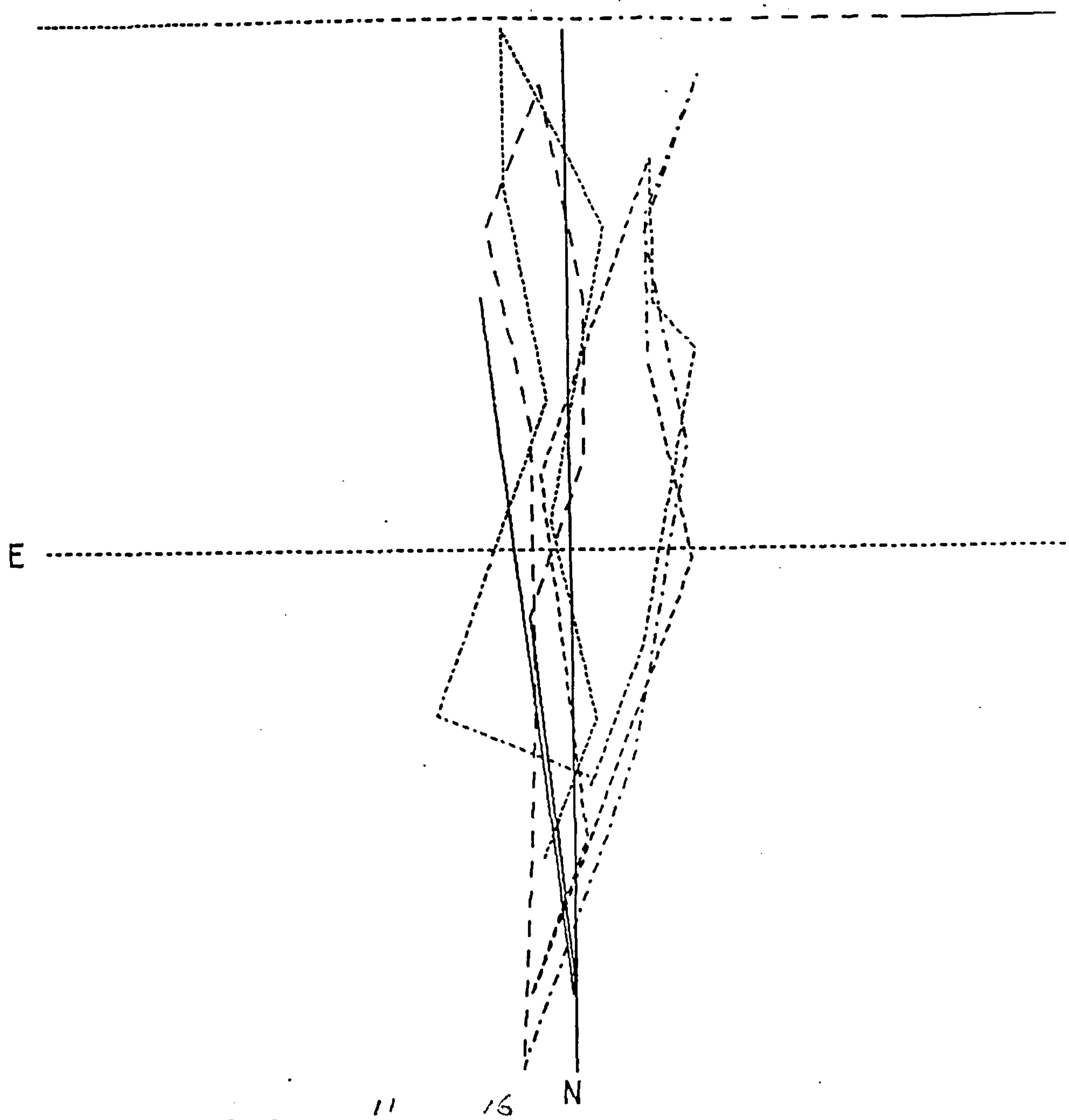

120
96
72
48
24
0
P
E
N
:11:00
:12:00
:13:00
:14:00
:15:00
:16:00
10/9 7:11-7:16 MAGNETIC FIELDS(GAMMAS)

Figure 1.



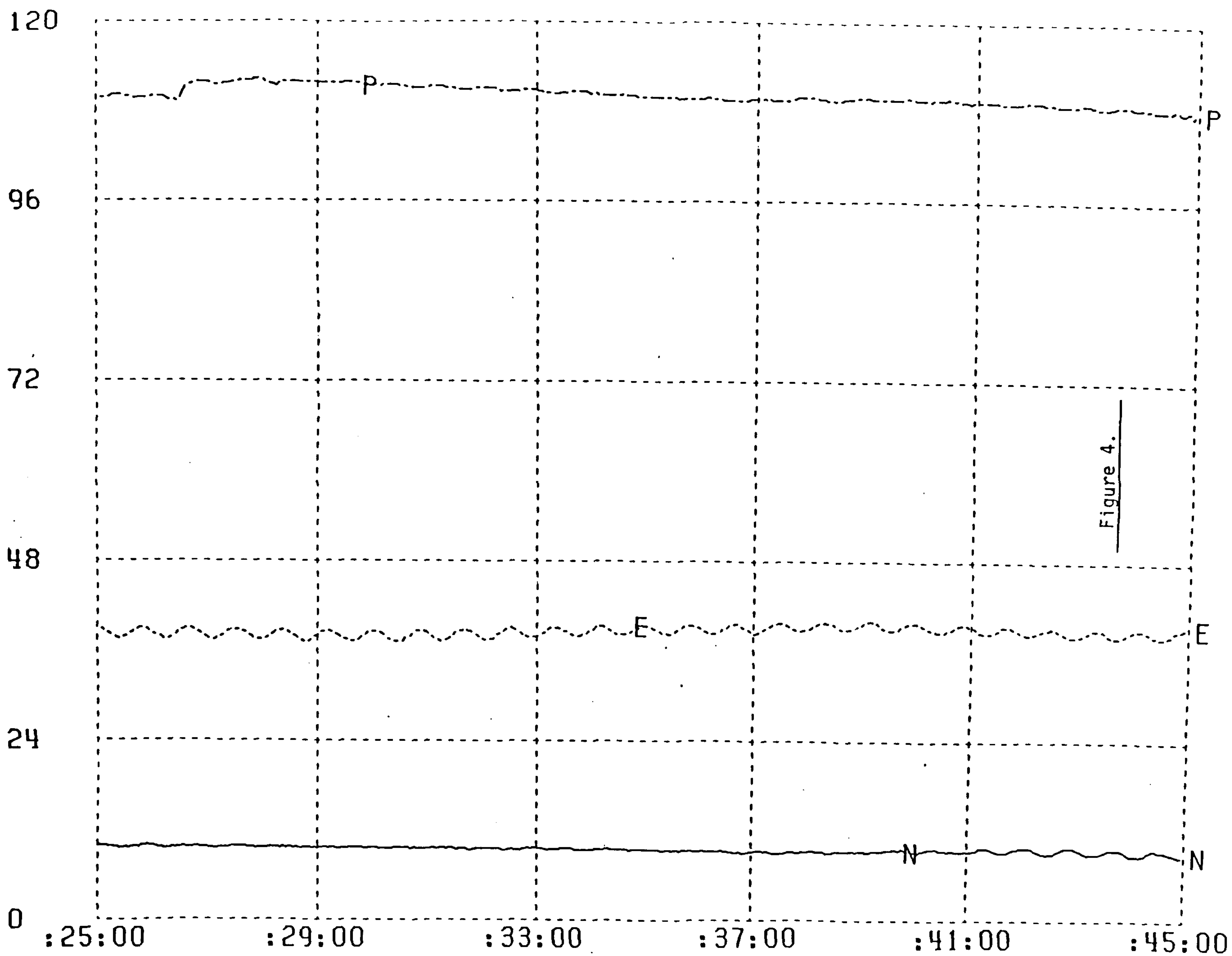
30.3
27.8
25.6
23.8
22.2 SEC
EN 0.640
NP 0.650
PE 0.562
50
40
30
20
10
0
N
P
E
0.0300
0.0330
0.0360
0.0390
0.0420
0.0450 HZ
10/9 7:00-7:00 POWER SPECTRA - TREND

Figure 2.



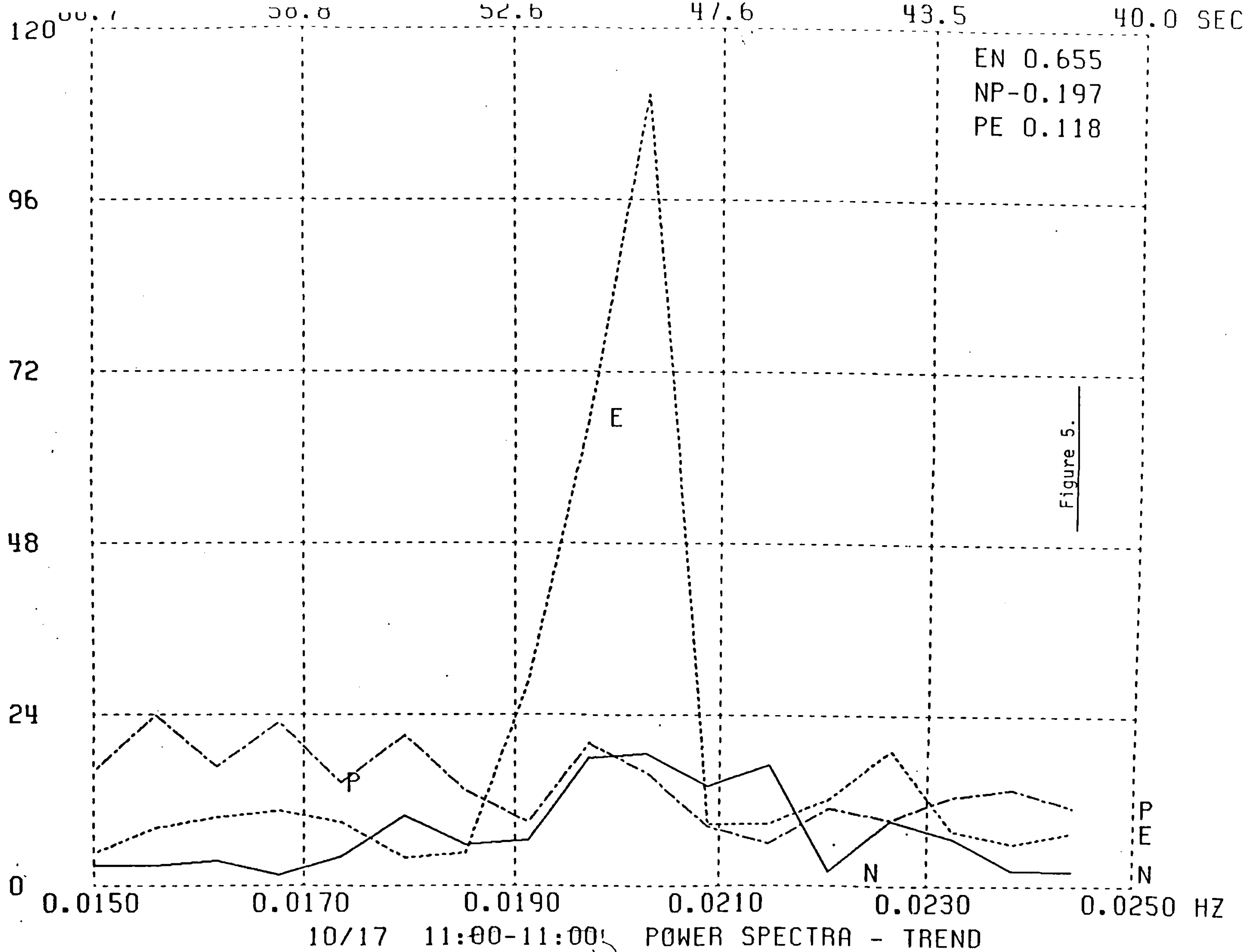
10/9 7:~~00~~ 11-7:~~00~~ 16 LISSAJOUS FIGURES

Figure 3.



10/17 11:25-11:45 MAGNETIC FIELDS (GAMMAS)

Figure 4.



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43.5
47.6
52.6
56.8
EN 0.655
NP-0.197
PE 0.118
120
96
72
48
24
0
E
P
N
P
E
N
0.0150
0.0170
0.0190
0.0210
0.0230
0.0250 HZ
10/17 11:00-11:00 POWER SPECTRA - TREND

Figure 5.

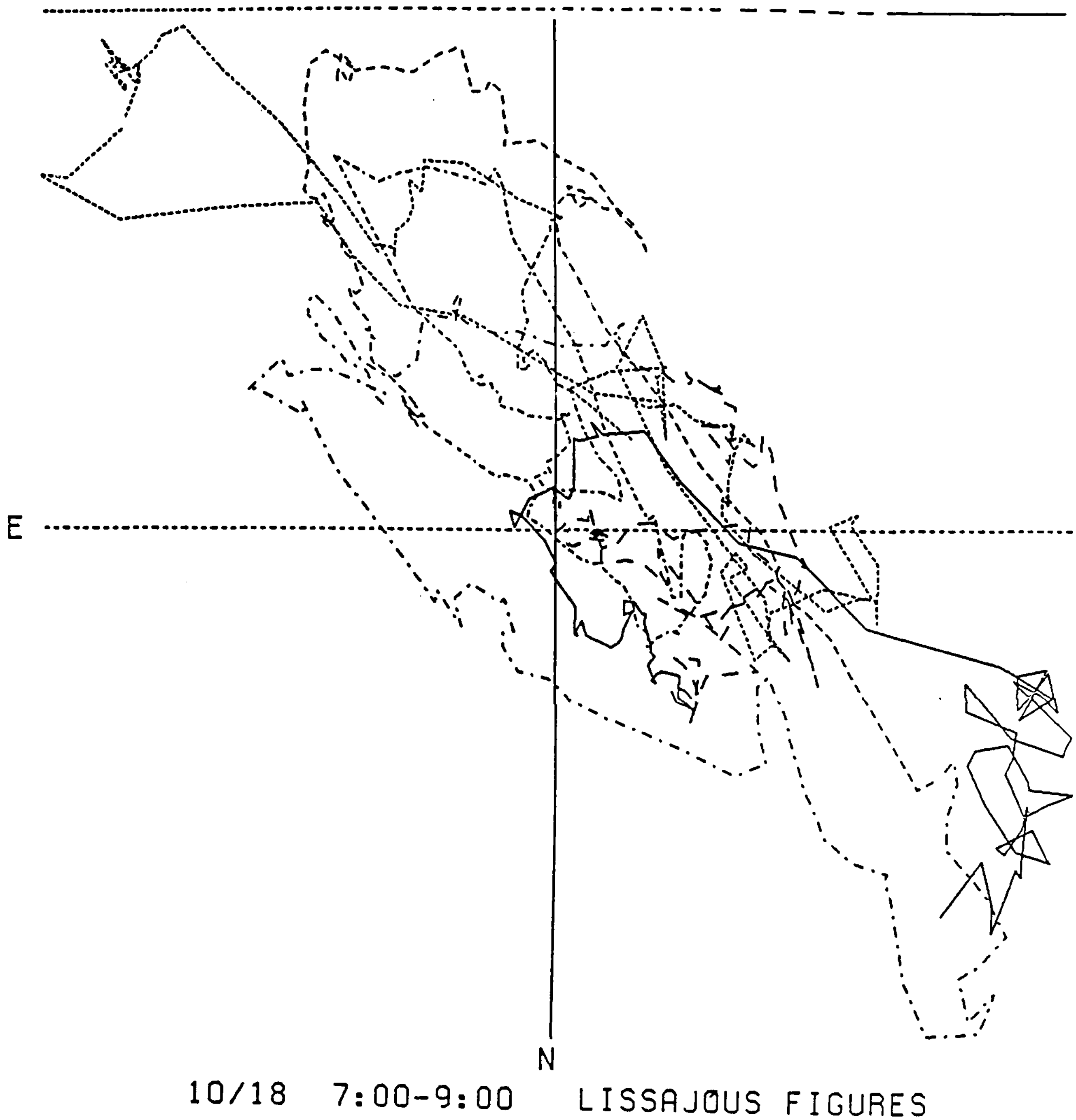

Figure 6.

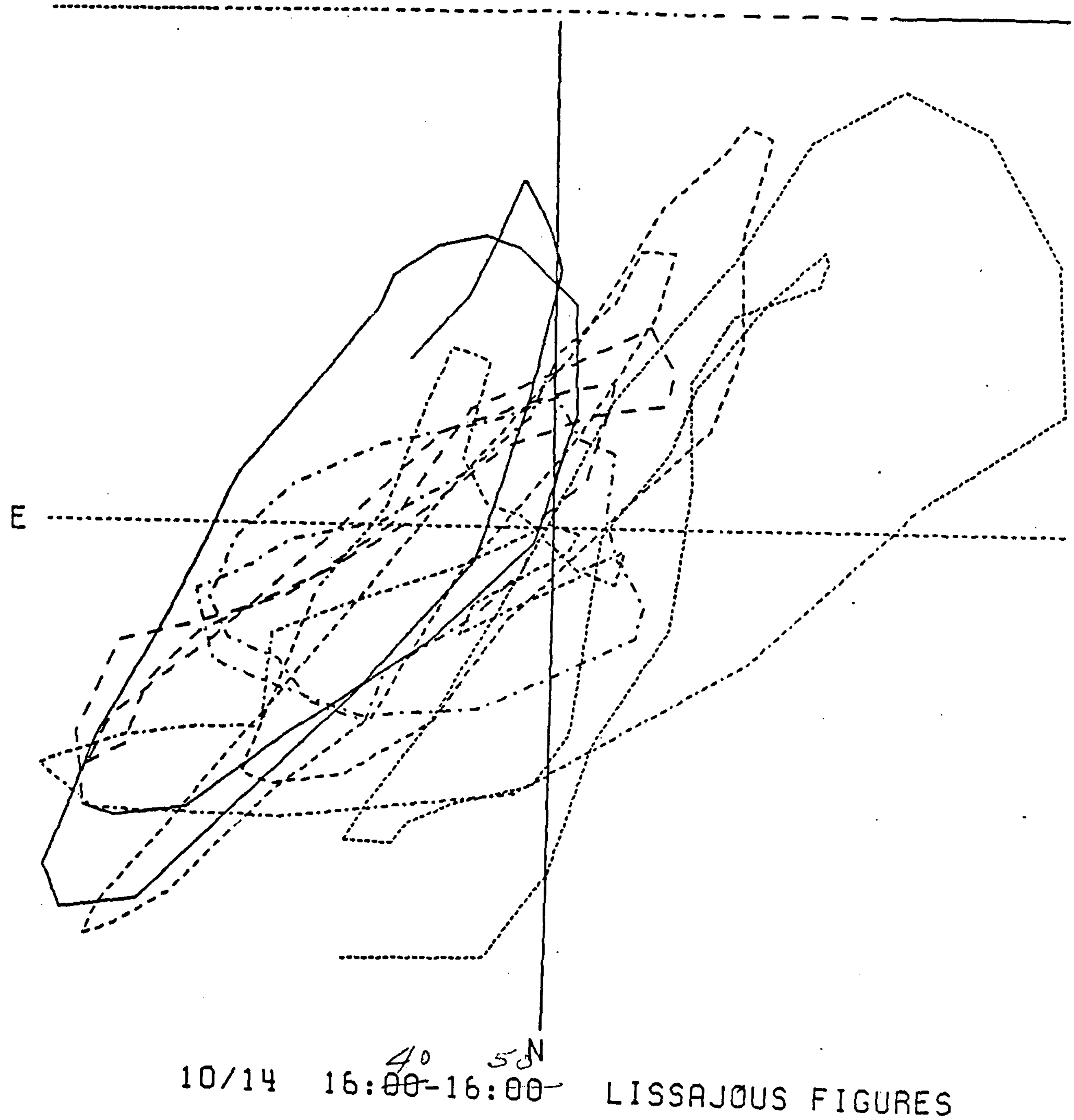

Figure 7.

APPENDIX 3

Particle Dynamics in Ultra Low Frequency Field

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We studied the behavior of particle flux in ultra low frequency (ULF) electromagnetic fields superimposed on the reference dipole field. Choosing a model hamiltonian for the perturbing field we have traced the particle motion numerically in two dimensional phase space. These calculations are performed for a wide range of initial energies of the particle typical of those in radiation belts. The particle velocity is studied at various times as a function of the amplitude and the frequency of the electromagnetic wave. The results highlight the effects due to nonlinearity and wave-particle resonance. Our calculations will lead us to study particle diffusion at Landau resonance.

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2. 005491587
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