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Generation of a Crowned Pinion Tooth Surface by a Plane

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SUMMARY

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The topology of a crowned spur pinion tooth surface that reduces the level of transmission errors due to misalignment is described. The geometry of the modified pinion tooth surface and of the regular involute gear tooth surface is discussed. The tooth contact analysis between the meshing surfaces is also described. Generating a modified pinion tooth surface by a plane whose motion is controlled by a 5-degree-of-freedom system is investigated. The included numerical results indicate that the transmission error remains low as the gears are misaligned. TRANSMISSIONS (mechanical), wear resistance. (JFS)E

INTRODUCTION

Spur involute gears are very sensitive to gear misalignment. Misalignment causes a shift of the bearing contact to the edge of the gear tooth surfaces and transmission errors that increase gear noise. Other investigators have improved the bearing contact of misaligned spur gears by crowning the pinion tooth surface. Reference 1 proposes various methods of crowning that can be achieved during gear generation. Reference 2 has used crowning to make (1) longitudinal corrections (fig. 1(a)), (2) a modified involute tooth profile uniform across the face width (fig. 1(b)), (3) a combination of longitudinal correction and uniform modified profile (fig. 1(c)), and (4) a topological modification (fig. 1(d)) that can provide any deviation of the crowned tooth surface from a regular involute surface.

The main goal of the other investigators was to crown the pinion tooth surface to improve the bearing contact of the misaligned gears. However, no attempt was made to control the transmission errors. The transmission errors of misaligned spur gears are a main source of noise. The problem is how to generate crowned spur gears with reduced transmission errors caused by gear misalignment. New methods are needed for generating gear tooth surfaces that can provide the required modification for the pinion tooth surface. It is sufficient to modify the pinion tooth surface only. The gear will be provided with a regular involute surface.

CROWNED PINION CONCEPTS

There are many methods of generating crowned pinion tooth surfaces. The method described in this study is based on using a five-axis numerically controlled machine for finishing a pinion tooth surface. The pinion tooth surface

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can be generated by a plane that performs the prescribed motions with respect to the pinion.

The determination of the required modifications for the pinion tooth surface is based on several considerations. Misaligned spur gears with a crowned pinion can provide two types of transmission errors $\Delta\phi_2(\phi_1)$ as shown in figures 2(a) and (b) (see more details in the next section). The transmission errors are determined by

$$\Delta\phi_2 = \phi_2(\phi_1) - \frac{N_1}{N_2} \phi_1 \quad (1)$$

where N_1 and N_2 are the numbers of gear teeth; ϕ_1 and ϕ_2 the angles of gear rotation; $\phi_2(\phi_1)$ the function that relates the angles of rotation of gears if the pinion is crowned and the gears are misaligned; $\phi_2^0 = (N_1/N_2)\phi_1$ the theoretical relation between the angles of rotation of the gears in the ideal case where the gears are not crowned nor misaligned and transmission errors do not exist. Transmission errors of type 1 (fig. 2(a)) are not acceptable because the tooth meshing is accompanied with an interruption of meshing or interference of tooth surfaces. The transmission errors of type 2 are preferred if the level of transmission errors does not exceed the prescribed limit.

On first glance, crowning should be directed at providing an exact involute shape in the middle cross section (fig. 3). In reality, this type of crowning is not acceptable because the misaligned gears will transform rotation with transmission errors of type 1 (fig. 2(a)). Because of this problem it is necessary to synthesize a specific crowned pinion tooth surface. Such a pinion in mesh with a gear having a regular involute surface is able to provide transformation of rotation with a parabolic type of transmission error function. This type of function of errors is synthesized for ideal gears that do not have any misalignment. Thus, the tendency to provide parabolic transmission errors can be extended for the misaligned gears and the discontinuance of meshing can be avoided. It is necessary to emphasize that the method of synthesis generates a shape in the middle cross section that deviates from the involute curve as shown in figure 3. The longitudinal deviation from a straight line is not related to the transmission errors but rather to the desired dimensions of the instantaneous contact ellipse for the mating tooth surface. The pinion tooth surface can be generated by a plane chosen as the tool surface. The motions of the plane with respect to the pinion must be controlled by a computer.

Tooth contact analysis (TCA) programs are utilized (1) to evaluate the bearing contact and the transmission errors for the misaligned gears and (2) to investigate the influence of errors of assembly. The programs developed are based on the following:

(1) The contacting gear tooth surfaces are represented in a fixed coordinate system S_f that is rigidly connected to the gear housing (fig. 4).

(2) The continuous tangency of gear tooth surfaces is provided if the position vectors and surface unit normals for the contacting surfaces coincide at the contact point at any instant. It is then possible to determine the path of contact on the gear tooth surfaces and the relations between the angles of rotation of the output and input gears. If the function $\phi_2(\phi_1)$ is known, the

$$\Delta\phi_2(\phi_1) = -d\left(\frac{N_1}{\pi}\right)^2 \phi_1^2 \quad (6)$$

where d is the level of transmission errors (fig. 2(b)). The following may be obtained since $\Delta\phi_2 = 0$ at $\phi_1 = 0$, $\Delta\phi_2 = -d$ at $\phi_1 = \pm\pi/N_1$ and equation (6) can be substituted into equation (4):

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} - d\left(\frac{N_1}{\pi}\right)^2 \phi_1^2 \quad (7)$$

Equation (7) will be used for the synthesis of the pinion tooth shape since d is given.

PINION TOOTH SURFACE

The determination of the pinion tooth surface is performed in two stages. First the pinion tooth shape in the middle cross section is determined, and then the entire tooth surface is determined.

Derivation of the Pinion Tooth Middle Cross Section

Consider that the gear and pinion shape are in mesh in the middle cross section. The gears must transform rotation with the prescribed function $\phi_2(\phi_1)$ represented by equation (7). The gear tooth shape is given as a regular involute curve. Three coordinate systems to be used are shown in figure 5. Coordinate systems S_2 , S_1 , and S_f are rigidly connected to the gear, the pinion, and the gear housing, respectively. The gear shape is represented in S_2 by the vector function $\underline{n}_2(\phi_G)$ where ϕ_G is the curve parameter (see the appendix). The gear shape unit normal is also represented in coordinate system S_2 by the vector function $\underline{n}_2(\phi_G)$. The procedure (ref. 3) for determining the pinion shape is now described.

Step 1: Determining the equation of meshing. - The equation of meshing is represented by

$$\underline{n}_2 \cdot \underline{v}_2^{(21)} = f(\phi_G, \phi_2) = 0 \quad (8)$$

where

$$\underline{v}_2^{(21)} = (\omega_2^{(2)} - \omega_2^{(1)}) \times \underline{r}_2 - \overline{O_2 O_1} \times \omega_2^{(1)} \quad (9)$$

is the sliding velocity. The subscript 2 means that the vectors are represented in coordinate system S_2 . Vectors $\omega_2^{(1)}$ and $\omega_2^{(2)}$ are the angular velocities of gears 1 and 2.

Step 2: Determining the locus of the gear shapes in coordinate system S_1 . - The locus of the gear shapes is represented by the matrix equation

$$[r_1] = [M_{12}] [r_2] \quad (10)$$

where $[M_{12}]$ is the matrix that describes the coordinate transformation from S_2 to S_1 . The elements of this matrix are expressed in terms of the angle of rotation of the gear ϕ_2 (angles ϕ_1 and ϕ_2 are related by eq. (4)). Equation (10) yields the vector equation

$$r_1 = r_1(\phi_G, \phi_2) \quad (11)$$

Step 3: Determining the pinion shape. - The pinion shape is determined by equations (8) and (10) that must be considered simultaneously. It should be emphasized that the pinion tooth shape deviates from a regular involute curve.

Derivation of Pinion Tooth Surface

The pinion and gear shapes are in contact in the middle cross section of the tooth surfaces if the gears are aligned. The gear tooth surface is a cylindrical involute surface, and the pinion tooth surface must be modified in the longitudinal direction from such a surface. There are various options for modifying the pinion tooth surface in that direction. The pinion tooth surface is chosen as a surface of revolution that is generated by rotating the pinion shape about a fixed axis. The modification of the pinion tooth surface is based on the following procedure:

Step 1. - The shape of the pinion tooth is represented in coordinate system S_1 by the vector equation

$$r_1(\phi_G, \phi_2), f(\phi_G, \phi_2) = 0 \quad (12)$$

To transfer the pinion tooth shape to an auxiliary coordinate system S_a (fig. 6(a)), use the matrix equation

$$[r_a] = [M_{a1}] [r_1] \quad (13)$$

Step 2. - Considering that the pinion tooth shape is rigidly connected to the coordinate system S_a , rotate S_a about the Y_b -axis of the second auxiliary coordinate system S_b (fig. 6(b)). This system S_b is rigidly connected to S_1 . The pinion tooth surface is generated by rotating the pinion tooth shape about Y_b . The coordinate transformation from S_a to S_b is represented by the matrix equation

$$[r_b] = [M_{ba}] [r_a] \quad (14)$$

Step 3. - To represent the pinion tooth surface in coordinate system S_1 , use the coordinate transformation from S_b to S_1 (fig. 6(c)). The final expressions for the pinion tooth surface are

$$\left. \begin{aligned} [r_1(\phi_G, \phi_2, \theta)] &= [M_{ba}] [M_{a1}] [r_1(\phi_G, \phi_2)] \\ f(\phi_G, \phi_2) &= 0 \end{aligned} \right\} \quad (15)$$

The pinion tooth surface is represented in three-parameter form in coordinate system S_1 ; however, only two of the parameters are independent since ϕ_G and ϕ_2 are related by the equation of meshing. Figure 7 shows the pinion tooth

surface with axis Y_a as the symmetric axis. Parameter R (fig. 6) affects one of the principal curvatures of the pinion tooth surface and the dimensions of the instantaneous contact ellipse.

TOOTH CONTACT ANALYSIS (TCA) PROGRAM

The TCA program is directed at determining the transmission errors caused by gear misalignment and the bearing contact. The gear and pinion tooth surfaces are in point contact. The path of contact and the transmission errors are determined from the equations of tangency of the contacting surfaces that are represented as follows:

$$r_f^{(1)}(\phi_G^*, \phi_2^*, \theta, \phi_1) = r_f^{(2)}(\phi_G, U_G, \phi_2) \quad (16)$$

$$n_f^{(1)}(\phi_G^*, \phi_2^*, \theta, \phi_1) = n_f^{(2)}(\phi_G, \phi_2) \quad (17)$$

$$f(\phi_G^*, \phi_2^*) = 0 \quad (18)$$

The subscript f indicates that the vectors of equations (16) and (17) are represented in the fixed coordinated system. The tangency of surfaces is illustrated in figure 4. Parameters ϕ_G and U_G are the surface coordinates for gear 2; parameters ϕ_G, ϕ_2 and θ with equation (18) represent the surface coordinates for pinion; ϕ_1 and ϕ_2 are the rotation angles of the pinion and the gear. It is necessary to distinguish parameters ϕ_G^* and ϕ_2^* that represent a point on the pinion shape from parameter ϕ_G that represents a point on the gear shape and parameter ϕ_2 that represents the angle of rotation of the gear being in mesh with the pinion. For the particular case when pinion and gear are aligned, $\phi_G^* = \phi_G$ and $\phi_2^* = \phi_2$. Equations (16) and (18) represent the contacting tooth surfaces that have a common point; equations (17) and (18) represent the tooth surfaces that have a common normal and are in tangency at the common point. Equations (16) to (18) represent a system of six independent scalar equations in seven unknowns, since $|n_f^{(1)}| = |n_f^{(2)}| = 1$. Following the Theorem of Implicit Function Existence (ref. 4), we are able to solve this system to get functions $\phi_2^*(\phi_1)$, $\phi_G^*(\phi_1)$, $\theta(\phi_1)$, $\phi_G(\phi_1)$, and $\phi_2(\phi_1)$. Function $\phi_2(\phi_1)$ relates the angles of rotation of misaligned gears. The transmission errors that are caused by the misalignment can be determined from equation (1).

To determine the contact path on the tooth surface, use the following expressions:

For the pinion:

$$r_1(\phi_G^*, \phi_2^*, \theta), f(\phi_G^*, \phi_2^*) = 0, \phi_2^*(\phi_1), \theta(\phi_1) \quad (19)$$

For the gear:

$$r_2(\phi_G, U_G), \phi_G(\phi_1), U_G(\phi_1) \quad (20)$$

The location of the contact points on the path of contact is related to the input parameter ϕ_1 , which is the angle of rotation of the pinion. The bearing contact is determined as the set of the contact ellipses on the tooth surface. The dimensions of the instantaneous contact ellipse depend on the principal curvatures of the contacting surfaces and the angle that is formed between the principal directions on both surfaces. Figure 8 illustrates the contact ellipses on the gear involute tooth surface.

A numerical example is now given with the following conditions: pinion and gear tooth numbers, $N_1 = 20$, $N_2 = 40$; diametral pitch, $P = 10 \text{ in.}^{-1}$; pressure angle, 20° ; predesigned function of kinematic errors (fig. 2(b)) is parabolic with $d = 2 \text{ arc sec}$; radius of rotation, $R_1 = 350 \text{ in.}$ The developed TCA program has been applied to determine kinematic errors for the following cases of misalignment:

Case 1: The change of the center distance is $\Delta c/c = 0.01$, the gear axes are crossed, and the twist angle is 5 arc min. The TCA results indicated that the kinematic error caused by gear misalignment is still parabolic and the maximum value of the function is 0.8 arc sec.

Case 2: The change of the center distance is $\Delta c/c = 0.01$, and the gear axes intersect and form an angle of 5 arc min. The function of kinematic errors is parabolic and the maximum value of the function is 2.2 arc sec.

GENERATION OF PINION SURFACE BY A PLANE

The pinion tooth surface with a prescribed topology can be generated by an automatic system having 5 degrees of freedom with a planar tool (research by Litvin and Shaheen to be published in the proceedings of USA/Japan Symposium on Flexible Automation held July 18-20, 1988, in Minneapolis, Minnesota). Consider that the pinion tooth surface is represented in coordinate system S_0 that is rigidly connected to the base link 0 of the automatic system (fig. 9). The automatic system, which consists of five movable links interconnected by three prismatic joints and two revolute joints, can provide three translational and two rotational motions. Link 5 is provided with the tool that can be a grinding wheel or a cutter whose generating surface is a plane. Coordinate system S_c is rigidly connected to the tool. The rotational motion of the tool about the Z_c -axis provides the desired velocity of cutting but is not related to the generating process.

The pinion surface Σ_0 and the tool surface Σ_c are in point contact at every instant. The control of motion of the tool with respect to the pinion is based on the following equations of tangency of surfaces Σ_c and Σ_0 :

$$r_0^{(0)}(U_0, \theta_0) = r_c^{(c)}(U_c, \theta_c, s_1, s_2, s_3, \phi_1, \phi_2) \quad (21)$$

$$n_0^{(0)}(U_0, \theta_0) = n_c^{(c)}(U_c, \theta_c, \phi_1, \phi_2) \quad (22)$$

where (U_o, θ_o) are the pinion tooth surface coordinates; (U_c, ϕ_c) the tool surface coordinates; and $s_1, s_2, s_3, \phi_1, \phi_2$ the parameters of the motion of the automatic system. Subscript o indicates that the vectors of equations (21) and (22) are represented in coordinate system S_o . The orientation of the tool surface normal does not depend on the surface coordinates for a plane; it depends only on one surface coordinate for a surface of revolution. Consider that at any instantaneous position the location of the point of contact on surfaces Σ_o and Σ_c is known. Then, the parameters of motion of the automatic system (s_1, s_2, s_3, ϕ_1 and ϕ_2) can be determined from equations (21) and (22). These equations provide five independent scalar equations since

$$[\tilde{n}_o^{(o)}] = [\tilde{n}_o^{(c)}] = 1$$

CONCLUSIONS

The topology of a crowned pinion tooth surface has been described. A method for generating the pinion surface has been derived. Also, the analytical tools have been developed that can simulate the meshing of the gears, determine the location and size of tooth contact as a function pinion rotation, and determine the level of transmission (kinematic) errors.

The overall conclusions that can be drawn from this study are the following:

1. Parabolic transmission errors can be attained for aligned and misaligned gears by the described pinion crowning method.
2. The maximum transmission error can be controlled based on the extent of pinion surface deviation.
3. Pinion crowning using a 5-degree-of-freedom automatic system with a planar tool that produces the prescribed parabolic function has been determined.

APPENDIX - PINION AND GEAR SURFACE EQUATIONS AND EQUATION OF MESHING BETWEEN THE GEARS

The regular spur gear shape in its middle cross section is an involute curve. The shape and its normal can be represented as

$$\underline{r}_2 = \begin{bmatrix} r_G [\sin \phi_G - \phi_G \cos \psi_c \cos (\phi_G - \psi_c)] \\ r_G [\cos \phi_G + \phi_G \cos \psi_c \sin (\phi_G - \psi_c)] \\ 0 \end{bmatrix} \quad (A1)$$

$$\underline{n}_2 = \begin{bmatrix} -\cos (\phi_G - \psi_c) \\ \sin (\phi_G - \psi_c) \\ 0 \end{bmatrix} \quad (A2)$$

where ϕ_G is the curve parameter and ψ_c the pressure angle.

Following the procedure described in the Pinion Tooth Surface section (p. 4), we can get the equation of meshing corresponding to equation (8) as

$$\phi_G = \psi_c + \frac{N_1}{N_2} \phi_1 - d \left(\frac{N_1}{\pi} \right)^2 \phi_1^2 + \arccos \left\{ \left[1 + \frac{-2d \left(\frac{N_1}{\pi} \right)^2 \phi_1}{\left(\frac{N_1}{N_2} + 1 \right)} \right] \cos \psi_c \right\} = 0 \quad (A3)$$

where ϕ_1 and ϕ_2 are related as shown in figure 2, and N_1 and N_2 are the teeth numbers of the pinion and the gear.

The shape of the middle cross section of the pinion can be determined by equations (8) and (11), and the explicit form can be written as

$$[r_1] = \begin{bmatrix} -c \sin \phi_1 + r_G \sin (\phi_1 - \psi_c + \Delta\phi_2') \\ + r_G \left(\frac{N_1}{N_2} \phi_1 + \Delta\phi_2 + \psi_c - \Delta\phi_2' \right) \cos \psi_c \cos (\phi_1 + \Delta\phi_2') \\ c \cos \phi_1 - r_G \cos (\phi_1 - \psi_c + \Delta\phi_2') \\ + r_G \left(\frac{N_1}{N_2} \phi_1 + \Delta\phi_2 + \psi_c - \Delta\phi_2' \right) \cos \psi_c \sin (\phi_1 + \Delta\phi_2') \\ 0 \end{bmatrix}$$

where $c = r_p + r_G = N_1/2p + N_2/2p$, p is the diametral pitch, $\Delta\phi_2(\phi_1)$ is represented by equation (6), and

$$\Delta\phi_2' = \frac{d}{d\phi_1} \Delta\phi_2(\phi_1)$$

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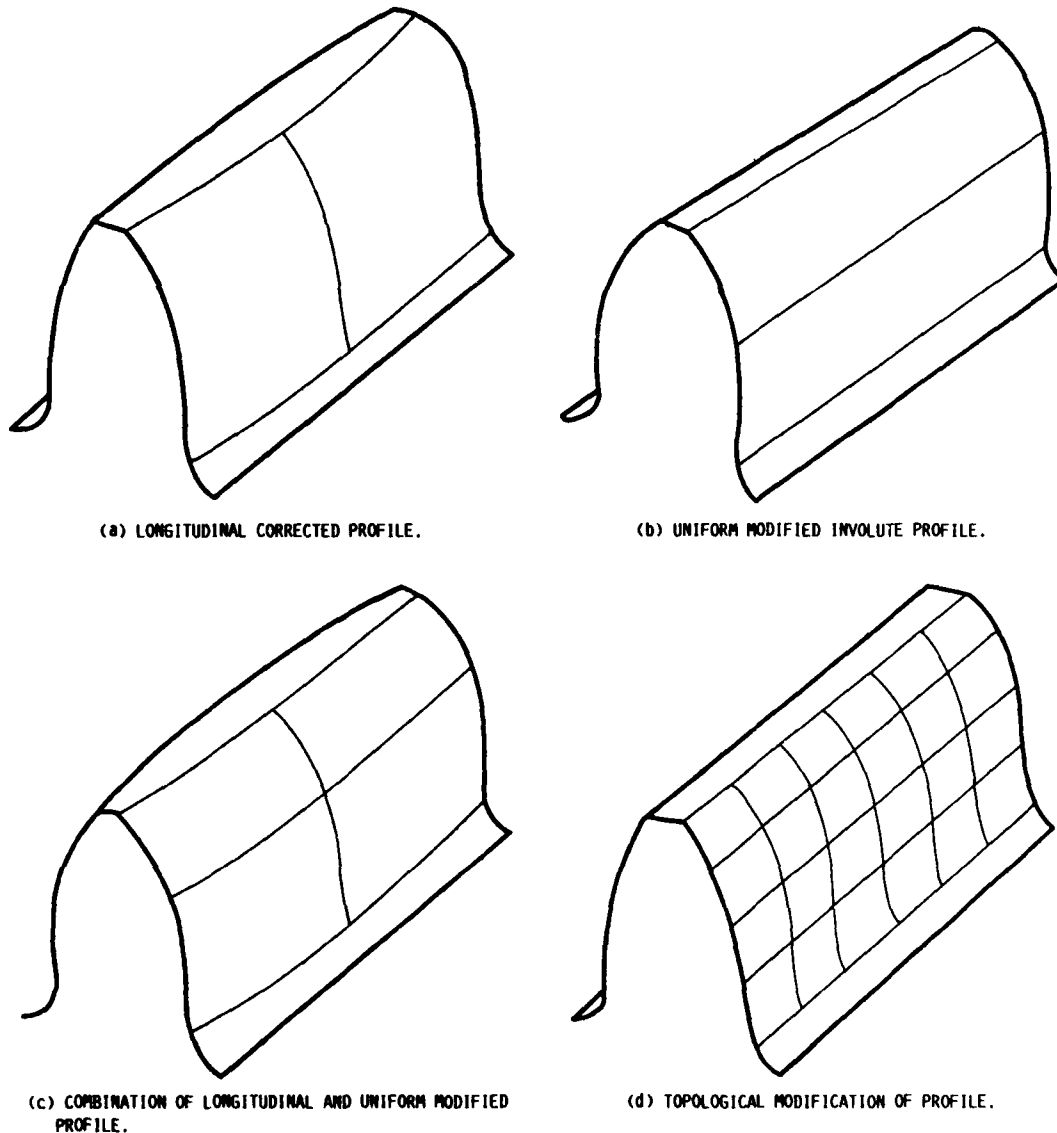


FIGURE 1. - SPUR GEARS WITH MODIFIED TOOTH SURFACES.

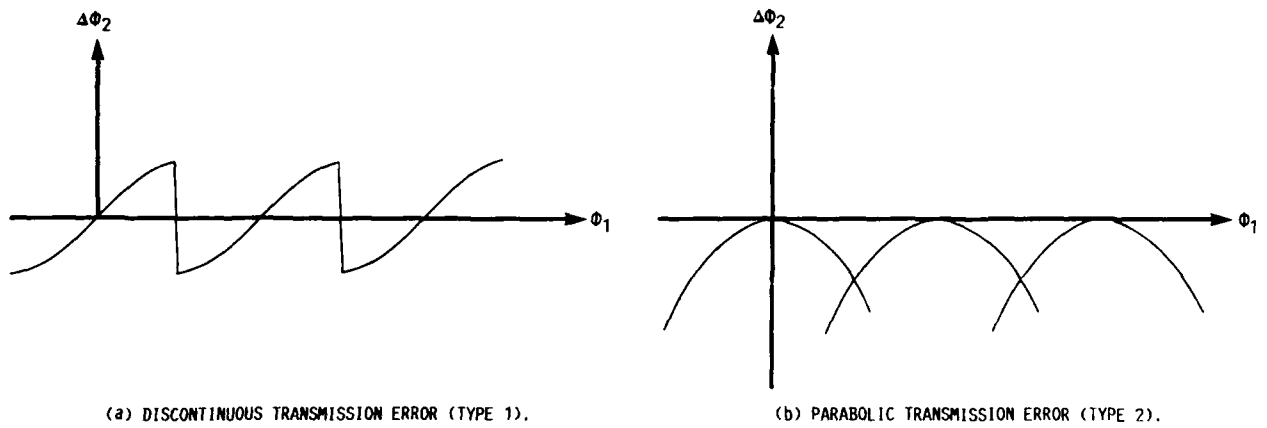


FIGURE 2. - TRANSMISSION ERRORS AS FUNCTION OF PINION ROTATION. PINION ROTATION, ϕ_1 ; TRANSMISSION ERROR, $\Delta\phi_2$.

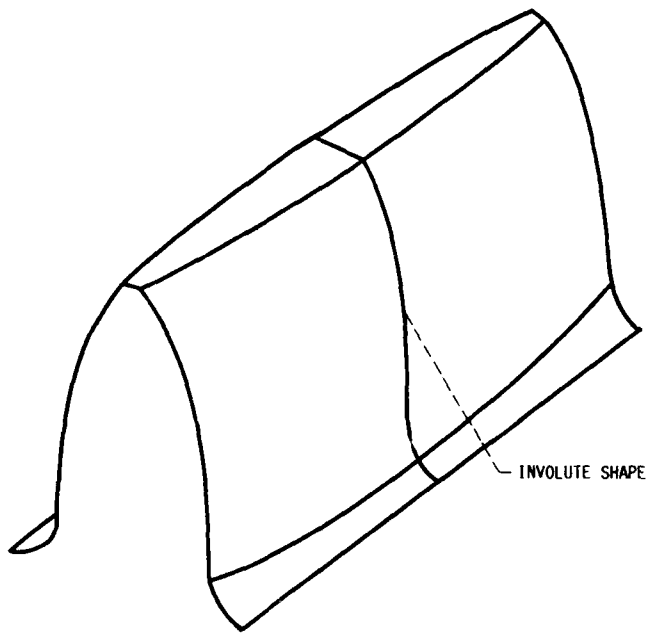


FIGURE 3. - CROWNED SPUR GEAR USING INVOLUTE PROFILE IN LONGITUDINAL DIRECTIONS.

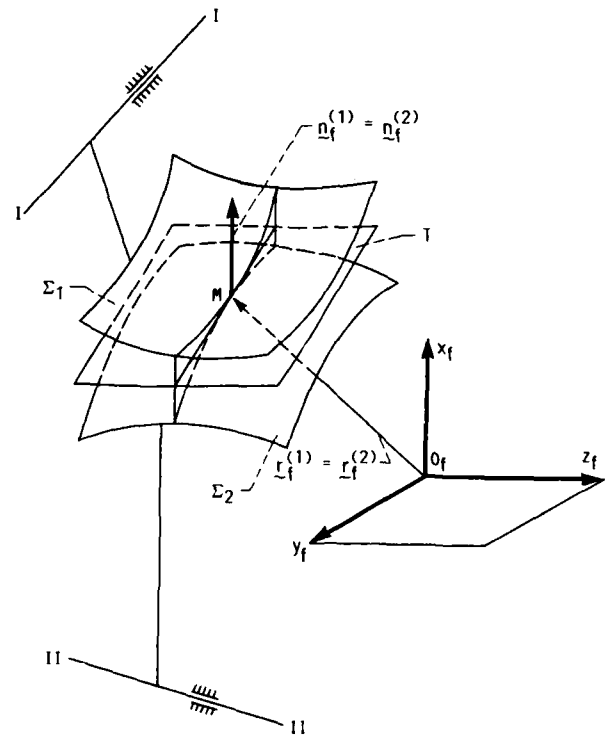


FIGURE 4. - FIXED COORDINATE SYSTEM USED FOR TOOTH CONTACT ANALYSIS (TCA).

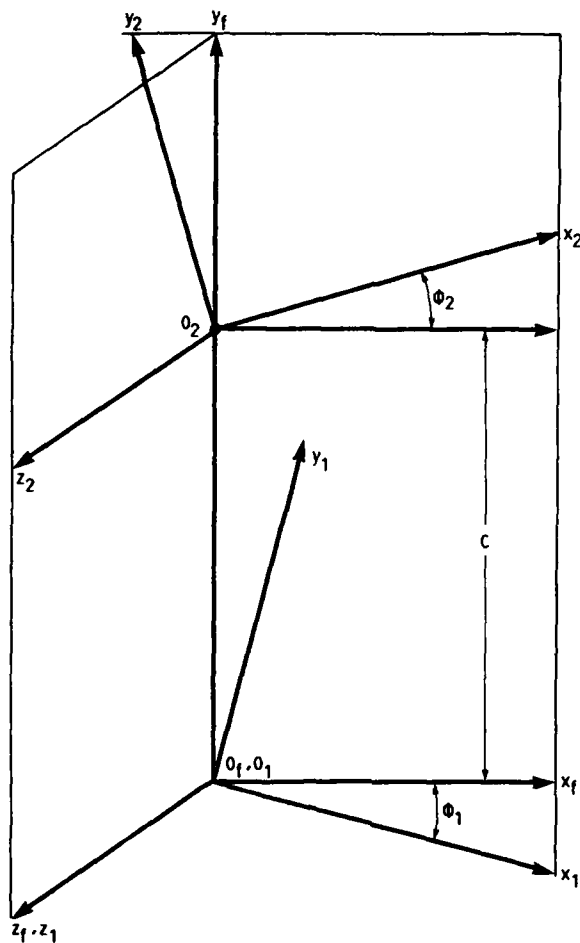


FIGURE 5. - COORDINATE SYSTEM REQUIRED TO DETERMINE PINION TOOTH SURFACE AT MIDDLE CROSS SECTION.

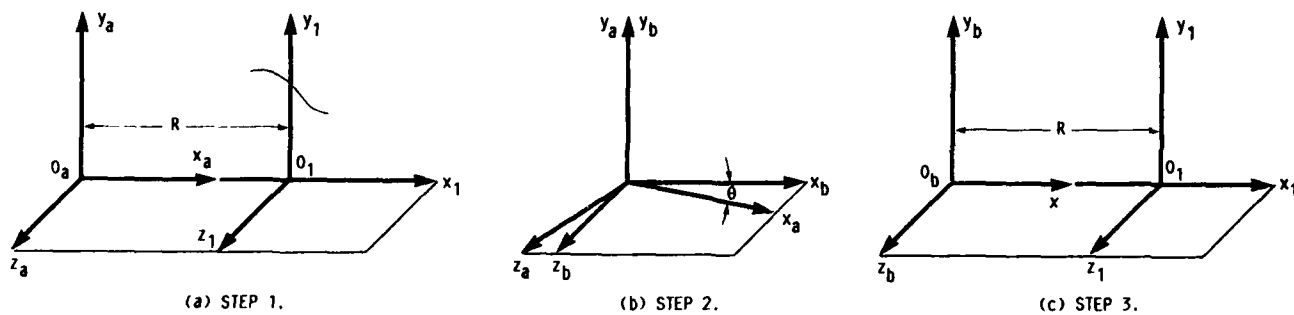


FIGURE 6. - COORDINATE SYSTEMS DESCRIBING REVOLUTION OF MIDDLE CROSS SECTION SURFACE ABOUT A FIXED AXIS.

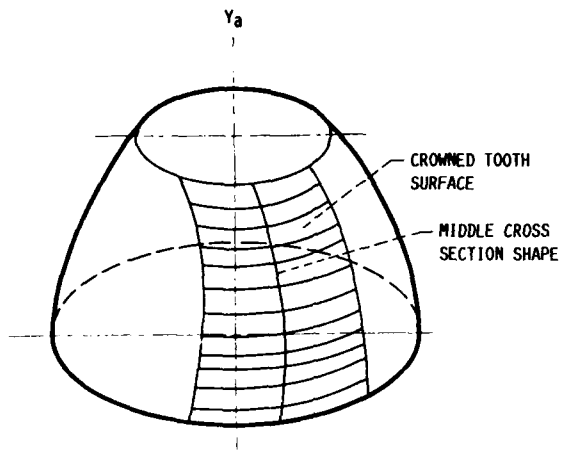


FIGURE 7. - PINION TOOTH SURFACE RELATIONSHIP TO SYMMETRIC AXIS Y_a .

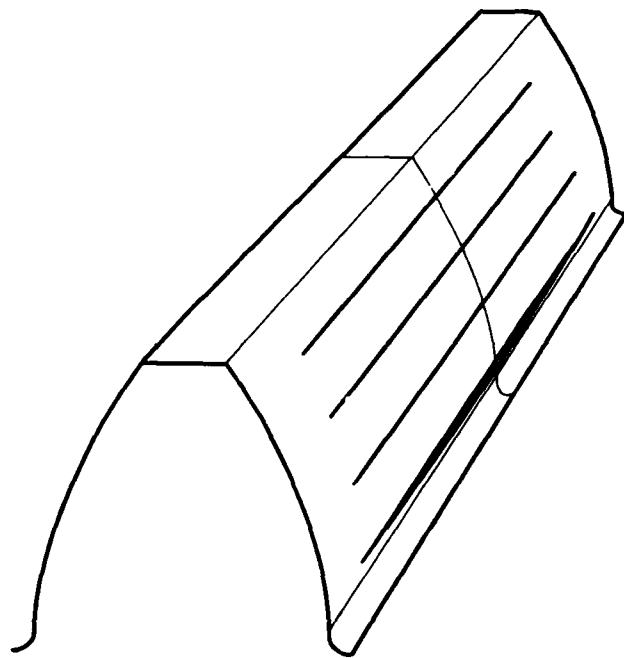


FIGURE 8. - THREE-DIMENSIONAL REPRESENTATION OF CONTACT ELLIPSE ON CROWNED SPUR GEAR SURFACE.

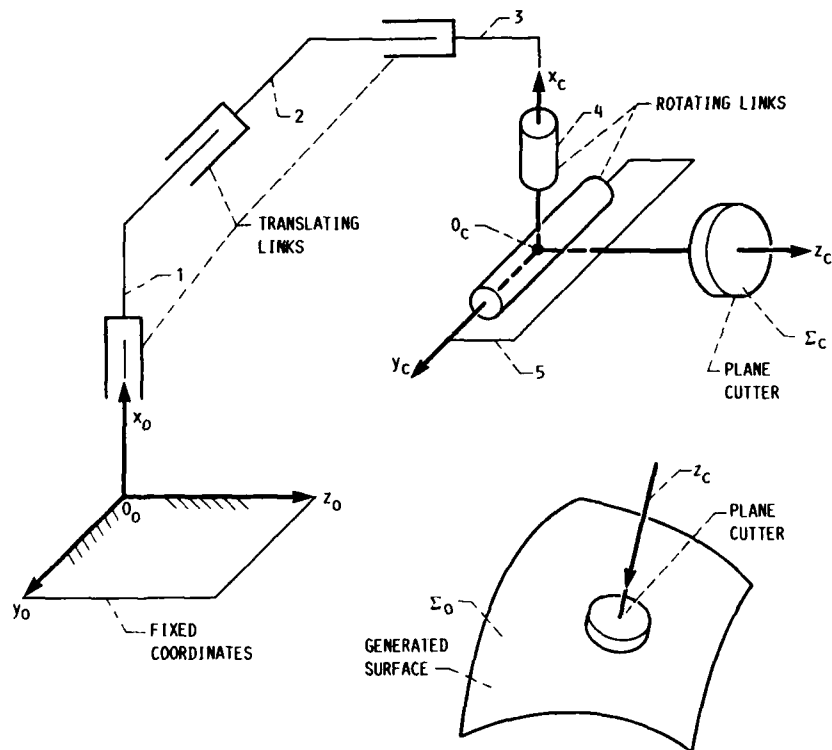


FIGURE 9. - FIVE DEGREE-OF-FREEDOM PLANAR TOOL FOR CROWNED SPUR GEAR TOOTH SURFACE GENERATION.

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