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ANALYSIS OF AIRCRAFT COMBAT SUSTAINABILITY USING A MARKOV CHAIN

by

Gregory C. Reuss

September 1988

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Analysis of Aircraft Combat Sustainability using a Markov Chain

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis develops a combat sustainability analysis which examines the effectiveness of the H-60 and V-22 in conducting assault support operations once ashore. An analytical model, represented as a finite state Markov chain in conjunction with first step analysis, is employed. Several measures of effectiveness are evaluated: survivability, productivity, and the build up of combat power. Sensitivity analysis is conducted on the parameters of sustainability, maintainability, and probability of detection.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND AND PROBLEM DEFINITION.

From Iwo Jima, during WWII, to Grenada in 1983, the U.S Marine Corps has been required to "hit the beach!". The Marine Corps still has capability to conduct amphibious assaults, that is, the landing of an Air-Ground Task Force against hostile shores. However, a large part of this mission, the air transport of troops and equipment, is being accomplished using "tired" medium assault helicopters. The aging CH-46 helicopter needs to be replaced.

The use of assault support aircraft provides a means of rapidly deploying forces ashore, avoiding the threat when possible, and redeploying forces to meet the threat during subsequent operations ashore. Once ashore, the Air Combat Element of the assault force will be expected to carry out the following types of missions in support of the Ground Combat Element: assault, resupply, reconnaissance, search and rescue, and medevac. Until recently, the only platform that provided the ability to accomplish this myriad of missions was the helicopter. Today, technology has provided another alternative which provides this integral VSTOL (vertical/short takeoff and landing) capability, the tilt rotor, MV-22 Osprey.

Numerous analyses have been performed to assist in the determination of the eventual replacement of the CH-46. In this thesis, a combat sustainability analysis is initiated to determine whether the V-22 is necessary to support and sustain the Marine Corps operations in the mid range period 1995-2010, [Ref. 1]. The two alternatives to be considered in the analysis include the H-60 Blackhawk and the V-22 Osprey.

The Blackhawk is a conventional helicopter currently in service in the fleet, primarily being used by the Navy for ASW (antisubmarine warfare), and by the Army for troop transport. When transporting 12 passengers, or 6300 pounds of cargo, it has a combat radius of approximately 110 nautical miles. Some advantages of choosing the Blackhawk alternative include low cost of the airframe, timeliness of delivery since the aircraft is still in production, and the added flexibility afforded the commander due to the greater number of aircraft available. Disadvantages include the greater numbers required to provide the same amount of lift, as well as the associated increases in required support: maintenance, supply, and pilots.

The V-22 Osprey is a tilt rotor aircraft. Its wing tip mounted engines and prop-rotor systems rotate, allowing it to take off and land like a helicopter and once airborne fly like a modern turbo-prop aircraft. The Osprey will transport 24 passengers or 8000 pounds of cargo with a combat radius of 320 nautical miles. Significant advantages will be seen in lift, range, and noise reduction. The greatest advantage to to be attained will be increased speed. The V-22 will be capable of cruising at 250 knots. compared to the Blackhawk's 130 knots. These improvements in overall system characteristics are anticipated to be reflected in decreased system vulnerability and increased system survivability. Potential disadvantages include the risks associated with new technology, the greater cost per airframe, the timeliness consideration imbedded in the acquisition process, and, finally, the significant degradation of the capability of the Air Combat Element due to the loss of one of these aircraft.

B. OVERVIEW.

This thesis will present a combat model suitable for conducting the required combat sustainability analysis. Chapter II develops an analytical model, for a reduced version of the given problem, using a Markov chain and first step analysis. An analytical model, rather than a large scale simulation, is appropriate for this analysis since the V-22 is currently a proposed system and the detailed data required for a high resolution model do not exist. Additionally, to adequately determine an alternative's combat effectiveness, based on its characteristics, extensive sensitivity analysis should be conducted. This desired sensitivity analysis is more easily accomplished with an analytical model than with a large scale simulation. Chapter III applies the model to the real problem.

Specific measures of effectiveness (MOEs) to be addressed in the sustainability analysis include the build up of combat power, sustainability, and the productivity of the assault support assets. The MOEs are designed to present some measure of combat effectiveness attributable to each alternative.

Chapter IV presents sensitivity analysis conducted by varying three different parameters: detection probability, maintainability, and survivability. The results, discussed in Chapter V, reflect the greater capability possessed by the V-22 in terms of the selected MOEs and suggest additional factors for consideration.

II. METHODOLOGY

An analysis of replacement aircraft is required, with alternatives and MOEs basically determined. The remaining challenge is to develop the appropriate model to be employed for this analysis. There are numerous types of combat models used for research and training; field exercises, wargames, and mathematical models are but a few. Based on the specific model, mathematical models may either high or low in resolution, and either stochastic or deterministic in nature.

The following is a list of several model properties, prepared by the Army Models Review Committee [Ref. 2: p. 7], which should be considered in model development, and which will reflect a model's usefulness or limitations.

- Transparency
- Reproducibility
- Consistency
- Enrichment potential
- Military realism
- Experimental validity
- Physical reasonableness
- Visibility to the analyst
- Credibility
- Flexibility
- Interface potential
- Resources required
- Responsiveness
- Sensitivity of the model
- Technical user capability
- Visibility to the user

Establishment of an appropriate weapons procurement model considers additional factors as well. The use of many models requires extensive knowledge of the weapon's characteristics as reflected in the data base. These data can be acquired from developmental or operational test and evaluation, high resolution models, historical data, etc. When developing a new weapon system utilizing new technology like the V-22, these data do not exist. Therefore, the development of expensive, highly detailed models incorporating specific probability distributions is premature. Thus the ability to conduct extensive sensitivity analysis on several parameters rapidly is essential because of the numerous uncertainties of the future system. Additionally an overall model goal is robustness, which reflects a wide range of conditions over which the model is effective.

This thesis presents such a model by developing a particular Semi-Markov Process. Specifically, let $\{X_n\}$ be a discrete time Markov chain with states labeled 0,1, ... , N. The states r, r+1, ... , N are absorbing states. The states 0, ... , r-1 are transient. Let P_{ij} be the ij element of the transition matrix **P** for $\{X_n\}$ which has the form (2.1)

$$P = \begin{bmatrix} Q & R \\ O & I \end{bmatrix} \tag{2.1}$$

where Q is an (r x r) matrix of transition probabilities, R is an r x (N-r) matrix of transition probabilities, O is a (N-r) x r matrix of zeros and I is an (N-r) x (N-r) identity matrix. Let $0 = T_0 < T_1 < T_2 < \ldots < T_n < \ldots$ be a sequence of random times with

$$P\{T_{n+1} - T_n \le t \mid T_1, \ldots, T_n, X_0, X_1, \ldots, X_n, X_{n+1}\} = P\{T_{n+1} - T_n < t \mid X_n, X_{n+1}\}$$

and put

$$F_{ij}(t) = P\{T_{n+1} - T_n \le t \mid X_n = i, X_{n+1} = j\}$$

Let $Y(t) = X_n$ if $T_n \le t < T_{n-1}$ then $\{Y(t); t \ge 0\}$ is an example of a semi-Markov process.

With this structure in place, several questions can now be answered utilizing first step analysis. What is the probability of absorption for a specific absorbing state? What is the mean time to absorption? What is the mean number of visits to a transient state prior to absorption? This same theory applied to the specific problem of interest yields a very robust model capable of providing a multitude of information for use in analysis.

In order to demonstrate the developing methodology a small example (a reduced version of the larger problem to be discussed in Chapter III) is presented. This example models an aircraft based ashore with the mission of resupplying ground units,located at various landing zones (LZ)s, upon request. Questions of interest include: How long will the aircraft survive? What is the probability of aircraft loss due to pilot error? What is the probability of aircraft loss due to catastrophic failure? How many times is the aircraft expected to resupply an LZ prior to absorption? There is no associated threat in this example, thus absorption is possible via pilot error or by catastrophic failure of the aircraft. The aircraft is capable of being in one of the following states:

- State 0--UP AWAITING MISSION: The aircraft is on the flightline in an up status, pending mission assignment.
- State 1--INBOUND TO LZ: With a resupply mission assigned, the aircraft now travels via an inbound route to an LZ. Speed of the aircraft and distance to the LZ determine travel times. From State 1 it is possible the aircraft lands at an LZ, or aborts the mission thus transitioning to State 3.
- State 2--IN LZ CONDUCTING RESUPPLY: The aircraft has landed in an LZ to discharge passengers or equipment. This unloading of passengers or equipment is expected to take approximately three minutes. From State 2 it is possible to transition to State 1, if flying to another LZ, or to to State 3 if flying back to base.
- State 3--EGRESS: Departure from the LZ or inbound state to return to base, the aircraft may now transition to travel to another zone (State 1) or continue to return to base, (State 0).
- State 4--PILOT ERROR: The aircraft is lost due to an error in judgement by the pilot.
- State 5--CATASTROPHIC FAILURE: The aircraft is lost due to a mechanical failure.

The **P** matrix. Figure 1 reflects the probabilities of all possible transitions, State i to State j, within this chain.

0	1	0	0	0	0
0	0	0.65	0.2	0.05	0.1
0	0.2	0	0.65	0.1	0.05
0.5	0.35	0	0	0.075	0.075
0	0	0	0	1	0
0	0	0	0	0	1

Figure 1. P matrix.

The **T** matrix, Figure 2, is made up of time, in hours, from entry into State i to transition to State j. These times would reflect the expected values of their associated f_{ij} distributions; for development of the model these times are estimates derived by the author.

0	18	0	0	0	0
0	0	1.7	1.1	0.58	0.58
0	0.05	0	0.05	0.05	0.05
2	0.5	0	0	0.23	0.23

Figure 2. T matrix.

Using the **P** and **T** matrices, first step analysis is employed to answer the questions of interest. If an absorbing state (States 4 or 5 in the example) is identified as State r1 or r2, then the probability of absorption in a particular state, r, given the initial state is v(i), given in equation (2.2) [Ref. 3: p. 82].

$$v(i) = P_{ir} + \sum_{j=0}^{r-1} P_{ij} v(j) \tag{2.2}$$

For the example, the probability of aircraft loss due to pilot error is found by solving for the probability of absorption in State 4. This is accomplished by solving the following set of simultaneous linear equations generated by the **P** matrix:

$$\begin{array}{lcccccccccccc}
v(0) = & 0 & + & 1v(1) & + & 0 & + & 0 & + & 0 & + & 0 \\
v(1) = & 0 & + & 0 & + & .65v(2) & + & .2v(3) & + & .05 & + & 0 \\
v(2) = & 0 & + & .2v(1) & + & 0 & + & .65v(3) & + & .1 & + & 0 \\
v(3) = & .5v(0) & + & .35v(1) & + & 0 & + & 0 & + & .075 & + & 0
\end{array}$$

This set of equations is easily solved using APL, and the output from a function utilizing the example's **T** matrix generates the display given in Figure 3.

```
PROBABILITY OF ABSORPTION IN STATE 4
FROM          IS
 0        0.4743307664
 1        0.4743307664
 2        0.5056839017
 3        0.4781811514

PROBABILITY OF ABSORPTION IN STATE 5
FROM          IS
 0        0.5256692336
 1        0.5256692336
 2        0.4943160983
 3        0.5218188486
```

Figure 3. Probability of absorption in a particular state.

Thus, in the current example, the probability an aircraft is lost due to pilot error (State 4), given that it starts awaiting a mission (State 0), is 0.4743307664. Likewise, given that an aircraft is inbound to the landing zone (State 1), the probability that it will be lost due to catastrophic failure of the airframe (State 5), is 0.5256692336.

Regardless of how the aircraft is ultimately lost, a determining measure of how successful or unsuccessful a particular weapon system performs is based on how long it is able to survive. Attrition information is conveniently generated from first step analysis by solving for mean time to absorption. For any transient state, i, the mean time to absorption, m(i), is computed using equation (2.3) in which $\int sF_{ij}(ds)$ is the mean wait time in state i when going to state j [Ref. 3: p. 83].

$$m(i) = \sum_{j=r}^{N} P_{ij}\int_{0}^{\infty} sF_{ij}(ds) + \sum_{j=0}^{r-1} P_{ij}\left[\int_{0}^{\infty} sF_{ij}(ds) + m(j)\right] \tag{2.3}$$

$E[S_i]$, the expected sojourn time in state i, is obtained using equation (2.4).

$$E[S_i] = \sum_{j=0}^{N} P_{ij} \int_{0}^{\infty} s F_{ij}(ds) \tag{2.4}$$

Equation (2.3) can be rewritten as

$$m(i) = E[S_i] + \sum_{j=0}^{r-1} P_{ij} m(j) \tag{2.5}$$

In applying this concept to the example, the **P** and **T** matrices formulate the set of linear equations:

$$\begin{array}{lcccccccccc}
m(0) = & 0 & + & 1m(1) & + & 0 & + & 0 & + & E[S_0] \\
m(1) = & 0 & + & 0 & + & .65m(2) & + & .2m(3) & + & E[S_1] \\
m(2) = & 0 & + & .2m(1) & + & 0 & + & .65m(3) & + & E[S_2] \\
m(3) = & .5m(0) & + & .35m(1) & + & 0 & + & 0 & + & E[S_3]
\end{array}$$

The solution to these simultaneous equations, as generated by an APL function, is presented in Figure 4. Referring to this figure, it may be seen that an aircraft is expected to survive approximately 41 hours if it starts awaiting a mission to be assigned. Given the aircraft is currently on a mission and is in the landing zone, it can expect to be attrited in 24 hours.

```
MEAN TIME TO ABSORPTION
FROM        IS
 0       40.8820352
 1       22.8820352
 2       23.90490649
 3       29.65922992
```

Figure 4. Mean time to absorption (in hours) from a given state.

Although a key MOE may be attrition related, such as survival time, more specific information reflecting the actual amount of work done within that survival time is of even more interest. How much time was spent effectively accomplishing what? By determining the number of times any particular state is visited prior to absorption, what specifically can be accomplished may be answered. First step analysis is tailored to solve for W_{ik}, the mean number of visits to state (k) prior to absorption, given the initial state (i), in equation (2.8) [Ref. 3: p. 117].

$$W_{ik} = \delta_{ik} + \sum_{j=0}^{r-1} P_{ij} W_{jk} \qquad where \quad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & otherwise \end{cases} \tag{2.8}$$

Employing a matrix format as per equation (2.1), in which Q is the matrix of the transient state transition probabilities, TAYLOR [Ref. 3: p. 118] develops the fundamental matrix, **W**, the matrix of all w_{ij}

$$W = (I - Q)^{-1}$$

Given the aircraft starts in state i, the mean number of visits to state j prior to absorption is W_{ij}. The fundamental matrix of the current example is given in Figure 5.

```
NUMBER OF VISITS TO STATE J FROM I BEFORE ABSORPTION
1.913091309  2.933626696  1.906857352  1.826182618
0.9130913091 2.933626696  1.906857352  1.826182618
1.01210121   2.207554089  2.434910158  2.02420242
1.276127613  2.493582692  1.62082875   2.552255226
```

Figure 5. Fundamental matrix.

For an aircraft that is initially flying outbound from the landing zone, that aircraft can expect to be in the inbound state 2.49 times. The ability to determine the number of times an aircraft is able to get into the landing zone to resupply is now obtainable. By incorporating the lift characteristics of a given weapon system (either the number of passengers or pounds of equipment) a better estimate of a particular measure of effectiveness is available.

The above calculations are for expected values conditioned on a specific initial state, Y = y. One can obtain the unconditional expected values by "unconditioning" [Ref. 4: p. 220]. For example, suppose the aircraft is initially in State 0 with probability 0.3, or in State 3 with probability 0.7, then the expectations in Figure 6 are generated.

Hours the aircraft is expected to survive prior to absorption:
0.3(40.882) + 0.7(23.905) = 28.998

Expected number of visits prior to absorption

STATE 0	STATE 1	STATE 2	STATE 3
1.28239824	2.425375871	2.276494316	1.96479648

Figure 6. Results based on probabilities associated with initial state.

As previously alluded to, by knowing specific state characteristics of the system, a better measure of effectiveness is obtainable. If the example aircraft is known to be resupplying the landing zone, State 2, with passengers and it is capable of transporting 35 passengers at a time, then it is reasonable to estimate that it is capable of transporting 35 x E(number of visits to State 2). Therefore, the modeled aircraft can transport 35 x 2.276494316 = 79.677 passengers in the 28.998 hours it is expected to survive.

As has been demonstrated by the use of this small example, the employment of the Semi-Markov chain and first step analysis in concert with a combat model application is a valuable analysis tool. This model is capable of providing a myriad of information easily transformable into quantiative MOEs. This simple, yet powerful concept will now be expanded in order to model the Osprey versus Blackhawk combat sustainability.

III. MODEL DEVELOPMENT

The concept developed and demonstrated in the previous chapter will now be expanded to provide insight into the real problem: determine whether the V-22 is necessary to support and sustain the Marine Corps operations in the mid-range period 1995-2010. This particular model focuses on subsequent operations ashore, following an amphibious assault.

The basis for and most essential element of this model is the establishment of the Markov chain. The development of the chain relies on the Markov assumption that the future is conditionally independent of the past given the present state. Each state space must be adequately defined to ensure that the desired combat process is accurately modeled. The degree of model resolution is determined by the detail of state definitions. The chain developed for this model reflects possible states one assault support aircraft could possibly be in while supporting a Ground Combat Element during subsequent operations ashore. This was accomplished by "walking through" the subject process step by step and defining the states necessary to reflect the numerous interactions that determine the effectiveness of an aircraft. The following is a listing of the specific states represented.

- State 0--AIRCRAFT UP AWAITING MISSION ASSIGNMENT
- State 1--MISSION ARRIVES AWAITING AIRCRAFT
- State 2--PREPARED ASSAULT MISSION ASSIGNED
- State 3--PREPARED RESUPPLY MISSION ASSIGNED
- State 4--PREPARED RECON MISSION ASSIGNED
- State 5--IMMEDIATE SAR MISSION ASSIGNED
- State 6--IMMEDIATE MEDEVAC MISSION ASSIGNED
- State 7--IMMEDIATE REACTION FORCE MISSION ASSIGNED
- State 8--IMMEDIATE EMERGENCY RESUPPLY ASSIGNED
- State 9--LOAD PASSENGERS (PAX)
- State 10--LOAD EQUIPMENT
- State 11--TAKE OFF AIRFIELD
- State 12--PICK UP EXTERNAL LOAD
- State 13--EXECUTE PRECAUTIONARY LANDING
- State 14--INGRESS LOW LEVEL
- State 15--INGRESS CONTOUR
- State 16--INGRESS NOE (NAP OF THE EARTH)
- State 17--LAND LOW FUEL
- State 18--DETECTED LOW LEVEL
- State 19--DETECTED CONTOUR
- State 20--DETECTED NOE
- State 21--SHOT AT

- State 22--EVASIVE MANEUVER
- State 23--MISSED SHOT
- State 24--HIT WITH NO DAMAGE
- State 25-- HIT WITH DAMAGE
- State 26--AIRCRAFT IN REPAIR
- State 27--EMERGENCY LANDING
- State 28--LAND IN LANDING ZONE (LZ)
- State 29--UNLOAD PAX
- State 30--UNLOAD EQUIP
- State 31--DELIVER EXTERNAL LOAD
- State 32--EGRESS NOE
- State 33--EGRESS CONTOUR
- State 34--EGRESS LOW LEVEL
- State 35--LANDED AIRFIELD
- State 36--REFUEL AIRCRAFT
- State 37--HOLD FOR MISSION SUPPORT
- State 38--ABORT MISSION
- State 39--AIRCRAFT DOWN AWP (AWAITING PARTS)
- State 40--AIRCRAFT DOWN AWM (AWAITING MAINTENANCE)
- State 41--TAKEOFF FROM LZ
- State 42--HIT WITH KILL AIRCRAFT LOSS
- State 43--PILOT ERROR AIRCRAFT LOSS
- State 44--CATASTROPHIC FAILURE AIRCRAFT LOSS

The specific values that are associated with the **P** and **T** matrices of this chain, for any alternative, are dictated by METTW (mission, enemy, terrain, troops, weather). METTW actually sets the constraints for a given scenario and a weapon's characteristics determine the appropriate probability and time transition values. For example, given the threat, with no terrain or weather limitations, the increased range and airspeed of the V-22 could enable it to take a route which would avoid threats to which the H-60 would be exposed. A greater lift capability can be translated into fewer exposures for the same lift mission. Based on the author's aviation experience, V-22 and H-60 characteristics, and the USMC Assault Support Manual [Ref. 5], **P** and **T** matrices for the alternatives have been created (see Appendix A). In Appendix A, State 0 is associated with Row 1, State 1 is associated with Row 2, etc. These are considered base case matrices in that modifications will be made to them to conduct various sensitivity analyses. More accurate data for these matrices could possibly be acquired from engineering data, historical data, expert judgement, or high resolution simulations (if they exist). However, for the purpose of model development the current values are representative.

The goal of this model is to allow the decision maker to evaluate, using meaningful MOEs, the effectiveness achieved by a particular alternative in a specific scenario. Certain weapon system configurations have been designated as in Figure 7. Based on the proposed characteristics of the tilt-rotor relative to the conventional helicopter, alternative base case matrices differ in ingress, egress, and detection transition probabilities. The following initial state probabilities hold: the aircraft is initially inbound to the LZ, flying low level (State 14) with probability 0.8 and the aircraft is on deck at the expeditionary airfield (State 35) with probability 0.2.

CAPABILITY	V-22	H-60
PASSENGERS	24	12
EQUIPMENT(LBS.)	10000	6300

Figure 7. Aircraft configurations.

As per the Combat Sustainability Model Directive [Ref. 1], particular MOEs have been identified for consideration: sustainability, productivity of the assault support asset, and the build-up of combat power. The reference fails to provide detailed definitions of these MOEs; thus for the purpose of this analysis, the defined measures of alternative performance will represent the respective MOE.

- Sustainability. Represented by an alternative's expected time to absorption.
- Productivity. Represented by the expected number of passengers and pounds of equipment transported prior to absorption.
- Build up of combat power. Represented by the number of passengers per hour delivered (the expected number of passengers transported divided by the expected number of hours the aircraft survives).

Figure 8 shows partial output of the model from base case runs for the H-60 as well as for the V-22. This output reflects the following expected values: the number of hours the aircraft survived, the number of passengers transported, and the pounds of equipment transported during that time. Additionally, the model generates the expected time, in hours, the aircraft is in repair, awaiting maintenance (AWM) and awaiting parts (AWP). These three values determine the "downtime" of the aircraft. The expected number of passengers transported is determined by multiplying the expected number of visits to the UNLOAD PAX state times the number of passengers the alternative is capable of carrying and likewise for the transport of equipment. The amount of time an aircraft spends in a particular state is obtained by multiplying the expected number of visits to the particular state times that state's sojurn time. Appendix B presents the APL functions which provide this information and detailed model output. Information regarding an activity related to any specific state such as amount of fuel required, number of emergency landings, the number of particular missions assigned, etc., would also be obtainable from this model.

```
THE FOLLOWING EXPECTED VALUES HAVE BEEN DETERMINED

HOURS MH-60 EXPECTED TO SURVIVE 360.8
PAX TRANS                 125.1
EQUIP TRANS               70876.1
TIME REPAIR               30.5
TIME AWM                  14.7
TIME AWP                    1.9

HOURS MV-22 EXPECTED TO SURVIVE 412.9
PAX TRANS                 275.9
EQUIP TRANS               124099.6
TIME REPAIR               32.0
TIME AWM                  15.6
TIME AWP                    1.7
```

Figure 8. Base case model outputs.

An attribute of an analytical model is its transparency. Sensitivity analysis has been conducted on three parameters: detection probability, survivability, and maintainability.

- Detection - The probability of an aircraft being detected during ingress and egress states is varied.
- Survivability - The probability of damage to an aircraft which would require it to land, given it has been shot at and hit, is varied.
- Maintainability - Repair time and time awaiting parts varies, as do the probabilities associated with transitioning to repair, awaiting maintenance, and awaiting parts states.

By incorporating the enhanced maintenance and survivability cases with reduced probability of detection, an upper estimate of performance run was made for each alternative. Likewise a lower estimate was achieved by combining degraded maintainability and survivability with an increased probability of detection condition. The following chapter examines each alternative's performance relative to the various MOEs, as well as identifying the impact resulting from varying different parameters using sensitivity analysis.

IV. ANALYSIS

The airland warfare process is complex. Predictions of combat results from any model are dependent on numerous factors such as METTW, and their interactions. Experimentally verified data associated with the combat process does not exist in reality, therefore a point estimate alone should not be presented to a decision maker for an evaluation [Ref. 6: p. 76]. Analytical models can be used to gain insight and to answer "what would happen if" questions and are also capable of identifying factors that are important as well as those that are not.

Sensitivity analysis has been performed on the base case of each alternative with respect to detection probability, survivability, and maintainability. By modifying the base case **P** and **T** matrices for each alternative, the following model runs have been formulated.

- MAINT(+): Decreases the probability of transition to AWM, AWP states, increasing the probability of transition to aircraft in repair. Decreases time associated with AWM, AWP and aircraft in repair states. Following a precautionary landing, the probability of requiring repair decreases.
- MAINT(-): Increases the probability of transition to AWM, AWP states, decreasing the probability of transition to aircraft in repair. Increases time associated with AWM, AWP and aircraft in repair states. Following a precautionary landing, the probability of requiring repair increases.
- SURV(+): Increases the probability of no damage given a hit, decreasing probability of damage given a hit.
- SURV(-): Decreases the probability of no damage given a hit, increasing probability of damage given a hit.
- PDET(+): Increases probability of detection from ingress and egress states, decreasing the probability of arrival at LZ and base.
- PDET(-): Decreases probability of detection from ingress and egress states, increasing the probability of arrival at LZ and base.
- LOWER ESTIMATE: Combines MAINT(-), SURV(-), and PDET(+) adjustments.
- UPPER ESTIMATE: Combines MAINT(+), SURV(+), and PDET(-) adjustments.

Appendix C lists the functions which assign the the specific values resulting in the associated improvement and degredation of the respective parameters. Tables 1 and 2 summarize the results for the H-60 and V-22, respectively. These results reflect the following expected values: the number of hours the aircraft survives, the number of passengers transported, the amount of equipment transported (thousands of pounds), the amount of downtime and the rate (passengers per hour) delivered.

Table 1. H-60 BLACKHAWK RESULTS.

H-60	HOURS SURVIVED	PAX TRANS	EQUIP TRANS	DOWN TIME	RATE PAX PER HOUR
BASE CASE	360.8	125.1	70.9	47.1	.35
MAINT(+)	319.2	124.8	70.7	21.8	.39
MAINT(-)	455.8	125.6	71.2	109.1	.28
SURV(+)	323.0	124.0	70.2	34.5	.38
SURV(-)	397.2	126.2	71.5	58.1	.32
PDET(+)	214.0	61.7	35.0	38.3	.29
PDET(-)	538.0	185.6	105.2	58.1	.35
LOWER ESTIMATE	346.4	62.0	35.1	115.6	.18
UPPER ESTIMATE	463.1	184.2	104.4	21.2	.40

Table 2. V-22 OSPREY RESULTS.

V-22	HOURS SURVIVED	PAX TRANS	EQUIP TRANS	DOWN TIME	RATE PAX PER HOUR
BASE CASE	412.9	275.9	124.1	49.3	.67
MAINT(+)	368.4	275.4	123.9	22.3	.75
MAINT(-)	514.4	277.1	124.6	114.4	.54
SURV(+)	378.0	273.5	123.0	38.7	.72
SURV(-)	446.5	278.3	125.2	59.2	.62
PDET(+)	234.7	134.5	60.5	39.2	.57
PDET(-)	566.3	371.4	167.0	58.2	.66
LOWER ESTIMATE	368.5	135.4	60.9	116.9	.33
UPPER ESTIMATE	491.1	368.5	165.8	21.3	.75

Information presented in these tables allows the decision maker to determine potential trade-offs. He or she can now identify which parameters are important and also determine which data are important and which are not. Sensitivity analysis also shows which MOEs are most sensitive to which parameters. The following are several specific comparisons of results provided to illustrate the scope of the analyses possible.

Figure 9 reflects the lower and upper estimates of alternative effectiveness, showing how very well or how poorly an alternative is capable of performing. As displayed by the sustainability graph, in the lower estimate case an aircraft survives fewer hours. More significantly the downtime associated with this case accounts for approximately 32% of the total survival time. The upper estimate reflects an improvement in sustainability and a reduction in downtime. This reduced downtime accounts for less than five percent of the improved total survival time.

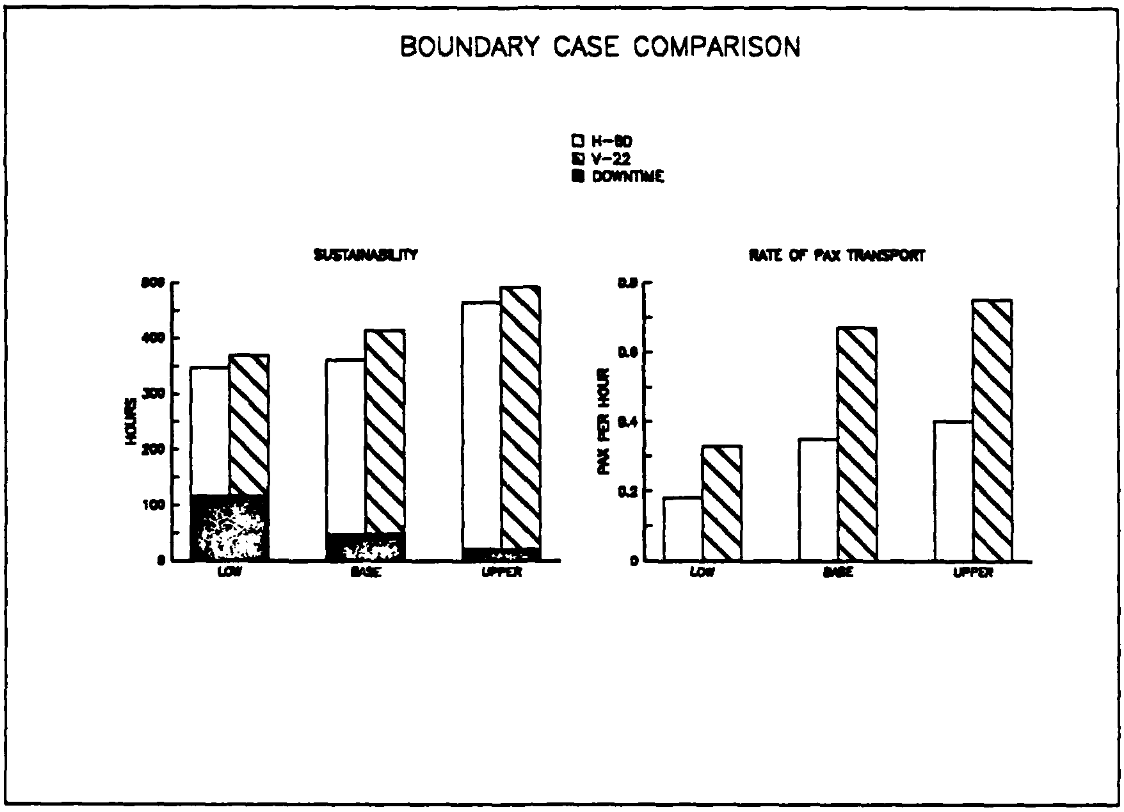

Figure 9. H-60 vs.V-22 Boundary Case Comparison.

The rate of passenger transport graph similarly reflects the range of performance which can be expected by each alternative. In viewing the rate graph in Figure 9, it can be seen that with the H-60 at its upper estimate and the V-22 at its lower estimate, the Blackhawk rate of passenger delivery actually exceeds that of the Osprey.

The probability of detection for a given scenario has the greatest impact on what can potentially be accomplished. Figure 10 shows a significant difference in the number of hours an aircraft is expected to survive between low and high probability of detection cases. Referring to Table 1, the Blackhawk is three times more productive when the probability of detection is low than when high. If it were possible to somehow decrease the probability of detection for the H-60, by lowering its infrared signature, decreasing its noise, or increasing its range, the Blackhawk could potentially transport approximately 186 passengers. Even with this marked H-60 improvement the Blackhawk would still transport 90 fewer passengers than the Osprey in its V-22 base case.

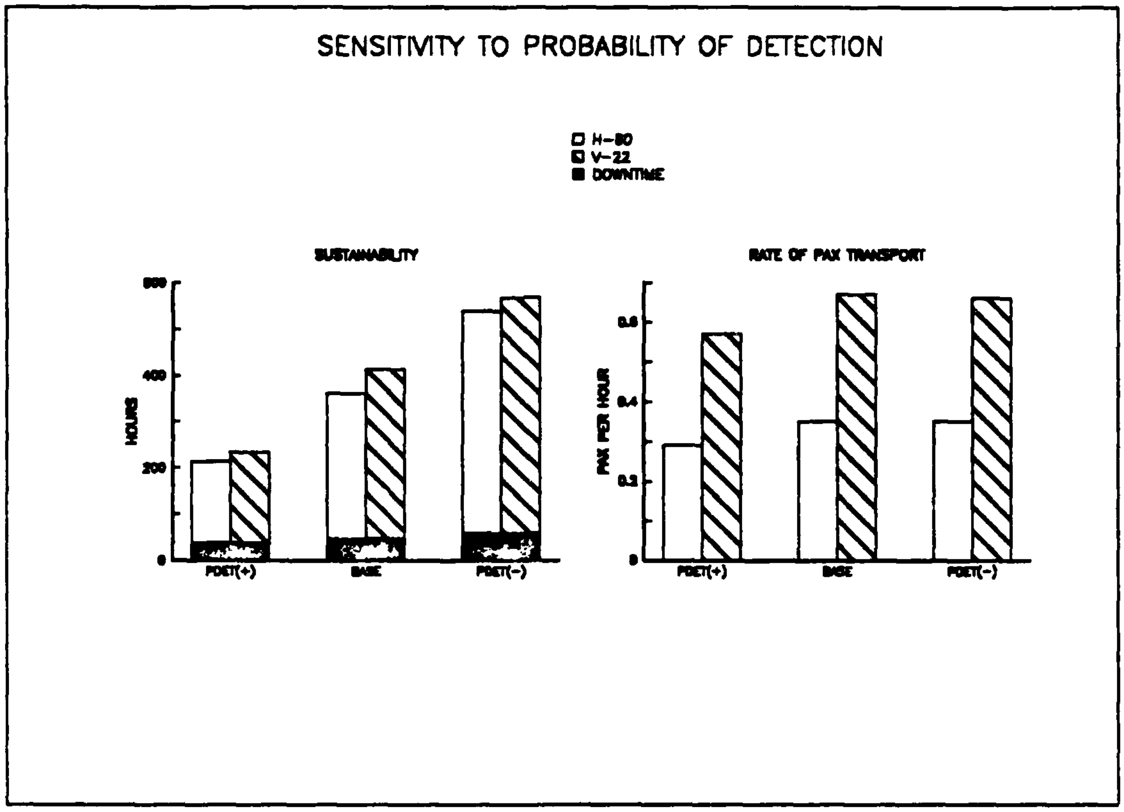

Figure 10. H-60 vs. V-22 Sensitivity to Probability of Detection.

As seen in Figure 11, the degraded maintenance case yields the greatest survival time. What is also shown by the sustainability graph is that approximately 23% of that time is downtime. Following a precautionary landing, the probability that significant maintenance is required decreases as the maintainability of the aircraft improves. The more time an aircraft spends in maintenance, the less time it is available to actually perform its mission and be subject to attrition. The sustainability graph additionally shows that as the maintenance improves, both the total survival time and downtime decrease. In the improved maintenance case only six percent of the total time is downtime.

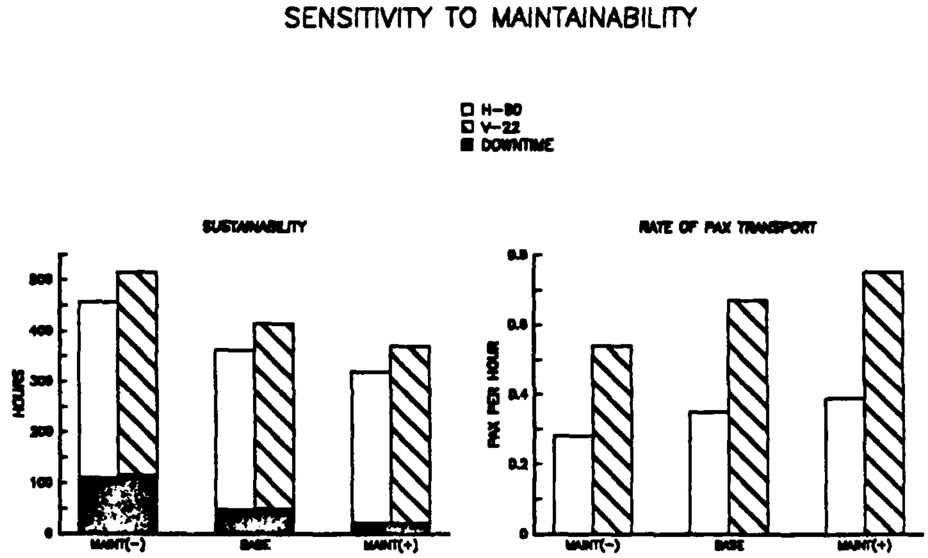

Figure 11. **H-60 vs. V-22 Sensitivity to Maintainability.**

The associated rate of passenger transport clearly reflects the results of increased maintainability. Referring to Table 2, the V-22 experiences a .21 passenger per hour difference within maintenance levels. Comparing the degraded maintenance case of the V-22 with the improved maintenance case of the H-60 (see Tables 1 and 2) the Osprey with twice the load carrying capability of the H-60 is only able to deliver "combat power" .15 passengers per hour better than the Blackhawk.

Improving the survivability of an aircraft results in its ability to tolerate more damage before being required to execute an emergency landing. Figure 12 shows as survivability increases, the sustainability (number of hours the aircraft survives) decreases. The more survivable the aircraft the less time it is associated with maintenance and the greater the time it is exposed to the threat. Although the aircraft survives less time, as the survivability improves, the downtime decreases from 15% to 10% of the total time. This translates into an improved rate of passenger transport as seen by the rate graph in Figure 12.

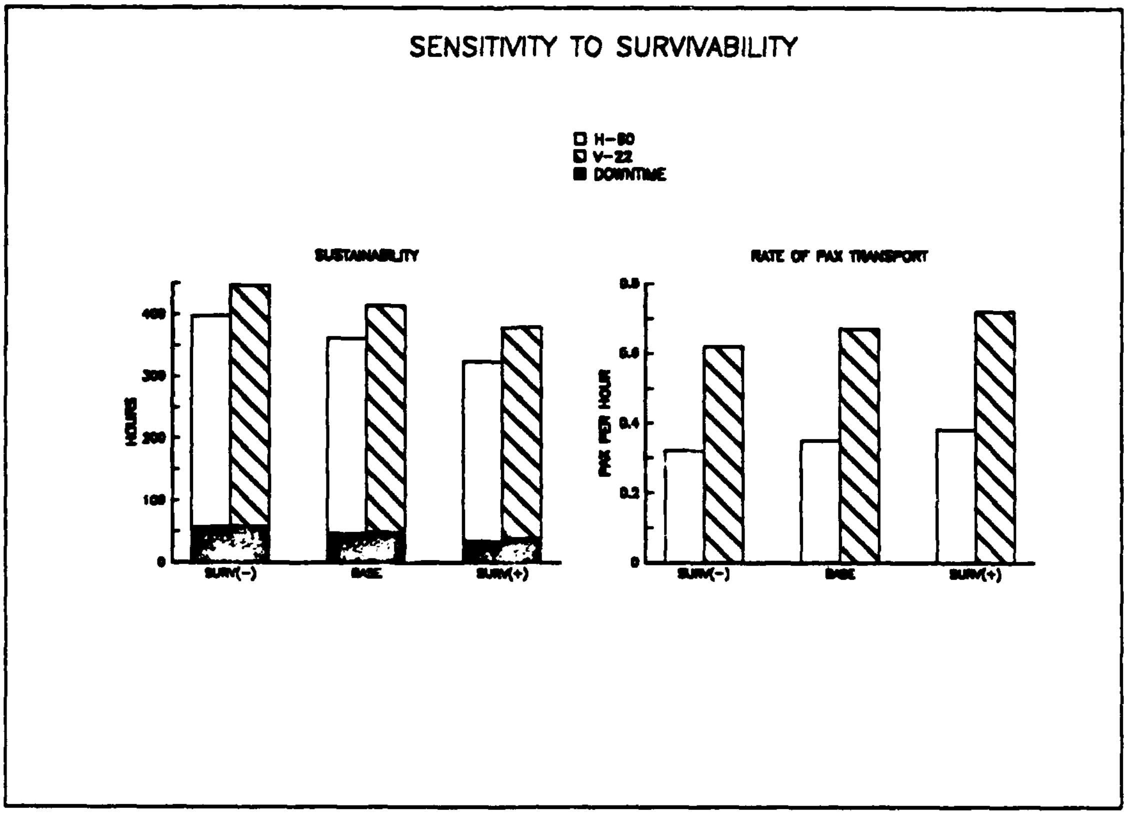

Figure 12. H-60 vs. V-22 Sensitivity to Survivability.

Based on the results of this analysis the V-22 provides better combat effectiveness than the H-60, in the majority of cases. The "how much better" is available for this particular scenario and is easily obtainable for any additional cases. This analysis also shows that there may be a combination of conditions with various MOEs where the H-60 is more effective. Comparing a squadron of 22 H-60s with a squadron of 11 V-22s, and applying the expected number of passengers transported per aircraft for each base case even more information can be obtained. By multiplying the number of aircraft available by the expected value, it can be determined that when the H-60 squadron loses four aircraft it is expected to transport 2252 passengers. However, when the V-22 squadron loses three aircraft its expected capability is reduced to 2207 passengers, and the loss of a fourth V-22 further reduces this to 1931 passengers.

The number of available comparisons of MOEs with varying parameters is extensive. The goal of the model is to present the decision maker with the information necessary to gain insight into the specific combat process and the performance of the specific alternatives being considered. The decision maker now has available additional information which may be incorporated with a myriad of other factors such as the number of aircraft available, cost considerations, timeliness of delivery, etc. to determine if the V-22 is "necessary" for the Marine Corps.

V. SUMMARY

The Marine Corps must find a replacement for the CH-46 assault support aircraft. Alternatives currently being considered are the H-60 Blackhawk and the V-22 Osprey. Having specified several MOEs including sustainability, productivity, and the ability to build up combat power, a study directive was initiated to determine whether the V-22 is "necessary" to support the Marine Corps in the 1995-2010 timeframe.

To provide insight into this problem, this thesis presents the development and implementation of a Markov chain analytical model. Runs were made to obtain a base case, as well as upper and lower performance estimates of the subject MOEs for each alternative. Additionally, sensitivity analysis was conducted on the parameters of maintainability, survivability and detection probability. The results of the analysis have identified shortcomings of various MOEs, reflected the sensitivity of the MOEs to different parameters, and given values of "how much better" one alternative is over the other for a given case.

By simply applying the results of this analysis, the greater capability possessed by the V-22 translates into better performance in the selected MOEs. In order to determine the necessity of the V-22 other factors need to be considered. The number of H-60s available could be twice the number of V-22s. Additional airframes afford the commander added flexibility. Also it must be determined how much of the total force capability is lost when one aircraft is attrited. This analysis allows the decision maker to gain an appreciation for what would happen if the proposed V-22 is unable to achieve its desired characteristics, and to see the results of potential trade-offs which could result in improved maintenance or survivability of the existing H-60. Due to the complex and dynamic nature of the combat process, neither one model nor analysis alone is able to determine the necessity of the V-22. Time of delivery, cost, and politics are other variables this analysis was unable to model; ultimately these factors will influence the determination of the eventual replacement for the CH-46.

The modeling technique developed for this analysis has extensive potential. In considering the modeling of flights of more than one aircraft, different transition matrices would be required to reflect such factors as the effect on maintenance, probability of detection, probability of hit, and expected number of sorties required. The characteristics of numerous threats may be incorporated as well. The significance of a particular threat is determined by the probability of absorption by that threat for a given initial state. Analysis of tactics could be conducted to see if different initial conditions result in significantly different outcomes. Markov chains can be established for any unit: infantry, artillery or armor. For any scenario, absorption could be determined for various thresholds, sensitivity conducted on numerous parameters including initial states, and representative MOEs evaluated. The robustness and transparency of this model are well suited for this type analysis. The degree of resolution is directly related to the degree in which the applicable Markov states are defined. As with any model the validity of the data will significantly impact on the validity of the model.

Perhaps the greatest potential use of this type model is in its ability to provide the framework for the incorporation of other combat models. Any combat process can be developed in the Markov chain structure from the platoon level to the division level. In such a framework, high resolution results could be integrated into the transition matrices of higher level processes and evaluated.

It is hoped that the analysis of the air assault support combat process contained in this thesis will prove beneficial to weapons systems analysts. Conducting thorough and thought provoking analyses have proven an integral part of the weapon system acquisition process which hopefully results in obtaining the most effective system for the money. Today's Marine Corps continues to do more with less, and quality analysis is an essential element necessary to ensure that this characteristic remains true in the future.

APPENDIX A. STATES OF THE MARKOV CHAIN

This appendix contains a listing of Markov chain states developed for the assault support aircraft combat sustainability model. Following the state listing are the probability transition matrix , P matrix, and the time transition matrix, T matrix, for the H-60 base case and Appendix C contains the APL function which transforms the H-60 base case to the V-22 base case.

- State 0--AIRCRAFT UP AWAITING MISSION ASSIGNMENT
- State 1--MISSION ARRIVES AWAITING AIRCRAFT
- State 2--PREPARED ASSAULT MISSION ASSIGNED
- State 3--PREPARED RESUPPLY MISSION ASSIGNED
- State 4--PREPARED RECON MISSION ASSIGNED
- State 5--IMMEDIATE SAR MISSION ASSIGNED
- State 6--IMMEDIATE MEDEVAC MISSION ASSIGNED
- State 7--IMMEDIATE REACTION FORCE MISSION ASSIGNED
- State 8--IMMEDIATE EMERGENCY RESUPPLY ASSIGNED
- State 9--LOAD PASSENGERS (PAX)
- State 10--LOAD EQUIPMENT
- State 11--TAKE OFF AIRFIELD
- State 12--PICK UP EXTERNAL LOAD
- State 13--EXECUTE PRECAUTIONARY LANDING
- State 14--INGRESS LOW LEVEL
- State 15--INGRESS CONTOUR
- State 16--INGRESS NOE (NAP OF THE EARTH)
- State 17--LAND LOW FUEL
- State 18--DETECTED LOW LEVEL
- State 19--DETECTED CONTOUR
- State 20--DETECTED NOE
- State 21--SHOT AT
- State 22--EVASIVE MANEUVER
- State 23--MISSED SHOT
- State 24--HIT WITH NO DAMAGE
- State 25-- HIT WITH DAMAGE
- State 26--AIRCRAFT IN REPAIR
- State 27--EMERGENCY LANDING
- State 28--LAND IN LANDING ZONE (LZ)
- State 29--UNLOAD PAX
- State 30--UNLOAD EQUIP
- State 31--DELIVER EXTERNAL LOAD
- State 32--EGRESS NOE
- State 33--EGRESS CONTOUR
- State 34--EGRESS LOW LEVEL
- State 35--LANDED AIRFIELD
- State 36--REFUEL AIRCRAFT
- State 37--HOLD FOR MISSION SUPPORT
- State 38--ABORT MISSION

- State 39--AIRCRAFT DOWN AWP (AWAITING PARTS)
- State 40--AIRCRAFT DOWN AWM (AWAITING MAINTENANCE)
- State 41--TAKEOFF FROM LZ
- State 42--HIT WITH KILL AIRCRAFT LOSS
- State 43--PILOT ERROR AIRCRAFT LOSS
- State 44--CATASTROPHIC FAILURE AIRCRAFT LOSS

```
H-60 BASE CASE
P MATRIX (45X45)
STATE 0 IS ROW 1, STATE 1 IS ROW 2, ...

ROW 1
        0
        0             0.2222222222  0.1666666667  0.1666666667  0.111111
        0.1111111111  0.1111111111  0.1111111111  0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0
ROW 2
        1
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0
ROW 3
        0
        0             0             0             0             0
        0             0             0             0.5           0.4
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0.1           0             0
        0             0             0             0
ROW 4
        0
        0             0             0             0             0
        0             0             0             0.3           0.6
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0             0             0
        0             0             0.1           0             0
        0             0             0             0
ROW 5
```

```
          0
          0         0         0         0         0
          0         0         0         0.5       0.4
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0.1       0         0
          0         0         0         0
   ROW 6
          0
          0         0         0         0         0
          0         0         0         0.5       0.5
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0
   ROW 7
          0
          0         0         0         0         0
          0         0         0         0.5       0.5
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0
   ROW 8
          0
          0         0         0         0         0
          0         0         0         0.75      0.25
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0
   ROW 9
          0
          0         0         0         0         0
          0         0         0         0.25      0.75
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0         0
          0         0         0         0
   ROW 10
          0
```

```
          0              0              0              0              0
          0              0              0              0              0.24
          0.5            0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.02           0              0
          0.24           0              0              0
      ROW 11
          0
          0              0              0              0              0
          0              0              0              0.3            0
          0.44           0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.02           0              0
          0.24           0              0              0
      ROW 12
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0.24           0.148          0.2            0.2
          0.2            0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0.004          0.004          0              0
          0              0              0.002          0.002
      ROW 13
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.092          0.3            0.3
          0.3            0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.004          0              0
          0              0              0.002          0.002
      ROW 14
          0
          0              0              0              0              0
          0              0              0              0              0
          0.34           0              0              0              0
          0              0              0              0              0
          0              0              0              0              0
          0.12           0              0              0              0
          0              0              0              0              0.08
          0              0              0.12           0              0
          0.34           0              0              0
      ROW 15
          0
          0              0              0              0              0
```

```
       0             0             0             0             0
       0             0             0.04          0             0.01
       0             0.02          0.2           0             0
       0             0             0             0             0
       0             0             0.5           0             0
       0.22          0             0             0             0
       0             0             0.006         0             0
       0             0             0.002         0.002
ROW 16
       0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0.04          0.01          0
       0.01          0.02          0             0.19          0
       0             0             0             0             0
       0             0             0.5           0             0
       0.22          0             0             0             0
       0             0             0.006         0             0
       0             0             0.002         0.002
ROW 17
       0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0.04          0             0.01
       0             0.02          0             0             0.18
       0             0             0             0             0
       0             0             0.51          0             0
       0.23          0             0             0             0
       0             0             0.006         0             0
       0             0             0.002         0.002
ROW 18
       0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0             0             0.1666666667
       0.6333333333 0             0.2           0             0
       0             0             0             0
ROW 19
       0
       0             0             0             0             0
       0             0             0             0             0
       0             0             0.02          0.37          0
       0             0             0             0             0
       0.4           0             0             0             0
       0             0             0.11          0             0
       0.09          0             0             0             0
       0             0             0.006         0             0
       0             0             0.002         0.002
ROW 20
       0
       0             0             0             0             0
       0             0             0             0             0
```

```
          0              0              0.02           0              0.41
          0              0              0              0              0
          0.38           0              0              0              0
          0              0              0.1            0              0
          0.08           0              0              0              0
          0              0              0.006          0              0
          0              0              0.002          0.002
ROW 21
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.02           0              0
          0.45           0              0              0              0
          0.32           0              0              0              0
          0              0              0.11           0              0
          0.09           0              0              0              0
          0              0              0.006          0              0
          0              0              0.002          0.002
ROW 22
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.02           0              0
          0              0              0              0              0
          0              0              0.3            0.3            0.28
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.006          0              0
          0              0.09           0.002          0.002
ROW 23
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.02           0.15           0.15
          0.15           0.02           0.08           0.08           0.08
          0              0              0              0              0
          0              0              0.13           0              0
          0.13           0              0              0              0
          0              0              0.006          0              0
          0              0              0.002          0.002
ROW 24
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.002          0.01           0.01
          0.01           0.002          0.15           0.15           0.15
          0              0.4            0              0              0
          0              0              0.06           0              0
          0.05           0              0              0              0
          0              0              0.002          0              0
          0              0              0.002          0.002
ROW 25
          0
          0              0              0              0              0
          0              0              0              0              0
          0              0              0.002          0.002          0.002
```

```
        0.002            0.002            0.1              0.1              0.1
        0                0.6              0                0                0
        0                0                0.044            0                0
        0.04             0                0                0                0
        0                0                0.002            0                0
        0                0                0.002            0.002
ROW 26
        0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0.99             0                0                0
        0                0                0                0                0
        0                0                0.006            0                0
        0                0                0.002            0.002
ROW 27
      0.8
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0.2
        0                0                0                0
ROW 28
        0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0.25             0                0                0                0
        0                0                0                0                0
        0                0                0                0.25             0.5
        0                0                0                0
ROW 29
        0
        0                0                0                0                0
        0                0                0                0.15             0.15
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0                0
        0                0                0                0.3              0.3
        0                0                0                0                0
        0                0                0.006            0                0
        0.09             0                0.002            0.002
ROW 30
        0
        0                0                0                0                0
        0                0                0                0.08             0.08
        0.034            0                0                0                0
        0                0                0                0                0
```

```
          0          0          0          0          0
          0          0          0          0          0.36
          0          0          0          0          0
          0.08       0          0.006      0          0
          0.36       0          0          0
ROW 31
          0
          0          0          0          0          0
          0          0          0          0.08       0.08
          0.034      0          0          0          0
          0          0          0          0          0
          0          0          0          0          0
          0          0          0          0.26       0
          0          0          0          0          0
          0.08       0          0.006      0          0
          0.46       0          0          0
ROW 32
          0
          0          0          0          0          0
          0          0          0          0          0
          0.1        0          0          0          0
          0          0          0          0          0
          0          0          0          0          0
          0          0          0.3        0          0
          0          0          0          0          0.2
          0          0          0          0          0
          0.4        0          0          0
ROW 33
          0
          0          0          0          0          0
          0          0          0          0          0
          0          0          0.04       0          0
          0          0.04       0          0          0.3
          0          0          0          0          0
          0          0          0          0          0
          0          0          0.02       0          0.59
          0          0          0.006      0          0
          0          0          0.002      0.002
ROW 34
          0
          0          0          0          0          0
          0          0          0          0          0
          0          0          0.04       0          0
          0          0.04       0          0.29       0
          0          0          0          0          0
          0          0          0          0          0
          0          0.01       0          0.02       0.59
          0          0          0.006      0          0
          0          0          0.002      0.002
ROW 35
          0
          0          0          0          0          0
          0          0          0          0          0
          0          0          0.04       0          0
          0          0.04       0.28       0          0
          0          0          0          0          0
```

```
     0           0           0           0           0
     0           0           0.02        0           0.61
     0           0           0.006       0           0
     0           0           0.002       0.002
ROW 36
    0.18
     0           0           0           0           0
     0           0           0           0.16        0.16
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0.16        0.16
     0           0           0           0           0
     0.17        0           0.006       0           0
     0           0           0.002       0.002
ROW 37
    0.6666666667
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0.3333333333
     0           0           0           0
ROW 38
     0
     0           0           0           0           0
     0           0           0           0           0
     1           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0
ROW 39
    0.25
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0.75
     0           0           0           0           0
     0           0           0           0
ROW 40
     0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0           0           0           0           0
     0.5         0           0           0           0
```

```
           0          0          0          0          0
           0          0          0          0          0.5
           0          0          0          0
   ROW 41
           0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           1          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0
   ROW 42
           0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0.09       0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0.3        0.3        0.3        0
           0          0          0.006      0          0
           0          0          0.002      0.002
   ROW 43
           0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          1          0          0
   ROW 44
           0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          1          0
   ROW 45
           0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
           0          0          0          0          0
```

```
      0            0            0            0            0
      0            0            0            1


H-60 BASE CASE
T MATRIX (42X45)
STATE 0 IS ROW 1, STATE 1 IS ROW 2, ...

ROW 1
        0            0            18           18
       18           18            18           18
       18            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0
ROW 2
        0.5          0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0
ROW 3
        0            0             0            0
        0            0             0            0
        0            5             5            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             3            0
        0            0             0            0
        0
ROW 4
        0            0             0            0
        0            0             0            0
        0            5             5            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
        0            0             0            0
```

```
           0              0              0              0
           0              0              3              0
           0              0              0              0
           0
ROW  5
           0              0              0              0
           0              0              0              0
           0              5              5              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              3              0
           0              0              0              0
           0
ROW  6
           0              0              0              0
           0              0              0              0
           0              0.5            0.5            0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0
ROW  7
           0              0              0              0
           0              0              0              0
           0              0.5            0.5            0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0
ROW  8
           0              0              0              0
           0              0              0              0
           0              0.5            0.5            0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0              0              0              0
           0
```

```
ROW 9
      0               0               0               0
      0               0               0               0
      0               0.5             0.5             0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0
ROW 10
      0               0               0               0
      0               0               0               0
      0               0               0.03333333333   0.05
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0.03333333333   0
      0               0.03333333333   0               0
      0
ROW 11
      0               0               0               0
      0               0               0               0
      0               0.03333333333   0               0.08333333333
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0.03333333333   0
      0               0.08333333333   0               0
      0
ROW 12
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0.05            0.03333333333   0.08333333333   0.08333333333
      0.08333333333   0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0               0               0
      0               0.1             0.03333333333   0
      0               0               0               0.01666666667
      0.03333333333
ROW 13
      0               0               0               0
      0               0               0               0
      0               0               0               0
```

```
          0               0.03333333333   0.08333333333   0.08333333333
          0.08333333333   0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0.03333333333   0
          0               0               0               0.01666666667
          0.03333333333
ROW 14
          0               0               0               0
          0               0               0               0
          0               0               0               0.3333333333
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0.08333333333   0
          0               0               0               0
          0               0               0               0.03333333333
          0               0               0.08333333333   0
          0               0.3333333333    0               0
          0
ROW 15
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0.5             0               0.3333333333
          0               1               0.5             0
          0               0               0               0
          0               0               0               0
          1               0               0               1
          0               0               0               0
          0               0               0.5             0
          0               0               0               0.5
          0.5
ROW 16
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0.5             1               0
          0.5             1               0               0.5
          0               0               0               0
          0               0               0               0
          1               0               0               1
          0               0               0               0
          0               0               0.5             0
          0               0               0               0.5
          0.5
ROW 17
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0.5             0               0.5
          0               0.5             0               0
          0.5             0               0               0
          0               0               0               0
```

```
          1.5              0               0               1.5
          0                0               0               0
          0                0               0.5             0
          0                0               0               0.5
          0.5
 ROW  18
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0               0               0.08333333333
          2                0               0.08333333333   0
          0                0               0               0
          0
 ROW  19
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0.03333333333   0.03333333333   0
          0                0               0               0
          0                0.01666666667   0               0
          0                0               0               0
          0.08333333333    0               0               0.08333333333
          0                0               0               0
          0                0               0.03333333333   0
          0                0               0               0.03333333333
          0.03333333333
 ROW  20
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0.03333333333   0               0.03333333333
          0                0               0               0
          0                0.01666666667   0               0
          0                0               0               0
          0.08333333333    0               0               0.08333333333
          0                0               0               0
          0                0               0.03333333333   0
          0                0               0               0.03333333333
          0.03333333333
 ROW  21
          0                0               0               0
          0                0               0               0
          0                0               0               0
          0                0.01666666667   0               0
          0.01666666667    0               0               0
          0                0.01666666667   0               0
          0                0               0               0
          0.03333333333    0               0               0.03333333333
          0                0               0               0
          0                0               0.01666666667   0
          0                0               0               0.01666666667
```

```
        0.01666666667
ROW 22
        0               0               0               0
        0               0               0               0
        0               0               0               0
        0               0.01666666667   0               0
        0               0               0               0
        0               0               0               0.01666666667
        0.01666666667   0.01666666667   0               0
        0               0               0               0
        0               0               0               0
        0               0               0.01666666667   0
        0               0               0.01666666667   0.01666666667
        0.01666666667
ROW 23
        0               0               0               0
        0               0               0               0
        0               0               0               0
        0               0               0.08333333333   0.08333333333
        0.08333333333   0.25            0.08333333333   0.08333333333
        0.08333333333   0               0               0
        0               0               0               0
        0.25            0               0               0.25
        0               0               0               0
        0               0               0.25            0
        0               0               0               0.25
        0.25
ROW 24
        0               0               0               0
        0               0               0               0
        0               0               0               0
        0               0.3333333333    0.008333333333  0.008333333333
        0.008333333333  0.3333333333    0.008333333333  0.008333333333
        0.008333333333  0               0.008333333333  0
        0               0               0               0
        0.3333333333    0               0               0.3333333333
        0               0               0               0
        0               0               0.3333333333    0
        0               0               0               0.3333333333
        0.3333333333
ROW 25
        0               0               0               0
        0               0               0               0
        0               0               0               0
        0               0.3333333333    0.008333333333  0.008333333333
        0.008333333333  0.3333333333    0.008333333333  0.008333333333
        0.008333333333  0               0.008333333333  0
        0               0               0               0
        0.3333333333    0               0               0.3333333333
        0               0               0               0
        0               0               0.3333333333    0
        0               0               0               0.3333333333
        0.3333333333
ROW 26
        0               0               0               0
        0               0               0               0
```

```
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0.08333333333
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0.08333333333      0
            0                  0                  0                  0.08333333333
            0.08333333333
     ROW 27
            5                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
           10                  0                  0                  0
            0
     ROW 28
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  3                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  3
            2                  0                  0                  0
            0
     ROW 29
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0.03333333333      0.03333333333      0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0.01666666667      0.01666666667      0
            0                  0                  0                  0
            0                  0.03333333333      0                  0
            0                  0.03333333333      0                  0.03333333333
            0.03333333333
     ROW 30
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0.03333333333      0.03333333333      0.03333333333
            0                  0                  0                  0
            0                  0                  0                  0
            0                  0                  0                  0
```

```
             0                 0                 0                 0
             0                 0                 0.03333333333     0
             0                 0                 0                 0
             0.03333333333     0                 0.03333333333     0
             0                 0.03333333333     0                 0
             0
      ROW 31
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0.08333333333     0.08333333333     0.08333333333
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0.08333333333     0                 0
             0                 0                 0                 0
             0.08333333333     0                 0.08333333333     0
             0                 0.08333333333     0                 0
             0
      ROW 32
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0.03333333333
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0.03333333333     0                 0                 0
             0                 0                 0                 0.03333333333
             0                 0                 0                 0
             0                 0.03333333333     0                 0
             0
      ROW 33
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0.5               0                 0
             0                 0.6666666667      0                 0
             0.08333333333     0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0.3333333333      0                 1.5
             0                 0                 0.6666666667      0
             0                 0                 0                 0.5
             0.5
      ROW 34
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0.5               0                 0
             0                 0.6666666667      0                 0.01666666667
             0                 0                 0                 0
             0                 0                 0                 0
             0                 0                 0                 0
             0.08333333333     0                 0.6666666667      1
             0                 0                 0.6666666667      0
```

```
        0                    0                    0                    0.5
        0.5
ROW 35
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0.5                  0                    0
        0                    0.6666666667         0.01666666667        0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0.5                  0                    1
        0                    0                    0.6666666667         0
        0                    0                    0                    0.08333333333
        0.5
ROW 36
        0.25                 0                    0                    0
        0                    0                    0                    0
        0                    0.25                 0.25                 0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0.25                 0.25                 0
        0                    0                    0                    0
        0.1666666667         0                    0.08333333333        0
        0                    0                    0                    0.25
        0.25
ROW 37
        0.3333333333         0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0.3333333333         0                    0                    0
        0
ROW 38
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0.25
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0                    0                    0                    0
        0
ROW 39
        0.25                 0                    0                    0
```

```
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0.5
          0               0               0               0
          0               0               0               0
          0
ROW 40
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               6               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          6               0               0               0
          0
ROW 41
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               4               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0
ROW 42
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0.08333333333   0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0               0               0               0
          0.08333333333   0.08333333333   0.08333333333   0
          0               0               0.08333333333   0
          0               0               0               0.08333333333
          0.08333333333
```

APPENDIX B. ANALYTICAL MODEL AND SAMPLE RESULTS

The two programs listed in this appendix model the combat sustainability of an assault support aircraft. By employing first step analysis on a finite state Markov chain the function PABIG determines the probability of absorption by a specific state given an initial state. The function TMBGABS determines the expected time to absorption and the expected number of visits to a given transient state prior to absorption. These values are then transformed into measures of performance. The listing of each function is followed by a sample output for the H-60 base case. (State 0 is Row 1, State 1 is Row 2, ... , when referring to the matrices.)

```
∇PABIG[⎕]∇
      - PABIG P;ABS42;ABS43;ABS44;B;C;D;A;AA
-1-   ⍝TAKES THE TRANSITION MATRIX OF A FINITE STATE MARKOV
-2-   ⍝CHAIN AND VIA FIRST STEP ANALYSIS PROVIDES PROB OF
-3-   ⍝ABSORPTION IN ONE STATE CONDITIONED ON INITIAL STATE
-4-   ⍝THIS MODEL SOLVES THIS PROBLEM FOR A 45 STATE CHAIN
-5-   ⍝WITH THREE ABSORBING STATES
[6]    A← 42 42 ↑P
[7]    IDEN← 42 42 ⍴1,42⍴0
[8]    AA←A-IDEN
[9]    B← 42 1 ⍴P[;43]
[10]   C← 42 1 ⍴P[;44]
[11]   D← 42 1 ⍴P[;45]
[12]   ABS42←-B⌹AA
[13]   ABS43←-C⌹AA
[14]   ABS44←-D⌹AA
[15]   Q← 42 1 ⍴0,⍳41
[16]   'PROB ABS DUE TO THE THREAT'
[17]   'FROM           IS '
[18]   SHOW1←Q,ABS42
[19]   ⍉SHOW1
[20]   ' '
[21]   ' '
[22]   'PROB OF ABS DUE TO PILOT ERROR'
[23]   'FROM           IS '
[24]   SHOW2←Q,ABS43
[25]   ⍉SHOW2
[26]   ' '
[27]   ' '
[28]   'PROB OF ABS DUE TO CATASTROPHIC FAILURE OF A/C'
[29]   'FROM           IS '
[30]   SHOW3←Q,ABS44
[31]   ⍉SHOW3
       ∇
```

```
          PABIG PBASE
PROB ABS DUE TO THE THREAT
FROM             IS
  0               0.399497916
  1               0.399497916
  2               0.3994496479
  3               0.3994558934
  4               0.3994496479
  5               0.3995559089
  6               0.3995559089
  7               0.3995481021
  8               0.3995637158
  9               0.3995402952
 10               0.3995715226
 11               0.3988949288
 12               0.3994186412
 13               0.399558875
 14               0.4017731502
 15               0.4013396551
 16               0.4004378249
 17               0.3990803369
 18               0.4212608539
 19               0.4200524344
 20               0.4163155474
 21               0.456078276
 22               0.4032473503
 23               0.408258715
 24               0.4060270441
 25               0.3978939903
 26               0.399497916
 27               0.399497916
 28               0.3985411905
 29               0.4003081312
 30               0.4003545575
 31               0.3994635109
 32               0.4022203334
 33               0.4031646317
 34               0.4032422932
 35               0.3981792451
 36               0.399497916
 37               0.3988949288
 38               0.3985089128
 39               0.399497916
 40               0.399497916
 41               0.4009395297


PROB OF ABS DUE TO PILOT ERROR
FROM             IS
  0               0.300251042
  1               0.300251042
  2               0.300275176
  3               0.3002720533
  4               0.300275176
  5               0.3002220455
  6               0.3002220455
```

```
  7             0.300225949
  8             0.3002181421
  9             0.3002298524
 10             0.3002142387
 11             0.3005525356
 12             0.3002906794
 13             0.3002205625
 14             0.2991134249
 15             0.2993301724
 16             0.2997810875
 17             0.3004598316
 18             0.2893695731
 19             0.2899737828
 20             0.2918422263
 21             0.271960862
 22             0.2983763249
 23             0.2958706425
 24             0.296986478
 25             0.3010530049
 26             0.300251042
 27             0.300251042
 28             0.3007294048
 29             0.2998459344
 30             0.2998227212
 31             0.3002682445
 32             0.2988898333
 33             0.2984176841
 34             0.2983788534
 35             0.3009103774
 36             0.300251042
 37             0.3005525356
 38             0.3007455436
 39             0.300251042
 40             0.300251042
 41             0.2995302351

PROB OF ABS DUE TO CATASTROPHIC FAILURE OF A/C
FROM           IS
  0             0.300251042
  1             0.300251042
  2             0.300275176
  3             0.3002720533
  4             0.300275176
  5             0.3002220455
  6             0.3002220455
  7             0.300225949
  8             0.3002181421
  9             0.3002298524
 10             0.3002142387
 11             0.3005525356
 12             0.3002906794
 13             0.3002205625
 14             0.2991134249
 15             0.2993301724
 16             0.2997810875
```

```
 17              0.3004598316
 18              0.2893695731
 19              0.2899737828
 20              0.2918422263
 21              0.271960862
 22              0.2983763249
 23              0.2958706425
 24              0.296986478
 25              0.3010530049
 26              0.300251042
 27              0.300251042
 28              0.3007294048
 29              0.2998459344
 30              0.2998227212
 31              0.3002682445
 32              0.2988898333
 33              0.2984176841
 34              0.2983788534
 35              0.3009103774
 36              0.300251042
 37              0.3005525356
 38              0.3007455436
 39              0.300251042
 40              0.300251042
 41              0.2995302351
```

```
∇TMBGABS[⎕]∇
      - P TMBGABS T;P;SOJ;A;AA;IDEN;TIMABS;SHOW4;VIZITS
-1-   ⍝FUNCTION USES TRANSITION AND TIME MATRICES ASSOCIATED
-2-   ⍝WITH A MARKOV CHAIN, SOLVES MEAN TIME TO ABSORPTION
-3-   ⍝AND MEAN NUMBER OF VISITS TO TRANSIENT STATES PRIOR
[4]   ⍝TO ABSORPTION GIVEN THE INITIAL STATE.
[5]    P← 42 45 ↑P
[6]    SOJ← 42 1 ⍴+/(P×T)
[7]    A← 42 42 ↑P
[8]    IDEN← 42 42 ⍴1,42⍴0
[9]    AA←A-IDEN
[10]   TIMABS←-SOJ⌹AA
[11]   'MEAN TIME TO ABS '
[12]   'FROM           IS '
[13]   Q← 42 1 ⍴0,⍳41
[14]   SHOW4←Q,TIMABS
[15]   ⍕SHOW4
[16]   VIZITS←IDEN-A
[17]   WW←⌹VIZITS
[18]  ⍝'NUMBER OF VISITS TO STATE J FROM I BEFORE ABS'
[19]  ⍝ ⍕WW
[20]   'THE FOLLOWING EXPECTED VALUES HAVE BEEN DETERMINED'
[21]   ' '
-22-   'HOURS MH-60 TO SURVIVE ',⍕(+⌿((42 1 ⍴PINTA)×TIMABS))
[23]   ' '
[24]   E←PINTA+.×WW
[25]   'PAX TRANS                     ',⍕(E[30]×12)
[26]   'EQUIP TRANS INT               ',⍕(E[31]×6300)
[27]   'TIME REPAIR                   ',⍕(E[27]×SOJ[27;])
```

```
[28]  'TIME AWM                     ',⍕(E[41]×SOJ[41;])
[29]  'TIME AWP                     ',⍕(E[40]×SOJ[40;])
      ∇


      PBASE TMBGABS TBASE
MEAN TIME TO ABS
FROM         IS
   0         381.6696153
   1         382.1696153
   2         366.1162477
   3         366.1324219
   4         366.1162477
   5         360.6052595
   6         360.6052595
   7         360.5850418
   8         360.6254772
   9         360.0648241
  10         360.1456949
  11         358.8212719
  12         358.6160232
  13         366.0932885
  14         358.8400292
  15         359.0781315
  16         359.9755671
  17         381.7447992
  18         347.5022718
  19         348.0763966
  20         350.1484738
  21         331.9415723
  22         356.7513292
  23         352.4262442
  24         353.4536437
  25         395.0197859
  26         390.1696153
  27         396.6696153
  28         361.2181907
  29         364.0898539
  30         363.9277455
  31         362.5418359
  32         362.8372862
  33         362.0578725
  34         362.1712315
  35         368.5342311
  36         386.1696153
  37         359.0712719
  38         372.2555771
  39         398.1696153
  40         394.1696153
  41         361.3851798

THE FOLLOWING EXPECTED VALUES HAVE BEEN DETERMINED

HOURS MH-60 EXPECTED TO SURVIVE 360.7788696
```

PAX TRANS	125.0755527
EQUIP TRANS INT	70876.1465
TIME REPAIR	30.49567063
TIME AWM	14.73997585
TIME AWP	1.852871186

APPENDIX C. SENSITIVITY ANALYSIS FUNCTIONS

The APL functions listed in this appendix reflect the specific values used in performing the sensitivity analyses on the parameters of probability of detection, survivability and maintainability. The H-60 functions are followed by the V-22 functions. (State 0 is Row 1, State 1 is Row 2, ... , when referring to the matrices.)

```
∇HDETECT[⎕]∇
     ∇ HDETECT P;PDET
-1-   ⍝FUNCTION VARIES PDET IN INBOUND AND EGRESS STATES
-2-   ⍝WITH PROBS OF GETTING TO LZ AND AEF FROM THOSE STATES
[3]    PDET←P
[4]    PDET[15;19]←300÷500
[5]    PDET[15;29]←150÷500
[6]    PDET[15;32]←10÷500
[7]    PDET[16;20]←295÷500
[8]    PDET[16;29]←150÷500
[9]    PDET[16;32]←10÷500
[10]   PDET[17;21]←290÷500
[11]   PDET[17;29]←155÷500
[12]   PDET[17;32]←15÷500
[13]   PDET[33;21]←250÷500
[14]   PDET[33;36]←195÷500
[15]   PDET[34;20]←245÷500
[16]   PDET[34;36]←195÷500
[17]   PDET[35;19]←240÷500
[18]   PDET[35;36]←205÷500
[19]   HBAD←PDET
[20]   'HELO DETECTION PROB HIGH '
[21]  ⍝⍕(+/PDET)
[22]   PDET TMBGABS TBASE
[23]   PDET[15;19]←50÷500
[24]   PDET[15;29]←275÷500
[25]   PDET[15;32]←135÷500
[26]   PDET[16;20]←45÷500
[27]   PDET[16;29]←275÷500
[28]   PDET[16;32]←135÷500
[29]   PDET[17;21]←40÷500
[30]   PDET[17;29]←280÷500
[31]   PDET[17;32]←140÷500
[32]   PDET[33;21]←50÷500
[33]   PDET[33;36]←395÷500
[34]   PDET[34;20]←45÷500
[35]   PDET[34;36]←395÷500
[36]   PDET[35;19]←40÷500
[37]   PDET[35;36]←405÷500
[38]   ' '
[39]   HGOOD←PDET
[40]   'HELO DETECTION PROB LOW '
```

```
[41] ⍝⍕(+/PDET)
[42]  PDET TMBGABS TBASE
      ∇

∇HSURVIVE[⎕]∇
      ∇ HSURVIVE P;HSURV
-1-  ⍝VARIES THE PROB DAMAGE TO A/C, HAVING BEEN HIT,
-2-  ⍝REQUIRES AN EMERGENCY LANDING VICE ABLE TO CONTINUE
[3]   HSURV←P
[4]   HSURV[22;26]←240÷500
[5]   HSURV[22;25]←50÷500
[6]   HBBAD←HSURV
[7]   'HELO SURVIVAL LOW'
[8]  ⍝⍕HSURV[22;]
[9]   HSURV TMBGABS TBASE
[10]  HSURV[22;26]←40÷500
[11]  HSURV[22;25]←250÷500
[12]  ' '
[13]  HGGOOD←HSURV
[14]  'HELO SURVIVAL HIGH'
[15] ⍝⍕HSURV[22;]
[16]  HSURV TMBGABS TBASE
      ∇

∇HMAINT[⎕]∇
      ∇ PROB HMAINT TIME
-1-  ⍝VARIES PROB OF REQUIRING MAINT, AWAITING PARTS AND
-2-  ⍝AWAITING MAINT AND THEIR EXPECTED SOJURN TIMES
[3]   PLOG←PROB
[4]   TLOG←TIME
[5]   PLOG[28;40]←0.6
[6]   PLOG[28;41]←0.3
[7]   PLOG[28;27]←0.1
[8]   TLOG[40;41]←18
[9]   TLOG[40;27]←18
[10]  PLOG[14;27]←32÷100
[11]  PLOG[14;12]←24÷100
[12]  PLOG[14;42]←24÷100
[13]  PLOG[37;1]←2÷5
[14]  PLOG[37;41]←3÷5
[15]  TLOG[27;1]←7
[16]  TLOG[27;41]←12
[17]
[18] ⍝⍕(+/PLOG)
[19]  'MAINT POOR'
[20]  PLOG TMBGABS TLOG
[21]
[22]  ' '
[23]  PLOG[28;40]←0.1
[24]  PLOG[28;41]←0.3
[25]  PLOG[28;27]←0.6
[26]  TLOG[40;41]←3
[27]  TLOG[40;27]←3
[28]  PLOG[14;27]←2÷100
[29]  PLOG[14;12]←39÷100
[30]  PLOG[14;42]←39÷100
```

```
[31]   PLOG[37;1]←4÷5
[32]   PLOG[37;41]←1÷5
[33]   TLOG[27;1]←3
[34]   TLOG[27;41]←8
[35]  ⍝⍕(+/TLOG)
[36]  ⍝⍕(+/PLOG)
[37]   ' '
[38]   'MAINT GOOD'
[39]   PLOG TMBGABS TLOG
       ∇

       ∇PVB[⎕]∇
       ∇ PVB P
-1-   ⍝FUNCTION CONVERTS BASE CASE PROBS TO V22 BASE CASE
-2-   ⍝DECREASING DET PROBS IN/OUT AND REDUCING TRIPS REQD
[3]    PVBS←P
[4]    PVBS[15;19]←80÷500
[5]    PVBS[15;29]←260÷500
[6]    PVBS[15;32]←120÷500
[7]    PVBS[16;20]←75÷500
[8]    PVBS[16;29]←260÷500
[9]    PVBS[16;32]←120÷500
[10]   PVBS[17;21]←70÷500
[11]   PVBS[17;29]←265÷500
[12]   PVBS[17;32]←125÷500
[13]   PVBS[33;21]←130÷500
[14]   PVBS[33;36]←315÷500
[15]   PVBS[34;20]←125÷500
[16]   PVBS[34;36]←315÷500
[17]   PVBS[35;19]←120÷500
[18]   PVBS[35;36]←325÷500
[19]   PVBS[36;1]←110÷500
[20]   PVBS[36;11]←70÷500
[21]   PVBS[36;10]←70÷500
[22]
       ∇
∇VDETECT[⎕]∇
       ∇ VDETECT P;VDET
[1]    VDET←P
[2]    VDET[15;19]←280÷500
[3]    VDET[15;29]←160÷500
[4]    VDET[15;32]←20÷500
[5]    VDET[16;20]←275÷500
[6]    VDET[16;29]←160÷500
[7]    VDET[16;32]←20÷500
[8]    VDET[17;21]←270÷500
[9]    VDET[17;29]←165÷500
[10]   VDET[17;32]←25÷500
[11]   VDET[33;21]←230÷500
[12]   VDET[33;36]←215÷500
[13]   VDET[34;20]←225÷500
[14]   VDET[34;36]←215÷500
[15]   VDET[35;19]←220÷500
[16]   VDET[35;36]←225÷500
[17]   VBAD←VDET
[18]   'V-22 DETECTION PROB HIGH '
```

```
[19]   ⍕(+/VDET)
[20]   VDET TMBVABS TBASE
[21]   VDET[15;19]←50÷500
[22]   VDET[15;29]←275÷500
[23]   VDET[15;32]←135÷500
[24]   VDET[16;20]←45÷500
[25]   VDET[16;29]←275÷500
[26]   VDET[16;32]←135÷500
[27]   VDET[17;21]←40÷500
[28]   VDET[17;29]←280÷500
[29]   VDET[17;32]←140÷500
[30]   VDET[33;21]←50÷500
[31]   VDET[33;36]←395÷500
[32]   VDET[34;20]←45÷500
[33]   VDET[34;36]←395÷500
[34]   VDET[35;19]←40÷500
[35]   VDET[35;36]←405÷500
[36]   ' '
[37]   'V-22 DETECTION PROB LOW '
[38]   VGOOD←VDET
[39]   ⍕(+/VDET)
[40]   VDET TMBVABS TBASE
       ∇

∇VSURVIVE[⎕]∇
       ∇ VSURVIVE P;VSURV
[1]    VSURV←P
[2]    VSURV[22;26]←240÷500
[3]    VSURV[22;25]←50÷500
[4]    'V-22 SURVIVAL LOW'
[5]    VBBAD←VSURV
[6]    ⍕VSURV[22;]
[7]    VSURV TMBVABS TBASE
[8]    VSURV[22;26]←40÷500
[9]    VSURV[22;25]←250÷500
[10]   ' '
[11]   'V-22 SURVIVAL HIGH'
[12]   ⍕VSURV[22;]
[13]   VGGOOD←VSURV
[14]   VSURV TMBVABS TBASE
       ∇

∇VMAINT[⎕]∇
       ∇ PROB VMAINT TIME
[1]    PLOG←PROB
[2]    TLOG←TIME
[3]    PLOG[28;40]←0.6
[4]    PLOG[28;41]←0.3
[5]    PLOG[28;27]←0.1
[6]    TLOG[40;41]←18
[7]    TLOG[40;27]←18
[8]    PLOG[14;27]←32÷100
[9]    PLOG[14;12]←24÷100
[10]   PLOG[14;42]←24÷100
[11]   PLOG[37;1]←2÷5
[12]   PLOG[37;41]←3÷5
```

```
[13]  TLOG[27;1]←7
[14]  TLOG[27;41]←12
[15]
[16] ⍝⍕(+/PLOG)
[17]  'V-22 MAINT POOR'
[18]  PLOG TMBVABS TLOG
[19]
[20]  ' '
[21]  PLOG[28;40]←0.1
[22]  PLOG[28;41]←0.3
[23]  PLOG[28;27]←0.6
[24]  TLOG[40;41]←3
[25]  TLOG[40;27]←3
[26]  PLOG[14;27]←2÷100
[27]  PLOG[14;12]←39÷100
[28]  PLOG[14;42]←39÷100
[29]  PLOG[37;1]←4÷5
[30]  PLOG[37;41]←1÷5
[31]  TLOG[27;1]←3
[32]  TLOG[27;41]←8
[33] ⍝⍕(+/TLOG)
[34] ⍝⍕(+/PLOG)
[35]  ' '
[36]  'V-22 MAINT GOOD'
[37]  PLOG TMBVABS TLOG
      ∇
```

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