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A SIMULATION STUDY OF ESTIMATES OF
SYSTEM AVAILABILITY

By

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A Simulation Study of Estimates of System Availability

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ABSTRACT

A system which is either working or under repair is modeled as an alternating renewal process. Data is in the form of a finite number of independent lifetimes and repair times. Several semi-parametric estimators of the probability that the system is up at a finite time t are studied via simulation. The estimators are based on an exponential approximation to the true system availability at time t and use empirical Laplace transforms of the lifetimes and repair times.

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I. INTRODUCTION

Assume a new system has been installed at t = 0 and is up. The system is subject to failure and repair. An initial simple model for this situation is an alternating renewal process.

Let U_i be the i^{th} time between failures and D_i be the i^{th} repair time. Assume $\{U_i\}$ are independent identically distributed random variables with distribution F and $\{D_i\}$ are independent identically distributed random variables independent of $\{U_i\}$ with distribution G. Let

$$X_t = \begin{cases} 1 & \text{if the system is up at time t} \\ 0 & \text{otherwise.} \end{cases}$$

A renewal argument yields

$$A(t) \equiv P\{X_t = 1 \mid X_0 = 1\}$$

$$= 1 - F(t) + \int_0^t (F^*G)(dy)A(t-y) \qquad (1.1)$$

where F * G denotes the convolution of F and G. It can be shown that the long run proportion of time the system is up is

$$A(\infty) \equiv \lim_{t\to\infty} A(t) = \frac{E(U_1)}{E(U_1) + E(D_1)} .$$

If F is exponential with mean λ^{-1} and G is exponential with mean μ^{-1} the equation (1.1) has the solution

$$A(t) = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}} + \frac{\frac{1}{\mu}}{\frac{1}{\lambda} + \frac{1}{\mu}} e^{-(\lambda+\mu)t}, \qquad t \geq 0 .$$

However, for most distributions F and G, equation (1.1) has a very complicated closed form solution.

Computing the Laplace transform of equation (1.1)

$$\hat{A}(s) = \int_0^\infty e^{-st} A(t)dt = \int_0^\infty e^{-st}\overline{F}(t)dt + \int_0^\infty e^{-st}\int_0^t (F*G)(ds)A(t-s)$$

$$= \frac{1}{s}[1 - \hat{F}(s)] + \hat{F}(s)\hat{G}(s)\hat{A}(s)$$

where

$$\hat{F}(s) = \int_0^\infty e^{-st}F(dt)$$

$$\hat{G}(s) = \int_0^\infty e^{-st}G(dt) .$$

Solving the equation results in

$$\hat{A}(s) = \frac{\frac{1}{s}[1 - \hat{F}(s)]}{1 - \hat{F}(s)\hat{G}(s)} . \tag{1.2}$$

The following approximation to A(t) is proposed

$$A_a(t) = A(\infty) + (1 - A(\infty))e^{-\beta t} \tag{1.3}$$

for some β . One choice for β is to set

$$\int_0^\infty se^{-st}A_a(t)dt = \int_0^\infty se^{-st}A(t)dt$$

and solve for β at $s = \frac{1}{t}$. More specifically let

$$P(t) = \frac{A(t) - A(\infty)}{1 - A(\infty)} \tag{1.4}$$

and set

$$s\hat{P}(s) \equiv \int_0^\infty se^{-st}P(t) = \int_0^\infty se^{-st}e^{-\beta t}dt \tag{1.5}$$

Thus

$$s\hat{P}(s) = \frac{s\hat{A}(s) - A(\infty)}{1 - A(\infty)} = \frac{1}{1 + \beta(\frac{1}{s})}$$

and

$$\beta = \frac{1 - s\hat{P}(s)}{\hat{P}(s)} \tag{1.6}$$

where

$$s\hat{A}(s) = \frac{1 - \hat{F}(s)}{1 - \hat{F}(s)\hat{G}(s)} \tag{1.7}$$

and $s = \frac{1}{t}$.

The choice of β can be "tuned" to t by use of a different weight function. For example, β can be chosen so that

$$\int_0^{\infty} w(t)A_a(t)dt = \int_0^{\infty} w(t)A(t)dt \tag{1.8}$$

where w(t) is the density function of the median of 5 independent exponential random variables each having mean $\frac{1}{s}$ and s is chosen so that the expected value of the median equals t. The resulting expression for β involves the Laplace transforms of F and G. Other approximations can be found in Gaver and Jacobs[Ref. 1].

Suppose that data are available on the successive up times and repair times of the system; say $u_1, u_2, ..., u_n$ and $d_1, d_2, ..., d_n$. A problem of practical interest is to use these data to estimate the probability the system will be up at a finite time t, often referred to as the availability of the system at time t. One estimate that is often used in practice is the ratio

$$\frac{\bar{u}}{(\bar{u} + \bar{d})}$$

where

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

and

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i \, .$$

This is an estimator of the long-run proportion of time the system is up. This estimator can be quite biased for small times, t.

In this thesis estimators of the availability of the system at time t are investigated by simulation. Three of the estimators are based on the exponential approximation (equation (1.3)) and use empirical Laplace transforms to estimate the Lapiace transforms of F and G. Another estimator that is investigated is one which simulates the alternating renewal process using the observed data and a bootstrap-like sampling scheme.

The specific estimators are described in Chapter 2. Chapter 3 contains details of the simulation experiment and results. Conclusions from the study are given in Chapter 4.

II. NATURE OF THE PROBLEM

A. PROBLEM

A system subject to failure and repair is modeled by an alternating renewal process. The problem is to estimate the system availability at time t from a sample of lifetimes, $u_1, u_2, ..., u_n$ and repair times $d_1, d_2, ..., d_n$. In this chapter four estimators will be described. The first is a computationally intensive bootstrap estimator. The last three estimators are based on the exponential approximation (equation(1.3)).

1. Empirical Estimator

This estimator simulates the alternating renewal process using the observed data $u_1, u_2, ..., u_n$ and $d_1, d_2, ..., d_n$. Specifically, a lifetime is drawn at random with replacement from $\{u_i\}$. If the lifetime is greater than t, the system is said to be up. If the lifetime is less than t, a repair time is drawn at random with replacement from $\{d_i\}$ and added to the lifetime. If the resulting sum is greater than t, the system is said to be down. If the sum is less than t another lifetime is drawn at random with replacement from $\{u_i\}$ and added to the sum. If the resulting sum is greater than t, the system is said to be up. If it is less than t, a repair time is drawn at random with replacement from $\{d_i\}$, etc. This procedure is repeated $N_e = 100$ times and the fraction of times the system is up is computed. This is the empirical estimate of availability $\hat{A}_{emp}(t)$. Confidence intervals are obtained for this estimate by repeating the whole procedure N_b times. The order statistics of the resulting N_b availability estimates are used to obtain confidence intervals. Specifically the $(\frac{\alpha}{2})N_b{}^{th}$ and $(1-\frac{\alpha}{2})N_b{}^{th}$ order statistic are used to compute two-sided $(1 - \alpha)$% confidence interval, and the $\alpha N_b{}^{th}$ order statistic and 1 are used to compute one-sided $(1 - \alpha$)% confidence interval. The procedure described is a Bootstrap, as discussed extensively by Efron [Refs. 2,3].

2. Exponential estimator

The exponential estimator is based on the exponential approximation (equation(1.3)) with weighting function the exponential density resulting in equation (1.6). The estimator is obtained by replacing $A(\infty)$, $\hat{F}(s)$ and $\hat{G}(s)$ by the following estimates.

$$\hat{A}(\infty) = \frac{\overline{u}}{\overline{u} + \overline{d}} \; ; \qquad (2.1)$$

$\hat{F}(s)$ is estimated by the empirical Laplace transform

$$\phi_U(s) = \frac{1}{n} \sum_{i=1}^{n} e^{-su_i} \; ; \qquad (2.2)$$

$\hat{G}(s)$ is estimated by

$$\phi_D(s) = \frac{1}{n} \sum_{i=1}^{n} e^{-sd_i} \; ; \qquad (2.3)$$

and s is taken to be $\frac{1}{t}$. The availability at time t is estimated by

$$\hat{A}_e(t) = \hat{A}(\infty) + (1 - \hat{A}(\infty)) e^{-\hat{\beta}_e t} \qquad (2.4)$$

where

$$\hat{\beta}_e = \frac{1 - \frac{1}{t} \hat{p}(\frac{1}{t})}{\hat{p}(\frac{1}{t})} \qquad (2.5)$$

with

$$\hat{p}(\frac{1}{t}) = \frac{\frac{1}{t} \hat{a}(\frac{1}{t}) - \hat{A}(\infty)}{\frac{1}{t}(1 - \hat{A}(\infty))} \qquad (2.6)$$

and

$$\frac{1}{t} \hat{a}(\frac{1}{t}) = \frac{1 - \phi_U(\frac{1}{t})}{1 - \phi_U(\frac{1}{t}) \phi_D(\frac{1}{t})} \; . \qquad (2.7)$$

A Taylor expansion of equation (1.6) for small s yields

$$\beta \simeq \frac{2E[C_1]\{E[C_1] - E[U_1]\}}{E[C_1^2]E[U_1] - E[C_1]E[U_1^2]} \qquad as\ s \to 0 \tag{2.8}$$

where

$$C_1 = U_1 + D_1 .$$

If $\hat{\beta}_e > 100$ then it is set equal to the sample version of equation (2.8); that is $E[U_1]$ is estimated by $\bar{u}$, $E[C_1]$ is estimated by $\bar{u} + \bar{d}$, $Var[C_1]$ is estimated by

$$\hat{\sigma}_c^2 = \frac{1}{N-1}\sum_{i=1}^{N}(u_i - \bar{u})^2 + \frac{1}{N-1}\sum_{i=1}^{N}(d_i - \bar{d})^2 \tag{2.9}$$

and $E[C_1^2]$ is estimated by $\hat{\sigma}_c^2 + (\bar{u} + \bar{d})^2$.

If $\hat{\beta}_e$ is negative it is set equal to $\frac{1}{\bar{u}} + \frac{1}{\bar{d}}$.

3. The Simplified Exponential Estimator

The simplified exponential estimator has the form

$$\hat{A}_s(t) = \hat{A}(\infty) + (1 - \hat{A}(\infty))e^{-\hat{\beta}_s t} \tag{2.10}$$

where $\hat{\beta}_s$ is as computed in equation (2.5). It differs from $\hat{\beta}_e$ in that, $\hat{\beta}_s$'s larger than 100 are not recomputed. However, if $\hat{\beta}_s < 0$, than it is set equal to $\frac{1}{\bar{u}} + \frac{1}{\bar{d}}$ as before.

4. Cubic Estimator (Median-of-5 Exponential Approximation)

Let Z have the distribution of the median of 5 exponentials each having mean $\frac{1}{r}$. Its density function is

$$w(t) = \frac{d}{dt} P\{Z \le t\} \tag{2.11}$$

$$= 5\binom{4}{2}[1 - e^{-rt}]^2 re^{-rt}e^{-2rt}$$

$$= 30r[e^{-3rt} - 2e^{-4rt} + e^{-5rt}] .$$

The expected value of Z is

$$E[Z] = \frac{1}{5r} + \frac{1}{4r} + \frac{1}{3r} = \frac{47}{60}\frac{1}{r} \tag{2.12}$$

set

$$r = \frac{47}{60}\frac{1}{t} \tag{2.13}$$

to ensure E[Z] = t. The cubic estimator is of the form

$$\hat{A}_c(t) = \hat{A}(\infty) + [1 - \hat{A}(\infty)]e^{-\hat{\beta}_c t} \tag{2.14}$$

where $\hat{\beta}_c$ is the solution of the sample version of equation (1.8) with weight function w(t) the density function of the median-of-5 exponentials (equation (2.11)). More specifically,

if

$$P(t) \equiv \frac{A(t) - A(\infty)}{1 - A(\infty)} \simeq e^{-\beta t} \tag{2.15}$$

then setting

$$\int_0^\infty w(t)P(t)dt = \int_0^\infty w(t)e^{-\beta t}dt \tag{2.16}$$

results in a cubic equation for β since

$$\int_0^\infty w(t)P(t)dt = 30r[\int_0^\infty e^{-3rt}P(t)dt - 2\int_0^\infty e^{-4rt}P(t)dt + \int_0^\infty e^{-5rt}P(t)dt] \tag{2.17}$$

$$= 30r[\hat{P}(3r) - 2\hat{P}(4r) + \hat{P}(5r)]$$

and

$$\int_0^\infty w(t)e^{-\beta t}dt = 30r[\int_0^\infty e^{(-3r-\beta)t}dt - 2\int_0^\infty e^{(-4r-\beta)t}dt + \int_0^\infty e^{(-5r-\beta)t}dt] \tag{2.18}$$

$$= 30r[\frac{2r^2}{(3r+\beta)(4r+\beta)(5r+\beta)}] .$$

Setting equations (2.17) and (2.18) equal results after some algebraic manipulation in the cubic equation for β

$$f(\beta) \equiv \beta^3 + 12r\beta^2 + 47r^2\beta + 60r^3 - \frac{2r^2}{\hat{P}(3r) - 2\hat{P}(4r) + \hat{P}(5r)} = 0 . \tag{2.19}$$

The first derivative of f(β) is

$$f'(\beta) = 3\beta^2 + 24r\beta + 47r^2 . \tag{2.20}$$

To solve the cubic equation (2.19) the quadratic equation $f'(\beta) = 0$ is solved for β to find the β 's associated with the minimum and maximum. The resulting β 's are

$$\beta_1 = (-4 - \frac{\sqrt{1}}{3})r \tag{2.21}$$

$$\beta_2 = (-4 + \frac{\sqrt{1}}{3})r .$$

Note that β_1 and β_2 are alway negative. Hence f(β) = 0 has at most one positive solution or all negative solutions. If f(0) > 0, there is no positive solution and

$$\beta_c = \frac{1}{\bar{u}} + \frac{1}{\bar{d}} . \tag{2.22}$$

If f(0) < 0, there is a positive solution. To find β_c for f(β_c) = 0, the Newton search method[Ref. 4] is used. The availability at time t is estimated by

$$\hat{A}_c(t) = \hat{A}(\infty) + (1 - \hat{A}(\infty))e^{-\hat{\beta}_c t} . \tag{2.23}$$

B. CONFIDENCE INTERVALS

Suppose X is a random variable whose probability law depends on an unknown parameter θ. Given a random sample of X ; $x_1, x_2, x_3, \ldots, x_n$, two statistics lower

($L = L(\underset{\sim}{x}) \equiv L(x_1, x_2, \ldots, x_n)$) and upper ($U = U(\underset{\sim}{x}) \equiv U(x_1, x_2, \ldots, x_n)$) form a 100(1- α)% confidence interval for θ, if in repeated random samples $L \leq \theta \leq U$ $100(1 - \alpha)$ % of the time.

To obtain confidence intervals for the exponential approximation point estimates for P[system up at time t], the Bootstrap Estimation method is used. Efron(1979) introduced the Bootstrap method to estimate the distribution of a statistic computed from observations [Refs. 2,3]. In this thesis the Bootstrap is implemented as follow:

Given a sample of n lifetimes $u_1, u_2, \ldots, u_n$ and n repair times $d_1, d_2, \ldots, d_n$, sample at random with replacement n times from $\{u_i\}$ and n times from $\{d_i\}$. This constitutes one bootstrap sample. Compute the estimate of interest $A_e(t)$, $A_s(t)$, or $A_c(t)$ from the bootstrap sample. Generate another independent bootstrap sample etc. until there are N_b estimates. Order the N_b estimates. The $(\frac{\alpha}{2})N_b^{th}$ and $(1 - \frac{\alpha}{2})N_b^{th}$ order statistics of the estimates form the two-sided (1 - α)% confidence interval of A(t). The one-sided (1 - α)% confidence interval has a lower interval point of the $\alpha N_b{}^{th}$ order statistic and 1 as the upper interval point. The number of bootstrap replications is $N_b = 100$.

III. ANALYSIS OF THE PROBLEM

A. SIMULATION

A Fortran program is written to generate and analyze the data for this problem. All simulations are carried out on an IBM3033AP computer at the Naval Postgraduate School using the LLRANDOM II random number generating package [Ref. 5].

The system lifetimes are independent identically distributed exponential with mean $\lambda^{-1} = 1$. The system repair times have one of three distributions in this study:

1. Exponential with mean $\frac{1}{2}$

   $$P\{R > t\} = e^{-2t} \qquad t > 0 \, ;$$

2. Gamma

   $$P\{R > t\} = e^{-4t} + 4te^{-4t} \qquad t > 0 \text{ ; short-tailed; increasing hazard;}$$

3. Mixed Exponential

   $$P\{R > t\} = 0.9e^{-9t} + 0.1e^{-2t} \qquad t > 0 \text{ ; long-tailed; decreasing hazard.}$$

Table 25(Appendix B) contains the true availability A(t) at various times t for each of these models. Tables 26 through 31(Appendix C) contain the mean, and square root of mean square error(SRMSE) of the estimated availability $\hat{A}_{emp}(t)$, $\hat{A}_e(t)$, $\hat{A}_t(t)$ and $\hat{A}_c(t)$ at various times t for each of these models; i.e. the mean is

$$\overline{A}_e(t) = \frac{1}{N_r} \sum_{k=1}^{N_r} \hat{A}_e(t)_k$$

and

$$SRMSE = \left(\frac{1}{N_r} \sum_{k=1}^{N_r} (\hat{A}_e(t)_k - A(t))^2\right)^{\frac{1}{2}}$$

where $\hat{A}_e(t)_k$ is the point estimate at time t in the k^{th} super-replication.

The simulation has $N_r = 500$ super-replications. For each super-replication N = 50 or 100 lifetimes and repair times are generated. Using this sample the Empirical Estimate, Exponential Estimate, Simplified Exponential Estimate, and Cubic Estimate are computed. $N_b = 100$ bootstrap replications are drawn and the estimates are recomputed. The 90 % two sided confidence intervals and 90 % one sided confidence intervals are computed using $N_b = 100$ bootstrap estimates. Recorded in Tables 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21 and 23 are the number of replications for which the confidence intervals cover the respective true probabilities; also recorded are the number of confidence intervals which are too low (true P[system up at time t] > upper bound) and too high (true P[system up at time t] < lower bound). The average length of the confidence interval is computed as well as the standard deviation of the lengths. The standard deviation is computed by subtracting the mean length from each length, squaring the results, summing over the 500 lengths and dividing by 499, and finally taking the square root of the result.

B. ANALYSIS & RESULTS

In this section results will be reported for the simulation experiment. Some true values of P[system up at time t] for these models can be found in Appendix A. The number of bootstrap replication is set at 100. The simulation is replicated 500 times. For each replication, 90 % two sided confidence intervals and 90 % one sided confidence intervals of P[system up at time t] are computed using each procedure of Chapter 2. The times considered are t = 0.2, 0.5, 1.0, 2.0 and 3.0. The sample size N is set at 50 and 100.

The confidence interval procedures for P[system up at time t] use the Bootstrap for all estimators. For each procedure, the number of intervals covering the true value of P[system up at time t] is recorded as well as the number of intervals that are too high or too low. These results are reported in tables 1, 3, 5, 7, 9 and 11 for two sided confidence intervals, and tables 13, 15, 17, 19, 21 and 23 for one sided confidence intervals. Next to each coverage count is given the corresponding coverage proportion in parenthesis. If a (1- α) % confidence interval procedure is performing well, then this interval should cover about (1- α) % of the time. In tables 2, 4, 6, 8, 10 and 12, the average length of the two sided confidence intervals for P[system up at time t] are recorded, and in tables 14, 16, 18, 20, 22 and 24, the average length of the one sided confidence intervals for P[system up at time t] are recorded. The estimated standard deviation of the length is below the average length. If an estimator performs well, its confidence interval should not only have the correct coverage rate but also a small average length. The simulation results recorded in tables 1 through 4 and 13 through 16 are for the Exponential repair time. Tables 5 through 8 and 17 through 20 for the Gamma repair time. Tables 9 through 12 and 21 through 24 for the Mixed exponential repair time. Coverage results for sample size N = 50 are presented in table 1, 5, 9 13, 17 and 21. Sample mean and standard deviation of the confidence interval lengths for sample size N = 50 are presented in table 2, 6, 10, 14, 18 and 22. Coverage results for sample size N = 100 are presented in table 3, 7, 11, 15, 19 and 23. Sample mean and standard deviation of the confidence interval lengths for sample size N = 100 are presented in table 4, 8, 12, 16, 20 and 24.

To assess the coverage results, recall that for a binomial random variable with parameters 500 and p = 0.9, a 95 % confidence interval for p is [0.874, 0.926] or for the number of successes is [436.9, 463].

The Empirical Estimator tends to undercover for the Exponential and Gamma repair time cases for sample size N = 50 and overcover for all repair time distributions for sample size N = 100. For all repair time distributions, the coverage rate and the confidence interval width of the Empirical Estimator tend to increase according to increasing time t. The Empirical Estimator tends to have the largest confidence interval for all repair time distributions. The simulation results for time t = 2.0, 3.0 at sample size N = 100 are not available because of the very large computational requirements of the procedure.

The two-sided 90 % confidence intervals for the Exponential Estimator tend to have the correct coverage for all repair time distributions at sample size N = 50; that is, the number that cover are within the Binomial 90 % confidence interval [437, 463]. Increasing the sample size to N = 100 results in under-coverage, suggesting the estimator is biased. For all repair time distributions, two-sided confidence intervals tend to have better coverage than one-sided confidence intervals at sample size N = 50, and the one-sided confidence intervals tend to have better coverage than the two-sided confidence intervals at sample size N = 100.

The Simplified Exponential Estimator tends to undercover for all repair time distributions. The two-sided coverage rate of the Simplified Exponential Estimator for Mixed exponential repair time tends to decrease and the confidence interval width tends to increase according to increasing time t at sample size N = 50. The Simplified Exponential Estimator tends to have the smallest confidence intervals for Exponential and

Gamma repair time. The best coverage for these confidence intervals is for the Exponential repair time at sample size N = 100.

The two-sided confidence intervals of the Cubic Estimator tend to undercover for all repair times. The coverage rate of the Cubic Estimator for exponential repair time tends to increase and for Mixed exponential repair time tends to decrease according to increasing time t at sample size N = 100. The confidence interval width tends to increase according to increasing time t.

For two-sided confidence intervals, the Exponential Estimator has the best coverage of the three estimators based on the exponential approximation. The Cubic Estimator confidence intervals tend to have slightly better coverage than the Simplified Exponential intervals. For all estimators, the average length of the confidence intervals is the largest for the Mixed exponential repair time.

The one-sided confidence interval coverage is about the same for all three estimators based on the exponential approximation. The results of the tables in Appendix B indicate that the means of all estimators tend to be close to the true availabilities, A(t), for all repair time distributions. The root-mean-square error of the Exponential Estimator is the smallest for most cases. Increasing the sample size from N = 50 to N = 100 results in more accurate estimates and decreases the root-mean-square error, as would be anticipated.

IV. CONCLUSIONS

This thesis considers the problem of estimating the availability of a system at a finite time t. Simulation is used to study four estimation procedures and associated confidence interval procedures. The basic model is the alternating renewal process; the successive lifetimes are independent identically distributed and the successive repair times are independent identically distributed. Three repair time distribution are used in the simulations: the exponential, the gamma, and the mixed exponential; the lifetime distribution is exponential.

Three of the estimators are based on an exponential approximation to the availability at time t. The fourth estimator, called the Empirical Estimator, simulates the underlying alternating renewal process using a bootstrap-like sampling scheme.

The following conclusions are drawn from the simulation experiment.

1. The two-sided confidence intervals of the Exponential Estimator have the best coverage among the two-sided confidence intervals.
2. When sample size is increased from $N = 50$ to $N = 100$, the two-sided confidence intervals tend to undercover. When a confidence interval does not cover, it tends to be too high indicating that the estimators based on the exponential approximation may be biased towards overestimating the availability.
3. For a sample size of $N = 50$, the two-sided confidence intervals for the Empirical Estimator tend to undercover. For $N = 100$, they tend to overcover. The one-sided confidence interval coverage for the Empirical Estimator is better than its two-sided confidence interval coverage. The coverage of the confidence intervals of the Empirical Estimator may be improved by increasing the number of bootstrap-like replications, N_e, used to compute the estimate.
4. The computational effort in computing confidence intervals for the Empirical Estimator is the greatest and may be prohibitive for larger times.
5. The average two-sided confidence interval lengths are the largest for Gamma repair times and the smallest for Mixed exponential repair times. This behavior is due to the difference in the true system availability under the different assumptions.
6. The Exponential Estimator has the smallest root-mean-square error for the Exponential repair time and Gamma repair time cases of all the estimators.
7. Of the four estimators, the Exponential Estimator appears to be the most promising.

APPENDIX A. COVERAGE RATIO & LENGTH OF C.I TABLES

Table 1. COVERAGE RATIO (EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	84(0.16)	51(0.10)	51(0.10)	46(0.09)
	cover	383(0.77)	428(0.86)	426(0.85)	431(0.86)
	too low	33(0.07)	21(0.04)	23(0.05)	23(0.05)
0.5	too high	48(0.10)	31(0.06)	42(0.08)	37(0.08)
	cover	396(0.79)	450(0.90)	436(0.87)	442(0.88)
	too low	56(0.11)	19(0.04)	22(0.05)	21(0.04)
1.0	too high	31(0.06)	47(0.10)	39(0.08)	33(0.07)
	cover	425(0.85)	441(0.88)	432(0.86)	439(0.88)
	too low	44(0.09)	12(0.02)	29(0.06)	28(0.05)
2.0	too high	45(0.09)	38(0.07)	38(0.08)	38(0.08)
	cover	426(0.85)	444(0.89)	422(0.84)	422(0.84)
	too low	29(0.06)	18(0.04)	40(0.08)	40(0.08)
3.0	too high	37(0.07)	38(0.08)	35(0.07)	35(0.07)
	cover	439(0.88)	442(0.88)	418(0.84)	420(0.84)
	too low	24(0.05)	20(0.04)	47(0.09)	45(0.09)

Table 2. LENGTH OF C.I (EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1104	0.1191	0.1167	0.1213
	std dev	0.0191	0.0244	0.024	0.0224
0.5	mean	0.1380	0.1354	0.1321	0.1351
	std dev	0.0156	0.0180	0.017	0.0175
1.0	mean	0.1476	0.1349	0.1324	0.134
	std dev	0.0150	0.0176	0.0176	0.018
2.0	mean	0.1476	0.1363	0.1337	0.1342
	std dev	0.0153	0.0196	0.0186	0.0185
3.0	mean	0.1493	0.1365	0.1335	0.1336
	std dev	0.0143	0.0201	0.0185	0.0184

Table 3. COVERAGE RATIO (EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	51(0.10)	39(0.08)	39(0.08)	40(0.08)
	cover	436(0.87)	431(0.86)	429(0.86)	430(0.86)
	too low	13(0.03)	30(0.06)	32(0.06)	30(0.06)
0.5	too high	16(0.03)	27(0.06)	31(0.06)	31(0.06)
	cover	469(0.94)	437(0.87)	442(0.88)	435(0.87)
	too low	15(0.03)	36(0.07)	27(0.06)	34(0.07)
1.0	too high	13(0.03)	30(0.06)	31(0.06)	28(0.06)
	cover	477(0.95)	440(0.88)	442(0.88)	443(0.88)
	too low	10(0.02)	30(0.06)	27(0.06)	29(0.06)
2.0	too high		25(0.05)	25(0.05)	25(0.05)
	cover		442(0.88)	436(0.87)	436(0.87)
	too low		33(0.07)	39(0.08)	39(0.08)
3.0	too high		20(0.04)	20(0.04)	20(0.04)
	cover		449(0.90)	444(0.89)	445(0.89)
	too low		31(0.06)	36(0.07)	35(0.07)

Table 4. LENGTH OF C.I (EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1116	0.0866	0.0846	0.0877
	std dev	0.0156	0.013	0.0127	0.012
0.5	mean	0.1384	0.0982	0.0949	0.0964
	std dev	0.0139	0.0105	0.010	0.0098
1.0	mean	0.1475	0.0974	0.0948	0.0962
	std dev	0.0142	0.0103	0.0108	0.011
2.0	mean		0.0995	0.0971	0.0973
	std dev		0.0121	0.0118	0.0117
3.0	mean		0.0994	0.0969	0.0969
	std dev		0.0113	0.0112	0.0112

Table 5. COVERAGE RATIO (GAMMA REPAIR TIME : TWO-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	76(0.15)	42(0.08)	39(0.08)	38(0.08)
	cover	377(0.76)	433(0.87)	438(0.88)	444(0.89)
	too low	47(0.09)	25(0.05)	23(0.04)	18(0.03)
0.5	too high	69(0.14)	59(0.12)	72(0.14)	61(0.12)
	cover	385(0.77)	432(0.86)	410(0.82)	429(0.86)
	too low	46(0.09)	9(0.02)	18(0.04)	10(0.02)
1.0	too high	43(0.09)	47(0.09)	66(0.13)	61(0.12)
	cover	422(0.84)	445(0.89)	411(0.82)	420(0.84)
	too low	35(0.07)	8(0.02)	23(0.05)	19(0.04)
2.0	too high	31(0.06)	29(0.06)	31(0.06)	32(0.07)
	cover	451(0.90)	452(0.90)	420(0.84)	422(0.84)
	too low	18(0.04)	19(0.04)	49(0.10)	46(0.09)
3.0	too high	22(0.05)	29(0.06)	26(0.05)	26(0.05)
	cover	461(0.92)	450(0.90)	431(0.86)	431(0.86)
	too low	17(0.03)	21(0.04)	43(0.09)	43(0.09)

Table 6. LENGTH OF C.I (GAMMA REPAIR TIME : TWO-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1164	0.1297	0.1284	0.1345
	std dev	0.0187	0.0249	0.0241	0.023
0.5	mean	0.1425	0.1328	0.1284	0.1352
	std dev	0.0152	0.0159	0.0147	0.0167
1.0	mean	0.1490	0.1234	0.1198	0.1222
	std dev	0.0147	0.0155	0.015	0.016
2.0	mean	0.1486	0.1200	0.1164	0.1169
	std dev	0.0142	0.0168	0.0154	0.0155
3.0	mean	0.149	0.1196	0.1171	0.1173
	std dev	0.0144	0.0171	0.0159	0.0159

Table 7. COVERAGE RATIO (GAMMA REPAIR TIME : TWO-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	38(0.07)1	35(0.07)	35(0.07)	36(0.07)
	cover	443(0.89)	440(0.88)	430(0.86)	434(0.87)
	too low	19(0.04)	25(0.05)	35(0.07)	30(0.06)
0.5	too high	23(0.05)	53(0.1)	60(0.12)	42(0.09)
	cover	463(0.92)	433(0.87)	427(0.85)	442(0.88)
	too low	14(0.03)	14(0.03)	13(0.03)	16(0.03)
1.0	too high	18(0.04)	45(0.09)	51(0.1)	49(0.1)
	cover	471(0.94)	435(0.87)	433(0.87)	433(0.87)
	too low	11(0.02)	20(0.04)	16(0.03)	18(0.03)
2.0	too high		26(0.05)	35(0.07)	35(0.07)
	cover		440(0.88)	431(0.86)	431(0.86)
	too low		34(0.07)	34(0.07)	34(0.07)
3.0	too high		34(0.07)	25(0.05)	25(0.05)
	cover		438(0.88)	436(0.87)	436(0.87)
	too low		28(0.05)	39(0.08)	39(0.08)

Table 8. LENGTH OF C.I (GAMMA REPAIR TIME : TWO-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1177	0.0948	0.0925	0.0964
	std dev	0.015	0.0135	0.0132	0.0127
0.5	mean	0.1436	0.096	0.0937	0.0969
	std dev	0.0141	0.0098	0.0095	0.0102
1.0	mean	0.1485	0.0879	0.0857	0.0878
	std dev	0.0142	0.009	0.0095	0.0104
2.0	mean		0.0862	0.0833	0.0837
	std dev		0.0104	0.0096	0.0096
3.0	mean		0.0852	0.0832	0.0832
	std dev		0.0098	0.0097	0.0097

Table 9. COVERAGE RATIO (MIXED EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	70(0.14)	49(0.1)	49(0.1)	50(0.1)
	cover	405(0.81)	444(0.89)	441(0.88)	441(0.88)
	too low	25(0.05)	7(0.01)	10(0.02)	9(0.02)
0.5	too high	48(0.1)	30(0.06)	50(0.1)	51(0.1)
	cover	425(0.85)	455(0.91)	423(0.85)	430(0.86)
	too low	27(0.05)	15(0.03)	27(0.05)	19(0.04)
1.0	too high	32(0.06)	38(0.08)	35(0.07)	38(0.08)
	cover	448(0.90)	446(0.89)	440(0.88)	439(0.88)
	too low	20(0.04)	16(0.03)	25(0.05)	23(0.04)
2.0	too high	22(0.04)	45(0.09)	41(0.08)	41(0.08)
	cover	451(0.90)	440(0.88)	426(0.85)	426(0.85)
	too low	27(0.06)	15(0.03)	33(0.07)	33(0.07)
3.0	too high	27(0.05)	45(0.09)	52(0.1)	52(0.1)
	cover	463(0.93)	442(0.88)	418(0.84)	418(0.84)
	too low	10(0.02)	13(0.03)	30(0.06)	30(0.06)

Table 10. LENGTH OF C.I (MIXED EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.0895	0.0727	0.0708	0.0715
	std dev	0.0181	0.0153	0.0148	0.0147
0.5	mean	0.0997	0.0751	0.0732	0.0728
	std dev	0.0157	0.016	0.0153	0.0147
1.0	mean	0.1043	0.0806	0.079	0.0787
	std dev	0.0149	0.0203	0.0197	0.0187
2.0	mean	0.1061	0.0844	0.085	0.0848
	std dev	0.0152	0.0246	0.0244	0.0238
3.0	mean	0.1066	0.0850	0.084	0.084
	std dev	0.0144	0.0256	0.0254	0.0253

Table 11. COVERAGE RATIO (MIXED EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	29(0.06)	33(0.07)	34(0.07)	34(0.07)
	cover	465(0.93)	441(0.88)	444(0.89)	447(0.89)
	too low	6(0.01)	26(0.05)	22(0.04)	19(0.04)
0.5	too high	12(0.02)	25(0.05)	29(0.06)	32(0.06)
	cover	483(0.97)	430(0.86)	431(0.86)	437(0.88)
	too low	5(0.01)	45(0.09)	40(0.08)	31(0.06)
1.0	too high	4(0.01)	31(0.06)	30(0.06)	35(0.07)
	cover	489(0.98)	433(0.87)	428(0.86)	428(0.86)
	too low	7(0.01)	36(0.07)	42(0.08)	37(0.07)
2.0	too high		50(0.10)	45(0.09)	46(0.09)
	cover		435(0.87)	431(0.86)	431(0.86)
	too low		15(0.03)	24(0.05)	23(0.06)
3.0	too high		41(0.08)	31(0.06)	31(0.06)
	cover		426(0.85)	436(0.87)	436(0.87)
	too low		33(0.07)	33(0.07)	33(0.07)

Table 12. LENGTH OF C.I (MIXED EXPONENTIAL REPAIR TIME : TWO-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.0904	0.0541	0.0526	0.0539
	std dev	0.0142	0.0078	0.0077	0.0078
0.5	mean	0.1020	0.0546	0.0533	0.0532
	std dev	0.0136	0.0095	0.0089	0.0086
1.0	mean	0.1051	0.0593	0.058	0.0573
	std dev	0.0133	0.0125	0.0115	0.0109
2.0	mean		0.0611	0.060	0.0596
	std dev		0.0146	0.0138	0.0132
3.0	mean		0.063	0.0617	0.0616
	std dev		0.015	0.0142	0.014

Table 13. COVERAGE RATIO (EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	123(0.25)	73(0.15)	73(0.15)	71(0.14)
	cover	377(0.75)	427(0.85)	427(0.85)	429(0.86)
0.5	too high	66(0.13)	70(0.14)	70(0.14)	66(0.13)
	cover	434(0.87)	430(0.86)	430(0.86)	434(0.87)
1.0	too high	61(0.12)	70(0.14)	64(0.13)	62(0.12)
	cover	439(0.88)	430(0.86)	436(0.87)	438(0.88)
2.0	too high	77(0.15)	66(0.13)	54(0.11)	56(0.11)
	cover	423(0.85)	434(0.87)	446(0.89)	444(0.89)
3.0	too high	62(0.12)	65(0.13)	55(0.11)	55(0.11)
	cover	438(0.88)	435(0.87)	445(0.89)	445(0.89)

Table 14. LENGTH OF C.I (EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1247	0.1286	0.1286	0.135
	std dev	0.0241	0.0281	0.0281	0.0267
0.5	mean	0.1585	0.1521	0.1511	0.1542
	std dev	0.0219	0.0233	0.0224	0.0226
1.0	mean	0.1726	0.1529	0.1533	0.1553
	std dev	0.0229	0.0226	0.0231	0.0238
2.0	mean	0.1712	0.1544	0.1552	0.1559
	std dev	0.0228	0.0248	0.0237	0.0237
3.0	mean	0.1729	0.1547	0.154	0.1543
	std dev	0.0213	0.0253	0.0248	0.0247

Table 15. COVERAGE RATIO (EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	78(0.16)	58(0.12)	58(0.12)	62(0.12)
	cover	422(0.84)	442(0.88)	442(0.88)	438(0.88)
0.5	too high	30(0.06)	58(0.12)	63(0.13)	62(0.12)
	cover	470(0.94)	442(0.88)	437(0.87)	438(0.88)
1.0	too high	35(0.07)	59(0.12)	59(0.12)	56(0.11)
	cover	465(0.93)	441(0.88)	441(0.88)	444(0.89)
2.0	too high		56(0.11)	56(0.11)	56(0.11)
	cover		444(0.89)	444(0.89)	444(0.89)
3.0	too high		49(0.10)	49(0.10)	50(0.10)
	cover		451(0.90)	451(0.90)	450(0.90)

Table 16. LENGTH OF C.I (EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1257	0.0959	0.0959	0.1003
	std dev	0.0192	0.0152	0.0152	0.0147
0.5	mean	0.1585	0.1117	0.1105	0.1126
	std dev	0.0196	0.0135	0.0146	0.0145
1.0	mean	0.1703	0.1104	0.1105	0.1123
	std dev	0.0218	0.014	0.0143	0.0143
2.0	mean		0.113	0.113	0.1134
	std dev		0.0158	0.0158	0.0158
3.0	mean		0.1135	0.1135	0.1136
	std dev		0.015	0.015	0.015

Table 17. COVERAGE RATIO (GAMMA REPAIR TIME : ONE-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	103(0.21)	77(0.15)	61(0.12)	57(0.11)
	cover	397(0.79)	423(0.85)	439(0.88)	443(0.89)
0.5	too high	99(0.20)	95(0.19)	106(0.21)	94(0.19)
	cover	401(0.80)	405(0.81)	394(0.79)	406(0.81)
1.0	too high	69(0.14)	81(0.16)	96(0.19)	95(0.19)
	cover	431(0.86)	419(0.84)	404(0.81)	405(0.81)
2.0	too high	54(0.11)	50(0.10)	51(0.10)	53(0.11)
	cover	446(0.89)	450(0.90)	449(0.90)	447(0.89)
3.0	too high	47(0.09)	47(0.09)	51(0.10)	50(0.10)
	cover	453(0.91)	453(0.91)	449(0.90)	450(0.90)

Table 18. LENGTH OF C.I (GAMMA REPAIR TIME : ONE-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1302	0.1412	0.1429	0.1518
	std dev	0.0234	0.0296	0.0289	0.0286
0.5	mean	0.1642	0.1518	0.151	0.1575
	std dev	0.0215	0.0212	0.022	0.0234
1.0	mean	0.1741	0.1398	0.1395	0.1407
	std dev	0.0226	0.0201	0.0199	0.0203
2.0	mean	0.1731	0.135	0.1335	0.1341
	std dev	0.0231	0.0209	0.02	0.0201
3.0	mean	0.1729	0.1344	0.1331	0.1333
	std dev	0.0198	0.0212	0.02	0.02

Table 19. COVERAGE RATIO (GAMMA REPAIR TIME : ONE-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	59(0.12)	52(0.10)	52(0.10)	60(0.12)
	cover	441(0.88)	448(0.90)	448(0.90)	440(0.88)
0.5	too high	49(0.10)	88(0.18)	108(0.22)	86(0.17)
	cover	451(0.90)	412(0.82)	392(0.78)	414(0.83)
1.0	too high	35(0.07)	75(0.15)	83(0.17)	80(0.16)
	cover	465(0.93)	425(0.85)	417(0.83)	420(0.84)
2.0	too high		52(0.10)	60(0.12)	63(0.13)
	cover		448(0.90)	440(0.88)	437(0.87)
3.0	too high		55(0.11)	53(0.11)	53(0.11)
	cover		445(0.89)	447(0.89)	447(0.89)

Table 20. LENGTH OF C.I (GAMMA REPAIR TIME : ONE-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.1322	0.1055	0.1055	0.111
	std dev	0.0206	0.0165	0.0165	0.0166
0.5	mean	0.1664	0.1113	0.11	0.1148
	std dev	0.0204	0.0139	0.0137	0.0149
1.0	mean	0.1717	0.0998	0.1001	0.1028
	std dev	0.0197	0.0126	0.0128	0.0133
2.0	mean		0.0982	0.0975	0.0979
	std dev		0.0134	0.0126	0.0126
3.0	mean		0.0974	0.0966	0.0967
	std dev		0.0125	0.0126	0.0126

Table 21. COVERAGE RATIO (MIXED EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 50)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	102(0.20)	83(0.17)	82(0.16)	81(0.16)
	cover	398(0.80)	417(0.83)	418(0.84)	419(0.84)
0.5	too high	78(0.16)	61(0.12)	60(0.12)	76(0.15)
	cover	422(0.84)	439(0.88)	440(0.88)	424(0.85)
1.0	too high	54(0.11)	60(0.12)	53(0.11)	57(0.11)
	cover	446(0.89)	440(0.88)	447(0.89)	443(0.89)
2.0	too high	38(0.08)	70(0.14)	70(0.14)	70(0.14)
	cover	462(0.92)	430(0.86)	430(0.86)	430(0.86)
3.0	too high	59(0.12)	70(0.14)	77(0.15)	78(0.16)
	cover	441(0.88)	430(0.86)	423(0.85)	422(0.84)

Table 22. LENGTH OF C.I (MIXED EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 50)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.0986	0.0783	0.0778	0.0785
	std dev	0.0222	0.0166	0.0166	0.0161
0.5	mean	0.1105	0.0806	0.08	0.08
	std dev	0.0201	0.0176	0.0174	0.0168
1.0	mean	0.1166	0.0862	0.0865	0.0861
	std dev	0.0194	0.0217	0.0215	0.0209
2.0	mean	0.1191	0.0898	0.0921	0.092
	std dev	0.0188	0.0254	0.0263	0.0257
3.0	mean	0.1198	0.0903	0.0911	0.0911
	std dev	0.0188	0.0262	0.0266	0.0265

Table 23. COVERAGE RATIO (MIXED EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 100)

time	coverage	EMP	EXP	SIM EXP	CUBIC
0.2	too high	54(0.11)	68(0.14)	69(0.14)	66(0.13)
	cover	446(0.89)	432(0.86)	431(0.86)	434(0.87)
0.5	too high	24(0.05)	48(0.10)	43(0.09)	59(0.12)
	cover	476(0.95)	452(0.90)	457(0.91)	441(0.88)
1.0	too high	26(0.05)	56(0.11)	55(0.11)	59(0.12)
	cover	474(0.95)	444(0.89)	445(0.89)	441(0.88)
2.0	too high		73(0.15)	78(0.16)	78(0.16)
	cover		427(0.85)	422(0.84)	422(0.84)
3.0	too high		61(0.12)	56(0.11)	56(0.11)
	cover		439(0.88)	444(0.89)	444(0.89)

Table 24. LENGTH OF C.I (MIXED EXPONENTIAL REPAIR TIME : ONE-SIDED, N = 100)

time	C.I	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.099	0.0604	0.0597	0.0609
	std dev	0.0171	0.0096	0.0098	0.0097
0.5	mean	0.1141	0.0601	0.0603	0.0601
	std dev	0.0168	0.0103	0.0108	0.0104
1.0	mean	0.1164	0.0647	0.0649	0.0642
	std dev	0.0159	0.0136	0.0135	0.0126
2.0	mean		0.0663	0.0668	0.0666
	std dev		0.0157	0.0153	0.015
3.0	mean		0.0684	0.0683	0.0682
	std dev		0.0161	0.0158	0.0156

APPENDIX B. TRUE AVAILABILITY TABLE

Table 25. TRUE AVAILABILITY OF SYSTEM AT A FINITE TIME T

time	Exponential $\lambda = 1.0, \mu_1 = 2.0$	Gamma $\lambda = 1.0, \mu_1 = 4.0, \mu_2 = 4.0$	Mixed Exponential $\lambda = 1.0, \mu_1 = 9.0, \mu_2 = 2.0$, p = 0.9
0.2	0.8496	0.8318	0.9070
0.5	0.7410	0.7090	0.8834
1.0	0.6833	0.6680	0.8742
2.0	0.6675	0.6666	0.8701
3.0	0.6667	0.6666	0.8696

APPENDIX C. ESTIMATED AVAILABILITY TABLES

Table 26. ESTIMATED AVAILABILITY OF SYSTEM (EXPONENTIAL REPAIR TIME : N = 50)

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.852	0.8506	0.8506	0.8511
	SRMSE	0.045	0.0383	0.0383	0.04
0.5	mean	0.7396	0.7444	0.7455	0.7458
	SRMSE	0.0533	0.042	0.0418	0.0418
1.0	mean	0.6826	0.6901	0.6836	0.6836
	SRMSE	0.0523	0.0412	0.0437	0.0439
2.0	mean	0.6667	0.6704	0.6659	0.6663
	SRMSE	0.0498	0.0417	0.0466	0.0467
3.0	mean	0.6665	0.6683	0.6646	0.6647
	SRMSE	0.0469	0.0418	0.0451	0.0452

Table 27. ESTIMATED AVAILABILITY OF SYSTEM(EXPONENTIAL REPAIR TIME : N = 100)

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.8519	0.848	0.848	0.8481
	SRMSE	0.0335	0.0272	0.0272	0.0282
0.5	mean	0.741	0.74	0.742	0.7411
	SRMSE	0.0365	0.0313	0.0307	0.031
1.0	mean	0.6834	0.6831	0.684	0.6842
	SRMSE	0.0387	0.031	0.0308	0.031
2.0	mean		0.6654	0.6654	0.6656
	SRMSE		0.0318	0.0318	0.0319
3.0	mean		0.6651	0.6651	0.6652
	SRMSE		0.0305	0.0305	0.0305

Table 28. ESTIMATED AVAILABILITY OF SYSTEM (GAMMA REPAIR TIME : N = 50)

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.8346	0.8319	0.8302	0.8307
	SRMSE	0.0485	0.0422	0.0423	0.0436
0.5	mean	0.7111	0.7232	0.7250	0.7233
	SRMSE	0.0562	0.0433	0.0463	0.0454
1.0	mean	0.6669	0.6779	0.6775	0.6780
	SRMSE	0.0507	0.0375	0.0415	0.042
2.0	mean	0.6656	0.6664	0.6629	0.6630
	SRMSE	0.0457	0.0351	0.0391	0.0392
3.0	mean	0.6655	0.6656	0.6634	0.6635
	SRMSE	0.0407	0.0351	0.0379	0.0379

Table 29. ESTIMATED AVAILABILITY OF SYSTEM (GAMMA REPAIR TIME : N = 100)

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.8337	0.8292	0.8292	0.8297
	SRMSE	0.0363	0.0296	0.0296	0.0308
0.5	mean	0.7079	0.7177	0.7194	0.7157
	SRMSE	0.0387	0.0316	0.0315	0.031
1.0	mean	0.6672	0.6735	0.6741	0.6741
	SRMSE	0.0388	0.0287	0.0285	0.0291
2.0	mean		0.6648	0.6661	0.6663
	SRMSE		0.028	0.0282	0.0282
3.0	mean		0.6656	0.6664	0.6664
	SRMSE		0.0282	0.0276	0.0276

Table 30. ESTIMATED AVAILABILITY OF SYSTEM (MIXED EXP REPAIR TIME : N = 50)

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.9097	0.9116	0.9114	0.9127
	SRMSE	0.0319	0.0224	0.0224	0.0219
0.5	mean	0.8846	0.884	0.8837	0.8858
	SRMSE	0.034	0.0222	0.0232	0.0228
1.0	mean	0.8738	0.8734	0.8712	0.8726
	SRMSE	0.0319	0.0245	0.0256	0.0251
2.0	mean	0.8692	0.8703	0.8682	0.8686
	SRMSE	0.0318	0.0261	0.0283	0.0282
3.0	mean	0.8688	0.87	0.868	0.8681
	SRMSE	0.0301	0.0265	0.029	0.029

Table 31. **ESTIMATED AVAILABILITY OF SYSTEM (MIXED EXP REPAIR TIME : N = 100)**

SRMSE : square root of mean square error

time	SYS AVAIL	EMP	EXP	SIM EXP	CUBIC
0.2	mean	0.9094	0.907	0.9077	0.9081
	SRMSE	0.0231	0.0169	0.0165	0.0164
0.5	mean	0.8831	0.8797	0.8805	0.8824
	SRMSE	0.023	0.018	0.0174	0.0171
1.0	mean	0.8752	0.8709	0.8708	0.8723
	SRMSE	0.0237	0.0196	0.02	0.0195
2.0	mean		0.8707	0.8706	0.8709
	SRMSE		0.0195	0.0197	0.0195
3.0	mean		0.8677	0.8684	0.8685
	SRMSE		0.0206	0.0197	0.0196

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