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Sounds in One-Dimensional Superfluid Helium

by

C. I. Um, W. H. Kahng, M. H. Whang, S K. Hong, H. G. Oh
and Thomas F. George

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Sounds in one-dimensional superfluid helium

C. I. Um*, W. H. Kahng, M. H. Whang and S. K. Hong
Department of Physics
College of Science
Korea University
Seoul 136, Korea

H. G. Oh and Thomas F. George
Departments of Chemistry and Physics
239 Fronczak Hall
State University of New York at Buffalo
Buffalo, New York 14260

Abstract

The temperature variations of first-, second- and third-sound velocity and attenuation coefficients in one-dimensional superfluid helium are evaluated explicitly for very low temperatures and frequencies ($\omega_s \tau \ll 1$, where ω_s is the sound frequency and τ the characteristic time), by using the collisionless kinetic equation and superfluid hydrodynamic equations with the energy dissipation effect. The leading terms for these sounds vary as T^2, and the ratio of second sound to first sound becomes unity as the temperature decreases to absolute zero.

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* To whom correspondence should be addressed.

I. Introduction

The propagation of sounds has been an important subject in the study of liquid helium, and especially the temperature variations of the various sounds are interesting because they are very closely related to the elementary excitation dispersion relation. It is well established that at low temperatures and pressures, where roton excitation can be neglected, the thermal properties of superfluid helium are dominated by low-momentum acoustic phonons[1] which do not possess a normal, but rather an anomalous excitation spectrum. These two cases lead to different microscopic processes. In the former case four-photon processes (4PP) are dominant, while in the latter case 3PP become important.[2] Andreev and Khalatnikov[3] calculated the temperature variation of sound in bulk liquid helium using a collisionless kinetic equation together with an equation for the superfluid velocity. Singh and Prakash[4] investigated the behavior of the retarded single-particle Green's function for a weakly-interacting Bose gas and found a new temperature-dependent term. However, they adopted the normal excitation spectrum in their calculations. We have obtained microscopically anomalous excitation dispersions[5] in two-and three-dimensional liquid helium which are based on the ring diagram approximation, and using these dispersions[5] we have not only improved on their results,[6] but have also obtained first, second and third sound in thin helium films.[7]

Concerning second sound in bulk liquid helium, the velocity of this mode is $c_0/\sqrt{3}$, where c_0 is the velocity of first sound in the low-frequency limit and low-temperature phonon dominant region. However, Maris[8] has predicted a new type of second sound which increases towards to c_0 in the regime of collinear phonon processes. More recently, using pulsed time-of-flight and cw resonance techniques, Eisenstein and Narayanamurti[9] have observed this new

type of collective mode as a function of frequency, the so-called one-dimensional second sound, in superfluid helium at low temperatures and pressures. (The term "one-dimensional" means simply that the thermal phonons in a wave motion have wave vectors almost parallel to the propagation direction of the wave in bulk superfluid helium.) This mode was predicted by Maris.[9]

In many-body theory there are several one-dimensional models[10] of Bose or Fermi gases with two-body interactions among particles. Although these models can describe the ground-state energy, eigenfunction and other quantities, they are not appropriate for application to this new mode. However, we have recently developed a model[11] of a one-dimensional Bose liquid based on the ring diagram approximation. Through this model we obtained the pair distribution function of a one-dimensional Bose liquid and then its anomalous excitation spectrum and thermodynamic functions. In this mode, we assumed that the interaction potential between helium atoms is given by a soft potential with a Lennard-Jones type tail as

$$\phi(x) = \begin{cases} V_o , & |x| \le a \\ \\ \epsilon_o [(\frac{a}{x})^{12} - (\frac{a}{x})^6] , & |x| \ge a . \end{cases} \tag{1.1}$$

In this paper we introduce a one-dimensional model for superfluid helium to derive the temperature variations of first, second and third sound for theory's sake, and also show that the above three sounds vary as T^2 and that second sound reduces to first sound at absolute zero temperature. In the long wavelength limit, the anomalous excitation spectrum is given by

,

$$E(q) = A_0 q + Bq^3 + Cq^5 + Dq^6 + \ldots \quad . \tag{1.2}$$

where the coefficients are

$$A_0^2 = 4na(V_0 - \frac{6}{55}\epsilon_0) \quad , \quad B = \frac{1+A_1^2}{2A_0}$$

$$C = \frac{A_2^2}{2A_0} - \frac{3(1+A_1^2)}{8A_0^3} \quad , \quad D = \frac{A_3^2}{2A_0} \quad ,$$

$$A_1^2 = 4na^3(-\frac{V_0}{3!} + \frac{\epsilon_0}{9}) \quad , \quad A_2^2 = 4na^5(\frac{V_0}{5!} - \frac{\epsilon_0}{28}) \quad ,$$

$$A_3^2 = 2na^b \frac{\pi}{5!}\epsilon_0 \quad , \tag{1.3}$$

where n is number density. We note that Eq. (1.2) represents an anomalous dispersion because B is positive. In the case of third sound, the excitation spectrum involves not only the anomalous excitation but also the roton dispersion, which corresponds to large momentum. For large momentum we can express the roton excitation spectrum as

$$E(q) = q(q^2 + 4naV_0 \frac{\sin(qa)}{qa})^{\frac{1}{2}} \quad . \tag{1.4}$$

The energy E(q) is oscillatory. Around the first minimum q_0, we can replace E(q) approximately by a Landau form,

$$E(q) = \Delta + \frac{\hbar^2}{2m^*}(q-q_0)^2 \quad , \tag{1.5}$$

where Δ and m* are the energy gap and effective mass, respectively.

In Sec. II we first evaluate the temperature variation of first sound near absolute zero following the method of Andreev and Khalatnikov, and then at low frequencies such that $\omega_s \tau \ll 1$, where ω_s is the sound frequency and τ the characteristic time, we solve the superfluid hydrodynamic equations to obtain first and second sound in Sec. III. In Sec. IV the derivation of one-dimensional third sound will be given, and we finally present numerical results and discussion with a table and graphs in Sec. V. Throughout this paper we take the units such that $\hbar = 1$ and $2m = 1$, where m is the particle mass, except for the case in which explicit restoration of $\hbar$ or $2m$ is required.

II. Kinetic approach

The temperature variation of first sound can be obtained by solving the equations of motion of the fluid assuming that roton excitation is negligible where the phonon excitation is dominant. In order to obtain the sound velocity, we solve the kinetic equation for the phonon distribution function $n(r,p,t)$ without the collision term together with the equation of continuity and the equation for the superfluid velocity v_s. Since these equations were given in our previous paper,[6] we shall not repeat them here. When the liquid is slightly perturbed from equilibrium, we can assume that the deviations of the distribution function, mass density and superfluid velocity (represented by n', ρ' and v_s', respectively) are small and proportional to $\exp[i(kx - \omega_s t)]$, where k is the complex wave vector, $k_1 + ik_2$, which characterizes the attenuation of sound. Linearizing the above-mentioned three equations for n', ρ' and v_s', we obtain

$$[\frac{\omega_s}{k} - \frac{\partial c_o}{\partial \rho_o} \int dq \; q^2 \; \frac{\partial n_o}{\partial E} \frac{kv}{\omega_s + kv}]\rho' - [\rho_o + \int d\rho \; q^2 \; \frac{\partial n_o}{\partial E} \frac{kv}{\omega_s + kv}] \; v_s' = 0 \quad ,$$

$$(2.1)$$

$$[\frac{c_o}{\rho_o} + \int dq \; \frac{\partial^2 E(q)}{\partial \rho_o^2} \; n_o + (\frac{\partial c_o}{\partial \rho_o})^2 \int dq \; q^2 \; \frac{\partial n_o}{\partial E} \frac{kv}{\omega_s + kv}]\rho'$$

$$+ [- \frac{\omega_s}{k} + \frac{\partial c_o}{\partial \rho_o} \int dq \; q^2 \; \frac{\partial n_o}{\partial E} \frac{kv}{\omega_s + kv}] \; v_s' = 0 \quad , \qquad (2.2)$$

where ρ_o is the equilibrium density of liquid. The nontrivial solution can be derived by setting the corresponding determinant equal to zero, and then making use of Eq. (1.2) we obtain the sound velocity expression

$$\frac{\omega}{q} = c + \delta c$$

$$\delta c = - \frac{c_o}{4}\{[(1+u)^2 + \frac{1}{2} \frac{\rho_o}{c_o} \frac{\partial^2 c_o}{\partial \rho_o^2} \; \rho_n][1 + \frac{3}{4} \frac{B}{A_0}(\frac{k_B t}{c_o})^2] - \frac{\rho_o^2}{c_o} \frac{\partial^2 c_o}{\partial \rho_o^2}\} \frac{\rho_n}{\rho_o} \quad , \quad (2.3)$$

where $u = (\rho_o/c)(\partial c/\partial \rho_o)$ is the Grüneisen constant, and the normal fluid density[11] in Eq. (2.3) is given by

$$\rho_n(T) = \frac{1}{2\pi}[\frac{2!\zeta(2)}{A_0^3}(k_B T)^2 - \frac{5!\zeta(4)}{A_0^6} B(k_B T)^4 + \frac{7\zeta(6)}{A_0^8} C(k_B T)^6 + \ldots] \quad . (2.4)$$

We note that at very low temperatures δc decreases as temperature increases.

III. Hydrodynamic method

For low frequencies ($\omega_s \tau \ll 1$), we can apply the superfluid hydrodynamic equations with the dissipation term to the problem of first and second sound propagation in one-dimensional liquid helium. These equations are also given in our previous paper,[7] and thus they will not be discussed here. As mentioned earlier, we also assume that deviations of the pressure, temperature and velocity of the normal component (P, T and v_n) from equilibrium (expressed by P', T' and v_n') are small and proportional to $\exp[i(kx-\omega_s t)]$. Linearizing these hydrodynamic equations, we obtain four hydrodynamic equations,

$$\frac{\omega_s}{k}[(\frac{\partial \rho}{\partial p})_T \, P' + (\frac{\partial \rho}{\partial T})_p T'] - \rho_n v_n' - \rho_s v_s' = 0 \quad , \tag{3.1}$$

$$\frac{\omega_s}{k}[\frac{\partial \rho S}{\partial p}]_T \, P' + [\frac{\omega_s}{k}(\frac{\partial \rho S}{\partial T})_p + \frac{\bar{\kappa}}{T}ik]T' - s_0 \rho_0 v_n' = 0 \quad , \tag{3.2}$$

$$P' - \rho_0 S_0 T' + ik\rho_0(\rho_s \xi_3 - \xi_4)v_n' - \rho_0(\frac{\omega_s}{k} + ik\rho_s\xi_3)v_s' = 0 \quad , \tag{3.3}$$

$$P' - (\frac{\omega_s}{k} - ik\xi_1\rho_s + ik\xi_2)v_n' - (\frac{\omega_s}{k} + ik\xi_1\rho_s)v_s' = 0 \quad , \tag{3.4}$$

where S is the entropy per unit mass, ρ_n and ρ_s are the mass densities of the two fluid components, $\bar{\kappa}$ is the thermal conductivity, and ξ_1, ξ_2, ξ_3 and ξ_4 are coefficients of second viscosity. These coefficients must be positive, and $(\xi_1+\xi_2)^2 \leq 4\xi_2\xi_3$ must be satisfied. Also, we must take $\xi_1 = \xi_4$ in view of Onsager's reciprocal theorem.

Evaluating the variables P' and T' from Eqs. (3.1)-(3.2) and combining them with Eqs. (3.3)-(3.4) together with the relations

$$(\frac{\partial S}{\partial P})_T - \frac{1}{\rho^2}(\frac{\partial \rho}{\partial T})_P \quad , \tag{3.5}$$

$$\{(\frac{\partial \rho}{\partial P})_T(\frac{\partial S}{\partial T})_P - (\frac{\partial \rho}{\partial T})_P(\frac{\partial S}{\partial P})_T\}^{-1} - (\frac{\partial P}{\partial \rho})_S(\frac{\partial S}{\partial T})_P^{-1} - (\frac{\partial s}{\partial T})_P^{-1}(\frac{\partial \rho}{\partial P})_T^{-1} \quad , \tag{3.6}$$

we obtain

$$(\frac{\omega_s}{k})^4 - [c_{10}^2 + c_{20}^2 - i\omega_s \frac{1}{\rho_o} \frac{\bar{\kappa}}{C_V} - i\omega_s \frac{1}{\rho_n}(-2\rho_o\xi_1 + \xi_2 - \rho_o\rho_s\xi_3)](\frac{\omega_s}{k})^2$$

$$+ c_{10}^2 c_{20}^2 \frac{C_V}{C_P} - i\omega_s c_{10}^2 \frac{\rho_s}{\rho_o\rho_n}(-2\rho_o\xi_1 + \xi_2 + \rho_o^2\xi_3 + \frac{\rho_n}{\rho_s} \frac{\bar{\kappa}}{C_P})$$

$$- i\omega_s \frac{\rho_s\rho_o}{\rho_o^2\rho_m}\{c_{10}^2 \frac{T}{C_P}[2(\frac{\partial \rho}{\partial T})_P](-\rho\xi_1 + \xi_2) + \rho_o\xi_2 S \frac{T}{C_V}\} = 0 \quad , \tag{3.7}$$

where

$$c_{10}^2 = (\frac{\partial P}{\partial \rho})_S \quad , \quad c_{20}^2 = \frac{\rho_s}{\rho_n} S^2 \frac{T}{C_V}$$

$$C_V = T(\frac{\partial S}{\partial T})_\rho \quad , \quad C_P = T(\frac{\partial S}{\partial T})_P \quad . \tag{3.8}$$

Equation (3.7) is a quadratic equation in ω_s/k, and its two solutions correspond to two modes of sound propagation known as first and second sound with the corresponding attenuation coefficients. Solving Eq. (3.7) we obtain

$$c_1^2 = c_{10}^2 - i\frac{\omega_s}{\rho_o}[\xi_2 + \frac{\bar{\kappa}}{C_V} - \frac{\bar{\kappa}}{C_P} - 2 \frac{\rho_s S}{\rho_o\rho_n} \frac{T}{C_P}(\frac{\partial \rho}{\partial T})_P(-\rho_o\xi_1 + \xi_2)] \quad , \tag{3.9}$$

$$c_2^2 - C_{20}^2 - i\frac{\omega_s \rho_s}{\rho_o \rho_n}[\xi_2 - 2\rho_o\xi_1 + \rho_o^2\xi_3 + \frac{\rho_n}{\rho_s}\frac{\bar{\kappa}}{C_P} + 2\frac{S}{\rho_o}\frac{T}{C_P}(\frac{\partial\rho}{\partial T})_P(-\rho_o\xi_1+\xi_2)] ,$$

$$(3.10)$$

$$c_1 - Im(\frac{\omega_s}{c_1}) - \frac{\omega_s}{2\rho_o c_{10}^2}[\xi_2 + \frac{\bar{\kappa}}{C_V} - \frac{\bar{\kappa}}{C_P} - \frac{\rho_s S}{\rho_o \rho_n}\frac{T}{C_P}(\frac{\partial\rho}{\partial T})_P(-\rho_o\xi_1+\xi_2)] , \quad (3.11)$$

$$\alpha_2 - Im(\frac{\omega_s}{c_2}) - \frac{\omega_s^2 \rho_s}{2\rho_o\rho_n c_{20}^2}[\xi_2 - 2\rho_o\xi_1 + \rho_o^2\xi_3 + \frac{\rho_n}{\rho_o}\frac{\bar{\kappa}}{C_P}$$

$$+ \frac{S}{\rho_o}\frac{T}{C_P}(\frac{\partial\rho}{\partial T})_P(-\rho_o\xi_1+\xi_2)] . \qquad (3.12)$$

The expressions for first and second sound and their attenuation coefficients are very similar to those for two-dimensional films.

IV. Third sound

When sound waves with wavelength longer than the film thickness propagate along a helium film, the normal fluid is clamped to the substrate, while the superfluid shows density fluctuations. Since third sound in superfluid helium films has been observed in thick and thin films,[12] there are several theoretical approaches.[13] Rutledge, McMillan, Mochel and Washburn[14] (RMMW) have analyzed third-sound data in thin helium films in terms of a quantum hydrodynamic approach with some phenomenological assumptions about the surface energy and roton excitation. We have also investigated thin-film data on the basis of our microscopic theory without any of the assumptions made by RMMW.

For the velocity of one-dimensional third sound, we make use of the formula

$$c_3^2(T) = \frac{\rho_s(T)}{m} \kappa(T) \quad ,$$ (4.1)

where m is the helium mass, $\rho_s(T)$ is the superfluid number density, and $\kappa(T)$ is the adiabatic elastic constant, which can be obtained from the second derivative of the internal energy with respect to the one-dimensional number density. The phonon and roton energies can be obtained from Eqs. (1.2) and (1.5) as

$$E_{ph}(T) = \frac{1}{2\pi}\{\frac{\zeta(2)}{D_1 n^{\frac{1}{2}}}(k_B T)^2 - \frac{3 \cdot 3! \zeta(4)}{2D_1^5 n^2}(n^{-\frac{1}{2}}+D_2^2 n^{\frac{1}{2}})(k_B T)^4$$

$$+ \frac{5! \zeta(6)}{2D_1^7 n^3}[D_3^2 n^{\frac{1}{2}} - \frac{1}{4}\frac{(D_2^2 n+1)^2}{D_1^2 n^{3/2}}](k_B T)^6 + \ldots\} \quad ,$$ (4.2)

$$E_{rot}(T) = (\frac{m^* k_B T}{2\pi \mu^2})^{\frac{1}{2}}(\Delta + \frac{1}{2}k_B T)e^{-\Delta/k_B T} \quad ,$$ (4.3)

and the ground state energy is given by

$$E_g = naV^*(1 - \frac{1}{\pi an} - \frac{\gamma a^2}{\pi V^*}V_o^2) \quad ,$$ (4.4)

where $V^* = (V_o - 6\epsilon_o/55)$ and $\gamma = 1.3290$, and for convenience we have used the following:

$$A_0^2 = D_1^2 n \quad , \quad A_1^2 = D_2^2 n \quad , \quad A_2^2 = D_3^2 n \quad . \tag{4.5}$$

Neglecting the variation of the roton energy Δ with temperature, we arrive at

$$\kappa(T) = \frac{d^2}{dn^2} [E_g + E_{ph}(T) + E_{rot}(T)]$$

$$= 2aV^*(1 - \frac{2m^*\gamma a^2 V_0^2}{\pi \hbar^2 V^*} + \frac{3\zeta(3)}{8\pi} \frac{1}{D_1 n^{5/2}} (k_B T)^2$$

$$- \frac{5!\zeta(4)}{8\pi}(\frac{3D_2^2}{8D_1^5 n^{7/2}} + \frac{7}{8D_1^5 n^{9/2}})(k_B T)^4$$

$$+ \frac{5!\zeta(6)}{2\pi}(\frac{35D_1 D_3^2}{8n^{\frac{1}{2}}} - \frac{35D_2^2}{32(D_1^2 n)^{9/2}} - \frac{63D_2^2}{16D_1^9 n^{11/2}} - \frac{99}{32D_1^9 n^{13/2}})(k_B T)^6$$

$$+ \ldots \tag{4.6}$$

Thus we obtain third sound as

$$c_3^2(T) = c^2(0)[1 - a_1 T^2 + a_2 T^4 + a_3 T^6 - a_4 T^{-\frac{1}{2}} e^{-\Delta/k_B T}$$

$$- a_5 T^{5/2} e^{-\Delta/k_B T} + a_6 T^{7/2} e^{-\Delta/k_B T} - a_7 T^{11/2} e^{-\Delta/k_B T}] \quad , \tag{4.7}$$

where the coefficients are given by

$$c^2(0) = \frac{nb}{m^*} \quad , \quad b = 2ak_B V^*(1 - \frac{2m^*\gamma a^2 v_o^2 k_B}{\pi \hbar^2 V^*}) \quad ,$$

$$a_1 = \frac{\zeta(2)k_B^2}{\pi D_1^3 n^{5/2}} (2 - \frac{3D_1^2}{8b}) \quad ,$$

$$a_2 = \frac{\zeta(4)k_B^4}{2\pi D_1^5 n^{7/2}} [\frac{5!}{D_1^2}(\frac{1}{n} + D_2^2) - \frac{9!!}{4b}(\frac{D_2^2}{7} + \frac{1}{3n}) - \frac{3\zeta(2)^2 D_1}{\pi\zeta(4)bn^{3/2}}] \quad ,$$

$$a_3 = \frac{\zeta(6)k_B^6}{\pi D_1^7 n^{9/2}} [\frac{7!}{2D_1^2}(D_3^2 - \frac{1}{4D_1^2 n^2} - \frac{D_2^2}{2D_1^2 n} - \frac{D_2^2}{4})$$

$$- \frac{45\zeta(2)\zeta(4)}{45\zeta(6)\pi b D_1 n^{5/2}} (\frac{8}{n} + D_2^2 + 3D_3^2)$$

$$+ \frac{1}{2bn} (7!!D_3^2 - \frac{7!!D_2^4}{4D_1^2} - \frac{9!!D_2^2}{2n} - \frac{11!!}{28D_1^2 n^2}] \quad ,$$

$$a_4 = 2q_o(\frac{m^*}{2\pi\hbar^2 k_B})^{\frac{1}{2}} \quad , \quad a_5 = \frac{3q_o^2\zeta(2)}{4\pi b D_1 n^{7/2}} (\frac{m^* k_B^3}{2\pi\hbar^2})^{\frac{1}{2}} \quad ,$$

$$a_6 = \frac{45\zeta(4)q_o^2}{4\pi b D_1^5 n^{9/2}} (\frac{m^* k_B}{2\pi\hbar^2})^{\frac{1}{2}}(3D_2^2 + \frac{7}{n}) \quad ,$$

$$a_7 = \frac{\zeta(6)q_o^2}{4\pi b D_1^9 n^{\frac{1}{2}}} (\frac{m^* k_B^{11}}{2\pi\hbar^2})^{\frac{1}{2}}(4\cdot 7!!D_1^2 D_3^2 - 7!!D_2^4 - \frac{2\cdot 9!!D_2^2}{n} - \frac{11!!}{7n^2}) \quad . \quad (4.8)$$

In Eq. (4.7) the first, next three and fifth terms are associated with the ground state, phonon and roton energy, respectively. The remaining terms are coupled with the phonon and roton energy.

V. Numerical results and discussion

In this section we present calculations based on the formulas obtained in the previous sections. Before going further, we first look at the excitation spectrum, because all sounds we have obtained are closely related to the spectrum. We adopted a soft potential and obtained the excitation spectrum as a function of the dimensionless parameter qa shown in Fig. 1 with the potential and roton parameters. As mentioned earlier, our theory is characterized by the potential parameters V_o, ϵ_o and a. For given values of these parameters, we can obtain the roton parameters Δ and q_o. Therefore, we have determined the potential parameters such that the excitation spectrum becomes anomalous phonon-like and roton-like for small and large momenta, respectively, and reproduces the same form as that of two-dimensional third sound (Fig. 2) because of the lack of the experimental data in one-dimensional superfluid helium. For the case of helium films, we have determined the parameters so as to reproduce the specific heat data of Bretz et al,[15] and with this choice we have obtained c_o = 84.06 m/s, which is close to the experimental value of (76 ± 2) m/s of Washburn et al.[16] Hence, the numerical parameters in one dimension have been chosen in association with the previous values, which we have taken earlier in two- and three-dimensional liquid helium. They are listed in Table 1. However, the effective mass and roton energy are much lower and a little higher than those of thin helium films, respectively. The behavior of the excitation spectrum, which is analogous to the Landau spectrum, is anomalous phonon-like for small momenta and roton-like

at around q_o for large momenta. By increasing (or decreasing) the numerical values of n, a and V_o, we can make the phonon slope steeper (or slow) and also deplete the roton minimum progressively. By appropriate increases of parameters, the roton minimum may disappear and eventually become an inflection point. Then the spectrum reduces to that of a pseudopotential or Coulomic potential.[17] Neglecting the Lennard-Jones-type tail, the excitation spectrum becomes the three-dimensional excitation spectrum of Brueckner and Sawada.[18] In comparison of the excitation with Lieb's results,[19] ours just corresponds to type-I excitation (the so-called Bogoliubov spectrum).

The thermodynamic functions obtained from phonon excitation (Eq. (1.2) agree with Padmore's results.[20] Due to the dimensionality, all thermodynamic functions are one power lower in temperature than those for two dimensions. For the roton part, they have the same form as those for other dimensions except for small modifications.

In Sec. 2 we have evaluated first sound (Eq. (2.3)) by solving the collisionless kinetic equation. The leading term for first sound increases as T^2, in contrast to T^3 and $T^4 \ln T^{-1}$ in two- and three-dimensional liquid helium, respectively. In one and two dimensions the absence of a logarithmic term is due to the fact that the angle integrals become zero or do not appear in both dimensions. The first-sound expression (Eq. (2.3)) decreases monotonically, while first sounds obtained by considering of the collision term as in the single-collision-time model[21] in two and three dimensions increase gently, and then decrease as temperature increases from absolute zero. We note that the single-collision-time model does not hold in one dimension. The collisionless kinetic equation does not yield second sound, which means that the collision integral and second sound are closely related to each other.

In the low-frequency region, first- and second-sound propagations are obtained by treating superfluid hydrodynamic equations including the energy dissipation effect. The results are given by Eqs. (3.9)-(3.12). Excluding attenuation terms and substituting thermodynamic functions and fluid densities (see the Appendix), first and second sound at absolute zero temperature reduce to

$$c_1(0) = c_2(0) = A_0 = [2na(V_0 - \tfrac{6}{55}\epsilon_0)/m^*]^{\frac{1}{2}} \quad .$$

We thus note that A_0 in Eq. (1.2) represents the sound velocity at absolute zero temperature, such that the ratio of c_2 to c_1 becomes unity at absolute zero. We have shown that the magnitude of second sound is $1/\sqrt{2}$ of the first-sound velocity in thin helium films[7] and find that c_2 is $1/\sqrt{3}$ times that of first sound in the bulk case[9] at absolute zero temperature in the low-frequency region. Figure 3 illustrates our theoretical sounds based on Eq. (3.8) at the density of $n = 0.167$ Å^{-1}, which corresponds to 2.18×10^{-2} Å^{-3} in three dimensions. The numerical value of $C_1(0)$ is given by 201.4 m/s, which is close to that of three dimensions rather than that in two dimensions. This figure confirms that the two sound modes show different temperature variations. However, the overall variations of the two sounds are very similar to those of two dimensions. As temperature increases, second sound passes through a first gentle maximum and arrives at a minimum at around 0.8 K. It then reaches another peak, after which it decreases rapdily and vanishes at the critical point. This variation is mainly due to the thermodynamic functions given in the Appendix. As temperature increases from absolute zero, first sound increases moderately and then decreaes monotonically. This indicates that the excitation spectrum is anomalous at

low momentum. As mentioned in preceding sections, we note that using a phonon-Boltzmann-equation method, Maris showed that second sound at high temperatures and low frequencies increases gradually to first sound in the opposite limit, and further calculations given by Benin[22] predicted that this one-dimensional second sound propagates far into the regime $\omega_s r \gg 1$.

In conclusion, we have two remarks: (1) The leading terms in first, second and third sound vary as T^2. (2) Second sound reduces to first sound at absolute zero temperature.

Acknowledgments

Appendix

The thermodynamic functions and normal fluid density for the phonon part and roton density[11] are given by

$$S_{ph}(T) = \frac{k_B}{2\pi} \left(\frac{2!\zeta(2)}{A_0} (k_B T) - \frac{4!\zeta(4)}{A_0^4} B(k_B T)^3 + \frac{6!\zeta(6)}{A_0^6} C(k_B T)^5 + \ldots \right) \quad ,$$

$$P_{ph}(T) = \frac{1}{2\pi} \left(\frac{3}{2} \frac{\zeta(2)}{A_0} (k_B T)^2 - \frac{3!\zeta(4)}{A_0^4} B_1 (k_B T)^4 + \ldots \right); \quad B_1 = \frac{5A_1^2 + 7}{2A_0} \quad ,$$

$$C_{V,ph}(T) = \frac{k_B}{2\pi} \left(\frac{2!\zeta(2)}{A_0} (k_B T) - \frac{3 \times 4!\zeta(4)}{A_0^4} B(k_B T)^3 + \frac{6!\zeta(6)}{A_0^6} C(k_B T)^5 + \ldots \right) \quad ,$$

$$\rho_{n,ph}(T) = \frac{1}{2\pi} \left(\frac{2!\zeta(2)}{A_0^3} (k_B T)^2 - \frac{5!\zeta(4)}{A_0^6} B(k_B T)^4 + \frac{7!\zeta(6)}{A_0^8} C(k_B T)^6 + \ldots \right) \quad ,$$

$$\rho_{n,rot}(T) = \left(\frac{m^*}{2\pi k_B T} \right)^{\frac{1}{2}} q_o^2 \, e^{-\Delta/k_B T} \quad .$$

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Table 1. Potential and roton parameters.

$n(\text{Å}^{-1})$	$a(\text{Å})$	$V_0(K)$	$\epsilon_0(K)$	$q_0(\text{Å}^{-1})$	m^*/m	$\Delta(K)$
0.167	3.40	8	26	1.19	0.30	5.36

Figure Captions

Figure 1.   Elementary excitation spectrum $\epsilon(q)/k_B$ for one-dimensional liquid helium plotted against the dimensionless parameter qa.

Figure 2.   Temperature variation of third sound at the density 0.167 Å^{-1}.

Figure 3.   Overall temperature variation of first- and second-sound velocity at the density 0.167 Å^{-1}.

Figure 4.   Temperature dependence of the ratio $(c_1-c_2)/c_1$ at the density 0.167 Å^{-1} at temperatures below about 0.3 K.
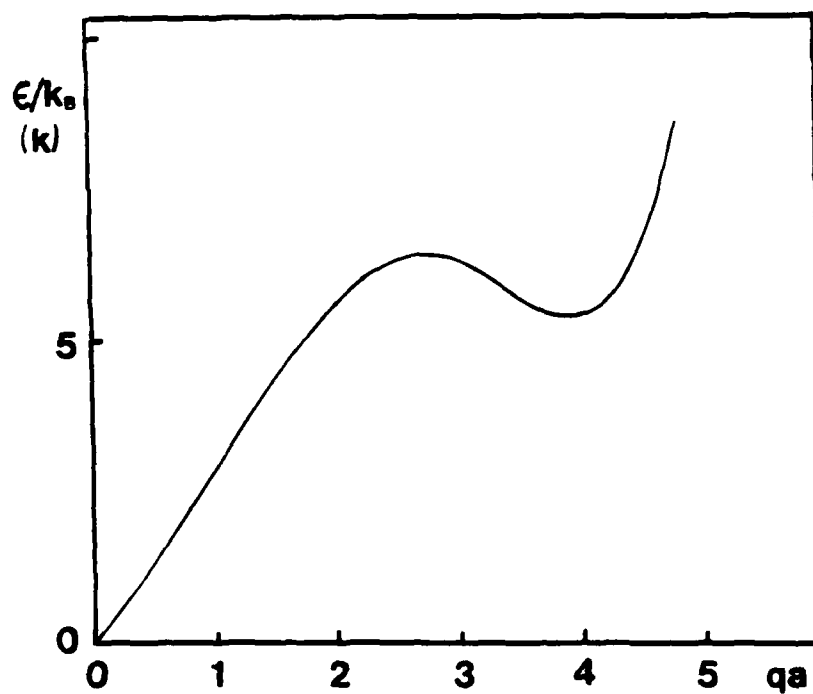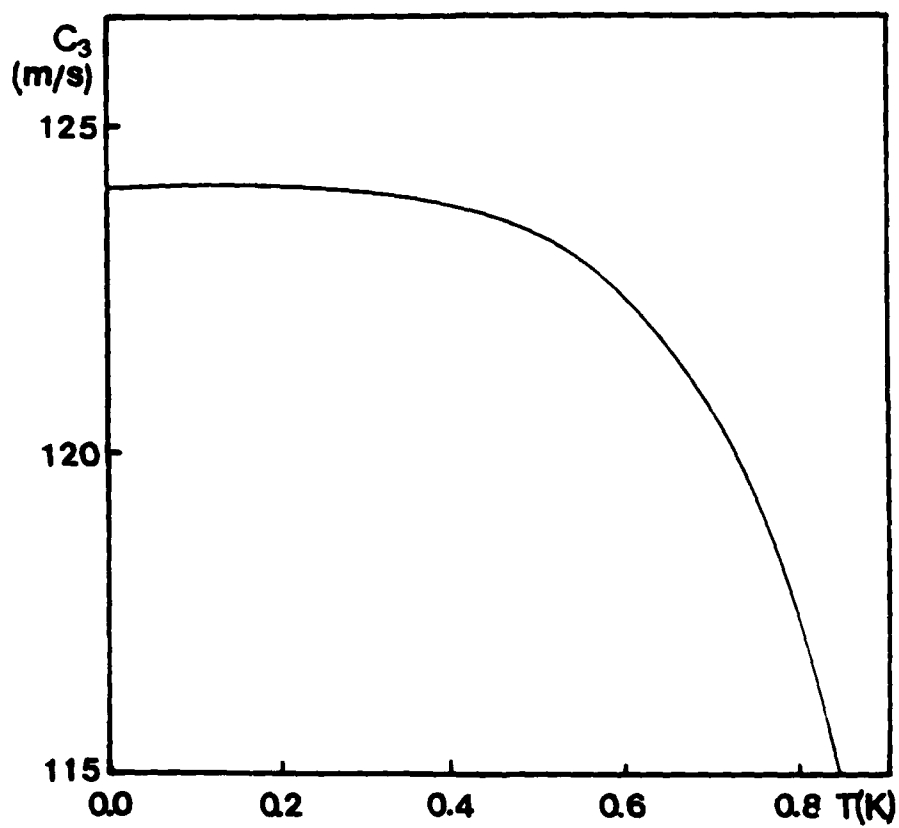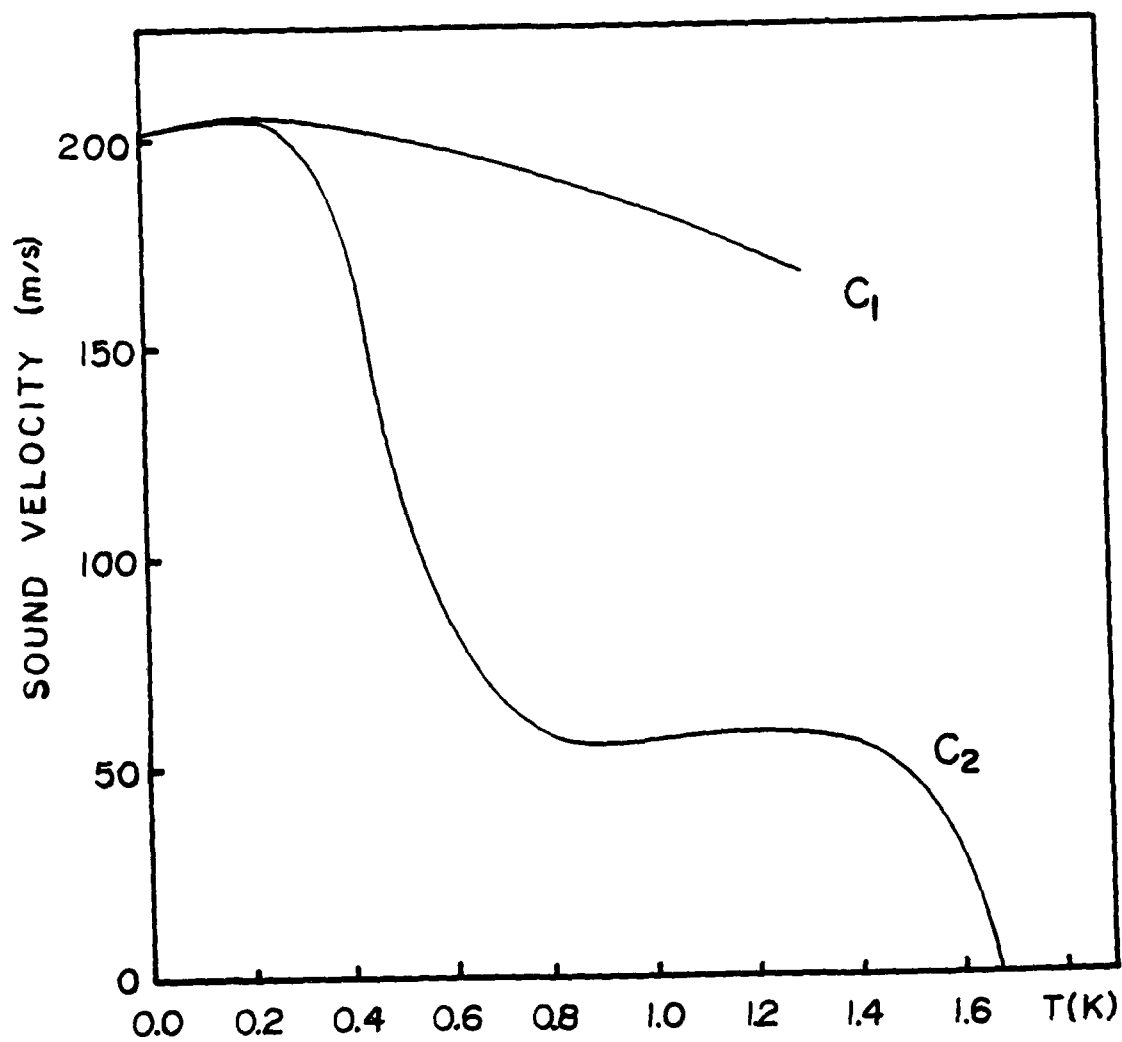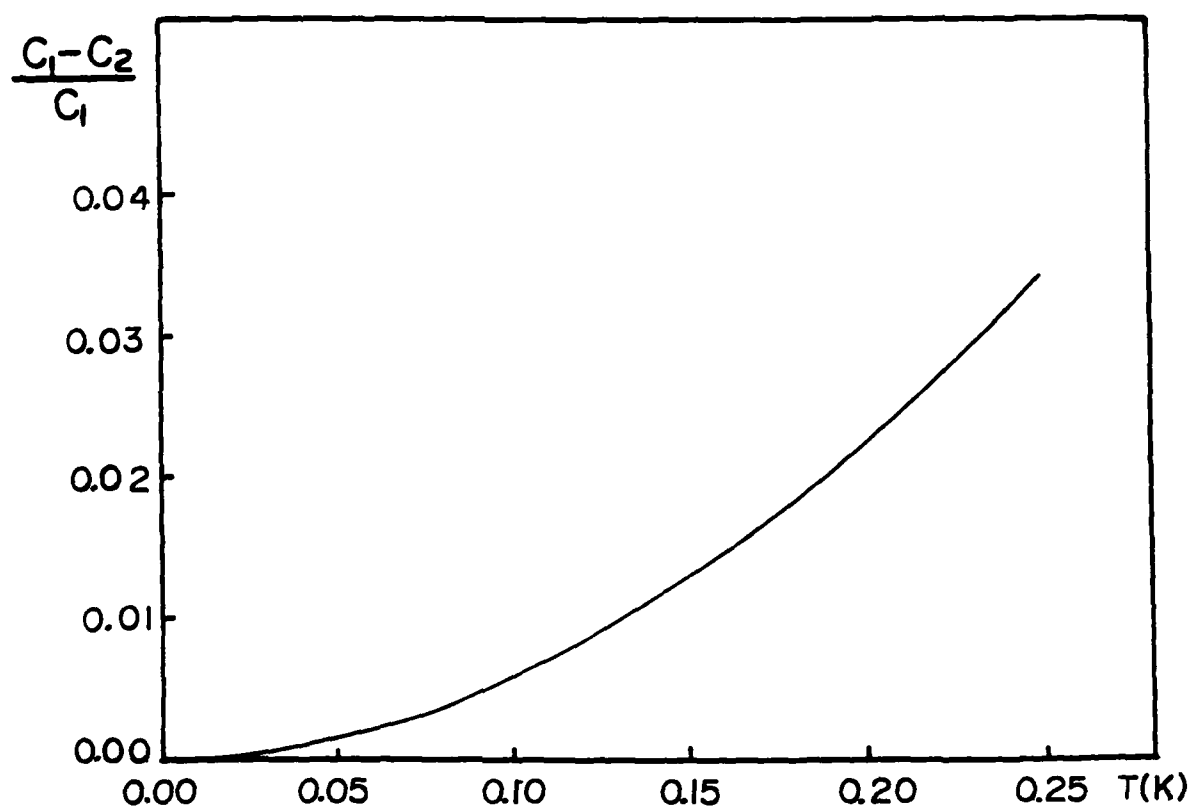
Fig. 1

Fig. 1

Fig. 2

Fig. 3

Fig. 4

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Dr. J. E. Jensen
Hughes Research Laboratory
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University of Minnesota
Minneapolis, Minnesota 55455

Dr. A. Reisman
Microelectronics Center of North Carolina
Research Triangle Park, North Carolina
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Department of Chemistry
Kansas State University
Manhattan, Kansas 66506

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Department of Chemistry
University of California
Berkeley, California 94720

Dr. F. Kutzler
Department of Chemistry
Box 5055
Tennessee Technological University
Cookesville, Tennessee 38501

Dr. D. DiLella
Chemistry Department
George Washington University
Washington D.C. 20052

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Chemistry Department
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Stanford University
Department of Chemistry
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Department of Chemistry
University of Richmond
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Philadelphia, Pennsylvania 19104

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Department of Chemistry
Case Western Reserve University
Cleveland, Ohio 41106

Dr. N. Winograd
Department of Chemistry
Pennsylvania State University
University Park, Pennsylvania 16802

Dr. Roald Hoffmann
Department of Chemistry
Cornell University
Ithaca, New York 14853

Dr. A. Steckl
Department of Electrical and
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Rensselaer Polytechnic Institute
Troy, New York 12181

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Department of Chemistry
Cornell University
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