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# CAPABILITIES OF TRACKING MOUNTS

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The result of the work reported on herein was an algorithm for establishing points on a surface partitioning the space around a tracking mount into two disjoint regions - one where tracking a constant velocity, constant altitude target is possible and the one where tracking cannot be accomplished. An idealized output torque versus shaft angular velocity is assumed for the drives. The method is general and can be extended to accommodate other driver characteristics.

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# I. INTRODUCTION

## A. Background

There is a logical sequence of events in the design of any defensive weapons system. Initially, it is recognized that a known or potential threat exists for which one or more countermeasures are desired. There are specific threat capabilities that existing systems cannot defeat, necessitating either modification of the existing system or the development of a new one. Three groups of people - strategists, designers, and analysts - participate in the development and/or modification process.

Initially, strategists and planners develop the general concepts of a weapons system to defeat the threat. Their decisions are based on knowledge of flight capabilities of the threat obtained from intelligence reports and other sources. Defensive system specifications are established to ensure defeat of the threat while minimizing its impact on military operations. Designers then assemble hardware in accordance with system specifications. Analysts have a two-fold role in the process: (1) they develop techniques for testing and comparing alternate designs and (2) they translate the results of simulation studies into usable design criteria for hardware designers. It is this role which provided the impetus for the work reported here.

This report addresses one particular facet of an analyst's contribution: namely, the development of an algorithm for calculating and plotting inner tracking limits of a tracking system. This work is an extension of the work of Campbell and Christensen (Report No. RL-TN-70-3, "Tracking Mount Relationships for a Constant Velocity Maneuvering Target"). The notation and basic kinematical relationships are retained.

The term "tracking" refers to the process of positioning a rigid body so that for all instants of time a line in the rigid body passes through a moving point. Common examples of such bodies are guns, guidance seekers, and missile launchers. The rigid body is typically constrained to rotate about horizontal and vertical axes (elevation and traversing). The torques and/or forces necessary for tracking are interrelated functions of object point movement, rigid body mass distribution, rigid body kinematics, and friction. Since it is impossible to design a system to track for all positions and velocities, some knowledge of the limitations and capabilities of a proposed tracking system is necessary in the initial design phase. Equally important is the ability to estimate the locus of points in space where a tentative tracking system can be expected to track a specified moving point.

Estimates of the rotational speeds and accelerations essential to an acceptable system must be available before design can begin. Obviously, these data are influenced by the kinematics of the moving point and significantly impact the choice of system driver(s). Equations for calculating these data as functions of point position and velocity were derived as part of this effort and can be used independently of the plotting segments of the program to check the adequacy of a proposed design.

The referenced report of Dean and Campbell considered the same problem treated here, but their procedure yielded larger regions where tracking is possible than those generated when more realistic characteristics of the driver are considered. It is more realistic and more accurate to examine each point in space in relation to the driver angular velocity and angular acceleration associated with a target passing through the point to see if the combination of velocity and acceleration are within the capabilities of the system driver. Velocity and/or acceleration bounds may preclude tracking, i.e., tracking may require an angular velocity in excess of maximum driver speed or a torque larger than the driver is capable of supplying.

This report addresses this essential activity which resulted in an algorithm for establishing the inner tracking boundaries of a tracking mount. The algorithm was implemented on an ALPHA-MICRO computer system. A finite number of parallel constant speed and constant altitude paths are examined to identify points along the paths where necessary angular accelerations and velocities are not within the torque and speed capabilities of the driving mechanisms. The output of the program is a plot of those points on the curve formed by the intersection of the surface bounding points in space where tracking is not possible (Appendix B). The analysis discussed here is limited to an elevating mechanism. Modification of the program to investigate a traversing mechanism would be straightforward.

This work consisted of developing computational algorithms and digital computer codes for calculating and plotting boundaries which partition the space surrounding a tracking mount into disjoint regions where tracking is possible and where tracking is impossible. The limitations are based on an assumed linear relationship between the angular velocity and torque capacity of elevating and traversing motors (Appendix A). A maximum angular velocity for the driver is implied; i.e., the magnitude of the angular speed in any direction is bounded. A point on a tracking boundary is determined if the required torque exceeds that available from the driving motor and the accompanying angular rate has a magnitude and direction which exceeds the maximum.

The output of the program is a closed curve in a plane of constant elevation. The shape of the curve is a function of target speed, target elevation, and driver characteristics. Points belonging to the region enclosed by the curve represent points in space for which, under the conditions specified, a target cannot be tracked. If suitable graphics software is available, several plots for different altitudes can be plotted on a single sheet simulating a three-dimensional plot of tracking boundaries.

## II. MATHEMATICAL BACKGROUND

An xyz-coordinate system was chosen where the y-axis is parallel to the flight path of an approaching target, with the positive y-axis directed toward the oncoming target. The positive z-axis is upward, and the positive x-axis is chosen to form a right-handed coordinate system (Fig. 1).

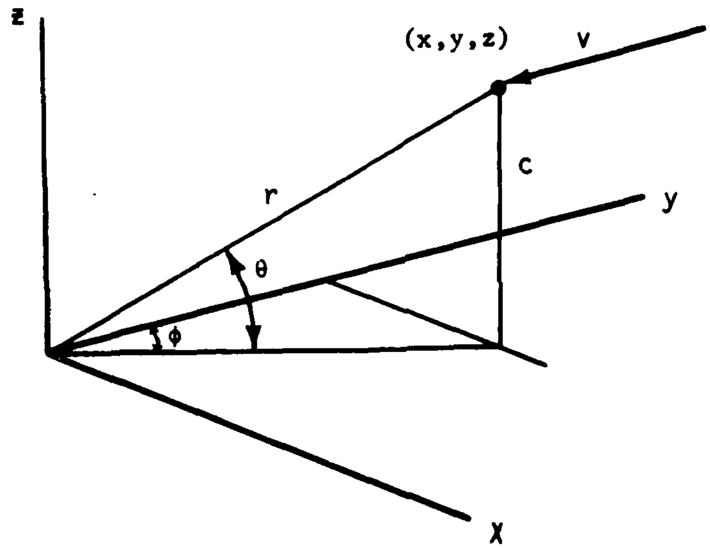


Figure 1. Illustration of an xyz - coordinate system.

Definitions

$\phi$ = mount to target heading (azimuth)
$\theta$ = mount to target altitude (elevation)
a = target x-coordinate (standoff distance)
c = target altitude
v = target speed

Using Figure 1, it is easy to establish the geometric relationships

$$x = r \cos\theta \sin\phi$$
$$y = r \cos\theta \cos\phi,$$

where r is the distance from the origin to the moving point. Under the conditions of the problem the altitude, c, is given by the equation

$$c = r \cos\theta. \qquad (1)$$

From

$$y = - v \qquad (2)$$

and

$$x/y = \tan\phi \tag{3}$$

it follows that

$$\dot{\phi} = (xv/y^2)\cos^2\phi \tag{4}$$

$$= (v/a)\sin^2\phi. \tag{5}$$

Also

$$\dot{\theta} = (v/c)\cos\phi\sin^2\theta. \tag{6}$$

Equations (5) and (6) can be differentiated to obtain expressions for angular accelerations as functions of position only.

Consider $\ddot{\theta}$. It is given by

$$\ddot{\theta} = -(\dot{\phi}v/c)\sin\phi\sin^2\theta + 2(\dot{\theta}v/c)\cos\phi\sin\theta\cos\theta . \tag{7}$$

Any rigid body experiencing an angular acceleration must be acted upon by a torque about the axis of rotation given by the relationship

$$T = I\ddot{\theta} \tag{8}$$

where I is the moment of inertia of the mount with respect to the axis of rotation. For tracking, this torque must be less than the maximum torque output available for the associated angular velocity. Driving motor characteristics (Appendix A) dictate that the maximum torque available for a given angular velocity $\dot{\theta}$ is given by:

$$I\ddot{\theta} = T = T_s - k\dot{\theta}. \tag{9}$$

Substituting the expressions for $\dot{\theta}$ and $\ddot{\theta}$ from equations (6) and (7) above and performing simple algebraic manipulations yield the conditions

$$f(\theta,\phi) = (v^2I/c^2T_s)\sin^2\theta[-\sin^2\phi\tan\theta + 2\cos^2\phi\cos\theta\sin\theta + (ck/vI)\cos\phi] = +1 \tag{10}$$

as necessary to establish a point on the tracking boundary. If $|f(\theta,\phi)| < 1$, the point in the plane z = c specified by the three parameters $\theta$, $\phi$, and c is a point in space for which tracking is possible. Pairs $(\theta,\phi)$ for which $|f(\theta,\phi)| > 1$ locate points in the plane z = c where tracking is not possible.

Before describing the procedure to establish tracking boundaries, some of the important properties of the function f are required. It is instructive to plot the function f for fixed $\phi$ and for $\theta$ varying between $\theta$ and (Fig. 2). With c also fixed, this variation in $\theta$ effectively generates points along the line of intersection of the planes $z = c$ and $\phi$ = constant, and the ordered triple $(\theta,\phi,c)$ locates a unique point along this line. Observe that for small $\phi$, there are two values of $\theta$ satisfying $f(\theta,\phi) = 1$ and one value of $\theta$ satisfying $f(\theta,\phi) = -1$. This property of f introduces a complexity into the algorithm for determining points on the tracking boundary and results in an unexpected shape for the area defined by this boundary.

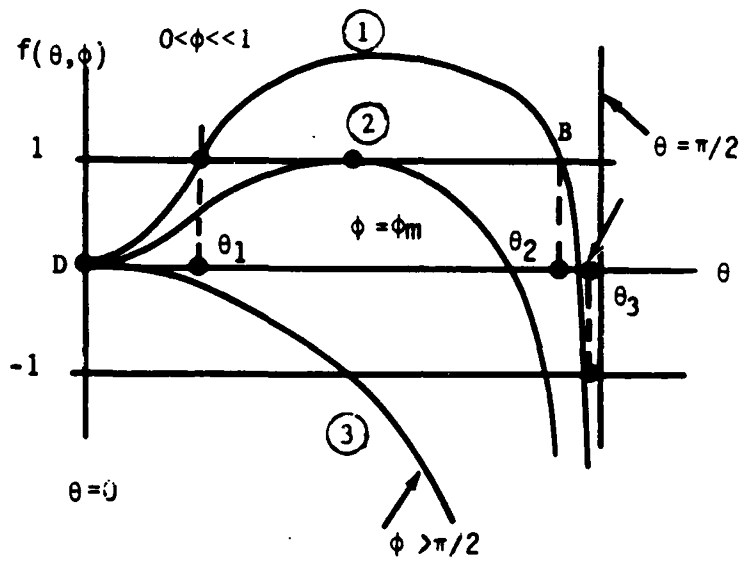


Figure 2. Level curves of $f(\theta,\phi)$.

Consider an ordered pair $(\theta,\phi)$ where the point $(\theta,f(\theta,\phi))$ belongs to the area in Figure 2 defined by the inequality

$$|f(\theta,\phi)| < 1. \tag{11}$$

Recall that the ordered pair $(\theta,\phi)$ defines a unique point $(x,y,c)$ in the plane $z = c$. For this point and given target velocity, $\ddot{\theta}$ and $\dot{\theta}$ can be calculated using equations (6) and (7) above. These are the acceleration and velocity of the elevation angle necessary for tracking a target passing through this point in space. Then a point in Figure A-1, Appendix A, whose coordinates are $(\dot{\theta}, I\ddot{\theta})$ lies between lines consisting of line segments AB and CD and their extensions. If in addition

$$|\dot{\theta}| < \frac{T_S}{K}, \tag{12}$$

then the given driver is capable of meeting the requirements for tracking a point moving through $(x,y,c)$ with speed v.

Figure 3 (valid for small $\phi$) is an aid in identifying points where tracking is not possible.

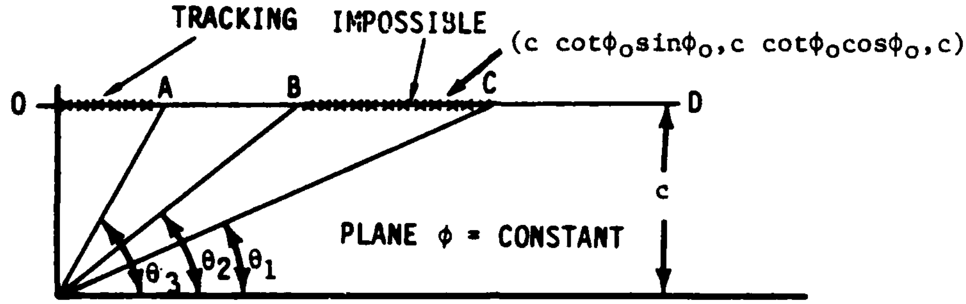


Figure 3. Tracking/non-tracking in plane $\phi$ = constant.

Line segment OD denotes the intersection of planes $z = c$ and $\phi = 0_0$, where $\phi_0$ is considered small, as above. The three angles $\theta_1$, $\theta_2$, $\theta_3$ are the angles defined in the previous paragraph. Points on the segment BC are associated with values of $\theta$ satisfying

$$\theta_1 < \theta < \theta_2 . \qquad (13)$$

For $\theta$ satisfying this condition, $f(\theta,\phi) > 1$ and tracking is impossible. Also, for

$$\theta_3 < \theta < \pi/2, \qquad (14)$$

$f(\theta,\phi_0) < -1$ and tracking is impossible for points along the segment OA. The tracking boundaries for fixed altitude consist of the locus of points A, B, and C for values of $\phi$ satisfying

$$0 < \phi < \pi. \qquad (15)$$

Because $f(\theta,\phi) = f(\theta, -\phi)$ the tracking boundary is symmetric to the plane $x = 0$. This symmetry is used in the plotting algorithm described below. It can be shown that the tracking boundary is not symmetric to the plane $y = 0$.

The algorithm for determining boundaries is straightforward. Parameters such as velocity, height, and torque-angular characteristics are assigned. The values of the function $f(\theta,\phi)$ are determined for different values of $\theta$ and $\phi$ as follows. The traversing angle, $\phi$, in incremented between 0 and $\pi$ radians starting at 0 radians. For each $\phi$, a value or values of $\theta$ are determined which satisfy

$$\left| f(0,o) \right| = +1. \qquad (16)$$

These angles identify limiting torque-velocity combinations and are used to define a point on the tracking boundary. The corresponding rectangular coordinates are determined, the point plotted, and the process repeated. The procedure is straightforward except for identifying the value of $\phi$, say $\phi_1$, for which two values of $\theta$ exist (Fig. 2). This value of $\phi$ is the upper bound of the set of $\phi$ for which there are two ordered pairs

$$(\theta_i, \phi_i), \; i = 1,2 \tag{17}$$

satisfying

$$f(\theta,\phi) = 1. \tag{18}$$

For $\phi_0 < \phi < \pi$, there are no real values of $\theta$ which satisfy

$$f(\theta,\phi) = 1, \tag{19}$$

and only one value of $\theta$ satisfying

$$f(\theta,\phi) = -1. \tag{20}$$

The most complex part of the computational algorithm is the logic for recognizing and handling the special situation arising when, to a single value of $\phi$, there are two values of $\theta$ for which $f(\theta,\phi) = +1$. Ordered pairs $(\theta,\phi)$ satisfying (19) are found using an increasing sequence of values of $\theta,\phi$ fixed. There will be a first $\phi$ for which there is no $\theta$ satisfying (19). Let this angle be called $\phi_1$ (Fig. 2). If $\phi$ is decreased, subsequent values of $\theta$ in the solution pairs $(\theta,\phi)$ of (19) will increase with $\pi/2$ as an upper bound. The azimuth angle $\phi$ is decremented to zero. As $\phi$ approaches 0 points on the tracking boundary approach, the z-axis tangent to the plane is x = 0. Finally, $\phi$ is incremented in the interval $(\phi_1,\pi)$ with the search for $\theta$ satisfying (20).

It should be clear that different combinations of system parameters will result in different tracking boundaries. In addition, many combinations of system parameters will yield the same plot. For example, consider the factor

$$v^2I/c^2T \tag{21}$$

in Equation (10). This factor is dimensionless and every choice of parameters yielding the same value for this ratio will result in the same tracking boundary.

The intersection of the tracking boundary and a plane z = constant is a closed curve and the area enclosed will approach zero with increasing height. The area cannot be zero for a finite altitude because of the rapid reversal in $\theta$ necessary for small standoff distance when $\phi$ is approximately 90°. As a special case, if the standoff distance, a, is zero, tracking requires an infinite $\theta$ as the target passes directly above the tracking mount. The time derivative of the elevation angle must reverse instantaneously to track a point moving directly overhead.

Additional insight into the relationship between the driver requirements and position can be obtained from a plot of a torque-angular velocity relationship with $\phi$ as a parameter varying between 0 and $\pi$. The plot will begin at the origin, since no torque or angular velocity is required for points at infinity. The larger torque and angular velocity requirements for tracking close-in targets will generate points more distant from the origin and, under some conditions, points on the tracking limit boundary. The $\theta$ and $\phi$ associated with these points determine a point on the spatial tracking boundary. The boundary surface for a given target (v = constant) can be determined by considering all possible standoff distances and altitudes. With a graphics system capable of manipulating and displaying 3-D geometric data, the bounding surface can be displayed to aid in visualizing its size and shape. An example obtained using an Evans and Sutherland PS-300 graphics system is shown in Figure 4.

As discussed above, a mount will lose the ability to track when the torque-angular velocity curve crosses the tracking limit boundary. Note that there are two limiting phenomena, those imposed by the sloping straight lines (torque) and those imposed by the vertical lines (velocity). For the sloping lines, the inability to track is characterized by a deficiency in torque, i.e., for the given angular velocity, the motor is incapable of generating the required torque. The vertical lines represent upper and lower bounds on the angular velocities of the motor. The driving motor is incapable of speeds outside these bounds.

Selected curves to illustrate several possibilities are shown in Figure 5. The ordinate and abscissa are dimensionless variables $T/T_s$ and $\dot{\theta}/\dot{\theta}_{max}$. Curve A is generated by a target flight path which does not intersect the inner tracking boundary surface. The locus of points representing required torque-angular velocity combinations is contained within the closed curve specifying driver characteristics. There are many sets of parameters which will generate such a torque-angular velocity curve. Large values of "a" and/or "c" or small values of "I" will contribute to generating similar curves. The trajectory generating curve B crosses the limiting boundary at four points. Points outside the boundary correspond to points in space where tracking is not possible. It can be concluded from curve B that the trajectory which generates it contains two intervals where the target is incapable of being tracked. The segments 1-2 and 3-4 lie outside the characteristic curve and correspond to points on the associated trajectory where tracking is not possible. Curve C is generated by a trajectory for which velocity as well as acceleration limitations preclude tracking.

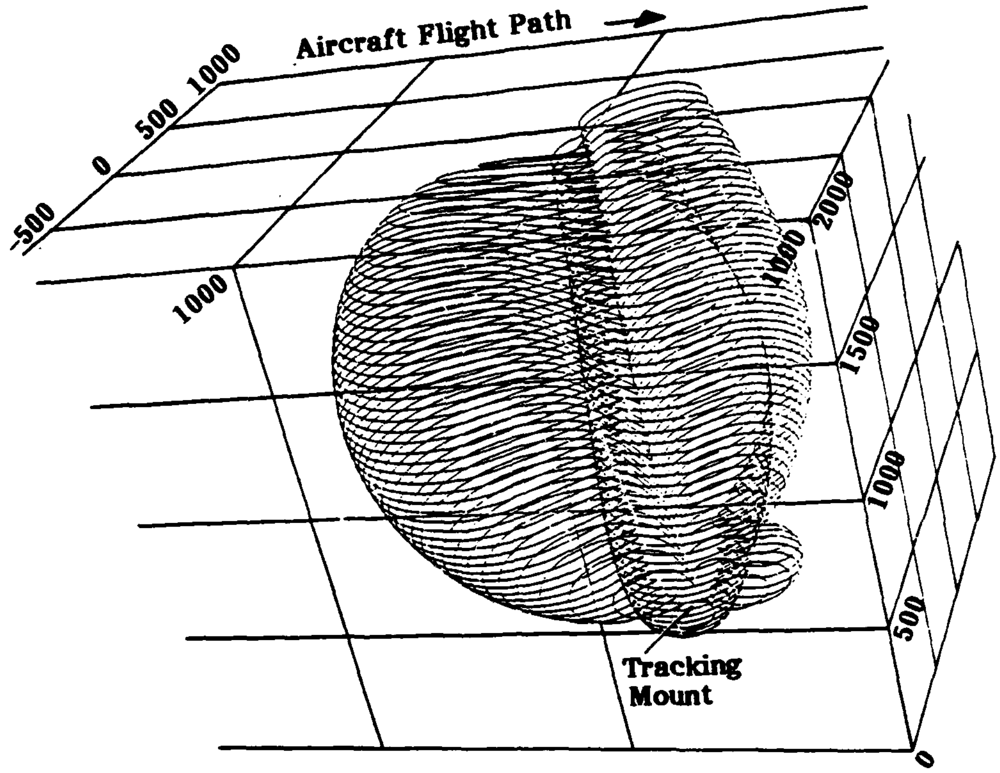


Figure 4. 3-D geometric display of boundary surface.

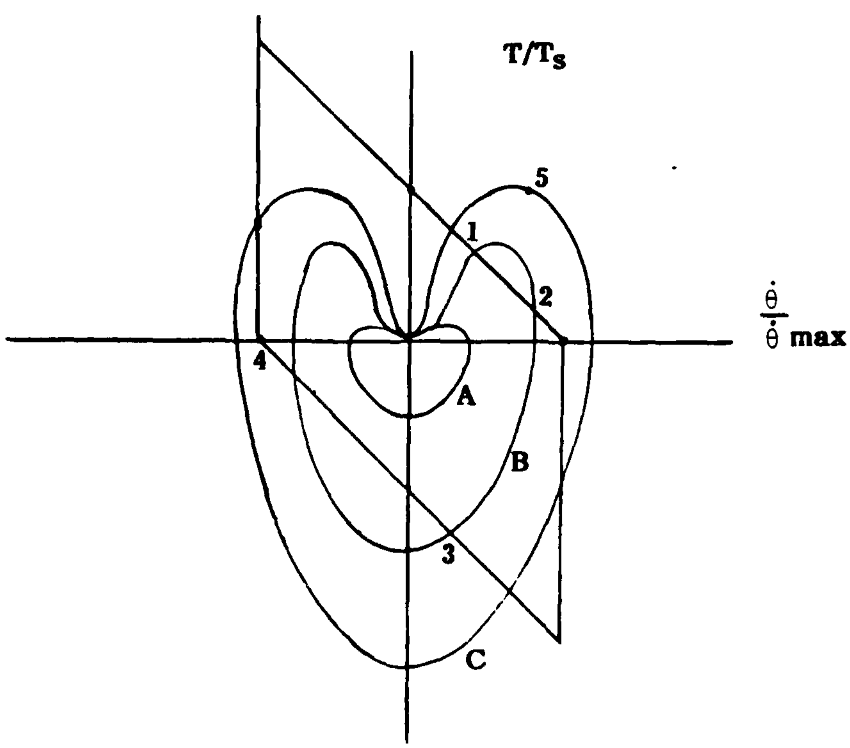


Figure 5. Typical torque-angular velocity curves.

## III. SHORTCOMINGS AND DEFICIENCIES

Throughout this report, the rotary inertias have been assumed independent of position, i.e., "I" has been assumed constant. In reality, the moments of inertia are time and position dependent. For example, the moments of inertia of a missile launcher are a function of the number of missiles in the launcher, and the moment of inertia of a mount about a traverse axis depends on the elevation angle of the mount. The error introduced by assuming them constant will depend on the total variation of I over the elevation limits. The variation of I with elevation, as well as static moments of system weights, could be accounted for. Other torque-speed relationships may be more realistic than the linear relationship treated herein.

The plots indicate that there may be trajectory-elevation combinations which determine disjoint sets of points where tracking is impossible, indicating that the ability to track a specific target can be lost, regained, and lost again before being acquired for the last time. (If the range is large enough, any system is capable of tracking a moving target, assuming adequate electronic acquisition and tracking capability.) This information is of little value unless the control system is capable, after losing the ability to track, of moving the tracking mount to the next available point to resume tracking when the target reaches the point. From a practical point of view, the most useful knowledge available from the algorithm is the location of the target when tracking capability is first lost.

## IV. CONCLUSIONS

The techniques described here are useful for visualizing the regions in space where a given tracking mount can track a target. They are too time consuming to be used in a real-time situation and are not suitable as a basis for electing alternate procedures when tracking becomes impossible. However, through the use of predefined tables, the location of the next point at which tracking is possible can be determined if tracking capability is lost. A control algorithm can be designed to position the mount to reacquire the target at some subsequent position when tracking again becomes possible.

This work demonstrates the possibility of creating a plot of the inner tracking boundaries for a simple tracking mount tracking a constant velocity target. The techniques can be extended to treat more complex situations where the target changes velocity or rotary inertias are functions of position.

The acceleration and velocity functions can be used during preliminary design for sizing motors to meet minimal ranges of engagement.

# APPENDIX A

# MOTOR CHARACTERISTICS

The assumed torque-speed relationship for the elevation and traversing drivers is shown in Figure A-1. The two inclined lines express the output torque as a function of rotational speed for a constant applied external voltage. Note for each angular speed there are two torques possible depending on direction of rotation and the sign of the voltage. One corresponds to a situation in which the applied voltage creates a torque in the direction of rotation. The other describes the situation when the torque acts as a brake, providing an angular acceleration opposite to the direction of rotation and reducing the speed of rotation. Line AB graphs the relationship when the external voltage induces rotation in one direction. Line CD graphs the relationship if voltage is reversed. Therefore, for a given angular speed there can be two output torques, depending on the sign of the applied voltage.

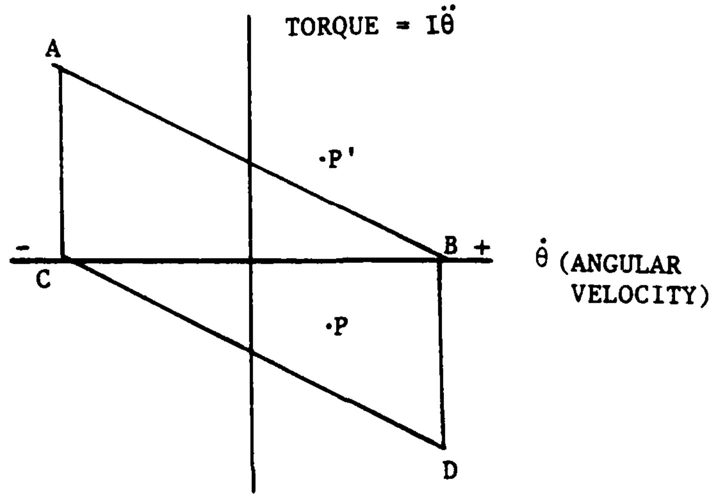


Figure A-1. Torque-speed relationship for elevation and traversing drivers.

It is further assumed that if the magnitude of the applied voltage is changed, the area enclosed by the characteristic curves changes proportionately. For example, if the magnitude of the applied voltage is halved, the maximum magnitude of the angular speed is also halved, reducing by a factor of four the torque-angular velocity combinations accessible to the control system. The coordinates of each point in the plane are a pair of numbers representing the mechanical state of a motor. Those points in the interior region represent physically possible states, i.e., through the use of appropriate control algorithms it is possible for the motor to operate at those states. At the given rotational speed it is possible to adjust the applied voltage so that the required torque is available. Combinations of torque and angular velocity associated with points in the exterior are physically impossible, and target velocities and/or positions yielding such combinations are positions for which tracking is impossible.

It is assumed that any combination of torque and speed which will plot within the boundaries of the parallelogram can be met by the elevating or traversing driver, i.e., a control system can be designed to meet the torque-speed requirement represented by point P. The figure above shows that a requirement represented by P cannot be met. The torque associated with point P is larger than can be provided by the driver at the corresponding angular velocity.

# APPENDIX B

# ALGORITHM FOR TRACKING BOUNDARIES

The algorithm for calculating and plotting tracking boundaries was implemented on an ALPHA MICRO minicomputer interfaced with a Hewlett-Packard digital plotter. The programming language is BASIC, but the program can be converted with minimal difficulty to some other language.

```
REM     THIS PROGRAM DETERMINES THE TRACKING LIMITS IMPOSED BY LIMITATIONS
REM     OF THE DRIVERS OF ELEVATING AND TRAVERSING MECHANISMS.
REM     IT IS ASSUMED THAT THERE IS A LINEAR RELATIONSHIP BETWEEN THE TORQUE
REM     OUTPUT AND ANGULAR VELOCITY OF A DRIVING MOTOR OF THE FORM:
REM     T = (+/-) TS + K * O, WHERE
REM        T  = TORQUE,
REM        TS = STALL TORQUE,
REM        K  = SLOPE OF THE TORQUE-ANGULAR VELOCITY LINE,
REM        O  = THE ANGULAR VELOCITY.
REM     THE ABSOLUTE VALUE OF THE ANGULAR VELOCITY MUST REMAIN LESS THAN TS/K.

REM ****** DEFINE PARAMETERS AND DIMENSIONLESS VARIABLES ******
        SIGNIFICANCE 11 : REM ESTABLISHES FIXED POINT ACCURACY
        GOSUB 600 : REM READ DEFAULT VARIABLES AND INTIALIZE PLOTTER
        CALL HP'INI

REM  **** MENU ****

MENU:   PRINT TAB(-1,0)
        ?"          MENU OF MTHETA "
        ?:?:?
        ?"   1-NEW DATA"
        ?"   2-NEW PLOT"
        ?"   3-RUN ACCEL. PLOT"
        ?"   4-RUN VELOCITY PLOT"
        ?"   5-END"
        ?"   6-GRID"
        ?""
        INPUT"",NUM
        ?TAB(-1,0)
        ON NUM GOTO 500,300,100,150,3000,900
        GO TO MENU


REM **** PLOT LIMITS BASED ON TORQUE ****

100     DT = DTI:B=1:GOSUB 200
        DT = DTI:B=-1:GOSUB 200
        GOTO MENU

REM **** PLOT LIMITS BASED ON VELOCITY ****
```

```
150     ZZ=1/DR1:IF ZZ > 1 THEN ? TAB(-1,0):FOR I=1 TO 10:?:NEXT I:&
        ?"NO BOUNDARY LIMITS EXIST FOR THIS CASE!!":FOR I=1 TO 2000:NEXT I:&
        GOTO MENU
        T=ASN(SQR(ZZ)): CP=1:SP=0:B=1:? PON$:CALL PL'T:? PD$
        PH=0:GOSUB 400
        PH=PI-PH:CP=COS(PH):SP=SIN(PH):?PON$:CALL PL'T:? PD$
        GOSUB 400
        T=ASN(SQR(ZZ)):CP=1:SP=0:B=-1:? PON$:CALL PL'T:? PD$
        PH=0:GOSUB 400
        GOTO MENU

REM ****** PLOTTING ROUTINE FOR BOUNDARY LIMITS BASED ON ATTAINABLE TORQUES

REM ****** INITIALIZE AZIMUTH AND ELEVATION(PH AND T)
REM ****** LOOK FOR VALUE OT THETA WHICH MAKES ABS(AC)=1

200     T=0:PH=0
        GOSUB 1100:CP=1:SP=0:?PON$:?PU$:CALL PL'T:?PD$

REM ****** IF AC>0 THERE ARE THREE VALUES OF THETA FOR WHICH ABS(AC))=1
REM ****** THERE ALSO EXISTS A PH(PH=PH*) FOR WHICH THERE ARE ONLY TWO ROOTS
REM ****** THIS OCCURS IF THERE EXIST THETA SUCH THAT AC = 1.

        IF AC > 1 THEN T=T-DTI:FLAG =1:GOTO 1200 : REM CASE FOR THREE ROOTS

        FLAG = 0 : REM IDENTIFIES CASE HAVING ONLY ONE ROOT FOR GIVEN PHI
        PH = 0 : GOTO 1305

REM **** PLOT VELOCITY IMPOSED BOUNDARIES ****


400     PH=PH+DP:CP=COS(PH):SP=SIN(PH)
420     ZZ1=ABS(ZZ/CP):IF ZZ1>1 THEN ? PU$:?POFF$:RETURN
440     T=ASN(SQR(ZZ1))
        CALL PL'T: GOTO 400


1000    REM ******  ROOT FINDING ALGORITHM ******

1100 REM ***** FIND SMALLEST THETA FOR WHICH ABS(AC)=1
1105    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(2.*S*CT+DR2)
        IF ABS(AC) < 1THEN T=T+DT:GOTO 1105
        IF ABS(AC-SGN(AC))<.01 THEN ?PON$:CALL PL'T:?PD$:RETURN
        T=T-DT:DT=DT*.1:T=T+DT:GOTO 1105

1200 REM ***** CASE FOR THREE ROOTS ******
REM ***** ENTER THIS ROUTINE WITH PHI(AZIMUTH) FIXED AND SMALLEST THETA FOR AC=1
```

```
REM ***** GET PRECISE VALUE OF THETA AND PLOT INITIAL POINT ******

        T=T-DT
1205    DT=DTI:PH=PH+DP
        CP=COS(PH):SP=SIN(PH)
1210    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC <0 THEN GOTO 1248:REM THETA ROOT DOES NOT EXIST
        IF AC <1THEN T=T+DT:GOTO 1210
        IF (AC-1)<.01 THEN TT=T:CALL PL'T:?PD$:GOTO 1205
        T=T-DT:DT=.1*DT:T=T+DT:GOTO 1210

REM DECREMENT PH UNTIL PH<DP
1248    T = TT
1250    DT=DTI:PH=PH-DP:IF ( PH < DP ) THEN GOTO 1300
        CP=COS(PH):SP=SIN(PH)
1255    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC>1 THEN T=T+DT:IF T > PI/2 THEN GOTO 1300: ELSE GOTO 1255
        IF 1-AC < .01 THEN CALL PL'T:GOTO 1250
1260    T=T-DT:DT=.1*DT:T=T+DT:GOTO 1255


1300    T=PI/2-DT
1305    DT=DTI:PH=PH+DP:IF PH>PI/2 THEN GOTO 1350
        CP=COS(PH):SP=SIN(PH)
1310    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC < -1 THEN T = T-DT:GOTO 1310
        IF AC+1 < .01 THEN CALL PL'T:GOTO 1305
        T=T+DT:DT=.1*DT:T=T-DT:GOTO 1310

REM ****  LET PH INCREASE TO PI
1350    T=T-DT
1355    DT=DTI:PH=PH+DP:IF PH>PI-DP THEN GOTO 1400
        CP=COS(PH):SP=SIN(PH)
1360    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC > -1 THEN T=T+DT:IF T > PI/2 THEN GOTO 1380: ELSE GOTO 1360
        IF -1-AC<.01 THEN CALL PL'T:GOTO 1355
1380    T=T-DT:DT=.1*DT:IF DT < 1E-10 THEN GOTO 1400 : ELSE T=T+DT:GOTO 1360

REM **** LET PHI DECREASE TO DP LOOK FOR THETA SUCH THAT AC = 1

1400    IF FLAG = 0 THEN GOTO 1600: REM CASE FOR WHICH AC DOES NOT EQUAL 1
        DT=DTI:PH=PH-DP
        CP=COS(PH):SP=SIN(PH)
1410    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC < 1 THEN T=T-DT:IF T > 0 THEN GOTO 1410
        IF T < 0 THEN GOTO 1600:REM NO VALUES OF THETA EXIST
```

```
        T=T+DT:PH=PH+DP
1430    DT=DTI:PH=PH-DP
        CP=COS(PH):SP=SIN(PH)

1440    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF T < 0 THEN GOTO 1500:REM NO THETA CAN BE FOUND
        IF AC < 1 THEN T= T-DT:GOTO 1440
        IF AC - 1 < .01 THEN TT=T:CALL PL'T:GOTO 1430
        T=T+DT:DT=.1*DT:T=T-DT:GOTO 1440

REM **** INCREASE PHI TO PI

1500    T=TT
1530    DT=DTI:PH=PH+DP:IF PH>PI THEN GOTO 1600
        CP=COS(PH):SP=SIN(PH)
1550    S=SIN(T):CT=COS(T)
        AC=DR3*S*S*(-SP*SP*S/CT+2.*CP*CP*S*CT+DR2*CP)
        IF AC > 1 THEN T=T-DT:GOTO 1550
        IF 1-AC < .01 THEN CALL PL'T:GOTO 1530
        T=T+DT:DT=.1*DT:T=T-DT:GOTO 1550


REM ***** PLOT COMPLETED

1600    ?PU$:?POFF$:REM TURNS PRINTER OFF
        RETURN

REM     THE FOLLOWING ARE SUB PROGRAMS WHICH CONTROL THE PLOTTER
HP'PL:
        ? PRT
        PRT=""
        RETURN
HP'INI:
        ? E$+".("
        ? "IN;"
        ? "DF;"
        ? E$+".M:"
        ? E$+".H;;17:"
        ? E$+".N;19:"
        ? E$+".@:"
        ? E$+".)"
        RETURN

300     INPUT"ORIGIN COORDINATES (X,Y) IN INCHES? ---------- ";XXX,YYY:?
330     INPUT"SCALE FACTORS FOR X AND Y(UNITS PER INCH)? --- ";SFXX,SFYY
        IF SFYY <= 0 CR SFXX <= 0 THEN 330
        ?
        INPUT" SELECT PEN 0/1/2/3/4 ----";P
        ?
350     KK=1016
        YO=KK*YYY
```

```
        XO=KK*XXX
        SFY=KK/SFYY
        SFX=KK/SFXX


        ? E$+".("
        ? "SP"P";"
        ? E$+".)"
        GOTO MENU

PL'T:
        R=C*COS(T)/SIN(T)
        YP=R*CP
        XP=B*R*SP
        XPI=INT(XP*SFX+XO)
        YPI=INT(YP*SFY+YO)
        ?"PA"STR(XPI)","STR(YPI)";"
        RETURN

HP'END:
        ? E$+".("
        ? "SP"0";"
        ? E$+".)"
        RETURN


REM     THIS SUBROUTINE DRAWS THE GRID
900     ?TAB(-1,0)
        ?"        ***** SELECTION OF GRID PARAMETERS *****":?:?:?
        ?" NOTE: All coordinates of the gird must satisfy 0<=X<=15 and"
        ?"0<=Y<=11, all dimensions inches.":?:?

        INPUT"ENTER ORIGIN IN INCHES - X,Y      ";X00,Y00
        INPUT"ENTER SIZE IN INCHES -- X,Y       ";PX,PY
        INPUT"ENTER NO. OF DIVISIONS- X,Y       ";DPX,DPY
        INPUT"ENTER PEN NUMBER 0/1/2/3/4        ";P

REM  ******  CONVERT TO PLOTTER COORDINATES  ******

        XO=X00*KK
        YO=Y00*KK
        DX=PX/DPX*KK
        DY=PY/DPY*KK
        X1=XO+PX*KK
        Y1=YO+PY*KK
        ?PON$
        ? "SP"P";"

REM ****** PLOT GRID ******
REM ****** PLOT HORIZONTAL LINES ******

        Z=X1-XO:YP=YO:N$=STR(DY)
```

```
        ?PU$"PA"STR(XO)","STR(YO)";" : REM MOVE PEN TO ORIGIN
920     ?PD$"PR"STR(Z)",0;"PU$:Z=-Z :YP=YP+DY
        IF Y1 >= YP THEN ?"PRO,"N$";":GO TO 920
```

REM ****** PLOT VERTICAL LINES ******

```
940     IF Z < 0 THEN N$=STR(-DX):XP=-X1:X1=-X0:&
                  ELSE N$=STR(DX):XP=X0
        Z=Y0-Y1
960     ?PD$"PRO,"STR(Z)";"PU$:Z=-Z:XP=XP+DX
        IF X1 >= XP THEN ?"PR"N$",0;": GOTO 960

980     ? E$+".)" : REM TURN OFF PLOTTER
        GO TO MENU : REM RETURN TO MENU
```

REM ***** MODIFICATION OF SYSTEM DATA *****

```
500     ?TAB(-1,0):?:?:?
        PRINT"INPUT STANDOFF DISTANCE (";A;") ";:INPUT"", A
        PRINT"INPUT TARGET  ALTITUDE  (";C;") ";:INPUT"", C
        PRINT"INPUT TARGET  VELOCITY  (";V;") ";:INPUT"", V
        PRINT"INPUT MOTOR STALL TORQUE          (";ST;") ";
        INPUT "",ST
        PRINT"INPUT SLOPE OF TORQUE SPEED CURVE (";K;") ";
        INPUT "",K
        PRINT"INPUT MOUNT MOMENT OF INERTIA     (";MI;") ";
        INPUT "",MI
        PRINT"INPUT INCREMENT IN THETA FOR PLOT (";DTH;") ";
        INPUT "",DTH:DTI=DTH/CF
        PRINT"INPUT MAXIMUM VALUE FOR THETA     (";TTM;") ";
        INPUT "",TTM:TMAX=TTM/CF
        PRINT"INPUT VALUE FOR THE ANGLE PHI (";PHI;") ";
        INPUT "",PHI:PHMIN=PHI/CF
        PRINT"INPUT INCREMENT IN PHI FOR PLOT (";DPD;")  ";
        INPUT "",DPD:DP=DPD/CF
        GOSUB 650
        GOTO MENU

600     E$=CHR(27):PON$=E$+".(":POFF$=E$+".)":PU$="PU;"
        PD$="PD;":CF=180./3.14159
        PI = 3.141592654:K=1016:REM CONV. FROM INCHES TO PLOTTER COORDINATES
        READ XXX,YYY,SFXX,SFYY,P,A,C,V,ST,K,MI,DTH,TTM,PHI,DPD
        DATA 1,1,1,1,0,1000,1000,1000,700,1140,450,5,90,0,1
```

REM ***** CALCULATE PARAMETERS *****

```
650     DTI=DTH/CF:DP=DPD/CF:PHIMIN=PHI/CF:TMAX=TTM/CF
        DR1=V*K/C/ST:DR2=K*C/V/MI:DR3=V*V*MI/C/C/ST
        RETURN
```

```
REM **** END OF PROGRAM ****
3000    CALL HP'END
        ?TAB(-1,0)
        END
```

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