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MODELLING COMBAT AS A SERIES OF MINI-BATTLES

by

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United States Army

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This report is concerned with investigating the possibility of decomposing a main battle into a number of smaller engagements, or minibattles. The main sources of data were armour/anti-armour combat trials held in Europe and the USA. Results of the data analysis are presented, together with conclusions as to how these might be used in the formulation of a network combat model. Various network and attrition methodologies are investigated with a view to finding appropriate methods for incorporation in such a model. Finally, the development of a prototype combat model is discussed.   to p. 3

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Abstract

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Results of the data analysis are presented together with
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Keywords: Combat modelling, network, minibattle, CHINESE EYE,
ARCOMS, attrition.

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<u>MODELLING COMBAT AS A SERIES OF MINIBATTLES</u>

CHAPTER 1

INTRODUCTION

1.1. This is the final report on the first phase of a study investigating the feasibility of modelling battalion level combat as a series of minibattles. The work is sponsored by the US Army Research, Development and Standardisation Group (UK) under contract number DAJA45-86-C-0053, and is also supported by the Directorate of Science (Land) of the UK Ministry of Defence under contract number D/ER1/9/4/2004/02/DSc(L).

<u>Objective of the Study</u>

1.2. The current interest in network battle modelling arose from the analysis of the trial 'CHINESE EYE III', [1,2] carried out by David Rowland and others at the UK Defence Operational Analysis Establishment (DOAE). The objective of the current investigation is to assess the utility of the networking concept as the basis for a model of battalion level combat. Such a model could be designed to be fast running and easy to set up - like many current highly aggregated Lanchester based models - and at the same time provide a more detailed and accurate representation of combat than is currently possible in the Lanchester based models.

1.3. The original study proposal envisaged that a programme of work would be required which would cover data collection and analysis, the investigation of modelling methodologies and the development of a model. The work that has been undertaken has covered:

    a. the collection and analysis of data,

    b. derivation of an appropriate methodology for generating networks,

    c. the investigation of attrition methodologies,

    d. development of a prototype combat model,

This report will discuss each of these aspects in turn.

<u>Data Collection and Analysis</u>

1.4. The objectives of this part of the study were twofold:

    a. To establish the relationship - if any - between network structure and the terrain, mix of forces and tactics employed.

    b. To assess the sensitivity of network structure to changes in the rules used to derive the network.

1.5. Results of this analysis are discussed in Chapter 2. Preliminary conclusions to be drawn from this analysis are

   a. The relation between scenario parameters and network structure is most significant for force ratio PDF's, and derivation of the force ratio PDF from relatively crude scenario data is clearly possible.

   b. The relationship between scenario data and other network parameters is less pronounced and this suggests that quite detailed scenario data will be required in order to generate a representative network in a combat model.

Methodology for Generating a Network

1.6. In the course of the study, a number of alternative methods have been considered for generating a network. It must be borne in mind, however, that it is a simpler matter to find a network which describes a battle that has already taken place than it is to generate one from scratch in order to effect a battle simulation.

1.7. This work area is discussed in Chapter 3.

Attrition Methodology

1.8. In a network based combat model, the forces will fight a number of small engagements. A logical approach, therefore, is to attempt to represent the decomposition process taking place while at the same time using an attrition methodology appropriate to small force-on-force engagements. Some progress in representing this decomposition was made by Sassenfeld in his ELAN model [3], but Deterministic and Exponential Lanchester models were still used for combat resolution.

1.9. This work area is discussed in Chapter 4.

Development of a Prototype Combat Model

1.10. Work is nearing completion on the development of a prototype network combat model. This model will be flexible enough to allow a variety of network generation and combat resolution methodologies to be employed.

1.11. This work area is discussed in Chapter 5.

DATA COLLECTION AND ANALYSIS

Introduction

2.1. The trial Chinese Eye III took place in Germany in the 1970's and consisted of a number of armour/anti-armour battles at Red Battalion, Blue Combat Team level. The trials involved tanks and guided weapons only.

2.2. The ARCOMS trials held in the USA involved an attacking force of three tank platoons, one APC platoon and a section of ATGWS with a defending force of one tank platoon and a single guided weapon. Repeated battles were fought over the same ground, with the avenue of attack, the attack and defence tactics being varied from battle to battle.

2.3. For each battle, each round fired was recorded, together with the firer's callsign and position, the target's callsign and position, the time of the event and the outcome of the engagement.

2.4. The objective in analysing this data was to determine how the decomposition of a large battle into a series of smaller engagements is determined by the detail of a given scenario (ie. terrain, deployment, objectives) and to assess the extent to which this decomposition could be modelled statistically. To this end, a number of FORTRAN programs were constructed to produce statistics relating to battle structure and decomposition from the Chinese Eye data. The resulting output has been analysed using a PC-based statistical package and a discussion of the results of this analysis follows. Throughout this report, the attacking force is referred to as red and the defending force is referred to as blue.

(Snew)

Data Analysis

2.5. The output from the analysis programs consisted of the following for each minibattle:

1. node number
2. start time
3. end time
4. duration
5. total number of shots fired in minibattle
6. average range
7. initial number of blue weapons
8. initial number of red weapons
9. final number of blue weapons
10. final number of red weapons
11. callsign of each weapon involved
12. last minibattle this weapon took part in

and, for each weapon, the sequence of minibattles in which that weapon was involved.

Force Ratio Data

2.6. The force ratio statistics are summarised below, for each battle in the Chinese Eye trials. Ten separate battle scenarios were studied and are identified by a separate unique battle number.

TABLE 2-1   Force Ratio Statistics by Battle Number (Chinese Eye)

battle	nodes	mean	mode	median	variance	st. dev.
4	29	1.42	1	1	0.681	0.825
5	19	2.29	1	2	2.23	1.49
6	23	1.04	1	1	0.260	0.510
7	22	1.79	1	1.5	0.912	0.955
8	26	1.79	1	1.5	1.612	1.270
12	26	2.41	1	2	2.918	1.708
13	8	2.63	1	2	3.411	1.847
14	14	2.14	2	2	1.363	1.167
18	38	1.64	1	1	1.445	1.202
19	50	1.65	1	1	1.138	1.067

2.7. The overall mean force ratio was 1.78 with a standard deviation of 1.24. The median was 1.25 and the mode was 1.

2.8. The same analysis was then performed on the ARCOMS data. Seventeen separate battles were studied and the results are presented in Table 2-2.

TABLE 2-2   Force Ratio Statistics by Battle Number (ARCOMS)

battle	nodes	mean	mode	median	variance	st. dev.
11	26	1.71	1	1	1.674	1.294
12	7	1.04	1	1	0.276	0.525
13	17	1.63	1	2	0.548	0.740
14	20	1.13	1	1	0.437	0.661
15	21	1.43	1	1	0.539	0.734
16	26	1.59	1	1	1.253	1.119
17	18	1.61	2	1.5	0.670	0.818
18	18	1.19	1	1	0.322	0.567
19	20	1.50	1	1	1.110	1.054
20	23	1.35	1	1	0.346	0.588
21	24	2.01	1	2	1.448	1.203
22	14	1.18	1	1	0.240	0.490
23	17	1.99	1	2	1.051	1.025
24	16	1.42	1	1	0.486	0.697
25	37	1.61	1	1.5	0.709	0.842
26	21	1.24	1	1	0.471	0.686
27	16	1.55	1	1	1.328	1.152

2.9. The overall mean force ratio was 1.50 with a standard deviation of 0.916. The median and mode were both 1.

2.10. The above data and the distributions of force ratio suggest that different scenarios do indeed result in different distributions of force ratios. In order to test this assertion, a series of statistical tests were conducted using the null hypothesis that force ratios in battles x and y (from the same set of trials) are identically distributed. Using the Kolmogorov-Smirnov 2-Sample Test, this hypothesis was rejected at the 5% significance level for each independent trial. In other words, none of the scenarios can be assumed to have identically distributed force ratios.

2.11. Rowland [1], in his original paper on the analysis of the Chinese Eye data, relates the mean local odds to the density ratio of red and blue forces. It is also possible to relate the mean engaged force ratio in a minibattle (EFR) to the density of blue forces - defining this to be the average separation of blue weapons systems, calculated using the Euclidean metric. Figure 2-1 shows this relationship.

2.12. Rowland pointed out that the relation between density of forces and mean local odds was strongly influenced by phenomena which he described as lateral division of defence (LDD) and longitudinal division of attack (LDA). LDD occurs when the attacking red thrust is concentrated at a particular point - usually on a flank - and the blue defending force is divided by an obstacle or terrain feature. This results in a portion of the blue defenders being unable to engage the attacking units and hence in an increase in local odds. LDA occurs in scenarios where the attacking force is advancing across a series of transverse ridges when engaged by the defenders. This results in individual red weapons, or at most red platoons, being engaged by the defending force. The effect of this phenomenon on local odds will also be a function of red force density. The effect in the two scenarios considered is obscured by the fact that the red force density is similar for both scenarios.
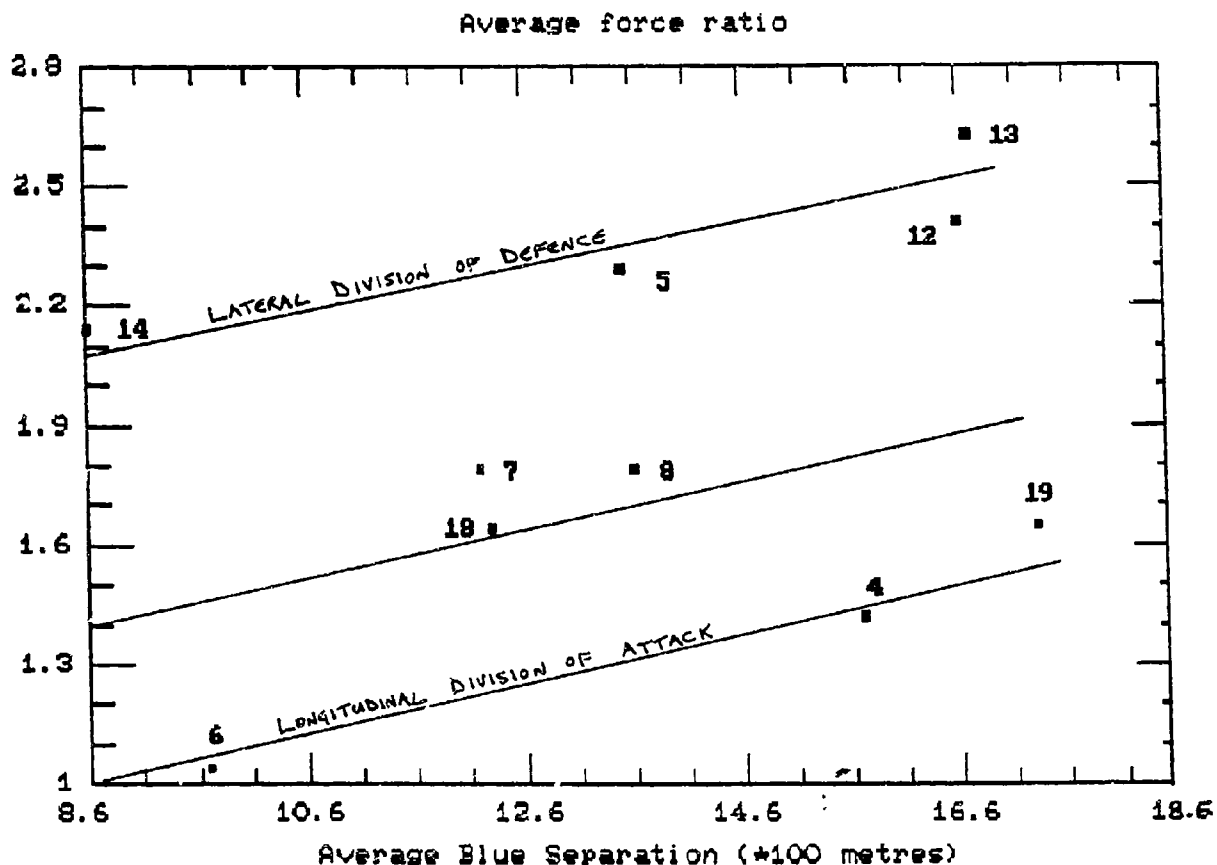
5

Figure 2-1

2.13. This influence is also apparent for the relation between density of blue forces and EFR, the upper dotted line in Figure 2-1 representing LDD and the lower, LDA. Comparison of the variance of the force ratio in a minibattle with the blue force density reveals a similar relation, although variances show a less consistent dependency on LDD and LDA.

2.14. It is also possible to predict the expected force ratio, using a multiple linear regression procedure with the expected number of shots fired per weapon for blue and red as the independent variables. In addition, the variance of force ratio can be predicted - although less accurately - in terms of the variance of the number of shots fired per weapon.

Expected Minibattle Duration

2.15. Some variation in the expected duration of minibattles from scenario to scenario was noted, and this data is plotted against blue force density in Figure 2-2.
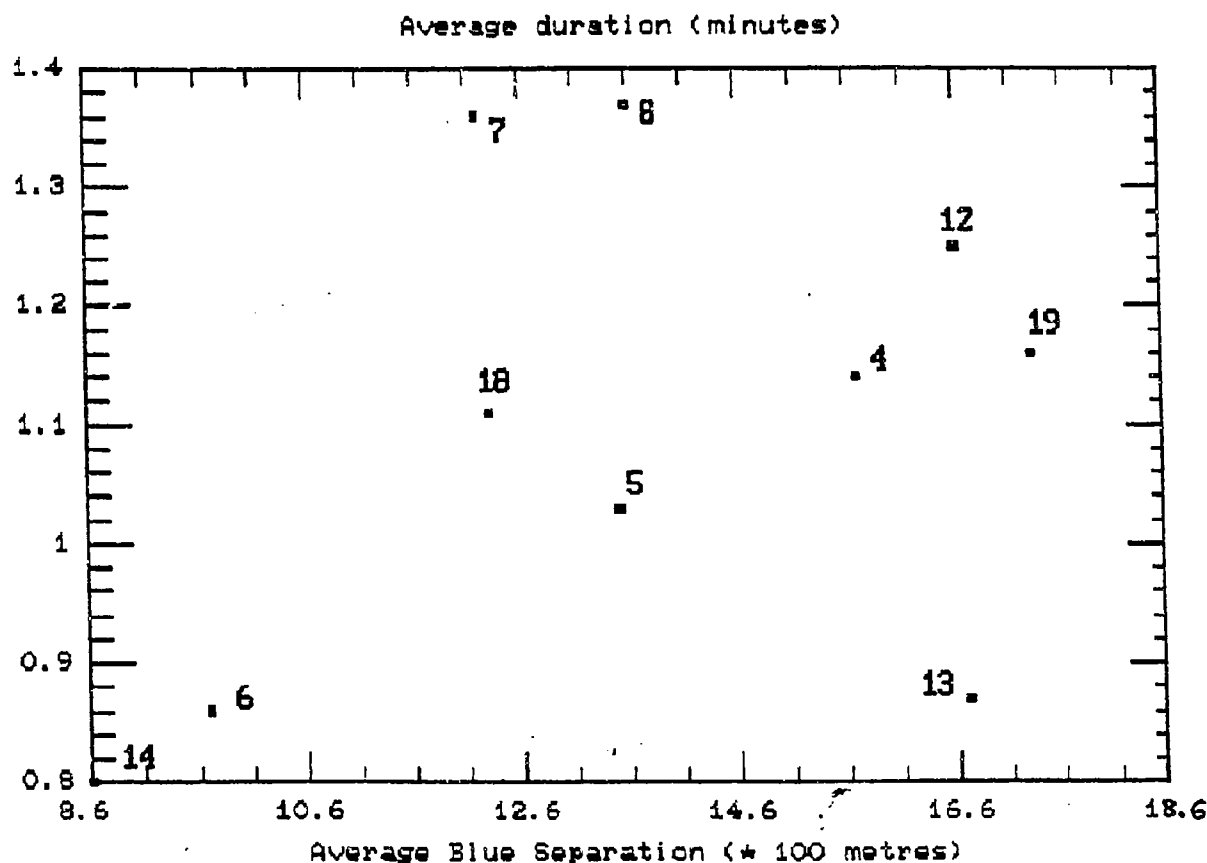
Figure 2-2

2.16. The particularly long expected minibattle durations for scenarios 7 and 8 are associated with battles fought over open, gently rolling countryside. Scenarios 13 and 14 took place in relatively poor visibility, and therefore the length and duration of LOS is not a function of terrain only. Scenario 6 is an example of a reverse slope defensive deployment, which seems to account for the short average minibattle duration.

2.17. No significant trend in minibattle duration as a function of battle time was apparent.

2.18. Minibattle durations appear to be well described by negative exponential distributions.

Minibattle Initiation Times

2.19. Although there is a clear variation in the shape of the start time pdf from scenario to scenario, there seems to be little relation between this and identifiable features of the scenarios themselves. Most of the pdf's are distinctly bi- or tri-modal and show distinct phases where no minibattles are initiated at all. The absence of an obvious relation between start time and scenario characteristics is explained by the fact that the initiation of a minibattle depends on decisions taken by commanders of individual weapon systems, and this is a function of a number of random factors, in addition to terrain and the tactical situation.

Network Parameters

2.20. The main network parameters are the number of distinct minibattles or nodes, in the network and the number of links between nodes.

2.21. The number of nodes is largely a function of the rules used to derive the network from the raw data, and not surprisingly, the number of nodes generated proves to be sensitive to certain variations in these rules. The selection of rules for generating a network is a subjective process, and appropriate rules can only be derived by analysis of the networks generated by a variety of different assumptions. As a result of analysis, some modifications have been made to the network generation rules employed, to allow more representative decompositions to be produced.

2.22. The number of links between nodes in a network depends on both the scenario characteristics and on the network generation assumptions. Again, no pattern that relates in an obvious way to the scenario being analysed emerges, although the number of links per node appears to follow a binomial distribution for each scenario studied.

Force Sizes in Minibattles

2.23. Some recent analysis has concentrated on red and blue force sizes in minibattles and some very interesting results were obtained.

2.24. It was found that the distributions of the number of red and the number of blue weapons in a minibattle follows the negative binomial distribution very closely. In fact, they usually follow the geometric distribution which is a special case of the negative binomial distribution.

2.25. The negative binomial distribution is a discrete distribution with a pdf given by:

$$p(j) = \begin{pmatrix} k+j-1 \\ j \end{pmatrix} p^x (1-p)^j \; ; \; j = 0,1,2,\ldots$$

where j is the discrete variable,

k and p are the parameters of the distribution known as the "number of successes" and the "event probability", respectively.

2.26. The geometric distribution occurs when k = 1 and its pdf is therefore given by:

$$p(j) = p(1-p)^j \; ; \; j = 0,1,2,\ldots$$

2.27. When the data from the Chinese Eye trials was used to construct a series of minibattles, the distributions of red and blue force sizes followed the geometric distribution with the variable j in the equation replaced by the force size minus one (obviously, if the force size was 0 for either side, there would be no minibattle). With the event probabilities (parameter p) for red and blue estimated from the sample data to be 0.381 and 0.634 respectively, Figure 2-3 was obtained,

comparing histograms of the actual distribution of red and blue force sizes with overlaid straight line plots showing the expected distributions if the force sizes followed the geometric distribution. As can be seen, the fit is very good and the results of chi-squared tests shown in Figure 2-4 confirm the goodness of fit.
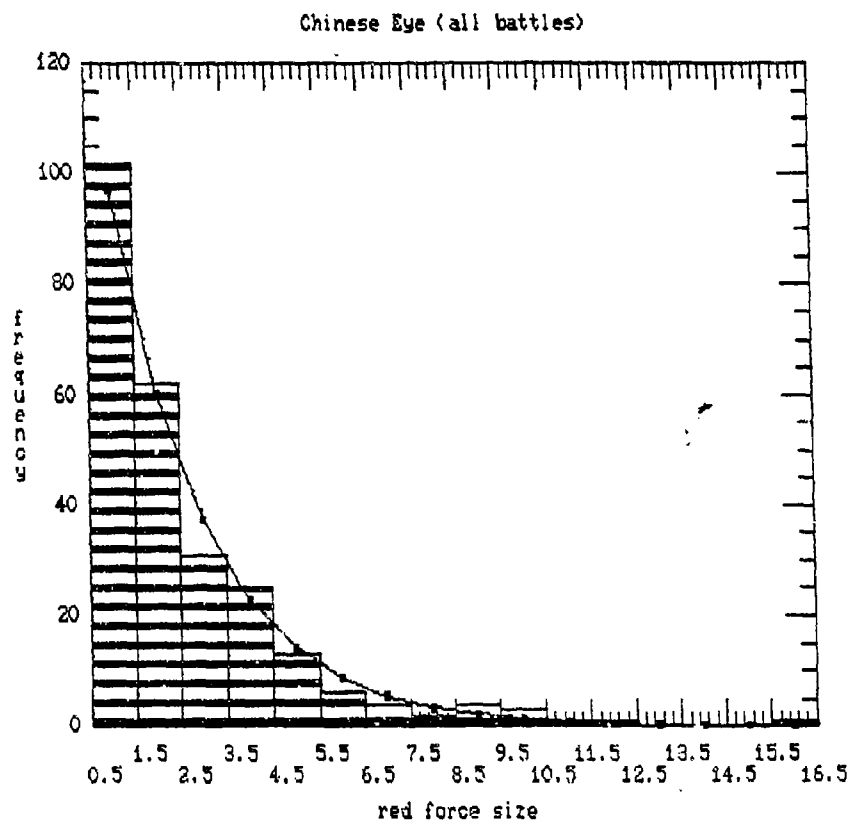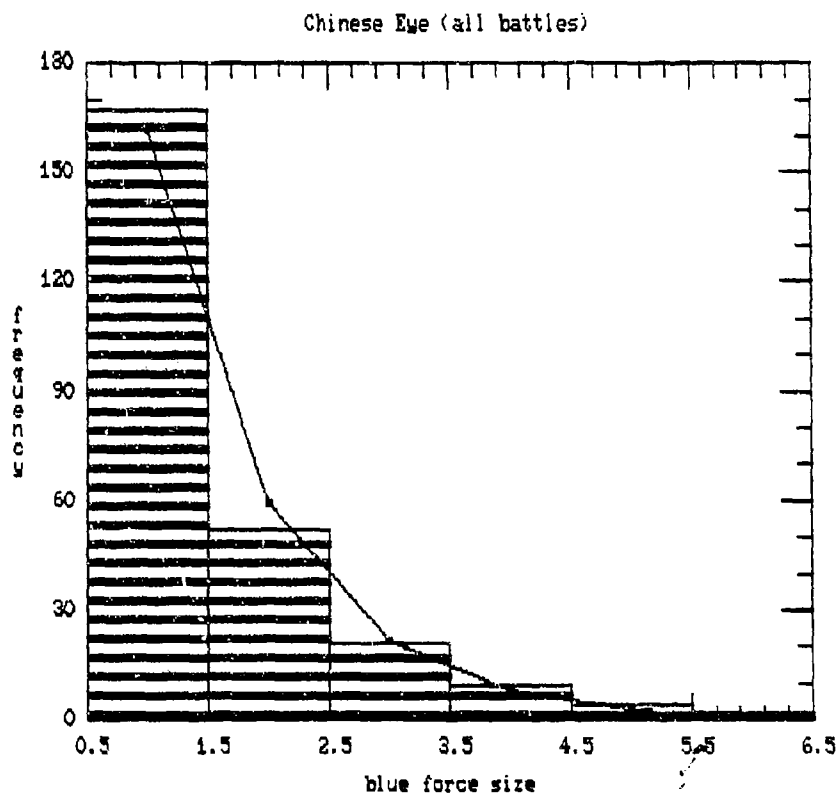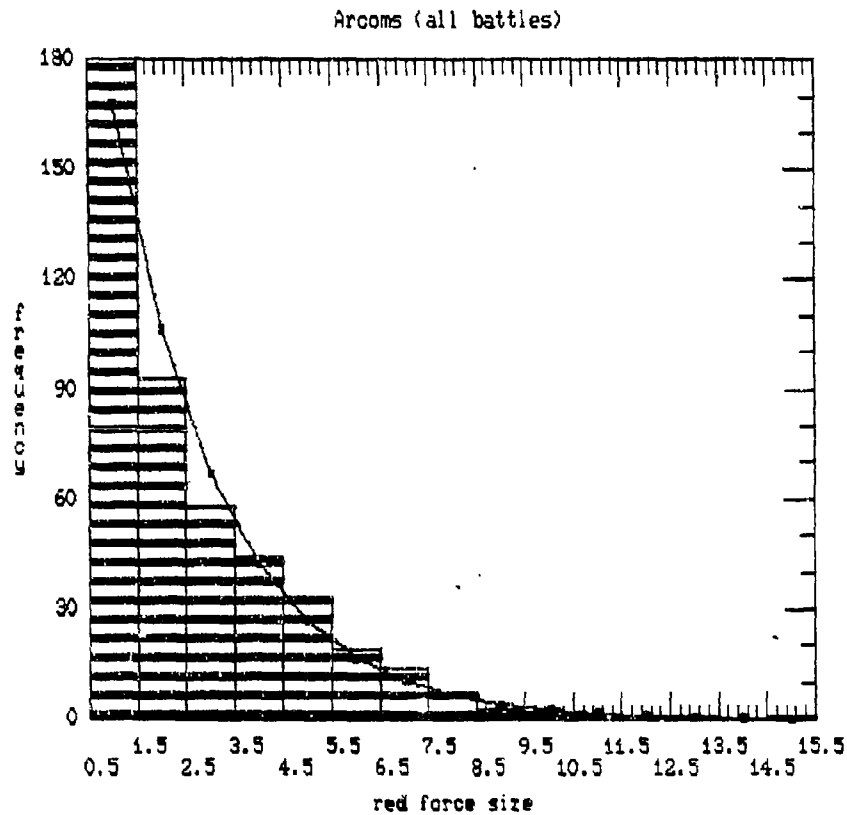
Chinese Eye (all battles)

red force size

Figure 2-3a

Chinese Eye (all battles)

Figure 2-3b

Arcoms (all battles)

Figure 2-3c

Arcoms (all battles)
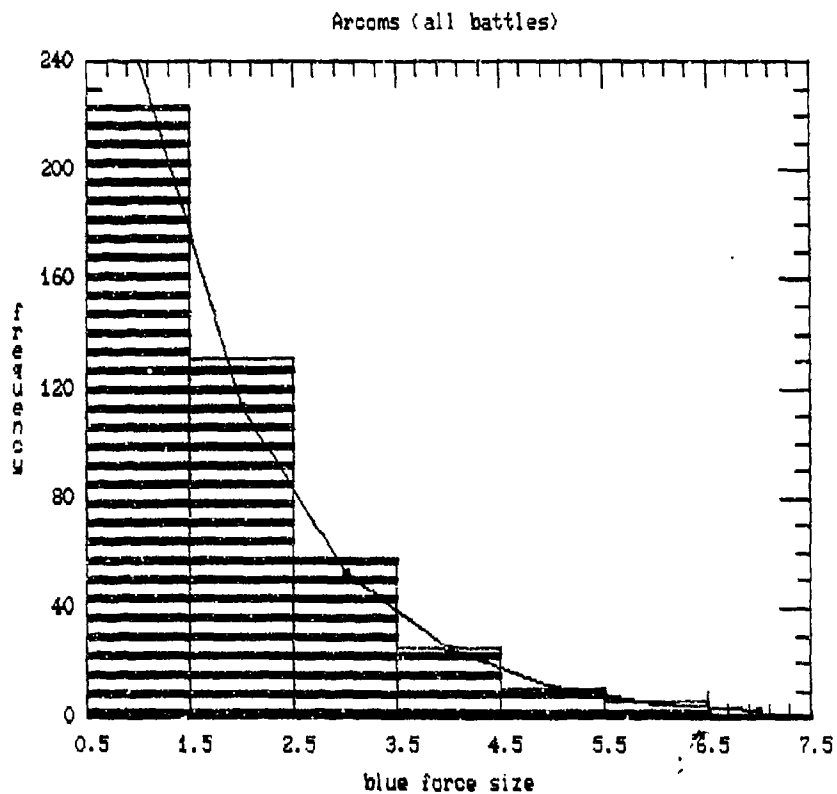

Figure 2-3d

Chisquare Test (Chinese Eye red forces)

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below		1.50	102	97	.2373
	1.50	2.50	62	60	.0570
	2.50	3.50	31	37	1.0401
	3.50	4.50	25	23	.1677
	4.50	5.50	13	14	.1104
	5.50	6.50	6	9	.9023
	6.50	7.50	4	5	.3899
above	7.50		12	9	1.1107

Chisquare = 4.01534 with 6 d.f.   Sig. level = 0.674601

Figure 2-4a

Chisquare Test (Chinese Eye blue forces)

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below		1.50	167	162	.1702
	1.50	2.50	52	59	.8640
	2.50	3.50	21	22	.0183
above	3.50		15	12	.5140

Chisquare = 1.56643 with 2 d.f. Sig. level = 0.456936

Figure 2-4b

Chisquare Test (Arcoms red forces)

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below		1.50	180	169	.7841
	1.50	2.50	93	106	1.6495
	2.50	3.50	58	67	1.2039
	3.50	4.50	44	42	.0743
	4.50	5.50	33	27	1.5270
	5.50	6.50	19	17	.2921
	6.50	7.50	14	11	1.1034
	7.50	8.50	7	7	.0161
above	8.50		8	11	1.0057

Chisquare = 7.6561 with 7 d.f. Sig. level = 0.363893

Figure 2-4c

Chisquare Test (Arcoms blue forces)

	Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below	1.50	1.50	224	242	1.389386
	1.50	2.50	131	114	2.682182
	2.50	3.50	58	53	.432902
	3.50	4.50	25	25	.000218
	4.50	5.50	11	12	.039447
above	5.50		7	10	1.054976

Chisquare = 5.59911 with 4 d.f.   Sig. level = 0.231154

Figure 2-4d

2.28. The next step was to analyse individual scenarios from both sets of trials and a summary of the results is presented in Tables 2-3 and 2-4 below.

TABLE 2-3   Event Probabilities for Individual Scenarios (Chinese Eye)

Battle	Sample	Blue event prob	chi-squared blue	Red event prob	chi-squared red
4	29	.527	.34(1)	.426	.09(1)
5	19	.792	–	.339	.61(1)
6	23	.590	–	.590	–
7	22	.537	–	.310	.38(1)
8	26	.650	–	.356	.16(1)
12	26	.578	–	.239	.74(2)
13	8	1.000	–	.381	–
14	14	1.000	–	.467	–
18	38	.613	.04(1)	.469	.24(2)
19	50	.676	.06(1)	.413	.91(2)

13

TABLE 2-4   Event Probabilities for Individual Scenarios (ARCOMS)

battle	sample	blue event prob	chi-squared blue	red event prob	chi-squared red
11	26	.667	-	.400	.727(1)
12	7	.318	-	.259	-
13	17	.586	-	.340	-
14	20	.541	-	.526	-
15	21	.568	-	.368	.007(1)
16	26	.591	-	.371	.099(1)
17	18	.450	-	.310	.214(1)
18	18	.545	-	.486	-
19	20	.488	-	.264	.082(1)
20	23	.561	-	.411	.474(1)
21	24	.585	-	.304	.641(2)
22	14	.341	-	.292	-
23	17	.567	-	.309	.231(1)
24	16	.533	-	.400	-
25	37	.529	.491(1)	.359	.202(2)
26	21	.512	-	.429	.247(1)
27	16	.432	-	.314	.913(1)

2.29. The degrees of freedom for each chi-squared test is shown in brackets after the test result. In several cases, there was insufficient data to carry out a test.

2.30. The tables show the extent of the variations in blue and red event probabilities from scenario to scenario. The blue event probabilities of 1.0 in Chinese Eye scenarios 13 and 14 arise because every minibattle in those two scenarios had only one blue weapon system present. This can be attributed to the small sample sizes.

2.31. The tables also show that where it was possible to conduct a chi-squared test, the result confirmed that the data was well fitted by the geometric distribution.

2.32. Figure A-1 in the Appendix compares histograms of the observed distributions of blue and red force sizes in each Chinese Eye scenario with overlaid straight line plots showing the expected distributions if the force sizes followed the geometric distribution.

Conditional Force Sizes

2.33. If we examine the distribution of red force sizes in a set of minibattles with the same size of blue force, we find that it still follows the geometric distribution, but the red event probability varies with the size of the blue force. The results are presented in Tables 2-5 and 2-6 and are supplemented by histograms in Figure 2-5.

TABLE 2-5   Red Distribution Parameter Variation (Chinese Eye)

blue force	sample	k	red event prob	chi-squared sig level
1	167	1	.525	0.661
2	52	1	.304	0.195
3	21	1	.223	0.307
4	9	1	.237	-
5	4	3	.375	-
6	2	5	.625	-

TABLE 2-6   Red Distribution Parameter Variation (ARCOMS)

blue force	sample	k	red event prob	chi-squared sig level
1	224	1	.627	0.333(2)
2	131	1	.344	0.0005(5)
3	58	1	.232	5.5×10^{-6}(5)
4	25	2	.427	0.713(1)
5	11	3	.425	-
6	6	4	.585	-

2.34. The tables show that the red force size distribution is conditional on the blue force size. As the blue force size increases, so does the probability of finding a large red force in the same minibattle. This will obviously have consequences for our modelling of minibattles.
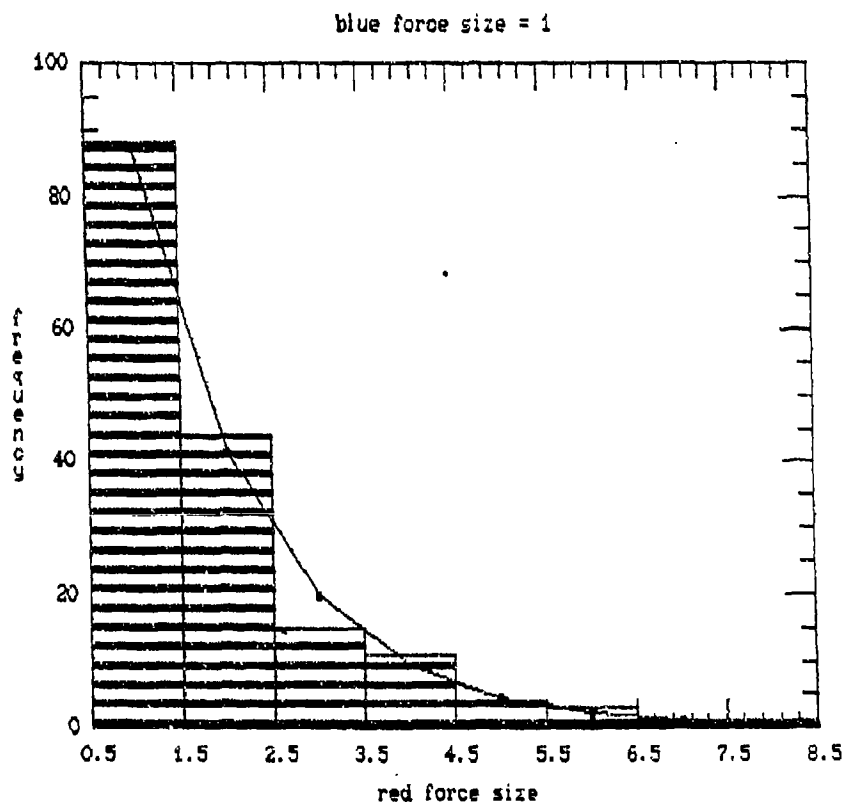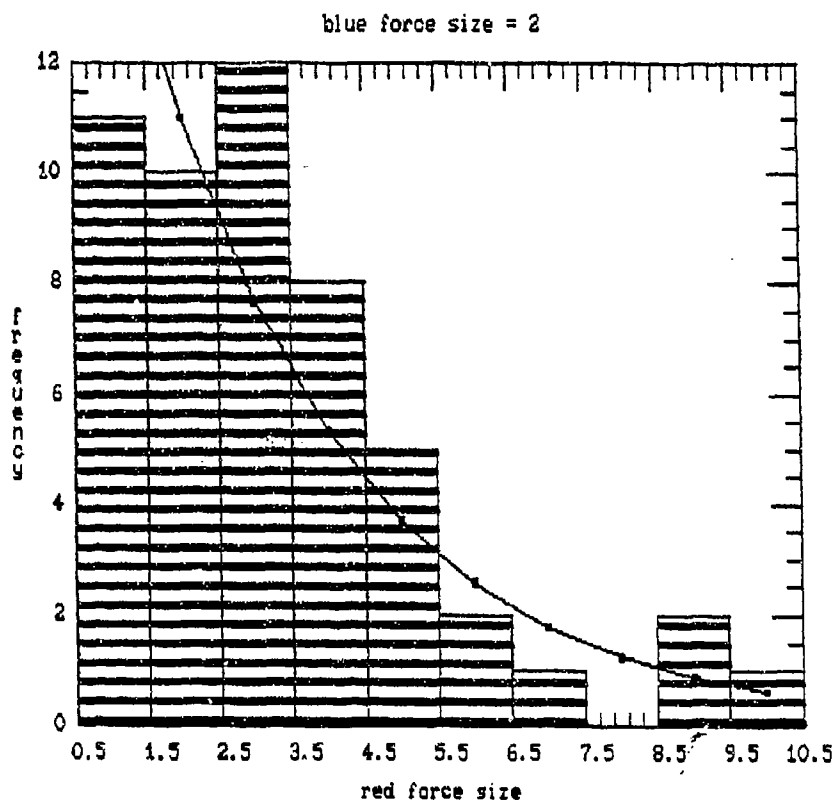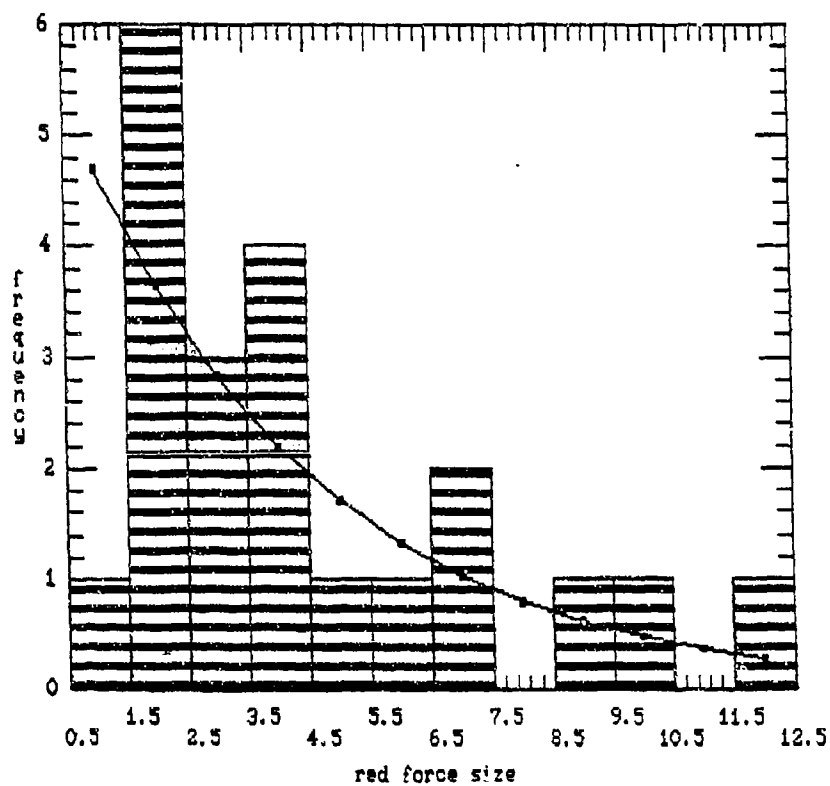
blue force size = 1

Figure 2-5a

15

blue force size = 2

Figure 2-5b

blue force size = 3

Figure 2-5c

16

blue force size = 4

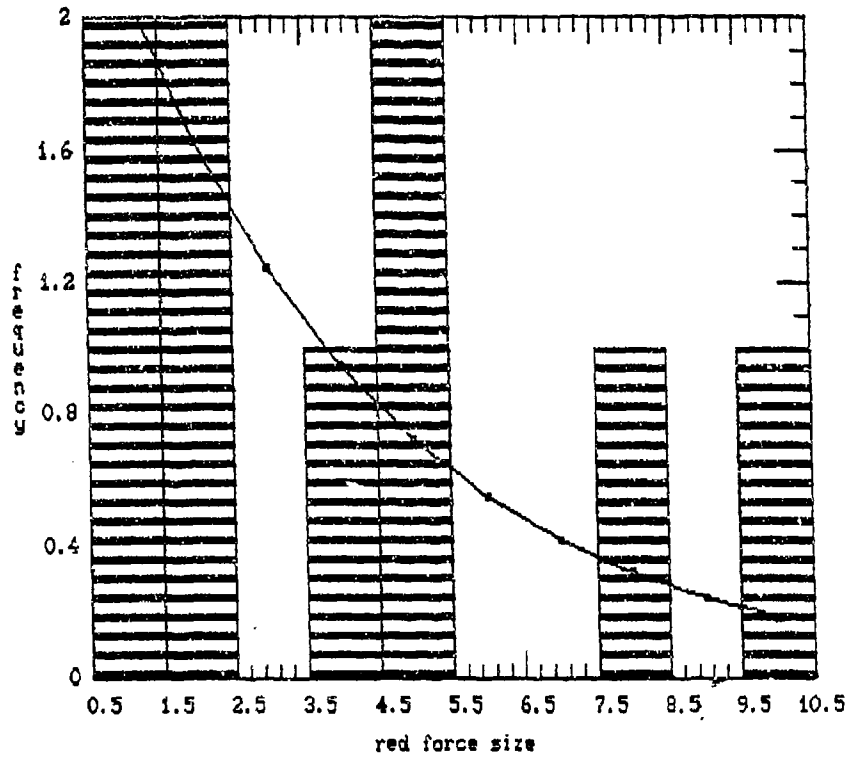red force size

Figure 2-5d

blue force size = 5

red force size

Figure 2-5e

17

blue force size = 6
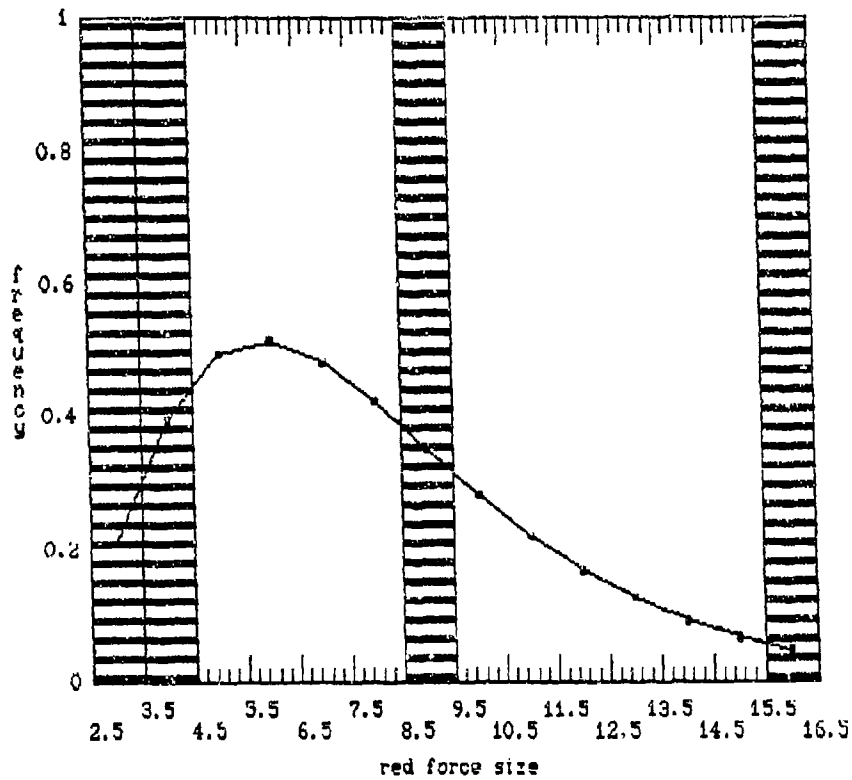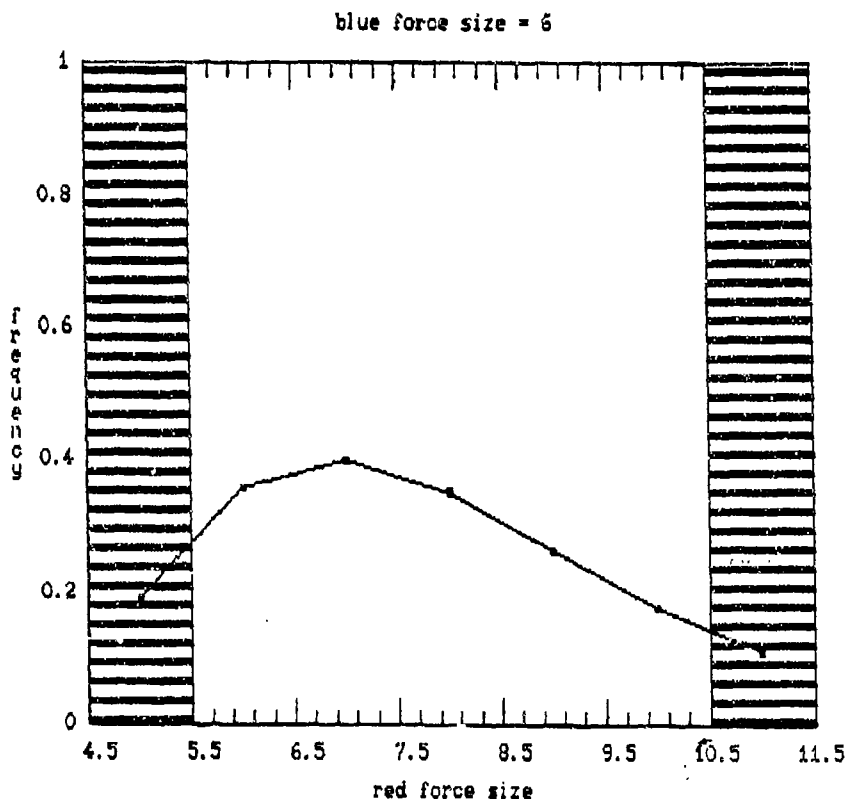

Figure 2-5f

2.35. While good results are obtained for small blue force sizes, the estimates of the parameters we obtain when the blue force is 4, 5 or 6 must be questioned. This is because of the small samples associated with these force sizes. Of particular note in Table 2-5 is the estimated red event probability associated with a blue force of 4 which is slightly larger than that for a blue force of 3 when we might expect it to be slightly smaller. This should be no cause for alarm, however, as nine data values are hardly sufficient to base such an estimate on. Moreover, the distribution parameter k changes from 1 when the blue force is greater than 4. The distribution is then no longer the geometric distribution but a more general form of the negative binomial distribution. Given that the red force distribution shifts to the right as the blue force size increases, this is to be expected, but the exact size of the blue force when this occurs may not be 5 although it appears to be in this region. The change occurs when the blue force equals 4 in the ARCOMS data. There is simply not enough data to be able to make good estimates of the distribution parameters for a blue force size of 4 or above but we can be fairly confident that the parameter k will become greater than 1 when the blue force size is in the region of 4 or 5.

Relation Between Distribution Parameters and Blue Force Density

2.36. Figure 2-6 shows how the ratio of the blue and red event probabilities varies with average blue separation of weapon systems. The graph is very similar to that obtained with the average force ratio and the effects of lateral division of defence and longitudinal division of attack are just as evident.
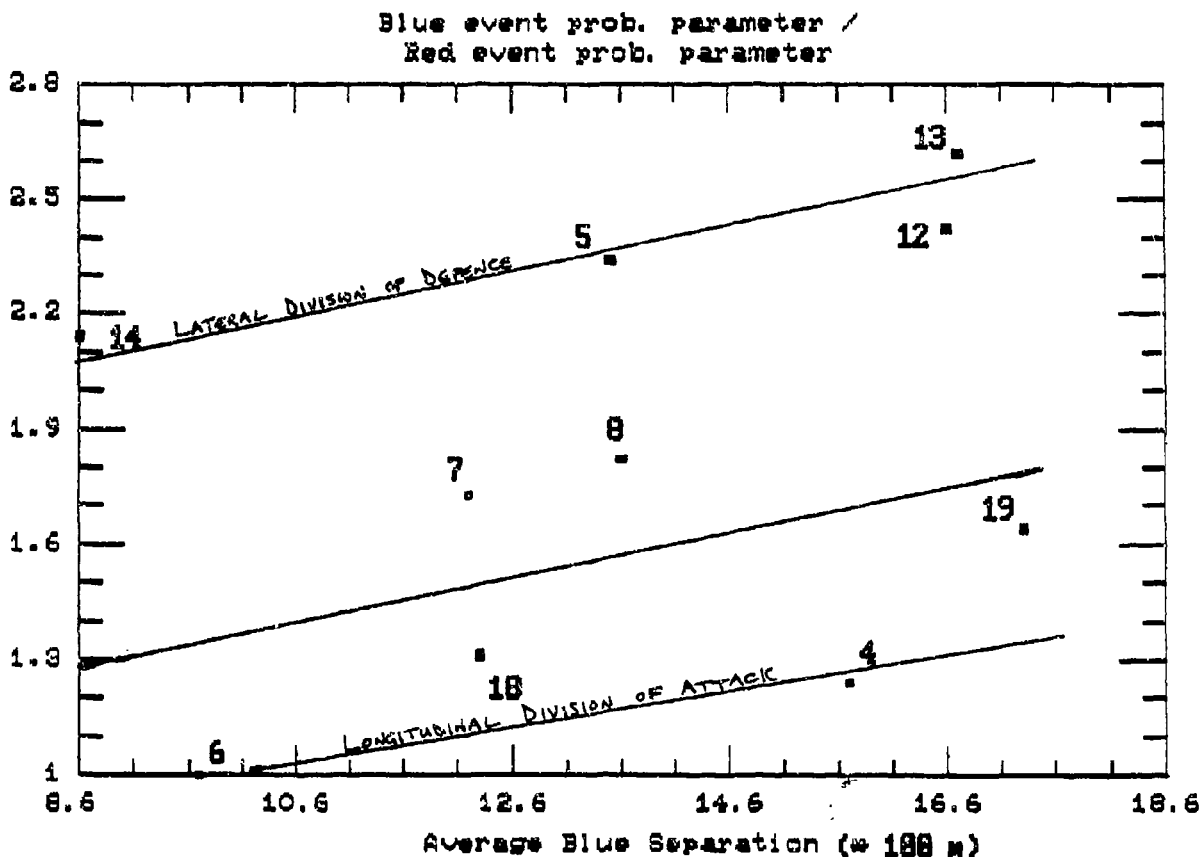
18

Figure 2-6

Conclusions

2.37. The results of the ARCOMS analysis back up those from Chinese Eye.

2.38. The relation between force ratios and blue force density and the expected number of shots fired by a weapon suggests that pdf of force ratio can be specified for a given scenario by the use of a small number of relatively crude parameters. The force ratio pdf is closely linked to the pdf's of the red and blue force sizes which appear to be well fitted by geometric distributions. Minibattle durations appear to be negative exponentially distributed with a scenario dependent parameter. The remainder of the network parameters, however, show little simple dependency on such parameters. The implications of this for the development of a combat model are twofold:

   a. If only a crude tactical/terrain description can be given, it will only be possible to specify the pdf's of force ratio and of minibattle duration with any certainty. Therefore, any combat model using such a terrain description will have to employ the network generation methodology based on force ratio pdf's discussed in Chapter 3.

   b. Any of the alternative network generation methodologies described in Chapter 3 will require a fairly detailed scenario description in order to function. This implies that an efficient

method for scenario generation and editing will have to be developed, and a data base of scenarios, and perhaps also of derived networks, will have to be established.

2.39. Further investigation of the relationship between battle structure and scenario type could be achieved by analysis of data from a computerised wargame such as JANUS. This has two advantages over the use of trials data. Firstly, a number of replications of the same battle could be played, and this would allow a rather better picture of the statistical properties of battle group level combat to be built up. Secondly, the effects of commander decision making could be more readily analysed. It is not feasible to undertake such a analysis as part of the current study, but a thorough statistical study of the results of battle group level combat could provide some useful information.

CHAPTER 3

NETWORK GENERATION METHODOLOGIES

Introduction

3.1. This section discusses some network generation methodologies and issues relating to their implementation in a combat model.

3.2. In previous progress reports, a number of different network generation methodologies have been discussed. However, in the light of the results of data analysis, some of these suggested methodologies have had to be reassessed.

Effects of Splitting Battles into Minibattles

3.3. At an early stage of the project, we looked at what would happen if a main battle was split up into a series of smaller battles using a standard stochastic Lanchester simulation.

3.4. Each minibattle had fixed initial force sizes and was fought to annihilation, with the survivors being returned to their main force. The procedure was repeated until one entire side was killed.

3.5. Although there were many simplifying assumptions such as constant initial force sizes in each minibattle, only one type of weapon, and the minibattles being fought to annihilation; nevertheless the comparisons which have been carried out give some idea of the magnitude of the changes which could be produced by considering a large battle to take place as a series of smaller battles.

3.6. Results for several sets of data have been produced. In each case, the main battle was split into all possible sizes of minibattle having integer initial force sizes and the same ratio of red to blue force sizes as the main battle. The ratio of attrition coefficients was changed until the sides had an equal chance of winning, ie the parity condition was reached. The change in this ratio required to achieve parity was taken as a measure of the effect of splitting the main battle.

3.7. For a main battle of size 30 vs 90, an attrition coefficient ratio of 9 gives parity. If, however, the main battle is split into a series of 2 vs 6 minibattles, then the attrition coefficient ratio decreases to 7.4 to achieve this condition. Figure 3-1 shows the ratio required for parity in a split battle as a proportion of that required for parity in the main battle.
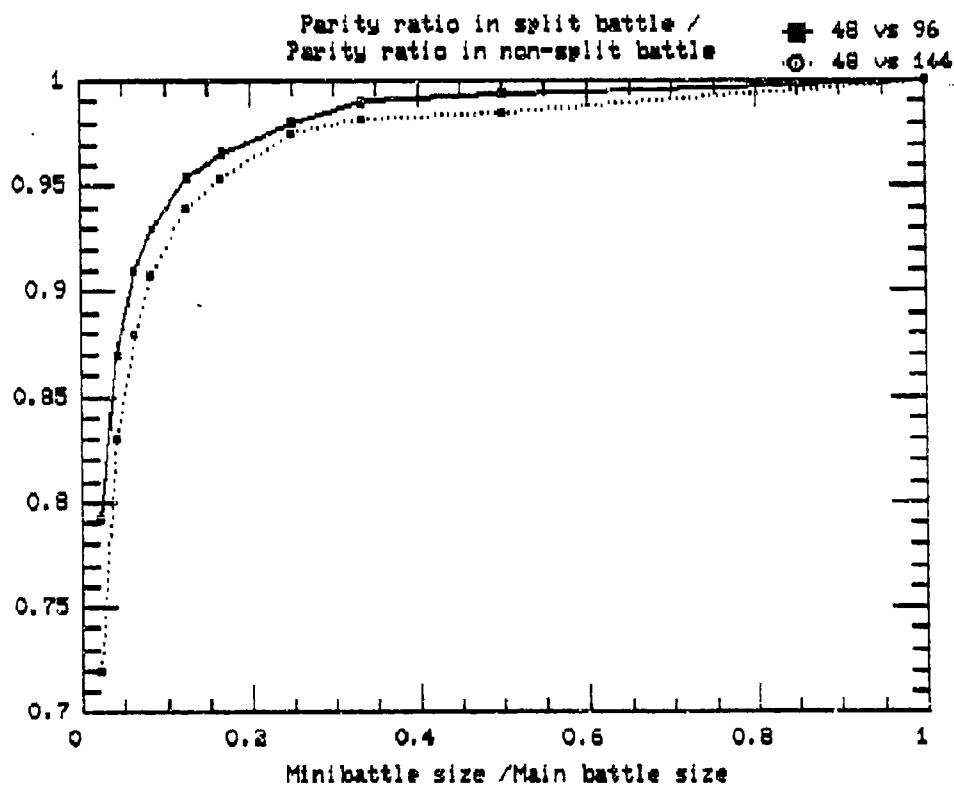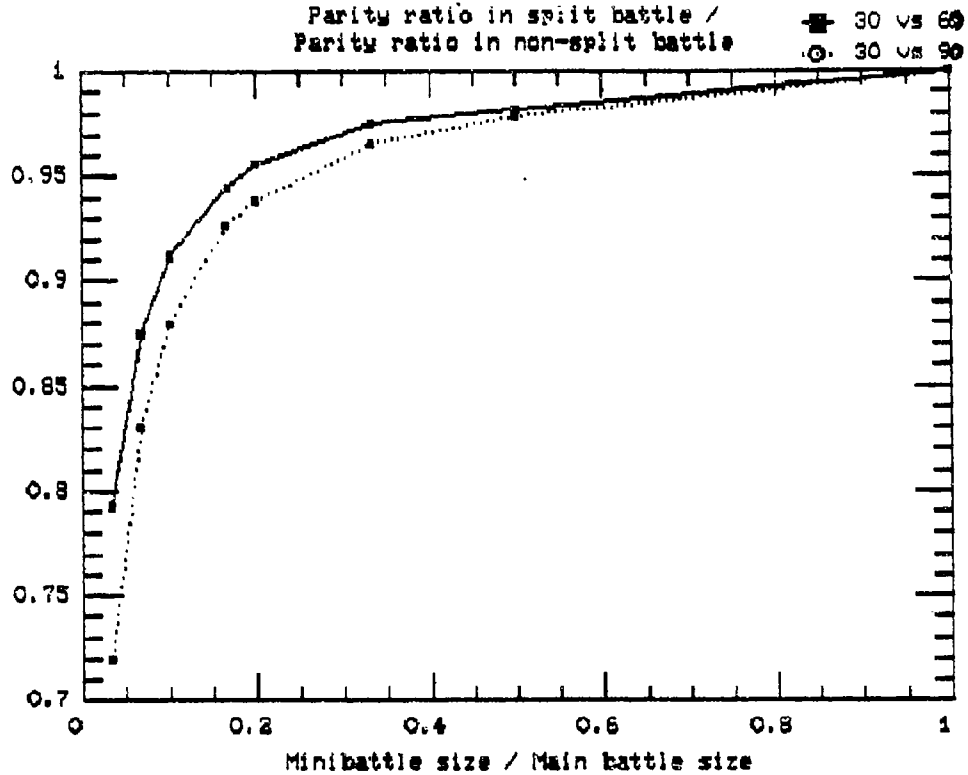
Parity ratio in split battle /
Parity ratio in non-split battle

30 vs 60
30 vs 90

Minibattle size / Main battle size

Parity ratio in split battle /
Parity ratio in non-split battle

48 vs 96
48 vs 144

Minibattle size /Main battle size

Figure 3-1

Properties of Network Models

3.8. In order to assess the effect of using a network based attrition methodology, a number of models were constructed, using a pre-defined network structure.

3.9. Results using a variety of networks were compared to those obtained from conventional force-on-force Exponential Lanchester models. The same battle was fought over a variety of networks, and in a simple force-on-force mode. A node was terminated when the number of blue or red survivors reached a given percentage of the original number of weapon systems at that node. The number of survivors at battle termination and the probability of a win were compared, and the terminating conditions for a node were varied. Some of the results are presented in Table 3-1.

Table 3-1 Effects of Resolving Combat Over a Network

The ratio of number of links to number of nodes is kept constant at 2.

No. of nodes	Terminating condition	Probability of a blue win	Av. % surviving Blue	Red
1	30%	0.11	0.31	0.63
5	50%	0.07	0.34	0.41
5	30%	0.09	0.20	0.29
15	50%	0.10	0.51	0.44
15	30%	0.11	0.57	0.26
25	50%	0.25	0.73	0.31
25	30%	0.29	0.41	0.24
35	50%	0.69	0.40	0.07
35	30%	0.41	0.29	0.12

3.10. It is clear from the above results that a network based combat model can produce very different results from a conventional force-on-force model, particularly where the smaller force has a superior weapon. The outcome of the battle in the network model is also influenced by the number of links between nodes, by the outcome of engagements at preceding nodes and by the terminating conditions for each node. The network structure means that a local advantage can propagate through the network, and thus have a significant effect on the overall outcome of the combat.

Methods Based on the Force Ratio PDF

3.11. One method for generating a series of minibattles for a given scenario would be to sample from a distribution of force ratios, specific to the scenario being analysed, over a series of time frames. The force ratio, however, is not a continuous variable, being the quotient of two small integers, and so it would not seem to be appropriate to sample values of the force ratio from a continuous distribution such as the Lognormal distribution. In any case, even if this was a suitable method, the derivation of a force ratio still does not fix the individual red and blue force sizes in a minibattle. It may be more practical, then, to try and sample these individual force sizes directly, but in such a way that the resulting distribution of force ratios is as we would expect it to be.

3.12. It must be remembered, however, that data is currently only available for combat at Red Battalion - Blue Combat Team level and lower. Furthermore, characteristics of force ratio and force size pdf's are related to terrain and deployment properties, but it is difficult to define consistent measures of these properties that give adequate results over a variety of terrain types.

Statistical Methods

3.13. In practical terms, the effect of a particular terrain and deployment type is that the blue and red inter-kill times (IKT) observed in a particular battle are different from those which would result from a battle where all the weapons are exposed to each other all the time. This observation suggests that it may be possible, using data from simulations, to construct a family of IKT distributions whose parameters are determined by factors related to the scenario under study. This could be achieved by defining the cumulative IKT density to be a finite mixture of normal (or perhaps Gamma) distributions.

ie

$$k(t) = \sum_{i=1}^{m} pi \; \Phi i(t, \theta i)$$

with parameters $pi, \theta i$ $i=1,...,m$ determined by the specific scenario.

3.14. This approach has the advantage that it would be simple to implement as a fast running simulation (and perhaps could be implemented analytically) and would not require a distinct attrition methodology to be developed. In addition, a large variety of shapes of IKT distribution could be modelled using this technique.

3.15. The potential disadvantages are that data from a more highly resolved model would be required as an input and also that it would be necessary to estimate up to 3m parameters, together with m itself. In practice, although methods for estimating the parameters of a mixture have been extensively studied, methods for the estimation of the number of components in a mixture are relatively poorly developed.

3.16. Furthermore, it may be necessary to estimate not $k(t)$, but $k(t,b,r)$, the IKT conditioned on there being b,r weapons on the blue and red sides. This means that, fixing m, a total of $L=3m.b.r$ parameters would need to be estimated, although for large values of b,r the change in $k(t,b,r)$ seems likely to be small.

3.17. A number of simulation experiments are currently underway to study the statistical properties of combat models, primarily in relation to the variations in IKT as a function of inter-firing time (IFT) pdf's and force sizes. The feasibility of this approach will depend on the outcome of these studies.

A Probabilistic Approach

3.18. An alternative to the above approaches is to develop a purely probabilistic model of the combat process.

3.19. Describe the combat state by the vector

$$s = (n, f1, f2, \ldots, fn)$$

at time t where   n is the number of active minibattles at time t. f1, ... ,fn are the forces engaged at nodes 1,2, ... ,n.   ie. the pair (bi , ri).

3.20. It is transitions affecting the number of active nodes which concern us rather than those which affect the force ratio at a node, which is a function of the combat process.

3.21. Transitions into the state s are possible from those states with n+1 and n-1 nodes when a node is terminated or initiated, respectively. The transition rates here depend on weapon characteristics and on scenario parameters such as length and duration of LOS, attack tactics, mobile or stationary defence.

3.22. This state representation can serve as the basis of a set of differential equations for the combat process, or more plausibly as the basis for a simulation model.   A higher level of aggregation can be achieved by allowing the basic units to be groups of red or blue weapons rather than individual weapon systems.

3.23. Alternatively, represent the battle state at time t by

$$s = (N, B, R)$$

the total number of active nodes, and the blue and red engaged force sizes respectively, and treat the battle as consisting of N nodes each involving either a number of weapons sampled from an appropriate probability distribution, or even B/N and R/N weapons.   This is clearly a much more highly aggregated approach, and is closer to the statistical procedures outlined above.

3.24. As a third alternative, model the activation and termination of nodes in this way:
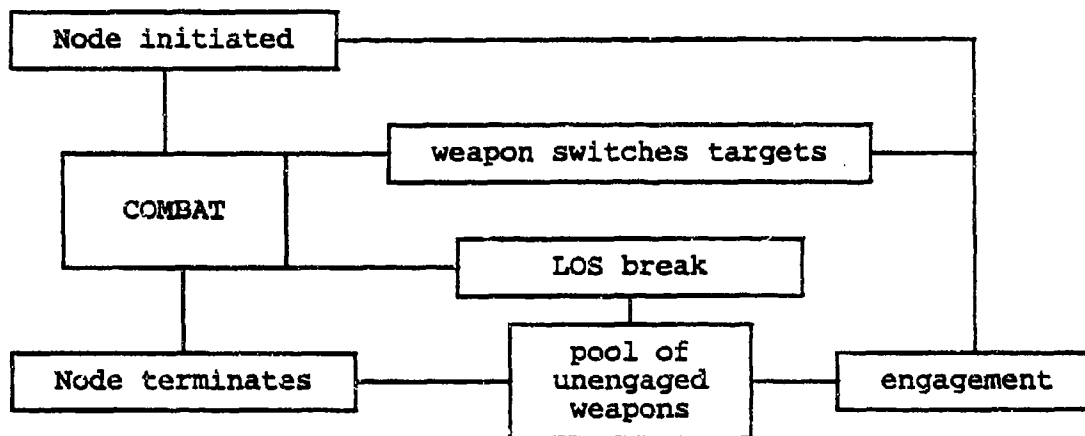
find

p(node initiated at t with b and r weapons involved)

and

p(duration of combat = x)

use simulation to set up a network, and resolve combat at a node

separately. The relation between events is given by

```
┌──────────────────┐
│  Node initiated  │──────────────────────────────────────┐
└────────┬─────────┘                                       │
         │                                                 │
┌────────┴─────────┐    ┌──────────────────────────┐       │
│                  │────│  weapon switches targets │───────┤
│      COMBAT      │    └──────────────────────────┘       │
│                  │         ┌──────────────────┐          │
│                  │─────────│    LOS break     │          │
└────────┬─────────┘         └────────┬─────────┘          │
         │                            │                    │
┌────────┴─────────┐      ┌───────────┴──────┐   ┌─────────┴────────┐
│  Node terminates │──────│     pool of      │───│   engagement     │
│                  │      │    unengaged     │   │                  │
└──────────────────┘      │     weapons      │   └──────────────────┘
                          └──────────────────┘
```

3.25. Any of the above methods will require much the same data as the computational approach, discussed below.

<u>A Computational Approach</u>

3.26. This method takes a specific scenario and calculates the most likely decomposition arising from it. This is achieved by computing the path of each group of weapons - a group in this context could be an individual weapon, platoon or troop - over the terrain of interest and deriving a decomposition from this information and from weapon system parameters. This approach is, in many ways, close to that of a resolved simulation. The difference is that no resolution of combat takes place at this stage, only the occurrence of engagement opportunities is assessed. The resolution of combat takes place after the decomposition has been computed.

<u>Data Requirements</u>

3.27. The computational and probabilistic approaches outlined above will require quite detailed data relating to terrain, deployments and tactics.

3.28. At this stage it is envisaged that the data requirements will be:

1. Definition of major terrain features such as hills and ridges, together with urban areas, crossings etc.

2. A deployment of defending forces at troop level together with arcs of responsibility.

3. Definition of at least, the starting positions and orientation of attacking forces - but preferably the route that the forces are to take over the defined terrain - at company level or lower.

4. The normal data relating to force composition, weapon system capabilities and basic rules of engagement.

Conclusions

3.29. All the above approaches have difficulties associated with them, mostly relating to the volume of data required to allow the model to operate. The statistical and probabilistic approaches have the advantage that no distinct attrition methodology is required to resolve combat, while the computational method is perhaps more attractive if the model is to be actively used as an assessment tool for studying force mixes, effects of terrain and tactical issues.

3.30. The construction of prototype models will allow the relative merits of each of these methods to be assessed.

ATTRITION METHODOLOGIES

Introduction

4.1. A number of attrition methodologies for minibattles have been considered in the course of the work. Some aspects of the attrition problem have been discussed in the previous section, where it can be seen that certain network generation methodologies make the development of a distinct attrition methodology unnecessary.

4.2. The main difficulty in attempting to assess attrition in battles involving small numbers of combatants is that the traditional Lanchester based approach is inappropriate for this type of situation - as Ancker and Gafarian [2] have shown - and high resolution Monte Carlo simulations are, in general, too slow for our purposes. This means that an alternative method for the resolution of few-on-few combat is required.

Exponential Lanchester

4.3. Initially an 'extended state space' Exponential Lanchester methodology was considered. The aim of this approach was to attempt to take explicit account of the detection processes which are usually ignored or incorporated in the kill rate in conventional Lanchester based approaches.

4.4. A weapon is allowed to be in two states, waiting to detect a target, or attempting to kill it. Time in each of these states is assumed to be negative exponentially distributed (NED). Transitions between these states are then modelled in the usual way.

4.5. Initial investigations suggested that the model might give good approximations to the outcomes of the general (ie non-NED interfiring times) few-on-few combat. Later investigations revealed that although the model worked well for 1-on-1 and 2-on-1 situations, the results were poor for casees involving larger forces.

M-on-N Stochastic Duel

4.6. The next approach was to attempt to model the m-on-n stochastic duel where inter-firing times followed an Erlang distribution, this being in many ways a natural extension of the previous approach. The approach is also related to techniques used in the study of transient stochastic networks [4,5].

4.7. The assumptions are

    a. The forces are homogeneous.

    b. There are initially $B0$ and $R0$ weapons on the blue and red sides.

c. For blue and red weapons:

IFT   blue: Er(α,n)   red: Er(β,m)
SSKP   p.   q

d. Target selection is random.

e. Weapons switch targets instantaneously, and complete their
current firing cycle if their target is killed by another weapon.

4.8. The system state is then given by

$$(b1,\ldots,bn;r1,\ldots,rm) = (b,r) , \text{ say}$$

ie. the numbers of blue (red) weapons at stage 1,2,...,n (m) of their
firing process. This then leads to a set of differential difference
equations for pt(b,r), the probability that the system is in state (b,r)
at time t, of the form:

$$\frac{dpt(b,r)}{dt} = -(n\alpha+m\beta)pt(b,r) + \alpha(\sum_{i=1}^{n-1} (bi+1)pt(b+ei-ei+1,r) \ \delta(bi+1))$$

$$+ \beta(\sum_{i=1}^{m-1} (ri+1)pt(b,r+ei-ei+1) \ \delta(ri+1))$$

$$+ (1-q)\beta(rn+1)pt(b,r+em-e1)\delta(r1)$$

$$+ (1-p)\alpha(bn+1)pt(b+en-e1,r)\delta(b1)$$

$$+ \alpha p \ \delta(b1,0)(bn+1)(\sum_{i=1}^{m-1} pt(b+en-e1,r+ei)(ri/R))$$

$$+ \beta q \ \delta(r1,0)(rm+1)(\sum_{i=1}^{n-1} pt(b+ei,r+em-e1)(bi/B))$$

where R $= \Sigma$ ri , B $= \Sigma$ bi , $\delta(x) = 0$ if x=0 and 1 otherwise

and   ei $= (0,0, \ldots,1, \ldots,0)$ ie. the zero vector, with a 1 in the
ith position

4.9. The probability of having a total of B and R survivors at time t,
Pt(B,R) is then given by

$$\Sigma\Sigma \ pt(b,r)$$

where the summation is over all values of b and r that sum to B,R.

4.10. The above equation can be reduced to a second order PDE in m+n+1
dimensions, for the probability generating function of the process.

4.11. The complexity of the above system of equations and the resulting PDE suggests that efficient analytical or numerical solutions are not a real possibility. However a simulation has been constructed based on this approach, which gives good agreement with results from the BAGSIM simulation [6], while being substantially faster for force sizes of less than about 20 per side.

Approximate Methods

4.12. The final approach so far considered is the use of approximate methods.

4.13. A simulation has been constructed to provide data for comparison of approximation methods and this simulation may also be used to determine those factors which have a significant effect on the outcome of a battle.

4.14. Gafarian has studied approximate methods involving the use of an inhomogeneous Poisson Process approximation to the inter-kill time and has obtained good results, although these appear not to have been published at the time of writing.

4.15. Other methods being studied are those which approximate the conditional IKT distribution by series expansion methods or by data fitting. Such an approach would allow a fast running simulation or an analytical model to be constructed, and relate to the IKT network generation method discussed in the previous section.

Conclusion

4.16. In conclusion, it appears that an alternative to the traditional Lanchester methods is required to resolve attrition in minibattles. While an IKT-based approach would be very fast, a lot more data analysis and related research would need to be undertaken before it could be implemented. Consequently, the stochastic duel methodology of Ancker and Gafarian [7,8] seems to be the most promising for the majority of minibattles, since so many minibattles involve two or fewer weapon systems on each side.

CHAPTER 5

DEVELOPMENT OF A PROTOTYPE MODEL

Methodology

5.1. The prototype model we are now completing envisages splitting the main battle into two lower levels (for battlegroup/battalion level combat).

5.2. The first level would split the main battle into a number of sub-battles, typically of company versus platoon size. Where and when each sub-battle would occur would depend on the scenario under study, the deployment of forces, the attack paths and the tactics to be employed. This, would therefore be a matter for military judgement.

5.3. The second level statistically decomposes each sub-battle into a number of minibattles and it is at this level where the resolution of combat occurs.

5.4. The sub-battles will be fought out in order of their starting times. Survivors from each side are passed on to other sub-battles in a manner designated by the user that is appropriate to the scenario under study; thus turning the main battle into a network of sub-battles.

5.5. Sub-battles will occur over a wide range of space and time. The more dispersed they are, the easier they will be to identify. Weapon systems will only be able to take part in one sub-battle at a time, however, so where there are large concentrations of forces at a particular place and time, one large sub-battle may have to be defined rather than several smaller ones. Each sub-battle will end either when one side is annihilated or when a maximum duration time is reached. These maximum durations will vary from sub-battle to sub-battle and will be dependent on the scenario in question. They may represent, for example, the length of time that a fast moving attacking force is able to exchange fire with a static defensive group before passing out of the effective range of that group and carrying on either towards its objective or towards another defensive group.

5.6. Each sub-battle will consist of a set of time frames of random length - the lengths of the time frames summing to the sub-battle duration and within each time frame a set of minibattles will take place.

5.7. The time frame lengths will be sampled from the negative exponential distribution and at the start of each time frame, the forces still present in the sub-battle will be re-configured into a new set of minibattles.

5.8. Each minibattle will be fought in turn until one side in the minibattle is annihilated or until the end of the time frame. The distribution of time frame lengths will match that of minibattle durations for minibattles which end due to line of sight breaks. The maximum duration of each minibattle within a given timeframe of a given sub-battle is therefore the same. While this is an artificiality, we do not believe it to be a serious one. While the duplication of some

minibattle durations means that the shape of the distribution of simulated durations will not precisely match that of observed durations, the means and variances of the two should still be the same.

5.9. Any weapon systems destroyed in a minibattle will be removed from the pool of weapons in the parent sub-battle so that they will not re-appear in any future time frame.

5.10. At the beginning of a new time frame, we have all of the surviving weapons in the sub-battle to sample from in order to make up a set of minibattles. As a result of our earlier data analysis, we can use the geometric distribution to decompose these survivors into a set of red and blue force sizes, sampling by rejection from a pair of distributions with the appropriate parameters. This will ensure that the distributions of red and blue force sizes in the minbattles as well as the distribution of force ratios will be the same as those we have observed in the trials data. See Table 5-1 for a comparison of force ratios produced by the above method and those observed in the CHINESE EYE trials.

TABLE 5-1   A Comparison of Observed and Generated Force Ratios

lower limit	upper limit	observed frequency	simulated frequency
-	0.64	17	22
0.64	0.96	8	3
0.96	1.28	104	106
1.28	1.60	16	5
1.60	1.92	3	2
1.92	2.24	54	60
2.24	2.56	8	1
2.56	2.88	0	0
2.88	3.20	19	30
3.20	3.52	2	1
2.52	4.16	12	15
4.16	5.12	7	7
5.12	-	5	3

Attrition in the Model

5.11. Initially, we intend resolving attrition at minibattle level using a Monte Carlo type simulation. However, we will experiment with other attrition methodologies such as stochastic Lanchester, and will also incorporate a network structure at sub-battle level into an existing stochastic Lanchester model - SLEW - which was developed at RMCS.

5.12. Our main aim, though, will be to employ the m-on-n stochastic duel methodology developed by Ancker and Gafarian as this seems to be the most appropriate for engagements involving only small numbers of weapon systems.

5.13. We estimate that approximately 45% of minibattles generated by the model will be 1 on 1, 25% will be 2 on 1 and 5% will be 2 on 2. Minibattles involving higher force sizes than these could still be resolved by simulation, Stochastic Lanchester or some other technique.

5.14. The advent of parallel processing techniques means that several minibattles could be fought out at once, thus reducing the time it takes for one replication.

Conclusion

5.15. The model's two-tier structure allows us to take account of different deployments and tactics at the higher level while the lower level utilises the results from our data analysis to effect a statistical decomposition of the combat. How successful this approach will be and how sensitive results will be to changes in the various input parameters will only become apparent after a considerable amount of testing.

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Chinese Eye Battle 4
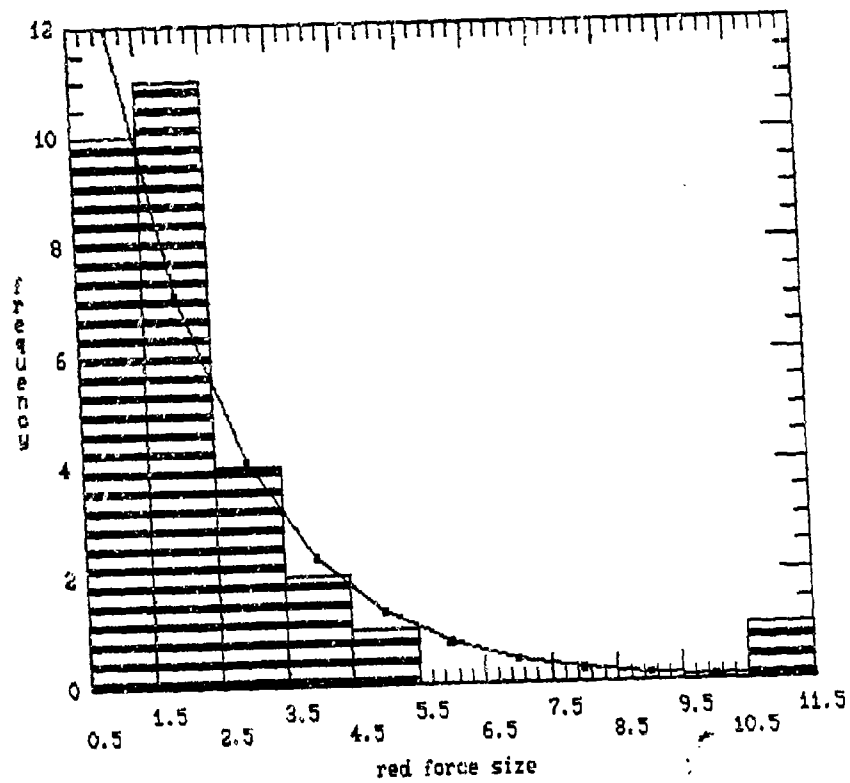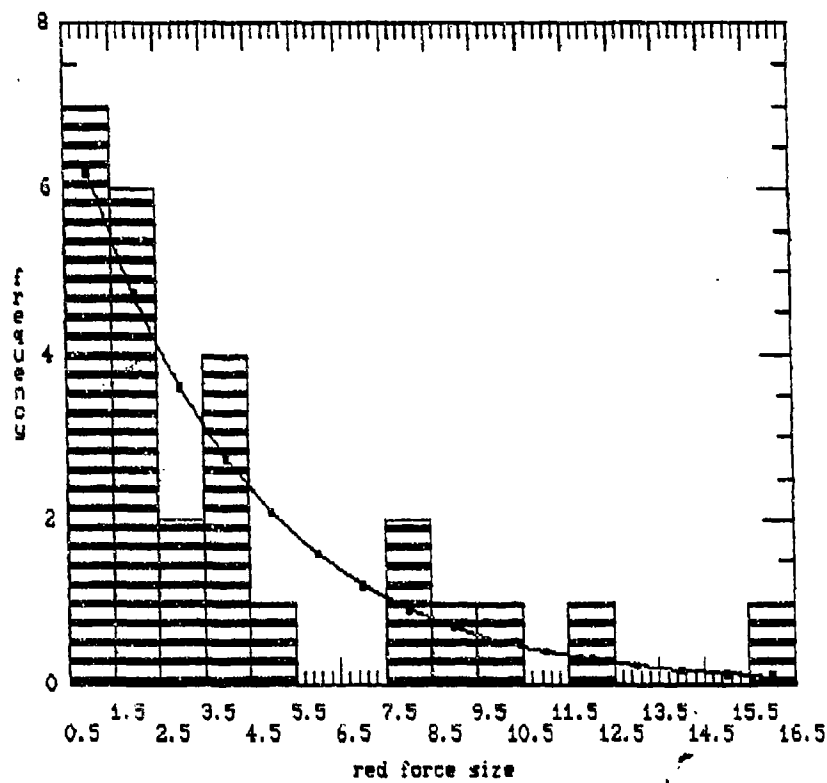

Figure A-1a

Chinese Eye Battle 4

Figure A-1b

A-1
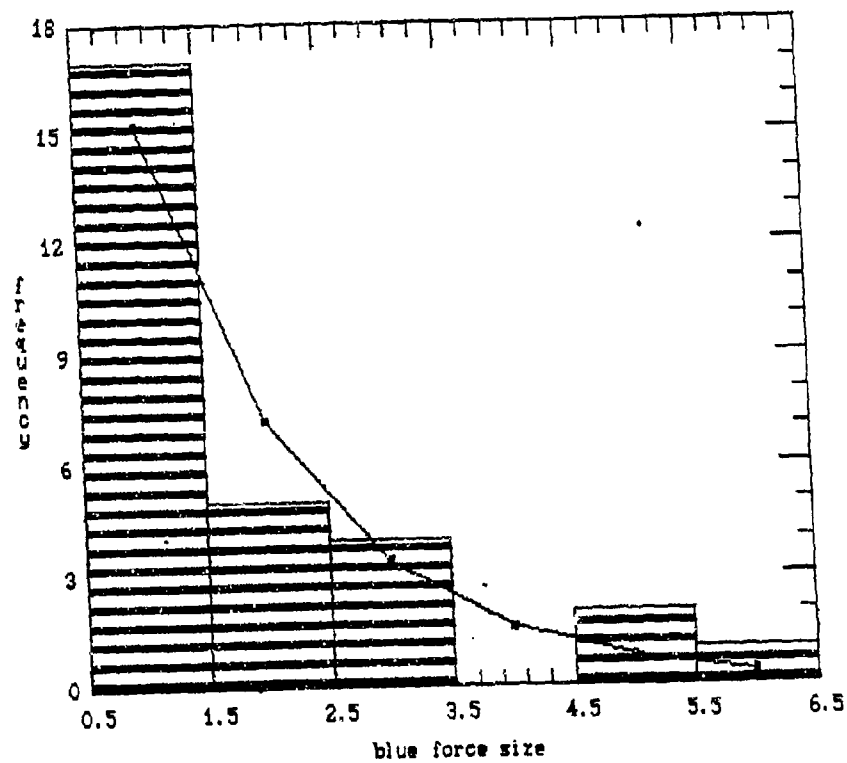
Chinese Eye Battle 5
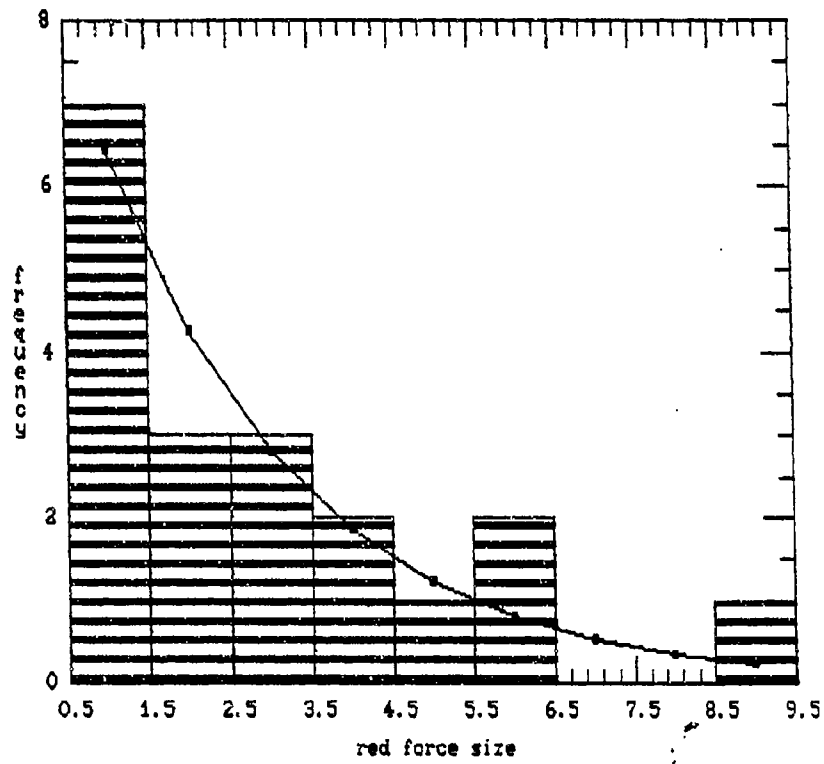
Figure A-1c

Chinese Eye Battle 5
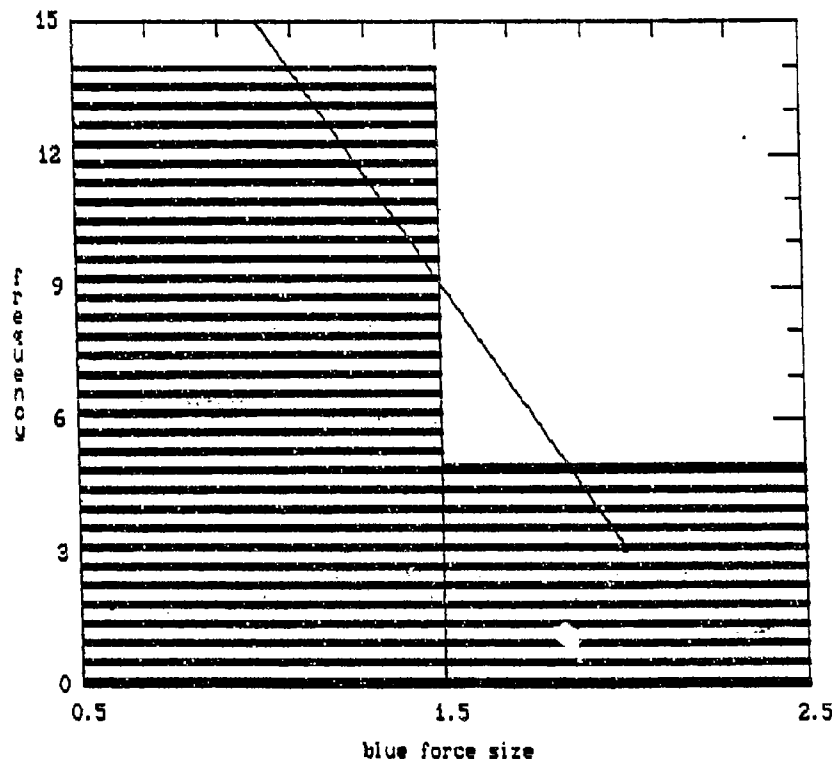
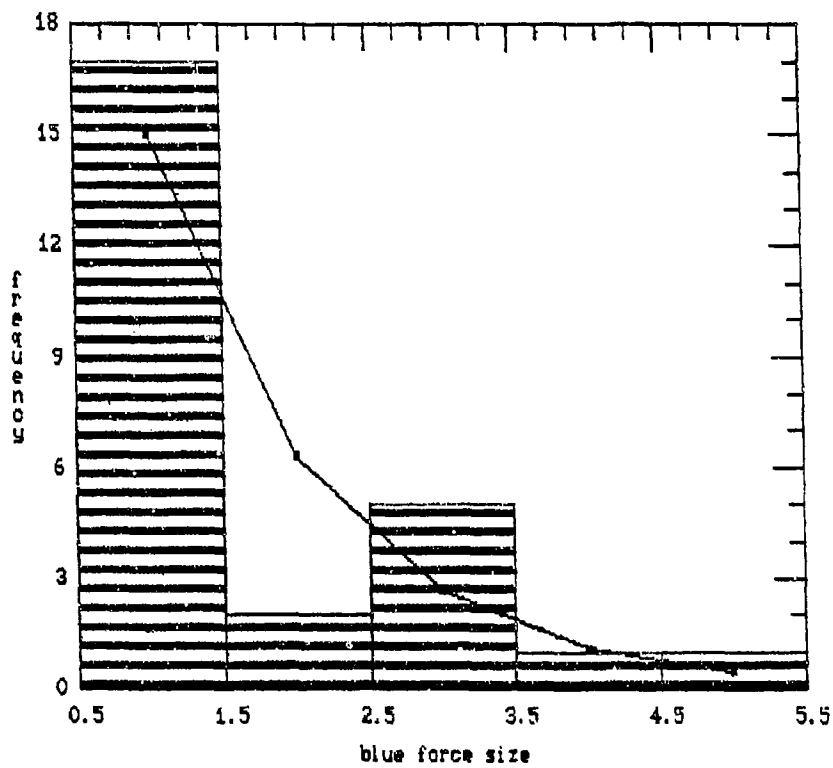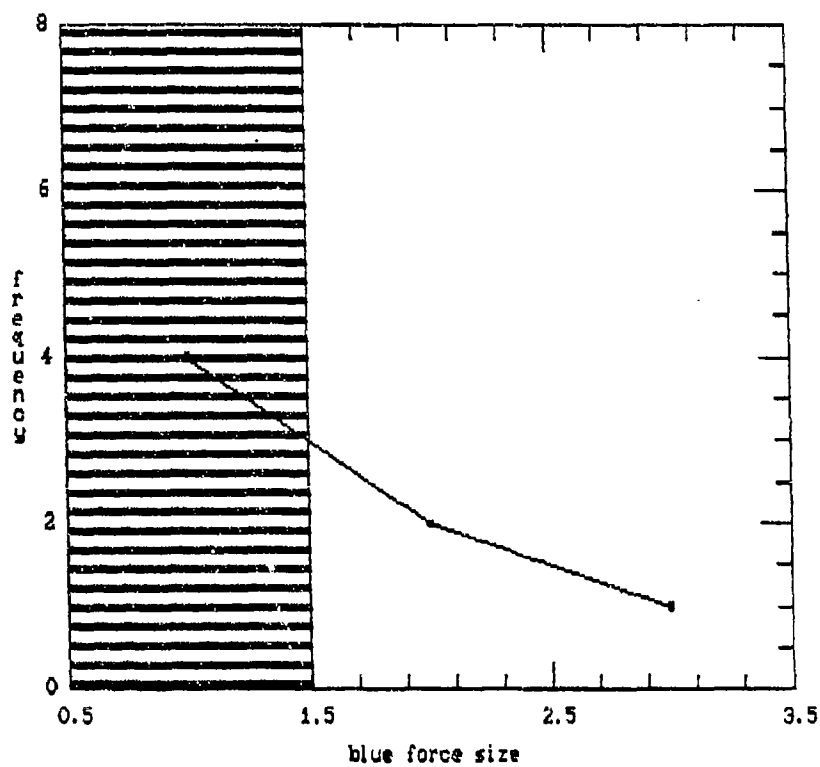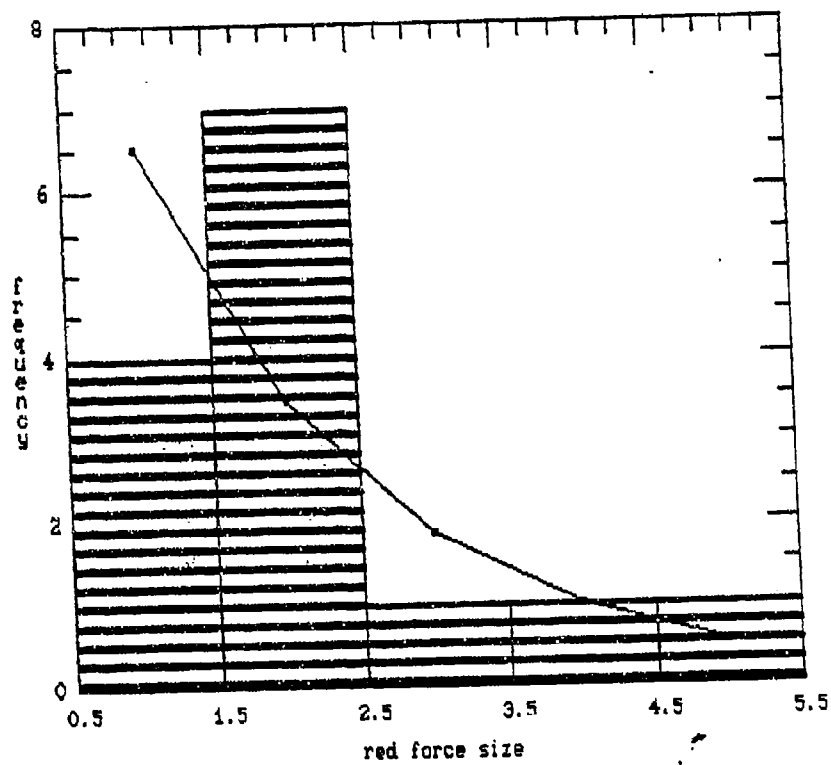Figure A-1d

Chinese Eye Battle 6
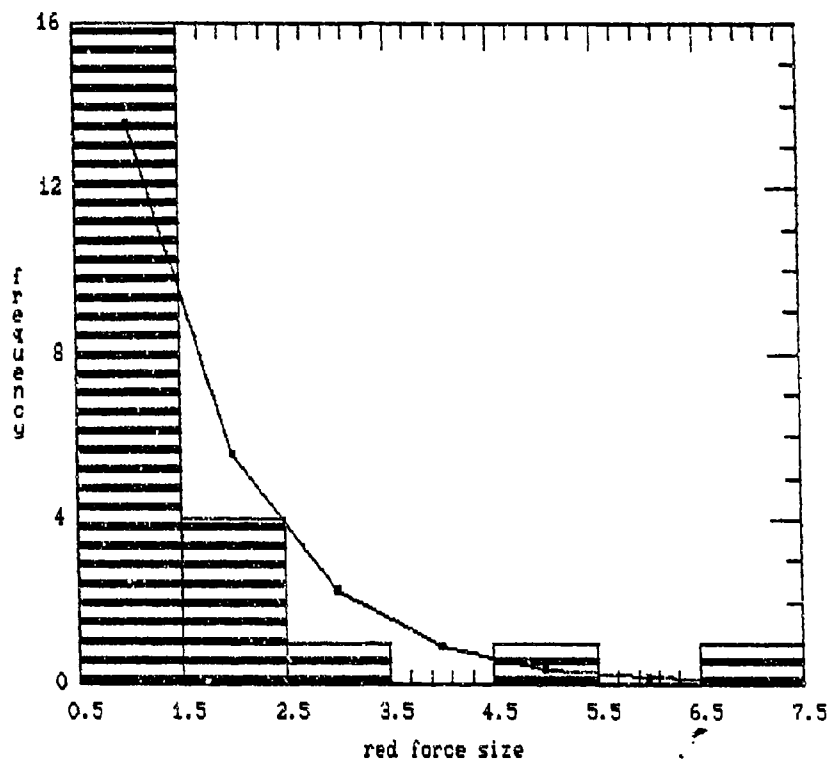
Figure A-1e

Chinese Eye Rattle 6
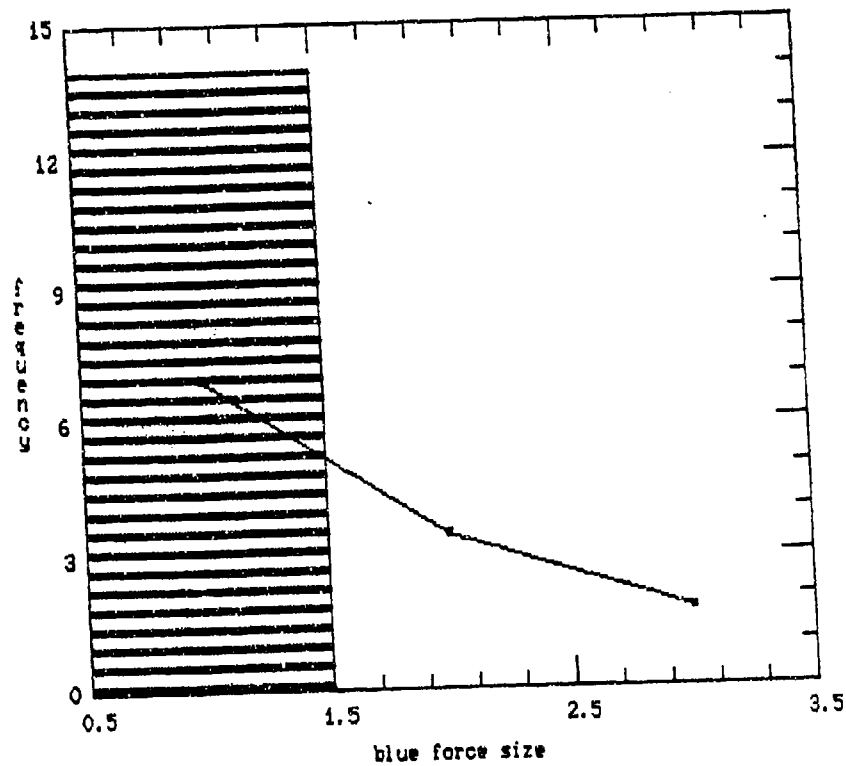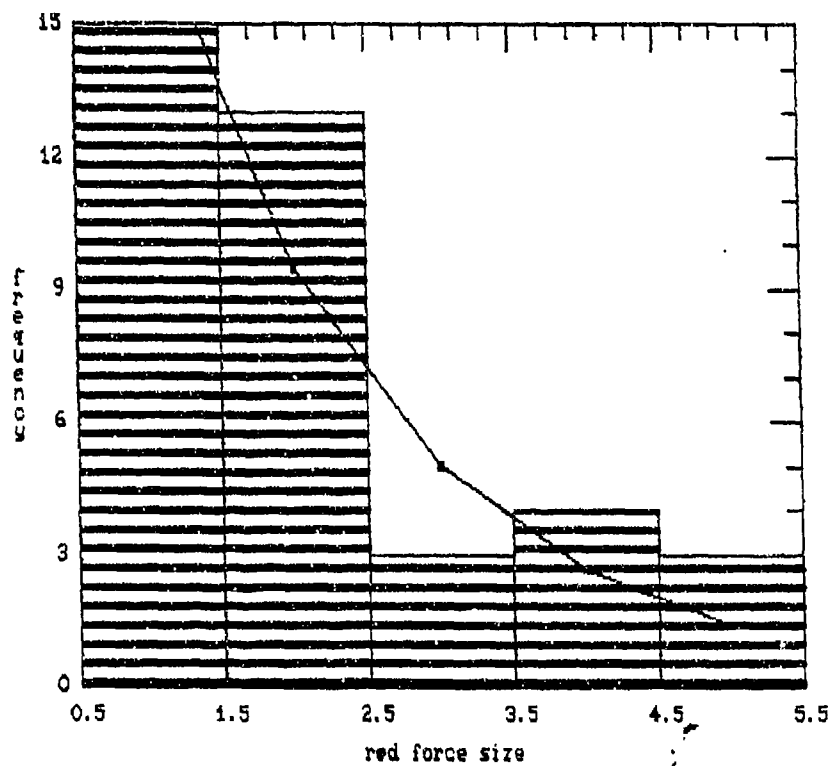

Figure A-1f

A-3
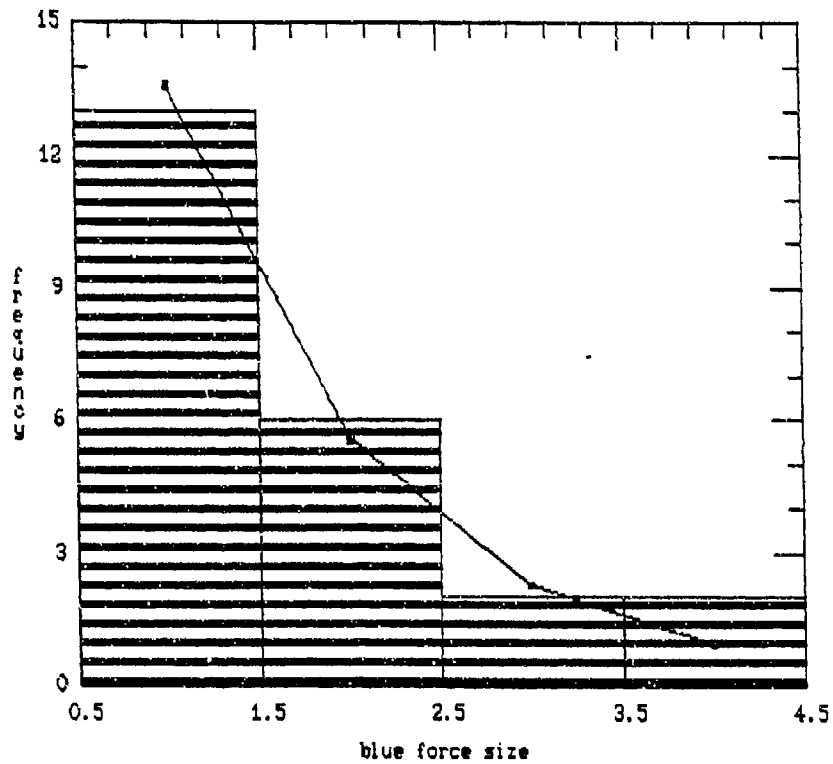
Chinese Eye Battle 7
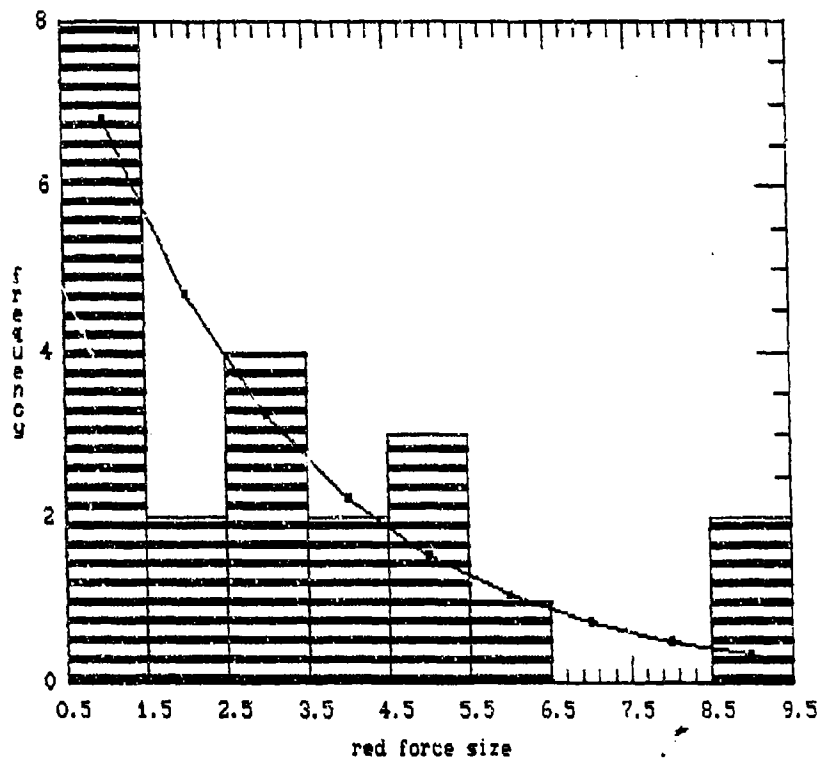
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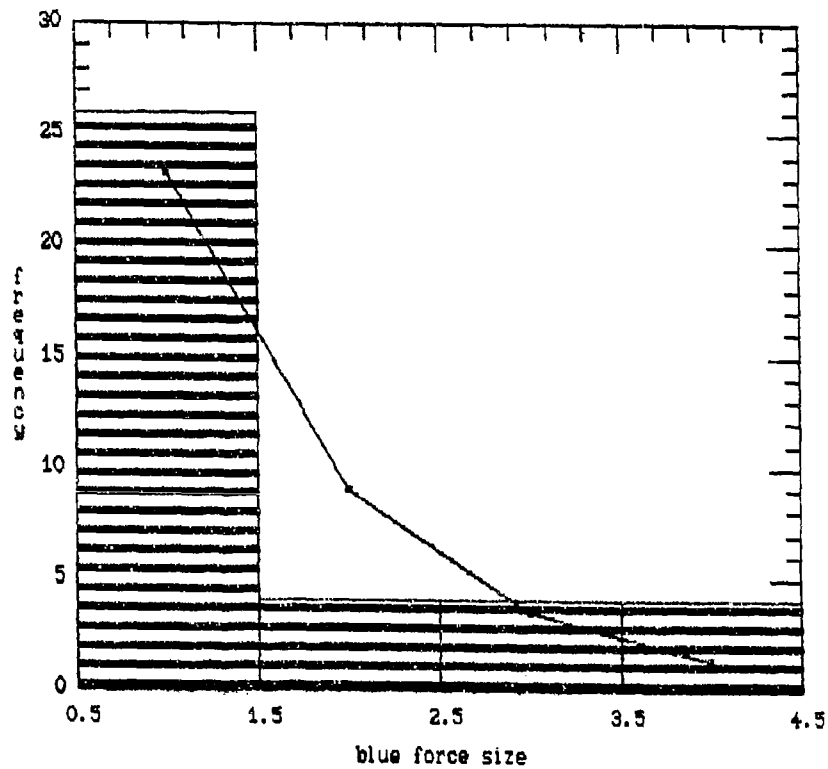
Chinese Eye Battle 7

Figure A-1h

A-4

Chinese Eye Battle 8

Figure A-1i

Chinese Eye Battle 8

Figure A-1j

A-5
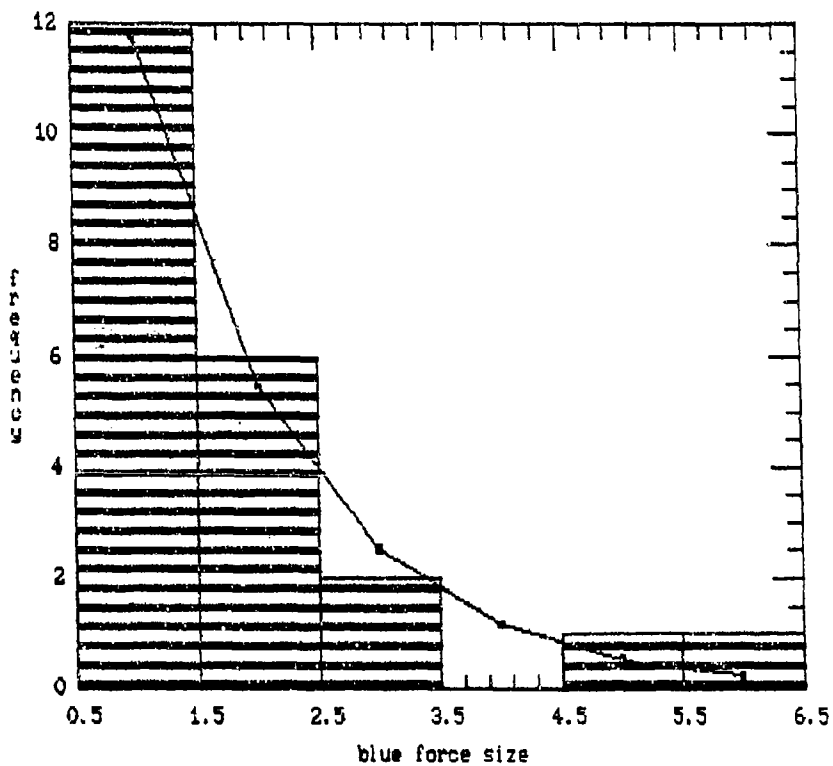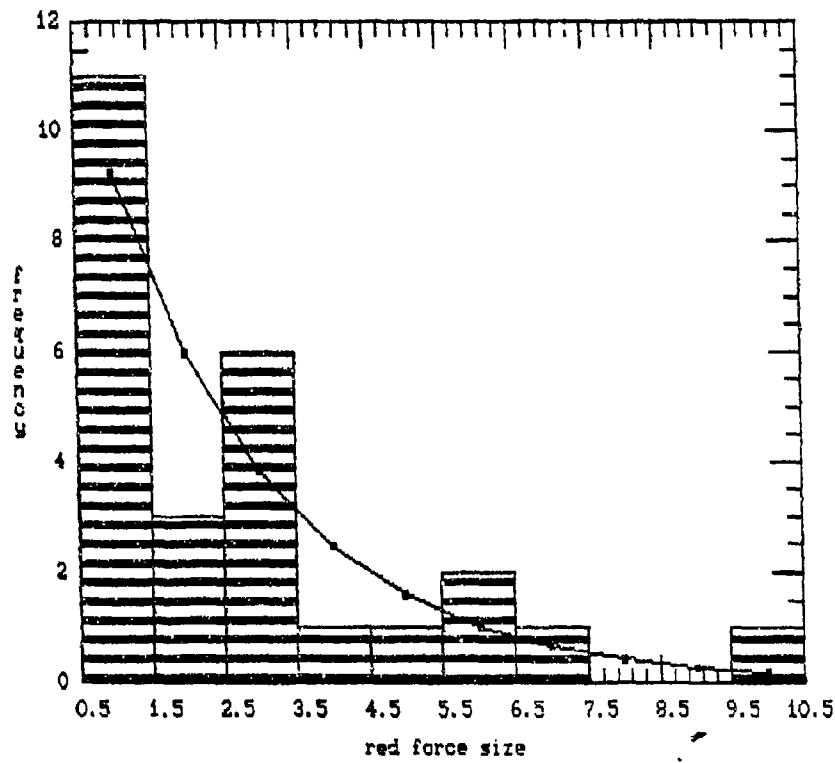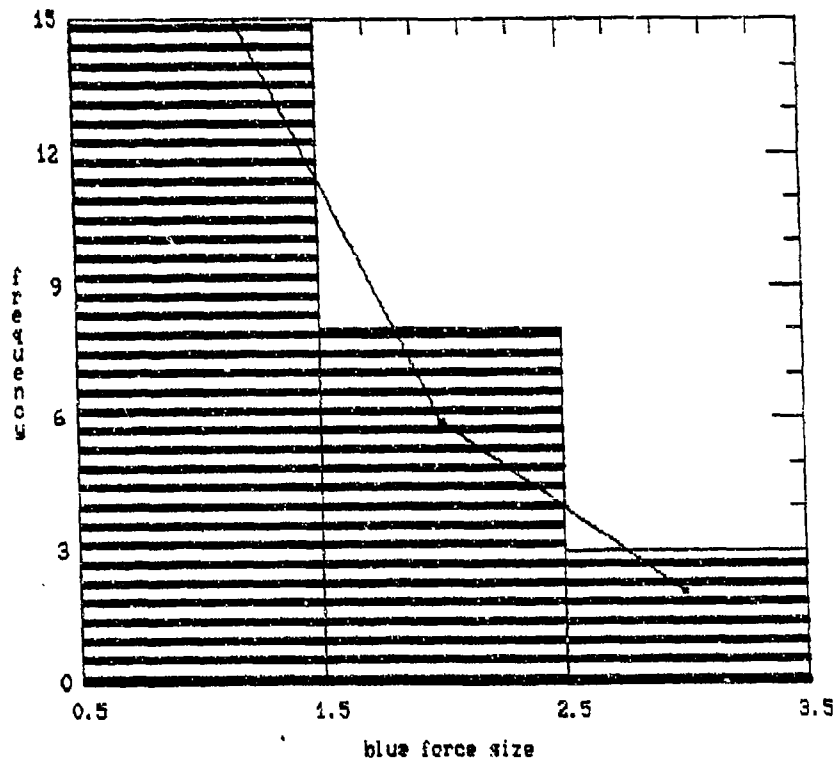
Figure A-1k

Chinese Eye Battle 12

Figure A-11

A-6

Chinese Eye Battle 13
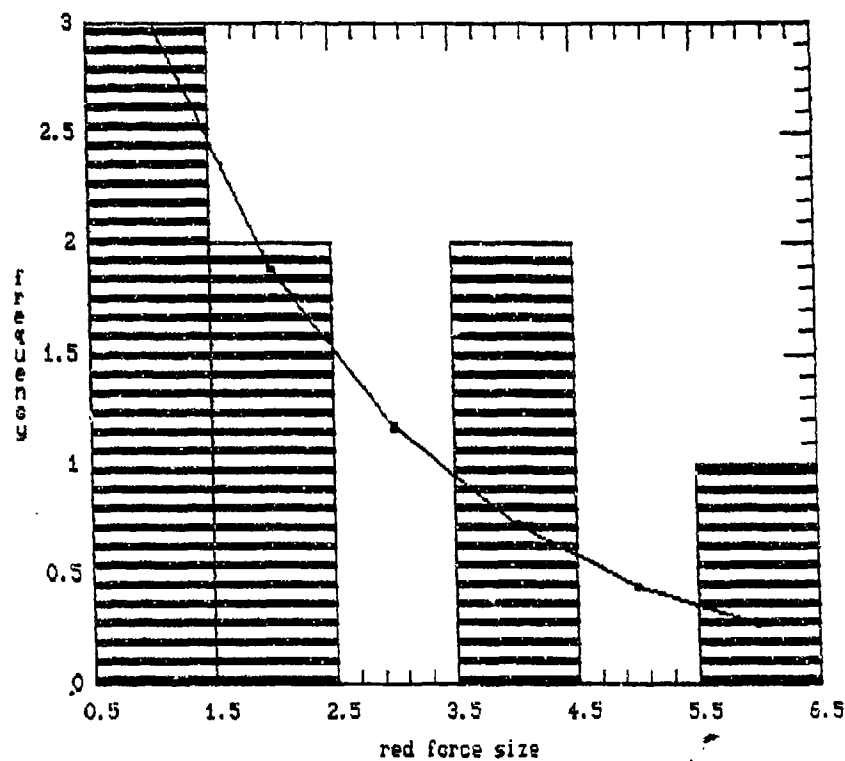
Figure A-1m

Chinese Eye Battle 13

Figure A-1n

A-7

Chinese Eye Battle 14

Figure A-1o

Chinese Eye Battle 14

Figure A-1p

A-8

Chinese Eye Battle 18

Figure A-1q

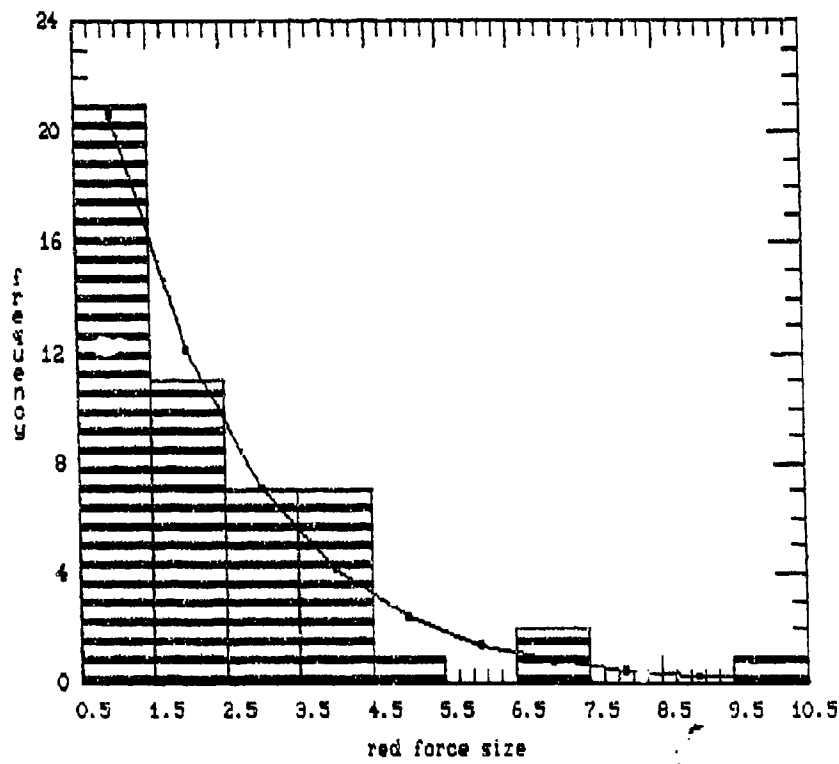Chinese Eye Battle 18

Figure A-1r

A-9

Chinese Eye Battle 19

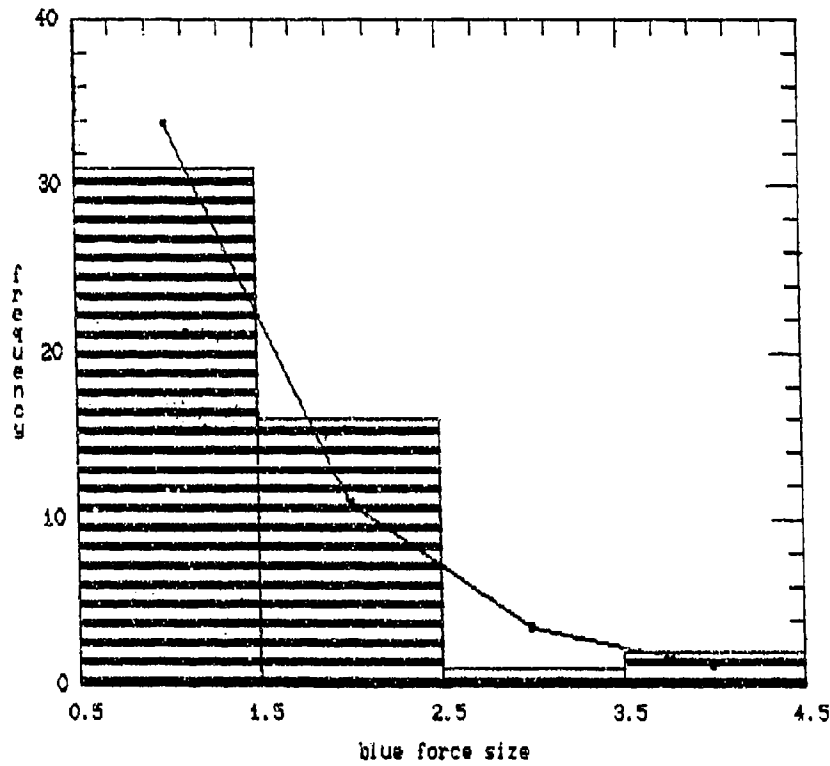Figure A-1s

Chinese Eye Battle 19

Figure A-1t

A-10