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FORCE-FREE MAGNETIC FIELDS: IS THERE A 'LOSS OF EQUILIBRIUM'?

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ABSTRACT

In this paper ~~we~~ examines the concept in solar physics that ~~has come to be~~ known as "loss of equilibrium" in which a sequence of force-free magnetic fields, said to represent a possible quasi-static evolution of solar magnetic fields, reaches a critical configuration beyond which no acceptable solution of the prescribed form exists. This concept has been used to explain eruptive phenomena ranging from solar flares to coronal mass ejections. ~~We here discuss~~ certain sequences of force-free configurations [are discussed] that ~~are said to~~ exhibit a loss of equilibrium, and ~~we~~ [its] argue that the concept is devoid of physical significance since each sequence is defined ~~in such~~ a way that ~~it~~ does not respresent an acceptable "thought experiment." For example, the sequence may be defined in terms of a global constraint on the magnetic field, rather than simply a reasonable constraint on the boundary conditions, or the evolution of the sequence may require the creation of magnetic flux that is not connected to the photosphere and is not present in the original configuration. The global constraints typically occur in using the so-called "generating function" method. We propose that an acceptable thought experiment is to specify the field configuration in terms of photospheric boundary conditions comprising the normal component of the field and the field-line connectivity. ~~We~~ consider a magnetic-field sequence that, when described in terms of a generating function, exhibits a loss of equilibrium and show that, when one instead defines the sequence in terms of the corresponding boundary conditions, the sequence is well behaved.

I. INTRODUCTION

Many phenomena that fall under the general heading of "solar activity" are believed to involve the sudden release of stored free magnetic energy. Such phenomena include flares, prominence eruptions, coronal mass ejections, and possibly also surges and sprays. Since these phenomena appear to involve only processes occurring above the photosphere, and since the photosphere is comparatively massive and is reasonably highly conducting, there is good reason to believe that the distribution of magnetic flux at the photospheric level is not changed by the activity. Hence the only energy that can be derived from the magnetic field, referred to as the "free" magnetic energy, is that which is due to currents distributed in the region above the photosphere that is predominantly the corona.

In the corona, the density is sufficiently low that for many configurations one may neglect the effects of the pressure gradient and the gravitational force of the material trapped in the magnetic field. In these circumstances, the field will to good approximation be in a "force-free" state for which the Lorentz force is zero, i.e.,

$$\mathbf{j} \times \mathbf{B} = 0 \ . \tag{1.1}$$

Since this force is zero only if the current is parallel to the magnetic field, we see that, for a force-free field,

$$\mathbf{j}(\mathbf{x}) = \kappa(\mathbf{x})\mathbf{B}(\mathbf{x}) \ . \tag{1.2}$$

Since $\mathbf{j}$ and $\mathbf{B}$ are both divergenceless, it follows that

$$\mathbf{B}\cdot\nabla\kappa = 0\,, \tag{1.3}$$

and therefore that κ is constant along any field line.

If κ is assumed to be constant throughout space, then equation (1.2) leads to the linear equation

$$\nabla\times\mathbf{B} = 4\pi\kappa\mathbf{B}\,, \tag{1.4}$$

determining $\mathbf{B}(\mathbf{x})$. However, this assumption is very restrictive so that there are strong reasons for seeking solutions of the nonlinear equation

$$(\nabla\times\mathbf{B})\times\mathbf{B} = 0\,. \tag{1.5}$$

Procedures have been developed to calculate such solutions by numerical methods (see, for instance, Sturrock and Woodbury 1967; Sakurai 1979; Yang, Sturrock, and Antiochos 1986; Craig and Sneyd 1986; and Zwingmann 1987). In addition, a certain procedure--that we refer to as the "generating function" procedure--has been developed and studied extensively for cases involving translational symmetry. See for instance the review articles by Birn and Schindler (1981) and Low (1982) and references cited therein. This method has led to the concept of "loss of equilibrium," that is the subject of this article.

II. THE GENERATIN^~ FUNCTION METHOD

We consider a magnetic-field configuration that is uniform in the z-direction of Cartesian coordinates x, y, z. Since $\nabla \cdot \mathbf{B} = 0$, we see that the magnetic field may be expressed as

$$\mathbf{B} = \left(\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, B_z \right). \tag{2.1}$$

We then find that equation (1.5) is satisfied if

$$\nabla B_z \times \nabla A = 0 , \tag{2.2}$$

which implies that B_z is expressible as a function of A, and if

$$\nabla^2 A + B_z \frac{dB_z}{dA} = 0 . \tag{2.3}$$

So far, no assumptions have been made about the magnetic field except that it is force free and that it has translational symmetry in the z-direction.

At this point, we note that it is possible to define a *family* of solutions of equation (2.3) by assuming that

$$B_z = \lambda F(A) \tag{2.4}$$

or, equivalently,

$$\frac{d}{dA}\left(\frac{1}{2} B_z^2 \right) = \lambda^2 f(A) , \tag{2.5}$$

where

$$f(A) = F(A)\, F'(A) \ . \tag{2.6}$$

(Some authors use λ rather than λ^2 in equation [2.5]). In terms of the generating function f(A), equation (2.3) now takes the form

$$\nabla^2 A = -\lambda^2 f(A) \, . \tag{2.7}$$

In any specific problem, the magnetic field is determined not only by the appropriate "field equation," but also by the boundary conditions. Since we are discussing problems of solar activity, it is appropriate to consider the plane y = 0 as the "photosphere" and to consider boundary conditions on that plane. If we were dealing with a potential field, it would be sufficient to specify $B_y(x,0)$. However, specifying the normal value of the magnetic field is not a sufficient boundary condition to determine a force-free field. From a physical point of view, the appropriate choice is to specify the normal value of magnetic field and the magnetic connectivity. In the present situation, this amounts to specifying the distance in the z-direction between the two end points of each field line.

From a mathematical point of view, these conditions can be specified by describing the magnetic field in terms of Clebsch variables,

$$\mathbf{B} = \nabla\alpha \times \nabla\beta \, , \tag{2.8}$$

and adopting the form

$$\alpha = \alpha(x,y), \quad \beta = z - \gamma(x,y) . \tag{2.9}$$

We then have

$$B_x = \frac{\partial\alpha}{\partial y}, \quad B_y = -\frac{\partial\alpha}{\partial x}, \quad B_z = -\frac{\partial\alpha}{\partial x}\frac{\partial\gamma}{\partial y} + \frac{\partial\alpha}{\partial y}\frac{\partial\gamma}{\partial x}, \tag{2.10}$$

showing that α and A are identical. Since **B** is divergenceless, α and β are constant along any field line. Hence γ measures the displacement of any field line in the z-direction. Specifying α and γ as functions of x in the plane y = 0 therefore specifies both the normal value of the magnetic field and the connectivity at that plane. This is the approach that has been taken by Sturrock and Woodbury (1967), Barnes and Sturrock (1972), Yang, Sturrock, and Antiochos (1986), and Klimchuk, Sturrock, and Yang (1988).

In the generating-function approach, on the other hand, the boundary conditions are *not* specified independently of the field equation. The procedure adopted is to specify A(x,0) at the photosphere and to assume a form for f(A); then to find a solution of equation (2.7); and, finally, to *deduce* the connectivity of the field from the solution. The connectivity is found by noting that

$$\frac{dz}{B_z} = \frac{ds}{|B_\perp|}, \tag{2.11}$$

where $B_\perp$ is the projection of the magnetic field vector into the x-y plane. The total displacement of the field line in the z-direction is then given by

$$Z = \int \frac{B_z}{|\nabla A|} ds \,, \tag{2.12}$$

where the integral is taken along the projection of a field line in the x-y plane, that is to say, along a curve A = constant.

The case $\lambda = 0$ always corresponds to a potential field, for which $B_z = 0$ and $Z = 0$. As λ is increased from zero, B_z increases and Z increases. As long as the field remains well behaved, one can place a simple physical interpretation on the sequence of solutions that is developed by varying λ. One can say that each field configuration represents the appropriate force-free field for the normal magnetic field specified by A(x,0) and for the connectivity determined, for each solution, by equation (2.12).

The important question to be addressed is the following. If we find that the solutions of equation (2.7) are well behaved only for a finite range of λ, say $0 \leq \lambda < \lambda^*$, does the limiting state $\lambda = \lambda^*$ have some physical significance and, if so, what is it?

III. "LOSS OF EQUILIBRIUM"

In the review articles referred to in Section I, and in a number of articles quoted in those review articles, the authors take the position that the existence of a limit to the sequence of solutions of equation (2.7) does have physical significance. For instance, Birn and Schindler (1981) state that the critical points $\lambda = \lambda^*$ "represent the onset points of eruptive phenomena such as solar flares, since no neighbouring equilibrium state could be achieved if λ was forced to exceed the critical value λ^*." Similarly, Low (1982) asserts that "the abrupt termination of a sequence of force-free fields" that occurs at the critical point $\lambda = \lambda^*$ may result in "a transition to a dynamical state" and suggests that this may explain "how a solar magnetic field would occasionally break into a flare or other eruptions." In yet another article, Birn, Goldstein and Schindler (1978) point out that "Schindler (1976) argued that a system driven into a situation, where a static state is no more possible, should become dynamic and interpreted the point $\lambda = \lambda^*$ as the onset of an eruptive phenomenon such as a solar flare."

On the other hand, such an interpretation of the calculations has been called into question by other authors. For instance, Hagyard and Rabin (1986) state, "In terms of the sequence of equilibria parametrized by λ, it makes no sense to speak of solutions with $\lambda > \lambda_{crit}$. Physically, the question must be whether the system can continue to evolve smoothly through some sequence of force-free states, whatever their mathematical origin." Priest and Milne (1980) state, "The non-existence of solutions for $\lambda > \lambda_{max}$ is therefore no evidence for the onset of instability, since it is the shear d(x) that must be prescribed rather than f(A)."

As a specific example of a generating-function model, we consider the case discussed originally by Low (1977) and later by Birn, Goldstein, and Schindler (1978) and Priest and Milne (1980). In our notation, the generating function is given by

$$f(A) = -k^2 \exp(-2A) \,, \tag{3.1}$$

corresponding to the following *global* constraint on the B_z component of the field:

$$B_z(A) = \lambda F(A) = \lambda k \exp(-A) \,. \tag{3.2}$$

The boundary conditions on the plane $y = 0$ are

$$A(x,0) = \ln\left(1 + k^2x^2\right), \tag{3.3}$$

and above the plane, at large distances from the origin, it is assumed that

$$\left(x^2 + y^2\right)^{1/2} \to \infty, \qquad A(x,y) \to \ln\left[k^2x^2 + (1 + ky)^2\right]. \tag{3.4}$$

Low finds that the solution of equation (2.6), subject to the boundary conditions (3.3) and (3.4) is

$$A(x,y) = \ln\left[1 + k^2x^2 + 2\left(\frac{1 - \mu^2}{1 + \mu^2}\right)ky + k^2y^2\right], \tag{3.5}$$

where λ and μ are related by

$$\lambda = \frac{4\mu}{1 + \mu^2}. \tag{3.6}$$

It may be noted that $A(x,y)$ given by equation (3.5) in fact does not satisfy the boundary condition (3.4) unless $\mu = 0$.

Low considers the sequence of magnetic-field configurations formed by allowing μ to increase from 0 to ∞. Then λ increases from 0 to a maximum of 2 (when $\mu = 1$) and then decreases back down to 0. For $0 < \mu \leq 1$, the magnetic-field configuration is that of a simple arcade in which the distribution of footpoints on the photosphere is given by

$$kz = \frac{2\mu}{1+\mu^2}\arcsin\left\{\left[\left(\frac{1-\mu^2}{1+\mu^2}\right)^2 + k^2x^2\right]^{-1/2} kx\right\}. \tag{3.7}$$

For $\mu > 1$, the magnetic field configuration is no longer that of a simple arcade. It contains a flux tube that runs above and parallel to the photosphere, that we here refer to as "floating flux." The distribution of footpoints is now given by[1]

$$kz = \frac{2\mu}{1+\mu^2}\left(\pi\frac{x}{|x|} - \arcsin\left\{\left[\left(\frac{1-\mu^2}{1+\mu^2}\right)^2 + k^2x^2\right]^{-1/2} kx\right\}\right). \tag{3.8}$$

Low takes the position that the creation of floating flux is forbidden by the assumption of infinite electrical conductivity, and concludes that "field configurations with $\mu > 1$ are not available to the evolving magnetic field." He suggests that "the quasi-steady evolution of the force-free field ceases at $\mu = 1$, whereupon explosive events take over."

Our approach to this problem is to consider the thought experiment described by the mathematical model, and then to ask whether that thought experiment is relevant to processes that can occur on the sun. Over the range $0 < \mu \le 1$, the variation of μ may be interpreted simply as a

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[1] There is a typographical error in equation (38) of Low (1977): the quantity in square brackets should be raised to the minus 1/2 power, not the plus 1/2 power. Also, the field lines are labeled incorrectly in Figures 1, 2, and 4 of Low (1977) and in Figures 3, 4, and 7 of Low (1982): the correct labeling sequence is log2, log5, log10, log17, rather than log2, log3, log4, log5. Note that these are natural logarithms.

displacement of the footpoints of magnetic-field lines according to equation (3.7), leaving the normal magnetic field strength B_y unchanged. This is a reasonable thought experiment that could possibly represent actual shearing motion on both sides of a magnetic-field polarity reversal line.

On the other hand, in increasing μ beyond the value of $\mu = 1$, we are requiring that two separate processes take place: (a) the footpoints should continue to move as described by equation (3.8); and (b) floating magnetic flux should be created. The second requirement invalidates this model as a reasonable thought experiment for solar physics. Rather than discuss the implications of a thought experiment which (as Low agrees) does not make physical sense, we hold that the appropriate course is to find a thought experiment that *does* make sense.

The thought experiment that does make sense has been alluded to already and has been noted in the literature by several authors. Jockers (1978) refers to the generating-function calculations as "a first step towards solving the more difficult problem of prescribing the photospheric shear rather than [F(A)]." Birn and Schindler (1981) state, "It is preferable to prescribe the position of footpoints of field lines rather than [F(A)], the reasons being that, in some cases, the Sun is more likely to do this . . . ." Finally, Low (1982) recommends, "It is physically more straightforward to ask: What happens to a magnetic field if its footpoints are subject to a prescribed displacement?" This is exactly the question that we now address.

IV. THE MAGNETO-FRICTIONAL METHOD

We calculate force-free magnetic field configurations using a numerical technique known as the magneto-frictional method. It was first proposed by Yang, Sturrock, and Antiochos (1986) and was further developed by Klimchuk, Sturrock, and Yang (1988). Essentially, it is a relaxation technique by which an initial guess at the field configuration relaxes to a force-free state subject to the constraint that the footpoints remain fixed at the photosphere. Quasi-static evolutionary sequences are then studied by varying the footpoint positions from one model calculation to the next.

The field is described in terms of Clebsch variables [equation (2.8)] and photospheric boundary conditions are specified by the distributions of α and γ at the $y = 0$ plane. There is no assumption made about the functional dependence of B_z upon α, as there is in the generating function approach. Since we wish to address the model of Low (1977), we take $\alpha(x,0)$ to have the form of equation (3.2) and take $\gamma(x,0)$ to have the form of either equation (3.7) or (3.8), depending on the value of μ. For simplicity, we assume $k = 1$. In Figure 1 we have plotted the shear function $\gamma(x,0)$ for several different values of μ.

The relaxation of the field is governed by the equations

$$\delta\alpha = -\nu^{-1}\mathbf{F}\cdot\nabla\alpha, \quad \delta\beta = -\nu^{-1}\mathbf{F}\cdot\nabla\beta, \tag{4.1}$$

where $\mathbf{F}$ is the Lorentz force and ν is analogous to a coefficient of friction. As discussed in Yang, Sturrock, and Antiochos (1986), it useful to take $\nu \propto B^2$. The calculations are performed on a two-dimensional grid using a second-order finite differencing scheme. Because the field is symmetric about the $x = 0$ plane, it is necessary to consider only the $x \geq 0$ half-space; our computational domain thus ranges from 0-60 in both the x- and the ydirections. A total of 289 x 289 grid points are spaced nonuniformly within this domain to provide the greatest resolution near the origin, where it is most important. The grid spacing there is only 0.11 compared with a maximum of 0.5 at the outer boundary. Note that, because of the finite spacing, we are unable to prescribe infinite shear at the origin, as given by equation (3.8). We feel that this has negligible effect on the results, however, since the width of the shear zone is trivial compared with all other scales of the problem. The reader may also note that infinite shear would never occur on the Sun.

Additional boundary conditions must be imposed at the remaining three sides of the computational domain. At $x = 0$ we impose symmetry boundary conditions, and at $x = 60$ and at $y = 60$ we impose the conditions on α specified by Low (1977) [equation (3.4)]. Although these outer boundary conditions are strictly appropriate only when $\mu = 0$, the boundaries are sufficiently far removed that their influence on the field should be negligible. The outer boundary conditions on γ are, at $x = 60$ and at $y = 60$,

$$\gamma(x,y) = \frac{2\mu}{1+\mu^2} \arctan\left(\frac{x}{y}\right) . \tag{4.2}$$

These boundary conditions are strictly appropriate only when $\mu = 0$ and $\mu = 1$, but once again the discrepancy should be negligible.

Each of the model calculations is performed in four separate stages, corresponding to grids of increasingly finer resolution. The calculations are terminated after about 2000 iterations, at which time the average angle between the current and magnetic field vectors (weighted by the product of the vector magnitudes) is only a fraction of a degree. The fields are thus force free to a very good approximation.

V. MODEL RESULTS

The results of six of our model calculations, corresponding to $\mu = 0.0$, 0.5, 1.0, 1.5, 2.0, and 2.5, are shown in Figure 2. The first three cases are essentially identical to the analytical solutions of Low (1977), given by equation (3.5). Slight differences are due in part to the finite difference nature of the numerical solutions. For example, the magnetic-field vectors are misaligned by about 0.1°, on average, for the $\mu = 0.0$ case and by 1.5°, on average, for the $\mu = 0.5$ and $\mu = 1.0$ cases. Such good agreement gives us confidence in our results.

On the other hand, the numerical solutions for the three cases with $\mu > 1.0$ are fundamentally different from the solutions of Low. Whereas Low's solutions show the appearance of floating flux, its quantity increasing with μ, the numerical solutions maintain the character of a simple arcade. This difference is made clear in Figure 3, where we have plotted both solutions together for the "supercritical" case $\mu = 2.5$.

The sequence of force-free fields in Figure 2 is fully consistent with the assumption of infinite electrical conductivity and therefore represents a valid thought experiment for the quasi-static evolution of magnetic fields on the Sun. It is interesting to note that as μ increases from 0.0 the field begins to inflate, but at larger values of μ the trend reverses and the field begins to depress. We can understand this behavior in terms of the shear profiles shown in Figure 1. As μ increases from 0.0 to 1.0, the shearing displacement increases everywhere along the arcade. Beyond $\mu = 1.0$, however, the shearing displacement decreases in all but the innermost region. Since greater displacement is associated with greater magnetic pressure, due to the enhancement of B_z, we would expect this behavior of the magnetic field.

VI. DISCUSSION

We have discussed and compared two methods for calculating sequences of force-free magnetic fields of translational symmetry: one is to use a generating-function description, which greatly simplifies the problem, and the other is to specify connectivity boundary conditions, a procedure that requires numerical solution. We have demonstrated that these two methods can produce fundamentally different results, even when the connectivity is the same. In the generating-function sequence of Low (1977), the magnetic field changes topology at the maximum value of the parameter λ (behavior that had led to the conjecture of a catastrophic loss of equilibrium), but in the corresponding numerical solution constrained only by boundary conditions, the field evolves smoothly and uneventfully beyond the critical point. This difference is due to the *additional global constraint* that the generating function places on the field, namely the dependence of B_z upon A given by equation (3.2). We have argued that this is an unacceptable constraint to adopt in modeling solar magnetic fields, and that, in consequence, the generating-function model does not represent a valid thought experiment.

Note that it is possible to think of the well behaved numerical sequence of Figure 2 in terms of generating functions if one realizes that the $B_z(A)$ functional dependence changes form (not just magnitude) as the field evolves beyond the critical point in response to the prescribed foot-point motions. By contrast, in Low's sequence (and in all generating-function sequences), the form of the dependence is held constant throughout the sequence. This difference was suggested some time ago by Priest and Milne (1980). To the best of our knowledge, no one has shown that physically acceptable constraints (such as acceptable boundary conditions) would automatically lead to the global constraint adopted in generating-function calculations, and we see no prospect that this global constraint can be shown to be physically acceptable. We expect that subphotospheric motions, which determine the locations of magnetic footpoints, will be unaffected by the form of the coronal magnetic field, so there is no reason to expect that the footpoint motions in a real solar situation will be such that the evolving magnetic field can be described by a generating function of constant form.

Not all generating-function sequences change topology at the maximum value of the parameter λ, as in Low's sequence. Some change topology for $\lambda < \lambda_{max}$, while others do not change topology at all (e.g., Jockers 1978; Birn, Goldstein, and Schindler 1978; Low 1977). It has been suggested that $\lambda = \lambda_{max}$ nevertheless determines a critical state of physical significance. For example, Birn, Goldstein, and Schindler (1978) write, "If now the system is forced into a situation where λ becomes larger than λ^*, a (two-dimensional) quasi-static evolution is no longer possible. It seems reasonable to expect that the system will assume a fast dynamic state."

As before, we argue that the above problem is not an acceptable thought experiment. The sequence of field configurations is described in terms of an unnacceptable global constraint on the field and, for $\lambda > \lambda^*$, it is not possible to replace this unacceptable global constraint by an equivalent acceptable constraint, such as the specification of connectivity boundary conditions. Our numerical studies indicate that, when a problem is formulated in terms of physically acceptable boundary conditions, a solution to the force-free equations can be found.

Birn, Goldstein, and Schindler (1978) have previously attempted to interpret the Low (1977) result in terms of the connectivity of the field. However, they conclude, "The maximum value $\lambda = \lambda^*$ corresponds to a maximum displacement $\Delta = \Delta^* = \pi / 2$. This means that no equilibrium solutions exists if the displacement of footpoints far away from the neutral line $x = 0$ exceeds the critical value Δ^*." This conclusion is incorrect. We show in Figure 4 the results of a model calculation for which the shear displacement has a magnitude of π for all field lines, demonstrating that equilibrium solutions do in fact exist for $\Delta > \Delta^*$.

In a recent related paper, Priest (1988) considers the simple model of a *cylindrical*, bounded arcade with uniform, but different, gas pressures within the arcade and in the outer field-free region. He shows that no equilibrium solutions *of the assumed form* exist when the pressure differential exceeds a critical value. This, he contends, "is suggestive that the magnetic arcade loses equilibrium and erupts catastrophically." Although the formulation of the problem does not involve a generating function, it nevertheless places a similar global constraint on the field (namely, that the field lines should be cylindrical in form) and therefore is a physically unacceptable thought experiment for the Sun. It is entirely possible that, as the pressure differential exceeds a critical value, the magnetic field would adjust to an equilibrium configuration that is no longer cylindrically symmetric. Priest himself points out this possibility, but states that he has so far been unable to find the non-cylindrical solutions.

In another recent paper, Zwingmann (1987) has undertaken a numerical study of magnetic and magnetohydrostatic configurations of translational symmetry. In agreement with the approach we advocate here, Zwingmann describes the field in terms of Clebsch variables. The variables are not required to satisfy any global constraints other than the appropriate field equations, and, for this reason, Zwingmann's procedure seems basically to be physically sound. However, we do have some reservations about the particular results that Zwingmann presents.

First, Zwingmann's boundary conditions on the upper boundary (but not the side boundary) require the field lines to be normal to this boundary. In our earlier work (Klimchuk, Sturrock, and Yang 1988), we required the field lines to be parallel to both the upper and side boundaries, a choice that is equivalent to considering the thought experiment of the evolution of a magnetic field that is contained in a box with highly conducting walls. It is not clear that the boundary conditions adopted by Zwingmann have any such simple interpretation. His boundary conditions could no doubt be achieved by an appropriate distribution of surface currents on the upper boundary, but then it is not obvious whether the opening of field lines that penetrate the upper boundary is due to the stresses internal to the box or to the imposed stresses (those associated with the currents) at the surface of the box.

As the shear increases in Zwingmann's model, more and more field lines intersect the upper boundary, where they are hypothesized to become open and lose their B_z component. Zwingmann attributes the fact that a nearly force-free equilibrium sequence remains well behaved as the footpoint shear is increased to large values to the fact that his boundary conditions allow field lines to become open. However, we have studied several similar sequences, with the difference that we require all field lines to remain closed, and find that these sequences also are well behaved even at large values of shear (Klimchuk, Sturrock, and Yang 1988). Hence it does not seem to be necessary to provide artificially for the opening of field lines in order to obtain well behaved sequences of magnetic-field configurations.

Zwingmann has also considered the effects of finite gas pressure and finds that some equilibrium sequences, corresponding to increasing gas pressure with fixed magnetic shear, have a "critical point" at which the topology and the energy change abruptly. It is difficult to evaluate this aspect of Zwingmann's article, since there is insufficient information concerning the gas and magnetic-field sequences. It appears (a) from Zwingmann's Figure 7, (b) from his specification of a pressure function "which is a constant outside the range of values for α" on the photosphere (implying that α has a greater range in the corona than on the photosphere), and (c) from his reference (on p. 322) to the existence of an "O-type" neutral point, that the topology of the magnetic field changes in going from one branch to another of the curve characterizing the solution, and that this change involves the introduction of new flux, analogous to the "floating flux" that appears in the configurations discussed in Section 3. On the other hand, Zwingmann makes no explicit mention of such a change of topology or of the introduction of additional flux. If, as it appears, Zwingmann's sequence does indeed require the creation of new flux, then the proposed evolution from one branch to another does not correspond to an acceptable thought experiment.

One additional, comparatively minor, concern regarding Zwingmann's article is that the critical points occur at what seem to us to be unreastically high values of the thermal pressure, corresponding to a plasma β of about 10^{-1} compared with values of order 10^{-3} inferred from active region observations.

To summarize, we believe that loss of equilibrium in sequences of force-free magnetic fields produced by the generating function method is an artifact brought about by stipulating physically unacceptable global constraints on the magnetic field. We propose that a physically acceptable description of the field is one in which the field is governed by the basic force-free equation and by the specification of appropriate photospheric boundary conditions, namely, the normal field component and the field-line connectivity. Our numerical calculations indicate that such a thought experiment results in well-behaved evolutionary sequences that do not exhibit any evidence of catastrophic behavior.

If our thesis is correct, that solar eruptive events cannot be explained by the concept of loss of equilibrium that features in the articles we have discussed, solar physicists are left with the important problem of determining the correct explanation of these events. It seems to us most likely that they are due to instabilities of the plasma system (Sturrock 1988). It is possible that the instability is an MHD instability that could be analyzed by an energy theorem (see, for instance, Bernstein 1973), but it is also possible that the instability involves microscopic plasma processes, such as field-line reconnection, in an essential manner. Recent numerical calculations by Mikic, Barnes and Schnack (1988) suggest that the latter possibility is a viable interpretation of solar eruptions.

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FIGURE CAPTIONS

Figure 1.--Shear functions showing the distribution of footpoint z-displacements for four different values of the parameter μ [from equations (3.7) and (3.8), with $k = 1$].

Figure 2.--Edge-on view of magnetic field lines for the numerical solutions with $\mu = 0.0, 0.5, 1.0, 1.5, 2.0$, and 2.5. These are projections of the field lines onto the x-y plane and correspond to contours of constant α. The contour increment is 0.5. Note that the domain of the computions (60 x 60) is much larger than shown.

Figure 3.--Comparison of the generating function solution (a) with the numerical solution (b) for the case $\mu = 2.5$. Shown are contours of constant α, corresponding to projections of field lines onto the x-y plane. The contour values range from -0.5 to +3.5 in (a) and from +0.5 to +3.5 in (b).

Figure 4.--The numerical solution for a model in which $\alpha(x,0)$ is given by equation (3.3), with $k = 1$, and in which $\gamma(x,0)$ has a constant magnitude of π and an opposite sign on either side of $x = 0$. Shown are contours of constant α ranging from +0.5 to +3.5.

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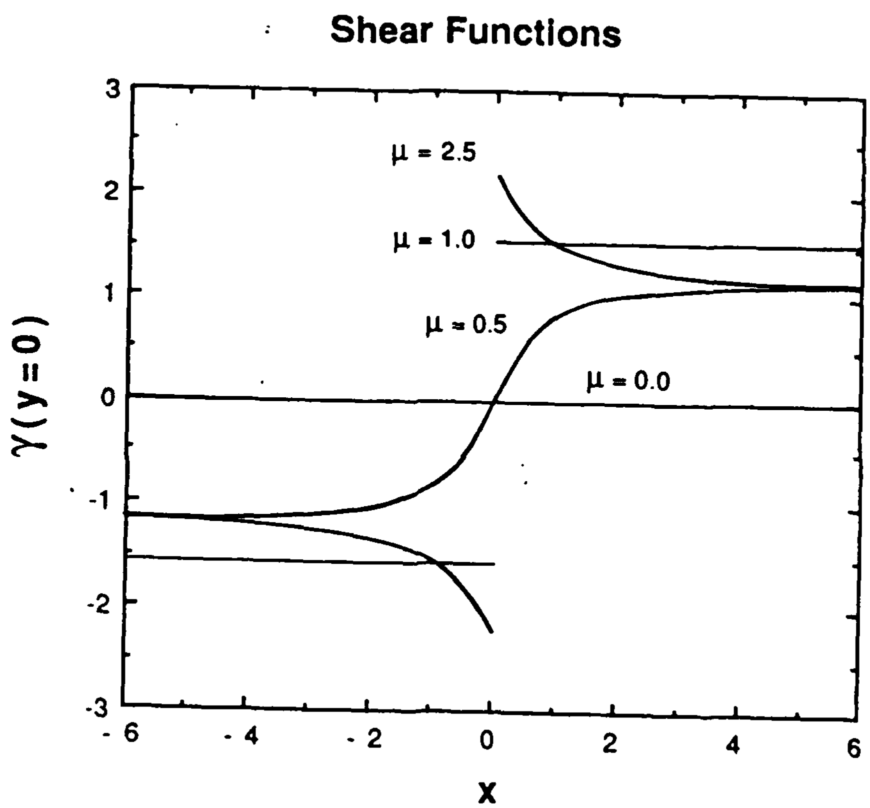

Figure 1

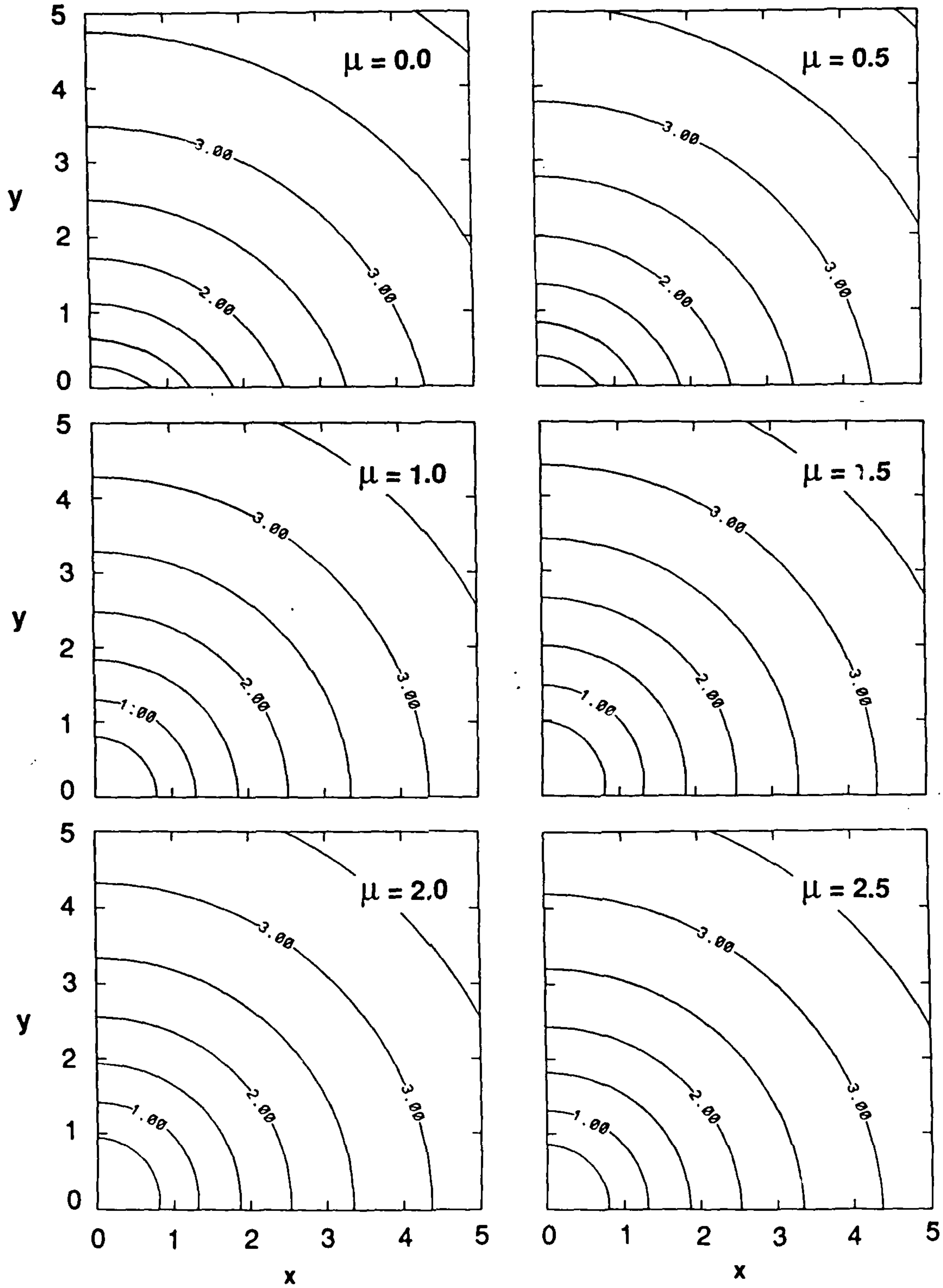

Figure 2

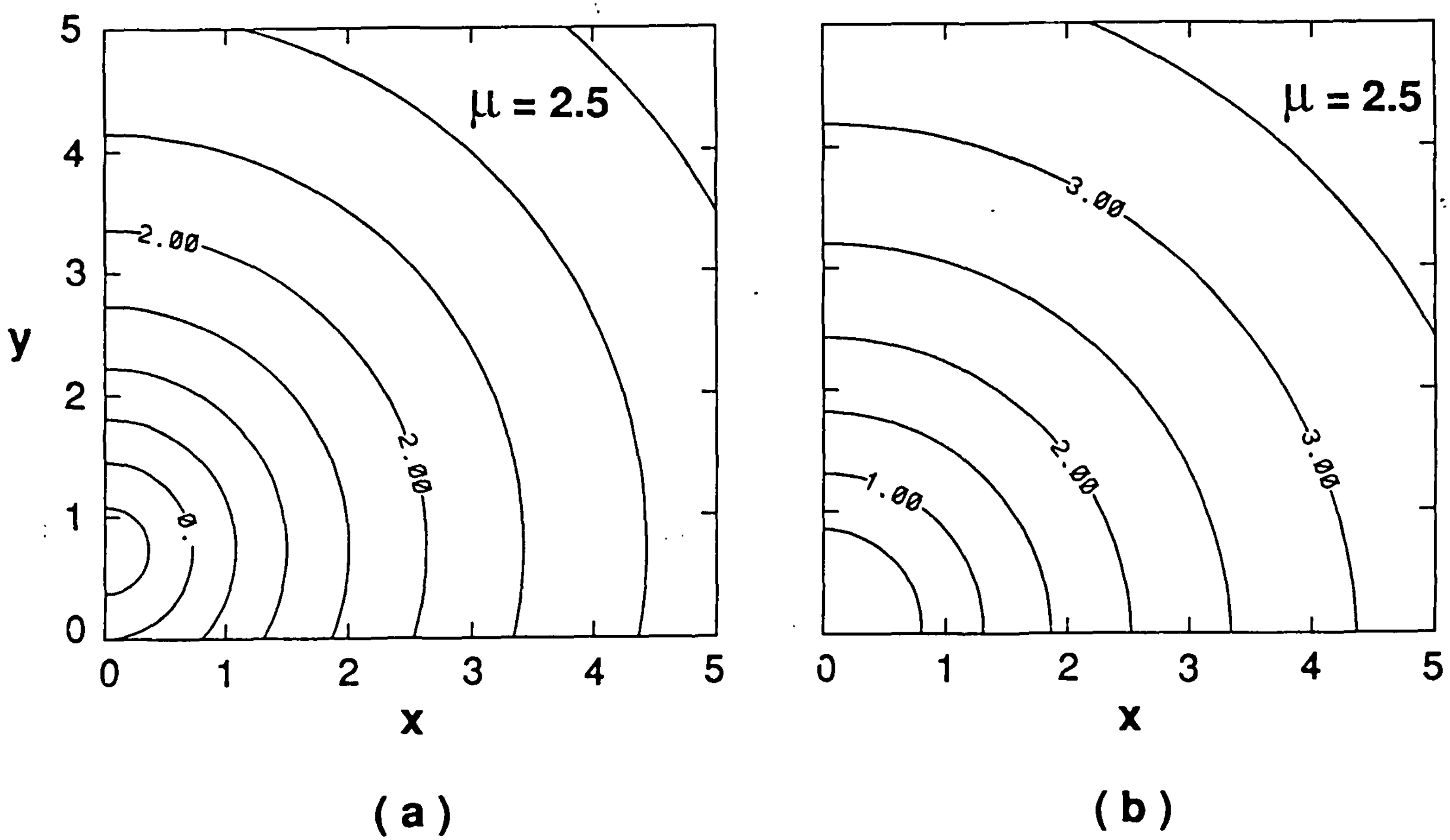

Figure 3

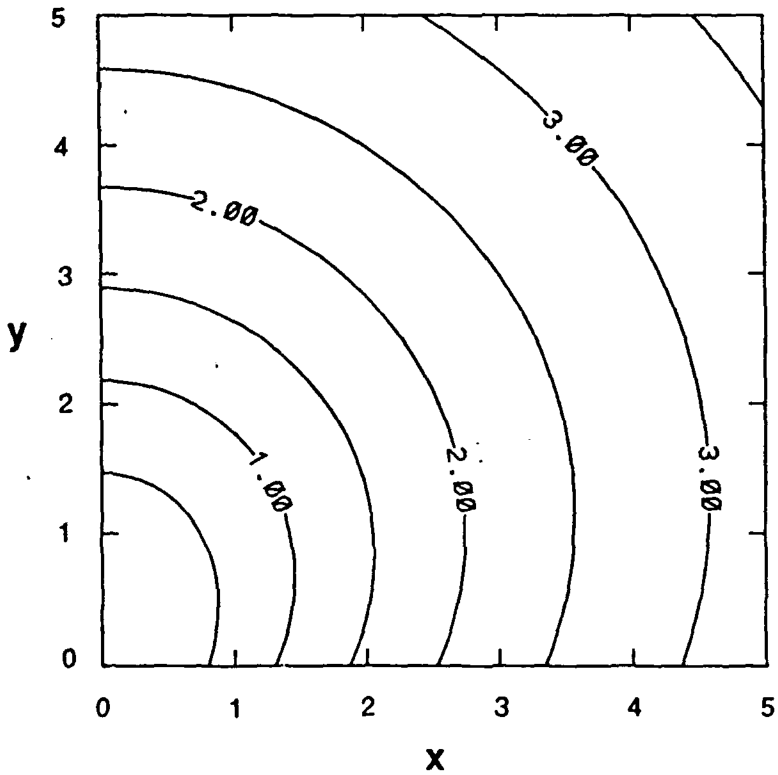

Figure 4